## Financial Toolbox ${ }^{T M}$

## User's Guide

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The MathWorks, Inc.
3 Apple Hill Drive
Natick, MA 01760-2098
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## Revision History

October 1995 First printing
January $1998 \quad$ Second printing
January 1999 Third printing
November 2000 Fourth printing
May 2003 Online only
June $2004 \quad$ Online only
August 2004 Online only
September 2005 Fifth printing
March 2006 Online only
September 2006 Sixth printing
March 2007 Online only
September 2007 Online only
March 2008 Online only
October 2008 Online only
March 2009 Online only
September 2009 Online only
March 2010 Online only
September 2010 Online only
April 2011 Online only
September 2011 Online only
March 2012 Online only
September 2012 Online only
March 2013 Online only
September 2013 Online only
March 2014 Online only
October 2014 Online only
March 2015 Online only
September 2015 Online only
March 2016 Online only
September 2016 Online only
March 2017 Online only
September 2017 Online only
March 2018 Online only
September 2018 Online only

Revised for Version 1.1
Revised for Version 2.0 (Release 11)
Revised for Version 2.1.2 (Release 12)
Revised for Version 2.3 (Release 13)
Revised for Version 2.4 (Release 14)
Revised for Version 2.4.1 (Release 14+)
Revised for Version 2.5 (Release 14SP3)
Revised for Version 3.0 (Release 2006a)
Revised for Version 3.1 (Release 2006b)
Revised for Version 3.2 (Release 2007a)
Revised for Version 3.3 (Release 2007b)
Revised for Version 3.4 (Release 2008a)
Revised for Version 3.5 (Release 2008b)
Revised for Version 3.6 (Release 2009a)
Revised for Version 3.7 (Release 2009b)
Revised for Version 3.7.1 (Release 2010a)
Revised for Version 3.8 (Release 2010b)
Revised for Version 4.0 (Release 2011a)
Revised for Version 4.1 (Release 2011b)
Revised for Version 4.2 (Release 2012a)
Revised for Version 5.0 (Release 2012b)
Revised for Version 5.1 (Release 2013a)
Revised for Version 5.2 (Release 2013b)
Revised for Version 5.3 (Release 2014a)
Revised for Version 5.4 (Release 2014b)
Revised for Version 5.5 (Release 2015a)
Revised for Version 5.6 (Release 2015b)
Revised for Version 5.7 (Release 2016a)
Revised for Version 5.8 (Release 2016b)
Revised for Version 5.9 (Release 2017a)
Revised for Version 5.10 (Release 2017b)
Revised for Version 5.11 (Release 2018a)
Revised for Version 5.12 (Release 2018b)

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## Getting Started

- "Financial Toolbox Product Description" on page 1-2
- "Expected Users" on page 1-3
- "Analyze Sets of Numbers Using Matrix Functions" on page 1-4
- "Matrix Algebra Refresher" on page 1-7
- "Using Input and Output Arguments with Functions" on page 1-17


## Financial Toolbox Product Description Analyze financial data and develop financial models

Financial Toolbox provides functions for the mathematical modeling and statistical analysis of financial data. You can perform portfolio optimization taking into account turnover, transaction costs, semi-continuous constraints, and minimum or maximum number of assets. The toolbox enables you to estimate risk, model credit scorecards, analyze yield curves, price fixed-income instruments and European options, and measure investment performance. Time series analysis functions let you perform transformations or regressions with missing data and convert between different trading calendars and day-count conventions.

## Key Features

- Mean-variance, MAD, and CVaR-based portfolio optimization
- Cash flow analysis, risk analysis, financial time-series modeling, date math, and calendar math
- Basic fixed-income analysis and option pricing
- Regression and estimation with missing data
- Technical indicators and financial charts
- Monte Carlo simulation of SDE models
- Credit scorecard modeling and analysis


## Expected Users

In general, this guide assumes experience working with financial derivatives and some familiarity with the underlying models.

In designing Financial Toolbox documentation, we assume that your title is like one of these:

- Analyst, quantitative analyst
- Risk manager
- Portfolio manager
- Asset allocator
- Financial engineer
- Trader
- Student, professor, or other academic

We also assume that your background, education, training, and responsibilities match some aspects of this profile:

- Finance, economics, perhaps accounting
- Engineering, mathematics, physics, other quantitative sciences
- Focus on quantitative approaches to financial problems


# Analyze Sets of Numbers Using Matrix Functions 

In this section...<br>"Introduction" on page 1-4<br>"Key Definitions" on page 1-4<br>"Referencing Matrix Elements" on page 1-5<br>"Transposing Matrices" on page 1-6

## Introduction

Many financial analysis procedures involve sets of numbers; for example, a portfolio of securities at various prices and yields. Matrices, matrix functions, and matrix algebra are the most efficient ways to analyze sets of numbers and their relationships. Spreadsheets focus on individual cells and the relationships between cells. While you can think of a set of spreadsheet cells (a range of rows and columns) as a matrix, a matrix-oriented tool like MATLAB ${ }^{\circledR}$ software manipulates sets of numbers more quickly, easily, and naturally. For more information, see "Matrix Algebra Refresher" on page 1-7.

## Key Definitions

## Matrix

A rectangular array of numeric or algebraic quantities subject to mathematical operations; the regular formation of elements into rows and columns. Described as a "m-by- $n$ " matrix, with $m$ the number of rows and $n$ the number of columns. The description is always "row-by-column." For example, here is a 2-by-3 matrix of two bonds (the rows) with different par values, coupon rates, and coupon payment frequencies per year (the columns) entered using MATLAB notation:

$$
\text { Bonds } \left.=\begin{array}{rll}
{[1000} & 0.06 & 2 \\
500 & 0.055 & 4
\end{array}\right]
$$

## Vector

A matrix with only one row or column. Described as a " 1 -by-n" or " $m$-by- 1 " matrix. The description is always "row-by-column." For example, here is a 1-by-4 vector of cash flows in MATLAB notation:

```
Cash = [1500 4470 5280 -1299]
```


## Scalar

A 1-by-1 matrix; that is, a single number.

## Referencing Matrix Elements

To reference specific matrix elements, use (row, column) notation. For example:
Bonds(1,2)
ans =
0.06

Cash(3)
ans $=$
5280.00

You can enlarge matrices using small matrices or vectors as elements. For example,
AddBond $=\left[\begin{array}{lll}1000 & 0.065 & 2\end{array}\right] ;$
Bonds = [Bonds; AddBond]
adds another row to the matrix and creates

## Bonds =

| 1000 | 0.06 | 2 |
| ---: | :--- | :--- |
| 500 | 0.055 | 4 |
| 1000 | 0.065 | 2 |

Likewise,
Prices $=[987.50$
475.00
995.00]

Bonds = [Prices, Bonds]
adds another column and creates
Bonds =

| 987.50 | 1000 | 0.06 | 2 |
| :--- | ---: | :--- | :--- |
| 475.00 | 500 | 0.055 | 4 |
| 995.00 | 1000 | 0.065 | 2 |

Finally, the colon (: ) is important in generating and referencing matrix elements. For example, to reference the par value, coupon rate, and coupon frequency of the second bond:

BondItems $=$ Bonds(2, 2:4)
BondItems =
$500.00 \quad 0.0554$

## Transposing Matrices

Sometimes matrices are in the wrong configuration for an operation. In MATLAB, the apostrophe or prime character (') transposes a matrix: columns become rows, rows become columns. For example,

Cash $=\left[\begin{array}{llll}1500 & 4470 & 5280 & -1299\end{array}\right]^{\prime}$
produces
Cash =
1500
4470
5280
-1299

## See Also

## More About

- "Matrix Algebra Refresher" on page 1-7
- "Using Input and Output Arguments with Functions" on page 1-17


## Matrix Algebra Refresher

In this section...<br>"Introduction" on page 1-7<br>"Adding and Subtracting Matrices" on page 1-7<br>"Multiplying Matrices" on page 1-8<br>"Dividing Matrices" on page 1-12<br>"Solving Simultaneous Linear Equations" on page 1-13<br>"Operating Element by Element" on page 1-16

## Introduction

The explanations in the sections that follow should help refresh your skills for using matrix algebra and using MATLAB functions.

In addition, Macro-Investment Analysis by William Sharpe also provides an excellent explanation of matrix algebra operations using MATLAB. It is available on the web at:
https://www.stanford.edu/~wfsharpe/mia/mia.htm

Tip When you are setting up a problem, it helps to "talk through" the units and dimensions associated with each input and output matrix. In the example under "Multiplying Matrices" on page 1-8, one input matrix has five days' closing prices for three stocks, the other input matrix has shares of three stocks in two portfolios, and the output matrix therefore has five days' closing values for two portfolios. It also helps to name variables using descriptive terms.

## Adding and Subtracting Matrices

Matrix addition and subtraction operate element-by-element. The two input matrices must have the same dimensions. The result is a new matrix of the same dimensions where each element is the sum or difference of each corresponding input element. For example, consider combining portfolios of different quantities of the same stocks ("shares of stocks $\mathrm{A}, \mathrm{B}$, and C [the rows] in portfolios P and Q [the columns] plus shares of $\mathrm{A}, \mathrm{B}$, and C in portfolios R and $\mathrm{S}^{\prime \prime}$ ).

```
Portfolios_PQ = [100 200
    500 400
    300 150];
Portfolios_RS = [175 125
    200 200
    100 500];
NewPortfolios = Portfolios_PQ + Portfolios_RS
NewPortfolios =
    275 325
    700 600
    400 650
```

Adding or subtracting a scalar and a matrix is allowed and also operates element-byelement.

SmallerPortf $=$ NewPortfolios-10
SmallerPortf =
$265.00 \quad 315.00$
$690.00 \quad 590.00$
$390.00 \quad 640.00$

## Multiplying Matrices

Matrix multiplication does not operate element-by-element. It operates according to the rules of linear algebra. In multiplying matrices, it helps to remember this key rule: the inner dimensions must be the same. That is, if the first matrix is $m$-by- 3 , the second must be 3-by-n. The resulting matrix is $m$-by- $n$. It also helps to "talk through" the units of each matrix, as mentioned in "Analyze Sets of Numbers Using Matrix Functions" on page 1-4.

Matrix multiplication also is not commutative; that is, it is not independent of order. A*B does not equal $\mathrm{B}^{*} \mathrm{~A}$. The dimension rule illustrates this property. If A is 1-by-3 matrix and B is 3-by-1 matrix, A * B yields a scalar (1-by-1) matrix but B * A yields a 3-by-3 matrix.

## Multiplying Vectors

Vector multiplication follows the same rules and helps illustrate the principles. For example, a stock portfolio has three different stocks and their closing prices today are:

```
ClosePrices = [42.5 15 78.875]
```

The portfolio contains these numbers of shares of each stock.
NumShares = [100
500
300]
To find the value of the portfolio, multiply the vectors

```
PortfValue = ClosePrices * NumShares
```

which yields:
PortfValue =

## $3.5413 \mathrm{e}+004$

The vectors are 1 -by- 3 and 3 -by-1; the resulting vector is 1-by-1, a scalar. Multiplying these vectors thus means multiplying each closing price by its respective number of shares and summing the result.

To illustrate order dependence, switch the order of the vectors

```
Values = NumShares * ClosePrices
```

Values =
$1.0 \mathrm{e}+004$ *

| 0.4250 | 0.1500 | 0.7887 |
| :--- | :--- | :--- |
| 2.1250 | 0.7500 | 3.9438 |
| 1.2750 | 0.4500 | 2.3663 |

which shows the closing values of 100,500 , and 300 shares of each stock, not the portfolio value, and meaningless for this example.

## Computing Dot Products of Vectors

In matrix algebra, if $X$ and $Y$ are vectors of the same length

$$
\begin{aligned}
& Y=\left[y_{1}, y_{2}, \ldots, y_{n}\right] \\
& X=\left[x_{1}, x_{2}, \ldots, x_{n}\right]
\end{aligned}
$$

then the dot product

$$
X \cdot Y=x_{1} y_{1}+x_{2} y_{2}+\ldots+x_{n} y_{n}
$$

is the scalar product of the two vectors. It is an exception to the commutative rule. To compute the dot product in MATLAB, use sum (X . * Y) or sum (Y .* X). Be sure that the two vectors have the same dimensions. To illustrate, use the previous vectors.

```
Value = sum(NumShares .* ClosePrices')
Value =
    3.5413e+004
Value = sum(ClosePrices .* NumShares')
Value =
    3.5413e+004
```

As expected, the value in these cases matches the PortfValue computed previously.

## Multiplying Vectors and Matrices

Multiplying vectors and matrices follows the matrix multiplication rules and process. For example, a portfolio matrix contains closing prices for a week. A second matrix (vector) contains the stock quantities in the portfolio.

| WeekClosePr = [42.5 | 15 | 78.875 |
| :---: | :---: | :---: |
| 42.125 | 15.5 | 78.75 |
| 42.125 | 15.125 | 79 |
| 42.625 | 15.25 | 78.875 |
| 43 | 15.25 | 78.625]; |
| PortQuan $=$ [100 |  |  |
| 500 |  |  |
| 300]; |  |  |

To see the closing portfolio value for each day, simply multiply
WeekPortValue $=$ WeekClosePr * PortQuan
WeekPortValue =
$1.0 \mathrm{e}+004$ *
3.5412

The prices matrix is 5 -by-3, the quantity matrix (vector) is 3 -by- 1 , so the resulting matrix (vector) is 5-by-1.

## Multiplying Two Matrices

Matrix multiplication also follows the rules of matrix algebra. In matrix algebra notation, if $A$ is an $m$-by- $n$ matrix and $B$ is an $n$-by- $p$ matrix

$$
A=\left[\begin{array}{cccc}
a_{11} & a_{12} & \cdots & a_{1 n} \\
\vdots & \vdots & & \vdots \\
a_{i 1} & a_{i 2} & \cdots & a_{i n} \\
\vdots & \vdots & & \vdots \\
a_{m 1} & a_{m 2} & \cdots & a_{m n}
\end{array}\right], \quad B=\left[\begin{array}{ccccc}
b_{11} & \cdots & b_{1 j} & \cdots & b_{1 p} \\
b_{21} & \cdots & b_{2 j} & \cdots & b_{2 p} \\
\vdots & & \vdots & & \vdots \\
b_{n 1} & \cdots & b_{n j} & \cdots & b_{n p}
\end{array}\right]
$$

then $C=A^{*} B$ is an $m$-by- $p$ matrix; and the element $c_{i \mathrm{ij}}$ in the $i$ th row and $j$ th column of $C$ is

$$
c_{i j}=a_{i 1} b_{1 j}+a_{i 2} b_{12}+\ldots+a_{i n} b_{n j} .
$$

To illustrate, assume that there are two portfolios of the same three stocks previously mentioned but with different quantities.

```
Portfolios = [100 200
    500 400
    300 150];
```

Multiplying the 5 -by- 3 week's closing prices matrix by the 3 -by- 2 portfolios matrix yields a 5-by-2 matrix showing each day's closing value for both portfolios.

PortfolioValues = WeekClosePr * Portfolios
PortfolioValues =
$1.0 \mathrm{e}+004$ *

| 3.5412 | 2.6331 |
| :--- | :--- |
| 3.5587 | 2.6437 |


| 3.5475 | 2.6325 |
| :--- | :--- |
| 3.5550 | 2.6456 |
| 3.5513 | 2.6494 |

Monday's values result from multiplying each Monday closing price by its respective number of shares and summing the result for the first portfolio, then doing the same for the second portfolio. Tuesday's values result from multiplying each Tuesday closing price by its respective number of shares and summing the result for the first portfolio, then doing the same for the second portfolio. And so on, through the rest of the week. With one simple command, MATLAB quickly performs many calculations.

## Multiplying a Matrix by a Scalar

Multiplying a matrix by a scalar is an exception to the dimension and commutative rules. It just operates element-by-element.

```
Portfolios = [100 200
    500 400
    300 150];
DoublePort = Portfolios * 2
DoublePort =
    200 400
    1000 800
    600 300
```


## Dividing Matrices

Matrix division is useful primarily for solving equations, and especially for solving simultaneous linear equations (see "Solving Simultaneous Linear Equations" on page 113). For example, you want to solve for $X$ in $A^{*} X=B$.

In ordinary algebra, you would divide both sides of the equation by $A$, and $X$ would equal $B / A$. However, since matrix algebra is not commutative ( $A^{*} X \neq X^{*} A$ ), different processes apply. In formal matrix algebra, the solution involves matrix inversion. MATLAB, however, simplifies the process by providing two matrix division symbols, left and right ( $\backslash$ and /). In general,
$X=A \backslash B$ solves for $X$ in $A * X=B$ and
$X=B / A$ solves for $X$ in $X * A=B$.

In general, matrix A must be a nonsingular square matrix; that is, it must be invertible and it must have the same number of rows and columns. (Generally, a matrix is invertible if the matrix times its inverse equals the identity matrix. To understand the theory and proofs, consult a textbook on linear algebra such as Elementary Linear Algebra by Hill listed in "Bibliography" on page A-2.) MATLAB gives a warning message if the matrix is singular or nearly so.

## Solving Simultaneous Linear Equations

Matrix division is especially useful in solving simultaneous linear equations. Consider this problem: Given two portfolios of mortgage-based instruments, each with certain yields depending on the prime rate, how do you weight the portfolios to achieve certain annual cash flows? The answer involves solving two linear equations.

A linear equation is any equation of the form

$$
a_{1} x+a_{2} y=b
$$

where $a_{1}, a_{2}$, and $b$ are constants (with $a_{1}$ and $a_{2}$ not both 0 ), and $x$ and $y$ are variables. (It is a linear equation because it describes a line in the $x y$-plane. For example, the equation $2 x+y=8$ describes a line such that if $x=2$, then $y=4$.)

A system of linear equations is a set of linear equations that you usually want to solve at the same time; that is, simultaneously. A basic principle for exact answers in solving simultaneous linear equations requires that there be as many equations as there are unknowns. To get exact answers for $x$ and $y$, there must be two equations. For example, to solve for $x$ and $y$ in the system of linear equations

$$
\begin{aligned}
& 2 x+y=13 \\
& x-3 y=-18
\end{aligned}
$$

there must be two equations, which there are. Matrix algebra represents this system as an equation involving three matrices: $A$ for the left-side constants, $X$ for the variables, and $B$ for the right-side constants

$$
A=\left[\begin{array}{cc}
2 & 1 \\
1 & -3
\end{array}\right], \quad X=\left[\begin{array}{l}
x \\
y
\end{array}\right], \quad B=\left[\begin{array}{c}
13 \\
-18
\end{array}\right],
$$

where $A^{*} X=B$.

Solving the system simultaneously means solving for $X$. Using MATLAB,

```
A = [\begin{array}{ll}{2}&{1}\end{array}]
    1 -3];
B = [13
    -18];
X=A\B
```

solves for X in $\mathrm{A} * \mathrm{X}=\mathrm{B}$.
$X=\left[\begin{array}{ll}3 & 7\end{array}\right]$

So $x=3$ and $y=7$ in this example. In general, you can use matrix algebra to solve any system of linear equations such as

$$
\begin{gathered}
a_{11} x_{1}+a_{12} x_{2}+\ldots+a_{1 n} x_{n}=b_{1} \\
a_{21} x_{1}+a_{22} x_{2}+\ldots+a_{2 n} x_{n}=b_{2} \\
\vdots \\
a_{m 1} x_{1}+a_{m 2} x_{2}+\ldots+a_{m n} x_{n}=b_{m}
\end{gathered}
$$

by representing them as matrices

$$
A=\left[\begin{array}{cccc}
a_{11} & a_{12} & \cdots & a_{1 n} \\
a_{21} & a_{22} & \cdots & a_{2 n} \\
\vdots & \vdots & & \vdots \\
a_{m 1} & a_{m 2} & \cdots & a_{m n}
\end{array}\right], \quad X=\left[\begin{array}{c}
x_{1} \\
x_{2} \\
\vdots \\
x_{n}
\end{array}\right], \quad B=\left[\begin{array}{c}
b_{1} \\
b_{2} \\
\vdots \\
b_{m}
\end{array}\right]
$$

and solving for $X$ in $A^{*} X=B$.
To illustrate, consider this situation. There are two portfolios of mortgage-based instruments, M1 and M2. They have current annual cash payments of $\$ 100$ and $\$ 70$ per unit, respectively, based on today's prime rate. If the prime rate moves down one percentage point, their payments would be $\$ 80$ and $\$ 40$. An investor holds 10 units of M1 and 20 units of M2. The investor's receipts equal cash payments times units, or $\mathrm{R}=\mathrm{C} * \mathrm{U}$, for each prime-rate scenario. As word equations:

|  | M1 | M2 |
| :--- | :--- | :--- |


| Prime flat: | $\$ 100 * 10$ units | $+\$ 70 * 20$ units $=\$ 2400$ <br> receipts |
| :--- | :--- | :--- |
| Prime down: | $\$ 80 * 10$ units | $+\$ 40 * 20$ units $=\$ 1600$ <br> receipts |

As MATLAB matrices:

```
Cash = [100 70
    80 40];
```

Units = [10
20];
Receipts $=$ Cash * Units
Receipts =
2400
1600

Now the investor asks this question: Given these two portfolios and their characteristics, how many units of each should they hold to receive $\$ 7000$ if the prime rate stays flat and $\$ 5000$ if the prime drops one percentage point? Find the answer by solving two linear equations.

|  | M1 | M2 |
| :--- | :--- | :--- |
| Prime flat: | $\$ 100 * x$ units | $+\$ 70 * y$ units $=\$ 7000$ <br> receipts |
| Prime down: | $\$ 80 * x$ units | $+\$ 40 * y$ units $=\$ 5000$ <br> receipts |

In other words, solve for U (units) in the equation R (receipts) $=\mathrm{C}$ (cash) $* \mathrm{U}$ (units). Using MATLAB left division

```
Cash = [100 70
    80 40];
Receipts = [7000
    5000];
Units = Cash \ Receipts
```


## Units =

43.7500
37.5000

The investor should hold 43.75 units of portfolio M1 and 37.5 units of portfolio M2 to achieve the annual receipts desired.

## Operating Element by Element

Finally, element-by-element arithmetic operations are called operations. To indicate a MATLAB array operation, precede the operator with a period (.). Addition and subtraction, and matrix multiplication and division by a scalar, are already array operations so no period is necessary. When using array operations on two matrices, the dimensions of the matrices must be the same. For example, given vectors of stock dividends and closing prices

```
Dividends = [1.90 0.40 1.56 4.50];
Prices = [\begin{array}{llll}{25.625 17.75 26.125 60.50];}\end{array};
Yields = Dividends ./ Prices
Yields =
\(0.0741 \quad 0.0225 \quad 0.0597 \quad 0.0744\)
```


## See Also

## More About

- "Analyze Sets of Numbers Using Matrix Functions" on page 1-4
- "Using Input and Output Arguments with Functions" on page 1-17


# Using Input and Output Arguments with Functions 

In this section...<br>"Input Arguments" on page 1-17<br>"Output Arguments" on page 1-19<br>"Interest Rate Arguments" on page 1-20

## Input Arguments

## Matrix Input

MATLAB software was designed to be a large-scale array (vector or matrix) processor. In addition to its linear algebra applications, the general array-based processing facility can perform repeated operations on collections of data. When MATLAB code is written to operate simultaneously on collections of data stored in arrays, the code is said to be vectorized. Vectorized code is not only clean and concise, but is also efficiently processed by MATLAB.

Because MATLAB can process vectors and matrices easily, most Financial Toolbox functions allow vector or matrix input arguments, rather than single (scalar) values. For example, the irr function computes the internal rate of return of a cash flow stream. It accepts a vector of cash flows and returns a scalar-valued internal rate of return. However, it also accepts a matrix of cash flow streams, a column in the matrix representing a different cash flow stream. In this case, irr returns a vector of internal rates of return, each entry in the vector corresponding to a column of the input matrix. Many other Financial Toolbox functions work similarly.

As an example, suppose that you make an initial investment of $\$ 100$, from which you then receive by a series of annual cash receipts of $\$ 10, \$ 20, \$ 30, \$ 40$, and $\$ 50$. This cash flow stream may be stored in a vector

CashFlows $=\left[\begin{array}{lllll}-100 & 10 & 20 & 30 & 40 \\ 50\end{array}\right]$
which MATLAB displays as

```
CashFlows =
```

- 100

10
20
30

The irr function can compute the internal rate of return of this stream.

```
Rate = irr(CashFlows)
```

The internal rate of return of this investment is

## Rate =

0.1201
or $12.01 \%$.
In this case, a single cash flow stream (written as an input vector) produces a scalar output - the internal rate of return of the investment.

Extending this example, if you process a matrix of identical cash flow streams

```
Rate = irr([CashFlows CashFlows CashFlows])
```

you should expect to see identical internal rates of return for each of the three investments.

## Rate =

$$
\begin{array}{lll}
0.1201 & 0.1201 & 0.1201
\end{array}
$$

This simple example illustrates the power of vectorized programming. The example shows how to collect data into a matrix and then use a toolbox function to compute answers for the entire collection. This feature can be useful in portfolio management, for example, where you might want to organize multiple assets into a single collection. Place data for each asset in a different column or row of a matrix, then pass the matrix to a Financial Toolbox function. MATLAB performs the same computation on all of the assets at once.

## Matrices of Character Vector Input

Enter MATLAB character vectors surrounded by single quotes ('string').
A character vector is stored as a character array, one ASCII character per element. Thus, the date character vector

DateString = '9/16/2001'
is actually a 1-by-9 vector. Character vectors making up the rows of a matrix or vector all must have the same length. To enter several date character vectors, therefore, use a column vector and be sure that all character vectors are the same length. Fill in with spaces or zeros. For example, to create a vector of dates corresponding to irregular cash flows

```
DateFields = ['01/12/2001'
    '02/14/2001'
    '03/03/2001'
    '06/14/2001'
    '12/01/2001'];
```

DateFields actually becomes a 5-by-10 character array.
Do not mix numbers and character vectors in a matrix. If you do, MATLAB treats all entries as characters. For example,

```
Item = [l83 90 99 '14-Sep-1999']
```

becomes a 1-by-14 character array, not a 1-by-4 vector, and it contains

## Item =

SZc14-Sep-1999

## Output Arguments

Some functions return no arguments, some return just one, and some return multiple arguments. Functions that return multiple arguments use the syntax
[A, B, C] = function(variables...)
to return arguments $A, B$, and $C$. If you omit all but one, the function returns the first argument. Thus, for this example if you use the syntax

X = function(variables...)
function returns a value for A , but not for B or C .
Some functions that return vectors accept only scalars as arguments. Why could such functions not accept vectors as arguments and return matrices, where each column in the output matrix corresponds to an entry in the input vector? The answer is that the output vectors can be variable length and thus will not fit in a matrix without some convention to indicate that the shorter columns are missing data.

Functions that require asset life as an input, and return values corresponding to different periods over that life, cannot generally handle vectors or matrices as input arguments. Those functions are:

| amortize | Amortization |
| :--- | :--- |
| depfixdb | Fixed declining-balance depreciation |
| depgendb | General declining-balance depreciation |
| depsoyd | Sum of years' digits depreciation |

For example, suppose that you have a collection of assets such as automobiles and you want to compute the depreciation schedules for them. The function depfixdb computes a stream of declining-balance depreciation values for an asset. You might want to set up a vector where each entry is the initial value of each asset. depfixdb also needs the lifetime of an asset. If you were to set up such a collection of automobiles as an input vector, and the lifetimes of those automobiles varied, the resulting depreciation streams would differ in length according to the life of each automobile, and the output column lengths would vary. A matrix must have the same number of rows in each column.

## Interest Rate Arguments

One common argument, both as input and output, is interest rate. All Financial Toolbox functions expect and return interest rates as decimal fractions. Thus an interest rate of $9.5 \%$ is indicated as 0.095 .

## See Also

## More About

- "Analyze Sets of Numbers Using Matrix Functions" on page 1-4
- "Matrix Algebra Refresher" on page 1-7


## Performing Common Financial Tasks

- "Handle and Convert Dates" on page 2-2
- "Charting Financial Data" on page 2-14
- "Bollinger Chart" on page 2-15
- "Analyzing and Computing Cash Flows" on page 2-18
- "Pricing and Computing Yields for Fixed-Income Securities" on page 2-22
- "Treasury Bills Defined" on page 2-37
- "Computing Treasury Bill Price and Yield" on page 2-38
- "Term Structure of Interest Rates" on page 2-42
- "Pricing and Analyzing Equity Derivatives" on page 2-45
- "About Life Tables" on page 2-50
- "Case Study for Life Tables Analysis" on page 2-53


## Handle and Convert Dates

```
In this section...
"Date Formats" on page 2-2
"Date Conversions" on page 2-3
"Current Date and Time" on page 2-9
"Determining Specific Dates" on page 2-10
"Determining Holidays" on page 2-11
"Determining Cash-Flow Dates" on page 2-12
```


## Date Formats

Virtually all financial data derives from a time series, functions in Financial Toolbox have extensive date-handling capabilities. The toolbox functions support date or date-and-time formats as character vectors, datetime arrays, or serial date numbers.

- Date character vectors are text that represent date and time, which you can use with multiple formats. For example, 'dd-mmm-yyyy HH:MM:SS', 'dd-mmm-yyyy', and 'mm/dd/yyyy' are all supported text formats for a date character vector. Most often, you work with date character vectors (such as 14-Sep-1999) when dealing with dates.
- Datetime arrays, created using datetime, are the best data type for representing points in time. datetime values have flexible display formats and up to nanosecond precision, and can account for time zones, daylight saving time, and leap seconds. When datetime objects are used as inputs to other Financial Toolbox functions, the format of the input datetime object is preserved. For example:

```
originalDate = datetime('now','Format','yyyy-MM-dd HH:mm:ss');
    % Find the next business day
    b = busdate(originalDate)
    b =
    datetime
        2017-04-20 14:47:39
```

- Serial date numbers represent a calendar date as the number of days that have passed since a fixed base date. In MATLAB software, serial date number 1 is January 1,0000 A.D. Financial Toolbox works internally with serial date numbers (such as, 730377). MATLAB also uses serial time to represent fractions of days beginning at midnight. For
example, 6 p.m. equals 0.75 serial days, so 6:00 p.m. on 14-Sep-1999, in MATLAB, is serial date number 730377.75

Note If you specify a two-digit year, MATLAB assumes that the year lies within the 100year period centered on the current year. See the function datenum for specific information. MATLAB internal date handling and calculations generate no ambiguous values. However, whenever possible, use serial date numbers or date character vectors containing four-digit years.

Many Financial Toolbox functions that require dates as input arguments accept date character vectors, datetime arrays, or serial date numbers. If you are dealing with a few dates at the MATLAB command-line level, date character vectors are more convenient. If you are using Financial Toolbox functions on large numbers of dates, as in analyzing large portfolios or cash flows, performance improves if you use datetime arrays or serial date numbers. For more information, see "Represent Dates and Times in MATLAB" (MATLAB).

## Date Conversions

Financial Toolbox provides functions that convert date character vectors to or from serial date numbers. In addition, you can convert character vectors or serial date numbers to datetime arrays.

Functions that convert between date formats are:

| datedisp | Displays a numeric matrix with date entries formatted as date <br> character vectors. |
| :--- | :--- |
| datenum | Converts a date character vector to a serial date number. |
| datestr | Converts a serial date number to a date character vector. |
| datetime | Converts from date character vectors or serial date numbers to <br> create a datetime array. |
| datevec | Converts a serial date number or date character vector to a date <br> vector whose elements are <br> $[$ Year Month Day Hour Minute Second $]$. |
| m2xdate | Converts MATLAB serial date number to Excel ${ }^{\circledR}$ serial date number. |
| x2mdate | Converts Microsoft ${ }^{\circledR}$ Excel serial date number to MATLAB serial <br> date number. |

For more information, see "Convert Between Datetime Arrays, Numbers, and Text" (MATLAB).

## Convert Between Datetime Arrays and Character Vectors

A date can be a character vector composed of fields related to a specific date and time. There are several ways to represent dates and times in several text formats. For example, all the following are character vectors represent August 23, 2010 at 04:35:42 PM:

```
'23-Aug-2010 04:35:06 PM'
'Wednesday, August 23'
'08/23/10 16:35'
'Aug 23 16:35:42.946'
```

A date character vector includes characters that separate the fields, such as the hyphen, space, and colon used here:

```
d = '23-Aug-2010 16:35:42'
```

Convert one or more date character vectors to a datetime array using the datetime function. For the best performance, specify the format of the input character vectors as an input to datetime.

Note The specifiers that datetime uses to describe date and time formats differ from the specifiers that the datestr, datevec, and datenum functions accept.

```
t = datetime(d,'InputFormat','dd-MMM-yyyy HH:mm:ss')
```

$\mathrm{t}=$

## 23-Aug-2010 16:35:42

Although the date string, d , and the datetime scalar, t , look similar, they are not equal. View the size and data type of each variable.

```
whos d t
```

| Name | Size | Bytes Class | Attributes |  |
| :--- | :--- | ---: | :--- | ---: |
|  |  |  |  |  |
| d | $1 \times 20$ | 40 | char |  |
| t | $1 \times 1$ | 121 | datetime |  |

Convert a datetime array to a character vector that uses char or cellstr. For example, convert the current date and time to a timestamp to append to a file name.

```
t = datetime('now','Format','yyyy-MM-dd''T''HHmmss')
t =
    datetime
```

    2016-12-11T125628
    S = char(t);
filename = ['myTest_',S]
filename =
'myTest_2016-12-11T125628'

## Convert Serial Date Numbers to Datetime Arrays

Serial time can represent fractions of days beginning at midnight. For example, 6 p.m. equals 0.75 serial days, so the character vector '31-0ct-2003, 6:00 PM' in MATLAB is date number 731885.75.

Convert one or more serial date numbers to a datetime array using the datetime function. Specify the type of date number that is being converted:

```
t = datetime(731885.75,'ConvertFrom','datenum')
t =
```

    datetime
    31-Oct-2003 18:00:00
    
## Convert Datetime Arrays to Numeric Values

Some MATLAB functions accept numeric data types but not datetime values as inputs. To apply these functions to your date and time data, first, convert datetime values to meaningful numeric values, and then call the function. For example, the log function accepts double inputs but not datetime inputs. Suppose that you have a datetime array of dates spanning the course of a research study or experiment.
$\mathrm{t}=$ datetime(2014,6,18) + calmonths(1:4)

```
t =
    1\times4 datetime array
    18-Jul-2014 18-Aug-2014 18-Sep-2014 18-0ct-2014
```

Subtract the origin value. For example, the origin value can be the starting day of an experiment.

```
dt = t - datetime(2014,7,1)
dt =
    1\times4 duration array
    408:00:00 1152:00:00 1896:00:00 2616:00:00
```

$d t$ is a duration array. Convert $d t$ to a double array of values in units of years, days, hours, minutes, or seconds by using the years, days, hours, minutes, or seconds function, respectively.

```
x = hours(dt)
x =
    408 1152 1896 2616
```

Pass the double array as the input to the log function.

```
y = log(x)
y =
```

    \(\begin{array}{llll}6.0113 & 7.0493 & 7.5475 & 7.8694\end{array}\)
    
## Input Conversions with datenum

The datenum function is important for using Financial Toolbox software efficiently. datenum takes an input date character vector in any of several formats, with 'dd-mmmyyyy','mm/dd/yyyy', or 'dd-mmm-yyyy, hh:mm:ss.ss' formats being the most common. The input date character vector can have up to six fields formed by letters and numbers separated by any other characters, such that:

- The day field is an integer from 1 through 31.
- The month field is either an integer from 1 through 12 or an alphabetical character vector with at least three characters.
- The year field is a nonnegative integer. If only two numbers are specified, then the year is assumed to lie within the 100-year period centered on the current year. If the year is omitted, the current year is the default.
- The hours, minutes, and seconds fields are optional. They are integers separated by colons or followed by 'am' or 'pm'.

For example, if the current year is 1999, then all these dates are equivalent:

```
' 17 -May-1999 '
'17-May-99'
'17-may'
'May 17, 1999'
'5/17/99'
'5/17'
```

Also, both of these formats represent the same time.

```
'17-May-1999, 18:30'
'5/17/99/6:30 pm'
```

The default format for numbers-only input follows the US convention. Therefore, $3 / 6$ is March 6, not June 3.

With datenum, you can convert dates into serial date format, store them in a matrix variable, and then later pass the variable to a function. Alternatively, you can use datenum directly in a function input argument list.

For example, consider the function bndprice that computes the price of a bond given the yield to maturity. First set up variables for the yield to maturity, coupon rate, and the necessary dates.

```
Yield = 0.07;
CouponRate = 0.08;
Settle = datenum('17-May-2000');
Maturity = datenum('01-0ct-2000');
```

Then call the function with the variables.

```
bndprice(Yield,CouponRate,Settle,Maturity)
```


## ans =

100.3503

Alternatively, convert date character vectors to serial date numbers directly in the function input argument list.

```
bndprice(0.07,0.08,datenum('17-May-2000'),...
datenum('01-0ct-2000'))
ans =
```

    100.3503
    bndprice is an example of a function designed to detect the presence of date character vectors and make the conversion automatically. For functions like bndprice, date character vectors can be passed directly.
bndprice(0.07,0.08,'17-May-2000','01-0ct-2000')
ans =
100.3503

The decision to represent dates as either date character vectors or serial date numbers is often a matter of convenience. For example, when formatting data for visual display or for debugging date-handling code, you can view dates more easily as date character vectors because serial date numbers are difficult to interpret. Alternately, serial date numbers are just another type of numeric data, which you can place in a matrix along with any other numeric data for convenient manipulation.

Remember that if you create a vector of input date character vectors, use a column vector, and be sure that all character vectors are the same length. To ensure that the character vectors are the same length, fill the character vectors with spaces or zeros. For more information, see "Matrices of Character Vector Input" on page 1-18.

## Output Conversions with datestr

The datestr function converts a serial date number to one of 19 different date character vector output formats showing date, time, or both. The default output for dates is a day-month-year character vector, for example, 24-Aug-2000. The datestr function is useful for preparing output reports.

| datestr Format | Description |
| :---: | :---: |
| 01-Mar-2000 15:45:17 | day-month-year hour:minute:second |
| 01-Mar-2000 | day-month-year |
| 03/01/00 | month/day/year |
| Mar | month, three letters |
| M | month, single letter |
| 3 | month number |
| 03/01 | month/day |
| 1 | day of month |
| Wed | day of week, three letters |
| W | day of week, single letter |
| 2000 | year, four numbers |
| 99 | year, two numbers |
| Mar01 | month year |
| 15:45:17 | hour:minute:second |
| 03:45:17 PM | hour:minute:second AM or PM |
| 15:45 | hour:minute |
| 03:45 PM | hour:minute AM or PM |
| Q1-99 | calendar quarter-year |
| Q1 | calendar quarter |
| Current Date and Time |  |
| The today and now functions return serial date numbers for the current date, and the current date and time, respectively. |  |
| today |  |
| ans = |  |
| 736675 |  |
| now |  |

## ans =

$7.3668 e+05$
The MATLAB function date returns a character vector for the current date.
date
ans $=$
'11-Dec-2016'

## Determining Specific Dates

Financial Toolbox provides many functions for determining specific dates. For example, assume that you schedule an accounting procedure for the last Friday of every month. Use the lweekdate function to return those dates for the year 2000. The input argument 6 specifies Friday.

```
Fridates = lweekdate(6,2000,1:12);
Fridays = datestr(Fridates)
Fridays =
    12\times11 char array
    '28-Jan-2000'
    '25-Feb-2000'
    '31-Mar-2000'
    '28-Apr-2000'
    '26-May-2000'
    '30-Jun-2000'
    '28-Jul-2000'
    '25-Aug-2000'
    '29-Sep-2000'
    '27-0ct-2000'
    '24-Nov-2000'
    '29-Dec-2000'
```

Another example of needing specific dates could be that your company closes on Martin Luther King Jr. Day, which is the third Monday in January. You can use thenweekdate function to determine those specific dates for 2011 through 2014.

```
MLKDates = nweekdate(3,2,2011:2014,1);
MLKDays = datestr(MLKDates)
MLKDays =
4\times11 char array
    '17-Jan-2011'
    '16-Jan-2012'
    '21-Jan-2013'
    '20-Jan-2014'
```


## Determining Holidays

Accounting for holidays and other nontrading days is important when you examine financial dates. Financial Toolbox provides the holidays function, which contains holidays and special nontrading days for the New York Stock Exchange from 1950 through 2030, inclusive. In addition, you can use nyseclosures to evaluate all known or anticipated closures of the New York Stock Exchange from January 1, 1885, to December 31, 2050. nyseclosures returns a vector of serial date numbers corresponding to market closures between the dates StartDate and EndDate, inclusive.

In this example, use holidays to determine the standard holidays in the last half of 2012.

```
LHHDates = holidays('1-Jul-2012','31-Dec-2012');
LHHDays = datestr(LHHDates)
LHHDays =
    6x11 char array
    '04-Jul-2012'
    '03-Sep-2012'
    '29-0ct-2012'
    '30-0ct-2012'
    '22-Nov-2012'
    '25-Dec-2012'
```

You can then use the busdate function to determine the next business day in 2012 after these holidays.

```
LHNextDates = busdate(LHHDates);
```

LHNextDays = datestr(LHNextDates)

LHNextDays =
$6 \times 11$ char array
'05-Jul-2012'
'04-Sep-2012'
'31-0ct-2012'
'31-0ct-2012'
' 23 -Nov-2012'
'26-Dec-2012'

## Determining Cash-Flow Dates

To determine cash-flow dates for securities with periodic payments, use cfdates. This function accounts for the coupons per year, the day-count basis, and the end-of-month rule. For example, you can determine the cash-flow dates for a security that pays four coupons per year on the last day of the month using an actual/365 day-count basis. To do so, enter the settlement date, the maturity date, and the parameters for Period, Basis, and EndMonthRule.

```
PayDates = cfdates('14-Mar-2000','30-Nov-2001',4,3,1);
PayDays = datestr(PayDates)
PayDays =
7x11 char array
'31-May-2000'
'31-Aug-2000'
'30-Nov-2000'
'28-Feb-2001'
'31-May-2001'
'31-Aug-2001'
'30-Nov-2001'
```


## See Also

busdate | cfdates | date | datedisp|datenum | datestr|datetime | datevec | format | holidays | lweekdate|m2xdate | nweekdate | nyseclosures |x2mdate

## Related Examples

- "Convert Between Datetime Arrays, Numbers, and Text" (MATLAB)
- "Read Collection or Sequence of Spreadsheet Files" (MATLAB)
- "Trading Calendars User Interface" on page 16-2
- "UICalendar User Interface" on page 16-4


## More About

- "Convert Dates Between Microsoft Excel and MATLAB" (Spreadsheet Link)


## Charting Financial Data

## Introduction

The following toolbox financial charting functions plot financial data and produce presentation-quality figures quickly and easily.

| bolling | Bollinger band chart |
| :--- | :--- |
| candle | Candlestick chart |
| candle | Time series candle plot |
| pointfig | Point and figure chart |
| highlow | High, low, open, close chart |
| highlow | Time series High-Low plot |
| movavg | Leading and lagging moving averages chart |

These functions work with standard MATLAB functions that draw axes, control appearance, and add labels and titles. The toolbox also provides a comprehensive set of charting functions that work with timetable objects, see "Chart Technical Indicators".

Here is a Bollinger band on page 2-15 chart. This example loads data from an external file (ibm. dat) and then calls the function using subsets of the data. The MATLAB variable ibm, which is created by loading ibm. dat, is a six-column matrix where each row is a trading day's data and where columns 2, 3, and 4 contain the high, low, and closing prices, respectively. The data in ibm. dat is fictional and for illustrative use only.

## See Also

bolling|bollinger|busdate|candle|candle|cfdates|cur2frac|cur2str| date | dateaxis|datedisp|datenum | datestr|datevec|frac2cur| highlow| highlow| holidays | load|lweekdate | m2xdate | movavg | nweekdate | nyseclosures | pointfig|size| $\times 2$ mdate

## Related Examples

- "Bollinger Chart" on page 2-15
- "Handle and Convert Dates" on page 2-2


## Bollinger Chart

The bolling function in Financial Toolbox ${ }^{\text {TM }}$ software produces a Bollinger band chart using all the closing prices in an IBM® stock price matrix. A Bollinger band chart plots actual data along with three other bands of data. The upper band is two standard deviations above a moving average; the lower band is two standard deviations below that moving average; and the middle band is the moving average itself. This example uses a 15 -day moving average. First, load the data using the ibm.dat data file and then execute the bolling function to plot the Bollinger bands.

```
load ibm.dat;
[ro, co] = size(ibm);
bolling(ibm(:,4), 15, 0);
```

Warning: BOLLING will be removed in a future release. Use BOLLINGER instead.

```
axis([0 ro min(ibm(:,4)) max(ibm(:,4))]);
ylabel('Price ($)');
title(['International Business Machines']);
dateaxis('x', 6,'31-Dec-1994')
```



Specify the axes, labels, and titles. Use dateaxis to add the $x$-axis dates.
For help using MATLAB ${ }^{\circledR}$ plotting functions, see "Create 2-D Line Plot" (MATLAB) in the MATLAB documentation. See the MATLAB documentation for details on the axis, title, xlabel, and ylabel functions.

## See Also

bolling | bollinger | busdate | candle | candle |cfdates |cur2frac|cur2str| date | dateaxis | datedisp | datenum| datestr|datevec|frac2cur|highlow| highlow| holidays| load|lweekdate|m2xdate|movavg|nweekdate| nyseclosures | pointfig|size|x2mdate

## Related Examples

- "Handle and Convert Dates" on page 2-2


## Analyzing and Computing Cash Flows

In this section...<br>"Introduction" on page 2-18<br>"Interest Rates/Rates of Return" on page 2-18<br>"Present or Future Values" on page 2-19<br>"Depreciation" on page 2-20<br>"Annuities" on page 2-20

## Introduction

Financial Toolbox cash-flow functions compute interest rates and rates of return, present or future values, depreciation streams, and annuities.

Some examples in this section use this income stream: an initial investment of \$20,000 followed by three annual return payments, a second investment of $\$ 5,000$, then four more returns. Investments are negative cash flows, return payments are positive cash flows.

```
Stream = [-20000, 2000, 2500, 3500, -5000, 6500,...
    9500, 9500, 9500];
```


## Interest Rates/Rates of Return

Several functions calculate interest rates involved with cash flows. To compute the internal rate of return of the cash stream, execute the toolbox function irr

```
ROR = irr(Stream)
```

which gives a rate of return of $11.72 \%$.
The internal rate of return of a cash flow may not have a unique value. Every time the sign changes in a cash flow, the equation defining irr can give up to two additional answers. An irr computation requires solving a polynomial equation, and the number of real roots of such an equation can depend on the number of sign changes in the coefficients. The equation for internal rate of return is

$$
\frac{c f_{1}}{(1+r)}+\frac{c f_{2}}{(1+r)^{2}}+\ldots+\frac{c f_{n}}{(1+r)^{n}}+\text { Investment }=0
$$

where Investment is a (negative) initial cash outlay at time $0, c f_{\mathrm{n}}$ is the cash flow in the $n$th period, and $n$ is the number of periods. irr finds the rate $r$ such that the present value of the cash flow equals the initial investment. If all the $c f_{\mathrm{n}} \mathrm{s}$ are positive there is only one solution. Every time there is a change of sign between coefficients, up to two additional real roots are possible.

Another toolbox rate function, effrr, calculates the effective rate of return given an annual interest rate (also known as nominal rate or annual percentage rate, APR) and number of compounding periods per year. To find the effective rate of a $9 \%$ APR compounded monthly, enter

Rate $=$ effrr(0.09, 12)
The answer is $9.38 \%$.
A companion function nomrr computes the nominal rate of return given the effective annual rate and the number of compounding periods.

## Present or Future Values

The toolbox includes functions to compute the present or future value of cash flows at regular or irregular time intervals with equal or unequal payments: fvfix, fvvar, pvfix, and pvvar. The - fix functions assume equal cash flows at regular intervals, while the -var functions allow irregular cash flows at irregular periods.

Now compute the net present value of the sample income stream for which you computed the internal rate of return. This exercise also serves as a check on that calculation because the net present value of a cash stream at its internal rate of return should be zero. Enter

NPV = pvvar(Stream, ROR)
which returns an answer very close to zero. The answer usually is not exactly zero due to rounding errors and the computational precision of the computer.

Note Other toolbox functions behave similarly. The functions that compute a bond's yield, for example, often must solve a nonlinear equation. If you then use that yield to compute the net present value of the bond's income stream, it usually does not exactly equal the purchase price, but the difference is negligible for practical applications.

## Depreciation

The toolbox includes functions to compute standard depreciation schedules: straight line, general declining-balance, fixed declining-balance, and sum of years' digits. Functions also compute a complete amortization schedule for an asset, and return the remaining depreciable value after a depreciation schedule has been applied.

This example depreciates an automobile worth $\$ 15,000$ over five years with a salvage value of $\$ 1,500$. It computes the general declining balance using two different depreciation rates: $50 \%$ (or 1.5), and $100 \%$ (or 2.0, also known as double declining balance). Enter

```
Decline1 = depgendb(15000, 1500, 5, 1.5)
Decline2 = depgendb(15000, 1500, 5, 2.0)
which returns
\begin{tabular}{rrrrr} 
Decline1 \(=\) \\
4500.00 & 3150.00 & 2205.00 & 1543.50 & 2101.50 \\
Decline2 \(=\) & 3600.00 & 2160.00 & 1296.00 & 444.00 \\
6000.00 & 360
\end{tabular}
```

These functions return the actual depreciation amount for the first four years and the remaining depreciable value as the entry for the fifth year.

## Annuities

Several toolbox functions deal with annuities. This first example shows how to compute the interest rate associated with a series of loan payments when only the payment amounts and principal are known. For a loan whose original value was $\$ 5000.00$ and which was paid back monthly over four years at $\$ 130.00 /$ month:

Rate $=$ annurate (4*12, 130, 5000, 0, 0)
The function returns a rate of 0.0094 monthly, or about $11.28 \%$ annually.
The next example uses a present-value function to show how to compute the initial principal when the payment and rate are known. For a loan paid at $\$ 300.00$ /month over four years at $11 \%$ annual interest

Principal = pvfix(0.11/12, $4 * 12,300,0,0)$
The function returns the original principal value of $\$ 11,607.43$.

The final example computes an amortization schedule for a loan or annuity. The original value was $\$ 5000.00$ and was paid back over 12 months at an annual rate of $9 \%$.

```
[Prpmt, Intpmt, Balance, Payment] = ...
    amortize(0.09/12, 12, 5000, 0, 0);
```

This function returns vectors containing the amount of principal paid,

```
Prpmt = [lllla9.76 402.76 405.78 408.82 411.89 414.97
    418.09 421.22 424.38 427.56 430.77 434.00]
```

the amount of interest paid,

```
Intpmt = [lllllllll}37.50 34.50 31.48 28.44 25.37 22.28
    19.17 16.03 12.88 9.69 6.49 3.26]
```

the remaining balance for each period of the loan,

```
Balance = [lll600.24 4197.49 3791.71 3382.89 2971.01
    2556.03 2137.94 1716.72 1292.34 864.77
        4 3 4 . 0 0 ~ 0 . 0 0 ] ~
```

and a scalar for the monthly payment.

```
Payment = 437.26
```


## See Also

effrr|fvfix|fvvar|irr|nomrr|pvfix|pvvar

## Related Examples

- "Handle and Convert Dates" on page 2-2
- "Pricing and Computing Yields for Fixed-Income Securities" on page 2-22


## Pricing and Computing Yields for Fixed-Income Securities

In this section...<br>"Introduction" on page 2-22<br>"Fixed-Income Terminology" on page 2-22<br>"Framework" on page 2-27<br>"Default Parameter Values" on page 2-28<br>"Coupon Date Calculations" on page 2-31<br>"Yield Conventions" on page 2-31<br>"Pricing Functions" on page 2-32<br>"Yield Functions" on page 2-32<br>"Fixed-Income Sensitivities" on page 2-33

## Introduction

The Financial Toolbox product provides functions for computing accrued interest, price, yield, convexity, and duration of fixed-income securities. Various conventions exist for determining the details of these computations. The Financial Toolbox software supports conventions specified by the Securities Industry and Financial Markets Association (SIFMA), used in the US markets, the International Capital Market Association (ICMA), used mainly in the European markets, and the International Swaps and Derivatives Association (ISDA). For historical reasons, SIFMA is referred to in Financial Toolbox documentation as SIA and ISMA is referred to as International Capital Market Association (ICMA).

## Fixed-Income Terminology

Since terminology varies among texts on this subject, here are some basic definitions that apply to these Financial Toolbox functions. The Glossary contains additional definitions.

The settlement date of a bond is the date when money first changes hands; that is, when a buyer pays for a bond. It need not coincide with the issue date, which is the date a bond is first offered for sale.

The first coupon date and last coupon date are the dates when the first and last coupons are paid, respectively. Although bonds typically pay periodic annual or semiannual coupons, the length of the first and last coupon periods may differ from the standard coupon period. The toolbox includes price and yield functions that handle these odd first and/or last periods.

Successive quasi-coupon dates determine the length of the standard coupon period for the fixed income security of interest, and do not necessarily coincide with actual coupon payment dates. The toolbox includes functions that calculate both actual and quasicoupon dates for bonds with odd first and/or last periods.

Fixed-income securities can be purchased on dates that do not coincide with coupon payment dates. In this case, the bond owner is not entitled to the full value of the coupon for that period. When a bond is purchased between coupon dates, the buyer must compensate the seller for the pro-rata share of the coupon interest earned from the previous coupon payment date. This pro-rata share of the coupon payment is called accrued interest. The purchase price, the price paid for a bond, is the quoted market price plus accrued interest.

The maturity date of a bond is the date when the issuer returns the final face value, also known as the redemption value or par value, to the buyer. The yield-to-maturity of a bond is the nominal compound rate of return that equates the present value of all future cash flows (coupons and principal) to the current market price of the bond.

The period of a bond refers to the frequency with which the issuer of a bond makes coupon payments to the holder.

## Period of a Bond

| Period Value | Payment Schedule |
| :--- | :--- |
| 0 | No coupons (Zero coupon bond) |
| 1 | Annual |
| 2 | Semiannual |
| 3 | Tri-annual |
| 4 | Quarterly |
| 6 | Bi-monthly |
| 12 | Monthly |

The basis of a bond refers to the basis or day-count convention for a bond. Basis is normally expressed as a fraction in which the numerator determines the number of days
between two dates, and the denominator determines the number of days in the year. For example, the numerator of actual/actual means that when determining the number of days between two dates, count the actual number of days; the denominator means that you use the actual number of days in the given year in any calculations (either 365 or 366 days depending on whether the given year is a leap year).

The day count convention determines how accrued interest is calculated and determines how cash flows for the bond are discounted, by that means effecting price and yield calculations. Furthermore, the SIA convention is to use the actual/actual day count convention for discounting cash flows in all cases.

## Basis of a Bond

| Basis Value | Meaning | Description |
| :---: | :---: | :---: |
| 0 (default) | actual/actual | Actual days held over actual days in coupon period. Denominator is 365 in most years and 366 in a leap year. |
| 1 | 30/360 (SIA) | Each month contains 30 days; a year contains 360 days. Payments are adjusted for bonds that pay coupons on the last day of February. |
| 2 | actual/360 | Actual days held over 360. |
| 3 | actual/365 | Actual days held over 365, even in leap years. |
| 4 | 30/360 PSA | Number of days in every month is set to 30 (including February). If the start date of the period is either the 31st of a month or the last day of February, the start date is set to the 30th. If the start date is the 30th of a month and the end date is the 31st, the end date is set to the 30th. The number of days in a year is 360 . |
| 5 | 30/360 ISDA (International Swap Dealers Association) | Variant of $30 / 360$ with slight differences for calculating number of days in a month. |
| 6 | 30E/360 European | Variant of 30/360 used primarily in Europe. |
| 7 | actual/365 Japanese | All years contain 365 days. Leap days are ignored. |


| Basis Value | Meaning | Description |
| :--- | :--- | :--- |
| 8 | actual/actual (ICMA) | Actual days held over actual <br> days in coupon period. <br> Denominator is 365 in most <br> years and 366 in a leap year. <br> This basis assumes an <br> annual compounding period. |
| 9 | actual/360 (ICMA) | Actual days held over 360. <br> This basis assumes an <br> annual compounding period. |
| 10 | actual/365 (ICMA) | Actual days held over 365, <br> even in leap years. This <br> basis assumes an annual <br> compounding period. |
| 11 | $30 / 360$ (ICMA) | The number of days in every <br> month is set to 30. If the <br> start date or the end date of <br> the period is the 31st of a <br> month, that date is set to <br> the 30th. The number of <br> days in a year is 360. |
| 12 | actual/365 (ISDA) | This day count fraction is <br> equal to the sum of number <br> of interest accrual days <br> falling with a leap year <br> divided by 366 and the <br> number of interest accrual <br> days not falling within a <br> leap year divided by 365. |


| Basis Value | Meaning | Description |
| :--- | :--- | :--- |
| 13 | BUS/252 | The number of business <br> days between the previous <br> coupon payment and the <br> settlement data divided by <br> 252. BUS/252 business days <br> are non-weekend, non- <br> holiday days. The <br> holidays.m file defines <br> holidays. |

For more information, see basis on page Glossary-0

Note Although the concept of day count sounds deceptively simple, the actual calculation of day counts can be complex. You can find a good discussion of day counts and the formulas for calculating them in Chapter 5 of Stigum and Robinson, Money Market and Bond Calculations in "Bibliography" on page A-2.

The end-of-month rule affects a bond's coupon payment structure. When the rule is in effect, a security that pays a coupon on the last actual day of a month will always pay coupons on the last day of the month. This means, for example, that a semiannual bond that pays a coupon on February 28 in nonleap years will pay coupons on August 31 in all years and on February 29 in leap years.

## End-of-Month Rule

| End-of-Month Rule Value | Meaning |
| :--- | :--- |
| 1 (default) | Rule in effect. |
| 0 | Rule not in effect. |

## Framework

Although not all Financial Toolbox functions require the same input arguments, they all accept the following common set of input arguments.

## Common Input Arguments

| Input | Meaning |
| :--- | :--- |
| Settle | Settlement date |
| Maturity | Maturity date |
| Period | Coupon payment period |
| Basis | Day-count basis |
| EndMonthRule | End-of-month payment rule |
| IssueDate | Bond issue date |
| FirstCouponDate | First coupon payment date |
| LastCouponDate | Last coupon payment date |

Of the common input arguments, only Settle and Maturity are required. All others are optional. They are set to the default values if you do not explicitly set them. By default, the FirstCouponDate and LastCouponDate are nonapplicable. In other words, if you do not specify FirstCouponDate and LastCouponDate, the bond is assumed to have no odd first or last coupon periods. In this case, the bond is a standard bond with a coupon payment structure based solely on the maturity date.

## Default Parameter Values

To illustrate the use of default values in Financial Toolbox functions, consider the cfdates function, which computes actual cash flow payment dates for a portfolio of fixed income securities regardless of whether the first and/or last coupon periods are normal, long, or short.

The complete calling syntax with the full input argument list is

```
CFlowDates = cfdates(Settle, Maturity, Period, Basis, ...
EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate)
```

while the minimal calling syntax requires only settlement and maturity dates
CFlowDates = cfdates(Settle, Maturity)

## Single Bond Example

As an example, suppose that you have a bond with these characteristics:

| Settle | $=' 20-$ Sep-1999' |
| :--- | :--- |
| Maturity | $=' 15-0 c t-2007$ |
| Period | $=2$ |
| Basis | $=0$ |
| EndMonthRule | $=1$ |
| IssueDate | $=\mathrm{NaN}$ |
| FirstCouponDate | $=\mathrm{NaN}$ |
| LastCouponDate | $=\mathrm{NaN}$ |

Period, Basis, and EndMonthRule are set to their default values, and IssueDate, FirstCouponDate, and LastCouponDate are set to NaN.

Formally, a NaN is an IEEE ${ }^{\circledR}$ arithmetic standard for Not-a-Number and is used to indicate the result of an undefined operation (for example, zero divided by zero). However, NaN is also a convenient placeholder. In the SIA functions of Financial Toolbox software, NaN indicates the presence of a nonapplicable value. It tells the Financial Toolbox functions to ignore the input value and apply the default. Setting IssueDate, FirstCouponDate, and LastCouponDate to NaN in this example tells cfdates to assume that the bond has been issued before settlement and that no odd first or last coupon periods exist.

Having set these values, all these calls to cfdates produce the same result.

```
cfdates(Settle, Maturity)
cfdates(Settle, Maturity, Period)
cfdates(Settle, Maturity, Period, [])
cfdates(Settle, Maturity, [], Basis)
cfdates(Settle, Maturity, [], [])
cfdates(Settle, Maturity, Period, [], EndMonthRule)
cfdates(Settle, Maturity, Period, [], NaN)
cfdates(Settle, Maturity, Period, [], [], IssueDate)
cfdates(Settle, Maturity, Period, [], [], IssueDate, [], [])
cfdates(Settle, Maturity, Period, [], [], [], [],LastCouponDate)
cfdates(Settle, Maturity, Period, Basis, EndMonthRule, ...
IssueDate, FirstCouponDate, LastCouponDate)
```

Thus, leaving a particular input unspecified has the same effect as passing an empty matrix ([]) or passing a NaN - all three tell cfdates (and other Financial Toolbox functions) to use the default value for a particular input parameter.

## Bond Portfolio Example

Since the previous example included only a single bond, there was no difference between passing an empty matrix or passing a NaN for an optional input argument. For a portfolio
of bonds, however, using NaN as a placeholder is the only way to specify default acceptance for some bonds while explicitly setting nondefault values for the remaining bonds in the portfolio.

Now suppose that you have a portfolio of two bonds.

```
Settle = '20-Sep-1999'
Maturity = ['15-0ct-2007'; '15-0ct-2010']
```

These calls to cfdates all set the coupon period to its default value (Period $=2$ ) for both bonds.

```
cfdates(Settle, Maturity, 2)
cfdates(Settle, Maturity, [2 2])
cfdates(Settle, Maturity, [])
cfdates(Settle, Maturity, NaN)
cfdates(Settle, Maturity, [NaN NaN])
cfdates(Settle, Maturity)
```

The first two calls explicitly set Period $=2$. Since Maturity is a 2 -by-1 vector of maturity dates, cfdates knows that you have a two-bond portfolio.

The first call specifies a single (that is, scalar) 2 for Period. Passing a scalar tells cfdates to apply the scalar-valued input to all bonds in the portfolio. This is an example of implicit scalar-expansion. The settlement date has been implicit scalar-expanded as well.

The second call also applies the default coupon period by explicitly passing a two-element vector of 2's. The third call passes an empty matrix, which cfdates interprets as an invalid period, for which the default value is used. The fourth call is similar, except that a NaN has been passed. The fifth call passes two NaN 's, and has the same effect as the third. The last call passes the minimal input set.

Finally, consider the following calls to cfdates for the same two-bond portfolio.

```
cfdates(Settle, Maturity, [4 NaN])
cfdates(Settle, Maturity, [4 2])
```

The first call explicitly sets Period $=4$ for the first bond and implicitly sets the default Period $=2$ for the second bond. The second call has the same effect as the first but explicitly sets the periodicity for both bonds.

The optional input Period has been used for illustrative purpose only. The defaulthandling process illustrated in the examples applies to any of the optional input arguments.

## Coupon Date Calculations

Calculating coupon dates, either actual or quasi dates, is notoriously complicated. Financial Toolbox software follows the SIA conventions in coupon date calculations.

The first step in finding the coupon dates associated with a bond is to determine the reference, or synchronization date (the sync date). Within the SIA framework, the order of precedence for determining the sync date is:

## 1 The first coupon date

2 The last coupon date
3 The maturity date
In other words, a Financial Toolbox function first examines the FirstCouponDate input. If FirstCouponDate is specified, coupon payment dates and quasi-coupon dates are computed with respect to FirstCouponDate; if FirstCouponDate is unspecified, empty ([]), or NaN, then the LastCouponDate is examined. If LastCouponDate is specified, coupon payment dates and quasi-coupon dates are computed with respect to LastCouponDate. If both FirstCouponDate and LastCouponDate are unspecified, empty ([]), or NaN, the Maturity (a required input argument) serves as the synchronization date.

## Yield Conventions

There are two yield and time factor conventions that are used in the Financial Toolbox software - these are determined by the input basis. Specifically, bases 0 to 7 are assumed to have semiannual compounding, while bases 8 to 12 are assumed to have annual compounding regardless of the period of the bond's coupon payments (including zero-coupon bonds). In addition, any yield-related sensitivity (that is, duration and convexity), when quoted on a periodic basis, follows this same convention. (See bndconvp, bndconvy, bnddurp, bnddury, and bndkrdur.)

## Pricing Functions

This example shows how easily you can compute the price of a bond with an odd first period using the function bndprice. Assume that you have a bond with these characteristics:

```
Settle = '11-Nov-1992';
Maturity = '01-Mar-2005';
IssueDate = '15-Oct-1992';
FirstCouponDate = '01-Mar-1993';
CouponRate = 0.0785;
Yield = 0.0625;
```

Allow coupon payment period (Period $=2$ ), day-count basis (Basis $=0$ ), and end-ofmonth rule (EndMonthRule = 1) to assume the default values. Also, assume that there is no odd last coupon date and that the face value of the bond is $\$ 100$. Calling the function
[Price, AccruedInt] = bndprice(Yield, CouponRate, Settle, ... Maturity, [], [], [], IssueDate, FirstCouponDate)
returns a price of $\$ 113.60$ and accrued interest of $\$ 0.59$.
Similar functions compute prices with regular payments, odd first and last periods, and prices of Treasury bills and discounted securities such as zero-coupon bonds.

Note bndprice and other functions use nonlinear formulas to compute the price of a security. For this reason, Financial Toolbox software uses Newton's method when solving for an independent variable within a formula. See any elementary numerical methods textbook for the mathematics underlying Newton's method.

## Yield Functions

To illustrate toolbox yield functions, compute the yield of a bond that has odd first and last periods and settlement in the first period. First set up variables for settlement, maturity date, issue, first coupon, and a last coupon date.

| Settle | $=' 12-J a n-2000 ' ;$ |
| :--- | :--- |
| Maturity | $=$ |
| IssueDate | $=1-01-$ Oct-2001'; |
| FirstCouponDate | $=$ '15-Jan-2000'; |
| LastCouponDate | $=15-\operatorname{Apr}-2000 ' ;$ |

Assume a face value of $\$ 100$. Specify a purchase price of $\$ 95.70$, a coupon rate of $4 \%$, quarterly coupon payments, and a 30/360 day-count convention (Basis $=1$ ).

```
Price = 95.7;
CouponRate = 0.04;
Period = 4;
Basis = 1;
EndMonthRule = 1;
```

Calling the function

```
Yield = bndyield(Price, CouponRate, Settle, Maturity, Period,...
```

Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate)
returns

Yield = 0.0659 (6.60\%).

## Fixed-Income Sensitivities

Financial Toolbox software supports the following options for managing interest-rate risk for one or more bonds:

- bnddurp and bnddury support duration and convexity analysis based on market quotes and assume parallel shifts in the bond yield curve.
- bndkrdur supports key rate duration based on a market yield curve and can model nonparallel shifts in the bond yield curve.


## Calculating Duration and Convexity for Bonds

The toolbox includes functions to perform sensitivity analysis such as convexity and the Macaulay and modified durations for fixed-income securities. The Macaulay duration of an income stream, such as a coupon bond, measures how long, on average, the owner waits before receiving a payment. It is the weighted average of the times payments are made, with the weights at time $T$ equal to the present value of the money received at time T. The modified duration is the Macaulay duration discounted by the per-period interest rate; that is, divided by (1+rate/frequency).

To illustrate, the following example computes the annualized Macaulay and modified durations, and the periodic Macaulay duration for a bond with settlement (12-Jan-2000) and maturity (01-Oct-2001) dates as above, a $5 \%$ coupon rate, and a $4.5 \%$ yield to maturity. For simplicity, any optional input arguments assume default values (that is,
semiannual coupons, and day-count basis = 0 (actual/actual), coupon payment structure synchronized to the maturity date, and end-of-month payment rule in effect).

CouponRate = 0.05;
Yield = 0.045;
[ModDuration, YearDuration, PerDuration] = bnddury(Yield,...
CouponRate, Settle, Maturity)
The durations are

```
ModDuration = 1.6107 (years)
YearDuration = 1.6470 (years)
PerDuration = 3.2940 (semiannual periods)
```

Note that the semiannual periodic Macaulay duration (PerDuration) is twice the annualized Macaulay duration (YearDuration).

## Calculating Key Rate Durations for Bonds

Key rate duration enables you to evaluate the sensitivity and price of a bond to nonparallel changes in the spot or zero curve by decomposing the interest rate risk along the spot or zero curve. Key rate duration refers to the process of choosing a set of key rates and computing a duration for each rate. Specifically, for each key rate, while the other rates are held constant, the key rate is shifted up and down (and intermediate cash flow dates are interpolated), and then the present value of the security given the shifted curves is computed.

The calculation of bndkrdur supports:

$$
k r d u r_{i}=\frac{\left(P V_{\text {down }}-P V_{u p}\right)}{(P V \times \text { ShiftValue } \times 2)}
$$

Where $P V$ is the current value of the instrument, $P V_{-} u p$ and $P V_{-}$down are the new values after the discount curve has been shocked, and ShiftValue is the change in interest rate. For example, if key rates of 3 months, $1,2,3,5,7,10,15,20,25,30$ years were chosen, then a 30 -year bond might have corresponding key rate durations of:

| 3 M | 1 Y | 2 Y | 3 Y | 5 Y | 7 Y | 10 Y | 15 Y | 20 Y | 25 Y | 30 Y |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| .01 | .04 | .09 | .21 | .4 | .65 | 1.27 | 1.71 | 1.68 | 1.83 | 7.03 |

The key rate durations add up to approximately equal the duration of the bond.

For example, compute the key rate duration of the US Treasury Bond with maturity date of August 15, 2028 and coupon rate of $5.5 \%$.

```
Settle = datenum('18-Nov-2008');
CouponRate = 5.500/100;
Maturity = datenum('15-Aug-2028');
Price = 114.83;
```

For the ZeroData information on the current spot curve for this bond, refer to https:// www.treas.gov/offices/domestic-finance/debt-management/interest-rate/yield.shtml:

```
ZeroDates = daysadd(Settle ,[30 90 180 360 360*2 360*3 360*5 ...
360*7 360*10 360*20 360*30]);
ZeroRates =([0.06 0.12 0.81 1.08 1.22 1.53 2.32 2.92 3.68 4.42 4.20]/100)';
```

Compute the key rate duration for a specific set of rates (choose this based on the maturities of the available hedging instruments):

```
krd = bndkrdur([ZeroDates ZeroRates],CouponRate,Settle,Maturity,'keyrates',[2 5 10 20])
krd =
    0.2865 0.8729 2.6451 8.5778
```

Note, the sum of the key rate durations approximately equals the duration of the bond:
[sum(krd) bnddurp(Price,CouponRate, Settle,Maturity)]
ans $=$
$12.3823 \quad 12.3919$

## See Also

bndconvp | bndconvy | bnddurp | bnddury | bndkrdur

## Related Examples

- "Handle and Convert Dates" on page 2-2
- "Term Structure of Interest Rates" on page 2-42
- "Computing Treasury Bill Price and Yield" on page 2-38


## More About

- "Treasury Bills Defined" on page 2-37


## Treasury Bills Defined

Treasury bills are short-term securities (issued with maturities of one year or less) sold by the United States Treasury. Sales of these securities are frequent, usually weekly. From time to time, the Treasury also offers longer duration securities called Treasury notes and Treasury bonds.

A Treasury bill is a discount security. The holder of the Treasury bill does not receive periodic interest payments. Instead, at the time of sale, a percentage discount is applied to the face value. At maturity, the holder redeems the bill for full face value.

The basis for Treasury bill interest calculation is actual/360. Under this system, interest accrues on the actual number of elapsed days between purchase and maturity, and each year contains 360 days.

## See Also

tbilldisc2yield|tbillprice|tbillrepo|tbillval01|tbillyield| tbillyield2disc|tbl2bond|tr2bonds|zbtprice|zbtyield

## Related Examples

- "Handle and Convert Dates" on page 2-2
- "Term Structure of Interest Rates" on page 2-42
- "Computing Treasury Bill Price and Yield" on page 2-38


## Computing Treasury Bill Price and Yield

In this section...<br>"Introduction" on page 2-38<br>"Treasury Bill Repurchase Agreements" on page 2-38<br>"Treasury Bill Yields" on page 2-40

## Introduction

Financial Toolbox software provides the following suite of functions for computing price and yield on Treasury bills.

## Treasury Bill Functions

| Function | Purpose |
| :--- | :--- |
| tbilldisc2yield | Convert discount rate to yield. |
| tbillprice | Price Treasury bill given its yield or discount rate. |
| tbillrepo | Break-even discount of repurchase agreement. |
| tbillyield | Yield and discount of Treasury bill given its price. |
| tbillyield2disc | Convert yield to discount rate. |
| tbillval01 | The value of 1 basis point given the characteristics of the <br> Treasury bill, as represented by its settlement and <br> maturity dates. You can relate the basis point to discount, <br> money-market, or bond-equivalent yield. |

For all functions with yield in the computation, you can specify yield as money-market or bond-equivalent yield. The functions all assume a face value of $\$ 100$ for each Treasury bill.

## Treasury Bill Repurchase Agreements

The following example shows how to compute the break-even discount rate. This is the rate that correctly prices the Treasury bill such that the profit from selling the tail equals 0 .

Maturity = '26-Dec-2002';
InitialDiscount = 0.0161;

```
PurchaseDate = '26-Sep-2002';
SaleDate = '26-0ct-2002';
RepoRate = 0.0149;
BreakevenDiscount = tbillrepo(RepoRate, InitialDiscount, ...
PurchaseDate, SaleDate, Maturity)
BreakevenDiscount =
    0.0167
```

You can check the result of this computation by examining the cash flows in and out from the repurchase transaction. First compute the price of the Treasury bill on the purchase date (September 26).

```
PriceOnPurchaseDate = tbillprice(InitialDiscount, ...
```

PurchaseDate, Maturity, 3)

PriceOnPurchaseDate =
99.5930

Next compute the interest due on the repurchase agreement.
RepoInterest = ...
RepoRate*Price0nPurchaseDate*days360(PurchaseDate, SaleDate)/360
RepoInterest =
0.1237

RepoInterest for a 1.49\% 30-day term repurchase agreement (30/360 basis) is 0.1237 .
Finally, compute the price of the Treasury bill on the sale date (October 26).
PriceOnSaleDate = tbillprice(BreakevenDiscount, SaleDate, ...
Maturity, 3)
PriceOnSaleDate =
99.7167

Examining the cash flows, observe that the break-even discount causes the sum of the price on the purchase date plus the accrued 30-day interest to be equal to the price on sale date. The next table shows the cash flows.

## Cash Flows from Repurchase Agreement

| Date | Cash Out Flow |  | Cash In Flow |  |
| :--- | :--- | :--- | :--- | :--- |
| $9 / 26 / 2002$ | Purchase T-bill | 99.593 | Repo money | 99.593 |
| $10 / 26 / 2002$ | Payment of repo | 99.593 | Sell T-bill | 99.7168 |
|  | Repo interest | 0.1238 |  |  |
|  | Total | 199.3098 |  | 199.3098 |

## Treasury Bill Yields

Using the same data as before, you can examine the money-market and bond-equivalent yields of the Treasury bill at the time of purchase and sale. The function tbilldisc2yield can perform both computations at one time.

```
Maturity = '26-Dec-2002';
InitialDiscount = 0.0161;
PurchaseDate = '26-Sep-2002';
SaleDate = '26-0ct-2002';
RepoRate = 0.0149;
BreakevenDiscount = tbillrepo(RepoRate, InitialDiscount, ...
PurchaseDate, SaleDate, Maturity)
[BEYield, MMYield] = ...
tbilldisc2yield([InitialDiscount; BreakevenDiscount], ...
[PurchaseDate; SaleDate], Maturity)
BreakevenDiscount =
    0.0167
BEYield =
    0.0164
    0.0170
```

MMYield =
0.0162
0.0168

For the short Treasury bill (fewer than 182 days to maturity), the money-market yield is 360/365 of the bond-equivalent yield, as this example shows.

## See Also

tbilldisc2yield|tbillprice|tbillrepo|tbillval01|tbillyield| tbillyield2disc|tbl2bond|tr2bonds|zbtprice|zbtyield

## Related Examples

- "Handle and Convert Dates" on page 2-2
- "Term Structure of Interest Rates" on page 2-42


## More About

- "Treasury Bills Defined" on page 2-37


## Term Structure of Interest Rates

```
In this section...
"Introduction" on page 2-42
"Deriving an Implied Zero Curve" on page 2-43
```


## Introduction

The Financial Toolbox product contains several functions to derive and analyze interest rate curves, including data conversion and extrapolation, bootstrapping, and interest-rate curve conversion functions.

One of the first problems in analyzing the term structure of interest rates is dealing with market data reported in different formats. Treasury bills, for example, are quoted with bid and asked bank-discount rates. Treasury notes and bonds, on the other hand, are quoted with bid and asked prices based on $\$ 100$ face value. To examine the full spectrum of Treasury securities, analysts must convert data to a single format. Financial Toolbox functions ease this conversion. This brief example uses only one security each; analysts often use 30, 100, or more of each.

First, capture Treasury bill quotes in their reported format

```
% Maturity Days Bid Ask AskYield
TBill = [datenum('12/26/2000') 53 0.0503 0.0499 0.0510];
```

then capture Treasury bond quotes in their reported format

```
% Coupon Maturity Bid Ask AskYield
TBond = [0.08875 datenum(2001,11,5) 103+4/32 103+6/32 0.0564];
```

and note that these quotes are based on a November 3, 2000 settlement date.

```
Settle = datenum('3-Nov-2000');
```

Next use the toolbox tbl2bond function to convert the Treasury bill data to Treasury bond format.

```
TBTBond = tbl2bond(TBill)
```

TBTBond =

| 0 | 730846 | 99.26 | 99.27 | 0.05 |
| :--- | :--- | :--- | :--- | :--- |

(The second element of TBTBond is the serial date number for December 26, 2000.)
Now combine short-term (Treasury bill) with long-term (Treasury bond) data to set up the overall term structure.

```
TBondsAll = [TBTBond; TBond]
TBondsAll =
\begin{tabular}{rrrrr}
0 & 730846 & 99.26 & 99.27 & 0.05 \\
0.09 & 731160 & 103.13 & 103.19 & 0.06
\end{tabular}
```

The Financial Toolbox software provides a second data-preparation function,tr2bonds, to convert the bond data into a form ready for the bootstrapping functions. tr2bonds generates a matrix of bond information sorted by maturity date, plus vectors of prices and yields.
[Bonds, Prices, Yields] = tr2bonds(TBondsAll);

## Deriving an Implied Zero Curve

Using this market data, you can use one of the Financial Toolbox bootstrapping functions to derive an implied zero curve. Bootstrapping is a process whereby you begin with known data points and solve for unknown data points using an underlying arbitrage theory. Every coupon bond can be valued as a package of zero-coupon bonds which mimic its cash flow and risk characteristics. By mapping yields-to-maturity for each theoretical zero-coupon bond, to the dates spanning the investment horizon, you can create a theoretical zero-rate curve. The Financial Toolbox software provides two bootstrapping functions: zbtprice derives a zero curve from bond data and prices, and zbtyield derives a zero curve from bond data and yields. Using zbtprice
[ZeroRates, CurveDates] = zbtprice(Bonds, Prices, Settle)
ZeroRates =
0.05
0.06

CurveDates =
730846
731160
CurveDates gives the investment horizon.

```
datestr(CurveDates)
ans =
26-Dec-2000
05-Nov-2001
```

Additional Financial Toolbox functions construct discount, forward, and par yield curves from the zero curve, and vice versa.

```
[DiscRates, CurveDates] = zero2disc(ZeroRates, CurveDates,...
Settle);
[FwdRates, CurveDates] = zero2fwd(ZeroRates, CurveDates, Settle):
[PYldRates, CurveDates] = zero2pyld(ZeroRates, CurveDates,...
Settle);
```


## See Also

tbilldisc2yield|tbillprice|tbillrepo|tbillval01|tbillyield| tbillyield2disc|tbl2bond|tr2bonds|zbtprice|zbtyield

## Related Examples

- "Handle and Convert Dates" on page 2-2
- "Computing Treasury Bill Price and Yield" on page 2-38


## More About

- "Treasury Bills Defined" on page 2-37


## Pricing and Analyzing Equity Derivatives

## In this section...

"Introduction" on page 2-45
"Sensitivity Measures" on page 2-45
"Analysis Models" on page 2-46

## Introduction

These toolbox functions compute prices, sensitivities, and profits for portfolios of options or other equity derivatives. They use the Black-Scholes model for European options and the binomial model for American options. Such measures are useful for managing portfolios and for executing collars, hedges, and straddles.

## Sensitivity Measures

There are six basic sensitivity measures associated with option pricing: delta, gamma, lambda, rho, theta, and vega - the "greeks." The toolbox provides functions for calculating each sensitivity and for implied volatility.

## Delta

Delta of a derivative security is the rate of change of its price relative to the price of the underlying asset. It is the first derivative of the curve that relates the price of the derivative to the price of the underlying security. When delta is large, the price of the derivative is sensitive to small changes in the price of the underlying security.

## Gamma

Gamma of a derivative security is the rate of change of delta relative to the price of the underlying asset; that is, the second derivative of the option price relative to the security price. When gamma is small, the change in delta is small. This sensitivity measure is important for deciding how much to adjust a hedge position.

## Lambda

Lambda, also known as the elasticity of an option, represents the percentage change in the price of an option relative to a $1 \%$ change in the price of the underlying security.

## Rho

Rho is the rate of change in option price relative to the risk-free interest rate.

## Theta

Theta is the rate of change in the price of a derivative security relative to time. Theta is usually small or negative since the value of an option tends to drop as it approaches maturity.

## Vega

Vega is the rate of change in the price of a derivative security relative to the volatility of the underlying security. When vega is large the security is sensitive to small changes in volatility. For example, options traders often must decide whether to buy an option to hedge against vega or gamma. The hedge selected usually depends upon how frequently one rebalances a hedge position and also upon the standard deviation of the price of the underlying asset (the volatility). If the standard deviation is changing rapidly, balancing against vega is preferable.

## Implied Volatility

The implied volatility of an option is the standard deviation that makes an option price equal to the market price. It helps determine a market estimate for the future volatility of a stock and provides the input volatility (when needed) to the other Black-Scholes functions.

## Analysis Models

Toolbox functions for analyzing equity derivatives use the Black-Scholes model for European options and the binomial model for American options. The Black-Scholes model makes several assumptions about the underlying securities and their behavior. The binomial model, on the other hand, makes far fewer assumptions about the processes underlying an option. For further explanation, see Options, Futures, and Other Derivatives by John Hull in "Bibliography" on page A-2.

## Black-Scholes Model

Using the Black-Scholes model entails several assumptions:

- The prices of the underlying asset follow an Ito process. (See Hull on page A-3, page 222.)
- The option can be exercised only on its expiration date (European option).
- Short selling is permitted.
- There are no transaction costs.
- All securities are divisible.
- There is no riskless arbitrage.
- Trading is a continuous process.
- The risk-free interest rate is constant and remains the same for all maturities.

If any of these assumptions is untrue, Black-Scholes may not be an appropriate model.
To illustrate toolbox Black-Scholes functions, this example computes the call and put prices of a European option and its delta, gamma, lambda, and implied volatility. The asset price is $\$ 100.00$, the exercise price is $\$ 95.00$, the risk-free interest rate is $10 \%$, the time to maturity is 0.25 years, the volatility is 0.50 , and the dividend rate is 0 . Simply executing the toolbox functions

```
[OptCall, OptPut] = blsprice(100, 95, 0.10, 0.25, 0.50, 0);
[CallVal, PutVal] = blsdelta(100, 95, 0.10, 0.25, 0.50, 0);
GammaVal = blsgamma(100, 95, 0.10, 0.25, 0.50, 0);
VegaVal = blsvega(100, 95, 0.10, 0.25, 0.50, 0);
[LamCall, LamPut] = blslambda(100, 95, 0.10, 0.25, 0.50, 0);
```

yields:

- The option call price OptCall = $\$ 13.70$
- The option put price OptPut $=\$ 6.35$
- delta for a call CallVal $=0.6665$ and delta for a put PutVal $=-0.3335$
- gamma GammaVal = 0.0145
- vega VegaVal = 18.1843
- lambda for a call LamCall = 4.8664 and lambda for a put LamPut $=-5.2528$

Now as a computation check, find the implied volatility of the option using the call option price from blsprice.

```
Volatility = blsimpv(100, 95, 0.10, 0.25, OptCall);
```

The function returns an implied volatility of 0.500 , the original bl sprice input.

## Binomial Model

The binomial model for pricing options or other equity derivatives assumes that the probability over time of each possible price follows a binomial distribution. The basic assumption is that prices can move to only two values, one up and one down, over any short time period. Plotting the two values, and then the subsequent two values each, and then the subsequent two values each, and so on over time, is known as "building a binomial tree." This model applies to American options, which can be exercised any time up to and including their expiration date.

This example prices an American call option using a binomial model. Again, the asset price is $\$ 100.00$, the exercise price is $\$ 95.00$, the risk-free interest rate is $10 \%$, and the time to maturity is 0.25 years. It computes the tree in increments of 0.05 years, so there are $0.25 / 0.05=5$ periods in the example. The volatility is 0.50 , this is a call $(f l a g=1)$, the dividend rate is 0 , and it pays a dividend of $\$ 5.00$ after three periods (an ex-dividend date). Executing the toolbox function
[StockPrice, OptionPrice] = binprice(100, 95, 0.10, 0.25,... $0.05,0.50,1,0,5.0,3) ;$
returns the tree of prices of the underlying asset
StockPrice =

| 100.00 | 111.27 | 123.87 | 137.96 | 148.69 | 166.28 |
| ---: | ---: | ---: | ---: | ---: | ---: |
| 0 | 89.97 | 100.05 | 111.32 | 118.90 | 132.96 |
| 0 | 0 | 81.00 | 90.02 | 95.07 | 106.32 |
| 0 | 0 | 0 | 72.98 | 76.02 | 85.02 |
| 0 | 0 | 0 | 0 | 60.79 | 67.98 |
| 0 | 0 | 0 | 0 | 0 | 54.36 |

and the tree of option values.
OptionPrice =

| 12.10 | 19.17 | 29.35 | 42.96 | 54.17 | 71.28 |
| ---: | ---: | ---: | ---: | ---: | ---: |
| 0 | 5.31 | 9.41 | 16.32 | 24.37 | 37.96 |
| 0 | 0 | 1.35 | 2.74 | 5.57 | 11.32 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |

The output from the binomial function is a binary tree. Read the StockPrice matrix this way: column 1 shows the price for period 0 , column 2 shows the up and down prices for
period 1, column 3 shows the up-up, up-down, and down-down prices for period 2, and so on. Ignore the zeros. The OptionPrice matrix gives the associated option value for each node in the price tree. Ignore the zeros that correspond to a zero in the price tree.

## See Also

binprice|blkimpv|blkprice|blsdelta|blsgamma|blsimpv|blslambda| blsprice|blsrho|blstheta|blsvega|opprofit

## Related Examples

- "Handle and Convert Dates" on page 2-2
- "Greek-Neutral Portfolios of European Stock Options" on page 10-18
- "Plotting Sensitivities of an Option" on page 10-30
- "Plotting Sensitivities of a Portfolio of Options" on page 10-33


## About Life Tables

Life tables are used for life insurance and work with the probability distribution of human mortality. This distribution, which is age-dependent, has several characteristic features that are consequences of biological, cultural, and behavioral factors. Usually, the practitioners of life studies use life tables that contain age-dependent series for specific demographics. The tables are in a standard format with standard notation that is specific to the life studies field. An example of a life table is shown in Table 1 from CDC life tables for the United States.

| Age (years) | $\begin{gathered} \text { Probability } \\ \text { of fying } \\ \text { between } \\ \text { ages } x \text { and } x+1 \end{gathered}$ | $\begin{aligned} & \text { Number } \\ & \text { suviving to } \\ & \text { age } x \end{aligned}$ | $\underset{\substack{\text { Number } \\ \text { dyinge } \\ \text { besen } \\ \text { agand } x+1}}{ }$ | $\begin{gathered} \text { Person-years } \\ \text { bied } \\ \text { between } \\ \text { ages } x \text { and } x+1 \end{gathered}$ | Total number of person-years lived above age $x$ | $\begin{aligned} & \text { Expectation } \\ & \text { of 值e } \\ & \text { at age } x \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $q_{4}$ | $L_{5}$ | $d_{x}$ | $L_{4}$ | $T_{x}$ | $e_{x}$ |
| $0-1$. | 0.006372 | 100,000 | 637 | 99,444 | 7,846,926 | 78.5 |
| 1-2. | 0.000407 | ${ }^{99,363}$ | 40 | 99,343 | 7,747,481 | 78.0 |
| 2-3. | 0.000274 | ${ }^{99,322}$ | ${ }^{27}$ | ${ }^{99,309}$ | 7,648,139 | 77.0 |
| ${ }^{3-4} 4$ | ${ }^{0.0002099}$ | ${ }^{99,295}$ | 21 | ${ }^{99,285}$ | 7,544.830 | 76.0 |
| 4-5. | 0.000160 | 99,274 | 16 | 99,266 | 7,449.545 | 75.0 |
| ${ }^{5-6}$. | 0.000150 | 99,259 | 15 13 | ${ }^{99,251}$ | $7,355,279$ 7 | 74.1 |
| ${ }_{7-8} 6-7$. | 0.000135 0.000122 | 99.244 99230 | 13 12 | 999237 99.224 | $7,251,028$ $7,151,791$ 7 | ${ }_{72.1}^{73.1}$ |
| ${ }_{8-9}$ | 0.0.000109 | ${ }_{9}^{99,2318}$ | ${ }_{11}^{12}$ | ${ }_{99}^{99,213}$ | ${ }^{7,051,566}$ | 72.1 71.1 |
| $9-10$ | 0.000095 | 99,207 | 9 | 99,203 | 6,953,354 | 70.1 |
| 10-11 | 0.000087 | 99,198 | 9 | 99,194 | 6,854,151 | 69.1 |
| $11-12$ | 0.000093 | 99,189 | 9 | 99,185 | 6,754,957 | 68.1 |
| 12-13 | ${ }^{0.0000127}$ | ${ }_{\text {99, } 9,180}$ | 13 | 99,174 | ${ }_{\text {c, }}^{6,655,773}$ | ${ }_{66.1}^{67.1}$ |
| ${ }_{1}^{13-14}$ | 0.0000193 0.000279 | ${ }_{99,148}^{99,167}$ | 19 28 | ${ }_{99,134}^{99,158}$ | 6.556 .599 6.457 .41 | ${ }_{65.1}^{66.1}$ |
| 14515 $15-16$ | 0.000279 0.000370 | ${ }_{99,121}^{99,18}$ | 28 38 | 99,102 | ${ }_{6,358,307}^{6,454,4{ }^{\text {en }}}$ | ${ }_{64.1}^{65.1}$ |
| 16-17 | 0.000454 | 99,084 | 45 | 99,061 | 6,259,205 | 63.2 |
| 17-18 | 0.000537 | 99,039 | 53 | 99,012 | 6,160,143 | 62.2 |
| 18-19 | 0.000615 0.00699 | ${ }_{9}^{98,986}$ | ${ }_{68}^{61}$ | 98.955 98.891 | ${ }_{5}^{6,96662,1175}$ | ${ }_{60.3}^{61.2}$ |
| $19-20$ | 0.000691 | 98,925 | $\infty$ | 98,891 | 5,962,175 | 60.3 |

Often, these life tables can have numerous variations such as abridged tables (which pose challenges due to the granularity of the data) and different termination criteria (that can make it difficult to compare tables or to compute life expectancies).

Most raw life tables have one or more of the first three series in this table ( $q_{x}, l_{x}$, and $d_{x}$ ) and the notation for these three series is standard in the field.

- The $q_{x}$ series is basically the discrete hazard function for human mortality.
- The $l_{x}$ series is the survival function multiplied by a radix of 100,000 .
- The $d_{x}$ series is the discrete probability density for the distribution as a function of age.

Financial Toolbox can handle arbitrary life table data supporting several standard models of mortality and provides various interpolation methods to calibrate and analyze the life table data.

Although primarily designed for life insurance applications, the life tables functions (lifetableconv, lifetablefit, and lifetablegen can also be used by social scientists, behavioral psychologists, public health officials, and medical researchers.

## Life Tables Theory

Life tables are based on hazard functions and survival functions which are, in turn, derived from probability distributions. Specifically, given a continuous probability distribution, its cumulative distribution function is $\mathrm{F}(x)$ and its probability density function is $\mathrm{f}(x)=\mathrm{dF}(x) / \mathrm{d} x$.

For the analysis of mortality, the random variable of interest $X$ is the distribution of ages at which individuals die within a population. So, the probability that someone dies by age $x$ is

$$
\operatorname{Pr}[X \leq x]=F(x)
$$

The survival function, $(s(x))$, which characterizes the probability that an individual lives beyond a specified age $x>0$, is

$$
\begin{aligned}
& s(x)=\operatorname{Pr}[X>x] \\
& =1-F(x)
\end{aligned}
$$

For a continuous probability distribution, the hazard function is a function of the survival function with

$$
\begin{aligned}
& h(x)=\lim _{\Delta x \rightarrow 0} \frac{\operatorname{Pr}[x \leq X<x+\Delta x \mid X \geq x]}{\Delta x} \\
& =-\frac{1}{s(x)} \frac{d(s(x)}{d x}
\end{aligned}
$$

and the survival functions is a function of the hazard function with

$$
s(x)=\exp \left(-\int_{0}^{x} h(\xi) d \xi\right)
$$

Life table models generally specify either the hazard function or the survival function. However, life tables are discrete and work with discrete versions of the hazard and survival functions. Three series are used for life tables and the notation is the convention. The discrete hazard function is denoted as

$$
\begin{aligned}
& q_{x} \approx h(x) \\
& =1-\frac{s(x+1)}{s(x)}
\end{aligned}
$$

which is the probability a person at age $x$ dies by age $x+1$ (where $x$ is in years). The discrete survival function is presented in terms of an initial number of survivors at birth called the life table radix (which is usually 100,000 individuals) and is denoted as

$$
l_{x}=l_{0} s(x)
$$

with radix $l_{0}=100000$. This number, $l_{x}$, represents the number of individuals out of 100,000 at birth who are still alive at age $x$.

A third series is related to the probability density function which is the number of "standardized" deaths in a given year denoted as

$$
d_{x}=l_{x}-l_{x+1}
$$

Based on a few additional rules about how to initialize and terminate these series, any one series can be derived from any of the other series.

## See Also

lifetableconv|lifetablefit|lifetablegen

## Related Examples

- "Case Study for Life Tables Analysis" on page 2-53


## Case Study for Life Tables Analysis

This example shows how to use the basic workflow for life tables.
Load the life table data file.
load us lifetable 2009
Calibrate life table from survival data with the default heligman-pollard parametric model.

```
a = lifetablefit(x, lx);
```

Generate life table series from the calibrated mortality model.

```
qx = lifetablegen((0:100), a);
display(qx(1:40,:))
```

| 0.0063 | 0.0069 | 0.0057 |
| :--- | :--- | :--- |
| 0.0005 | 0.0006 | 0.0004 |
| 0.0002 | 0.0003 | 0.0002 |
| 0.0002 | 0.0002 | 0.0002 |
| 0.0001 | 0.0001 | 0.0001 |
| 0.0001 | 0.0001 | 0.0001 |
| 0.0001 | 0.0001 | 0.0001 |
| 0.0001 | 0.0001 | 0.0001 |
| 0.0001 | 0.0001 | 0.0001 |
| 0.0001 | 0.0001 | 0.0001 |
| 0.0001 | 0.0001 | 0.0001 |
| 0.0001 | 0.0001 | 0.0001 |
| 0.0002 | 0.0002 | 0.0001 |
| 0.0002 | 0.0002 | 0.0002 |
| 0.0002 | 0.0003 | 0.0002 |
| 0.0003 | 0.0004 | 0.0002 |
| 0.0004 | 0.0005 | 0.0002 |
| 0.0005 | 0.0006 | 0.0003 |
| 0.0006 | 0.0008 | 0.0003 |
| 0.0007 | 0.0009 | 0.0003 |
| 0.0008 | 0.0011 | 0.0003 |
| 0.0008 | 0.0012 | 0.0004 |
| 0.0009 | 0.0013 | 0.0004 |
| 0.0009 | 0.0014 | 0.0005 |
| 0.0010 | 0.0014 | 0.0005 |
| 0.0010 | 0.0015 | 0.0005 |


| 0.0010 | 0.0015 | 0.0006 |
| :--- | :--- | :--- |
| 0.0010 | 0.0015 | 0.0006 |
| 0.0010 | 0.0015 | 0.0007 |
| 0.0010 | 0.0014 | 0.0007 |
| 0.0011 | 0.0014 | 0.0007 |
| 0.0011 | 0.0014 | 0.0008 |
| 0.0011 | 0.0014 | 0.0008 |
| 0.0011 | 0.0014 | 0.0009 |
| 0.0011 | 0.0014 | 0.0009 |
| 0.0012 | 0.0015 | 0.0010 |
| 0.0012 | 0.0015 | 0.0011 |
| 0.0013 | 0.0016 | 0.0011 |
| 0.014 | 0.0017 | 0.0012 |
| 0.0015 | 0.0018 | 0.0013 |

Plot the qx series and display the legend. The series qx is the conditional probability that a person at age ${ }^{x}$ will die between age ${ }^{x}$ and the next age in the series
plot((0:100), log(qx));
legend(series, 'location', 'southeast');
title('Conditional Probability of Dying within One Year of Current Age'); xlabel('Age');
ylabel('Log Probability');


## See Also

lifetableconv|lifetablefit|lifetablegen

## More About

- "About Life Tables" on page 2-50


## Portfolio Analysis

- "Analyzing Portfolios" on page 3-2
- "Portfolio Optimization Functions" on page 3-4
- "Portfolio Construction Examples" on page 3-7
- "Portfolio Selection and Risk Aversion" on page 3-9
- "portopt Migration to Portfolio Object" on page 3-14
- "frontcon Migration to Portfolio Object" on page 3-25
- "Constraint Specification Using a Portfolio Object" on page 3-34
- "Active Returns and Tracking Error Efficient Frontier" on page 3-43


## Analyzing Portfolios

Portfolio managers concentrate their efforts on achieving the best possible trade-off between risk and return. For portfolios constructed from a fixed set of assets, the risk/ return profile varies with the portfolio composition. Portfolios that maximize the return, given the risk, or, conversely, minimize the risk for the given return, are called optimal. Optimal portfolios define a line in the risk/return plane called the efficient frontier.

A portfolio may also have to meet additional requirements to be considered. Different investors have different levels of risk tolerance. Selecting the adequate portfolio for a particular investor is a difficult process. The portfolio manager can hedge the risk related to a particular portfolio along the efficient frontier with partial investment in risk-free assets. The definition of the capital allocation line, and finding where the final portfolio falls on this line, if at all, is a function of:

- The risk/return profile of each asset
- The risk-free rate
- The borrowing rate
- The degree of risk aversion characterizing an investor

Financial Toolbox software includes a set of portfolio optimization functions designed to find the portfolio that best meets investor requirements.

Warning frontcon has been removed. Use Portfolio instead.
portopt has been partially removed and will no longer accept ConSet or varargin arguments. portopt will only solve the portfolio problem for long-only fully invested portfolios. Use Portfolio instead.

## See Also

Portfolio|abs2active|active2abs|frontier|pcalims | pcgcomp|pcglims |
pcpval|portalloc|portcons|portopt|portvrisk

## Related Examples

- "Portfolio Optimization Functions" on page 3-4
- "Portfolio Construction Examples" on page 3-7
- "Portfolio Selection and Risk Aversion" on page 3-9
- "Active Returns and Tracking Error Efficient Frontier" on page 3-43
- "Plotting an Efficient Frontier Using portopt" on page 10-26


## More About

- "Portfolio Object Workflow" on page 4-21


## Portfolio Optimization Functions

The portfolio optimization functions assist portfolio managers in constructing portfolios that optimize risk and return.

| Capital Allocation | Description |
| :--- | :--- |
| portalloc | Computes the optimal risky portfolio on the efficient frontier, based <br> on the risk-free rate, the borrowing rate, and the investor's degree <br> of risk aversion. Also generates the capital allocation line, which <br> provides the optimal allocation of funds between the risky portfolio <br> and the risk-free asset. |
| Efficient Frontier <br> Computation | Description |
| frontcon | Computes portfolios along the efficient frontier for a given group of <br> assets. The computation is based on sets of constraints representing <br> the maximum and minimum weights for each asset, and the <br> maximum and minimum total weight for specified groups of assets. |
|  | Warning frontcon has been removed. Use Portfolio instead. <br> For more information on migrating frontcon code to Port folio, <br> see "frontcon Migration to Portfolio Object" on page 3-25. |
| frontier | Computes portfolios along the efficient frontier for a given group of <br> assets. Generates a surface of efficient frontiers showing how asset <br> allocation influences risk and return over time. |
| portopt | Computes portfolios along the efficient frontier for a given group of <br> assets. The computation is based on a set of user-specified linear <br> constraints. Typically, these constraints are generated using the <br> constraint specification functions described below. |
|  | Warning portopt has been partially removed and will no longer <br> accept ConSet or varargin arguments. portopt will only solve <br> the portfolio problem for long-only fully invested portfolios. Use <br> Portfolio instead. For more information on migrating portopt <br> code to Portfolio, see "portopt Migration to Portfolio Object" on <br> page 3-14. |


| Constraint <br> Specification | Description |
| :--- | :--- |
| portcons | Generates the portfolio constraints matrix for a portfolio of asset <br> investments using linear inequalities. The inequalities are of the <br> type A*Wts ' < b, where Wts is a row vector of weights. |
| portvrisk | Portfolio value at risk (VaR) returns the maximum potential loss in <br> the value of a portfolio over one period of time, given the loss <br> probability level RiskThreshold. |
| pcalims | Asset minimum and maximum allocation. Generates a constraint set <br> to fix the minimum and maximum weight for each individual asset. |
| pcgcomp | Group-to-group ratio constraint. Generates a constraint set <br> specifying the maximum and minimum ratios between pairs of <br> groups. |
| pcglims | Asset group minimum and maximum allocation. Generates a <br> constraint set to fix the minimum and maximum total weight for <br> each defined group of assets. |
| pcpval | Total portfolio value. Generates a constraint set to fix the total value <br> of the portfolio. |
| Constraint | Description |
| Conversion | Transforms a constraint matrix expressed in absolute weight format <br> to an equivalent matrix expressed in active weight format. |
| abs2active | Transforms a constraint matrix expressed in active weight format to <br> an equivalent matrix expressed in absolute weight format. |
| active2abs |  |

Note An alternative to using these portfolio optimization functions is to use the Portfolio object (Portfolio) for mean-variance portfolio optimization. This object supports gross or net portfolio returns as the return proxy, the variance of portfolio returns as the risk proxy, and a portfolio set that is any combination of the specified constraints to form a portfolio set. For information on the workflow when using Portfolio objects, see "Portfolio Object Workflow" on page 4-21.

## See Also

Portfolio| abs2active| active2abs|frontier|pcalims | pcgcomp|pcglims | pcpval| portalloc|portcons|portopt|portvrisk

## Related Examples

- "Portfolio Construction Examples" on page 3-7
- "Portfolio Selection and Risk Aversion" on page 3-9
- "Active Returns and Tracking Error Efficient Frontier" on page 3-43
- "Plotting an Efficient Frontier Using portopt" on page 10-26
- "portopt Migration to Portfolio Object" on page 3-14
- "frontcon Migration to Portfolio Object" on page 3-25


## More About

- "Analyzing Portfolios" on page 3-2
- "Portfolio Object Workflow" on page 4-21


## Portfolio Construction Examples

In this section...<br>"Introduction" on page 3-7<br>"Efficient Frontier Example" on page 3-7

## Introduction

The efficient frontier computation functions require information about each asset in the portfolio. This data is entered into the function via two matrices: an expected return vector and a covariance matrix. The expected return vector contains the average expected return for each asset in the portfolio. The covariance matrix is a square matrix representing the interrelationships between pairs of assets. This information can be directly specified or can be estimated from an asset return time series with the function ewstats.

Note An alternative to using these portfolio optimization functions is to use the Portfolio object (Portfolio) for mean-variance portfolio optimization. This object supports gross or net portfolio returns as the return proxy, the variance of portfolio returns as the risk proxy, and a portfolio set that is any combination of the specified constraints to form a portfolio set. For information on the workflow when using Portfolio objects, see "Portfolio Object Workflow" on page 4-21.

## Efficient Frontier Example

frontcon has been removed. To model the efficient frontier, use the Portfolio object instead. For example, using the Portfolio object, you can model an efficient frontier:

- "Obtaining Portfolios Along the Entire Efficient Frontier" on page 4-115
- "Obtaining Endpoints of the Efficient Frontier" on page 4-118
- "Obtaining Efficient Portfolios for Target Returns" on page 4-121
- "Obtaining Efficient Portfolios for Target Risks" on page 4-125
- "Efficient Portfolio That Maximizes Sharpe Ratio" on page 4-128
- "Estimate Efficient Frontiers for Portfolio Object" on page 4-140
- "Plotting the Efficient Frontier for a Portfolio Object" on page 4-143


## See Also

Portfolio|abs2active| active2abs | frontier| pcalims | pcgcomp | pcglims | pcpval|portalloc|portcons|portopt|portvrisk

## Related Examples

- "Portfolio Optimization Functions" on page 3-4
- "Portfolio Selection and Risk Aversion" on page 3-9
- "Active Returns and Tracking Error Efficient Frontier" on page 3-43
- "Plotting an Efficient Frontier Using portopt" on page 10-26
- "portopt Migration to Portfolio Object" on page 3-14
- "frontcon Migration to Portfolio Object" on page 3-25


## More About

- "Analyzing Portfolios" on page 3-2
- "Portfolio Object Workflow" on page 4-21


## Portfolio Selection and Risk Aversion

In this section...<br>"Introduction" on page 3-9<br>"Optimal Risky Portfolio" on page 3-10

## Introduction

One of the factors to consider when selecting the optimal portfolio for a particular investor is the degree of risk aversion. This level of aversion to risk can be characterized by defining the investor's indifference curve. This curve consists of the family of risk/ return pairs defining the trade-off between the expected return and the risk. It establishes the increment in return that a particular investor requires to make an increment in risk worthwhile. Typical risk aversion coefficients range from 2.0 through 4.0, with the higher number representing lesser tolerance to risk. The equation used to represent risk aversion in Financial Toolbox software is

$$
U=E(r)-0.005 * A^{*} \operatorname{sig}^{\wedge}{ }^{\wedge} 2
$$

where:
U is the utility value.
$E(r)$ is the expected return.
A is the index of investor's aversion.
sig is the standard deviation.


Note An alternative to using these portfolio optimization functions is to use the Portfolio object (Portfolio) for mean-variance portfolio optimization. This object supports gross or net portfolio returns as the return proxy, the variance of portfolio returns as the risk proxy, and a portfolio set that is any combination of the specified constraints to form a portfolio set. For information on the workflow when using Portfolio objects, see "Portfolio Object Workflow" on page 4-21.

## Optimal Risky Portfolio

This example computes the optimal risky portfolio on the efficient frontier based on the risk-free rate, the borrowing rate, and the investor's degree of risk aversion. You do this with the function portalloc.

First generate the efficient frontier data using portopt.
ExpReturn = [0.1 0.2 0.15];
ExpCovariance $=\left[\begin{array}{lll}0.005 & -0.010 & 0.004 ;\end{array}\right.$

| -0.010 | 0.040 | $-0.002 ;$ |
| ---: | ---: | ---: |
| 0.004 | -0.002 | $0.023] ;$ |

Consider 20 different points along the efficient frontier.

```
NumPorts = 20;
```

[PortRisk, PortReturn, PortWts] = portopt(ExpReturn,...
ExpCovariance, NumPorts);

Calling portopt, while specifying output arguments, returns the corresponding vectors and arrays representing the risk, return, and weights for each of the portfolios along the efficient frontier. Use these as the first three input arguments to the function portalloc.

Now find the optimal risky portfolio and the optimal allocation of funds between the risky portfolio and the risk-free asset, using these values for the risk-free rate, borrowing rate, and investor's degree of risk aversion.

RisklessRate = 0.08
BorrowRate $=0.12$
RiskAversion = 3
Calling portalloc without specifying any output arguments gives a graph displaying the critical points.

```
portalloc (PortRisk, PortReturn, PortWts, RisklessRate,...
```

BorrowRate, RiskAversion);


Calling portalloc while specifying the output arguments returns the variance (RiskyRisk), the expected return (RiskyReturn), and the weights (RiskyWts) allocated to the optimal risky portfolio. It also returns the fraction (RiskyFraction) of the complete portfolio allocated to the risky portfolio, and the variance (OverallRisk) and expected return (OverallReturn) of the optimal overall portfolio. The overall portfolio combines investments in the risk-free asset and in the risky portfolio. The actual proportion assigned to each of these two investments is determined by the degree of risk aversion characterizing the investor.

```
[RiskyRisk, RiskyReturn, RiskyWts,RiskyFraction, OverallRisk,...
OverallReturn] = portalloc (PortRisk, PortReturn, PortWts,...
RisklessRate, BorrowRate, RiskAversion)
RiskyRisk = 0.1288
RiskyReturn = 0.1791
RiskyWts = 0.0057 0.5879 0.4064
RiskyFraction = 1.1869
OverallRisk = 0.1529
OverallReturn = 0.1902
```

The value of RiskyFraction exceeds 1 (100\%), implying that the risk tolerance specified allows borrowing money to invest in the risky portfolio, and that no money is invested in the risk-free asset. This borrowed capital is added to the original capital available for investment. In this example, the customer tolerates borrowing $18.69 \%$ of the original capital amount.

## See Also

Portfolio| abs2active| active2abs|frontier|pcalims | pcgcomp|pcglims | pcpval|portalloc|portcons|portopt|portvrisk

## Related Examples

- "Portfolio Optimization Functions" on page 3-4
- "Portfolio Selection and Risk Aversion" on page 3-9
- "Active Returns and Tracking Error Efficient Frontier" on page 3-43
- "Plotting an Efficient Frontier Using portopt" on page 10-26
- "portopt Migration to Portfolio Object" on page 3-14
- "frontcon Migration to Portfolio Object" on page 3-25


## More About

- "Analyzing Portfolios" on page 3-2
- "Portfolio Object Workflow" on page 4-21


## portopt Migration to Portfolio Object

```
In this section...
"Migrate portopt Without Output Arguments" on page 3-14
"Migrate portopt with Output Arguments" on page 3-16
"Migrate portopt for Target Returns Within Range of Efficient Portfolio Returns" on page
3-18
"Migrate portopt for Target Return Outside Range of Efficient Portfolio Returns" on page
3-19
"Migrate portopt Using portcons Output for ConSet" on page 3-20
"Integrate Output from portcons, pcalims, pcglims, and pcgcomp with a Portfolio Object"
on page 3-22
```


## Migrate portopt Without Output Arguments

This example shows how to migrate portopt without output arguments to a Portfolio object.

The basic portopt functionality is represented as:

```
ExpReturn = [ 0.0054; 0.0531; 0.0779; 0.0934; 0.0130 ];
ExpCovariance = [ 0.0569, 0.0092, 0.0039, 0.0070, 0.0022;
    0.0092, 0.0380, 0.0035, 0.0197, 0.0028;
    0.0039, 0.0035, 0.0997, 0.0100, 0.0070;
    0.0070, 0.0197, 0.0100, 0.0461, 0.0050;
    0.0022, 0.0028, 0.0070, 0.0050, 0.0573 ];
NumPorts = 10;
```

portopt(ExpReturn, ExpCovariance, NumPorts);


To migrate a portopt syntax without output arguments to a Portfolio object:
ExpReturn = [ 0.0054; 0.0531; 0.0779; 0.0934; 0.0130 ];
ExpCovariance = [ 0.0569, 0.0092, 0.0039, 0.0070, 0.0022;
0.0092, 0.0380, 0.0035, 0.0197, 0.0028;
$0.0039,0.0035,0.0997,0.0100,0.0070$;
$0.0070,0.0197,0.0100,0.0461,0.0050$;
$0.0022,0.0028,0.0070,0.0050,0.0573$ ];
NumPorts = 10;
p = Portfolio;
p = setAssetMoments(p, ExpReturn, ExpCovariance);
p = setDefaultConstraints(p);
plotFrontier(p, NumPorts);


Without output arguments, portopt plots the efficient frontier. The Portfolio object has similar behavior although the Portfolio object writes to the current figure window rather than create a new window each time a plot is generated.

## Migrate portopt with Output Arguments

This example shows how to migrate portopt with output arguments to a Portfolio object.
With output arguments, the basic functionality of portopt returns portfolio moments and weights. Once the Portfolio object is set up, moments and weights are obtained in separate steps.

```
ExpReturn = [ 0.0054; 0.0531; 0.0779; 0.0934; 0.0130 ];
ExpCovariance = [ 0.0569, 0.0092, 0.0039, 0.0070, 0.0022;
    0.0092, 0.0380, 0.0035, 0.0197, 0.0028;
    0.0039, 0.0035, 0.0997, 0.0100, 0.0070;
    0.0070, 0.0197, 0.0100, 0.0461, 0.0050;
    0.0022, 0.0028, 0.0070, 0.0050, 0.0573 ];
```

```
NumPorts = 10;
[PortRisk, PortReturn, PortWts] = portopt(ExpReturn, ExpCovariance, NumPorts);
display(PortWts);
PortWts =
\begin{tabular}{rrrrr}
0.2103 & 0.2746 & 0.1157 & 0.1594 & 0.2400 \\
0.1744 & 0.2657 & 0.1296 & 0.2193 & 0.2110 \\
0.1386 & 0.2567 & 0.1436 & 0.2791 & 0.1821 \\
0.1027 & 0.2477 & 0.1575 & 0.3390 & 0.1532 \\
0.0668 & 0.2387 & 0.1714 & 0.3988 & 0.1242 \\
0.0309 & 0.2298 & 0.1854 & 0.4587 & 0.0953 \\
0 & 0.2168 & 0.1993 & 0.5209 & 0.0629 \\
0 & 0.1791 & 0.2133 & 0.5985 & 0.0091 \\
0 & 0.0557 & 0.2183 & 0.7260 & 0 \\
0 & 0 & 0 & 1.0000 & 0
\end{tabular}
```

To migrate a portopt syntax with output arguments:

```
ExpReturn = [ 0.0054; 0.0531; 0.0779; 0.0934; 0.0130 ];
```

ExpCovariance = [ 0.0569, 0.0092, 0.0039, 0.0070, 0.0022;
0.0092, 0.0380, 0.0035, 0.0197, 0.0028;
0.0039, 0.0035, 0.0997, 0.0100, 0.0070;
$0.0070,0.0197,0.0100,0.0461,0.0050$;
$0.0022,0.0028,0.0070,0.0050,0.0573$ ];
NumPorts = 10;
p = Portfolio;
$p$ = setAssetMoments(p, ExpReturn, ExpCovariance);
p = setDefaultConstraints(p);
PortWts = estimateFrontier(p, NumPorts);
[PortRisk, PortReturn] = estimatePortMoments(p, PortWts);
display(PortWts);
PortWts =

| 0.2103 | 0.1744 | 0.1386 | 0.1027 | 0.0668 | 0.0309 | 0 | 0 | 0 | 0 |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 0.2746 | 0.2657 | 0.2567 | 0.2477 | 0.2387 | 0.2298 | 0.2168 | 0.1791 | 0.0557 | 0 |
| 0.1157 | 0.1296 | 0.1436 | 0.1575 | 0.1714 | 0.1854 | 0.1993 | 0.2133 | 0.2183 | 0 |
| 0.1594 | 0.2193 | 0.2791 | 0.3390 | 0.3988 | 0.4587 | 0.5209 | 0.5985 | 0.7260 | 1.0000 |
| 0.2400 | 0.2110 | 0.1821 | 0.1532 | 0.1242 | 0.0953 | 0.0629 | 0.0091 | 0 | 0 |

The Portfolio object returns PortWts with portfolios going down columns, not across rows. Portfolio risks and returns are still in column format.

## Migrate portopt for Target Returns Within Range of Efficient Portfolio Returns

This example shows how to migrate portopt target returns within range of efficient portfolio returns to a Portfolio object.
portopt can obtain portfolios with specific targeted levels of return but requires that the targeted returns fall within the range of efficient returns. The Portfolio object handles this by selecting portfolios at the ends of the efficient frontier.

```
ExpReturn = [ 0.0054; 0.0531; 0.0779; 0.0934; 0.0130 ];
ExpCovariance = [ 0.0569, 0.0092, 0.0039, 0.0070, 0.0022;
    0.0092, 0.0380, 0.0035, 0.0197, 0.0028;
    0.0039, 0.0035, 0.0997, 0.0100, 0.0070;
    0.0070, 0.0197, 0.0100, 0.0461, 0.0050;
    0.0022, 0.0028, 0.0070, 0.0050, 0.0573 ];
NumPorts = 10;
TargetReturn = [ 0.05; 0.06; 0.07; 0.08; 0.09 ];
[PortRisk, PortReturn, PortWts] = portopt(ExpReturn, ExpCovariance, [], TargetReturn);
disp(' Efficient Target');
disp([PortReturn, TargetReturn]);
    Efficient Target
    0.0500 0.0500
    0.0600 0.0600
    0.0700 0.0700
    0.0800 0.0800
    0.0900 0.0900
```

To migrate a portopt syntax for target returns within range of efficient portfolio returns to a Portfolio object:

```
ExpReturn = [ 0.0054; 0.0531; 0.0779; 0.0934; 0.0130 ];
ExpCovariance = [ 0.0569, 0.0092, 0.0039, 0.0070, 0.0022;
    0.0092, 0.0380, 0.0035, 0.0197, 0.0028;
    0.0039, 0.0035, 0.0997, 0.0100, 0.0070;
    0.0070, 0.0197, 0.0100, 0.0461, 0.0050;
    0.0022, 0.0028, 0.0070, 0.0050, 0.0573 ];
NumPorts = 10;
```

```
TargetReturn = [ 0.05; 0.06; 0.07; 0.08; 0.09 ];
p = Portfolio;
p = setAssetMoments(p, ExpReturn, ExpCovariance);
p = setDefaultConstraints(p);
PortWts = estimateFrontierByReturn(p, TargetReturn);
[PortRisk, PortReturn] = estimatePortMoments(p, PortWts);
disp(' Efficient Target');
disp([PortReturn, TargetReturn]);
Efficient Target
    0.0500 0.0500
    0.0600 0.0600
    0.0700 0.0700
    0.0800 0.0800
    0.0900 0.0900
```


## Migrate portopt for Target Return Outside Range of Efficient Portfolio Returns

This example shows how to migrate portopt target returns outside of range of efficient portfolio returns to a Portfolio object.

When the target return is outside of the range of efficient portfolio returns, portopt generates an error. The Portfolio object handles this effectively by selecting portfolios at the ends of the efficient frontier.

```
ExpReturn = [ 0.0054; 0.0531; 0.0779; 0.0934; 0.0130 ];
ExpCovariance = [ 0.0569, 0.0092, 0.0039, 0.0070, 0.0022;
    0.0092, 0.0380, 0.0035, 0.0197, 0.0028;
    0.0039, 0.0035, 0.0997, 0.0100, 0.0070;
    0.0070, 0.0197, 0.0100, 0.0461, 0.0050;
    0.0022, 0.0028, 0.0070, 0.0050, 0.0573 ];
NumPorts = 10;
TargetReturn = [ 0.05; 0.06; 0.07; 0.08; 0.09; 0.10 ];
[PortRisk, PortReturn, PortWts] = portopt(ExpReturn, ExpCovariance, [], TargetReturn);
disp(' Efficient Target');
disp([PortReturn, TargetReturn]);
In portopt at 85
Error using portopt (line 297)
One or more requested returns are greater than the maximum achievable return of 0.093400.
```

To migrate a portopt syntax for target returns outside of the range of efficient portfolio returns to a Portfolio object:

```
ExpReturn = [ 0.0054; 0.0531; 0.0779; 0.0934; 0.0130 ];
ExpCovariance = [ 0.0569, 0.0092, 0.0039, 0.0070, 0.0022;
    0.0092, 0.0380, 0.0035, 0.0197, 0.0028;
    0.0039, 0.0035, 0.0997, 0.0100, 0.0070;
    0.0070, 0.0197, 0.0100, 0.0461, 0.0050;
NumPorts = 10;
TargetReturn = [ 0.05; 0.06; 0.07; 0.08; 0.09; 0.10 ];
p = Portfolio;
p = setAssetMoments(p, ExpReturn, ExpCovariance);
p = setDefaultConstraints(p);
PortWts = estimateFrontierByReturn(p, TargetReturn);
[PortRisk, PortReturn] = estimatePortMoments(p, PortWts);
disp(' Efficient Target');
disp([PortReturn, TargetReturn]);
Warning: One or more target return values are outside the feasible range [
0.0427391, 0.0934 ].
    Will return portfolios associated with endpoints of the range for these
    values.
> In Portfolio/estimateFrontierByReturn (line 106)
    Efficient Target
    0.0500 0.0500
    0.0600 0.0600
    0.0700 0.0700
    0.0800 0.0800
    0.0900 0.0900
    0.0934 0.1000
```


## Migrate portopt Using portcons Output for ConSet

This example shows how to migrate portopt when the ConSet output from portcons is used with portopt.
portopt accepts as input the outputs from portcons, pcalims, pcglims, and pcgcomp. This example focuses on portcons. portcons sets up linear constraints for portopt in the form $A *$ Port $<=$ b. In a matrix ConSet $=[\mathrm{A}, \mathrm{b}]$ and break into separate $A$ and $b$ arrays with $A=C o n S e t(:, 1: e n d-1) ;$ and $b=C o n S e t(:, e n d) ;$ In addition, to illustrate default problem with additional group constraints, consider three groups. Assets 2, 3, and 4 can constitute up to $80 \%$ of portfolio, Assets 1 and 2 can constitute up to $70 \%$ of portfolio, and Assets 3, 4, and 5 can constitute up to $90 \%$ of portfolio.

```
ExpReturn = [ 0.0054; 0.0531; 0.0779; 0.0934; 0.0130 ];
```

```
ExpCovariance = [ 0.0569, 0.0092, 0.0039, 0.0070, 0.0022;
    0.0092, 0.0380, 0.0035, 0.0197, 0.0028;
    0.0039, 0.0035, 0.0997, 0.0100, 0.0070;
    0.0070, 0.0197, 0.0100, 0.0461, 0.0050;
    0.0022, 0.0028, 0.0070, 0.0050, 0.0573 ];
NumPorts = 10;
Groups = [ 0 1 1 1 0; 1 1 0 0 0; 0 0 1 1 1 ];
GroupBounds = [ 0, 0.8; 0, 0.7; 0, 0.9 ];
LowerGroup = GroupBounds(:,1);
UpperGroup = GroupBounds(:,2);
ConSet = portcons('default', 5, 'grouplims', Groups, LowerGroup, UpperGroup);
[PortRisk, PortReturn, PortWts] = portopt(ExpReturn, ExpCovariance, NumPorts, [], ConSet);
disp([PortRisk, PortReturn]);
Error using portopt (line 83)
In the current and future releases, portopt will no longer accept ConSet or varargin arguments.
'It will only solve the portfolio problem for long-only fully-invested portfolios.
To solve more general problems, use the Portfolio object.
See the release notes for details, including examples to make the conversion.
```

To migrate portopt to a Portfolio object when the ConSet output from portcons is used with portopt:

```
ExpReturn = [ 0.0054; 0.0531; 0.0779; 0.0934; 0.0130 ];
ExpCovariance = [ 0.0569, 0.0092, 0.0039, 0.0070, 0.0022;
    0.0092, 0.0380, 0.0035, 0.0197, 0.0028;
    0.0039, 0.0035, 0.0997, 0.0100, 0.0070;
    0.0070, 0.0197, 0.0100, 0.0461, 0.0050;
    0.0022, 0.0028, 0.0070, 0.0050, 0.0573 ];
NumPorts = 10;
Groups = [ 0 1 1 1 0; 1 1 0 0 0; 0 0 1 1 1 ];
GroupBounds = [ 0, 0.8; 0, 0.7; 0, 0.9 ];
LowerGroup = GroupBounds(:,1);
UpperGroup = GroupBounds(:,2);
ConSet = portcons('default', 5, 'grouplims', Groups, LowerGroup, UpperGroup);
A = ConSet(:,1:end-1);
b = ConSet(:,end);
p = Portfolio;
p = setAssetMoments(p, ExpReturn, ExpCovariance);
p = setInequality(p, A, b); % implement group constraints here
```

PortWts $=$ estimateFrontier(p, NumPorts);
[PortRisk, PortReturn] = estimatePortMoments(p, PortWts);
disp([PortRisk, PortReturn]);
$0.1288 \quad 0.0427$
0.12920 .0465

| 0.1306 | 0.0503 |
| :--- | :--- |
| 0.1328 | 0.0540 |
| 0.1358 | 0.0578 |
| 0.1395 | 0.0615 |
| 0.1440 | 0.0653 |
| 0.1504 | 0.0690 |
| 0.1590 | 0.0728 |
| 0.1806 | 0.0766 |

The constraints are entered directly into the Portfolio object with the setInequality or addInequality functions.

## Integrate Output from portcons, pcalims, pcglims, and pcgcomp with a Portfolio Object

This example shows how to integrate output from pcalims, pcalims, pcglims, or pcgcomp with a Portfolio object implementation.
portcons, pcalims, pcglims, and pcgcomp setup linear constraints for portopt in the form $A *$ Port <= b. Although some functions permit two outputs, assume that the output is a single matrix ConSet. Break into separate $A$ and $b$ arrays with:

- $A=C o n S e t(:, 1: e n d-1)$;
- b = ConSet(: ,end);

In addition, to illustrate default problem with additional group constraints, consider three groups:

- Assets 2, 3, and 4 can constitute up to $80 \%$ of portfolio.
- Assets 1 and 2 can constitute up to $70 \%$ of portfolio.
- Assets 3, 4, and 5 can constitute up to $90 \%$ of portfolio.

Groups = [ $011110 ; 11000 ; 00111] ;$
GroupBounds = [ 0, 0.8; 0, 0.7; 0, 0.9 ];
To integrate the ConSet output of portcons with a Portfolio object implementation:

```
ExpReturn = [ 0.0054; 0.0531; 0.0779; 0.0934; 0.0130 ];
ExpCovariance = [ 0.0569, 0.0092, 0.0039, 0.0070, 0.0022;
    0.0092, 0.0380, 0.0035, 0.0197, 0.0028;
    0.0039, 0.0035, 0.0997, 0.0100, 0.0070;
    0.0070, 0.0197, 0.0100, 0.0461, 0.0050;
    0.0022, 0.0028, 0.0070, 0.0050, 0.0573 ];
```

```
NumPorts = 10;
Groups = [ 0 1 1 1 0; 1 1 0 0 0; 0 0 1 1 1 ];
GroupBounds = [ 0, 0.8; 0, 0.7; 0, 0.9 ];
LowerGroup = GroupBounds(:,1);
UpperGroup = GroupBounds(:,2);
ConSet = portcons('default', 5, 'grouplims', Groups, LowerGroup, UpperGroup);
A = ConSet(:,1:end-1);
b = ConSet(:,end);
p = Portfolio;
p = setAssetMoments(p, ExpReturn, ExpCovariance);
p = setDefaultConstraints(p); % implement default constraints here
p = setInequality(p, A, b); % implement group constraints here
PortWts = estimateFrontier(p, NumPorts);
[PortRisk, PortReturn] = estimatePortMoments(p, PortWts);
disp([PortRisk, PortReturn]);
\begin{tabular}{ll}
0.1288 & 0.0427 \\
0.1292 & 0.0465 \\
0.1306 & 0.0503 \\
0.1328 & 0.0540 \\
0.1358 & 0.0578 \\
0.1395 & 0.0615 \\
0.1440 & 0.0653 \\
0.1504 & 0.0690 \\
0.1590 & 0.0728 \\
0.1806 & 0.0766
\end{tabular}
```

To integrate the output of pcalims and pcglims with a Portfolio object implementation:

```
ExpReturn = [ 0.0054; 0.0531; 0.0779; 0.0934; 0.0130 ];
ExpCovariance = [ 0.0569, 0.0092, 0.0039, 0.0070, 0.0022;
    0.0092, 0.0380, 0.0035, 0.0197, 0.0028;
    0.0039, 0.0035, 0.0997, 0.0100, 0.0070;
    0.0070, 0.0197, 0.0100, 0.0461, 0.0050;
    0.0022, 0.0028, 0.0070, 0.0050, 0.0573 ];
NumPorts = 10;
Groups = [ 0 1 1 1 0; 1 1 0 0 0; 0 0 1 1 1 ];
GroupBounds = [ 0, 0.8; 0, 0.7; 0, 0.9 ];
LowerGroup = GroupBounds(:,1);
UpperGroup = GroupBounds(:,2);
AssetMin = [ 0; 0; 0; 0; 0 ];
AssetMax = [ 0.8; 0.8; 0.8; 0.8; 0.8 ];
[Aa, ba] = pcalims(AssetMin, AssetMax);
[Ag, bg] = pcglims(Groups, LowerGroup, UpperGroup);
```

```
p = Portfolio;
disp([PortRisk, PortReturn]);
0.1288 0.0427
0.1292 0.0465
0.1306 0.0503
0.1328 0.0540
0.1358 0.0578
0.1395 0.0615
0.1440 0.0653
0.1504 0.0690
0.1590 0.0728
0.1806 0.0766
```

p = setAssetMoments(p, ExpReturn, ExpCovariance);
$p=$ setDefaultConstraints(p); $\quad$ implement default constraints first
p = addInequality(p, Aa, ba); \% implement bound constraints here
$\mathrm{p}=\operatorname{addInequality}(\mathrm{p}, \mathrm{Ag}, \mathrm{bg})$; $\quad$ \% implement group constraints here
PortWts = estimateFrontier(p, NumPorts);
[PortRisk, PortReturn] = estimatePortMoments(p, PortWts);

## See Also

Portfolio| addInequality | estimateFrontier |estimateFrontierByReturn | estimatePortMoments | pcalims | pcgcomp| pcglims | portcons | portopt | setAssetMoments| setDefaultConstraints| setInequality

## Related Examples

- "frontcon Migration to Portfolio Object" on page 3-25


## More About

- "Portfolio Object Workflow" on page 4-21


## frontcon Migration to Portfolio Object

In this section...<br>"Migrate frontcon Without Output Arguments" on page 3-25<br>"Migrate frontcon with Output Arguments" on page 3-26<br>"Migrate frontcon for Target Returns Within Range of Efficient Portfolio Returns" on page 3-27<br>"Migrate frontcon for Target Returns Outside Range of Efficient Portfolio Returns" on page 3-29<br>"Migrate frontcon Syntax When Using Bounds" on page 3-30<br>"Migrate frontcon Syntax When Using Groups" on page 3-31

## Migrate frontcon Without Output Arguments

This example shows how to migrate frontcon without output arguments to a Portfolio object.

The basic frontcon functionality is represented as:

```
ExpReturn = [ 0.0054; 0.0531; 0.0779; 0.0934; 0.0130 ];
ExpCovariance = [ 0.0569, 0.0092, 0.0039, 0.0070, 0.0022;
    0.0092, 0.0380, 0.0035, 0.0197, 0.0028;
    0.0039, 0.0035, 0.0997, 0.0100, 0.0070;
    0.0070, 0.0197, 0.0100, 0.0461, 0.0050;
    0.0022, 0.0028, 0.0070, 0.0050, 0.0573 ];
NumPorts = 10
frontcon(ExpReturn, ExpCovariance, NumPorts);
Undefined function or variable 'frontcon'.
To migrate a frontcon syntax without output arguments to a Portfolio object:
```

```
ExpReturn = [ 0.0054; 0.0531; 0.0779; 0.0934; 0.0130 ];
```

ExpReturn = [ 0.0054; 0.0531; 0.0779; 0.0934; 0.0130 ];
ExpCovariance = [ 0.0569, 0.0092, 0.0039, 0.0070, 0.0022;
ExpCovariance = [ 0.0569, 0.0092, 0.0039, 0.0070, 0.0022;
0.0092, 0.0380, 0.0035, 0.0197, 0.0028;
0.0092, 0.0380, 0.0035, 0.0197, 0.0028;
0.0039, 0.0035, 0.0997, 0.0100, 0.0070;
0.0039, 0.0035, 0.0997, 0.0100, 0.0070;
0.0070, 0.0197, 0.0100, 0.0461, 0.0050;
0.0070, 0.0197, 0.0100, 0.0461, 0.0050;
0.0022, 0.0028, 0.0070, 0.0050, 0.0573 ];
0.0022, 0.0028, 0.0070, 0.0050, 0.0573 ];
NumPorts = 10;

```
NumPorts = 10;
```

```
p = Portfolio;
p = setAssetMoments(p, ExpReturn, ExpCovariance);
p = setDefaultConstraints(p);
plotFrontier(p, NumPorts);
```



The Portfolio object writes to the current figure window rather than create a new window each time a plot is generated.

## Migrate frontcon with Output Arguments

This example shows how to migrate frontcon with output arguments to a Portfolio object.

The basic frontcon functionality is represented as:

```
ExpReturn = [ 0.0054; 0.0531; 0.0779; 0.0934; 0.0130 ];
ExpCovariance = [ 0.0569, 0.0092, 0.0039, 0.0070, 0.0022;
    0.0092, 0.0380, 0.0035, 0.0197, 0.0028;
    0.0039, 0.0035, 0.0997, 0.0100, 0.0070;
    0.0070, 0.0197, 0.0100, 0.0461, 0.0050;
    0.0022, 0.0028, 0.0070, 0.0050, 0.0573 ];
NumPorts = 10;
[PortRisk, PortReturn, PortWts] = frontcon(ExpReturn, ExpCovariance, NumPorts);
display(PortWts);
Undefined function or variable 'frontcon'.
```

To migrate a frontcon syntax with output arguments:

```
ExpReturn = [ 0.0054; 0.0531; 0.0779; 0.0934; 0.0130 ];
```

ExpCovariance $=[0.0569,0.0092,0.0039,0.0070,0.0022$;
$0.0092,0.0380,0.0035,0.0197,0.0028$;
0.0039, 0.0035, 0.0997, 0.0100, 0.0070;
0.0070, 0.0197, 0.0100, 0.0461, 0.0050;
$0.0022,0.0028,0.0070,0.0050,0.0573$ ];
NumPorts = 10;
p = Portfolio;
$\mathrm{p}=$ setAssetMoments(p, ExpReturn, ExpCovariance);
$p=$ setDefaultConstraints(p);
PortWts $=$ estimateFrontier(p, NumPorts);
[PortRisk, PortReturn] = estimatePortMoments(p, PortWts);
display(PortWts);
PortWts =

| 0.2103 | 0.1744 | 0.1386 | 0.1027 | 0.0668 | 0.0309 | 0 | 0 | 0 | 0 |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 0.2746 | 0.2657 | 0.2567 | 0.2477 | 0.2387 | 0.2298 | 0.2168 | 0.1791 | 0.0557 | 0 |
| 0.1157 | 0.1296 | 0.1436 | 0.1575 | 0.1714 | 0.1854 | 0.1993 | 0.2133 | 0.2183 | 0 |
| 0.1594 | 0.2193 | 0.2791 | 0.3390 | 0.3988 | 0.4587 | 0.5209 | 0.5985 | 0.7260 | 1.0000 |
| 0.2400 | 0.2110 | 0.1821 | 0.1532 | 0.1242 | 0.0953 | 0.0629 | 0.0091 | 0 | 0 |

The Portfolio object returns PortWts with portfolios going down columns, not across rows. Portfolio risks and returns are still in column format.

## Migrate frontcon for Target Returns Within Range of Efficient Portfolio Returns

This example shows how to migrate frontcon target returns within range of efficient portfolio returns to a Portfolio object.
frontcon can obtain portfolios with specific targeted levels of return but requires that the targeted returns fall within the range of efficient returns. The Portfolio object handles this by selecting portfolios at the ends of the efficient frontier.

```
ExpReturn = [ 0.0054; 0.0531; 0.0779; 0.0934; 0.0130 ];
ExpCovariance = [ 0.0569, 0.0092, 0.0039, 0.0070, 0.0022;
    0.0092, 0.0380, 0.0035, 0.0197, 0.0028;
    0.0039, 0.0035, 0.0997, 0.0100, 0.0070;
    0.0070, 0.0197, 0.0100, 0.0461, 0.0050;
    0.0022, 0.0028, 0.0070, 0.0050, 0.0573 ];
NumPorts = 10;
TargetReturn = [ 0.05; 0.06; 0.07; 0.08; 0.09 ];
[PortRisk, PortReturn, PortWts] = frontcon(ExpReturn, ExpCovariance, [], TargetReturn);
disp(' Efficient Target');
disp([PortReturn, TargetReturn]);
Undefined function or variable 'frontcon'.
```

To migrate a frontcon syntax for target returns within range of efficient portfolio returns to a Portfolio object:

```
ExpReturn = [ 0.0054; 0.0531; 0.0779; 0.0934; 0.0130 ];
ExpCovariance = [ 0.0569, 0.0092, 0.0039, 0.0070, 0.0022;
    0.0092, 0.0380, 0.0035, 0.0197, 0.0028;
    0.0039, 0.0035, 0.0997, 0.0100, 0.0070;
    0.0070, 0.0197, 0.0100, 0.0461, 0.0050;
    0.0022, 0.0028, 0.0070, 0.0050, 0.0573 ];
NumPorts = 10;
TargetReturn = [ 0.05; 0.06; 0.07; 0.08; 0.09 ];
p = Portfolio;
p = setAssetMoments(p, ExpReturn, ExpCovariance);
p = setDefaultConstraints(p);
PortWts = estimateFrontierByReturn(p, TargetReturn);
[PortRisk, PortReturn] = estimatePortMoments(p, PortWts);
disp(' Efficient Target');
disp([PortReturn, TargetReturn]);
Efficient Target
    0.0500 0.0500
    0.0600 0.0600
    0.0700 0.0700
```

| 0.0800 | 0.0800 |
| :--- | :--- |
| 0.0900 | 0.0900 |

## Migrate frontcon for Target Returns Outside Range of Efficient Portfolio Returns

This example shows how to migrate frontcon target returns outside of range of efficient portfolio returns to a Portfolio object.

When the target return is outside of the range of efficient portfolio returns, frontcon generates an error. The Portfolio object handles this effectively by selecting portfolios at the ends of the efficient frontier.

```
ExpReturn = [ 0.0054; 0.0531; 0.0779; 0.0934; 0.0130 ];
ExpCovariance = [ 0.0569, 0.0092, 0.0039, 0.0070, 0.0022;
    0.0092, 0.0380, 0.0035, 0.0197, 0.0028;
    0.0039, 0.0035, 0.0997, 0.0100, 0.0070;
    0.0070, 0.0197, 0.0100, 0.0461, 0.0050;
    0.0022, 0.0028, 0.0070, 0.0050, 0.0573 ];
NumPorts = 10;
TargetReturn = [ 0.05; 0.06; 0.07; 0.08; 0.09; 0.10 ];
[PortRisk, PortReturn, PortWts] = frontcon(ExpReturn, ExpCovariance, [], TargetReturn);
disp(' Efficient Target');
disp([PortReturn, TargetReturn]);
Undefined function or variable 'frontcon'.
```

To migrate a frontcon syntax for target returns outside of the range of efficient portfolio returns to a Portfolio object:

```
ExpReturn = [ 0.0054; 0.0531; 0.0779; 0.0934; 0.0130 ];
ExpCovariance = [ 0.0569, 0.0092, 0.0039, 0.0070, 0.0022;
    0.0092, 0.0380, 0.0035, 0.0197, 0.0028;
    0.0039, 0.0035, 0.0997, 0.0100, 0.0070;
    0.0070, 0.0197, 0.0100, 0.0461, 0.0050;
    0.0022, 0.0028, 0.0070, 0.0050, 0.0573 ];
NumPorts = 10;
TargetReturn = [ 0.05; 0.06; 0.07; 0.08; 0.09; 0.10 ];
p = Portfolio;
p = setAssetMoments(p, ExpReturn, ExpCovariance);
p = setDefaultConstraints(p);
PortWts = estimateFrontierByReturn(p, TargetReturn);
[PortRisk, PortReturn] = estimatePortMoments(p, PortWts);
```

```
disp(' Efficient Target');
disp([PortReturn, TargetReturn]);
Warning: One or more target return values are outside the feasible range [
0.0427391, 0.0934 ].
    Will return portfolios associated with endpoints of the range for these
    values.
    In Portfolio/estimateFrontierByReturn (line 106)
    Efficient Target
    0.0500 0.0500
    0.0600 0.0600
    0.0700 0.0700
    0.0800 0.0800
    0.0900 0.0900
    0.0934 0.1000
```


## Migrate frontcon Syntax When Using Bounds

This example shows how to migrate frontcon syntax for AssetBounds to a Portfolio object.

Use frontcon with an input specification for AssetBounds that contains the lower and upper bounds on the weight allocated to each asset in the portfolio:

```
ExpReturn = [ 0.0054; 0.0531; 0.0779; 0.0934; 0.0130 ];
ExpCovariance = [ 0.0569, 0.0092, 0.0039, 0.0070, 0.0022;
    0.0092, 0.0380, 0.0035, 0.0197, 0.0028;
    0.0039, 0.0035, 0.0997, 0.0100, 0.0070;
    0.0070, 0.0197, 0.0100, 0.0461, 0.0050;
    0.0022, 0.0028, 0.0070, 0.0050, 0.0573 ];
NumPorts = 10;
AssetBounds = [ 0.1, 0.1, 0.1, 0.1, 0.1; 0.5, 0.5, 0.5, 0.5, 0.5 ];
[PortRisk, PortReturn, PortWts] = frontcon(ExpReturn, ExpCovariance, NumPorts, [], AssetBounds);
disp([PortRisk, PortReturn]);
Undefined function or variable 'frontcon'
To migrate a frontcon syntax using AssetBounds to a Portfolio object:
ExpReturn = [ 0.0054; 0.0531; 0.0779; 0.0934; 0.0130 ];
ExpCovariance = [ 0.0569, 0.0092, 0.0039, 0.0070, 0.0022;
0.0092, 0.0380, 0.0035, 0.0197, 0.0028;
0.0039, 0.0035, 0.0997, 0.0100, 0.0070;
0.0070, 0.0197, 0.0100, 0.0461, 0.0050;
\(0.0022,0.0028,0.0070,0.0050,0.0573 \mathrm{~J}\);
NumPorts = 10;
AssetBounds = [ 0.1, 0.1, 0.1, 0.1, 0.1; 0.5, 0.5, 0.5, 0.5, 0.5 ];
```

```
LowerBound = AssetBounds(1,:);
UpperBound = AssetBounds(2,:);
p = Portfolio;
p = setAssetMoments(p, ExpReturn, ExpCovariance);
p = setDefaultConstraints(p);
p = setBounds(p, LowerBound, UpperBound);
PortWts = estimateFrontier(p, NumPorts);
[PortRisk, PortReturn] = estimatePortMoments(p, PortWts);
disp([PortRisk, PortReturn]);
0.1288 0.0427
0.1291 0.0457
0.1299 0.0487
0.1313 0.0516
0.1332 0.0546
0.1356 0.0576
0.1385 0.0605
0.1419 0.0635
0.1461 0.0665
0.1519 0.0694
```


## Migrate frontcon Syntax When Using Groups

This example shows how to migrate frontcon syntax for Groups and GroupBounds to a Portfolio object.

Use frontcon with an input specification for Groups (asset groups or classes.) and GroupBounds (the lower and upper bounds of the total weights of all assets in a group). Consider three groups: Assets 2, 3, and 4 can constitute up to $80 \%$ of a portfolio, Assets 1 and 2 can constitute up to $70 \%$ of a portfolio, and Assets 3, 4, and 5 can constitute up to $90 \%$ of a portfolio.

```
ExpReturn = [ 0.0054; 0.0531; 0.0779; 0.0934; 0.0130 ];
ExpCovariance = [ 0.0569, 0.0092, 0.0039, 0.0070, 0.0022;
    0.0092, 0.0380, 0.0035, 0.0197, 0.0028;
    0.0039, 0.0035, 0.0997, 0.0100, 0.0070;
    0.0070, 0.0197, 0.0100, 0.0461, 0.0050;
    0.0022, 0.0028, 0.0070, 0.0050, 0.0573 ];
NumPorts = 10;
Groups = [ 0 1 1 1 0; 1 1 0 0 0; 0 0 1 1 1 ];
```

```
GroupBounds = [ 0, 0.8; 0, 0.7; 0, 0.9 ];
[PortRisk, PortReturn, PortWgts] = frontcon(ExpReturn, ExpCovariance, NumPorts, [], [], ...
    Groups, GroupBounds);
disp([PortRisk, PortReturn]);
Undefined function or variable 'frontcon'.
```

To migrate a frontcon syntax using Groups and GroupBounds to a Portfolio object:

```
ExpReturn = [ 0.0054; 0.0531; 0.0779; 0.0934; 0.0130 ];
ExpCovariance = [ 0.0569, 0.0092, 0.0039, 0.0070, 0.0022;
    0.0092, 0.0380, 0.0035, 0.0197, 0.0028;
    0.0039, 0.0035, 0.0997, 0.0100, 0.0070;
    0.0070, 0.0197, 0.0100, 0.0461, 0.0050;
    0.0022, 0.0028, 0.0070, 0.0050, 0.0573 ];
NumPorts = 10;
Groups = [ 0 1 1 1 0; 1 1 0 0 0; 0 0 1 1 1 ];
GroupBounds = [ 0, 0.8; 0, 0.7; 0, 0.9 ];
LowerGroup = GroupBounds(:,1);
UpperGroup = GroupBounds(:,2);
p = Portfolio;
p = setAssetMoments(p, ExpReturn, ExpCovariance);
p = setDefaultConstraints(p);
p = setGroups(p, Groups, LowerGroup, UpperGroup);
PortWts = estimateFrontier(p, NumPorts);
[PortRisk, PortReturn] = estimatePortMoments(p, PortWts);
disp([PortRisk, PortReturn]);
0.1288 0.0427
0.1292 0.0465
0.1306 0.0503
0.1328 0.0540
0.1358 0.0578
0.1395 0.0615
0.1440 0.0653
0.1504 0.0690
0.1590 0.0728
0.1806 0.0766
```


## See Also

Portfolio| addInequality | estimateFrontier |estimateFrontierByReturn | estimatePortMoments | pcalims | pcgcomp| pcglims | portcons | portopt | setAssetMoments | setBounds | setDefaultConstraints | setGroups | setInequality

## Related Examples

- "portopt Migration to Portfolio Object" on page 3-14


## More About

- "Portfolio Object Workflow" on page 4-21


## Constraint Specification Using a Portfolio Object

## In this section...

"Constraints for Efficient Frontier" on page 3-34
"Linear Constraint Equations" on page 3-36
"Specifying Group Constraints" on page 3-39

## Constraints for Efficient Frontier

This example computes the efficient frontier of portfolios consisting of three different assets, INTC, XON, and RD, given a list of constraints. The expected returns for INTC, XON, and RD are respectively as follows:

```
ExpReturn = [0.1 0.2 0.15];
```

The covariance matrix is
ExpCovariance $\left.=\left[\begin{array}{rrr}0.005 & -0.010 & 0.004 ; \\ & -0.010 & 0.040 \\ & -0.002 ; \\ & 0.004 & -0.002\end{array}\right) 0.023\right] ;$

- Constraint 1
- Allow short selling up to $10 \%$ of the portfolio value in any asset, but limit the investment in any one asset to $110 \%$ of the portfolio value.
- Constraint 2
- Consider two different sectors, technology and energy, with the following table indicating the sector each asset belongs to.

| Asset | INTC | XON | RD |
| :--- | :--- | :--- | :--- |
| Sector | Technology | Energy | Energy |

Constrain the investment in the Energy sector to $80 \%$ of the portfolio value, and the investment in the Technology sector to 70\%.

To solve this problem, use Portfolio, passing in a list of asset constraints.
Consider eight different portfolios along the efficient frontier:

```
NumPorts = 8;
```

To introduce the asset bounds constraints specified in Constraint 1, create the matrix AssetBounds, where each column represents an asset. The upper row represents the lower bounds, and the lower row represents the upper bounds. Since the bounds are the same for each asset, only one pair of bounds is needed because of scalar expansion.

AssetBounds = [-0.1, 1.1];
Constraint 2 must be entered in two parts, the first part defining the groups, and the second part defining the constraints for each group. Given the information above, you can build a matrix of 1 s and 0 s indicating whether a specific asset belongs to a group. Each column represents an asset, and each row represents a group. This example has two groups: the technology group, and the energy group. Create the matrix Groups as follows.

```
Groups = [\begin{array}{ccc}{0}&{1}&{1;}\\{1}&{0}&{0];}\end{array};\mp@code{;};
```

The GroupBounds matrix allows you to specify an upper and lower bound for each group. Each row in this matrix represents a group. The first column represents the minimum allocation, and the second column represents the maximum allocation to each group. Since the investment in the Energy sector is capped at $80 \%$ of the portfolio value, and the investment in the Technology sector is capped at 70\%, create the GroupBounds matrix using this information.

```
GroupBounds = [0 0.80;
    0 0.70];
```

Now use Portfolio to obtain the vectors and arrays representing the risk, return, and weights for each of the eight portfolios computed along the efficient frontier. A budget constraint is added to ensure that the portfolio weights sum to 1.

```
p = Portfolio('AssetMean', ExpReturn, 'AssetCovar', ExpCovariance);
p = setBounds(p, AssetBounds(1), AssetBounds(2));
p = setBudget(p, 1, 1);
p = setGroups(p, Groups, GroupBounds(:,1), GroupBounds(:,2));
PortWts = estimateFrontier(p, NumPorts);
[PortRisk, PortReturn] = estimatePortMoments(p, PortWts);
PortRisk
PortReturn
PortWts
PortRisk =
```

> 0.0416
> 0.0499
> 0.0624
> 0.0767
> 0.0920
> 0.1100
> 0.1378
> 0.1716

PortReturn =
0.1279
0.1361
0.1442
0.1524
0.1605
0.1687
0.1768
0.1850

PortWts =

| 0.7000 | 0.6031 | 0.4864 | 0.3696 | 0.2529 | 0.2000 | 0.2000 | 0.2000 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 0.2582 | 0.3244 | 0.3708 | 0.4172 | 0.4636 | 0.5738 | 0.7369 | 0.9000 |
| 0.0418 | 0.0725 | 0.1428 | 0.2132 | 0.2835 | 0.2262 | 0.0631 | -0.1000 |

The outputs are represented as columns for the portfolio's risk and return. Portfolio weights are identified as corresponding column vectors in a matrix.

## Linear Constraint Equations

While the Portfolio object allows you to enter a fixed set of constraints related to minimum and maximum values for groups and individual assets, you often need to specify a larger and more general set of constraints when finding the optimal risky portfolio. Portfolio also addresses this need, by accepting an arbitrary set of constraints.

This example requires specifying the minimum and maximum investment in various groups.

Maximum and Minimum Group Exposure

| Group | Minimum Exposure | Maximum Exposure |
| :--- | :--- | :--- |
| North America | 0.30 | 0.75 |
| Europe | 0.10 | 0.55 |
| Latin America | 0.20 | 0.50 |
| Asia | 0.50 | 0.50 |

The minimum and maximum exposure in Asia is the same. This means that you require a fixed exposure for this group.

Also assume that the portfolio consists of three different funds. The correspondence between funds and groups is shown in the table below.

## Group Membership

| Group | Fund 1 | Fund 2 | Fund 3 |
| :--- | :--- | :--- | :--- |
| North America | X | X |  |
| Europe |  |  | X |
| Latin America | X |  |  |
| Asia |  | X | X |

Using the information in these two tables, build a mathematical representation of the constraints represented. Assume that the vector of weights representing the exposure of each asset in a portfolio is called Wts $=$ [W1 W2 W3].

Specifically

| 1. | $W 1+W 2$ | $\geq$ | 0.30 |
| :--- | :--- | :--- | :--- |
| 2. | $W 1+W 2$ | $\leq$ | 0.75 |
| 3. | $W 3$ | $\geq$ | 0.10 |
| 4. | $W 3$ | $\leq$ | 0.55 |
| 5. | $W 1$ | $\geq$ | 0.20 |
| 6. | $W 1$ | $\leq$ | 0.50 |
| 7. | $W 2+W 3$ | $=$ | 0.50 |

Since you must represent the information in the form $A * W t s<=b$, multiply equations 1 , 3 and 5 by -1 . Also turn equation 7 into a set of two inequalities: $W 2+W 3 \geq 0.50$ and $W 2$ $+W 3 \leq 0.50$. (The intersection of these two inequalities is the equality itself.) Thus

| 1. | $-W 1-W 2$ | $\leq$ | -0.30 |
| :--- | :--- | :--- | :--- |
| 2. | $W 1+W 2$ | $\leq$ | 0.75 |
| 3. | $-W 3$ | $\leq$ | -0.10 |
| 4. | $W 3$ | $\leq$ | 0.55 |
| 5. | $-W 1$ | $\leq$ | -0.20 |
| 6. | $W 1$ | $\leq$ | 0.50 |
| 7. | $-W 2-W 3$ | $\leq$ | -0.50 |
| 8. | $W 2+W 3$ | $\leq$ | 0.50 |

Bringing these equations into matrix notation gives
$A=\left[\begin{array}{rrr}-1 & -1 & 0 ; \\ 1 & 1 & 0 ; \\ 0 & 0 & -1 ; \\ 0 & 0 & 1 ; \\ -1 & 0 & 0 ; \\ 1 & 0 & 0 ; \\ 0 & -1 & -1 ; \\ 0 & 1 & 1]\end{array}\right.$
b = [-0.30;
0.75 ;
-0.10;
0.55 ;
-0.20;
0.50 ;
-0.50;
0.50 ]

One approach to solving this portfolio problem is to explicitly use the setInequality function:
p = Portfolio('AssetMean', ExpReturn, 'AssetCovar', ExpCovariance);
$p=$ setBounds( $p$, AssetBounds(1), AssetBounds(2));
$\mathrm{p}=\operatorname{setBudget}(\mathrm{p}, 1,1)$;
$\mathrm{p}=\operatorname{set} \operatorname{Inequality}(\mathrm{p}, \mathrm{A}, \mathrm{b})$;

```
PortWts = estimateFrontier(p, NumPorts);
[PortRisk, PortReturn] = estimatePortMoments(p, PortWts);
PortRisk
PortReturn
PortWts
PortRisk =
    0.0586
    0.0586
    0.0586
    0.0586
    0.0586
    0.0586
    0.0586
    0.0586
PortReturn =
    0.1375
    0.1375
    0.1375
    0.1375
    0.1375
    0.1375
    0.1375
    0.1375
PortWts =
\begin{tabular}{llllllll}
0.5000 & 0.5000 & 0.5000 & 0.5000 & 0.5000 & 0.5000 & 0.5000 & 0.5000 \\
0.2500 & 0.2500 & 0.2500 & 0.2500 & 0.2500 & 0.2500 & 0.2500 & 0.2500 \\
0.2500 & 0.2500 & 0.2500 & 0.2500 & 0.2500 & 0.2500 & 0.2500 & 0.2500
\end{tabular}
```

In this case, the constraints allow only one optimum portfolio. Since eight portfolios were requested, all eight portfolios are the same. Note that the solution to this portfolio problem using the setInequality function is the same as using the setGroups function in the next example ("Specifying Group Constraints" on page 3-39).

## Specifying Group Constraints

The example above ("Linear Constraint Equations" on page 3-36) defines a constraint matrix that specifies a set of typical scenarios. It defines groups of assets, specifies upper and lower bounds for total allocation in each of these groups, and it sets the total allocation of one group to a fixed value. Constraints like these are common occurrences. Portfolio object enables you to simplify the creation of the constraint matrix for these and other common portfolio requirements.

An alternative approach for solving the portfolio problem is to use the Portfolio object to define:

- A Group matrix, indicating the assets that belong to each group.
- A GroupMin vector, indicating the minimum bounds for each group.
- A GroupMax vector, indicating the maximum bounds for each group.

Based on the table Group Membership, build the Group matrix, with each row representing a group, and each column representing an asset.

```
Group = [1 1 0;
    0 0 1;
    1 0 0;
    0 1 1];
```

The table Maximum and Minimum Group Exposure has the information to build GroupMin and GroupMax.

```
GroupMin = [l0.30 0.10 0.20 0.50];
GroupMax = [l0.75 0.55 0.50 0.50];
```

Now use Portfolio and the setInequality function to obtain the vectors and arrays representing the risk, return, and weights for the portfolios computed along the efficient frontier.

```
p = Portfolio('AssetMean', ExpReturn, 'AssetCovar', ExpCovariance);
p = setBounds(p, AssetBounds(1), AssetBounds(2));
p = setBudget(p, 1, 1);
p = setGroups(p, Group, GroupMin, GroupMax);
PortWts = estimateFrontier(p, NumPorts);
[PortRisk, PortReturn] = estimatePortMoments(p, PortWts);
```

PortRisk
PortReturn
PortWts
PortRisk $=$
0.0586
0.0586
0.0586
0.0586
0.0586
0.0586
0.0586
0.0586

## PortReturn =

0.1375
0.1375
0.1375
0.1375
0.1375
0.1375
0.1375
0.1375

PortWts =

| 0.5000 | 0.5000 | 0.5000 | 0.5000 | 0.5000 | 0.5000 | 0.5000 | 0.5000 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.2500 | 0.2500 | 0.2500 | 0.2500 | 0.2500 | 0.2500 | 0.2500 | 0.2500 |
| 0.2500 | 0.2500 | 0.2500 | 0.2500 | 0.2500 | 0.2500 | 0.2500 | 0.2500 |

In this case, the constraints allow only one optimum portfolio. Since eight portfolios were requested, all eight portfolios are the same. Note that the solution to this portfolio problem using the setGroups function is the same as using the setInequality function in the previous example ("Linear Constraint Equations" on page 3-36).

## See Also

Portfolio|estimateFrontier|estimatePortMoments | setGroups | setInequality

## Related Examples

- "Setting Default Constraints for Portfolio Weights Using Portfolio Object" on page 468
- "Working with 'Simple' Bound Constraints Using Portfolio Object" on page 4-73
- "Working with Budget Constraints Using Portfolio Object" on page 4-77
- "Working with Group Constraints Using Portfolio Object" on page 4-80
- "Working with Group Ratio Constraints Using Portfolio Object" on page 4-84
- "Working with Linear Equality Constraints Using Portfolio Object" on page 4-88
- "Working with Linear Inequality Constraints Using Portfolio Object" on page 4-91
- "Working with Average Turnover Constraints Using Portfolio Object" on page 4-98
- "Working with One-Way Turnover Constraints Using Portfolio Object" on page 4-102
- "Working with Tracking Error Constraints Using Portfolio Object" on page 4-106
- "Asset Allocation Case Study" on page 4-196
- "Portfolio Optimization Examples" on page 4-167


## More About

- "Portfolio Set for Optimization Using Portfolio Object" on page 4-10
- "Portfolio Object Workflow" on page 4-21
- "Setting Up a Tracking Portfolio" on page 4-45


## Active Returns and Tracking Error Efficient Frontier

Suppose that you want to identify an efficient set of portfolios that minimize the variance of the difference in returns with respect to a given target portfolio, subject to a given expected excess return. The mean and standard deviation of this excess return are often called the active return and active risk, respectively. Active risk is sometimes referred to as the tracking error. Since the objective is to track a given target portfolio as closely as possible, the resulting set of portfolios is sometimes referred to as the tracking error efficient frontier.

Specifically, assume that the target portfolio is expressed as an index weight vector, such that the index return series may be expressed as a linear combination of the available assets. This example illustrates how to construct a frontier that minimizes the active risk (tracking error) subject to attaining a given level of return. That is, it computes the tracking error efficient frontier.

One way to construct the tracking error efficient frontier is to explicitly form the target return series and subtract it from the return series of the individual assets. In this manner, you specify the expected mean and covariance of the active returns, and compute the efficient frontier subject to the usual portfolio constraints.

This example works directly with the mean and covariance of the absolute (unadjusted) returns but converts the constraints from the usual absolute weight format to active weight format.

Consider a portfolio of five assets with the following expected returns, standard deviations, and correlation matrix based on absolute weekly asset returns.


Convert the correlations and standard deviations to a covariance matrix using corr2cov.

```
ExpCovariance = corr2cov(Sigmas, Correlations);
```

Next, assume that the target index portfolio is an equally weighted portfolio formed from the five assets. The sum of index weights equals 1, satisfying the standard full investment budget equality constraint.

Index = ones(NumAssets, 1)/NumAssets;
Generate an asset constraint matrix using portcons. The constraint matrix AbsConSet is expressed in absolute format (unadjusted for the index), and is formatted as [A b], corresponding to constraints of the form $A * W<=b$. Each row of AbsConSet corresponds to a constraint, and each column corresponds to an asset. Allow no short-selling and full investment in each asset (lower and upper bounds of each asset are 0 and 1, respectively). In particular, note that the first two rows correspond to the budget equality constraint; the remaining rows correspond to the upper/lower investment bounds.

```
AbsConSet = portcons('PortValue', 1, NumAssets, ...
'AssetLims', zeros(NumAssets,1), ones(NumAssets,1));
```

Now transform the absolute constraints to active constraints with abs2active.

```
ActiveConSet = abs2active(AbsConSet, Index);
```

An examination of the absolute and active constraint matrices reveals that they differ only in the last column (the columns corresponding to the $b$ in $A^{*} w<=b$ ).

```
[AbsConSet(:,end) ActiveConSet(:,end)]
```

ans $=$

| 1.0000 | 0 |
| ---: | ---: |
| -1.0000 | 0 |
| 1.0000 | 0.8000 |
| 1.0000 | 0.8000 |
| 1.0000 | 0.8000 |
| 1.0000 | 0.8000 |
| 1.0000 | 0.8000 |
| 0 | 0.2000 |
| 0 | 0.2000 |
| 0 | 0.2000 |
| 0 | 0.2000 |
| 0 | 0.2000 |

In particular, note that the sum-to-one absolute budget constraint becomes a sum-to-zero active budget constraint. The general transformation is as follows:

$$
b_{\text {active }}=b_{\text {absolute }}-A \times \text { Index } .
$$

Now construct the Portfolio object and plot the tracking error efficient frontier with 21 portfolios.

```
p = Portfolio('AssetMean', ExpReturn, 'AssetCovar', ExpCovariance);
p = p.setInequality(ActiveConSet(:,1:end-1), ActiveConSet(:,end));
[ActiveRisk, ActiveReturn] = p.plotFrontier(21);
plot(ActiveRisk*100, ActiveReturn*100, 'blue')
grid('on')
xlabel('Active Risk (Standard Deviation in Percent)')
ylabel('Active Return (Percent)')
title('Tracking Error Efficient Frontier')
```



Of particular interest is the lower-left portfolio along the frontier. This zero-risk/zeroreturn portfolio has a practical economic significance. It represents a full investment in the index portfolio itself. Each tracking error efficient portfolio (each row in the array ActiveWeights) satisfies the active budget constraint, and thus represents portfolio investment allocations with respect to the index portfolio. To convert these allocations to absolute investment allocations, add the index to each efficient portfolio.

ActiveWeights = p.estimateFrontier(21);
AbsoluteWeights = ActiveWeights + repmat(Index, 1, 21);

## See Also

Portfolio|abs2active|active2abs|estimateFrontier|frontier|pcalims pcgcomp| pcglims | pcpval|plotFrontier|portalloc|portcons|portvrisk| setInequality

## Related Examples

- "Portfolio Optimization Functions" on page 3-4
- "Portfolio Selection and Risk Aversion" on page 3-9
- "Plotting an Efficient Frontier Using portopt" on page 10-26


## More About

- "Analyzing Portfolios" on page 3-2
- "Portfolio Object Workflow" on page 4-21


## Mean-Variance Portfolio Optimization Tools

- "Portfolio Optimization Theory" on page 4-3
- "Portfolio Set for Optimization Using Portfolio Object" on page 4-10
- "Default Portfolio Problem" on page 4-19
- "Portfolio Object Workflow" on page 4-21
- "Portfolio Object" on page 4-23
- "Creating the Portfolio Object" on page 4-28
- "Common Operations on the Portfolio Object" on page 4-37
- "Setting Up an Initial or Current Portfolio" on page 4-42
- "Setting Up a Tracking Portfolio" on page 4-45
- "Asset Returns and Moments of Asset Returns Using Portfolio Object" on page 4-48
- "Working with a Riskless Asset" on page 4-61
- "Working with Transaction Costs" on page 4-63
- "Working with Portfolio Constraints Using Defaults" on page 4-68
- "Working with 'Simple' Bound Constraints Using Portfolio Object" on page 4-73
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- "Working with Average Turnover Constraints Using Portfolio Object" on page 4-98
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- "Validate the Portfolio Problem for Portfolio Object" on page 4-110
- "Estimate Efficient Portfolios for Entire Efficient Frontier for Portfolio Object" on page 4-115
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- "Efficient Portfolio That Maximizes Sharpe Ratio" on page 4-128
- "Choosing and Controlling the Solver for Mean-Variance Portfolio Optimization" on page 4-131
- "Estimate Efficient Frontiers for Portfolio Object" on page 4-140
- "Plotting the Efficient Frontier for a Portfolio Object" on page 4-143
- "Postprocessing Results to Set Up Tradable Portfolios" on page 4-149
- "Troubleshooting Portfolio Optimization Results" on page 4-152
- "Portfolio Optimization Examples" on page 4-167
- "Asset Allocation Case Study" on page 4-196
- "Portfolio Optimization with Semicontinuous and Cardinality Constraints" on page 4-210
- "Portfolio Optimization Against a Benchmark" on page 4-226


## Portfolio Optimization Theory

```
In this section...
"Portfolio Optimization Problems" on page 4-3
"Portfolio Problem Specification" on page 4-3
"Return Proxy" on page 4-4
"Risk Proxy" on page 4-6
```


## Portfolio Optimization Problems

Portfolio optimization problems involve identifying portfolios that satisfy three criteria:

- Minimize a proxy for risk.
- Match or exceed a proxy for return.
- Satisfy basic feasibility requirements.

Portfolios are points from a feasible set of assets that constitute an asset universe. A portfolio specifies either holdings or weights in each individual asset in the asset universe. The convention is to specify portfolios in terms of weights, although the portfolio optimization tools work with holdings as well.

The set of feasible portfolios is necessarily a nonempty, closed, and bounded set. The proxy for risk is a function that characterizes either the variability or losses associated with portfolio choices. The proxy for return is a function that characterizes either the gross or net benefits associated with portfolio choices. The terms "risk" and "risk proxy" and "return" and "return proxy" are interchangeable. The fundamental insight of Markowitz (see "Portfolio Optimization" on page A-7) is that the goal of the portfolio choice problem is to seek minimum risk for a given level of return and to seek maximum return for a given level of risk. Portfolios satisfying these criteria are efficient portfolios and the graph of the risks and returns of these portfolios forms a curve called the efficient frontier.

## Portfolio Problem Specification

To specify a portfolio optimization problem, you need the following:

- Proxy for portfolio return ( $\mu$ )
- Proxy for portfolio risk ( $\Sigma$ )
- Set of feasible portfolios ( $X$ ), called a portfolio set

Financial Toolbox has three objects to solve specific types of portfolio optimization problems:

- The Portfolio object (Portfolio) supports mean-variance portfolio optimization (see Markowitz [46], [47] at "Portfolio Optimization" on page A-7). This object has either gross or net portfolio returns as the return proxy, the variance of portfolio returns as the risk proxy, and a portfolio set that is any combination of the specified constraints to form a portfolio set.
- The PortfolioCVaR object (PortfolioCVaR) implements what is known as conditional value-at-risk portfolio optimization (see Rockafellar and Uryasev [48], [49] at "Portfolio Optimization" on page A-7), which is referred to as CVaR portfolio optimization. CVaR portfolio optimization works with the same return proxies and portfolio sets as mean-variance portfolio optimization but uses conditional value-at-risk of portfolio returns as the risk proxy.
- The PortfolioMAD object (PortfolioMAD) implements what is known as meanabsolute deviation portfolio optimization (see Konno and Yamazaki [50] at "Portfolio Optimization" on page A-7), which is referred to as MAD portfolio optimization. MAD portfolio optimization works with the same return proxies and portfolio sets as mean-variance portfolio optimization but uses mean-absolute deviation portfolio returns as the risk proxy.


## Return Proxy

The proxy for portfolio return is a function $\mu: X \rightarrow R$ on a portfolio set $X \subset R^{n}$ that characterizes the rewards associated with portfolio choices. Usually, the proxy for portfolio return has two general forms, gross and net portfolio returns. Both portfolio return forms separate the risk-free rate $r_{0}$ so that the portfolio $x \in X$ contains only risky assets.

Regardless of the underlying distribution of asset returns, a collection of $S$ asset returns $y_{1}, \ldots, y_{S}$ has a mean of asset returns

$$
m=\frac{1}{S} \sum_{s=1}^{S} y_{s}
$$

and (sample) covariance of asset returns

$$
C=\frac{1}{S-1} \sum_{s=1}^{S}\left(y_{s}-m\right)\left(y_{s}-m\right)^{T} .
$$

These moments (or alternative estimators that characterize these moments) are used directly in mean-variance portfolio optimization to form proxies for portfolio risk and return.

## Gross Portfolio Returns

The gross portfolio return for a portfolio $x \in X$ is

$$
\mu(x)=r_{0}+\left(m-r_{0} 1\right)^{T} x,
$$

where:
$r_{0}$ is the risk-free rate (scalar).
$m$ is the mean of asset returns ( $n$ vector).
If the portfolio weights sum to 1 , the risk-free rate is irrelevant. The properties in the Portfolio object to specify gross portfolio returns are:

- RiskFreeRate for $r_{0}$
- AssetMean for $m$


## Net Portfolio Returns

The net portfolio return for a portfolio $x \in X$ is

$$
\mu(x)=r_{0}+\left(m-r_{0} 1\right)^{T} x-b^{T} \max \left\{0, x-x_{0}\right\}-s^{T} \max \left\{0, x_{0}-x\right\},
$$

where:
$r_{0}$ is the risk-free rate (scalar).
$m$ is the mean of asset returns ( $n$ vector).
$b$ is the proportional cost to purchase assets ( $n$ vector).
$s$ is the proportional cost to sell assets ( $n$ vector).
You can incorporate fixed transaction costs in this model also. Though in this case, it is necessary to incorporate prices into such costs. The properties in the Portfolio object to specify net portfolio returns are:

- RiskFreeRate for $r_{0}$
- AssetMean for $m$
- InitPort for $x_{0}$
- BuyCost for b
- SellCost for s


## Risk Proxy

The proxy for portfolio risk is a function $\Sigma: X \rightarrow R$ on a portfolio set $X \subset R^{n}$ that characterizes the risks associated with portfolio choices.

## Variance

The variance of portfolio returns for a portfolio $x \in X$ is

$$
\Sigma(x)=x^{T} C x
$$

where $C$ is the covariance of asset returns ( $n$-by-n positive-semidefinite matrix).
The property in the Portfolio object to specify the variance of portfolio returns is AssetCovar for $C$.

Although the risk proxy in mean-variance portfolio optimization is the variance of portfolio returns, the square root, which is the standard deviation of portfolio returns, is often reported and displayed. Moreover, this quantity is often called the "risk" of the portfolio. For details, see Markowitz ("Portfolio Optimization" on page A-7).

## Conditional Value-at-Risk

The conditional value-at-risk for a portfolio $x \in X$, which is also known as expected shortfall, is defined as

$$
C \operatorname{VaR}_{\alpha}(x)=\frac{1}{1-\alpha} \int_{f(x, y) \geq V a R_{\alpha}(x)} f(x, y) p(y) d y
$$

where:
$\alpha$ is the probability level such that $0<\alpha<1$.
$f(x, y)$ is the loss function for a portfolio $x$ and asset return $y$.
$p(y)$ is the probability density function for asset return $y$.
$V a R_{\alpha}$ is the value-at-risk of portfolio $x$ at probability level $\alpha$.
The value-at-risk is defined as

$$
\operatorname{VaR}_{\alpha}(x)=\min \{\gamma: \operatorname{Pr}[f(x, Y) \leq \gamma] \geq \alpha\} .
$$

An alternative formulation for CVaR has the form:

$$
\operatorname{CVa}_{\alpha}(x)=\operatorname{VaR}_{\alpha}(x)+\frac{1}{1-\alpha} \int_{R^{n}} \max \left\{0,\left(f(x, y)-\operatorname{Va}_{\alpha}(x)\right)\right\} p(y) d y
$$

The choice for the probability level $\alpha$ is typically 0.9 or 0.95 . Choosing $\alpha$ implies that the value-at-risk $\operatorname{VaR}_{\alpha}(x)$ for portfolio $x$ is the portfolio return such that the probability of portfolio returns falling below this level is $(1-\alpha)$. Given $\operatorname{VaR}_{\alpha}(x)$ for a portfolio $x$, the conditional value-at-risk of the portfolio is the expected loss of portfolio returns above the value-at-risk return.

Note Value-at-risk is a positive value for losses so that the probability level $\alpha$ indicates the probability that portfolio returns are below the negative of the value-at-risk.

To describe the probability distribution of returns, the PortfolioCVaR object takes a finite sample of return scenarios $y_{s}$, with $s=1, \ldots, S$. Each $y_{s}$ is an $n$ vector that contains the returns for each of the $n$ assets under the scenario $s$. This sample of $S$ scenarios is stored as a scenario matrix of size $S$-by-n. Then, the risk proxy for CVaR portfolio optimization, for a given portfolio $x \in X$ and $\alpha \in(0,1)$, is computed as

$$
\sum(x)=V a R_{\alpha}(x)+\frac{1}{(1-\alpha) S} \sum_{s=1}^{S} \max \left\{0,-y_{s}^{T} x-V a R_{\alpha}(x)\right\}
$$

The value-at-risk, $\operatorname{Va}_{\alpha}(x)$, is estimated whenever the CVaR is estimated. The loss function is $f\left(x, y_{s}\right)=-y_{s}^{T} x$, which is the portfolio loss under scenario $s$.

Under this definition, VaR and CVaR are sample estimators for VaR and CVaR based on the given scenarios. Better scenario samples yield more reliable estimates of VaR and CVaR.

For more information, see Rockafellar and Uryasev [48], [49], and Cornuejols and Tütüncü, [51], at "Portfolio Optimization" on page A-7.

## Mean Absolute-Deviation

The mean-absolute-deviation (MAD) for a portfolio $x \in X$ is defined as

$$
\sum(x)=\frac{1}{S} \sum_{s=1}^{S}\left|\left(y_{s}-m\right)^{T} x\right|
$$

where:
$y_{s}$ are asset returns with scenarios $s=1, \ldots S$ ( $S$ collection of $n$ vectors).
$f(x, y)$ is the loss function for a portfolio $x$ and asset return $y$.
$m$ is the mean of asset returns ( $n$ vector).
such that

$$
m=\frac{1}{S} \sum_{s=1}^{S} y_{s}
$$

For more information, see Konno and Yamazaki [50] at "Portfolio Optimization" on page A7.

## See Also

Portfolio

## Related Examples

- "Creating the Portfolio Object" on page 4-28
- "Working with Portfolio Constraints Using Defaults" on page 4-68
- "Asset Allocation Case Study" on page 4-196
- "Portfolio Optimization Examples" on page 4-167


## More About

- Portfolio
- "Portfolio Object Workflow" on page 4-21
- "Portfolio Set for Optimization Using Portfolio Object" on page 4-10
- "Default Portfolio Problem" on page 4-19


## External Websites

- Using MATLAB to Optimize Portfolios with Financial Toolbox (33 min 24 sec )
- MATLAB for Advanced Portfolio Construction and Stock Selection Models ( 30 min 28 sec )


## Portfolio Set for Optimization Using Portfolio Object

The final element for a complete specification of a portfolio optimization problem is the set of feasible portfolios, which is called a portfolio set. A portfolio set $X \subset R^{n}$ is specified by construction as the intersection of sets formed by a collection of constraints on portfolio weights. A portfolio set necessarily and sufficiently must be a nonempty, closed, and bounded set.

When setting up your portfolio set, ensure that the portfolio set satisfies these conditions. The most basic or "default" portfolio set requires portfolio weights to be nonnegative (using the lower-bound constraint) and to sum to 1 (using the budget constraint). The most general portfolio set handled by the portfolio optimization tools can have any of these constraints:

- Linear inequality constraints
- Linear equality constraints
- Bound constraints
- Budget constraints
- Group constraints
- Group ratio constraints
- Average turnover constraints
- One-way turnover constraints
- Tracking error constraints


## Linear Inequality Constraints

Linear inequality constraints are general linear constraints that model relationships among portfolio weights that satisfy a system of inequalities. Use setInequality to set linear inequality constraints. Linear inequality constraints take the form

$$
A_{I} x \leq b_{I}
$$

where:
$x$ is the portfolio ( $n$ vector).
$A_{I}$ is the linear inequality constraint matrix ( $n_{I}$-by- $n$ matrix).
$b_{I}$ is the linear inequality constraint vector ( $n_{I}$ vector).
$n$ is the number of assets in the universe and $n_{I}$ is the number of constraints.
Portfolio object properties to specify linear inequality constraints are:

- AInequality for $A_{I}$
- bInequality for $b_{I}$
- NumAssets for $n$

The default is to ignore these constraints.

## Linear Equality Constraints

Linear equality constraints are general linear constraints that model relationships among portfolio weights that satisfy a system of equalities. Use setEquality to set linear equality constraints. Linear equality constraints take the form

$$
A_{E} x=b_{E}
$$

where:
$x$ is the portfolio ( $n$ vector).
$A_{E}$ is the linear equality constraint matrix ( $n_{E}$-by- $n$ matrix).
$b_{E}$ is the linear equality constraint vector ( $n_{E}$ vector).
$n$ is the number of assets in the universe and $n_{E}$ is the number of constraints.
Portfolio object properties to specify linear equality constraints are:

- AEquality for $A_{E}$
- bEquality for $b_{E}$
- NumAssets for $n$

The default is to ignore these constraints.

## 'Simple' Bound Constraints

'Simple' Bound constraints are specialized linear constraints that confine portfolio weights to fall either above or below specific bounds. Use setBounds to specify bound constraints with a 'Simple' BoundType. Since every portfolio set must be bounded, it is often a good practice, albeit not necessary, to set explicit bounds for the portfolio problem. To obtain explicit 'Simple' bounds for a given portfolio set, use the estimateBounds function. Bound constraints take the form

$$
l_{B} \leq x \leq u_{B}
$$

where:
$x$ is the portfolio ( $n$ vector).
$l_{B}$ is the lower-bound constraint ( $n$ vector).
$u_{B}$ is the upper-bound constraint ( $n$ vector).
$n$ is the number of assets in the universe.
Portfolio object properties to specify bound constraints are:

- LowerBound for $l_{B}$
- UpperBound for $u_{B}$
- NumAssets for $n$

The default is to ignore these constraints.
The default portfolio optimization problem (see "Default Portfolio Problem" on page 4-
19) has $l_{B}=0$ with $u_{B}$ set implicitly through a budget constraint.

## Budget Constraints

Budget constraints are specialized linear constraints that confine the sum of portfolio weights to fall either above or below specific bounds. Use setBudget to set budget constraints. The constraints take the form

$$
l_{S} \leq 1^{T} x \leq u_{S}
$$

where:
$x$ is the portfolio ( $n$ vector).
1 is the vector of ones ( $n$ vector).
$l_{S}$ is the lower-bound budget constraint (scalar).
$u_{S}$ is the upper-bound budget constraint (scalar).
$n$ is the number of assets in the universe.
Portfolio object properties to specify budget constraints are:

- LowerBudget for $l_{S}$
- UpperBudget for $u_{S}$
- NumAssets for $n$

The default is to ignore this constraint.
The default portfolio optimization problem (see "Default Portfolio Problem" on page 419) has $l_{S}=u_{S}=1$, which means that the portfolio weights sum to 1 . If the portfolio optimization problem includes possible movements in and out of cash, the budget constraint specifies how far portfolios can go into cash. For example, if $l_{S}=0$ and $u_{S}=1$, then the portfolio can have 0-100\% invested in cash. If cash is to be a portfolio choice, set RiskFreeRate ( $r_{0}$ ) to a suitable value (see "Return Proxy" on page 4-4 and "Working with a Riskless Asset" on page 4-61).

## Group Constraints

Group constraints are specialized linear constraints that enforce "membership" among groups of assets. Use setGroups to set group constraints. The constraints take the form

$$
l_{G} \leq G x \leq u_{G}
$$

where:
$x$ is the portfolio ( $n$ vector).
$l_{G}$ is the lower-bound group constraint ( $n_{G}$ vector).
$u_{G}$ is the upper-bound group constraint ( $n_{G}$ vector).
$G$ is the matrix of group membership indexes ( $n_{G}$-by-n matrix).
Each row of $G$ identifies which assets belong to a group associated with that row. Each row contains either 0 s or 1 s with 1 indicating that an asset is part of the group or 0 indicating that the asset is not part of the group.

Portfolio object properties to specify group constraints are:

- GroupMatrix for $G$
- LowerGroup for $l_{G}$
- UpperGroup for $u_{G}$
- NumAssets for $n$

The default is to ignore these constraints.

## Group Ratio Constraints

Group ratio constraints are specialized linear constraints that enforce relationships among groups of assets. Use setGroupRatio to set group ratio constraints. The constraints take the form

$$
l_{R i}\left(G_{B} x\right)_{i} \leq\left(G_{A} x\right)_{i} \leq u_{R i}\left(G_{B} x\right)_{i}
$$

for $i=1, \ldots, n_{R}$ where:
$x$ is the portfolio ( $n$ vector).
$l_{R}$ is the vector of lower-bound group ratio constraints ( $n_{R}$ vector).
$u_{R}$ is the vector matrix of upper-bound group ratio constraints ( $n_{R}$ vector).
$G_{A}$ is the matrix of base group membership indexes ( $n_{R}$-by- $n$ matrix).
$G_{B}$ is the matrix of comparison group membership indexes ( $n_{R}$-by- $n$ matrix).
$n$ is the number of assets in the universe and $n_{R}$ is the number of constraints.
Each row of $G_{A}$ and $G_{B}$ identifies which assets belong to a base and comparison group associated with that row.

Each row contains either 0 s or 1 s with 1 indicating that an asset is part of the group or 0 indicating that the asset is not part of the group.

Portfolio object properties to specify group ratio constraints are:

- GroupA for $G_{A}$
- GroupB for $G_{B}$
- LowerRatio for $l_{R}$
- UpperRatio for $u_{R}$
- NumAssets for $n$

The default is to ignore these constraints.

## Average Turnover Constraints

Turnover constraint is a linear absolute value constraint that ensures estimated optimal portfolios differ from an initial portfolio by no more than a specified amount. Although portfolio turnover is defined in many ways, the turnover constraints implemented in Financial Toolbox compute portfolio turnover as the average of purchases and sales. Use setTurnover to set average turnover constraints. Average turnover constraints take the form

$$
\frac{1}{2} 1^{T}\left|x-x_{0}\right| \leq \tau
$$

where:
$x$ is the portfolio ( $n$ vector).
1 is the vector of ones ( $n$ vector).
$x_{0}$ is the initial portfolio ( $n$ vector).
$\tau$ is the upper bound for turnover (scalar).
$n$ is the number of assets in the universe.
Portfolio object properties to specify the average turnover constraint are:

- Turnover for $\tau$
- InitPort for $x_{0}$
- NumAssets for $n$

The default is to ignore this constraint.

## One-Way Turnover Constraints

One-way turnover constraints ensure that estimated optimal portfolios differ from an initial portfolio by no more than specified amounts according to whether the differences are purchases or sales. Use setOneWayTurnover to set one-way turnover constraints. The constraints take the forms

$$
\begin{aligned}
& 1^{T} \max \left\{0, x-x_{0}\right\} \leq \tau_{B} \\
& 1^{T} \max \left\{0, x_{0}-x\right\} \leq \tau_{S}
\end{aligned}
$$

where:
$x$ is the portfolio ( $n$ vector)
1 is the vector of ones ( $n$ vector).
$x_{0}$ is the Initial portfolio ( $n$ vector).
$\tau_{B}$ is the upper bound for turnover constraint on purchases (scalar).
$\tau_{S}$ is the upper bound for turnover constraint on sales (scalar).
To specify one-way turnover constraints, use the following properties in the Portfolio or PortfolioCVaR object:

- BuyTurnover for $\tau_{B}$
- SellTurnover for $\tau_{S}$
- InitPort for $x_{0}$

The default is to ignore this constraint.

Note The average turnover constraint (see "Working with Average Turnover Constraints Using Portfolio Object" on page 4-98) with $\tau$ is not a combination of the one-way turnover constraints with $\tau=\tau_{B}=\tau_{S}$.

## Tracking Error Constraints

Tracking error constraint, within a portfolio optimization framework, is an additional constraint to specify the set of feasible portfolios known as a portfolio set. Use setTrackingError to set tracking error constraints. The tracking-error constraint has the form

$$
\left(x-x_{T}\right)^{T} C\left(x-x_{T}\right) \leq \tau_{T}{ }^{2}
$$

where:
$x$ is the portfolio ( $n$ vector).
$\chi_{T}$ is the tracking portfolio against which risk is to be measured ( $n$ vector).
$\tau_{T}$ is the upper bound for tracking error (scalar).
$n$ is the number of assets in the universe.
Portfolio object properties to specify the average turnover constraint are:

- TrackingPort for $x_{T}$
- TrackingError for $\tau_{T}$

The default is to ignore this constraint.

Note The tracking error constraints can be used with any of the other supported constraints in the Portfolio object without restrictions. However, since the portfolio set necessarily and sufficiently must be a non-empty compact set, the application of a tracking error constraint may result in an empty portfolio set. Use estimateBounds to confirm that the portfolio set is non-empty and compact.

## See Also

## Related Examples

- "Creating the Portfolio Object" on page 4-28
- "Working with Portfolio Constraints Using Defaults" on page 4-68
- "Asset Allocation Case Study" on page 4-196
- "Portfolio Optimization Examples" on page 4-167


## More About

- Portfolio
- "Portfolio Object Workflow" on page 4-21
- "Default Portfolio Problem" on page 4-19


## External Websites

- Using MATLAB to Optimize Portfolios with Financial Toolbox (33 min 24 sec )
- MATLAB for Advanced Portfolio Construction and Stock Selection Models (30 min 28 sec)


## Default Portfolio Problem

The default portfolio optimization problem has a risk and return proxy associated with a given problem, and a portfolio set that specifies portfolio weights to be nonnegative and to sum to 1 . The lower bound combined with the budget constraint is sufficient to ensure that the portfolio set is nonempty, closed, and bounded. The default portfolio optimization problem characterizes a long-only investor who is fully invested in a collection of assets.

- For mean-variance portfolio optimization, it is sufficient to set up the default problem. After setting up the problem, data in the form of a mean and covariance of asset returns are then used to solve portfolio optimization problems.
- For conditional value-at-risk portfolio optimization, the default problem requires the additional specification of a probability level that must be set explicitly. Generally, "typical" values for this level are 0.90 or 0.95 . After setting up the problem, data in the form of scenarios of asset returns are then used to solve portfolio optimization problems.
- For MAD portfolio optimization, it is sufficient to set up the default problem. After setting up the problem, data in the form of scenarios of asset returns are then used to solve portfolio optimization problems.


## See Also

Portfolio|PortfolioCVaR|PortfolioMAD

## Related Examples

- "Creating the Portfolio Object" on page 4-28
- "Working with Portfolio Constraints Using Defaults" on page 4-68
- "Asset Allocation Case Study" on page 4-196
- "Portfolio Optimization Examples" on page 4-167


## More About

- Portfolio
- "Portfolio Set for Optimization Using Portfolio Object" on page 4-10
- "Portfolio Object Workflow" on page 4-21


## External Websites

- Using MATLAB to Optimize Portfolios with Financial Toolbox (33 min 24 sec )
- MATLAB for Advanced Portfolio Construction and Stock Selection Models (30 min 28 sec)


## Portfolio Object Workflow

The Portfolio object workflow for creating and modeling a mean-variance portfolio is:
1 Create a Portfolio.
Create a Portfolio object for mean-variance portfolio optimization. For more information, see "Creating the Portfolio Object" on page 4-28.
2 Estimate the mean and covariance for returns.
Evaluate the mean and covariance for portfolio asset returns, including assets with missing data and financial time series data. For more information, see "Asset Returns and Moments of Asset Returns Using Portfolio Object" on page 4-48.
3 Specify the Portfolio Constraints.
Define the constraints for portfolio assets such as linear equality and inequality, bound, budget, group, group ratio, turnover, tracking error, 'Conditional' BoundType, and MinNumAssets, MaxNumAssets constraints. For more information, see "Working with Portfolio Constraints Using Defaults" on page 4-68 and "Working with 'Conditional' BoundType, MinNumAssets, and MaxNumAssets Constraints Using Portfolio Object" on page 4-94.
4 Validate the Portfolio.
Identify errors for the portfolio specification. For more information, see "Validate the Portfolio Problem for Portfolio Object" on page 4-110.
5 Estimate the efficient portfolios and frontiers.
Analyze the efficient portfolios and efficient frontiers for a portfolio. For more information, see "Estimate Efficient Portfolios for Entire Efficient Frontier for Portfolio Object" on page 4-115 and "Estimate Efficient Frontiers for Portfolio Object" on page 4-140.

6 Postprocess the results.
Use the efficient portfolios and efficient frontiers results to set up trades. For more information, see "Postprocessing Results to Set Up Tradable Portfolios" on page 4149.

For an example of this workflow, see "Asset Allocation Case Study" on page 4-196 and "Portfolio Optimization Examples" on page 4-167.

## See Also

## Related Examples

- "Asset Allocation Case Study" on page 4-196
- "Portfolio Optimization Examples" on page 4-167
- "Portfolio Optimization with Semicontinuous and Cardinality Constraints"


## More About

- "Portfolio Optimization Theory" on page 4-3


## External Websites

- Using MATLAB to Optimize Portfolios with Financial Toolbox (33 min 24 sec )
- MATLAB for Advanced Portfolio Construction and Stock Selection Models ( 30 min 28 sec )


## Portfolio Object

```
In this section...
"Portfolio Object Properties and Functions" on page 4-23
"Working with Portfolio Objects" on page 4-23
"Setting and Getting Properties" on page 4-24
"Displaying Portfolio Objects" on page 4-25
"Saving and Loading Portfolio Objects" on page 4-25
"Estimating Efficient Portfolios and Frontiers" on page 4-25
"Arrays of Portfolio Objects" on page 4-25
"Subclassing Portfolio Objects" on page 4-26
"Conventions for Representation of Data" on page 4-26
```


## Portfolio Object Properties and Functions

The Portfolio object implements mean-variance portfolio optimization. Every property and function of the Portfolio object is public, although some properties and functions are hidden. See Portfolio for the properties and functions of the Portfolio object. The Portfolio object is a value object where every instance of the object is a distinct version of the object. Since the Portfolio object is also a MATLAB object, it inherits the default functions associated with MATLAB objects.

## Working with Portfolio Objects

The Portfolio object and its functions are an interface for mean-variance portfolio optimization. So, almost everything you do with the Portfolio object can be done using the associated functions. The basic workflow is:

1 Design your portfolio problem.
2 Use Portfolio to create the Portfolio object or use the various set functions to set up your portfolio problem.
3 Use estimate functions to solve your portfolio problem.
In addition, functions are available to help you view intermediate results and to diagnose your computations. Since MATLAB features are part of a Portfolio object, you can save
and load objects from your workspace and create and manipulate arrays of objects. After settling on a problem, which, in the case of mean-variance portfolio optimization, means that you have either data or moments for asset returns and a collection of constraints on your portfolios, use Portfolio to set the properties for the Portfolio object. Portfolio lets you create an object from scratch or update an existing object. Since the Portfolio object is a value object, it is easy to create a basic object, then use functions to build upon the basic object to create new versions of the basic object. This is useful to compare a basic problem with alternatives derived from the basic problem. For details, see "Creating the Portfolio Object" on page 4-28.

## Setting and Getting Properties

You can set properties of a Portfolio object using either Portfolio or various set functions.

Note Although you can also set properties directly, it is not recommended since errorchecking is not performed when you set a property directly.

The Portfolio object supports setting properties with name-value pair arguments such that each argument name is a property and each value is the value to assign to that property. For example, to set the AssetMean and AssetCovar properties in an existing Portfolio object $p$ with the values $m$ and $C$, use the syntax:
$p=$ Portfolio(p, 'AssetMean', m, 'AssetCovar', C);
In addition to Portfolio, which lets you set individual properties one at a time, groups of properties are set in a Portfolio object with various "set" and "add" functions. For example, to set up an average turnover constraint, use the setTurnover function to specify the bound on portfolio average turnover and the initial portfolio. To get individual properties from a Portfolio object, obtain properties directly or use an assortment of "get" functions that obtain groups of properties from a Portfolio object. The Portfolio object and the set functions have several useful features:

- Portfolio and the set functions try to determine the dimensions of your problem with either explicit or implicit inputs.
- Portfolio and the set functions try to resolve ambiguities with default choices.
- Portfolio and the set functions perform scalar expansion on arrays when possible.
- The associated Portfolio object functions try to diagnose and warn about problems.


## Displaying Portfolio Objects

The Portfolio object uses the default display functions provided by MATLAB, where display and disp display a Portfolio object and its properties with or without the object variable name.

## Saving and Loading Portfolio Objects

Save and load Portfolio objects using the MATLAB save and load commands.

## Estimating Efficient Portfolios and Frontiers

Estimating efficient portfolios and efficient frontiers is the primary purpose of the portfolio optimization tools. A collection of "estimate" and "plot" functions provide ways to explore the efficient frontier. The "estimate" functions obtain either efficient portfolios or risk and return proxies to form efficient frontiers. At the portfolio level, a collection of functions estimates efficient portfolios on the efficient frontier with functions to obtain efficient portfolios:

- At the endpoints of the efficient frontier
- That attains targeted values for return proxies
- That attains targeted values for risk proxies
- Along the entire efficient frontier

These functions also provide purchases and sales needed to shift from an initial or current portfolio to each efficient portfolio. At the efficient frontier level, a collection of functions plot the efficient frontier and estimate either risk or return proxies for efficient portfolios on the efficient frontier. You can use the resultant efficient portfolios or risk and return proxies in subsequent analyses.

## Arrays of Portfolio Objects

Although all functions associated with a Portfolio object are designed to work on a scalar Portfolio object, the array capabilities of MATLAB enable you to set up and work with arrays of Portfolio objects. The easiest way to do this is with the repmat function. For example, to create a 3-by-2 array of Portfolio objects:

```
p = repmat(Portfolio, 3, 2);
disp(p)
```

After setting up an array of Portfolio objects, you can work on individual Portfolio objects in the array by indexing. For example:

```
p(i,j) = Portfolio(p(i,j), ... );
```

This example calls Portfolio for the (i,j) element of a matrix of Portfolio objects in the variable p .

If you set up an array of Portfolio objects, you can access properties of a particular Portfolio object in the array by indexing so that you can set the lower and upper bounds lb and $u b$ for the ( $\mathrm{i}, \mathrm{j}, \mathrm{k}$ ) element of a 3-D array of Portfolio objects with
$p(i, j, k)=\operatorname{setBounds}(p(i, j, k), l b, u b)$;
and, once set, you can access these bounds with
[lb, ub] = getBounds(p(i,j,k));
Portfolio object functions work on only one Portfolio object at a time.

## Subclassing Portfolio Objects

You can subclass the Portfolio object to override existing functions or to add new properties or functions. To do so, create a derived class from the Portfolio class. This gives you all the properties and functions of the Portfolio class along with any new features that you choose to add to your subclassed object. The Portfolio class is derived from an abstract class called AbstractPortfolio. Because of this, you can also create a derived class from AbstractPortfolio that implements an entirely different form of portfolio optimization using properties and functions of the AbstractPortfolio class.

## Conventions for Representation of Data

The portfolio optimization tools follow these conventions regarding the representation of different quantities associated with portfolio optimization:

- Asset returns or prices are in matrix form with samples for a given asset going down the rows and assets going across the columns. In the case of prices, the earliest dates must be at the top of the matrix, with increasing dates going down.
- The mean and covariance of asset returns are stored in a vector and a matrix and the tools have no requirement that the mean must be either a column or row vector.
- Portfolios are in vector or matrix form with weights for a given portfolio going down the rows and distinct portfolios going across the columns.
- Constraints on portfolios are formed in such a way that a portfolio is a column vector.
- Portfolio risks and returns are either scalars or column vectors (for multiple portfolio risks and returns).


## See Also

Portfolio

## Related Examples

- "Creating the Portfolio Object" on page 4-28
- "Working with Portfolio Constraints Using Defaults" on page 4-68
- "Asset Allocation Case Study" on page 4-196
- "Portfolio Optimization Examples" on page 4-167


## More About

- "Portfolio Optimization Theory" on page 4-3
- "Portfolio Object Workflow" on page 4-21


## External Websites

- Using MATLAB to Optimize Portfolios with Financial Toolbox (33 min 24 sec )


## Creating the Portfolio Object

## In this section...

"Syntax" on page 4-28
"Portfolio Problem Sufficiency" on page 4-29
"Portfolio Function Examples" on page 4-29
To create a fully specified mean-variance portfolio optimization problem, instantiate the Portfolio object using Portfolio. For information on the workflow when using Portfolio objects, see "Portfolio Object Workflow" on page 4-21.

## Syntax

Use Portfolio to create an instance of an object of the Portfolio class. You can use Portfolio in several ways. To set up a portfolio optimization problem in a Portfolio object, the simplest syntax is:

```
p = Portfolio;
```

This syntax creates a Portfolio object, p, such that all object properties are empty.
The Portfolio object also accepts collections of argument name-value pair arguments for properties and their values. The Portfolio object accepts inputs for public properties with the general syntax:

```
p = Portfolio('property1', value1, 'property2', value2, ... );
```

If a Portfolio object already exists, the syntax permits the first (and only the first argument) of Portfolio to be an existing object with subsequent argument name-value pair arguments for properties to be added or modified. For example, given an existing Portfolio object in $p$, the general syntax is:

```
p = Portfolio(p, 'property1', value1, 'property2', value2, ... );
```

Input argument names are not case-sensitive, but must be completely specified. In addition, several properties can be specified with alternative argument names (see "Shortcuts for Property Names" on page 4-33). The Portfolio object detects problem dimensions from the inputs and, once set, subsequent inputs can undergo various scalar or matrix expansion operations that simplify the overall process to formulate a problem.

In addition, a Portfolio object is a value object so that, given portfolio $p$, the following code creates two objects, p and q , that are distinct:
$\mathrm{q}=\operatorname{Portfolio(p,} . .$.

## Portfolio Problem Sufficiency

A mean-variance portfolio optimization is completely specified with the Portfolio object if these two conditions are met:

- The moments of asset returns must be specified such that the property AssetMean contains a valid finite mean vector of asset returns and the property AssetCovar contains a valid symmetric positive-semidefinite matrix for the covariance of asset returns.

The first condition is satisfied by setting the properties associated with the moments of asset returns.

- The set of feasible portfolios must be a nonempty compact set, where a compact set is closed and bounded.

The second condition is satisfied by an extensive collection of properties that define different types of constraints to form a set of feasible portfolios. Since such sets must be bounded, either explicit or implicit constraints can be imposed, and several functions, such as estimateBounds, provide ways to ensure that your problem is properly formulated.

Although the general sufficiency conditions for mean-variance portfolio optimization go beyond these two conditions, the Portfolio object implemented in Financial Toolbox implicitly handles all these additional conditions. For more information on the Markowitz model for mean-variance portfolio optimization, see "Portfolio Optimization" on page A7.

## Portfolio Function Examples

If you create a Portfolio object, $p$, with no input arguments, you can display it using disp:

```
p = Portfolio;
disp(p); Portfolio
```

```
            BuyCost: []
            SellCost: []
    RiskFreeRate: []
        AssetMean: []
        AssetCovar: []
TrackingError: []
    TrackingPort: []
            Turnover: []
        BuyTurnover: []
        SellTurnover: []
            Name: []
        NumAssets: []
        AssetList: []
            InitPort: []
        AInequality: []
        bInequality: []
            AEquality: []
            bEquality: []
        LowerBound: []
        UpperBound: []
    LowerBudget: []
    UpperBudget: []
    GroupMatrix: []
    LowerGroup: []
    UpperGroup: []
        GroupA: []
            GroupB: []
        LowerRatio: []
    UpperRatio: []
        BoundType: []
    MinNumAssets: []
    MaxNumAssets: []
```

The approaches listed provide a way to set up a portfolio optimization problem with the Portfolio object. The set functions offer additional ways to set and modify collections of properties in the Portfolio object.

## Using the Portfolio Function for a Single-Step Setup

You can use the Portfolio object to directly set up a "standard" portfolio optimization problem, given a mean and covariance of asset returns in the variables $m$ and $C$ :

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
```

```
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
p = Portfolio('assetmean', m, 'assetcovar', C, ...
'lowerbudget', 1, 'upperbudget', 1, 'lowerbound', 0);
```

The LowerBound property value undergoes scalar expansion since AssetMean and AssetCovar provide the dimensions of the problem.

You can use dot notation with the function plotFrontier.

```
p.plotFrontier;
```



## Using the Portfolio Function with a Sequence of Steps

An alternative way to accomplish the same task of setting up a "standard" portfolio optimization problem, given a mean and covariance of asset returns in the variables $m$ and C (which also illustrates that argument names are not case-sensitive):

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
p = Portfolio;
p = Portfolio(p, 'assetmean', m, 'assetcovar', C);
p = Portfolio(p, 'lowerbudget', 1, 'upperbudget', 1);
p = Portfolio(p, 'lowerbound', 0);
plotFrontier(p);
```



This way works because the calls to Portfolio are in this particular order. In this case, the call to initialize AssetMean and AssetCovar provides the dimensions for the problem. If you were to do this step last, you would have to explicitly dimension the LowerBound property as follows:

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
```

```
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
p = Portfolio;
p = Portfolio(p, 'LowerBound', zeros(size(m)));
p = Portfolio(p, 'LowerBudget', 1, 'UpperBudget', 1);
p = Portfolio(p, 'AssetMean', m, 'AssetCovar', C);
plotFrontier(p);
```



If you did not specify the size of LowerBound but, instead, input a scalar argument, the Portfolio object assumes that you are defining a single-asset problem and produces an error at the call to set asset moments with four assets.

## Shortcuts for Property Names

The Portfolio object has shorter argument names that replace longer argument names associated with specific properties of the Portfolio object. For example, rather than enter 'assetcovar', the Portfolio object accepts the case-insensitive name 'covar' to set
the AssetCovar property in a Portfolio object. Every shorter argument name corresponds with a single property in the Portfolio object. The one exception is the alternative argument name 'budget', which signifies both the LowerBudget and UpperBudget properties. When 'budget' is used, then the LowerBudget and UpperBudget properties are set to the same value to form an equality budget constraint.

Shortcuts for Property Names

| Shortcut Argument Name | Equivalent Argument / Property Name |
| :--- | :--- |
| ae | AEquality |
| ai | AInequality |
| covar | AssetCovar |
| assetnames or assets | AssetList |
| mean | AssetMean |
| be | bEquality |
| bi | bInequality |
| group | GroupMatrix |
| lb | LowerBound |
| n or num | NumAssets |
| rfr | RiskFreeRate |
| ub | UpperBound |
| budget | UpperBudget and LowerBudget |

For example, this call Portfolio uses these shortcuts for properties and is equivalent to the previous examples:

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
p = Portfolio('mean', m, 'covar', C, 'budget', 1, 'lb', 0);
plotFrontier(p);
```


## Direct Setting of Portfolio Object Properties

Although not recommended, you can set properties directly, however no error-checking is done on your inputs:

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
p = Portfolio;
p.NumAssets = numel(m);
p.AssetMean = m;
p.AssetCovar = C;
p.LowerBudget = 1;
p.UpperBudget = 1;
p.LowerBound = zeros(size(m));
plotFrontier(p);
```


## See Also

Portfolio|estimateBounds

## Related Examples

- "Common Operations on the Portfolio Object" on page 4-37
- "Working with Portfolio Constraints Using Defaults" on page 4-68
- "Asset Allocation Case Study" on page 4-196
- "Portfolio Optimization Examples" on page 4-167


## More About

- "Portfolio Object" on page 4-23
- "Portfolio Optimization Theory" on page 4-3
- "Portfolio Object Workflow" on page 4-21


## External Websites

- Using MATLAB to Optimize Portfolios with Financial Toolbox (33 min 24 sec )
- MATLAB for Advanced Portfolio Construction and Stock Selection Models (30 min 28 sec)


## Common Operations on the Portfolio Object

In this section...<br>"Naming a Portfolio Object" on page 4-37<br>"Configuring the Assets in the Asset Universe" on page 4-37<br>"Setting Up a List of Asset Identifiers" on page 4-38<br>"Truncating and Padding Asset Lists" on page 4-39

## Naming a Portfolio Object

To name a Portfolio object, use the Name property. Name is informational and has no effect on any portfolio calculations. If the Name property is nonempty, Name is the title for the efficient frontier plot generated by plotFrontier. For example, if you set up an asset allocation fund, you could name the Portfolio object Asset Allocation Fund:

```
p = Portfolio('Name','Asset Allocation Fund');
disp(p.Name);
```

Asset Allocation Fund

## Configuring the Assets in the Asset Universe

The fundamental quantity in the Portfolio object is the number of assets in the asset universe. This quantity is maintained in the NumAssets property. Although you can set this property directly, it is derived from other properties such as the mean of asset returns and the initial portfolio. In some instances, the number of assets may need to be set directly. This example shows how to set up a Portfolio object that has four assets:

```
p = Portfolio('NumAssets', 4);
disp(p.NumAssets);
```

4

After setting the NumAssets property, you cannot modify it (unless no other properties are set that depend on NumAssets). The only way to change the number of assets in an existing Portfolio object with a known number of assets is to create a new Portfolio object.

## Setting Up a List of Asset Identifiers

When working with portfolios, you must specify a universe of assets. Although you can perform a complete analysis without naming the assets in your universe, it is helpful to have an identifier associated with each asset as you create and work with portfolios. You can create a list of asset identifiers as a cell vector of character vectors in the property AssetList. You can set up the list using the next two functions.

## Setting Up Asset Lists Using the Portfolio Function

Suppose that you have a Portfolio object, p , with assets with symbols 'AA' ', 'BA', 'CAT', 'DD', and 'ETR'. You can create a list of these asset symbols in the object using the Portfolio object:

```
p = Portfolio('assetlist', { 'AA', 'BA', 'CAT', 'DD', 'ETR' });
disp(p.AssetList);
'AA' 'BA' 'CAT' 'DD' 'ETR'
```

Notice that the property AssetList is maintained as a cell array that contains character vectors, and that it is necessary to pass a cell array into the Portfolio object to set AssetList. In addition, notice that the property NumAssets is set to 5 based on the number of symbols used to create the asset list:

```
disp(p.NumAssets);
```

5

## Setting Up Asset Lists Using the setAssetList Function

You can also specify a list of assets using the setAssetList function. Given the list of asset symbols 'AA', 'BA', 'CAT', 'DD', and 'ETR', you can use setAssetList with:

```
p = Portfolio;
p = setAssetList(p, { 'AA', 'BA', 'CAT', 'DD', 'ETR' });
disp(p.AssetList);
    'AA' 'BA' 'CAT' 'DD' 'ETR'
```

setAssetList also enables you to enter symbols directly as a comma-separated list without creating a cell array of character vectors. For example, given the list of assets symbols 'AA', 'BA', 'CAT', 'DD', and 'ETR', use setAssetList:

```
p = Portfolio;
p = setAssetList(p,'AA', 'BA', 'CAT', 'DD', 'ETR');
disp(p.AssetList);
'AA' 'BA' 'CAT' 'DD' 'ETR'
```

setAssetList has many additional features to create lists of asset identifiers. If you use setAssetList with just a Portfolio object, it creates a default asset list according to the name specified in the hidden public property defaultforAssetList (which is 'Asset' by default). The number of asset names created depends on the number of assets in the property NumAssets. If NumAssets is not set, then NumAssets is assumed to be 1.

For example, if a Portfolio object p is created with NumAssets $=5$, then this code fragment shows the default naming behavior:

```
p = Portfolio('numassets',5);
p = setAssetList(p);
disp(p.AssetList);
'Asset1' 'Asset2' 'Asset3' 'Asset4' 'Asset5'
```

Suppose that your assets are, for example, ETFs and you change the hidden property defaultforAssetList to 'ETF', you can then create a default list for ETFs:

```
p = Portfolio('numassets',5);
p.defaultforAssetList = 'ETF';
p = setAssetList(p);
disp(p.AssetList);
'ETF1' 'ETF2' 'ETF3' 'ETF4' 'ETF5'
```


## Truncating and Padding Asset Lists

If the NumAssets property is already set and you pass in too many or too few identifiers, the Portfolio object, and the setAssetList function truncate or pad the list with numbered default asset names that use the name specified in the hidden public property defaultforAssetList. If the list is truncated or padded, a warning message indicates the discrepancy. For example, assume that you have a Portfolio object with five ETFs and you only know the first three CUSIPs '921937835', '922908769', and '922042775'. Use this syntax to create an asset list that pads the remaining asset identifiers with numbered 'UnknownCUSIP ' placeholders:

```
p = Portfolio('numassets',5);
p.defaultforAssetList = 'UnknownCUSIP';
```

```
p = setAssetList(p,'921937835', '922908769', '922042775');
disp(p.AssetList);
Warning: Input list of assets has 2 too few identifiers. Padding with numbered assets.
> In Portfolio.setAssetList at 121
    '921937835' '922908769' '922042775' 'UnknownCUSIP4' 'UnknownCUSIP5'
```

Alternatively, suppose that you have too many identifiers and need only the first four assets. This example illustrates truncation of the asset list using the Portfolio object:

```
p = Portfolio('numassets',4);
p = Portfolio(p, 'assetlist', { 'AGG', 'EEM', 'MDY', 'SPY', 'VEU' });
disp(p.AssetList);
Warning: AssetList has 1 too many identifiers. Using first 4 assets.
> In Portfolio.checkarguments at 434
    In Portfolio.Portfolio>Portfolio.Portfolio at 171
        'AGG' 'EEM' 'MDY' 'SPY'
```

The hidden public property uppercaseAssetList is a Boolean flag to specify whether to convert asset names to uppercase letters. The default value for uppercaseAssetList is false. This example shows how to use the uppercaseAssetList flag to force identifiers to be uppercase letters:

```
p = Portfolio;
p.uppercaseAssetList = true;
p = setAssetList(p,{ 'aa', 'ba', 'cat', 'dd', 'etr' });
disp(p.AssetList);
'AA' 'BA' 'CAT' 'DD' 'ETR'
```


## See Also

Portfolio| checkFeasibility|estimateBounds|setAssetList| setInitPort |setTrackingPort

## Related Examples

- "Setting Up an Initial or Current Portfolio" on page 4-42
- "Working with Portfolio Constraints Using Defaults" on page 4-68
- "Asset Returns and Moments of Asset Returns Using Portfolio Object" on page 4-48
- "Validate the Portfolio Problem for Portfolio Object" on page 4-110
- "Asset Allocation Case Study" on page 4-196
- "Portfolio Optimization Examples" on page 4-167


## More About

- "Portfolio Object" on page 4-23
- "Portfolio Optimization Theory" on page 4-3
- "Portfolio Object Workflow" on page 4-21


## External Websites

- Using MATLAB to Optimize Portfolios with Financial Toolbox (33 min 24 sec )
- MATLAB for Advanced Portfolio Construction and Stock Selection Models (30 min 28 sec)


## Setting Up an Initial or Current Portfolio

In many applications, creating a new optimal portfolio requires comparing the new portfolio with an initial or current portfolio to form lists of purchases and sales. The Portfolio object property InitPort lets you identify an initial or current portfolio. The initial portfolio also plays an essential role if you have either transaction costs or turnover constraints. The initial portfolio need not be feasible within the constraints of the problem. This can happen if the weights in a portfolio have shifted such that some constraints become violated. To check if your initial portfolio is feasible, use the checkFeasibility function described in "Validating Portfolios" on page 4-112. Suppose that you have an initial portfolio in $\times 0$, then use the Portfolio object to set up an initial portfolio:

```
x0 = [ 0.3; 0.2; 0.2; 0.0 ];
p = Portfolio('InitPort', x0);
disp(p.InitPort);
    0.3000
    0.2000
    0.2000
    0
```

As with all array properties, you can set InitPort with scalar expansion. This is helpful to set up an equally weighted initial portfolio of, for example, 10 assets:

```
p = Portfolio('NumAssets', 10, 'InitPort', 1/10);
disp(p.InitPort);
```

0.1000
0.1000
0.1000
0.1000
0.1000
0.1000
0.1000
0.1000
0.1000
0.1000

To clear an initial portfolio from your Portfolio object, use either the Portfolio object or the setInitPort function with an empty input for the InitPort property. If transaction costs or turnover constraints are set, it is not possible to clear the InitPort property in this way. In this case, to clear InitPort, first clear the dependent properties and then clear theInitPort property.

The InitPort property can also be set with setInitPort which lets you specify the number of assets if you want to use scalar expansion. For example, given an initial portfolio in x0, use setInitPort to set the InitPort property:

```
p = Portfolio;
x0 = [ 0.3; 0.2; 0.2; 0.0 ];
p = setInitPort(p, x0);
disp(p.InitPort);
0.3000
0.2000
0.2000
0
```

To create an equally weighted portfolio of four assets, use setInitPort:

```
p = Portfolio;
p = setInitPort(p, 1/4, 4);
disp(p.InitPort);
    0.2500
    0.2500
    0.2500
    0.2500
```

Portfolio object functions that work with either transaction costs or turnover constraints also depend on the InitPort property. So, the set functions for transaction costs or turnover constraints permit the assignment of a value for the InitPort property as part of their implementation. For details, see "Working with Average Turnover Constraints Using Portfolio Object" on page 4-98, "Working with One-Way Turnover Constraints Using Portfolio Object" on page 4-102, and "Working with Transaction Costs" on page 463 for details. If either transaction costs or turnover constraints are used, then the InitPort property must have a nonempty value. Absent a specific value assigned through the Portfolio object or various set functions, the Portfolio object sets InitPort to 0 and warns if BuyCost, SellCost, or Turnover properties are set. The following example illustrates what happens if an average turnover constraint is specified with an initial portfolio:

```
p = Portfolio('Turnover', 0.3, 'InitPort', [ 0.3; 0.2; 0.2; 0.0 ]);
disp(p.InitPort);
```

0.3000
0.2000
0.2000

0

In contrast, this example shows what happens if an average turnover constraint is specified without an initial portfolio:

```
p = Portfolio('Turnover', 0.3);
disp(p.InitPort);
```

Warning: InitPort and NumAssets are empty and either transaction costs or turnover constraints specified. Will set NumAssets $=1$ and InitPort $=0$.
> In Portfolio.checkarguments at 367
In Portfolio. Portfolio>Portfolio. Portfolio at 171
0

## See Also

Portfolio| checkFeasibility|estimateBounds|setAssetList| setInitPort

## Related Examples

- "Setting Up a Tracking Portfolio" on page 4-45
- "Common Operations on the Portfolio Object" on page 4-37
- "Working with Portfolio Constraints Using Defaults" on page 4-68
- "Asset Returns and Moments of Asset Returns Using Portfolio Object" on page 4-48
- "Validate the Portfolio Problem for Portfolio Object" on page 4-110
- "Asset Allocation Case Study" on page 4-196
- "Portfolio Optimization Examples" on page 4-167


## More About

- "Portfolio Object" on page 4-23
- "Portfolio Optimization Theory" on page 4-3
- "Portfolio Object Workflow" on page 4-21


## External Websites

- Using MATLAB to Optimize Portfolios with Financial Toolbox (33 min 24 sec )


## Setting Up a Tracking Portfolio

Given a benchmark or tracking portfolio, you can ensure that the risk of a portfolio relative to the benchmark portfolio is no greater than a specified amount. The Portfolio object property TrackingPort lets you identify a tracking portfolio. For more information on using a tracking portfolio with tracking error constraints, see "Working with Tracking Error Constraints Using Portfolio Object" on page 4-106.

The tracking error constraints can be used with any of the other supported constraints in the Portfolio object without restrictions. However, since the portfolio set necessarily and sufficiently must be a non-empty compact set, the application of a tracking error constraint can result in an empty portfolio set. Use estimateBounds to confirm that the portfolio set is non-empty and compact.

Suppose that you have an initial portfolio in x0, then use the Portfolio object to set up a tracking portfolio:

```
x0 = [ 0.3; 0.2; 0.2; 0.0 ];
p = Portfolio('TrackingPort', x0);
disp(p.TrackingPort);
0.3000
0.2000
0.2000
0
```

As with all array properties, you can set TrackingPort with scalar expansion. This is helpful to set up an equally weighted tracking portfolio of, for example, 10 assets:

```
p = Portfolio('NumAssets', 10, 'TrackingPort', 1/10);
disp(p.TrackingPort);
```

0.1000
0.1000
0.1000
0.1000
0.1000
0.1000
0.1000
0.1000
0.1000
0.1000

To clear a tracking portfolio from your Portfolio object, use either the Portfolio object or the setTrackingPort function with an empty input for the TrackingPort property. If transaction costs or turnover constraints are set, it is not possible to clear the TrackingPort property in this way. In this case, to clear TrackingPort, first clear the dependent properties and then clear theTrackingPort property.

The TrackingPort property can also be set with setTrackingPort which lets you specify the number of assets if you want to use scalar expansion. For example, given an initial portfolio in $\times 0$, use setTrackingPort to set the TrackingPort property:

```
p = Portfolio;
x0 = [ 0.3; 0.2; 0.2; 0.0 ];
p = setTrackingPort(p, x0);
disp(p.TrackingPort);
```

0.3000
0.2000
0.2000

0
To create an equally weighted portfolio of four assets, use setTrackingPort:

```
p = Portfolio;
p = setTrackingPort(p, 1/4, 4);
disp(p.TrackingPort);
    0.2500
    0.2500
    0.2500
    0.2500
```


## See Also

Portfolio| checkFeasibility|estimateBounds| setAssetList| setInitPort | setTrackingError|setTrackingPort

## Related Examples

- "Setting Up an Initial or Current Portfolio" on page 4-42
- "Common Operations on the Portfolio Object" on page 4-37
- "Working with Portfolio Constraints Using Defaults" on page 4-68
- "Asset Returns and Moments of Asset Returns Using Portfolio Object" on page 4-48
- "Validate the Portfolio Problem for Portfolio Object" on page 4-110
- "Asset Allocation Case Study" on page 4-196
- "Portfolio Optimization Examples" on page 4-167


## More About

- "Portfolio Object" on page 4-23
- "Portfolio Optimization Theory" on page 4-3
- "Portfolio Object Workflow" on page 4-21


## External Websites

- Using MATLAB to Optimize Portfolios with Financial Toolbox (33 min 24 sec )


# Asset Returns and Moments of Asset Returns Using Portfolio Object 

```
In this section...
"Assignment Using the Portfolio Function" on page 4-48
"Assignment Using the setAssetMoments Function" on page 4-50
"Scalar Expansion of Arguments" on page 4-50
"Estimating Asset Moments from Prices or Returns" on page 4-52
"Estimating Asset Moments with Missing Data" on page 4-55
"Estimating Asset Moments from Time Series Data" on page 4-57
```

Since mean-variance portfolio optimization problems require estimates for the mean and covariance of asset returns, the Portfolio object has several ways to set and get the properties AssetMean (for the mean) and AssetCovar (for the covariance). In addition, the return for a riskless asset is kept in the property RiskFreeRate so that all assets in AssetMean and AssetCovar are risky assets. For information on the workflow when using Portfolio objects, see "Portfolio Object Workflow" on page 4-21.

## Assignment Using the Portfolio Function

Suppose that you have a mean and covariance of asset returns in variables $m$ and $C$. The properties for the moments of asset returns are set using the Portfolio object:

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;
p = Portfolio('AssetMean', m, 'AssetCovar', C);
disp(p.NumAssets);
disp(p.AssetMean);
disp(p.AssetCovar);
```

    4
    0.0042
    ```
0.0083
0.0100
0.0150
\begin{tabular}{rrrr}
0.0005 & 0.0003 & 0.0002 & 0 \\
0.0003 & 0.0024 & 0.0017 & 0.0010 \\
0.0002 & 0.0017 & 0.0048 & 0.0028 \\
0 & 0.0010 & 0.0028 & 0.0102
\end{tabular}
```

Notice that the Portfolio object determines the number of assets in NumAssets from the moments. The Portfolio object enables separate initialization of the moments, for example:

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
        0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;
p = Portfolio;
p = Portfolio(p, 'AssetMean', m);
p = Portfolio(p, 'AssetCovar', C);
[assetmean, assetcovar] = p.getAssetMoments
assetmean =
```

    0.0042
    0.0083
    0.0100
    0.0150
    assetcovar =

| 0.0005 | 0.0003 | 0.0002 | 0 |
| ---: | ---: | ---: | ---: |
| 0.0003 | 0.0024 | 0.0017 | 0.0010 |
| 0.0002 | 0.0017 | 0.0048 | 0.0028 |
| 0 | 0.0010 | 0.0028 | 0.0102 |

The getAssetMoments function lets you get the values for AssetMean and AssetCovar properties at the same time.

## Assignment Using the setAssetMoments Function

You can also set asset moment properties using the setAssetMoments function. For example, given the mean and covariance of asset returns in the variables $m$ and $C$, the asset moment properties can be set:

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;
p = Portfolio;
p = setAssetMoments(p, m, C);
[assetmean, assetcovar] = getAssetMoments(p)
assetmean =
    0.0042
    0.0083
    0.0100
    0.0150
assetcovar =
\begin{tabular}{rrrr}
0.0005 & 0.0003 & 0.0002 & 0 \\
0.0003 & 0.0024 & 0.0017 & 0.0010 \\
0.0002 & 0.0017 & 0.0048 & 0.0028 \\
0 & 0.0010 & 0.0028 & 0.0102
\end{tabular}
```


## Scalar Expansion of Arguments

Both the Portfolio object and the setAssetMoments function perform scalar expansion on arguments for the moments of asset returns. When using the Portfolio object, the number of assets must be already specified in the variable NumAssets. If NumAssets has not already been set, a scalar argument is interpreted as a scalar with NumAssets set to 1 . setAssetMoments provides an additional optional argument to specify the number of assets so that scalar expansion works with the correct number of assets. In addition, if either a scalar or vector is input for the covariance of asset returns, a diagonal matrix is formed such that a scalar expands along the diagonal and a vector
becomes the diagonal. This example demonstrates scalar expansion for four jointly independent assets with a common mean 0.1 and common variance 0.03 :

```
p = Portfolio;
p = setAssetMoments(p, 0.1, 0.03, 4);
[assetmean, assetcovar] = getAssetMoments(p)
assetmean =
```

    0.1000
    0.1000
    0.1000
    0.1000
    assetcovar =

| 0.0300 | 0 | 0 | 0 |
| ---: | ---: | ---: | ---: |
| 0 | 0.0300 | 0 | 0 |
| 0 | 0 | 0.0300 | 0 |
| 0 | 0 | 0 | 0.0300 |

If at least one argument is properly dimensioned, you do not need to include the additional NumAssets argument. This example illustrates a constant-diagonal covariance matrix and a mean of asset returns for four assets:

```
p = Portfolio;
p = setAssetMoments(p, [ 0.05; 0.06; 0.04; 0.03 ], 0.03);
[assetmean, assetcovar] = getAssetMoments(p)
assetmean =
    0.0500
    0.0600
    0.0400
    0.0300
assetcovar =
\begin{tabular}{rrrr}
0.0300 & 0 & 0 & 0 \\
0 & 0.0300 & 0 & 0 \\
0 & 0 & 0.0300 & 0 \\
0 & 0 & 0 & 0.0300
\end{tabular}
```

In addition, scalar expansion works with the Portfolio object if NumAssets is known, or is deduced from the inputs.

## Estimating Asset Moments from Prices or Returns

Another way to set the moments of asset returns is to use the estimateAssetMoments function which accepts either prices or returns and estimates the mean and covariance of asset returns. Either prices or returns are stored as matrices with samples going down the rows and assets going across the columns. In addition, prices or returns can be stored in a table or timetable (see "Estimating Asset Moments from Time Series Data" on page 4-57). To illustrate using estimateAssetMoments, generate random samples of 120 observations of asset returns for four assets from the mean and covariance of asset returns in the variables m and C with portsim. The default behavior of portsim creates simulated data with estimated mean and covariance identical to the input moments m and C. In addition to a return series created by portsim in the variable $X$, a price series is created in the variable Y :

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;
X = portsim(m', C, 120);
Y = ret2tick(X);
```

Note Portfolio optimization requires that you use total returns and not just price returns. So, "returns" should be total returns and "prices" should be total return prices.

Given asset returns and prices in variables $X$ and $Y$ from above, this sequence of examples demonstrates equivalent ways to estimate asset moments for the Portfolio object. A Portfolio object is created in p with the moments of asset returns set directly in the Portfolio object, and a second Portfolio object is created in $q$ to obtain the mean and covariance of asset returns from asset return data in X using estimateAssetMoments:

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;
```

```
X = portsim(m', C, 120);
p = Portfolio('mean', m, 'covar', C);
q = Portfolio;
q = estimateAssetMoments(q, X);
[passetmean, passetcovar] = getAssetMoments(p)
[qassetmean, qassetcovar] = getAssetMoments(q)
passetmean =
    0.0042
    0.0083
    0.0100
    0.0150
passetcovar =
\begin{tabular}{rrrr}
0.0005 & 0.0003 & 0.0002 & 0 \\
0.0003 & 0.0024 & 0.0017 & 0.0010 \\
0.0002 & 0.0017 & 0.0048 & 0.0028 \\
0 & 0.0010 & 0.0028 & 0.0102
\end{tabular}
qassetmean =
    0.0042
    0.0083
    0.0100
    0.0150
qassetcovar =
\begin{tabular}{llll}
0.0005 & 0.0003 & 0.0002 & 0.0000 \\
0.0003 & 0.0024 & 0.0017 & 0.0010 \\
0.0002 & 0.0017 & 0.0048 & 0.0028 \\
0.0000 & 0.0010 & 0.0028 & 0.0102
\end{tabular}
```

Notice how either approach has the same moments. The default behavior of estimateAssetMoments is to work with asset returns. If, instead, you have asset prices in the variable $Y$, estimateAssetMoments accepts a name-value pair argument name 'DataFormat ' with a corresponding value set to 'prices' to indicate that the input to the function is in the form of asset prices and not returns (the default value for the 'DataFormat' argument is 'returns'). This example compares direct assignment of moments in the Portfolio object $p$ with estimated moments from asset price data in $Y$ in the Portfolio object q:

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;
X = portsim(m', C, 120);
Y = ret2tick(X);
p = Portfolio('mean',m,'covar',C);
q = Portfolio;
q = estimateAssetMoments(q, Y, 'dataformat', 'prices');
[passetmean, passetcovar] = getAssetMoments(p)
[qassetmean, qassetcovar] = getAssetMoments(q)
passetmean =
    0.0042
    0.0083
    0.0100
    0.0150
passetcovar =
\begin{tabular}{rrrr}
0.0005 & 0.0003 & 0.0002 & 0 \\
0.0003 & 0.0024 & 0.0017 & 0.0010 \\
0.0002 & 0.0017 & 0.0048 & 0.0028 \\
0 & 0.0010 & 0.0028 & 0.0102
\end{tabular}
qassetmean =
    0.0042
    0.0083
    0.0100
    0.0150
qassetcovar =
\begin{tabular}{llll}
0.0005 & 0.0003 & 0.0002 & 0.0000 \\
0.0003 & 0.0024 & 0.0017 & 0.0010
\end{tabular}
```

| 0.0002 | 0.0017 | 0.0048 | 0.0028 |
| :--- | :--- | :--- | :--- |
| 0.0000 | 0.0010 | 0.0028 | 0.0102 |

## Estimating Asset Moments with Missing Data

Often when working with multiple assets, you have missing data indicated by NaN values in your return or price data. Although "Multivariate Normal Regression" on page 9-2 goes into detail about regression with missing data, the estimateAssetMoments function has a name-value pair argument name 'MissingData' that indicates with a Boolean value whether to use the missing data capabilities of Financial Toolbox software. The default value for 'MissingData' is false which removes all samples with NaN values. If, however, 'MissingData' is set to true, estimateAssetMoments uses the ECM algorithm to estimate asset moments. This example illustrates how this works on price data with missing values:

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336
    0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;
X = portsim(m', C, 120);
Y = ret2tick(X);
Y(1:20,1) = NaN;
Y(1:12,4) = NaN;
p = Portfolio('mean',m,'covar',C);
q = Portfolio:
q = estimateAssetMoments(q, Y, 'dataformat', 'prices');
r = Portfolio;
r = estimateAssetMoments(r, Y, 'dataformat', 'prices', 'missingdata', true);
[passetmean, passetcovar] = getAssetMoments(p)
[qassetmean, qassetcovar] = getAssetMoments(q)
[rassetmean, rassetcovar] = getAssetMoments(r)
passetmean =
0.0042
0.0083
0.0100
0.0150
```

```
passetcovar =
\begin{tabular}{rrrr}
0.0005 & 0.0003 & 0.0002 & 0 \\
0.0003 & 0.0024 & 0.0017 & 0.0010 \\
0.0002 & 0.0017 & 0.0048 & 0.0028 \\
0 & 0.0010 & 0.0028 & 0.0102
\end{tabular}
qassetmean =
    0.0045
    0.0082
    0.0101
    0.0091
qassetcovar =
\begin{tabular}{rrrr}
0.0006 & 0.0003 & 0.0001 & -0.0000 \\
0.0003 & 0.0023 & 0.0017 & 0.0011 \\
0.0001 & 0.0017 & 0.0048 & 0.0029 \\
-0.0000 & 0.0011 & 0.0029 & 0.0112
\end{tabular}
rassetmean =
    0.0045
    0.0083
    0.0100
    0.0113
rassetcovar =
\begin{tabular}{rrrr}
0.0008 & 0.0005 & 0.0001 & -0.0001 \\
0.0005 & 0.0032 & 0.0022 & 0.0015 \\
0.0001 & 0.0022 & 0.0063 & 0.0040 \\
-0.0001 & 0.0015 & 0.0040 & 0.0144
\end{tabular}
```

The Portfolio object p contains raw moments, the object q contains estimated moments in which NaN values are discarded, and the object $r$ contains raw moments that accommodate missing values. Each time you run this example, you will get different estimates for the moments in $q$ and $r$, and these will also differ from the moments in $p$.

## Estimating Asset Moments from Time Series Data

The estimateAssetMoments function also accepts asset returns or prices stored in a table or timetable. estimateAssetMoments implicitly works with matrices of data or data in a table or timetable object using the same rules for whether the data are returns or prices.

To illustrate the use of a table and timetable, use array2timetable and array2timetable to create a table and a timetable that contain asset returns generated with portsim (see "Estimating Asset Moments from Prices or Returns" on page 4-52). Two portfolio objects are then created with the AssetReturns based on a table and a timetable object.

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;
assetRetnScenarios = portsim(m', C, 120);
dates = datetime(datenum(2001,1:120,31), 'ConvertFrom', 'datenum');
assetsName = {'Bonds', 'LargeCap', 'SmallCap', 'Emerging'};
assetRetnTimeTable = array2timetable(assetRetnScenarios,'RowTimes',dates, 'VariableNames', assetsName);
assetRetnTable = array2table(assetRetnScenarios, 'VariableNames', assetsName);
% Create two Portfolio objects:
% p with predefined mean and covar: q with asset return scenarios to estimate mean and covar.
p = Portfolio('mean', m, 'covar', C);
q = Portfolio;
% estimate asset moments with timetable
q = estimateAssetMoments(q, assetRetnTimeTable);
[passetmean, passetcovar] = getAssetMoments(p)
[qassetmean, qassetcovar] = getAssetMoments(q)
% estimate asset moments with table
q = estimateAssetMoments(q, assetRetnTable);
[passetmean, passetcovar] = getAssetMoments(p)
[qassetmean, qassetcovar] = getAssetMoments(q)
passetmean =
    0.0042
    0.0083
    0.0100
    0.0150
passetcovar =
```

```
    0.0005 0.0003 0.0002 0
    0.0003 0.0024 0.0017 0.0010
    0.0002 0.0017 0.0048 0.0028
    0.0010 0.0028 0.0102
qassetmean =
    0.0042
    0.0083
    0.0100
    0.0150
qassetcovar =
    0.0005 0.0003 0.0002 -0.0000
    0.0003 0.0024 0.0017 0.0010
    0.0002 0.0017 0.0048 0.0028
    -0.0000 0.0010 0.0028 0.0102
passetmean =
    0.0042
    0.0083
    0.0100
    0.0150
passetcovar =
    0.0005 0.0003 0.0002 0
    0.0003 0.0024 0.0017 0.0010
    0.0002 0.0017 0.0048 0.0028
        0 0.0010 0.0028 0.0102
qassetmean =
    0.0042
    0.0083
    0.0100
```


### 0.0150

```
qassetcovar =
```

| 0.0005 | 0.0003 | 0.0002 | -0.0000 |
| ---: | ---: | ---: | ---: |
| 0.0003 | 0.0024 | 0.0017 | 0.0010 |
| 0.0002 | 0.0017 | 0.0048 | 0.0028 |
| -0.0000 | 0.0010 | 0.0028 | 0.0102 |

As you can see, the moments match between th two portfolios. In addition, estimateAssetMoments also extracts asset names or identifiers from a table or timetable when the argument name 'GetAssetList' set to true (its default value is false). If the 'GetAssetList' value is true, the identifiers are used to set the AssetList property of the object. To show this, the formation of the Portfolio object $q$ is repeated from the previous example with the 'GetAssetList' flag set to true extracts the column labels from a table or timetable object:

```
q = estimateAssetMoments(q,assetRetnTable,'GetAssetList',true);
disp(q.AssetList)
'Bonds' 'LargeCap' 'SmallCap' 'Emerging'
```

Note if you set the 'GetAssetList ' flag set to true and your input data is in a matrix, estimateAssetMoments uses the default labeling scheme from setAssetList described in "Setting Up a List of Asset Identifiers" on page 4-38.

## See Also

Portfolio|estimateAssetMoments|getAssetMoments| setAssetMoments| setCosts

## Related Examples

- "Creating the Portfolio Object" on page 4-28
- "Working with Portfolio Constraints Using Defaults" on page 4-68
- "Validate the Portfolio Problem for Portfolio Object" on page 4-110
- "Estimate Efficient Portfolios for Entire Efficient Frontier for Portfolio Object" on page 4-115
- "Estimate Efficient Frontiers for Portfolio Object" on page 4-140
- "Asset Allocation Case Study" on page 4-196
- "Portfolio Optimization Examples" on page 4-167


## More About

- "Portfolio Object" on page 4-23
- "Portfolio Optimization Theory" on page 4-3
- "Portfolio Object Workflow" on page 4-21


## External Websites

- Using MATLAB to Optimize Portfolios with Financial Toolbox (33 min 24 sec )
- MATLAB for Advanced Portfolio Construction and Stock Selection Models (30 min 28 sec)


## Working with a Riskless Asset

You can specify a riskless asset with the mean and covariance of asset returns in the AssetMean and AssetCovar properties such that the riskless asset has variance of 0 and is completely uncorrelated with all other assets. In this case, the Portfolio object uses a separate RiskFreeRate property that stores the rate of return of a riskless asset. Thus, you can separate your universe into a riskless asset and a collection of risky assets. For example, assume that your riskless asset has a return in the scalar variable r0, then the property for the RiskFreeRate is set using the Portfolio object:

```
r0 = 0.01/12;
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
p = Portfolio('RiskFreeRate', r0, 'AssetMean', m, 'AssetCovar', C);
disp(p.RiskFreeRate);
```

8.3333e-004

Note If your problem has a budget constraint such that your portfolio weights must sum to 1 , then the riskless asset is irrelevant.

## See Also

Portfolio|estimateAssetMoments|getAssetMoments| setAssetMoments

## Related Examples

- "Creating the Portfolio Object" on page 4-28
- "Working with Portfolio Constraints Using Defaults" on page 4-68
- "Validate the Portfolio Problem for Portfolio Object" on page 4-110
- "Estimate Efficient Portfolios for Entire Efficient Frontier for Portfolio Object" on page 4-115
- "Estimate Efficient Frontiers for Portfolio Object" on page 4-140
- "Asset Allocation Case Study" on page 4-196
- "Portfolio Optimization Examples" on page 4-167


## More About

- "Portfolio Object" on page 4-23
- "Portfolio Optimization Theory" on page 4-3
- "Portfolio Object Workflow" on page 4-21


## External Websites

- Using MATLAB to Optimize Portfolios with Financial Toolbox (33 min 24 sec )


## Working with Transaction Costs

The difference between net and gross portfolio returns is transaction costs. The net portfolio return proxy has distinct proportional costs to purchase and to sell assets which are maintained in the Portfolio object properties BuyCost and SellCost. Transaction costs are in units of total return and, as such, are proportional to the price of an asset so that they enter the model for net portfolio returns in return form. For example, suppose that you have a stock currently priced $\$ 40$ and your usual transaction costs are five cents per share. Then the transaction cost for the stock is $0.05 / 40=0.00125$ (as defined in "Net Portfolio Returns" on page 4-5). Costs are entered as positive values and credits are entered as negative values.

## Setting Transaction Costs Using the Portfolio Function

To set up transaction costs, you must specify an initial or current portfolio in the InitPort property. If the initial portfolio is not set when you set up the transaction cost properties, InitPort is 0 . The properties for transaction costs can be set using thePortfolio object. For example, assume that purchase and sale transaction costs are in the variables bc and sc and an initial portfolio is in the variable $\times 0$, then transaction costs are set:

```
bc = [ 0.00125; 0.00125; 0.00125; 0.00125; 0.00125 ];
sc = [ 0.00125; 0.007; 0.00125; 0.00125; 0.0024 ];
x0 = [ 0.4; 0.2; 0.2; 0.1; 0.1 ];
p = Portfolio('BuyCost', bc, 'SellCost', sc, 'InitPort', x0);
disp(p.NumAssets);
disp(p.BuyCost);
disp(p.SellCost);
disp(p.InitPort);
```

5
0.0013
0.0013
0.0013
0.0013
0.0013
0.0013
0.0070
0.0013
0.0013
0.0024
0.4000
0.2000
0.2000
0.1000
0.1000

## Setting Transaction Costs Using the setCosts Function

You can also set the properties for transaction costs using setCosts. Assume that you have the same costs and initial portfolio as in the previous example. Given a Portfolio object $p$ with an initial portfolio already set, use setCosts to set up transaction costs:

```
bc = [ 0.00125; 0.00125; 0.00125; 0.00125; 0.00125 ];
sc = [ 0.00125; 0.007; 0.00125; 0.00125; 0.0024 ];
x0 = [ 0.4; 0.2; 0.2; 0.1; 0.1 ];
p = Portfolio('InitPort', x0);
p = setCosts(p, bc, sc);
disp(p.NumAssets);
disp(p.BuyCost);
disp(p.SellCost);
disp(p.InitPort);
```


## 5

0.0013
0.0013
0.0013
0.0013
0.0013
0.0013
0.0070
0.0013
0.0013
0.0024
0.4000
0.2000
0.2000
0.1000
0.1000

You can also set up the initial portfolio's InitPort value as an optional argument to setCosts so that the following is an equivalent way to set up transaction costs:

```
bc = [ 0.00125; 0.00125; 0.00125; 0.00125; 0.00125 ];
sc = [ 0.00125; 0.007; 0.00125; 0.00125; 0.0024 ];
x0 = [ 0.4; 0.2; 0.2; 0.1; 0.1 ];
p = Portfolio;
p = setCosts(p, bc, sc, x0);
disp(p.NumAssets);
disp(p.BuyCost);
disp(p.SellCost);
disp(p.InitPort);
    5
    0.0013
    0.0013
    0.0013
    0.0013
    0.0013
    0.0013
    0.0070
    0.0013
    0.0013
    0.0024
    0.4000
    0.2000
    0.2000
    0.1000
    0.1000
```

For an example of setting costs, see "Portfolio Analysis with Turnover Constraints".

## Setting Transaction Costs with Scalar Expansion

Both the Portfolio object and the setCosts function implement scalar expansion on the arguments for transaction costs and the initial portfolio. If the NumAssets property is
already set in the Portfolio object, scalar arguments for these properties are expanded to have the same value across all dimensions. In addition, setCosts lets you specify NumAssets as an optional final argument. For example, assume that you have an initial portfolio $x 0$ and you want to set common transaction costs on all assets in your universe. You can set these costs in any of these equivalent ways:

```
x0 = [ 0.4; 0.2; 0.2; 0.1; 0.1 ];
p = Portfolio('InitPort', x0, 'BuyCost', 0.002, 'SellCost', 0.002);
Or
x0 = [ 0.4; 0.2; 0.2; 0.1; 0.1 ];
p = Portfolio('InitPort', x0);
p = setCosts(p, 0.002, 0.002);
or
x0 = [ 0.4; 0.2; 0.2; 0.1; 0.1 ];
p = Portfolio;
p = setCosts(p, 0.002, 0.002, x0);
```

To clear costs from your Portfolio object, use either the Portfolio object or setCosts with empty inputs for the properties to be cleared. For example, you can clear sales costs from the Portfolio object $p$ in the previous example:
p = Portfolio(p, 'SellCost', []);

## See Also

Portfolio|estimateAssetMoments | getAssetMoments| setAssetMoments | setCosts

## Related Examples

- "Creating the Portfolio Object" on page 4-28
- "Working with Portfolio Constraints Using Defaults" on page 4-68
- "Validate the Portfolio Problem for Portfolio Object" on page 4-110
- "Estimate Efficient Portfolios for Entire Efficient Frontier for Portfolio Object" on page 4-115
- "Estimate Efficient Frontiers for Portfolio Object" on page 4-140
- "Portfolio Analysis with Turnover Constraints"
- "Asset Allocation Case Study" on page 4-196
- "Portfolio Optimization Examples" on page 4-167


## More About

- "Portfolio Object" on page 4-23
- "Portfolio Optimization Theory" on page 4-3
- "Portfolio Object Workflow" on page 4-21


## External Websites

- Using MATLAB to Optimize Portfolios with Financial Toolbox (33 min 24 sec )
- MATLAB for Advanced Portfolio Construction and Stock Selection Models ( 30 min 28 sec)


## Working with Portfolio Constraints Using Defaults

The final element for a complete specification of a portfolio optimization problem is the set of feasible portfolios, which is called a portfolio set. A portfolio set $X \subset R^{n}$ is specified by construction as the intersection of sets formed by a collection of constraints on portfolio weights. A portfolio set necessarily and sufficiently must be a nonempty, closed, and bounded set.

When setting up your portfolio set, ensure that the portfolio set satisfies these conditions. The most basic or "default" portfolio set requires portfolio weights to be nonnegative (using the lower-bound constraint) and to sum to 1 (using the budget constraint). For information on the workflow when using Portfolio objects, see "Portfolio Object Workflow" on page 4-21.

## Setting Default Constraints for Portfolio Weights Using Portfolio Object

The "default" portfolio problem has two constraints on portfolio weights:

- Portfolio weights must be nonnegative.
- Portfolio weights must sum to 1 .

Implicitly, these constraints imply that portfolio weights are no greater than 1, although this is a superfluous constraint to impose on the problem.

## Setting Default Constraints Using the Portfolio Function

Given a portfolio optimization problem with NumAssets $=20$ assets, use the Portfolio object to set up a default problem and explicitly set bounds and budget constraints:

```
p = Portfolio('NumAssets', 20, 'LowerBound', 0, 'Budget', 1);
disp(p);
    Portfolio with properties:
```

        BuyCost: []
        SellCost: []
    RiskFreeRate: []
        AssetMean: []
        AssetCovar: []
    ```
TrackingError: []
    TrackingPort: []
            Turnover: []
        BuyTurnover: []
    SellTurnover: []
                Name: []
        NumAssets: 20
        AssetList: []
            InitPort: []
        AInequality: []
        bInequality: []
            AEquality: []
            bEquality: []
        LowerBound: [20x1 double]
        UpperBound: []
    LowerBudget: 1
    UpperBudget: 1
    GroupMatrix: []
        LowerGroup: []
        UpperGroup: []
            GroupA: []
            GroupB: []
        LowerRatio: []
        UpperRatio: []
            BoundType: []
MinNumAssets: []
MaxNumAssets: []
```


## Setting Default Constraints Using the setDefaultConstraints Function

An alternative approach is to use the setDefaultConstraints function. If the number of assets is already known in a Portfolio object, use setDefaultConstraints with no arguments to set up the necessary bound and budget constraints. Suppose that you have 20 assets to set up the portfolio set for a default problem:

```
p = Portfolio('NumAssets', 20);
p = setDefaultConstraints(p);
disp(p);
Portfolio with properties:
BuyCost: []
SellCost: []
RiskFreeRate: []
AssetMean: []
```

```
    AssetCovar: []
TrackingError: []
    TrackingPort: []
            Turnover: []
        BuyTurnover: []
    SellTurnover: []
        Name: []
        NumAssets: 20
        AssetList: []
            InitPort: []
        AInequality: []
        bInequality: []
            AEquality: []
            bEquality: []
        LowerBound: [20×1 double]
        UpperBound: []
    LowerBudget: 1
    UpperBudget: 1
    GroupMatrix: []
        LowerGroup: []
        UpperGroup: []
            GroupA: []
            GroupB: []
        LowerRatio: []
        UpperRatio: []
        BoundType: [0×0 categorical]
    MinNumAssets: []
    MaxNumAssets: []
```

If the number of assets is unknown, setDefaultConstraints accepts NumAssets as an optional argument to form a portfolio set for a default problem. Suppose that you have 20 assets:
p = Portfolio;
$p=$ setDefaultConstraints(p, 20);
disp(p);
Portfolio with properties:
BuyCost: []
SellCost: []
RiskFreeRate: []
AssetMean: []
AssetCovar: []
TrackingError: []

```
TrackingPort: []
            Turnover: []
    BuyTurnover: []
SellTurnover: []
            Name: []
        NumAssets: 20
        AssetList: []
        InitPort: []
    AInequality: []
    bInequality: []
        AEquality: []
        bEquality: []
        LowerBound: [20\times1 double]
        UpperBound: []
    LowerBudget: 1
    UpperBudget: 1
    GroupMatrix: []
        LowerGroup: []
        UpperGroup: []
        GroupA: []
        GroupB: []
    LowerRatio: []
    UpperRatio: []
        BoundType: [0×0 categorical]
MinNumAssets: []
MaxNumAssets: []
```


## See Also

Portfolio| setBounds | setBudget | setDefaultConstraints| setEquality| setGroupRatio|setGroups| setInequality| setOneWayTurnover| setTrackingError|setTrackingPort|setTurnover

## Related Examples

- "Working with 'Simple' Bound Constraints Using Portfolio Object" on page 4-73
- "Working with Budget Constraints Using Portfolio Object" on page 4-77
- "Working with Group Constraints Using Portfolio Object" on page 4-80
- "Working with Group Ratio Constraints Using Portfolio Object" on page 4-84
- "Working with Linear Equality Constraints Using Portfolio Object" on page 4-88
- "Working with Linear Inequality Constraints Using Portfolio Object" on page 4-91
- "Working with Average Turnover Constraints Using Portfolio Object" on page 4-98
- "Working with One-Way Turnover Constraints Using Portfolio Object" on page 4-102
- "Working with Tracking Error Constraints Using Portfolio Object" on page 4-106
- "Working with 'Conditional' BoundType, MinNumAssets, and MaxNumAssets Constraints Using Portfolio Object" on page 4-94
- "Creating the Portfolio Object" on page 4-28
- "Validate the Portfolio Problem for Portfolio Object" on page 4-110
- "Estimate Efficient Portfolios for Entire Efficient Frontier for Portfolio Object" on page 4-115
- "Estimate Efficient Frontiers for Portfolio Object" on page 4-140
- "Constraint Specification Using a Portfolio Object" on page 3-34
- "Asset Allocation Case Study" on page 4-196
- "Portfolio Optimization Examples" on page 4-167


## More About

- "Portfolio Object" on page 4-23
- "Portfolio Optimization Theory" on page 4-3
- "Portfolio Object Workflow" on page 4-21
- "Setting Up a Tracking Portfolio" on page 4-45


## External Websites

- Using MATLAB to Optimize Portfolios with Financial Toolbox (33 min 24 sec )
- MATLAB for Advanced Portfolio Construction and Stock Selection Models (30 min 28 $\mathrm{sec})$


## Working with 'Simple' Bound Constraints Using Portfolio Object


#### Abstract

'Simple' bound constraints are optional linear constraints that maintain upper and lower bounds on portfolio weights (see "'Simple' Bound Constraints" on page 4-12). Although every portfolio set must be bounded, it is not necessary to specify a portfolio set with explicit bound constraints. For example, you can create a portfolio set with an implicit upper bound constraint or a portfolio set with average turnover constraints. The bound constraints have properties LowerBound for the lower-bound constraint and UpperBound for the upper-bound constraint. Set default values for these constraints using the setDefaultConstraints function (see "Setting Default Constraints for Portfolio Weights Using Portfolio Object" on page 4-68).


## Setting 'Simple' Bounds Using the Portfolio Function

The properties for bound constraints are set through the Portfolio object. Suppose that you have a balanced fund with stocks that can range from $50 \%$ to $75 \%$ of your portfolio and bonds that can range from $25 \%$ to $50 \%$ of your portfolio. The bound constraints for a balanced fund are set with:

```
lb = [ 0.5; 0.25 ];
ub = [ 0.75; 0.5 ];
p = Portfolio('LowerBound', lb, 'UpperBound', ub, 'BoundType', 'Simple');
disp(p.NumAssets);
disp(p.LowerBound);
disp(p.UpperBound);
2
0.5000
0.2500
    0.7500
    0.5000
```

To continue with this example, you must set up a budget constraint. For details, see "Working with Budget Constraints Using Portfolio Object" on page 4-77.

## Setting 'Simple' Bounds Using the setBounds Function

You can also set the properties for bound constraints using setBounds. Suppose that you have a balanced fund with stocks that can range from $50 \%$ to $75 \%$ of your portfolio and bonds that can range from $25 \%$ to $50 \%$ of your portfolio. Given a Portfolio object p, use setBounds to set the bound constraints:

```
lb = [ 0.5; 0.25 ];
ub = [ 0.75; 0.5 ];
p = Portfolio;
p = setBounds(p, lb, ub,'BoundType', 'Simple');
disp(p.NumAssets);
disp(p.LowerBound);
disp(p.UpperBound);
```

    2
    0.5000
0.2500
0.7500
0.5000

## Setting 'Simple' Bounds Using the Portfolio Function or setBounds Function

Both the Portfolio object and setBounds function implement scalar expansion on either the LowerBound or UpperBound properties. If the NumAssets property is already set in the Portfolio object, scalar arguments for either property expand to have the same value across all dimensions. In addition, setBounds lets you specify NumAssets as an optional argument. Suppose that you have a universe of 500 assets and you want to set common bound constraints on all assets in your universe. Specifically, you are a long-only investor and want to hold no more than $5 \%$ of your portfolio in any single asset. You can set these bound constraints in any of these equivalent ways:

```
p = Portfolio('NumAssets', 500, 'LowerBound', 0, 'UpperBound', 0.05,'BoundType', 'Simple');
or
p = Portfolio('NumAssets', 500);
p = setBounds(p, 0, 0.05,'BoundType','Simple');
```

or

```
p = Portfolio;
p = setBounds(p, 0, 0.05, 500,'BoundType','Simple');
```

To clear bound constraints from your Portfolio object, use either the Portfolio object or setBounds with empty inputs for the properties to be cleared. For example, to clear the upper-bound constraint from the Portfolio object $p$ in the previous example:

```
p = Portfolio(p, 'UpperBound', []);
```


## See Also

Portfolio| setBounds | setBudget | setDefaultConstraints| setEquality| setGroupRatio| setGroups | setInequality| setOneWayTurnover| setTrackingError|setTrackingPort|setTurnover

## Related Examples

- "Creating the Portfolio Object" on page 4-28
- "Working with 'Conditional' BoundType, MinNumAssets, and MaxNumAssets Constraints Using Portfolio Object" on page 4-94
- "Working with Portfolio Constraints Using Defaults" on page 4-68
- "Validate the Portfolio Problem for Portfolio Object" on page 4-110
- "Estimate Efficient Portfolios for Entire Efficient Frontier for Portfolio Object" on page 4-115
- "Estimate Efficient Frontiers for Portfolio Object" on page 4-140
- "Constraint Specification Using a Portfolio Object" on page 3-34
- "Asset Allocation Case Study" on page 4-196
- "Portfolio Optimization Examples" on page 4-167


## More About

- "Portfolio Object" on page 4-23
- "Portfolio Optimization Theory" on page 4-3
- "Portfolio Object Workflow" on page 4-21
- "Setting Up a Tracking Portfolio" on page 4-45


## External Websites

- Using MATLAB to Optimize Portfolios with Financial Toolbox (33 min 24 sec )
- MATLAB for Advanced Portfolio Construction and Stock Selection Models (30 min 28 sec)


## Working with Budget Constraints Using Portfolio Object

The budget constraint is an optional linear constraint that maintains upper and lower bounds on the sum of portfolio weights (see "Budget Constraints" on page 4-12). Budget constraints have properties LowerBudget for the lower budget constraint and UpperBudget for the upper budget constraint. If you set up a portfolio optimization problem that requires portfolios to be fully invested in your universe of assets, you can set LowerBudget to be equal to UpperBudget. These budget constraints can be set with default values equal to 1 using setDefaultConstraints (see "Setting Default Constraints for Portfolio Weights Using Portfolio Object" on page 4-68).

## Setting Budget Constraints Using the Portfolio Function

The properties for the budget constraint can also be set using the Portfolio object. Suppose that you have an asset universe with many risky assets and a riskless asset and you want to ensure that your portfolio never holds more than $1 \%$ cash, that is, you want to ensure that you are 99-100\% invested in risky assets. The budget constraint for this portfolio can be set with:

```
p = Portfolio('LowerBudget', 0.99, 'UpperBudget', 1);
disp(p.LowerBudget);
disp(p.UpperBudget);
```

0.9900

1

## Setting Budget Constraints Using the setBudget Function

You can also set the properties for a budget constraint using setBudget. Suppose that you have a fund that permits up to $10 \%$ leverage which means that your portfolio can be from $100 \%$ to $110 \%$ invested in risky assets. Given a Portfolio object p, use setBudget to set the budget constraints:

```
p = Portfolio;
p = setBudget(p, 1, 1.1);
disp(p.LowerBudget);
disp(p.UpperBudget);
```

1

If you were to continue with this example, then set the RiskFreeRate property to the borrowing rate to finance possible leveraged positions. For details on the RiskFreeRate property, see "Working with a Riskless Asset" on page 4-61. To clear either bound for the budget constraint from your Portfolio object, use either the Portfolio object or setBudget with empty inputs for the properties to be cleared. For example, clear the upper-budget constraint from the Portfolio object p in the previous example with:

```
p = Portfolio(p, 'UpperBudget', []);
```


## See Also

Portfolio| setBounds | setBudget | setDefaultConstraints| setEquality| setGroupRatio | setGroups | setInequality | setOneWayTurnover | setTrackingError|setTrackingPort|setTurnover

## Related Examples

- "Creating the Portfolio Object" on page 4-28
- "Working with Portfolio Constraints Using Defaults" on page 4-68
- "Validate the Portfolio Problem for Portfolio Object" on page 4-110
- "Estimate Efficient Portfolios for Entire Efficient Frontier for Portfolio Object" on page 4-115
- "Estimate Efficient Frontiers for Portfolio Object" on page 4-140
- "Constraint Specification Using a Portfolio Object" on page 3-34
- "Asset Allocation Case Study" on page 4-196
- "Portfolio Optimization Examples" on page 4-167


## More About

- "Portfolio Object" on page 4-23
- "Portfolio Optimization Theory" on page 4-3
- "Portfolio Object Workflow" on page 4-21
- "Setting Up a Tracking Portfolio" on page 4-45


## External Websites

- Using MATLAB to Optimize Portfolios with Financial Toolbox (33 min 24 sec )
- MATLAB for Advanced Portfolio Construction and Stock Selection Models ( 30 min 28 sec)


## Working with Group Constraints Using Portfolio Object

Group constraints are optional linear constraints that group assets together and enforce bounds on the group weights (see "Group Constraints" on page 4-13). Although the constraints are implemented as general constraints, the usual convention is to form a group matrix that identifies membership of each asset within a specific group with Boolean indicators (either true or false or with 1 or 0) for each element in the group matrix. Group constraints have properties GroupMatrix for the group membership matrix, LowerGroup for the lower-bound constraint on groups, and UpperGroup for the upper-bound constraint on groups.

## Setting Group Constraints Using the Portfolio Function

The properties for group constraints are set through the Portfolio object. Suppose that you have a portfolio of five assets and want to ensure that the first three assets constitute no more than $30 \%$ of your portfolio, then you can set group constraints:

```
G = [ 1 1 1 0 0 ];
p = Portfolio('GroupMatrix', G, 'UpperGroup', 0.3);
disp(p.NumAssets);
disp(p.GroupMatrix);
disp(p.UpperGroup);
5
1 
0.3000
```

The group matrix G can also be a logical matrix so that the following code achieves the same result.

```
G = [ true true true false false ];
p = Portfolio('GroupMatrix', G, 'UpperGroup', 0.3);
disp(p.NumAssets);
disp(p.GroupMatrix);
disp(p.UpperGroup);
5
1
0.3000
```


## Setting Group Constraints Using the setGroups and addGroups Functions

You can also set the properties for group constraints using setGroups. Suppose that you have a portfolio of five assets and want to ensure that the first three assets constitute no more than $30 \%$ of your portfolio. Given a Portfolio object p, use setGroups to set the group constraints:

```
G = [ true true true false false ];
p = Portfolio;
p = setGroups(p, G, [], 0.3);
disp(p.NumAssets);
disp(p.GroupMatrix);
disp(p.UpperGroup);
5
1 1 1 1 1 0 0
```


### 0.3000

In this example, you would set the LowerGroup property to be empty ([]).
Suppose that you want to add another group constraint to make odd-numbered assets constitute at least $20 \%$ of your portfolio. Set up an augmented group matrix and introduce infinite bounds for unconstrained group bounds or use the addGroups function to build up group constraints. For this example, create another group matrix for the second group constraint:

```
p = Portfolio;
G = [ true true true false false ]; % group matrix for first group constraint
p = setGroups(p, G, [], 0.3);
G = [ true false true false true ]; % group matrix for second group constraint
p = addGroups(p, G, 0.2);
disp(p.NumAssets);
disp(p.GroupMatrix);
disp(p.LowerGroup);
disp(p.UpperGroup);
```

5
$\begin{array}{lllll}1 & 1 & 1 & 0 & 0\end{array}$
$\begin{array}{lllll}1 & 0 & 1 & 0 & 1\end{array}$
- Inf
0.2000

### 0.3000

Inf
addGroups determines which bounds are unbounded so you only need to focus on the constraints that you want to set.

The Portfolio object and setGroups and addGroups implement scalar expansion on either the LowerGroup or UpperGroup properties based on the dimension of the group matrix in the property GroupMatrix. Suppose that you have a universe of 30 assets with six asset classes such that assets $1-5$, assets $6-12$, assets $13-18$, assets $19-22$, assets $23-$ 27, and assets $28-30$ constitute each of your six asset classes and you want each asset class to fall from $0 \%$ to $25 \%$ of your portfolio. Let the following group matrix define your groups and scalar expansion define the common bounds on each group:

```
p = Portfolio;
G = blkdiag(true(1,5), true(1,7), true(1,6), true(1,4), true(1,5), true(1,3));
p = setGroups(p, G, 0, 0.25);
disp(p.NumAssets);
disp(p.GroupMatrix);
disp(p.LowerGroup);
disp(p.UpperGroup);
3 0
Columns 1 through 16
\begin{tabular}{llllllllllllllll}
1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{tabular}
Columns 17 through 30
\begin{tabular}{llllllllllllll}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1
\end{tabular}
```


## See Also

Portfolio| setBounds | setBudget | setDefaultConstraints| setEquality| setGroupRatio | setGroups | setInequality | setOneWayTurnover | setTrackingError| setTrackingPort|setTurnover

## Related Examples

- "Creating the Portfolio Object" on page 4-28
- "Working with Portfolio Constraints Using Defaults" on page 4-68
- "Validate the Portfolio Problem for Portfolio Object" on page 4-110
- "Estimate Efficient Portfolios for Entire Efficient Frontier for Portfolio Object" on page 4-115
- "Estimate Efficient Frontiers for Portfolio Object" on page 4-140
- "Constraint Specification Using a Portfolio Object" on page 3-34
- "Asset Allocation Case Study" on page 4-196
- "Portfolio Optimization Examples" on page 4-167


## More About

- "Portfolio Object" on page 4-23
- "Portfolio Optimization Theory" on page 4-3
- "Portfolio Object Workflow" on page 4-21
- "Setting Up a Tracking Portfolio" on page 4-45


## External Websites

- Using MATLAB to Optimize Portfolios with Financial Toolbox (33 min 24 sec )
- MATLAB for Advanced Portfolio Construction and Stock Selection Models ( 30 min 28 sec )


## Working with Group Ratio Constraints Using Portfolio Object

Group ratio constraints are optional linear constraints that maintain bounds on proportional relationships among groups of assets (see "Group Ratio Constraints" on page 4-14). Although the constraints are implemented as general constraints, the usual convention is to specify a pair of group matrices that identify membership of each asset within specific groups with Boolean indicators (either true or false or with 1 or 0 ) for each element in each of the group matrices. The goal is to ensure that the ratio of a base group compared to a comparison group fall within specified bounds. Group ratio constraints have properties:

- GroupA for the base membership matrix
- GroupB for the comparison membership matrix
- LowerRatio for the lower-bound constraint on the ratio of groups
- UpperRatio for the upper-bound constraint on the ratio of groups


## Setting Group Ratio Constraints Using the Portfolio Function

The properties for group ratio constraints are set using the Portfolio object. For example, assume that you want the ratio of financial to nonfinancial companies in your portfolios to never go above $50 \%$. Suppose that you have six assets with three financial companies (assets 1-3) and three nonfinancial companies (assets 4-6). To set group ratio constraints:

```
GA = [ 1 1 1 0 0 0 ]; % financial companies
GB}=[\begin{array}{llllllll}{0}&{0}&{0}&{1}&{1}&{1}\end{array}]; % nonfinancial companies
p = Portfolio('GroupA', GA, 'GroupB', GB, 'UpperRatio', 0.5);
disp(p.NumAssets);
disp(p.GroupA);
disp(p.GroupB);
disp(p.UpperRatio);
6
\begin{tabular}{llllll}
1 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & 1
\end{tabular}
0.5000
```

Group matrices GA and GB in this example can be logical matrices with true and false elements that yield the same result:

```
GA = [ true true true false false false ]; % financial companies
GB = [ false false false true true true ]; % nonfinancial companies
p = Portfolio('GroupA', GA, 'GroupB', GB, 'UpperRatio', 0.5);
disp(p.NumAssets);
disp(p.GroupA);
disp(p.GroupB)
disp(p.UpperRatio);
```

6

| 1 | 1 | 1 | 0 | 0 | 0 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 1 | 1 | 1 |

0.5000

## Setting Group Ratio Constraints Using the setGroupRatio and addGroupRatio Functions

You can also set the properties for group ratio constraints using setGroupRatio. For example, assume that you want the ratio of financial to nonfinancial companies in your portfolios to never go above 50\%. Suppose that you have six assets with three financial companies (assets 1-3) and three nonfinancial companies (assets 4-6). Given a Portfolio object $p$, use setGroupRatio to set the group constraints:

```
GA = [ true true true false false false ]; % financial companies
GB = [ false false false true true true ]; % nonfinancial companies
p = Portfolio;
p = setGroupRatio(p, GA, GB, [], 0.5);
disp(p.NumAssets);
disp(p.GroupA);
disp(p.GroupB);
disp(p.UpperRatio);
6
1 1 1 1 1 0
0
0.5000
```

In this example, you would set the LowerRatio property to be empty ([ ]).

Suppose that you want to add another group ratio constraint to ensure that the weights in odd-numbered assets constitute at least $20 \%$ of the weights in nonfinancial assets your portfolio. You can set up augmented group ratio matrices and introduce infinite bounds for unconstrained group ratio bounds, or you can use the addGroupRatio function to build up group ratio constraints. For this example, create another group matrix for the second group constraint:

```
p = Portfolio;
GA = [ true true true false false false ]; % financial companies
GB = [ false false false true true true ]; % nonfinancial companies
p = setGroupRatio(p, GA, GB, [], 0.5);
GA = [ true false true false true false ]; % odd-numbered companies
GB = [ false false false true true true ]; % nonfinancial companies
p = addGroupRatio(p, GA, GB, 0.2);
disp(p.NumAssets);
disp(p.GroupA);
disp(p.GroupB);
disp(p.LowerRatio);
disp(p.UpperRatio);
6
\begin{tabular}{llllll}
1 & 1 & 1 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 & 1 & 0 \\
& & & & & \\
0 & 0 & 0 & 1 & 1 & 1 \\
0 & 0 & 0 & 1 & 1 & 1
\end{tabular}
    -Inf
0.2000
0.5000
Inf
```

Notice that addGroupRatio determines which bounds are unbounded so you only need to focus on the constraints you want to set.

The Portfolio object, setGroupRatio, and addGroupRatio implement scalar expansion on either the LowerRatio or UpperRatio properties based on the dimension of the group matrices in GroupA and GroupB properties.

## See Also

Portfolio| setBounds | setBudget | setDefaultConstraints| setEquality| setGroupRatio | setGroups | setInequality | setOneWayTurnover | setTrackingError| setTrackingPort|setTurnover

## Related Examples

- "Creating the Portfolio Object" on page 4-28
- "Working with Portfolio Constraints Using Defaults" on page 4-68
- "Validate the Portfolio Problem for Portfolio Object" on page 4-110
- "Estimate Efficient Portfolios for Entire Efficient Frontier for Portfolio Object" on page 4-115
- "Estimate Efficient Frontiers for Portfolio Object" on page 4-140
- "Constraint Specification Using a Portfolio Object" on page 3-34
- "Asset Allocation Case Study" on page 4-196
- "Portfolio Optimization Examples" on page 4-167


## More About

- "Portfolio Object" on page 4-23
- "Portfolio Optimization Theory" on page 4-3
- "Portfolio Object Workflow" on page 4-21
- "Setting Up a Tracking Portfolio" on page 4-45


## External Websites

- Using MATLAB to Optimize Portfolios with Financial Toolbox (33 min 24 sec )
- MATLAB for Advanced Portfolio Construction and Stock Selection Models ( 30 min 28 sec )


## Working with Linear Equality Constraints Using Portfolio Object

Linear equality constraints are optional linear constraints that impose systems of equalities on portfolio weights (see "Linear Equality Constraints" on page 4-11). Linear equality constraints have properties AEquality, for the equality constraint matrix, and bEquality, for the equality constraint vector.

## Setting Linear Equality Constraints Using the Portfolio Function

The properties for linear equality constraints are set using the Portfolio object. Suppose that you have a portfolio of five assets and want to ensure that the first three assets are $50 \%$ of your portfolio. To set this constraint:

```
A = [ 1 1 1 0 0 ];
b = 0.5;
p = Portfolio('AEquality', A, 'bEquality', b);
disp(p.NumAssets);
disp(p.AEquality);
disp(p.bEquality);
5
1 
0.5000
```


## Setting Linear Equality Constraints Using the setEquality and addEquality Functions

You can also set the properties for linear equality constraints using setEquality. Suppose that you have a portfolio of five assets and want to ensure that the first three assets are $50 \%$ of your portfolio. Given a Portfolio object p, use setEquality to set the linear equality constraints:
$\mathrm{A}=\left[\begin{array}{lllll}1 & 1 & 1 & 0 & 0\end{array}\right] ;$
b $=0.5$;
p = Portfolio;
$\mathrm{p}=\operatorname{setEquality}(\mathrm{p}, \mathrm{A}, \mathrm{b})$;

```
disp(p.NumAssets);
disp(p.AEquality);
disp(p.bEquality);
5
1 1 1 1 0
0.5000
```

Suppose that you want to add another linear equality constraint to ensure that the last three assets also constitute $50 \%$ of your portfolio. You can set up an augmented system of linear equalities or use addEquality to build up linear equality constraints. For this example, create another system of equalities:

```
p = Portfolio;
A = [ 1 1 1 0 0 ]; % first equality constraint
b = 0.5;
p = setEquality(p, A, b);
A = [ 0 0 1 1 1 ]; % second equality constraint
b = 0.5;
p = addEquality(p, A, b);
disp(p.NumAssets);
disp(p.AEquality);
disp(p.bEquality);
5
\begin{tabular}{lllll}
1 & 1 & 1 & 0 & 0
\end{tabular}
0.5000
0.5000
```

The Portfolio object, setEquality, and addEquality implement scalar expansion on the bEquality property based on the dimension of the matrix in the AEquality property.

## See Also

Portfolio | setBounds | setBudget | setDefaultConstraints| setEquality| setGroupRatio | setGroups | setInequality | setOneWayTurnover | setTrackingError|setTrackingPort|setTurnover

## Related Examples

- "Creating the Portfolio Object" on page 4-28
- "Working with Portfolio Constraints Using Defaults" on page 4-68
- "Validate the Portfolio Problem for Portfolio Object" on page 4-110
- "Estimate Efficient Portfolios for Entire Efficient Frontier for Portfolio Object" on page 4-115
- "Estimate Efficient Frontiers for Portfolio Object" on page 4-140
- "Constraint Specification Using a Portfolio Object" on page 3-34
- "Asset Allocation Case Study" on page 4-196
- "Portfolio Optimization Examples" on page 4-167


## More About

- "Portfolio Object" on page 4-23
- "Portfolio Optimization Theory" on page 4-3
- "Portfolio Object Workflow" on page 4-21
- "Setting Up a Tracking Portfolio" on page 4-45


## External Websites

- Using MATLAB to Optimize Portfolios with Financial Toolbox (33 min 24 sec )
- MATLAB for Advanced Portfolio Construction and Stock Selection Models ( 30 min 28 sec )


## Working with Linear Inequality Constraints Using Portfolio Object

Linear inequality constraints are optional linear constraints that impose systems of inequalities on portfolio weights (see "Linear Inequality Constraints" on page 4-10). Linear inequality constraints have properties AInequality for the inequality constraint matrix, and bInequality for the inequality constraint vector.

## Setting Linear Inequality Constraints Using the Portfolio Function

The properties for linear inequality constraints are set using the Portfolio object. Suppose that you have a portfolio of five assets and you want to ensure that the first three assets are no more than $50 \%$ of your portfolio. To set up these constraints:

```
A = [ 1 1 1 0 0 ];
b = 0.5;
p = Portfolio('AInequality', A, 'bInequality', b);
disp(p.NumAssets);
disp(p.AInequality);
disp(p.bInequality);
5
1 1 1 1 1 0 0
0.5000
```


## Setting Linear Inequality Constraints Using the setInequality and addInequality Functions

You can also set the properties for linear inequality constraints using setInequality. Suppose that you have a portfolio of five assets and you want to ensure that the first three assets constitute no more than $50 \%$ of your portfolio. Given a Portfolio object p, use setInequality to set the linear inequality constraints:
$\mathrm{A}=\left[\begin{array}{lllll}1 & 1 & 1 & 0\end{array}\right] ;$
b $=0.5$;
p = Portfolio;
p = setInequality(p, A, b);

```
disp(p.NumAssets);
disp(p.AInequality);
disp(p.bInequality);
5
1 1 1 1 0
0.5000
```

Suppose that you want to add another linear inequality constraint to ensure that the last three assets constitute at least 50\% of your portfolio. You can set up an augmented system of linear inequalities or use the addInequality function to build up linear inequality constraints. For this example, create another system of inequalities:

```
p = Portfolio;
A = [ 1 1 1 0 0 ]; % first inequality constraint
b = 0.5;
p = setInequality(p, A, b);
A = [ 0 0 -1 -1 -1 ]; % second inequality constraint
b = -0.5;
p = addInequality(p, A, b);
disp(p.NumAssets);
disp(p.AInequality);
disp(p.bInequality);
5
\begin{tabular}{rrrrr}
1 & 1 & 1 & 0 & 0 \\
0 & 0 & -1 & -1 & -1
\end{tabular}
0.5000
-0.5000
```

The Portfolio object, setInequality, and addInequality implement scalar expansion on the bInequality property based on the dimension of the matrix in the AInequality property.

## See Also

Portfolio| setBounds | setBudget | setDefaultConstraints| setEquality| setGroupRatio | setGroups | setInequality | setOneWayTurnover | setTrackingError|setTrackingPort|setTurnover

## Related Examples

- "Creating the Portfolio Object" on page 4-28
- "Working with Portfolio Constraints Using Defaults" on page 4-68
- "Validate the Portfolio Problem for Portfolio Object" on page 4-110
- "Estimate Efficient Portfolios for Entire Efficient Frontier for Portfolio Object" on page 4-115
- "Estimate Efficient Frontiers for Portfolio Object" on page 4-140
- "Constraint Specification Using a Portfolio Object" on page 3-34
- "Asset Allocation Case Study" on page 4-196
- "Portfolio Optimization Examples" on page 4-167


## More About

- "Portfolio Object" on page 4-23
- "Portfolio Optimization Theory" on page 4-3
- "Portfolio Object Workflow" on page 4-21
- "Setting Up a Tracking Portfolio" on page 4-45


## External Websites

- Using MATLAB to Optimize Portfolios with Financial Toolbox (33 min 24 sec )
- MATLAB for Advanced Portfolio Construction and Stock Selection Models ( 30 min 28 sec )


# Working with 'Conditional' BoundType, MinNumAssets, and MaxNumAssets Constraints Using Portfolio Object 

> When any one, or any combination of 'Conditional ' BoundType, MinNumAssets, or MaxNumAssets constraints are active, the portfolio problem is formulated by adding NumAssets binary variables, where 0 indicates not invested, and 1 is invested. For example, to explain the 'Conditional' BoundType and MinNumAssets and MaxNumAssets constraints, assume that your portfolio has a universe of 100 assets that you want to invest:
> - 'Conditional' BoundType (also known as semicontinuous constraints), set by setBounds, is often used in situations where you do not want to invest small values. A standard example is a portfolio optimization problem where many small allocations are not attractive because of transaction costs. Instead, you prefer fewer instruments in the portfolio with larger allocations. This situation can be handled using 'Conditional' BoundType constraints for a Portfolio object.

For example, the weight you invest in each asset is either 0 or between [0.01, 0.5]. Generally, a semicontinuous variable $x$ is a continuous variable between bounds [lb, $u b]$ that also can assume the value 0 , where $l b>0, l b \leq u b$. Applying this to portfolio optimization requires that very small or large positions should be avoided, that is values that fall in ( $0, \mathrm{lb}$ ) or are more than ub.

- MinNumAssets and MaxNumAssets (also known as cardinality constraints), set by setMinMaxNumAssets, limit the number of assets in a Portfolio object. For example, if you have 100 assets in your portfolio and you want the number of assets allocated in the portfolio to be from 40 through 60. Using MinNumAssets and MaxNumAssets you can limit the number of assets in the optimized portfolio, which allows you to limit transaction and operational costs or to create an index tracking portfolio.


## Setting 'Conditional' BoundType Constraints Using the setBounds Function

Use setBounds with a 'conditional' BoundType to set $x i=0$ or $0.02<=x i<=0.5$ for all $i=1, \ldots$ NumAssets:
p = Portfolio('AssetMean', AssetMean, 'AssetCovar', AssetCovar); p = setBounds(p, 0.02, 0.5,'BoundType', 'Conditional', 'NumAssets', 3)

```
p =
    Portfolio with properties:
            BuyCost: []
            SellCost: []
        RiskFreeRate: []
            AssetMean: [3\times1 double]
            AssetCovar: [ }3\times3\mathrm{ double]
    TrackingError: []
        TrackingPort: []
            Turnover: []
        BuyTurnover: []
    SellTurnover: []
                Name: []
            NumAssets: 3
            AssetList: []
            InitPort: []
        AInequality: []
        bInequality: []
            AEquality: []
            bEquality: []
            LowerBound: [3\times1 double]
            UpperBound: [3\times1 double]
            LowerBudget: []
            UpperBudget: []
            GroupMatrix: []
            LowerGroup: []
            UpperGroup: []
            GroupA: []
            GroupB: []
            LowerRatio: []
            UpperRatio: []
            BoundType: [3\times1 categorical]
MinNumAssets: []
MaxNumAssets: []
```


## Setting the Limits on the Number of Assets Invested Using the setMinMaxNumAssets Function

You can also set the MinNumAssets and MaxNumAssets properties to define a limit on the number of assets invested using setMinMaxNumAssets. For example, by setting MinNumAssets=MaxNumAssets=2, only two of the three assets are invested in the portfolio.

```
AssetMean = [ 0.0101110; 0.0043532; 0.0137058 ];
AssetCovar = [ 0.00324625 0.00022983 0.00420395;
                                    0.00022983 0.00049937 0.00019247;
                                0.00420395 0.00019247 0.00764097 ];
p = Portfolio('AssetMean', AssetMean, 'AssetCovar', AssetCovar);
p = setMinMaxNumAssets(p, 2, 2)
    Portfolio with properties:
            BuyCost: []
            SellCost: []
        RiskFreeRate: []
            AssetMean: [3\times1 double]
            AssetCovar: [3\times3 double]
TrackingError: []
    TrackingPort: []
            Turnover: []
        BuyTurnover: []
    SellTurnover: []
            Name: []
            NumAssets: 3
            AssetList: []
            InitPort: []
    AInequality: []
    bInequality: []
            AEquality: []
            bEquality: []
            LowerBound: []
            UpperBound: []
            LowerBudget: []
            UpperBudget: []
            GroupMatrix: []
            LowerGroup: []
            UpperGroup: []
            GroupA: []
            GroupB: []
            LowerRatio: []
            UpperRatio: []
            BoundType: []
    MinNumAssets: 2
    MaxNumAssets: 2
```


## See Also

Portfolio| setBounds| setBounds| setBudget | setDefaultConstraints| setEquality|setGroupRatio|setGroups|setInequality|
setMinMaxNumAssets | setOneWayTurnover| setTrackingError| setTrackingPort|setTurnover

## Related Examples

- "Creating the Portfolio Object" on page 4-28
- "Working with Portfolio Constraints Using Defaults" on page 4-68
- "Validate the Portfolio Problem for Portfolio Object" on page 4-110
- "Estimate Efficient Portfolios for Entire Efficient Frontier for Portfolio Object" on page 4-115
- "Estimate Efficient Frontiers for Portfolio Object" on page 4-140
- "Constraint Specification Using a Portfolio Object" on page 3-34
- "Portfolio Optimization with Semicontinuous and Cardinality Constraints"


## More About

- "Portfolio Object" on page 4-23
- "Portfolio Optimization Theory" on page 4-3
- "Portfolio Object Workflow" on page 4-21
- "Setting Up a Tracking Portfolio" on page 4-45


## External Websites

- Using MATLAB to Optimize Portfolios with Financial Toolbox (33 min 24 sec )
- MATLAB for Advanced Portfolio Construction and Stock Selection Models (30 min 28 sec)


## Working with Average Turnover Constraints Using Portfolio Object

The turnover constraint is an optional linear absolute value constraint (see "Average Turnover Constraints" on page 4-15) that enforces an upper bound on the average of purchases and sales. The turnover constraint can be set using the Portfolio object or the setTurnover function. The turnover constraint depends on an initial or current portfolio, which is assumed to be zero if not set when the turnover constraint is set. The turnover constraint has properties Turnover, for the upper bound on average turnover, and InitPort, for the portfolio against which turnover is computed.

## Setting Average Turnover Constraints Using the Portfolio Function

The properties for the turnover constraints are set using the Portfolio object. Suppose that you have an initial portfolio of 10 assets in a variable $x 0$ and you want to ensure that average turnover is no more than $30 \%$. To set this turnover constraint:

```
x0 = [ 0.12; 0.09; 0.08; 0.07; 0.1; 0.1; 0.15; 0.11; 0.08; 0.1 ];
p = Portfolio('Turnover', 0.3, 'InitPort', x0);
disp(p.NumAssets);
disp(p.Turnover);
disp(p.InitPort);
```

10
0.3000
0.1200
0.0900
0.0800
0.0700
0.1000
0.1000
0.1500
0.1100
0.0800
0.1000

Note if the NumAssets or InitPort properties are not set before or when the turnover constraint is set, various rules are applied to assign default values to these properties (see "Setting Up an Initial or Current Portfolio" on page 4-42).

## Setting Average Turnover Constraints Using the setTurnover Function

You can also set properties for portfolio turnover using setTurnover to specify both the upper bound for average turnover and an initial portfolio. Suppose that you have an initial portfolio of 10 assets in a variable $x 0$ and want to ensure that average turnover is no more than $30 \%$. Given a Portfolio object p, use setTurnover to set the turnover constraint with and without the initial portfolio being set previously:

```
x0 = [ 0.12; 0.09; 0.08; 0.07; 0.1; 0.1; 0.15; 0.11; 0.08; 0.1 ];
p = Portfolio('InitPort', x0);
p = setTurnover(p, 0.3);
disp(p.NumAssets);
disp(p.Turnover);
disp(p.InitPort);
1 0
0.3000
0.1200
0.0900
0.0800
0.0700
0.1000
0.1000
0.1500
0.1100
0.0800
0.1000
or
x0 = [ 0.12; 0.09; 0.08; 0.07; 0.1; 0.1; 0.15; 0.11; 0.08; 0.1 ];
p = Portfolio;
p = setTurnover(p, 0.3, x0);
disp(p.NumAssets);
disp(p.Turnover);
disp(p.InitPort);
1 0
0.3000
0.1200
0.0900
```

0.0800
0.0700
0.1000
0.1000
0.1500
0.1100
0.0800
0.1000

For an example of setting turnover, see "Portfolio Analysis with Turnover Constraints".
setTurnover implements scalar expansion on the argument for the initial portfolio. If the NumAssets property is already set in the Portfolio object, a scalar argument for InitPort expands to have the same value across all dimensions. In addition, setTurnover lets you specify NumAssets as an optional argument. To clear turnover from your Portfolio object, use the Portfolio object or setTurnover with empty inputs for the properties to be cleared.

## See Also

Portfolio| setBounds | setBudget | setDefaultConstraints| setEquality| setGroupRatio | setGroups | setInequality| setOneWayTurnover | setTrackingError|setTrackingPort|setTurnover

## Related Examples

- "Creating the Portfolio Object" on page 4-28
- "Working with Portfolio Constraints Using Defaults" on page 4-68
- "Validate the Portfolio Problem for Portfolio Object" on page 4-110
- "Estimate Efficient Portfolios for Entire Efficient Frontier for Portfolio Object" on page 4-115
- "Estimate Efficient Frontiers for Portfolio Object" on page 4-140
- "Constraint Specification Using a Portfolio Object" on page 3-34
- "Portfolio Analysis with Turnover Constraints"
- "Asset Allocation Case Study" on page 4-196
- "Portfolio Optimization Examples" on page 4-167
- "Portfolio Analysis with Turnover Constraints"


## More About

- "Portfolio Object" on page 4-23
- "Portfolio Optimization Theory" on page 4-3
- "Portfolio Object Workflow" on page 4-21
- "Setting Up a Tracking Portfolio" on page 4-45


## External Websites

- Using MATLAB to Optimize Portfolios with Financial Toolbox (33 min 24 sec )
- MATLAB for Advanced Portfolio Construction and Stock Selection Models ( 30 min 28 sec)


## Working with One-Way Turnover Constraints Using Portfolio Object

One-way turnover constraints are optional constraints (see "One-Way Turnover Constraints" on page 4-16) that enforce upper bounds on net purchases or net sales. Oneway turnover constraints can be set using the Portfolio object or the setOneWayTurnover function. One-way turnover constraints depend upon an initial or current portfolio, which is assumed to be zero if not set when the turnover constraints are set. One-way turnover constraints have properties BuyTurnover, for the upper bound on net purchases, SellTurnover, for the upper bound on net sales, and InitPort, for the portfolio against which turnover is computed.

## Setting One-Way Turnover Constraints Using the Portfolio Function

The Properties for the one-way turnover constraints are set using the Portfolio object. Suppose that you have an initial portfolio with 10 assets in a variable x 0 and you want to ensure that turnover on purchases is no more than $30 \%$ and turnover on sales is no more than $20 \%$ of the initial portfolio. To set these turnover constraints:

```
x0 = [ 0.12; 0.09; 0.08; 0.07; 0.1; 0.1; 0.15; 0.11; 0.08; 0.1 ];
p = Portfolio('BuyTurnover', 0.3, 'SellTurnover', 0.2, 'InitPort', x0);
disp(p.NumAssets);
disp(p.BuyTurnover);
disp(p.SellTurnover);
disp(p.InitPort);
    1 0
    0.3000
    0.2000
    0.1200
    0.0900
    0.0800
    0.0700
    0.1000
    0.1000
    0.1500
    0.1100
    0.0800
    0.1000
```

If the NumAssets or InitPort properties are not set before or when the turnover constraint is set, various rules are applied to assign default values to these properties (see "Setting Up an Initial or Current Portfolio" on page 4-42).

## Setting Turnover Constraints Using the setOneWayTurnover Function

You can also set properties for portfolio turnover using setOneWayTurnover to specify the upper bounds for turnover on purchases (BuyTurnover) and sales (SellTurnover) and an initial portfolio. Suppose that you have an initial portfolio of 10 assets in a variable x0 and want to ensure that turnover on purchases is no more than $30 \%$ and that turnover on sales is no more than $20 \%$ of the initial portfolio. Given a Portfolio object p, use setOneWayTurnover to set the turnover constraints with and without the initial portfolio being set previously:

```
x0 = [ 0.12; 0.09; 0.08; 0.07; 0.1; 0.1; 0.15; 0.11; 0.08; 0.1 ];
p = Portfolio('InitPort', x0);
p = setOneWayTurnover(p, 0.3, 0.2);
disp(p.NumAssets);
disp(p.BuyTurnover);
disp(p.SellTurnover);
disp(p.InitPort);
    1 0
    0.3000
    0.2000
    0.1200
    0.0900
    0.0800
    0.0700
    0.1000
    0.1000
    0.1500
    0.1100
    0.0800
    0.1000
Or
x0 = [ 0.12; 0.09; 0.08; 0.07; 0.1; 0.1; 0.15; 0.11; 0.08; 0.1 ];
p = Portfolio;
```

```
p = setOneWayTurnover(p, 0.3, 0.2, x0);
disp(p.NumAssets);
disp(p.BuyTurnover);
disp(p.SellTurnover);
disp(p.InitPort);
    10
    0.3000
    0.2000
    0.1200
    0.0900
    0.0800
    0.0700
    0.1000
    0.1000
    0.1500
    0.1100
    0.0800
    0.1000
```

setOneWayTurnover implements scalar expansion on the argument for the initial portfolio. If the NumAssets property is already set in the Portfolio object, a scalar argument for InitPort expands to have the same value across all dimensions. In addition, setOneWayTurnover lets you specify NumAssets as an optional argument. To remove one-way turnover from your Portfolio object, use the Portfolio object or setOneWayTurnover with empty inputs for the properties to be cleared.

## See Also

Portfolio| setBounds | setBudget | setDefaultConstraints| setEquality| setGroupRatio | setGroups | setInequality | setOneWayTurnover | setTrackingError|setTrackingPort|setTurnover

## Related Examples

- "Creating the Portfolio Object" on page 4-28
- "Working with Portfolio Constraints Using Defaults" on page 4-68
- "Validate the Portfolio Problem for Portfolio Object" on page 4-110
- "Estimate Efficient Portfolios for Entire Efficient Frontier for Portfolio Object" on page 4-115
- "Estimate Efficient Frontiers for Portfolio Object" on page 4-140
- "Constraint Specification Using a Portfolio Object" on page 3-34
- "Asset Allocation Case Study" on page 4-196
- "Portfolio Optimization Examples" on page 4-167


## More About

- "Portfolio Object" on page 4-23
- "Portfolio Optimization Theory" on page 4-3
- "Portfolio Object Workflow" on page 4-21
- "Setting Up a Tracking Portfolio" on page 4-45


## External Websites

- Using MATLAB to Optimize Portfolios with Financial Toolbox (33 min 24 sec )
- MATLAB for Advanced Portfolio Construction and Stock Selection Models (30 min 28 sec)


## Working with Tracking Error Constraints Using Portfolio Object

Tracking error constraints are optional constraints (see "Tracking Error Constraints" on page 4-17) that measure the risk relative to a portfolio called a tracking portfolio. Tracking error constraints can be set using the Portfolio object or the setTrackingError function.

The tracking error constraint is an optional quadratic constraint that enforces an upper bound on tracking error, which is the relative risk between a portfolio and a designated tracking portfolio. For more information, see "Tracking Error Constraints" on page 4-17.

The tracking error constraint can be set using the Portfolio object or the setTrackingPort and setTrackingError functions. The tracking error constraint depends on a tracking portfolio, which is assumed to be zero if not set when the tracking error constraint is set. The tracking error constraint has properties TrackingError, for the upper bound on tracking error, and TrackingPort, for the portfolio against which tracking error is computed.

Note The initial portfolio in the Portfolio object property InitPort is distinct from the tracking portfolio in the Portfolio object property TrackingPort.

## Setting Tracking Error Constraints Using the Portfolio Function

The properties for the tracking error constraints are set using the Portfolio object. Suppose that you have a tracking portfolio of 10 assets in a variable $x 0$ and you want to ensure that the tracking error of any portfolio on the efficient frontier is no more than $8 \%$ relative to this portfolio. To set this constraint:

```
x0 = [ 0.12; 0.09; 0.08; 0.07; 0.1; 0.1; 0.15; 0.11; 0.08; 0.1 ];
p = Portfolio('TrackingError', 0.08, 'TrackingPort', x0);
disp(p.NumAssets);
disp(p.TrackingError);
disp(p.TrackingPort);
```

10
0.0800
0.1200
0.0900
0.0800
0.0700
0.1000
0.1000
0.1500
0.1100
0.0800
0.1000

If the NumAssets or TrackingPort properties are not set before or when the tracking error constraint is set, various rules are applied to assign default values to these properties (see "Setting Up a Tracking Portfolio" on page 4-45).

## Setting Tracking Error Constraints Using thesetTrackingError Function

You can also set properties for portfolio tracking error using the setTrackingError function to specify both the upper bound for tracking error and a designated tracking portfolio. Suppose that you have a tracking portfolio of 10 assets in a variable x 0 and want to ensure that tracking error is no more than $8 \%$. Given a Portfolio object p, use setTrackingError to set the tracking error constraint with and without the initial portfolio being set previously:

```
x0 = [ 0.12; 0.09; 0.08; 0.07; 0.1; 0.1; 0.15; 0.11; 0.08; 0.1 ];
p = Portfolio('TrackingPort', x0);
p = setTrackingError(p, 0.08);
disp(p.NumAssets);
disp(p.TrackingError);
disp(p.TrackingPort);
```

10
0.0800
0.1200
0.0900
0.0800
0.0700
0.1000
0.1000
0.1500
0.1100

```
    0.0800
    0.1000
Or
x0 = [ 0.12; 0.09; 0.08; 0.07; 0.1; 0.1; 0.15; 0.11; 0.08; 0.1 ];
p = Portfolio('TrackingPort', x0);
p = setTrackingError(p, 0.08, x0);
disp(p.NumAssets);
disp(p.TrackingError);
disp(p.TrackingPort);
    1 0
    0.0800
    0.1200
    0.0900
    0.0800
    0.0700
    0.1000
    0.1000
    0.1500
    0.1100
    0.0800
    0.1000
```

If the NumAssets or TrackingPort properties are not set before or when the tracking error constraint is set, various rules are applied to assign default values to these properties (see "Setting Up a Tracking Portfolio" on page 4-45).
setTrackingError implements scalar expansion on the argument for the tracking portfolio. If the NumAssets property is already set in the Portfolio object, a scalar argument for TrackingPort expands to have the same value across all dimensions. In addition, setTrackingError lets you specify NumAssets as an optional argument. To clear tracking error from your Portfolio object, use the Portfolio object or setTrackingError with empty inputs for the properties to be cleared.

## See Also

Portfolio| setBounds | setBudget | setDefaultConstraints| setEquality| setGroupRatio | setGroups | setInequality | setOneWayTurnover | setTrackingError|setTrackingPort|setTurnover

## Related Examples

- "Creating the Portfolio Object" on page 4-28
- "Working with Portfolio Constraints Using Defaults" on page 4-68
- "Validate the Portfolio Problem for Portfolio Object" on page 4-110
- "Estimate Efficient Portfolios for Entire Efficient Frontier for Portfolio Object" on page 4-115
- "Estimate Efficient Frontiers for Portfolio Object" on page 4-140
- "Constraint Specification Using a Portfolio Object" on page 3-34
- "Asset Allocation Case Study" on page 4-196
- "Portfolio Optimization Examples" on page 4-167


## More About

- "Portfolio Object" on page 4-23
- "Portfolio Optimization Theory" on page 4-3
- "Portfolio Object Workflow" on page 4-21
- "Setting Up a Tracking Portfolio" on page 4-45


## External Websites

- Using MATLAB to Optimize Portfolios with Financial Toolbox (33 min 24 sec )
- MATLAB for Advanced Portfolio Construction and Stock Selection Models (30 min 28 sec)


## Validate the Portfolio Problem for Portfolio Object

In this section...<br>"Validating a Portfolio Set" on page 4-110<br>"Validating Portfolios" on page 4-112

Sometimes, you may want to validate either your inputs to, or outputs from, a portfolio optimization problem. Although most error checking that occurs during the problem setup phase catches most difficulties with a portfolio optimization problem, the processes to validate portfolio sets and portfolios are time consuming and are best done offline. So, the portfolio optimization tools have specialized functions to validate portfolio sets and portfolios. For information on the workflow when using Portfolio objects, see "Portfolio Object Workflow" on page 4-21.

## Validating a Portfolio Set

Since it is necessary and sufficient that your portfolio set must be a nonempty, closed, and bounded set to have a valid portfolio optimization problem, the estimateBounds function lets you examine your portfolio set to determine if it is nonempty and, if nonempty, whether it is bounded. Suppose that you have the following portfolio set which is an empty set because the initial portfolio at 0 is too far from a portfolio that satisfies the budget and turnover constraint:

```
p = Portfolio('NumAssets', 3, 'Budget', 1);
p = setTurnover(p, 0.3, 0);
```

If a portfolio set is empty, estimateBounds returns NaN bounds and sets the isbounded flag to []:
[lb, ub, isbounded] = estimateBounds(p)
lb =
NaN
NaN
NaN
$u b=$
NaN
NaN

NaN
isbounded =
[]
Suppose that you create an unbounded portfolio set as follows:

```
p = Portfolio('AInequality', [1 -1; 1 1 ], 'bInequality', 0);
[lb, ub, isbounded] = estimateBounds(p)
lb =
    -Inf
    -Inf
ub =
    1.0e-08
    -0.3712
        Inf
isbounded =
    logical
    0
```

In this case, estimateBounds returns (possibly infinite) bounds and sets the isbounded flag to false. The result shows which assets are unbounded so that you can apply bound constraints as necessary.

Finally, suppose that you created a portfolio set that is both nonempty and bounded. estimateBounds not only validates the set, but also obtains tighter bounds which are useful if you are concerned with the actual range of portfolio choices for individual assets in your portfolio set:

```
p = Portfolio;
p = setBudget(p, 1,1);
p = setBounds(p, [ -0.1; 0.2; 0.3; 0.2 ], [ 0.5; 0.3; 0.9; 0.8 ]);
[lb, ub, isbounded] = estimateBounds(p)
```

```
lb =
    -0.1000
        0.2000
        0.3000
        0.2000
ub =
    0.3000
    0.3000
    0.7000
    0.6000
isbounded =
    logical
    1
```

In this example, all but the second asset has tighter upper bounds than the input upper bound implies.

## Validating Portfolios

Given a portfolio set specified in a Portfolio object, you often want to check if specific portfolios are feasible with respect to the portfolio set. This can occur with, for example, initial portfolios and with portfolios obtained from other procedures. The checkFeasibility function determines whether a collection of portfolios is feasible. Suppose that you perform the following portfolio optimization and want to determine if the resultant efficient portfolios are feasible relative to a modified problem.

First, set up a problem in the Portfolio object p, estimate efficient portfolios in pwgt, and then confirm that these portfolios are feasible relative to the initial problem:

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
```

```
p = Portfolio;
p = setAssetMoments(p, m, C);
p = setDefaultConstraints(p);
pwgt = estimateFrontier(p);
checkFeasibility(p, pwgt)
ans =
```

| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

Next, set up a different portfolio problem that starts with the initial problem with an additional a turnover constraint and an equally weighted initial portfolio:

```
q = setTurnover(p, 0.3, 0.25);
checkFeasibility(q, pwgt)
ans =
```

    \(\begin{array}{llllllllll}0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0\end{array}\)
    In this case, only two of the 10 efficient portfolios from the initial problem are feasible relative to the new problem in Portfolio object q. Solving the second problem using checkFeasibility demonstrates that the efficient portfolio for Portfolio object q is feasible relative to the initial problem:

```
qwgt = estimateFrontier(q);
checkFeasibility(p, qwgt)
ans =
\begin{tabular}{llllllllll}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1
\end{tabular}
```


## See Also

Portfolio|checkFeasibility|estimateBounds

## Related Examples

- "Creating the Portfolio Object" on page 4-28
- "Working with Portfolio Constraints Using Defaults" on page 4-68
- "Estimate Efficient Portfolios for Entire Efficient Frontier for Portfolio Object" on page 4-115
- "Estimate Efficient Frontiers for Portfolio Object" on page 4-140
- "Asset Allocation Case Study" on page 4-196
- "Portfolio Optimization Examples" on page 4-167


## More About

- "Portfolio Object" on page 4-23
- "Portfolio Optimization Theory" on page 4-3
- "Portfolio Object Workflow" on page 4-21


## External Websites

- Using MATLAB to Optimize Portfolios with Financial Toolbox (33 min 24 sec )
- MATLAB for Advanced Portfolio Construction and Stock Selection Models (30 min 28 sec)


## Estimate Efficient Portfolios for Entire Efficient Frontier for Portfolio Object

There are two ways to look at a portfolio optimization problem that depends on what you are trying to do. One goal is to estimate efficient portfolios and the other is to estimate efficient frontiers. This section focuses on the former goal and "Estimate Efficient Frontiers for Portfolio Object" on page 4-140 focuses on the latter goal. For information on the workflow when using Portfolio objects, see "Portfolio Object Workflow" on page 421.

## Obtaining Portfolios Along the Entire Efficient Frontier

The most basic way to obtain optimal portfolios is to obtain points over the entire range of the efficient frontier. Given a portfolio optimization problem in a Portfolio object, the estimateFrontier function computes efficient portfolios spaced evenly according to the return proxy from the minimum to maximum return efficient portfolios. The number of portfolios estimated is controlled by the hidden property defaultNumPorts which is set to 10. A different value for the number of portfolios estimated is specified as input to estimateFrontier. This example shows the default number of efficient portfolios over the entire range of the efficient frontier:

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
```

p = Portfolio;
$p=\operatorname{set} A s s e t M o m e n t s(p, m, C)$;
$p=\operatorname{setDefaultConstraints(p);~}$
pwgt = estimateFrontier(p);
disp(pwgt);

| 0.8891 | 0.7215 | 0.5540 | 0.3865 | 0.2190 | 0.0515 | 0 | 0 | 0 | 0 |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 0.0369 | 0.1289 | 0.2209 | 0.3129 | 0.4049 | 0.4969 | 0.4049 | 0.2314 | 0.0579 | 0 |
| 0.0404 | 0.0567 | 0.0730 | 0.0893 | 0.1056 | 0.1219 | 0.1320 | 0.1394 | 0.1468 | 0 |
| 0.0336 | 0.0929 | 0.1521 | 0.2113 | 0.2705 | 0.3297 | 0.4630 | 0.6292 | 0.7953 | 1.0000 |

If you want only four portfolios in the previous example:

```
pwgt = estimateFrontier(p, 4);
disp(pwgt);
```

$$
\begin{array}{llll}
0.8891 & 0.3865 & 0 & 0
\end{array}
$$

| 0.0369 | 0.3129 | 0.4049 | 0 |
| ---: | ---: | ---: | ---: |
| 0.0404 | 0.0893 | 0.1320 | 0 |
| 0.0336 | 0.2113 | 0.4630 | 1.0000 |

Starting from the initial portfolio, estimateFrontier also returns purchases and sales to get from your initial portfolio to each efficient portfolio on the efficient frontier. For example, given an initial portfolio in pwgt0, you can obtain purchases and sales:

```
pwgt0 = [ 0.3; 0.3; 0.2; 0.1 ];
p = setInitPort(p, pwgt0);
[pwgt, pbuy, psell] = estimateFrontier(p);
display(pwgt);
display(pbuy);
display(psell);
pwgt =
\begin{tabular}{rrrrrrrrrr}
0.8891 & 0.7215 & 0.5540 & 0.3865 & 0.2190 & 0.0515 & 0 & 0 & 0 & 0 \\
0.0369 & 0.1289 & 0.2209 & 0.3129 & 0.4049 & 0.4969 & 0.4049 & 0.2314 & 0.0579 & 0 \\
0.0404 & 0.0567 & 0.0730 & 0.0893 & 0.1056 & 0.1219 & 0.1320 & 0.1394 & 0.1468 & 0 \\
0.0336 & 0.0929 & 0.1521 & 0.2113 & 0.2705 & 0.3297 & 0.4630 & 0.6292 & 0.7953 & 1.0000
\end{tabular}
```



If you do not specify an initial portfolio, the purchase and sale weights assume that your initial portfolio is 0 .

## See Also

Portfolio |estimateFrontier|estimateFrontierByReturn | estimateFrontierByRisk|estimateFrontierByRisk estimateFrontierLimits|estimateMaxSharpeRatio|estimatePortMoments| estimatePortReturn|estimatePortRisk|setSolver

## Related Examples

- "Obtaining Endpoints of the Efficient Frontier" on page 4-118
- "Obtaining Efficient Portfolios for Target Returns" on page 4-121
- "Obtaining Efficient Portfolios for Target Risks" on page 4-125
- "Efficient Portfolio That Maximizes Sharpe Ratio" on page 4-128
- "Plotting the Efficient Frontier for a Portfolio Object" on page 4-143
- "Creating the Portfolio Object" on page 4-28
- "Working with Portfolio Constraints Using Defaults" on page 4-68
- "Estimate Efficient Frontiers for Portfolio Object" on page 4-140
- "Asset Allocation Case Study" on page 4-196
- "Portfolio Optimization Examples" on page 4-167


## More About

- "Portfolio Object" on page 4-23
- "Portfolio Optimization Theory" on page 4-3
- "Portfolio Object Workflow" on page 4-21


## External Websites

- Using MATLAB to Optimize Portfolios with Financial Toolbox (33 min 24 sec )
- MATLAB for Advanced Portfolio Construction and Stock Selection Models (30 min 28 sec)


## Obtaining Endpoints of the Efficient Frontier

Often, you might be interested in the endpoint portfolios for the efficient frontier. Suppose that you want to determine the range of returns from minimum to maximum to refine a search for a portfolio with a specific target return. Use the estimateFrontierLimits function to obtain the endpoint portfolios:

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
p = Portfolio;
p = setAssetMoments(p, m, C);
p = setDefaultConstraints(p);
pwgt = estimateFrontierLimits(p);
disp(pwgt);
    0.8891 0
    0.0369 0
    0.0404 0
    0.0336 1.0000
```

The estimatePortMoments function shows the range of risks and returns for efficient portfolios:

```
[prsk, pret] = estimatePortMoments(p, pwgt);
disp([prsk, pret]);
0.0769 0.0590
0.3500 0.1800
```

Starting from an initial portfolio, estimateFrontierLimits also returns purchases and sales to get from the initial portfolio to the endpoint portfolios on the efficient frontier. For example, given an initial portfolio in pwgt0, you can obtain purchases and sales:

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
```

```
p = Portfolio;
p = setAssetMoments(p, m, C);
p = setDefaultConstraints(p);
pwgt0 = [ 0.3; 0.3; 0.2; 0.1 ];
p = setInitPort(p, pwgt0);
[pwgt, pbuy, psell] = estimateFrontierLimits(p);
display(pwgt);
display(pbuy);
display(psell);
pwgt =
    0.8891 0
    0.0369 0
    0.0404 0
    0.0336 1.0000
pbuy =
\begin{tabular}{rr}
0.5891 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0.9000
\end{tabular}
psell =
\begin{tabular}{rr}
0 & 0.3000 \\
0.2631 & 0.3000 \\
0.1596 & 0.2000 \\
0.0664 & 0
\end{tabular}
```

If you do not specify an initial portfolio, the purchase and sale weights assume that your initial portfolio is 0 .

## See Also

Portfolio|estimateFrontier|estimateFrontierByReturn | estimateFrontierByRisk|estimateFrontierByRisk|
estimateFrontierLimits|estimateMaxSharpeRatio|estimatePortMoments | estimatePortReturn|estimatePortRisk|setSolver

## Related Examples

- "Estimate Efficient Portfolios for Entire Efficient Frontier for Portfolio Object" on page 4-115
- "Creating the Portfolio Object" on page 4-28
- "Working with Portfolio Constraints Using Defaults" on page 4-68
- "Estimate Efficient Frontiers for Portfolio Object" on page 4-140
- "Asset Allocation Case Study" on page 4-196
- "Portfolio Optimization Examples" on page 4-167


## More About

- "Portfolio Object" on page 4-23
- "Portfolio Optimization Theory" on page 4-3
- "Portfolio Object Workflow" on page 4-21


## External Websites

- Using MATLAB to Optimize Portfolios with Financial Toolbox (33 min 24 sec )
- MATLAB for Advanced Portfolio Construction and Stock Selection Models ( 30 min 28 sec )


## Obtaining Efficient Portfolios for Target Returns

To obtain efficient portfolios that have targeted portfolio returns, the estimateFrontierByReturn function accepts one or more target portfolios returns and obtains efficient portfolios with the specified returns. For example, assume that you have a universe of four assets where you want to obtain efficient portfolios with target portfolio returns of $6 \%, 9 \%$, and $12 \%$ :

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
p = Portfolio;
p = setAssetMoments(p, m, C);
p = setDefaultConstraints(p);
pwgt = estimateFrontierByReturn(p, [0.06, 0.09, 0.12]);
display(pwgt);
pwgt =
\begin{tabular}{lll}
0.8772 & 0.5032 & 0.1293 \\
0.0434 & 0.2488 & 0.4541 \\
0.0416 & 0.0780 & 0.1143 \\
0.0378 & 0.1700 & 0.3022
\end{tabular}
```

Sometimes, you can request a return for which no efficient portfolio exists. Based on the previous example, suppose that you want a portfolio with a $5 \%$ return (which is the return of the first asset). A portfolio that is fully invested in the first asset, however, is inefficient. estimateFrontierByReturn warns if your target returns are outside the range of efficient portfolio returns and replaces it with the endpoint portfolio of the efficient frontier closest to your target return:

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336
    0 0.0119 0.0336 0.1225 ];
p = Portfolio;
p = setAssetMoments(p, m, C);
p = setDefaultConstraints(p);
pwgt = estimateFrontierByReturn(p, [0.05, 0.09, 0.12]);
```

```
display(pwgt);
Warning: One or more target return values are outside the feasible range [ 0.0590468, 0.18 ].
    Will return portfolios associated with endpoints of the range for these values.
> In Portfolio.estimateFrontierByReturn at 70
pwgt =
    0.8891 0.5032 0.1293
    0.0369 0.2488 0.4541
    0.0404 0.0780 0.1143
    0.0336 0.1700 0.3022
```

The best way to avoid this situation is to bracket your target portfolio returns with estimateFrontierLimits and estimatePortReturn (see "Obtaining Endpoints of the Efficient Frontier" on page 4-118 and "Obtaining Portfolio Risks and Returns" on page 4-140).

```
pret = estimatePortReturn(p, p.estimateFrontierLimits);
display(pret);
pret =
    0.0590
    0.1800
```

This result indicates that efficient portfolios have returns that range between $5.9 \%$ and 18\%.

If you have an initial portfolio, estimateFrontierByReturn also returns purchases and sales to get from your initial portfolio to the target portfolios on the efficient frontier. For example, given an initial portfolio in pwgt0, to obtain purchases and sales with target returns of $6 \%, 9 \%$, and $12 \%$ :

```
pwgt0 = [ 0.3; 0.3; 0.2; 0.1 ];
p = setInitPort(p, pwgt0);
[pwgt, pbuy, psell] = estimateFrontierByReturn(p, [0.06, 0.09, 0.12]);
display(pwgt);
display(pbuy);
display(psell);
pwgt =
\begin{tabular}{lll}
0.8772 & 0.5032 & 0.1293 \\
0.0434 & 0.2488 & 0.4541 \\
0.0416 & 0.0780 & 0.1143 \\
0.0378 & 0.1700 & 0.3022
\end{tabular}
```

pbuy =

| 0.5772 | 0.2032 | 0 |
| ---: | ---: | ---: |
| 0 | 0 | 0.1541 |
| 0 | 0 | 0 |
| 0 | 0.0700 | 0.2022 |

psell =

| 0 | 0 | 0.1707 |
| ---: | ---: | ---: |
| 0.2566 | 0.0512 | 0 |
| 0.1584 | 0.1220 | 0.0857 |
| 0.0622 | 0 | 0 |

If you do not have an initial portfolio, the purchase and sale weights assume that your initial portfolio is 0 .

## See Also

Portfolio|estimateFrontier|estimateFrontierByReturn | estimateFrontierByRisk|estimateFrontierByRisk| estimateFrontierLimits|estimateMaxSharpeRatio|estimatePortMoments| estimatePortReturn|estimatePortRisk|setSolver

## Related Examples

- "Estimate Efficient Portfolios for Entire Efficient Frontier for Portfolio Object" on page 4-115
- "Creating the Portfolio Object" on page 4-28
- "Working with Portfolio Constraints Using Defaults" on page 4-68
- "Estimate Efficient Frontiers for Portfolio Object" on page 4-140
- "Asset Allocation Case Study" on page 4-196
- "Portfolio Optimization Examples" on page 4-167


## More About

- "Portfolio Object" on page 4-23
- "Portfolio Optimization Theory" on page 4-3
- "Portfolio Object Workflow" on page 4-21


## External Websites

- Using MATLAB to Optimize Portfolios with Financial Toolbox (33 min 24 sec )
- MATLAB for Advanced Portfolio Construction and Stock Selection Models (30 min 28 sec)


## Obtaining Efficient Portfolios for Target Risks

To obtain efficient portfolios that have targeted portfolio risks, the estimateFrontierByRisk function accepts one or more target portfolio risks and obtains efficient portfolios with the specified risks. Suppose that you have a universe of four assets where you want to obtain efficient portfolios with target portfolio risks of $12 \%$, $14 \%$, and $16 \%$.

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
p = Portfolio;
p = setAssetMoments(p, m, C);
p = setDefaultConstraints(p);
pwgt = estimateFrontierByRisk(p, [0.12, 0.14, 0.16]);
display(pwgt);
pwgt =
\begin{tabular}{lll}
0.3984 & 0.2659 & 0.1416 \\
0.3064 & 0.3791 & 0.4474 \\
0.0882 & 0.1010 & 0.1131 \\
0.2071 & 0.2540 & 0.2979
\end{tabular}
```

Sometimes, you can request a risk for which no efficient portfolio exists. Based on the previous example, suppose that you want a portfolio with $7 \%$ risk (individual assets in this universe have risks ranging from $8 \%$ to $35 \%$ ). It turns out that a portfolio with $7 \%$ risk cannot be formed with these four assets. estimateFrontierByRisk warns if your target risks are outside the range of efficient portfolio risks and replaces it with the endpoint of the efficient frontier closest to your target risk:

```
pwgt = estimateFrontierByRisk(p, 0.07)
Warning: One or more target risk values are outside the feasible range [ 0.0769288, 0.35 ].
    Will return portfolios associated with endpoints of the range for these values.
> In Portfolio.estimateFrontierByRisk at }8
```

pwgt =
0.8891
0.0369
0.0404
0.0336

The best way to avoid this situation is to bracket your target portfolio risks with estimateFrontierLimits and estimatePortRisk (see "Obtaining Endpoints of the Efficient Frontier" on page 4-118 and "Obtaining Portfolio Risks and Returns" on page 4140).

```
prsk = estimatePortRisk(p, p.estimateFrontierLimits);
display(prsk);
prsk =
    0.0769
    0.3500
```

This result indicates that efficient portfolios have risks that range from $7.7 \%$ to $35 \%$.
Starting with an initial portfolio, estimateFrontierByRisk also returns purchases and sales to get from your initial portfolio to the target portfolios on the efficient frontier. For example, given an initial portfolio in pwgt0, you can obtain purchases and sales from the example with target risks of $12 \%, 14 \%$, and $16 \%$ :

```
pwgt0 = [ 0.3; 0.3; 0.2; 0.1 ];
p = setInitPort(p, pwgt0);
[pwgt, pbuy, psell] = estimateFrontierByRisk(p, [0.12, 0.14, 0.16]);
display(pwgt);
display(pbuy);
display(psell);
pwgt =
\begin{tabular}{lll}
0.3984 & 0.2659 & 0.1416 \\
0.3064 & 0.3791 & 0.4474 \\
0.0882 & 0.1010 & 0.1131 \\
0.2071 & 0.2540 & 0.2979
\end{tabular}
pbuy =
\begin{tabular}{rrr}
0.0984 & 0 & 0 \\
0.0064 & 0.0791 & 0.1474 \\
0 & 0 & 0 \\
0.1071 & 0.1540 & 0.1979
\end{tabular}
psell =
```

| 0 | 0.0341 | 0.1584 |
| ---: | ---: | ---: |
| 0 | 0 | 0 |
| 0.1118 | 0.0990 | 0.0869 |
| 0 | 0 | 0 |

If you do not specify an initial portfolio, the purchase and sale weights assume that your initial portfolio is 0 .

## See Also

Portfolio|estimateFrontier|estimateFrontierByReturn |
estimateFrontierByRisk|estimateFrontierLimits| estimateMaxSharpeRatio|estimatePortMoments|estimatePortReturn| estimatePortRisk|setSolver

## Related Examples

- "Estimate Efficient Portfolios for Entire Efficient Frontier for Portfolio Object" on page 4-115
- "Creating the Portfolio Object" on page 4-28
- "Working with Portfolio Constraints Using Defaults" on page 4-68
- "Estimate Efficient Frontiers for Portfolio Object" on page 4-140
- "Asset Allocation Case Study" on page 4-196
- "Portfolio Optimization Examples" on page 4-167


## More About

- "Portfolio Object" on page 4-23
- "Portfolio Optimization Theory" on page 4-3
- "Portfolio Object Workflow" on page 4-21


## External Websites

- Using MATLAB to Optimize Portfolios with Financial Toolbox (33 min 24 sec )
- MATLAB for Advanced Portfolio Construction and Stock Selection Models (30 min 28 sec)


## Efficient Portfolio That Maximizes Sharpe Ratio

The Sharpe ratio is defined as the ratio

$$
\frac{\mu(x)-r_{0}}{\sqrt{\sum(x)}}
$$

where $x \in R^{n}$ and $r_{0}$ is the risk-free rate ( $\mu$ and $\Sigma$ proxies for portfolio return and risk). For more information, see "Portfolio Optimization Theory" on page 4-3.

Portfolios that maximize the Sharpe ratio are portfolios on the efficient frontier that satisfy several theoretical conditions in finance. For example, such portfolios are called tangency portfolios since the tangent line from the risk-free rate to the efficient frontier taps the efficient frontier at portfolios that maximize the Sharpe ratio.

To obtain efficient portfolios that maximizes the Sharpe ratio, the estimateMaxSharpeRatio function accepts a Portfolio object and obtains efficient portfolios that maximize the Sharpe Ratio.

Suppose that you have a universe with four risky assets and a riskless asset and you want to obtain a portfolio that maximizes the Sharpe ratio, where, in this example, $r_{0}$ is the return for the riskless asset.

```
r0 = 0.03;
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
p = Portfolio('RiskFreeRate', r0);
p = setAssetMoments(p, m, C);
p = setDefaultConstraints(p);
pwgt = estimateMaxSharpeRatio(p);
display(pwgt);
pwgt =
    0.4251
    0.2917
```


### 0.0856 <br> 0.1977

If you start with an initial portfolio, estimateMaxSharpeRatio also returns purchases and sales to get from your initial portfolio to the portfolio that maximizes the Sharpe ratio. For example, given an initial portfolio in pwgt0, you can obtain purchases and sales from the previous example:

```
pwgt0 = [ 0.3; 0.3; 0.2; 0.1 ];
p = setInitPort(p, pwgt0);
[pwgt, pbuy, psell] = estimateMaxSharpeRatio(p);
display(pwgt);
display(pbuy);
display(psell);
pwgt =
    0.4251
    0.2917
    0.0856
    0.1977
pbuy =
    0.1251
            0
            0
    0.0977
psell =
    0
    0.0083
    0.1144
            0
```

If you do not specify an initial portfolio, the purchase and sale weights assume that you initial portfolio is 0 .

## See Also

Portfolio|estimateFrontier|estimateFrontierByReturn|
estimateFrontierByRisk|estimateFrontierByRisk|
estimateFrontierLimits|estimateMaxSharpeRatio|estimatePortMoments| estimatePortReturn|estimatePortRisk|setSolver

## Related Examples

- "Estimate Efficient Portfolios for Entire Efficient Frontier for Portfolio Object" on page 4-115
- "Creating the Portfolio Object" on page 4-28
- "Working with Portfolio Constraints Using Defaults" on page 4-68
- "Estimate Efficient Frontiers for Portfolio Object" on page 4-140
- "Asset Allocation Case Study" on page 4-196
- "Portfolio Optimization Examples" on page 4-167


## More About

- "Portfolio Object" on page 4-23
- "Portfolio Optimization Theory" on page 4-3
- "Portfolio Object Workflow" on page 4-21


## External Websites

- Using MATLAB to Optimize Portfolios with Financial Toolbox (33 min 24 sec )
- MATLAB for Advanced Portfolio Construction and Stock Selection Models ( 30 min 28 sec)


## Choosing and Controlling the Solver for Mean-Variance Portfolio Optimization

The default solver for mean-variance portfolio optimization is lcprog, which implements a linear complementarity programming (LCP) algorithm. Although lcprog works for most problems, you can adjust arguments to control the algorithm. Alternatively, the meanvariance portfolio optimization tools let you use any of the variations of quadprog from Optimization Toolbox ${ }^{\mathrm{TM}}$ software. Unlike Optimization Toolbox which uses the trust-region-reflective algorithm as the default algorithm for quadprog, the portfolio optimization tools use the interior-point-convex algorithm. For details about quadprog and quadratic programming algorithms and options, see "Quadratic Programming Algorithms" (Optimization Toolbox).

To modify either lcprog or to specify quadprog as your solver, use the setSolver function to set the hidden properties solverType and solverOptions that specify and control the solver. Since the solver properties are hidden, you cannot set these using the Portfolio object. The default solver is lcprog so you do not need to use setSolver to specify this solver. To use quadprog, you must set up the interior-point-convex version of quadprog using:

```
p = Portfolio;
p = setSolver(p, 'quadprog');
display(p.solverType);
quadprog
```

and you can switch back tol cprog with:

```
p = setSolver(p, 'lcprog');
display(p.solverType);
lcprog
```

In both cases, setSolver sets up default options associated with either solver. If you want to specify additional options associated with a given solver, setSolver accepts these options with argument name-value pair arguments in the function call. For example, if you intend to use quadprog and want to use the active-set algorithm, call setSolver with:

```
p = setSolver(p, 'quadprog', 'Algorithm', 'active-set');
```

display(p.solverOptions.Algorithm);
active-set

In addition, if you want to specify any of the options for quadprog that are normally set through optimoptions, setSolver accepts an optimoptions object as the second argument. For example, you can start with the default options for quadprog set by setSolver and then change the algorithm to 'trust-region-reflective' with no displayed output:

```
p = Portfolio;
options = optimoptions('quadprog', 'Algorithm', 'trust-region-reflective', 'Display', 'off');
p = setSolver(p, 'quadprog', options);
display(p.solverOptions.Algorithm);
display(p.solverOptions.Display);
trust-region-reflective
off
```

The mixed integer nonlinear programming (MINLP) solver, configured using setSolverMINLP, enables you to specify associated solver options for portfolio optimization for a Portfolio object. When any one, or any combination of 'Conditional ' BoundType, MinNumAssets, or MaxNumAssets constraints are active, the portfolio problem is formulated by adding NumAssets binary variables, where 0 indicates not invested, and 1 is invested. For more information on using 'Conditional' BoundType, see setBounds. For more information on specifying MinNumAssets and MaxNumAssets, see setMinMaxNumAssets.

When using the estimate functions with a portfolio object where 'Conditional' BoundType, MinNumAssets, or MaxNumAssets constraints are active, the mixed integer nonlinear programming (MINLP) solver is automatically used.

The following table provides guidelines for using setSolver and setSolverMINLP.

| Portfolio <br> Problem | Portfolio <br> Function | Type of <br> Optimization <br> Problem | Main Solver | Helper Solver |
| :--- | :--- | :--- | :--- | :--- |
| Portfolio without <br> tracking error <br> constraints | estimateFront <br> ierByRisk | Optimizing a <br> portfolio for a <br> certain risk level <br> introduces a <br> nonlinear <br> constraint. <br> Therefore, this <br> problem has a <br> linear objective <br> with linear and <br> nonlinear <br> constraints. | 'fmincon' <br> using <br> setSolver | 'quadprog' or <br> ' lcprog'' |
| Portfolio without <br> tracking error <br> constraints | estimateFront <br> ierByReturn | Quadratic <br> objective with <br> linear <br> constraints | 'quadprog' or <br> ' lcprog' using <br> setSolver | 'quadprog' or <br> 'lcprog'' |
| Portfolio without <br> tracking error <br> constraints | estimateFront <br> ierLimits | Quadratic or <br> linear objective <br> with linear <br> constraints | For 'min': <br> quadratic <br> objective, <br> 'quadprog' or <br> 'lcprog' using <br> setSolver | Not applicable |


| Portfolio <br> Problem | Portfolio <br> Function | Type of <br> Optimization <br> Problem | Main Solver | Helper Solver |
| :--- | :--- | :--- | :--- | :--- |
| Portfolio without <br> tracking error <br> constraints | estimateMaxSh <br> arpeRatio | Quadratic <br> objective with <br> linear <br> constraints | 'quadprog' <br> using <br> setSolver | Because <br> estimateMaxSh <br> arpeRatio <br> internally calls <br> estimateFront <br> ierLimits, all <br> solvers needed <br> by <br> estimateFront <br> ierLimits will <br> be the helper <br> solvers |
| Portfolio with <br> tracking error <br> constraints | estimateFront <br> ierByRisk | Linear objective <br> with linear and <br> nonlinear <br> constraints | 'fmincon' <br> using <br> setSolver | Not applicable |
| Portfolio with <br> tracking error <br> constraints | estimateFront <br> ierByReturn | Linear objective <br> with linear and <br> nonlinear <br> constraints | 'fmincon' <br> using <br> setSolver | Not applicable |
| Portfolio with <br> tracking error <br> constraints | estimateFront <br> ierLimits | Quadratic (min <br> risk problem) or <br> linear (max <br> return problem) <br> objective with <br> linear and <br> nonlinear <br> constraints | 'fmincon ' <br> using <br> setSolver | Not applicable |
| Portfolio with <br> tracking error <br> constraints | estimateMaxSh <br> arpeRatio | Quadratic <br> objective with <br> linear and <br> nonlinear <br> constraints | ' fmincon ' <br> using <br> setSolver | Not applicable |


| Portfolio <br> Problem | Portfolio <br> Function | Type of <br> Optimization <br> Problem | Main Solver | Helper Solver |
| :--- | :--- | :--- | :--- | :--- |
| Portfolio with <br> active <br> Conditional ' <br> BoundType, <br> MinNumAssets, <br> and <br> MaxNumAssets | estimateFront <br> ierByRisk | The problem is <br> formulated by <br> introducing <br> NumAssets <br> binary variables <br> to indicate <br> whether the <br> corresponding <br> asset is invested <br> or not. <br> Therefore, it <br> requires a mixed <br> integer <br> nonlinear <br> programming <br> solver. Three <br> types of MINLP <br> solvers are <br> offered, see <br> setSolverMINL <br> P. | Mixed integer <br> nonlinear <br> programming <br> using (MINLP) <br> PetSolverMINL | ' quadprog' or <br> ' fmincon ' are <br> used when the <br> estimate <br> functions reduce <br> the problem into |


| Portfolio Problem | Portfolio Function | Type of Optimization Problem | Main Solver | Helper Solver |
| :---: | :---: | :---: | :---: | :---: |
| Portfolio with active <br> 'Conditional' <br> BoundType, <br> MinNumAssets, <br> and <br> MaxNumAssets | estimateFront ierByReturn | The problem is formulated by introducing NumAssets binary variables to indicate whether the corresponding asset is invested or not. <br> Therefore, it requires a mixed integer nonlinear programming solver. Three types of MINLP solvers are offered, see setSolverMINL P. | Mixed integer nonlinear programming solver (MINLP) using setSolverMINL P | 'quadprog' or <br> 'fmincon' are used when the estimate functions reduce the problem into NLP. These two solvers can be configured through setSolver |


| Portfolio <br> Problem | Portfolio <br> Function | Type of <br> Optimization <br> Problem | Main Solver | Helper Solver |
| :--- | :--- | :--- | :--- | :--- |
| Portfolio with <br> active <br> Conditional ' <br> BoundType, <br> MinNumAssets, <br> and <br> MaxNumAssets | estimateFront <br> ierLimits | The problem is <br> formulated by <br> introducing <br> NumAssets <br> binary variables <br> to indicate <br> whether the <br> corresponding <br> asset is invested <br> or not. <br> Therefore, it <br> requires a mixed <br> integer <br> nonlinear <br> programming <br> solver. Three <br> types of MINLP <br> solvers are <br> offered, see <br> setSolverMINL <br> P. | Mixed integer <br> nonlinear <br> programming <br> solver (MINLP) <br> setSolverMINL | ' quadprog' or <br> ' fmincon ' are <br> used when the <br> estimate <br> functions reduce <br> the problem into |


| Portfolio <br> Problem | Portfolio <br> Function | Type of <br> Optimization <br> Problem | Main Solver | Helper Solver |
| :--- | :--- | :--- | :--- | :--- |
| Portfolio with <br> active <br> Conditional' <br> BoundType, <br> MinNumAssets, <br> and <br> MaxNumAssets | estimateMaxSh <br> arpeRatio | The problem is <br> formulated by <br> introducing <br> NumAssets <br> binary variables <br> to indicate <br> whether the <br> corresponding <br> asset is invested <br> or not. <br> Therefore, it <br> requires a mixed <br> integer <br> nonlinear <br> programming <br> solver. Three <br> types of MINLP <br> solvers are <br> offered, see <br> setSolverMINL <br> P. | Mixed integer <br> ponlinear <br> solver (MINLP) <br> using <br> setSolverMINL | ' quadprog' or <br> ' fmincon' are <br> used, when the <br> estimate <br> functions reduce <br> the problem into |

## See Also

Portfolio|estimateFrontier|estimateFrontierByReturn| estimateFrontierByRisk|estimateFrontierByRisk| estimateFrontierLimits |estimateMaxSharpeRatio|estimatePortMoments | estimatePortReturn|estimatePortRisk|setSolver|setSolverMINLP

## Related Examples

- "Estimate Efficient Portfolios for Entire Efficient Frontier for Portfolio Object" on page 4-115
- "Creating the Portfolio Object" on page 4-28
- "Working with Portfolio Constraints Using Defaults" on page 4-68
- "Estimate Efficient Frontiers for Portfolio Object" on page 4-140
- "Asset Allocation Case Study" on page 4-196
- "Portfolio Optimization Examples" on page 4-167
- "Portfolio Optimization with Semicontinuous and Cardinality Constraints"


## More About

- "Portfolio Object" on page 4-23
- "Portfolio Optimization Theory" on page 4-3
- "Portfolio Object Workflow" on page 4-21
- "Working with 'Conditional' BoundType, MinNumAssets, and MaxNumAssets Constraints Using Portfolio Object" on page 4-94


## External Websites

- Using MATLAB to Optimize Portfolios with Financial Toolbox (33 min 24 sec )
- MATLAB for Advanced Portfolio Construction and Stock Selection Models (30 min 28 sec)


## Estimate Efficient Frontiers for Portfolio Object

Whereas "Estimate Efficient Portfolios for Entire Efficient Frontier for Portfolio Object" on page 4-115 focused on estimation of efficient portfolios, this section focuses on the estimation of efficient frontiers. For information on the workflow when using Portfolio objects, see "Portfolio Object Workflow" on page 4-21.

## Obtaining Portfolio Risks and Returns

Given any portfolio and, in particular, efficient portfolios, the functions estimatePortReturn, estimatePortRisk, and estimatePortMoments provide estimates for the return (or return proxy), risk (or the risk proxy), and, in the case of mean-variance portfolio optimization, the moments of expected portfolio returns. Each function has the same input syntax but with different combinations of outputs. Suppose that you have this following portfolio optimization problem that gave you a collection of portfolios along the efficient frontier in pwgt:

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
pwgt0 = [ 0.3; 0.3; 0.2; 0.1 ];
p = Portfolio('AssetMean', m, 'AssetCovar', C, 'InitPort', pwgt0);
p = setDefaultConstraints(p);
pwgt = estimateFrontier(p);
```

Given pwgt0 and pwgt, use the portfolio risk and return estimation functions to obtain risks and returns for your initial portfolio and the portfolios on the efficient frontier:

```
[prsk0, pret0] = estimatePortMoments(p, pwgt0);
[prsk, pret] = estimatePortMoments(p, pwgt);
```

or
prsk0 = estimatePortRisk(p, pwgt0);
pret0 = estimatePortReturn(p, pwgt0);
prsk = estimatePortRisk(p, pwgt);
pret $=$ estimatePortReturn(p, pwgt);

In either case, you obtain these risks and returns:

```
display(prsk0);
```

display(pret0);

```
display(prsk);
display(pret);
prsk0 =
    0.1103
pret0 =
    0.0870
prsk =
    0.0769
    0.0831
    0.0994
    0.1217
    0.1474
    0.1750
    0.2068
    0.2487
    0.2968
    0.3500
pret =
    0.0590
    0.0725
    0.0859
    0.0994
    0.1128
    0.1262
    0.1397
    0.1531
    0.1666
    0.1800
```

The returns and risks are at the periodicity of the moments of asset returns so that, if you have values for AssetMean and AssetCovar in terms of monthly returns, the estimates for portfolio risk and return are in terms of monthly returns as well. In addition, the estimate for portfolio risk in the mean-variance case is the standard deviation of portfolio returns, not the variance of portfolio returns.

## See Also

Portfolio|estimatePortMoments|estimatePortReturn|plotFrontier

## Related Examples

- "Plotting the Efficient Frontier for a Portfolio Object" on page 4-143
- "Creating the Portfolio Object" on page 4-28
- "Working with Portfolio Constraints Using Defaults" on page 4-68
- "Estimate Efficient Portfolios for Entire Efficient Frontier for Portfolio Object" on page 4-115
- "Postprocessing Results to Set Up Tradable Portfolios" on page 4-149
- "Asset Allocation Case Study" on page 4-196
- "Portfolio Optimization Examples" on page 4-167


## More About

- "Portfolio Object" on page 4-23
- "Portfolio Optimization Theory" on page 4-3
- "Portfolio Object Workflow" on page 4-21


## External Websites

- Using MATLAB to Optimize Portfolios with Financial Toolbox (33 min 24 sec )
- MATLAB for Advanced Portfolio Construction and Stock Selection Models (30 min 28 sec)


## Plotting the Efficient Frontier for a Portfolio Object

The plotFrontier function creates a plot of the efficient frontier for a given portfolio optimization problem. This function accepts several types of inputs and generates a plot with an optional possibility to output the estimates for portfolio risks and returns along the efficient frontier. plotFrontier has four different ways that it can be used. In addition to a plot of the efficient frontier, if you have an initial portfolio in the InitPort property, plotFrontier also displays the return versus risk of the initial portfolio on the same plot. If you have a well-posed portfolio optimization problem set up in a Portfolio object and you use plotFrontier, you get a plot of the efficient frontier with the default number of portfolios on the frontier (the default number is 10 and is maintained in the hidden property defaultNumPorts). This example illustrates a typical use of plotFrontier to create a new plot:

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
pwgt0 = [ 0.3; 0.3; 0.2; 0.1 ];
p = Portfolio('Name', 'Asset Allocation Portfolio', 'InitPort', pwgt0);
p = setAssetMoments(p, m, C);
p = setDefaultConstraints(p);
plotFrontier(p);
```



The Name property appears as the title of the efficient frontier plot if you set it in the Portfolio object. Without an explicit name, the title on the plot would be "Efficient Frontier." If you want to obtain a specific number of portfolios along the efficient frontier, use plotFrontier with the number of portfolios that you want. Suppose that you have the Portfolio object from the previous example and you want to plot 20 portfolios along the efficient frontier and to obtain 20 risk and return values for each portfolio:
[prsk, pret] = plotFrontier(p, 20);
display([pret, prsk]);
$\begin{array}{cc}\text { ans }= & \\ 0.0590 & 0.0769 \\ 0.0654 & 0.0784 \\ 0.0718 & 0.0825 \\ 0.0781 & 0.0890 \\ 0.0845 & 0.0973 \\ 0.0909 & 0.1071 \\ 0.0972 & 0.1179 \\ 0.1036 & 0.1296\end{array}$

| 0.1100 | 0.1418 |
| :--- | :--- |
| 0.1163 | 0.1545 |
| 0.1227 | 0.1676 |
| 0.1291 | 0.1810 |
| 0.1354 | 0.1955 |
| 0.1418 | 0.2128 |
| 0.1482 | 0.2323 |
| 0.1545 | 0.2535 |
| 0.1609 | 0.2760 |
| 0.1673 | 0.2995 |
| 0.1736 | 0.3239 |
| 0.1800 | 0.3500 |



## Plotting Existing Efficient Portfolios

If you already have efficient portfolios from any of the "estimateFrontier" functions (see "Estimate Efficient Portfolios for Entire Efficient Frontier for Portfolio Object" on page 4115), pass them into plotFrontier directly to plot the efficient frontier:

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
pwgt0 = [ 0.3; 0.3; 0.2; 0.1 ];
p = Portfolio('Name', 'Asset Allocation Portfolio', 'InitPort', pwgt0);
p = setAssetMoments(p, m, C);
p = setDefaultConstraints(p);
pwgt = estimateFrontier(p, 20);
plotFrontier(p, pwgt);
```



## Plotting Existing Efficient Portfolio Risks and Returns

If you already have efficient portfolio risks and returns, you can use the interface to plotFrontier to pass them into plotFrontier to obtain a plot of the efficient frontier:

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
```

```
pwgt0 = [ 0.3; 0.3; 0.2; 0.1 ];
p = Portfolio('Name', 'Asset Allocation Portfolio', 'InitPort', pwgt0);
p = setAssetMoments(p, m, C);
p = setDefaultConstraints(p);
[prsk, pret] = estimatePortMoments(p, p.estimateFrontier(20));
plotFrontier(p, prsk, pret);
```



## See Also

Portfolio|estimatePortMoments|estimatePortReturn|plotFrontier

## Related Examples

- "Estimate Efficient Frontiers for Portfolio Object" on page 4-140
- "Creating the Portfolio Object" on page 4-28
- "Working with Portfolio Constraints Using Defaults" on page 4-68
- "Estimate Efficient Portfolios for Entire Efficient Frontier for Portfolio Object" on page 4-115
- "Postprocessing Results to Set Up Tradable Portfolios" on page 4-149
- "Asset Allocation Case Study" on page 4-196
- "Portfolio Optimization Examples" on page 4-167


## More About

- "Portfolio Object" on page 4-23
- "Portfolio Optimization Theory" on page 4-3
- "Portfolio Object Workflow" on page 4-21


## External Websites

- Using MATLAB to Optimize Portfolios with Financial Toolbox (33 min 24 sec )
- MATLAB for Advanced Portfolio Construction and Stock Selection Models (30 min 28 sec)


## Postprocessing Results to Set Up Tradable Portfolios

After obtaining efficient portfolios or estimates for expected portfolio risks and returns, use your results to set up trades to move toward an efficient portfolio. For information on the workflow when using Portfolio objects, see "Portfolio Object Workflow" on page 4-21.

## Setting Up Tradable Portfolios

Suppose that you set up a portfolio optimization problem and obtained portfolios on the efficient frontier. Use the dataset object from Statistics and Machine Learning Toolbox ${ }^{\mathrm{TM}}$ to form a blotter that lists your portfolios with the names for each asset. For example, suppose that you want to obtain five portfolios along the efficient frontier. You can set up a blotter with weights multiplied by 100 to view the allocations for each portfolio:

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
        0.00408 0.0289 0.0204 0.0119;
        0.00192 0.0204 0.0576 0.0336;
        0 0.0119 0.0336 0.1225 ];
pwgt0 = [ 0.3; 0.3; 0.2; 0.1 ];
    p = Portfolio('InitPort', pwgt0);
    p = setAssetList(p, 'Bonds','Large-Cap Equities','Small-Cap Equities','Emerging Equities');
    p = setAssetMoments(p, m, C);
    p = setDefaultConstraints(p);
    pwgt = estimateFrontier(p, 5);
    pnames = cell(1,5);
        for i = 1:5
        pnames{i} = sprintf('Port%d',i);
    end
```

    Blotter = dataset([\{100*pwgt\}, pnames],'obsnames',p.AssetList);
    display(Blotter);
    Blotter =

|  | Port1 | Port2 | Port3 | Port4 | Port5 |
| :--- | ---: | :--- | :--- | ---: | ---: |
| Bonds | 88.906 | 51.216 | 13.525 | 0 | 0 |
| Large-Cap Equities | 3.6875 | 24.387 | 45.086 | 27.479 | 0 |
| Small-Cap Equities | 4.0425 | 7.7088 | 11.375 | 13.759 | 0 |
| Emerging Equities | 3.364 | 16.689 | 30.014 | 58.762 | 100 |

This result indicates that you would invest primarily in bonds at the minimum-risk/ minimum-return end of the efficient frontier (Port1), and that you would invest completely in emerging equity at the maximum-risk/maximum-return end of the efficient frontier (Port5). You can also select a particular efficient portfolio, for example, suppose that you want a portfolio with $15 \%$ risk and you add purchase and sale weights outputs obtained from the "estimateFrontier" functions to set up a trade blotter:

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336
    0 0.0119 0.0336 0.1225 ];
pwgt0 = [ 0.3; 0.3; 0.2; 0.1 ];
p = Portfolio('InitPort', pwgt0);
p = setAssetList(p, 'Bonds','Large-Cap Equities','Small-Cap Equities','Emerging Equities');
p = setAssetMoments(p, m, C);
p = setDefaultConstraints(p)
[pwgt, pbuy, psell] = estimateFrontierByRisk(p, 0.15);
Blotter = dataset([{100*[pwgt0, pwgt, pbuy, psell]}, ...
    {'Initial','Weight', 'Purchases','Sales'}],'obsnames',p.AssetList);
display(Blotter);
Blotter =
\begin{tabular}{llrcr} 
& Initial & Weight & Purchases & Sales \\
Bonds & 30 & 20.299 & 0 & 9.7007 \\
Large-Cap Equities & 30 & 41.366 & 11.366 & 0 \\
Small-Cap Equities & 20 & 10.716 & 0 & 9.2838 \\
Emerging Equities & 10 & 27.619 & 17.619 & 0
\end{tabular}
```

If you have prices for each asset (in this example, they can be ETFs), add them to your blotter and then use the tools of the dataset object to obtain shares and shares to be traded. For an example, see "Asset Allocation Case Study" on page 4-196.

## See Also

Portfolio| checkFeasibility|estimateAssetMoments

## Related Examples

- "Troubleshooting Portfolio Optimization Results" on page 4-152
- "Creating the Portfolio Object" on page 4-28
- "Working with Portfolio Constraints Using Defaults" on page 4-68
- "Estimate Efficient Portfolios for Entire Efficient Frontier for Portfolio Object" on page 4-115
- "Estimate Efficient Frontiers for Portfolio Object" on page 4-140
- "Asset Allocation Case Study" on page 4-196
- "Portfolio Optimization Examples" on page 4-167


## More About

- "Portfolio Object" on page 4-23
- "Portfolio Optimization Theory" on page 4-3
- "Portfolio Object Workflow" on page 4-21


## External Websites

- Using MATLAB to Optimize Portfolios with Financial Toolbox (33 min 24 sec )
- MATLAB for Advanced Portfolio Construction and Stock Selection Models ( 30 min 28 $\mathrm{sec})$


## Troubleshooting Portfolio Optimization Results

## Portfolio Object Destroyed When Modifying

If a Portfolio object is destroyed when modifying, remember to pass an existing object into the Portfolio object if you want to modify it, otherwise it creates a new object. See "Creating the Portfolio Object" on page 4-28 for details.

## Optimization Fails with "Bad Pivot" Message

If the optimization fails with a "bad pivot" message from lcprog, try a larger value for tolpiv which is a tolerance for pivot selection in the lcprog algorithm (try 1.0e-7, for example) or try the interior-point-convex version of quadprog. For details, see "Choosing and Controlling the Solver for Mean-Variance Portfolio Optimization" on page 4-131, the help header for lcprog, and quadprog.

## Speed of Optimization

Although it is difficult to characterize when one algorithm is faster than the other, the default solver, lcprog is faster for smaller problems and the quadprog solver is faster for larger problems. If one solver seems to take too much time, try the other solver. To change solvers, use setSolver.

## Matrix Incompatibility and "Non-Conformable" Errors

If you get matrix incompatibility or "non-conformable" errors, the representation of data in the tools follows a specific set of basic rules described in "Conventions for Representation of Data" on page 4-26.

## Missing Data Estimation Fails

If asset return data has missing or NaN values, the estimateAssetMoments function with the 'missingdata' flag set to true may fail with either too many iterations or a singular covariance. To correct this problem, consider this:

- If you have asset return data with no missing or NaN values, you can compute a covariance matrix that may be singular without difficulties. If you have missing or NaN
values in your data, the supported missing data feature requires that your covariance matrix must be positive-definite, that is, nonsingular.
- estimateAssetMoments uses default settings for the missing data estimation procedure that might not be appropriate for all problems.

In either case, you might want to estimate the moments of asset returns separately with either the ECM estimation functions such as ecmnmle or with your own functions.

## mv_optim_transform Errors

If you obtain optimization errors such as:

```
Error using mv_optim_transform (line 233)
Portfolio set appears to be either empty or unbounded. Check constraints.
Error in Portfolio/estimateFrontier (line 63)
    [A, b, f0, f, H, g, lb] = mv_optim_transform(obj);
Or
Error using mv_optim_transform (line 238)
Cannot obtain \overline{finite}
Error in Portfolio/estimateFrontier (line 63)
    [A, b, f0, f, H, g, lb] = mv_optim_transform(obj);
```

Since the portfolio optimization tools require a bounded portfolio set, these errors (and similar errors) can occur if your portfolio set is either empty and, if nonempty, unbounded. Specifically, the portfolio optimization algorithm requires that your portfolio set have at least a finite lower bound. The best way to deal with these problems is to use the validation functions in "Validate the Portfolio Problem for Portfolio Object" on page 4110. Specifically, use estimateBounds to examine your portfolio set, and use checkFeasibility to ensure that your initial portfolio is either feasible and, if infeasible, that you have sufficient turnover to get from your initial portfolio to the portfolio set.

Tip To correct this problem, try solving your problem with larger values for turnover or tracking-error and gradually reduce to the value that you want.

## Efficient Portfolios Do Not Make Sense

If you obtain efficient portfolios that do not seem to make sense, this can happen if you forget to set specific constraints or you set incorrect constraints. For example, if you
allow portfolio weights to fall between 0 and 1 and do not set a budget constraint, you can get portfolios that are $100 \%$ invested in every asset. Although it may be hard to detect, the best thing to do is to review the constraints you have set with display of the object. If you get portfolios with $100 \%$ invested in each asset, you can review the display of your object and quickly see that no budget constraint is set. Also, you can use estimateBounds and checkFeasibility to determine if the bounds for your portfolio set make sense and to determine if the portfolios you obtained are feasible relative to an independent formulation of your portfolio set.

## Efficient Frontiers Do Not Make Sense

If you obtain efficient frontiers that do not seem to make sense, this can happen for some cases of mean and covariance of asset returns. It is possible for some mean-variance portfolio optimization problems to have difficulties at the endpoints of the efficient frontier. It is rare for standard portfolio problems, but this can occur. For example, this can occur when using unusual combinations of turnover constraints and transaction costs. Usually, the workaround of setting the hidden property enforcePareto produces a single portfolio for the entire efficient frontier, where any other solutions are not Pareto optimal (which is what efficient portfolios must be).

An example of a portfolio optimization problem that has difficulties at the endpoints of the efficient frontier is this standard mean-variance portfolio problem (long-only with a budget constraint) with the following mean and covariance of asset returns:

```
m = [ 1; 2; 3 ];
C = [ 1 1 0; 1 1 0; 0 0 1 ];
p = Portfolio;
p = Portfolio(p, 'assetmean', m, 'assetcovar', C);
p = Portfolio(p, 'lowerbudget', 1, 'upperbudget', 1);
p = Portfolio(p, 'lowerbound', 0);
plotFrontier(p);
```



To work around this problem, set the hidden Portfolio object property for enforcePareto. This property instructs the optimizer to perform extra steps to ensure a Pareto-optimal solution. This slows down the solver, but guarantees a Pareto-optimal solution.

```
p.enforcePareto = true;
plotFrontier(p);
```



## Troubleshooting for Setting 'Conditional' BoundType, MinNumAssets, and MaxNumAssets Constraints

When configuring a Portfolio object to include 'Conditional ' BoundType (semicontinuous) constraints using setBounds or MinNumAssets and MaxNumAssets (cardinality) constraints using setMinMaxNumAssets, the values of the inputs that you supply can result in warning messages.

## Conditional Bounds with LowerBound Defined as Empty or Zero

When using setBounds with the BoundType set to 'Conditional' and the LowerBound input argument is empty ( [ ] ) or 0, the Conditional bound is not effective and is equivalent to a Simple bound.

```
AssetMean = [ 0.0101110; 0.0043532; 0.0137058 ];
AssetCovar = [ 0.00324625 0.00022983 0.00420395;
    0.00022983 0.00049937 0.00019247;
    0.00420395 0.00019247 0.00764097 ];
p = Portfolio('AssetMean', AssetMean, 'AssetCovar', AssetCovar, 'Budget', 1);
p = setBounds(p, 0, 0.5, 'BoundType', 'Conditional');
```

```
p = setMinMaxNumAssets(p, 3, 3);
estimateFrontier(p, 10)
Warning: Conditional bounds with 'LowerBound' as zero are equivalent to simple bounds.
Consider either using strictly positive 'LowerBound' or 'simple' as the 'BoundType'
instead.
> In internal.finance.PortfolioMixedInteger/checkBoundType (line 46)
    In Portfolio/checkarguments (line 204)
    In Portfolio/setBounds (line 80)
Warning: The solution may have less than 'MinNumAssets' assets with nonzero weight. To
enforce 'MinNumAssets' requirement, set strictly positive lower conditional bounds.
> In internal.finance.PortfolioMixedInteger/hasIntegerConstraints (line 44)
    In Portfolio/estimateFrontier (line 51)
ans =
```

    Columns 1 through 8
    | 0.5000 | 0.3555 | 0.3011 | 0.3299 | 0.3585 | 0.3873 | 0.4160 | 0.4448 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.5000 | 0.5000 | 0.4653 | 0.3987 | 0.3322 | 0.2655 | 0.1989 | 0.1323 |
| 0.0000 | 0.1445 | 0.2335 | 0.2714 | 0.3093 | 0.3472 | 0.3850 | 0.4229 |

Columns 9 through 10
$0.4735 \quad 0.5000$
$0.0657 \quad 0$
$0.4608 \quad 0.5000$

In all the 10 optimal allocations, there are allocations (the first and last ones) that only have two assets, which is in conflict with the MinNumAssets constraint that three assets should be allocated. Also there are two warnings, which actually explain what happens. In this case, the 'Conditional' bound constraints are defined as $\mathrm{xi}=0$ or $0<=\mathrm{xi}<=$ 0.5 , which are internally modeled as $0 * v i<=x i<=0.5 * v i$, where vi is 0 or 1 , where 0 indicates not allocated, and 1 indicates allocated. Here, vi=1, which still allows the asset to have a weight of 0 . In other words, setting LowerBound as 0 or empty, doesn't clearly define the minimum allocation for an allocated asset. Therefore, a 0 weighted asset is also considered as an allocated asset. To fix this warning, follow the instructions in the warning message, and set a LowerBound value that is strictly positive.

```
AssetMean = [ 0.0101110; 0.0043532; 0.0137058 ];
AssetCovar = [ 0.00324625 0.00022983 0.00420395;
    0.00022983 0.00049937 0.00019247;
    0.00420395 0.00019247 0.00764097 ];
p = Portfolio('AssetMean', AssetMean, 'AssetCovar', AssetCovar, 'Budget', 1);
p = setBounds(p, 0.3, 0.5, 'BoundType', 'Conditional');
p = setMinMaxNumAssets(p, 3, 3);
estimateFrontier(p, 10)
ans =
```

    Columns 1 through 8
    | 0.3000 | 0.3180 | 0.3353 | 0.3489 | 0.3580 | 0.3638 | 0.3694 | 0.3576 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.4000 | 0.3820 | 0.3642 | 0.3479 | 0.3333 | 0.3199 | 0.3067 | 0.3001 |
| 0.3000 | 0.3000 | 0.3005 | 0.3032 | 0.3088 | 0.3163 | 0.3240 | 0.3423 |

Columns 9 through 10

| 0.3289 | 0.3000 |
| :--- | :--- |
| 0.3000 | 0.3000 |
| 0.3711 | 0.4000 |

## Length of 'BoundType' Must Be Conformable with NumAssets

The setBounds optional name-value argument for 'BoundType' must be defined for all assets in the Portfolio object. By default, the 'BoundType' is 'Simple' and applies to all assets. Using setBounds, you can choose to define a 'BoundType' for each asset. In this case, the number of 'BoundType' specifications must match the number of assets (NumAssets) in the Portfolio object. The following example demonstrates the error when the number of 'BoundType' specifications do not match the number of assets in the Portfolio object.

```
AssetMean = [ 0.0101110; 0.0043532; 0.0137058 ];
AssetCovar = [ 0.00324625 0.00022983 0.00420395;
    0.00022983 0.00049937 0.00019247;
    0.00420395 0.00019247 0.00764097 ];
p = Portfolio('AssetMean', AssetMean, 'AssetCovar', AssetCovar, 'Budget', 1);
p = setBounds(p, 0.1, 0.5, 'BoundType',["simple"; "conditional"])
Cannot create bound constraints.
Caused by:
    Error using internal.finance.PortfolioMixedInteger/checkBoundType (line 28)
    Length of 'BoundType' must be conformable with 'NumAssets'=3.
```

To correct this, modify the BoundType to include three specifications because the Portfolio object has three assets.

```
AssetMean = [ 0.0101110; 0.0043532; 0.0137058 ];
AssetCovar = [ 0.00324625 0.00022983 0.00420395
    0.00022983 0.00049937 0.00019247;
    0.00420395 0.00019247 0.00764097 ];
p = Portfolio('AssetMean', AssetMean, 'AssetCovar', AssetCovar, 'Budget', 1);
p = setBounds(p, 0.1, 0.5, 'BoundType',["simple"; "conditional";"conditional"])
p.BoundType
p =
    Portfolio with properties:
            BuyCost: [
            SellCost: []
        RiskFreeRate: []
            AssetMean: [3\times1 double]
            AssetCovar: [3\times3 double]
    TrackingError: []
        TrackingPort: []
            Turnover: [
        BuyTurnover: [
        SellTurnover: []
            Name: []
            NumAssets: 3
            AssetList: []
```

```
            InitPort: [
    AInequality:
    bInequality: [
        AEquality:
        bEquality: [
        LowerBound: [ 3x1 double]
        UpperBound: [ }3\times1\mathrm{ double]
        LowerBudget:
        UpperBudget: 1
        GroupMatrix: []
        LowerGroup: []
        UpperGroup: []
            GroupA:
            GroupB:
        LowerRatio: [
        UpperRatio: []
        BoundType: [3\times1 categorical]
    MinNumAssets: []
    MaxNumAssets: []
ans =
    3\times1 categorical array
    simple
    conditional
    conditional
```


## Redundant Constraints from'BoundType', 'MinNumAssets', 'MaxNumAssets' Constraints

When none of the constraints from 'BoundType', 'MinNumAssets', or 'MaxNumAssets' are active, the redundant constraints from 'BoundType', 'MinNumAssets', 'MaxNumAssets' warning occurs. This happens when you explicitly use setBounds and setMinMaxNumAssets but with values that are inactive. That is the 'Conditional' BoundType has a LowerBound = [ ] or 0, 'MinNumAssets' is 0, or 'MaxNumAssets ' is the same value as NumAssets. In other words, if any of the three is active, the warning will not show up when using the estimate functions or plotFrontier. The following two examples show the rationale.

The first example is when the BoundType is explicitly set as 'Conditional' but the LowerBound is 0 , and no 'MinNumAssets' and 'MaxNumAssets' constraints are defined using setMinMaxNumAssets.

```
AssetMean = [ 0.0101110; 0.0043532; 0.0137058 ];
AssetCovar = [ 0.00324625 0.00022983 0.00420395;
    0.00022983 0.00049937 0.00019247;
    0.00420395 0.00019247 0.00764097 ];
p = Portfolio('AssetMean', AssetMean, 'AssetCovar', AssetCovar, 'Budget', 1);
p = setBounds(p, 0, 0.5, 'BoundType', 'Conditional');
estimateFrontier(p, 10)
Warning: Redundant constraints from 'BoundType', 'MinNumAssets', 'MaxNumAssets'.
> In internal.finance.PortfolioMixedInteger/hasIntegerConstraints (line 24)
    In Portfolio/estimateFrontier (line 51)
```

| Columns 1 | through 8 |  |  |  |  |  |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
|  |  |  |  |  |  |  |  |
| 0.5000 | 0.3555 | 0.3011 | 0.3299 | 0.3586 | 0.3873 | 0.4160 | 0.4448 |
| 0.5000 | 0.5000 | 0.4653 | 0.3987 | 0.3321 | 0.2655 | 0.1989 | 0.1323 |
| 0 | 0.1445 | 0.2335 | 0.2714 | 0.3093 | 0.3471 | 0.3850 | 0.4229 |

Columns 9 through 10
$0.4735 \quad 0.5000$
$0.0657 \quad 0$

The second example is when you explicitly set the three constraints, but all with inactive values. In this example, the BoundType is 'Conditional' and the LowerBound is 0 , thus specifying ineffective 'Conditional' BoundType constraints, and the 'MinNumAssets' and 'MaxNumAssets' values are 0 and 3, respectively. The setMinMaxNumAssets function specifies ineffective 'MinNumAssets' and 'MaxNumAssets' constraints.

```
AssetMean = [ 0.0101110; 0.0043532; 0.0137058 ];
AssetCovar = [ 0.00324625 0.00022983 0.00420395
    0.00022983 0.00049937 0.00019247;
    0.00420395 0.00019247 0.00764097 ];
p = Portfolio('AssetMean', AssetMean, 'AssetCovar', AssetCovar, 'Budget', 1);
p = setBounds(p, 0, 0.5, 'BoundType', 'Conditional');
p = setMinMaxNumAssets(p, 0, 3);
estimateFrontier(p, 10)
Warning: Redundant constraints from 'BoundType', 'MinNumAssets', 'MaxNumAssets'.
> In internal.finance.PortfolioMixedInteger/hasIntegerConstraints (line 24)
    In Portfolio/estimateFrontier (line 51)
ans =
```

    Columns 1 through 8
    | 0.5000 | 0.3555 | 0.3011 | 0.3299 | 0.3586 | 0.3873 | 0.4160 | 0.4448 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 0.5000 | 0.5000 | 0.4653 | 0.3987 | 0.3321 | 0.2655 | 0.1989 | 0.1323 |
| 0 | 0.1445 | 0.2335 | 0.2714 | 0.3093 | 0.3471 | 0.3850 | 0.4229 |

Columns 9 through 10
$0.4735 \quad 0.5000$
$0.0657 \quad 0$

## Infeasible Portfolio Problem with 'BoundType', 'MinNumAssets', 'MaxNumAssets'

The Portfolio object performs validations of all the constraints that you set before solving any specific optimization problems. The Portfolio object first considers all constraints other than 'Conditional' BoundType, 'MinNumAssets', and 'MaxNumAssets' and issues an error message if they are not compatible. Then the Portfolio object adds the three constraints to check if they are compatible with the already checked constraints.

This separation is natural because 'Conditional' BoundType, 'MinNumAssets', and 'MaxNumAssets ' require additional binary variables in the mathematical formulation that leads to a MINLP, while other constraints only need continuous variables. You can follow the error messages to check when the infeasible problem occurs and take actions to fix the constraints.

One possible scenario is when the BoundType is 'Conditional' and Groups are defined for the Portfolio object. In this case, the Group definitions are themselves in conflict. Consequently, the 'Conditional' bound constraint cannot be applied when running estimateFrontierLimits.

```
AssetMean = [ 0.0101110; 0.0043532; 0.0137058 ]
AssetCovar = [ 0.00324625 0.00022983 0.00420395;
    0.00022983 0.00049937 0.00019247;
    0.00420395 0.00019247 0.00764097 ];
p = Portfolio('AssetMean', AssetMean, 'AssetCovar', AssetCovar, 'Budget', 1);
p = setBounds(p, 0.1, 0.5, 'BoundType','Conditional');
p = setGroups(p, [1,1,0], 0.3, 0.5);
p = addGroups(p, [0,1,0], 0.6, 0.7);
pwgt = estimateFrontierLimits(p)
Error using Portfolio/buildMixedIntegerProblem (line 31)
Infeasible portfolio problem prior to considering 'BoundType', 'MinNumAssets',
'MaxNumAssets'. Verify if constraints from groups, bounds, group ratios, inequality,
equality, etc. are compatible.
```

Error in Portfolio/estimateFrontierLimits>int frontierLimits (line 93)
ProbStruct = buildMixedIntegerProblem(obj);
Error in Portfolio/estimateFrontierLimits (line 73)
pwgt = int_frontierLimits(obj, minsolution, maxsolution);

To correct this error, change the LowerGroup in the addGroups function to also be 0.3 to match the GroupMatrix input from setGroups.

```
AssetMean = [ 0.0101110; 0.0043532; 0.0137058 ];
AssetCovar = [ 0.00324625 0.00022983 0.00420395;
    0.00022983 0.00049937 0.00019247;
    0.00420395 0.00019247 0.00764097 ];
p = Portfolio('AssetMean', AssetMean, 'AssetCovar', AssetCovar, 'Budget', 1);
p = setBounds(p, 0.1, 0.5, 'BoundType','Conditional');
p = setGroups(p, [1,1,0], 0.3, 0.5);
p = addGroups(p, [0,1,0], 0.3, 0.7);
pwgt = estimateFrontierLimits(p)
pwgt =
    0.0.2000
    0.5000 0.3000
    0.5000 0.5000
```

A second possible scenario is when the BoundType is 'Conditional' and the setEquality function is used with the bEquality parameter set to 0.04. This sets an
equality constraint to have $x 1+x 3=0.04$. At the same time, setBounds also set the semicontinuous constraints to have $x i=0$ or $0.1<=x i<=2.5$, which lead to $x 1+x 3=$ 0 or $0.1<=x 1+x 3<=5$. The semicontinuous constraints are not compatible with the equality constraint because there is no way to get $x 1+x 3$ to equal 0.04 . Therefore, the error message is displayed.

```
AssetMean = [ 0.05; 0.1; 0.12; 0.18 ];
AssetCovar = [ ll0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
p = Portfolio('AssetMean', AssetMean, 'AssetCovar', AssetCovar, 'Budget', 1);
A = [ 1 0 1 0 ];
b = 0.04;
p = setEquality(p, A, b);
p = setBounds(p, 0.1, 2.5, 'BoundType','Conditional');
p = setMinMaxNumAssets(p, 2, 2);
pwgt = estimateFrontierLimits(p)
Error using Portfolio/buildMixedIntegerProblem (line 109)
Infeasible portfolio problem when considering 'BoundType', 'MinNumAssets',
'MaxNumAssets'. Verify if these are compatible with constraints from groups, bounds,
group ratios, inequality, equality, etc.
Error in Portfolio/estimateFrontierLimits>int_frontierLimits (line 93)
ProbStruct = buildMixedIntegerProblem(obj);
Error in Portfolio/estimateFrontierLimits (line 73)
    pwgt = int_frontierLimits(obj, minsolution, maxsolution);
```

To correct this error, change the bEquality parameter from 0.04 to .4 .

```
AssetMean = [ 0.05; 0.1; 0.12; 0.18 ];
AssetCovar = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
p = Portfolio('AssetMean', AssetMean, 'AssetCovar', AssetCovar, 'Budget', 1);
A = [ 1 0 1 0 ];
b = 0.4;
p = setEquality(p, A, b);
p = setBounds(p, 0.1, 2.5, 'BoundType','Conditional');
p = setMinMaxNumAssets(p, 2, 2);
pwgt = estimateFrontierLimits(p)
pwgt =
\begin{tabular}{rr}
0.4000 & 0 \\
0.6000 & 0 \\
0 & 0.4000 \\
0 & 0.6000
\end{tabular}
```


## Unbounded Portfolio Problem

This error occurs when there is no UpperBound defined in setBounds and you are using setMinMaxNumAssets. In this case, this is formulated as a mixed integer programming problem and an UpperBound is required to enforce MinNumAssets and MaxNumAssets constraints.

The optimizer first attempts to estimate the upper bound of each asset, based on all the specified constraints. If the UpperBound cannot be found, an error message occurs which instructs you to set an explicit UpperBound. In most cases, as long as you set some upper bounds to the problem using any set function, the optimizer can successfully find a good estimation.

| AssetMean = [ 0.05; 0.1; 0.12; 0.18 ]; |  |  |  |
| :---: | :---: | :---: | :---: |
| AssetCovar = [ 0.0064 | 0.00408 | 0.00192 | 0 ; |
| 0.00408 | 0.0289 | 0.0204 | 0.0119; |
| 0.00192 | 0.0204 | 0.0576 | $0.0336 ;$ |
| 0 | 0.0119 | 0.0336 | 0.1225 ] |

```
p = Portfolio('AssetMean', AssetMean, 'AssetCovar', AssetCovar);
p = setBounds(p, 0.1, 'BoundType','Conditional');
p = setGroups(p, [1,1,1,0], 0.3, 0.5);
p = setMinMaxNumAssets(p, 3, 3);
pwgt = estimateFrontierLimits(p)
```

Error using Portfolio/buildMixedIntegerProblem (line 42)
Unbounded portfolio problem. Upper bounds cannot be inferred from the existing
constraints. Set finite upper bounds using 'setBounds'.
Error in Portfolio/estimateFrontierLimits>int frontierLimits (line 93)
ProbStruct = buildMixedIntegerProblem(obj);
Error in Portfolio/estimateFrontierLimits (line 73)
pwgt = int frontierLimits(obj, minsolution, maxsolution);

To correct this error, specify an UpperBound value for setBounds.

```
AssetMean = [ 0.05; 0.1; 0.12; 0.18 ];
AssetCovar = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 
p = Portfolio('AssetMean', AssetMean, 'AssetCovar', AssetCovar)
p = setBounds(p, 0.1, .9, 'BoundType','Conditional');
p = setGroups(p, [1,1,1,0], 0.3, 0.5);
p = setMinMaxNumAssets(p, 3, 3);
pwgt = estimateFrontierLimits(p)
pwgt =
    0.1000 0
    0.1000 0.1000
    0.1000 0.4000
        0.9000
```


## Total Number of Portfolio Weights with a Value > 0 Are Greater Than MaxNumAssets Specified

The optimal allocation $w$ may contain some very small values that leads to sum( $w>0$ ) larger than MaxNumAssets, even though the MaxNumAssets constraint is specified using setMinMaxNumAssets. For example, in the following code when setMinMaxNumAssets is used to set MaxNumAssets to 15, the sum(w>0) indicates that there are 19 assets. A close examination of the weights shows that the weights are extremely small and are actually 0 .

```
T = readtable('dowPortfolio.xlsx');
symbol = T.Properties.VariableNames(3:end);
assetReturn = tick2ret(T{:,3:end});
p = Portfolio('AssetList', symbol, 'budget', 1);
p = setMinMaxNumAssets(p, 10, 15);
p = estimateAssetMoments(p,assetReturn);
p = setBounds(p,0.01,0.5,'BoundType','Conditional','NumAssets',30);
p = setTrackingError(p,0.05,ones(1, p.NumAssets)/p.NumAssets);
w = estimateFrontierLimits(p,'min'); % minimum risk portfolio
sum(w>0) % Number of assets that are allocated in the optimal portfolio
w(w<eps) % Check the weights of the very small weighted assets
ans =
    1 9
ans =
    1.0e-20 *
    0.0000
        0
        . }029
    0.3626
    0.2494
    0.0926
    -0.0000
    0.0020
        0
        0
        0
```

This situation only happens when the OuterApproximation algorithm is used to solver a MINLP portfolio optimization problem. The OuterApproximation internally fixes the latest solved integer variables and runs an NLP with quadprog or fmincon, which introduces numerical issues and leads to weights that are very close to 0 .

If you do not want to deal with very small values, you can use setSolverMINLP to select a different algorithm. In this example, the 'TrustRegionCP' algorithm is specified.

```
T = readtable('dowPortfolio.xlsx');
symbol = T.Properties.VariableNames(3:end);
assetReturn = tick2ret(T{:,3:end});
p = Portfolio('AssetList', symbol, 'budget', 1);
p = setMinMaxNumAssets(p, 10, 15);
p = estimateAssetMoments(p,assetReturn);
p = setBounds(p,0.01,0.5,'BoundType','Conditional','NumAssets',30);
p = setTrackingError(p,0.05,ones(1, p.NumAssets)/p.NumAssets);
p = setSolverMINLP(p,'TrustRegionCP');
w = estimateFrontierLimits(p,'min'); % minimum risk portfolio
sum(w>0) % Number of assets that are allocated in the optimal portfolio
w(w<eps) % The weights of the very small weighted assets are strictly zeros
ans =
    14
```

ans $=$
0
0
0
0
0
0
0
0
0
0
0
0
0
0
0
0
0
0
0
0
0

## See Also

Portfolio| checkFeasibility|estimateAssetMoments| setBounds| setMinMaxNumAssets

## Related Examples

- "Postprocessing Results to Set Up Tradable Portfolios" on page 4-149
- "Creating the Portfolio Object" on page 4-28
- "Working with Portfolio Constraints Using Defaults" on page 4-68
- "Estimate Efficient Portfolios for Entire Efficient Frontier for Portfolio Object" on page 4-115
- "Estimate Efficient Frontiers for Portfolio Object" on page 4-140
- "Asset Allocation Case Study" on page 4-196
- "Portfolio Optimization Examples" on page 4-167
- "Portfolio Optimization with Semicontinuous and Cardinality Constraints"


## More About

- "Portfolio Object" on page 4-23
- "Portfolio Optimization Theory" on page 4-3
- "Portfolio Object Workflow" on page 4-21


## External Websites

- Using MATLAB to Optimize Portfolios with Financial Toolbox (33 min 24 sec )
- MATLAB for Advanced Portfolio Construction and Stock Selection Models ( 30 min 28 sec)


## Portfolio Optimization Examples

The following sequence of examples highlights features of the Portfolio object in the Financial Toolbox ${ }^{\mathrm{TM}}$. Specifically, the examples use the Portfolio object to show how to set up mean-variance portfolio optimization problems that focus on the two-fund theorem, the impact of transaction costs and turnover constraints, how to obtain portfolios that maximize the Sharpe ratio, and how to set up two popular hedge-fund strategies - dollarneutral and 130-30 portfolios.

## Set up the Data

Every example works with moments for monthly total returns of a universe of 30 "bluechip" stocks. Although derived from real data, these data are for illustrative purposes and are not meant to be representative of specific assets or of market performance. The data are contained in the file BlueChipStockMoments.mat with a list of asset identifiers in the variable AssetList, a mean and covariance of asset returns in the variables AssetMean and AssetCovar, and the mean and variance of cash and market returns in the variables CashMean, CashVar, MarketMean, and MarketVar. Since most of the analysis requires the use of the standard deviation of asset returns as the proxy for risk, cash and market variances are converted into standard deviations.
load BlueChipStockMoments
mret $=$ MarketMean;
mrsk = sqrt(MarketVar);
cret = CashMean;
crsk = sqrt(CashVar);

## Create a Portfolio Object

The first step is to create a "standard" Portfolio object with Portfolio to incorporate the list of assets, the risk-free rate, and the moments of asset returns into the object.

```
p = Portfolio('AssetList',AssetList,'RiskFreeRate',CashMean);
p = setAssetMoments(p,AssetMean,AssetCovar);
```

To provide a basis for comparison, set up an equal-weight portfolio and make it the initial portfolio in the Portfolio object. Keep in mind that the hedged portfolios to be constructed later will require a different initial portfolio. Once the initial portfolio is created, the estimatePortMoments function estimates the mean and standard deviation of equal-weight portfolio returns.

```
p = setInitPort(p,1/p.NumAssets);
[ersk,eret] = estimatePortMoments(p,p.InitPort);
```

A specialized "helper" function portfolioexamples_plot makes it possible to plot all results to be developed here. This first plot shows the distribution of individual assets according to their means and standard deviations of returns. In addition, the equalweight, market, and cash portfolios are plotted on the same plot. Note that the plot function converts monthly total returns into annualized total returns.

```
clf;
portfolioexamples_plot('Asset Risks and Returns', ...
    {'scatter', mrsk, mret, {'Market'}}, ...
    {'scatter', crsk, cret, {'Cash'}}, ...
    {'scatter', ersk, eret, {'Equal'}}, ...
    {'scatter', sqrt(diag(p.AssetCovar)), p.AssetMean, p.AssetList, '.r'});
```



## Set up a Portfolio Optimization Problem

Set up a "standard" or default mean-variance portfolio optimization problem with the setDefaultConstraints function that requires fully-invested long-only portfolios (nonnegative weights that must sum to 1). Given this initial problem, estimate the efficient frontier with the functions estimateFrontier and estimatePortMoments, where estimateFrontier estimates efficient portfolios and estimatePortMoments estimates risks and returns for portfolios. The next figure overlays the efficient frontier on the previous plot.

```
p = setDefaultConstraints(p);
pwgt = estimateFrontier(p,20);
[prsk,pret] = estimatePortMoments(p,pwgt);
```

\% Plot efficient frontier
clf;
portfolioexamples_plot('Efficient Frontier', ...
\{'line', prsk, pret\}, ...
\{'scatter', [mrsk, crsk, ersk], [mret, cret, eret], \{'Market', 'Cash', 'Equal'\}\},
\{'scatter', sqrt(diag(p.AssetCovar)), p.AssetMean, p.AssetList, '.r'\});
Efficient Frontier


## Illustrate the Tangent Line to the Efficient Frontier

Tobin's mutual fund theorem (Tobin 1958) says that the portfolio allocation problem can be viewed as a decision to allocate between a riskless asset and a risky portfolio. In the mean-variance framework, cash can serve as a proxy for a riskless asset and an efficient
portfolio on the efficient frontier serves as the risky portfolio such that any allocation between cash and this portfolio dominates all other portfolios on the efficient frontier. This portfolio is called a tangency portfolio because it is located at the point on the efficient frontier where a tangent line that originates at the riskless asset touches the efficient frontier.

Given that the Portfolio object already has the risk-free rate, obtain the tangent line by creating a copy of the Portfolio object with a budget constraint that permits allocation between $0 \%$ and $100 \%$ in cash. Since the Portfolio object is a value object, it is easy to create a copy by assigning the output of either Portfolio or the "set" functions to a new instance of the Portfolio object. The plot shows the efficient frontier with Tobin's allocations that form the tangent line to the efficient frontier.

```
q = setBudget(p, 0, 1);
qwgt = estimateFrontier(q,20);
[qrsk,qret] = estimatePortMoments(q,qwgt);
% Plot efficient frontier with tangent line (0 to 1 cash)
clf;
portfolioexamples_plot('Efficient Frontier with Tangent Line', ...
    {'line', prsk, pret}, ...
    {'line', qrsk, qret, [], [], l}, ...
    {'scatter', [mrsk, crsk, ersk], [mret, cret, eret], {'Market', 'Cash', 'Equal'}},
    {'scatter', sqrt(diag(p.AssetCovar)), p.AssetMean, p.AssetList, '.r'});
```



Note that cash actually has a small risk so that the tangent line does not pass through the cash asset.

## Obtain Range of Risks and Returns

To obtain efficient portfolios with target values of either risk or return, it is necessary to obtain the range of risks and returns among all portfolios on the efficient frontier. This can be accomplished with the estimateFrontierLimits function.

```
[rsk,ret] = estimatePortMoments(p,estimateFrontierLimits(p));
```

display(rsk);

```
rsk = 2x1
    0.0348
    0.0903
display(ret);
ret = 2x1
    0.0094
    0.0179
```

The range of monthly portfolio returns is between $0.9 \%$ and $1.8 \%$ and the range for portfolio risks is between $3.5 \%$ and $9.0 \%$. In annualized terms, the range of portfolio returns is $11.2 \%$ to $21.5 \%$ and the range of portfolio risks is $12.1 \%$ to $31.3 \%$.

## Find a Portfolio with a Targeted Return and Targeted Risk

Given the range of risks and returns, it is possible to locate specific portfolios on the efficient frontier that have target values for return and risk using the functions estimateFrontierByReturn and estimateFrontierByRisk.

```
TargetReturn = 0.20; % input target annualized return and risk here
TargetRisk = 0.15;
% Obtain portfolios with targeted return and risk
awgt = estimateFrontierByReturn(p,TargetReturn/12);
[arsk,aret] = estimatePortMoments(p,awgt);
bwgt = estimateFrontierByRisk(p,TargetRisk/sqrt(12));
[brsk,bret] = estimatePortMoments(p,bwgt);
% Plot efficient frontier with targeted portfolios
clf;
portfolioexamples_plot('Efficient Frontier with Targeted Portfolios', ...
    {'line', prsk, pret}, ...
    {'scatter', [mrsk, crsk, ersk], [mret, cret, eret], {'Market', 'Cash', 'Equal'}},
    {'scatter', arsk, aret, {sprintf('%g%% Return',100*TargetReturn)}}, ...
    {'scatter', brsk, bret, {sprintf('%g%% Risk',100*TargetRisk)}}
    {'scatter', sqrt(diag(p.AssetCovar)), p.AssetMean, p.AssetList, '.r'});
```



To see what these targeted portfolios look like, use the dataset object to set up "blotters" that contain the portfolio weights and asset names (which are obtained from the Portfolio object).

```
aBlotter = dataset({100*awgt(awgt > 0),'Weight'}, 'obsnames', p.AssetList(awgt > 0));
displayPortfolio(sprintf('Portfolio with %g%% Target Return', 100*TargetReturn), aBlot
Portfolio with 20% Target Return
    Weight
    CAT 1.1445
    INTC 0.17452
    MO 9.6521
    MSFT 0.85862
```

```
    UTX 56.918
WMT
31.253
bBlotter = dataset({100*bwgt(bwgt > 0),'Weight'}, 'obsnames', p.AssetList(bwgt > 0));
displayPortfolio(sprintf('Portfolio with %g%% Target Risk', 100*TargetRisk), bBlotter,
Portfolio with 15% Target Risk
    Weight
    AXP 2.6405e-21
    BA 1.1823e-20
    C 2.0029e-21
    DD 6.5342e-22
    DIS 4.7055e-21
    GM 6.9062e-22
    HD 1.447e-21
    HON 2.7353e-21
    HPQ 5.9348e-21
    IBM 3.7157e-20
    INTC 2.2585
    JNJ 9.2162
    JPM 2.8021e-22
    KO 2.4688e-21
    MMM 16.603
    MO 15.388
    MRK 1.4925e-21
    MSFT 4.4467
    PG 4.086
    UTX 10.281
    VZ 9.7895e-22
    WMT 25.031
    XOM 12.69
```


## Transactions Costs

The Portfolio object makes it possible to account for transaction costs as part of the optimization problem. Although individual costs can be set for each asset, use the scalar expansion features of the Portfolio object's functions to set up uniform transaction costs across all assets and compare efficient frontiers with gross versus net portfolio returns.

```
BuyCost = 0.0020;
SellCost = 0.0020;
q = setCosts(p,BuyCost,SellCost);
```

```
qwgt = estimateFrontier(q,20);
[qrsk,qret] = estimatePortMoments(q,qwgt);
% Plot efficient frontiers with gross and net returns
clf;
portfolioexamples_plot('Efficient Frontier with and without Transaction Costs', ...
{'line', prsk, pret, {'Gross'}, ':b'}, ...
{'line', qrsk, qret, {'Net'}}, ...
{'scatter', [mrsk, crsk, ersk], [mret, cret, eret], {'Market', 'Cash', 'Equal'}},
{'scatter', sqrt(diag(p.AssetCovar)), p.AssetMean, p.AssetList, '.r'});
```

Efficient Frontier with and without Transaction Costs


## Turnover Constraint

In addition to transaction costs, the Portfolio object can handle turnover constraints. The following example demonstrates that a turnover constraint produces an efficient frontier in the neighborhood of an initial portfolio that may restrict trading. Moreover, the introduction of a turnover constraint often implies that multiple trades may be necessary to shift from an initial portfolio to an unconstrained efficient frontier. Consequently, the turnover constraint introduces a form of time diversification that can spread trades out over multiple time periods. In this example, note that the sum of purchases and sales from the estimateFrontier function confirms that the turnover constraint has been satisfied.

```
BuyCost = 0.0020;
SellCost = 0.0020;
Turnover = 0.2;
q = setCosts(p, BuyCost,SellCost);
q = setTurnover(q,Turnover);
[qwgt,qbuy,qsell] = estimateFrontier(q,20);
[qrsk,qret] = estimatePortMoments(q,qwgt);
% Plot efficient frontier with turnover constraint
clf;
portfolioexamples plot('Efficient Frontier with Turnover Constraint', ...
    {'line', prsk, pret, {'Unconstrained'}, ':b'}, ...
    {'line', qrsk, qret, {sprintf('%g%% Turnover', 100*Turnover)}}, ...
    {'scatter', [mrsk, crsk, ersk], [mret, cret, eret], {'Market', 'Cash', 'Equal'}},
    {'scatter', sqrt(diag(p.AssetCovar)), p.AssetMean, p.AssetList, '.r'});
```



The Portfolio object can handle tracking-error constraints, where tracking-error is the relative risk of a portfolio compared with a tracking portfolio. In this example, a subcollection of nine assets forms an equally-weighted tracking portfolio. The goal is to find efficient portfolios with tracking errors that are within 5\% of this tracking portfolio.

```
ii = [15, 16, 20, 21, 23, 25, 27, 29, 30]; % indexes of assets to include in tracki
TrackingError = 0.05/sqrt(12);
TrackingPort = zeros(30, 1);
TrackingPort(ii) = 1;
TrackingPort = (1/sum(TrackingPort))*TrackingPort;
q = setTrackingError(p,TrackingError,TrackingPort);
qwgt = estimateFrontier(q,20);
[qrsk,qret] = estimatePortMoments(q,qwgt);
[trsk,tret] = estimatePortMoments(q,TrackingPort);
% Plot efficient frontier with tracking-error constraint
clf;
portfolioexamples_plot('Efficient Frontier with 5% Tracking-Error Constraint', ...
    {'line', prsk, pret, {'Unconstrained'}, ':b'}, ...
    {'line', qrsk, qret, {'Tracking'}}, ...
    {'scatter', [mrsk, crsk], [mret, cret], {'Market', 'Cash'}}, ...
    {'scatter', trsk, tret, {'Tracking'}, 'r'});
```



## Combined Turnover and Tracking-Error Constraints

This example illustrates the interactions that can occur with combined constraints. In this case, both a turnover constraint relative to an initial equal-weight portfolio and a tracking-error constraint relative to a tracking portfolio must be satisfied. The turnover constraint has maximum 30\% turnover and the tracking-error constraint has maximum $5 \%$ tracking error. Note that the turnover to get from the initial portfolio to the tracking portfolio is $70 \%$ so that an upper bound of $30 \%$ turnover means that the efficient frontier will lie somewhere between the initial portfolio and the tracking portfolio.

```
Turnover = 0.3;
InitPort = (1/q.NumAssets)*ones(q.NumAssets, 1);
ii = [15, 16, 20, 21, 23, 25, 27, 29, 30]; % indexes of assets to include in tracki,
```

```
TrackingError = 0.05/sqrt(12);
TrackingPort = zeros(30, 1);
TrackingPort(ii) = 1;
TrackingPort = (1/sum(TrackingPort))*TrackingPort;
q = setTurnover(q,Turnover,InitPort);
qwgt = estimateFrontier(q,20);
[qrsk,qret] = estimatePortMoments(q,qwgt);
[trsk,tret] = estimatePortMoments(q,TrackingPort);
[ersk,eret] = estimatePortMoments(q,InitPort);
% Plot efficient frontier with combined turnover and tracking-error constraint
clf;
portfolioexamples_plot('Efficient Frontier with Turnover and Tracking-Error Constraint
    {'line', prsk, pret, {'Unconstrained'}, ':b'}, ...
    {'line', qrsk, qret, {'Turnover & Tracking'}}, ...
    {'scatter', [mrsk, crsk], [mret, cret], {'Market', 'Cash'}}, ...
    {'scatter', trsk, tret, {'Tracking'}, 'r'}, ...
    {'scatter', ersk, eret, {'Initial'}, 'b'});
```



## Maximize the Sharpe Ratio

The Sharpe ratio (Sharpe 1966) is a measure of return-to-risk that plays an important role in portfolio analysis. Specifically, a portfolio that maximizes the Sharpe ratio is also the tangency portfolio on the efficient frontier from the mutual fund theorem. The maximum Sharpe ratio portfolio is located on the efficient frontier with the function estimateMaxSharpeRatio and the dataset object is used to list the assets in this portfolio.

```
p = setInitPort(p, 0);
swgt = estimateMaxSharpeRatio(p);
[srsk,sret] = estimatePortMoments(p,swgt);
```

\% Plot efficient frontier with portfolio that attains maximum Sharpe ratio
clf;
portfolioexamples_plot('Efficient Frontier with Maximum Sharpe Ratio Portfolio', ...
\{'line', prsk, pret\}, ...
\{'scatter', srsk, sret, \{'Sharpe'\}\}, ...
\{'scatter', [mrsk, crsk, ersk], [mret, cret, eret], \{'Market', 'Cash', 'Equal'\}\}, \{'scatter', sqrt(diag(p.AssetCovar)), p.AssetMean, p.AssetList, '.r'\});

Efficient Frontier with Maximum Sharpe Ratio Portfolio

\% Set up a dataset object that contains the portfolio that maximizes the Sharpe ratio
Blotter = dataset(\{100*swgt(swgt > 0),'Weight'\}, 'obsnames', AssetList(swgt > 0));

```
displayPortfolio('Portfolio with Maximum Sharpe Ratio', Blotter, false);
Portfolio with Maximum Sharpe Ratio
        Weight
    AA 1.9674e-15
    AIG 1.9059e-15
    AXP 1.0567e-15
    BA 5.6013e-16
    C 2.0055e-15
    CAT 2.9716e-15
    DD 3.1346e-16
    DIS 5.9556e-16
    GE 5.6145e-15
    GM 7.3246e-16
    HD 2.0342e-11
    HON 3.309e-16
    HPQ 2.0307e-15
    IBM 8.8746e-15
    INTC 2.6638
    JNJ 9.0044
    JPM 5.4026e-16
    KO 2.4602e-16
    MCD 8.7472e-16
    MMM 15.502
    MO 13.996
    MRK 3.2239e-16
    MSFT 4.4777
    PFE 7.3425e-16
    PG 7.4588
    SBC 3.8329e-16
    UTX 6.0056
    VZ 3.9079e-16
    WMT 22.051
    XOM 18.841
```


## Confirm that Maximum Sharpe Ratio is a Maximum

The following plot demonstrates that this portfolio (which is located at the dot on the plots) indeed maximizes the Sharpe ratio among all portfolios on the efficient frontier.

```
psratio = (pret - p.RiskFreeRate) ./ prsk;
ssratio = (sret - p.RiskFreeRate) / srsk;
clf;
```

```
subplot(2,1,1);
plot(prsk, pret, 'LineWidth', 2);
hold on
scatter(srsk, sret, 'g', 'filled');
title('\bfEfficient Frontier');
xlabel('Portfolio Risk');
ylabel('Portfolio Return');
hold off
subplot(2,1,2);
plot(prsk, psratio, 'LineWidth', 2);
hold on
scatter(srsk, ssratio, 'g', 'filled');
title('\bfSharpe Ratio');
xlabel('Portfolio Risk');
ylabel('Sharpe Ratio');
hold off
```



## Illustrate that Sharpe is the Tangent Portfolio

The next plot demonstrates that the portfolio that maximizes the Sharpe ratio is also a tangency portfolio (in this case, the budget constraint is opened up to permit between $0 \%$ and $100 \%$ in cash).
$\mathrm{q}=\operatorname{setBudget}(\mathrm{p}, 0,1)$;
qwgt = estimateFrontier(q,20);
[qrsk, qret] = estimatePortMoments(q,qwgt);
\% Plot that shows Sharpe ratio portfolio is the tangency portfolio clf;

```
portfolioexamples_plot('Efficient Frontier with Maximum Sharpe Ratio Portfolio', ...
    {'line', prsk, pret}, ...
    {'line', qrsk, qret, [], [], 1}, ...
    {'scatter', srsk, sret, {'Sharpe'}}, ...
    {'scatter', [mrsk, crsk, ersk], [mret, cret, eret], {'Market', 'Cash', 'Equal'}},
    {'scatter', sqrt(diag(p.AssetCovar)), p.AssetMean, p.AssetList, '.r'});
```

Efficient Frontier with Maximum Sharpe Ratio Portfolio


## Dollar-Neutral Hedge-Fund Structure

To illustrate how to use the portfolio optimization tools in hedge fund management, two popular strategies with dollar-neutral and 130-30 portfolios are examined. The dollarneutral strategy invests equally in long and short positions such that the net portfolio position is 0 . Such a portfolio is said to be "dollar-neutral."

To set up a dollar-neutral portfolio, start with the "standard" portfolio problem and set the maximum exposure in long and short positions in the variable Exposure. The bounds for individual asset weights are plus or minus Exposure. Since the net position must be dollar-neutral, the budget constraint is 0 and the initial portfolio must be 0. Finally, the one-way turnover constraints provide the necessary long and short restrictions to prevent "double-counting" of long and short positions. The blotter shows the portfolio weights for the dollar-neutral portfolio that maximizes the Sharpe ratio. The long and short positions are obtained from the buy and sell trades relative to the initial portfolio.

```
Exposure = 1;
q = setBounds(p, -Exposure, Exposure);
q = setBudget(q, 0, 0);
q = setOneWayTurnover(q, Exposure, Exposure, 0);
[qwgt,qlong,qshort] = estimateFrontier(q,20);
[qrsk,qret] = estimatePortMoments(q,qwgt);
[qswgt,qslong,qsshort] = estimateMaxSharpeRatio(q);
[qsrsk,qsret] = estimatePortMoments(q,qswgt);
% Plot efficient frontier for a dollar-neutral fund structure with tangency portfolio
clf;
portfolioexamples_plot('Efficient Frontier with Dollar-Neutral Portfolio', ...
    {'line', prsk, pret, {'Standard'}, 'b:'}, ...
    {'line', qrsk, qret, {'Dollar-Neutral'}, 'b'}, ...
    {'scatter', qsrsk, qsret, {'Sharpe'}}, ...
    {'scatter', [mrsk, crsk, ersk], [mret, cret, eret], {'Market', 'Cash', 'Equal'}},
    {'scatter', sqrt(diag(p.AssetCovar)), p.AssetMean, p.AssetList, '.r'});
```


## Efficient Frontier with Dollar-Neutral Portfolio


\% Set up a dataset object that contains the portfolio that maximizes the Sharpe ratio
Blotter = dataset(\{100*qswgt(abs(qswgt) > 1.0e-4), 'Weight'\}, ...
\{100*qslong(abs(qswgt) > 1.0e-4), 'Long'\}, ... \{100*qsshort(abs(qswgt) > 1.0e-4), 'Short'\}, ... 'obsnames', AssetList(abs(qswgt) > 1.0e-4));
displayPortfolio('Dollar-Neutral Portfolio with Maximum Sharpe Ratio', Blotter, true,
Dollar-Neutral Portfolio with Maximum Sharpe Ratio

|  | Weight | Long | Short |
| :--- | :--- | :--- | ---: |
| AA | 0.5088 | 0.5088 | 0 |


| AXP | 0.92797 | 0.92797 | 0 |
| :--- | ---: | ---: | ---: |
| BA | -3.4952 | 0 | 3.4952 |
| C | 14.003 | 14.003 | 0 |
| CAT | 3.7261 | 3.7261 | 0 |
| DD | -18.063 | 0 | 18.063 |
| DIS | -4.8236 | 0 | 4.8236 |
| GE | -3.6178 | 0 | 3.6178 |
| GM | -3.7211 | 0 | 3.7211 |
| HD | 1.101 | 1.101 | 0 |
| HON | -1.4349 | 0 | 1.4349 |
| HPQ | 0.09909 | 0.09909 | 0 |
| IBM | -8.0585 | 0 | 8.0585 |
| INTC | 1.7693 | 1.7693 | 0 |
| JNJ | 1.3696 | 1.3696 | 0 |
| JPM | -2.5271 | 0 | 2.5271 |
| KO | -14.205 | 0 | 14.205 |
| MCD | 3.91 | 3.91 | 0 |
| MMM | 7.5995 | 7.5995 | 0 |
| MO | 4.0856 | 4.0856 | 0 |
| MRK | 3.747 | 3.747 | 0 |
| MSFT | 4.0769 | 4.0769 | 0 |
| PFE | -9.096 | 0 | 9.096 |
| PG | 1.6493 | 1.6493 | 0 |
| SBC | -5.2547 | 0 | 5.2547 |
| UTX | 5.7454 | 5.7454 | 0 |
| VZ | -2.438 | 0 | 2.438 |
| WMT | 0.84844 | 0.84844 | 0 |
| XOM | 18.529 | 18.529 | 0 |

Confirm Dollar-Neutral Portfolio
(Net, Long, Short)
-0.0000 76.7350 76.7350

## 130/30 Fund Structure

Finally, the turnover constraints can be used to set up a 130-30 portfolio structure, which is a structure with a net long position but permits leverage with long and short positions up to a maximum amount of leverage. In the case of a 130-30 portfolio, the leverage is 30\%.

To set up a 130-30 portfolio, start with the "standard" portfolio problem and set the maximum value for leverage in the variable Leverage. The bounds for individual asset weights range between -Leverage and $1+$ Leverage. Since the net position must be long, the budget constraint is 1 and, once again, the initial portfolio is 0 . Finally, the one-
way turnover constraints provide the necessary long and short restrictions to prevent "double-counting" of long and short positions. The blotter shows the portfolio weights for the 130-30 portfolio that maximizes the Sharpe ratio. The long and short positions are obtained from the buy and sell trades relative to the initial portfolio.

```
Leverage = 0.3;
q = setBounds(p, -Leverage, 1 + Leverage);
q = setBudget(q, 1, 1);
q = setOneWayTurnover(q, 1 + Leverage, Leverage);
[qwgt,qbuy,qsell] = estimateFrontier(q,20);
[qrsk,qret] = estimatePortMoments(q,qwgt);
[qswgt,qslong,qsshort] = estimateMaxSharpeRatio(q);
[qsrsk,qsret] = estimatePortMoments(q,qswgt);
% Plot efficient frontier for a 130-30 fund structure with tangency portfolio
clf;
portfolioexamples_plot(sprintf('Efficient Frontier with %g-%g Portfolio', ...
    100*(1 + Leverage),100*Leverage), ...
    {'line', prsk, pret, {'Standard'}, 'b:'}, ...
    {'line', qrsk, qret, {'130-30'}, 'b'}, ...
    {'scatter', qsrsk, qsret, {'Sharpe'}}, ...
    {'scatter', [mrsk, crsk, ersk], [mret, cret, eret], {'Market', 'Cash', 'Equal'}},
    {'scatter', sqrt(diag(p.AssetCovar)), p.AssetMean, p.AssetList, '.r'});
```

Efficient Frontier with 130-30 Portfolio

\% Set up a dataset object that contains the portfolio that maximizes the Sharpe ratio
Blotter = dataset(\{100*qswgt(abs(qswgt) > 1.0e-4), 'Weight'\}, ...
\{100*qslong(abs(qswgt) > 1.0e-4), 'Long'\}, ... \{100*qsshort(abs(qswgt) > 1.0e-4), 'Short'\}, ... 'obsnames', AssetList(abs(qswgt) > 1.0e-4));
displayPortfolio(sprintf('\%g-\%g Portfolio with Maximum Sharpe Ratio', 100*(1 + Leverag

| 130-30 | Portfolio with |  |  |
| :---: | :---: | :---: | :---: |
| Weight | Long | Sharpe Ratio |  |
| DD | Short |  |  |
| HON | -9.5565 | 0 | 9.5565 |
|  | -6.0244 | 0 | 6.0244 |


| INTC | 4.0335 | 4.0335 | 0 |
| :--- | ---: | ---: | ---: |
| JNJ | 7.1234 | 7.1234 | 0 |
| JPM | -0.44583 | 0 | 0.44583 |
| KO | -13.646 | 0 | 13.646 |
| MMM | 20.908 | 20.908 | 0 |
| MO | 14.433 | 14.433 | 0 |
| MSFT | 4.5592 | 4.5592 | 0 |
| PG | 17.243 | 17.243 | 0 |
| SBC | -0.32712 | 0 | 0.32712 |
| UTX | 5.3584 | 5.3584 | 0 |
| WMT | 21.018 | 21.018 | 0 |
| XOM | 35.323 | 35.323 | 0 |

## Confirm 130-30 Portfolio

(Net, Long, Short)
100.0000 130.0000 30.0000

## References

1 R. C. Grinold and R. N. Kahn (2000), Active Portfolio Management, 2nd ed.
2 H. M. Markowitz (1952), "Portfolio Selection," Journal of Finance, Vol. 1, No. 1, pp. 77-91.

3 J. Lintner (1965), "The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets," Review of Economics and Statistics, Vol. 47, No. 1, pp. 13-37.

4 H. M. Markowitz (1959), Portfolio Selection: Efficient Diversification of Investments, John Wiley \& Sons, Inc.

5 W. F. Sharpe (1966), "Mutual Fund Performance," Journal of Business, Vol. 39, No. 1, Part 2, pp. 119-138.

6 J. Tobin (1958), "Liquidity Preference as Behavior Towards Risk," Review of Economic Studies, Vol. 25, No.1, pp. 65-86.

7 J. L. Treynor and F. Black (1973), "How to Use Security Analysis to Improve Portfolio Selection," Journal of Business, Vol. 46, No. 1, pp. 68-86.

## Utility Functions

function displaySumOfTransactions(Turnover, qbuy, qsell)
fprintf('Sum of Purchases by Portfolio along Efficient Frontier (Max. Turnover \% $9 \%$ ) \n 100*Turnover) ;
fprintf('\%.4f ', 100*sum(qbuy)), sprintf('\n\n');
fprintf('\n')

```
fprintf('Sum of Sales by Portfolio along Efficient Frontier (Max. Turnover %g%%)\n',
    100*Turnover);
fprintf('%.4f ', 100*sum(qsell));
end
function displayPortfolio(Description, Blotter, LongShortFlag, portfolioType)
fprintf('%s\n', Description);
disp(Blotter);
if (LongShortFlag)
    fprintf('Confirm %s Portfolio\n', portfolioType);
    fprintf(' (Net, Long, Short)\n');
    fprintf('%.4f ' , [ sum(Blotter.Weight), sum(Blotter.Long), sum(Blotter.Short) ]);
end
end
```


## See Also

Portfolio| addGroups | estimateAssetMoments |estimateBounds | estimateFrontierByRisk|estimateFrontierLimits |estimatePortRisk| plotFrontier|setAssetMoments| setBounds

## Related Examples

- "Creating the Portfolio Object" on page 4-28
- "Working with Portfolio Constraints Using Defaults" on page 4-68
- "Validate the Portfolio Problem for Portfolio Object" on page 4-110
- "Estimate Efficient Portfolios for Entire Efficient Frontier for Portfolio Object" on page 4-115
- "Estimate Efficient Frontiers for Portfolio Object" on page 4-140
- "Postprocessing Results to Set Up Tradable Portfolios" on page 4-149


## More About

- "Portfolio Object" on page 4-23
- "Portfolio Optimization Theory" on page 4-3


## External Websites

- Using MATLAB to Optimize Portfolios with Financial Toolbox (33 min 24 sec )
- MATLAB for Advanced Portfolio Construction and Stock Selection Models (30 min 28 sec )


## Asset Allocation Case Study

This example shows how to set up a basic asset allocation problem that uses meanvariance portfolio optimization with a Portfolio object to estimate efficient portfolios.

## Step 1. Defining the portfolio problem.

Suppose that you want to manage an asset allocation fund with four asset classes: bonds, large-cap equities, small-cap equities, and emerging equities. The fund is long-only with no borrowing or leverage, should have no more than $85 \%$ of the portfolio in equities, and no more than $35 \%$ of the portfolio in emerging equities. The cost to trade the first three assets is 10 basis points annualized and the cost to trade emerging equities is four times higher. Finally, you want to ensure that average turnover is no more than $15 \%$. To solve this problem, you will set up a basic mean-variance portfolio optimization problem and then slowly introduce the various constraints on the problem to get to a solution.

To set up the portfolio optimization problem, start with basic definitions of known quantities associated with the structure of this problem. Each asset class is assumed to have a tradeable asset with a real-time price. Such assets can be, for example, exchangetraded funds (ETFs). The initial portfolio with holdings in each asset that has a total of $\$ 7.5$ million along with an additional cash position of $\$ 60,000$. These basic quantities and the costs to trade are set up in the following variables with asset names in the cell array Asset, current prices in the vector Price, current portfolio holdings in the vector Holding, and transaction costs in the vector UnitCost.

To analyze this portfolio, you can set up a blotter in a dataset object to help track prices, holdings, weights, and so forth. In particular, you can compute the initial portfolio weights and maintain them in a new blotter field called InitPort.

```
Asset = { 'Bonds', 'Large-Cap Equities', 'Small-Cap Equities', 'Emerging Equities' };
Price = [ 52.4; 122.7; 35.2; 46.9 ];
Holding = [ 42938; 24449; 42612; 15991 ];
UnitCost = [ 0.001; 0.001; 0.001; 0.004 ];
Blotter = dataset({Price, 'Price'}, {Holding, 'InitHolding'},'obsnames',Asset);
Wealth = sum(Blotter.Price .* Blotter.InitHolding);
Blotter.InitPort = (1/Wealth)*(Blotter.Price .* Blotter.InitHolding);
Blotter.UnitCost = UnitCost;
disp(Blotter);
```

| Price | InitHolding | InitPort | UnitCost |
| ---: | :--- | :--- | :--- |
| 52.4 | 42938 | 0.3 | 0.001 |


| Large-Cap Equities | 122.7 | 24449 | 0.4 | 0.001 |
| :--- | ---: | ---: | ---: | ---: |
| Small-Cap Equities | 35.2 | 42612 | 0.2 | 0.001 |
| Emerging Equities | 46.9 | 15991 | 0.1 | 0.004 |

## Step 2. Simulating asset prices.

Since this is a hypothetical example, to simulate asset prices from a given mean and covariance of annual asset total returns for the asset classes, the portsim function is used to create asset returns with the desired mean and covariance. Specifically, portsim is used to simulate five years of monthly total returns and then plotted to show the log of the simulated total return prices

The mean and covariance of annual asset total returns are maintained in the variables AssetMean and AssetCovar. The simulated asset total return prices (which are compounded total returns) are maintained in the variable Y. All initial asset total return prices are normalized to 1 in this example.

```
AssetMean = [ 0.05; 0.1; 0.12; 0.18 ];
AssetCovar = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
```

X = portsim(AssetMean'/12, AssetCovar/12, 60); \% monthly total returns for 5 years (60
[Y, T] = ret2tick(X, [], 1/12); \% form total return prices
Warning: This syntax will be removed in a future release. See the documentation for re
plot(T, $\log (Y))$;
title('\bfSimulated Asset Class Total Return Prices');
xlabel('Year');
ylabel('Log Total Return Price');
legend(Asset,'Location', 'best');


## Step 3. Setting up the Portfolio object.

To explore portfolios on the efficient frontier, set up a Portfolio object using these specifications:

- Portfolio weights are nonnegative and sum to 1.
- Equity allocation is no more than $85 \%$ of the portfolio.
- Emerging equity is no more than $35 \%$ of the portfolio.

These specifications are incorporated into the Portfolio object p in the following sequence of using functions that starts with using the Portfolio object.

1 The specification of the initial portfolio from Blotter gives the number of assets in your universe so you do not need to specify the NumAssets property directly. Next, set up default constraints (long-only with a budget constraint). In addition, set up the group constraint that imposes an upper bound on equities in the portfolio (equities are identified in the group matrix with 1's) and the upper bound constraint on emerging equities. Although you could have set the upper bound on emerging equities using the setBounds function, notice how the addGroups function is used to set up this constraint.

2 To have a fully specified mean-variance portfolio optimization problem, you must specify the mean and covariance of asset returns. Since starting with these moments in the variables AssetMean and AssetCovar, you can use the setAssetMoments function to enter these variables into your Portfolio object (remember that you are assuming that your raw data are monthly returns which is why you divide your annual input moments by 12 to get monthly returns).
3 Use the total return prices with the estimateAssetMoments function with a specification that your data in Y are prices, and not returns, to estimate asset return moments for your Portfolio object.
4 Although the returns in your Portfolio object are in units of monthly returns, and since subsequent costs are annualized, it is convenient to specify them as annualized total returns with this direct transformation of the AssetMean and AssetCovar properties of your Portfolio object p.
5 Display the Portfolio object p.
p = Portfolio('Name', 'Asset Allocation Portfolio', ...
'AssetList', Asset, 'InitPort', Blotter.InitPort);
$p=$ setDefaultConstraints(p);
$p=\operatorname{set} \operatorname{roups}(p,[0,1,1,1],[], 0.85) ;$
$p=\operatorname{addGroups}(p,[0,0,0,1],[], 0.35) ;$
$p=\operatorname{set} A s s e t M o m e n t s(p, A s s e t M e a n / 12$, AssetCovar/12);
p = estimateAssetMoments(p, Y, 'DataFormat', 'Prices');
p.AssetMean $=12 *$ p.AssetMean;
p.AssetCovar = 12*p.AssetCovar;
display(p);
$\mathrm{p}=$
Portfolio with properties:

```
            BuyCost: []
            SellCost: []
    RiskFreeRate: []
        AssetMean: [4x1 double]
        AssetCovar: [4x4 double]
TrackingError: []
    TrackingPort: []
            Turnover: []
    BuyTurnover: []
SellTurnover: []
            Name: 'Asset Allocation Portfolio'
        NumAssets: 4
        AssetList: {1x4 cell}
            InitPort: [4x1 double]
    AInequality: []
    bInequality: []
            AEquality: []
            bEquality: []
            LowerBound: [4x1 double]
            UpperBound: []
    LowerBudget: 1
    UpperBudget: 1
    GroupMatrix: [2x4 double]
    LowerGroup: []
    UpperGroup: [2x1 double]
            GroupA: []
            GroupB: []
        LowerRatio: []
    UpperRatio: []
MinNumAssets: []
MaxNumAssets: []
    BoundType: [4x1 categorical]
```


## Step 4. Validate the portfolio problem.

An important step in portfolio optimization is to validate that the portfolio problem is feasible and the main test is to ensure that the set of portfolios is nonempty and bounded. Use the estimateBounds function to determine the bounds for the portfolio set. In this case, since both lb and ub are finite, the set is bounded.

```
[lb, ub] = estimateBounds(p);
display([lb, ub]);
```

| 0.1500 | 1.0000 |
| :--- | :--- |
| 0.0000 | 0.8500 |
| 0.0000 | 0.8500 |
| 0.0000 | 0.3500 |

## Step 5. Plotting the efficient frontier.

Given the constructed Portfolio object, use the plotFrontier function to view the efficient frontier. Instead of using the default of 10 portfolios along the frontier, you can display the frontier with 40 portfolios. Notice gross efficient portfolio returns fall between approximately $6 \%$ and $16 \%$ per years.
plotFrontier(p, 40);

Asset Allocation Portfolio


## Step 6. Evaluating gross vs. net portfolio returns.

The Portfolio object p does not include transaction costs so that the portfolio optimization problem specified in $p$ uses gross portfolio return as the return proxy. To handle net returns, create a second Portfolio object q that includes transaction costs.

```
q = setCosts(p, UnitCost, UnitCost);
display(q);
q =
    Portfolio with properties:
            BuyCost: [4x1 double]
            SellCost: [4x1 double]
        RiskFreeRate: []
            AssetMean: [4x1 double]
            AssetCovar: [4\times4 double]
        TrackingError: []
            TrackingPort: []
            Turnover: []
        BuyTurnover: []
        SellTurnover: []
            Name: 'Asset Allocation Portfolio'
            NumAssets: 4
            AssetList: {1x4 cell}
            InitPort: [4x1 double]
            AInequality: []
            bInequality: []
            AEquality: []
            bEquality: []
            LowerBound: [4x1 double]
            UpperBound: []
            LowerBudget: 1
            UpperBudget: 1
            GroupMatrix: [2x4 double]
            LowerGroup: []
            UpperGroup: [2x1 double]
                GroupA: []
            GroupB: []
            LowerRatio: []
            UpperRatio: []
            MinNumAssets: []
            MaxNumAssets: []
            BoundType: [4x1 categorical]
```


## Step 7. Analyzing descriptive properties of the Portfolio structures.

To be more concrete about the ranges of efficient portfolio returns and risks, use the estimateFrontierLimits function to obtain portfolios at the endpoints of the efficient frontier. Given these portfolios, compute their moments using the estimatePortMoments function. The following code generates a table that lists the risk and return of the initial portfolio as well as the gross and net moments of portfolio returns for the portfolios at the endpoints of the efficient frontier:

```
[prsk0, pret0] = estimatePortMoments(p, p.InitPort);
pret = estimatePortReturn(p, p.estimateFrontierLimits);
qret = estimatePortReturn(q, q.estimateFrontierLimits);
displayReturns(pret0, pret, qret);
Annualized Portfolio Returns ...
\begin{tabular}{lrr} 
& Gross & Net \\
Initial Portfolio Return & \(9.70 \%\) & \(9.70 \%\) \\
Minimum Efficient Portfolio Return & \(5.90 \%\) & \(5.77 \%\) \\
Maximum Efficient Portfolio Return & \(13.05 \%\) & \(12.86 \%\)
\end{tabular}
```

The results show that the cost to trade ranges from 14 to 19 basis points to get from the current portfolio to the efficient portfolios at the endpoints of the efficient frontier (these costs are the difference between gross and net portfolio returns.) In addition, notice that the maximum efficient portfolio return (13\%) is less than the maximum asset return (18\%) due to the constraints on equity allocations.

## Step 8. Obtaining a Portfolio at the specified return level on the efficient frontier.

A common approach to select efficient portfolios is to pick a portfolio that has a desired fraction of the range of expected portfolio returns. To obtain the portfolio that is $30 \%$ of the range from the minimum to maximum return on the efficient frontier, obtain the range of net returns in qret using the Portfolio object q and interpolate to obtain a $30 \%$ level with the interp1 function to obtain a portfolio qwgt.

```
Level = 0.3;
qret = estimatePortReturn(q, q.estimateFrontierLimits);
qwgt = estimateFrontierByReturn(q, interp1([0, 1], qret, Level));
[qrsk, qret] = estimatePortMoments(q, qwgt);
displayReturnLevel(Level, qret, qrsk);
```

```
Portfolio at 30% return level on efficient frontier ...
    Return Risk
    7.90 9.09
display(qwgt);
qwgt = 4×1
    0.6252
    0.1856
    0.0695
    0.1198
```

The target portfolio that is $30 \%$ of the range from minimum to maximum net returns has a return of $7.9 \%$ and a risk of $9.1 \%$.

## Step 9. Obtaining a Portfolio at the specified risk levels on the efficient frontier.

Although you could accept this result, suppose that you want to target values for portfolio risk. Specifically, suppose that you have a conservative target risk of $10 \%$, a moderate target risk of $15 \%$, and an aggressive target risk of $20 \%$ and you want to obtain portfolios that satisfy each risk target. Use the estimateFrontierByRisk function to obtain targeted risks specified in the variable TargetRisk. The resultant three efficient portfolios are obtained in qwgt.

```
TargetRisk = [ 0.10; 0.15; 0.20 ];
qwgt = estimateFrontierByRisk(q, TargetRisk);
display(qwgt);
qwgt = 4×3
    0.5407 0.2020 0.1500
    0.2332 0.4000 0.0318
    0.0788 0.1280 0.4682
    0.1474 0.2700 0.3500
```

Use the estimatePortRisk function to compute the portfolio risks for the three portfolios to confirm that the target risks have been attained:

```
display(estimatePortRisk(q, qwgt));
```

0.1000
0.1500
0.2000

Suppose that you want to shift from the current portfolio to the moderate portfolio. You can estimate the purchases and sales to get to this portfolio:
[qwgt, qbuy, qsell] = estimateFrontierByRisk(q, 0.15);
If you average the purchases and sales for this portfolio, you can see that the average turnover is $17 \%$, which is greater than the target of $15 \%$ :

```
disp(sum(qbuy + qsell)/2)
    0.1700
```

Since you also want to ensure that average turnover is no more than $15 \%$, you can add the average turnover constraint to the Portfolio object using setTurnover:

```
q = setTurnover(q, 0.15);
[qwgt, qbuy, qsell] = estimateFrontierByRisk(q, 0.15);
```

You can enter the estimated efficient portfolio with purchases and sales into the Blotter:

```
qbuy(abs(qbuy)< 1.0e-5) = 0;
qsell(abs(qsell) < 1.0e-5) = 0; % zero out near 0 trade weights
Blotter.Port = qwgt;
Blotter.Buy = qbuy;
Blotter.Sell = qsell;
display(Blotter);
Blotter =
\begin{tabular}{lrlll} 
& Price & InitHolding & InitPort & UnitCost \\
Bonds & 52.4 & 42938 & 0.3 & 0.001 \\
Large-Cap Equities & 122.7 & 24449 & 0.4 & 0.001 \\
Small-Cap Equities & 35.2 & 42612 & 0.2 & 0.001 \\
Emerging Equities & 46.9 & 15991 & 0.1 & 0.004
\end{tabular}
```

Port Buy Sell

| Bonds | 0.18787 | 0 | 0.11213 |
| :--- | ---: | ---: | ---: |
| Large-Cap Equities | 0.4 | 0 | 0 |
| Small-Cap Equities | 0.16213 | 0 | 0.037871 |
| Emerging Equities | 0.25 | 0.15 | 0 |

The Buy and Sell elements of the Blotter are changes in portfolio weights that must be converted into changes in portfolio holdings to determine the trades. Since you are working with net portfolio returns, you must first compute the cost to trade from your initial portfolio to the new portfolio. This can be accomplished as follows:

```
TotalCost = Wealth * sum(Blotter.UnitCost .* (Blotter.Buy + Blotter.Sell))
TotalCost = 5.6248e+03
```

The cost to trade is $\$ 5,625$, so that, in general, you would have to adjust your initial wealth accordingly before setting up your new portfolio weights. However, to keep the analysis simple, note that you have sufficient cash $(\$ 60,0000)$ set aside to pay the trading costs and that you will not touch the cash position to build up any positions in your portfolio. Thus, you can populate your blotter with the new portfolio holdings and the trades to get to the new portfolio without making any changes in your total invested wealth. First, compute portfolio holding:

```
Blotter.Holding = Wealth * (Blotter.Port ./ Blotter.Price);
```

Compute number of shares to Buy and Sell in your Blotter:

```
Blotter.BuyShare = Wealth * (Blotter.Buy ./ Blotter.Price);
Blotter.SellShare = Wealth * (Blotter.Sell ./ Blotter.Price);
```

Notice how you used an ad hoc truncation rule to obtain unit numbers of shares to buy and sell. Clean up the blotter by removing the unit costs and the buy and sell portfolio weights:

```
Blotter.Buy = [];
Blotter.Sell = [];
Blotter.UnitCost = [];
```


## Step 10. Displaying the final results.

The final result is a blotter that contains proposed trades to get from your current portfolio to a moderate-risk portfolio. To make the trade, you would need to sell 16,049 shares of your bond asset and 8,069 shares of your small-cap equity asset and would need to purchase 23,986 shares of your emerging equities asset.

```
display(Blotter);
```


## Blotter =

|  | Price | InitHolding | InitPort | Port | Holding |
| :--- | ---: | :--- | :--- | ---: | :--- |
| Bonds | 52.4 | 42938 | 0.3 | 0.18787 | 26889 |
| Large-Cap Equities | 122.7 | 24449 | 0.4 | 0.4 | 24449 |
| Small-Cap Equities | 35.2 | 42612 | 0.2 | 0.16213 | 34543 |
| Emerging Equities | 46.9 | 15991 | 0.1 | 0.25 | 39977 |


|  | BuyShare | SellShare |
| :--- | ---: | ---: |
| Bonds | 0 | 16049 |
| Large-Cap Equities | 0 | 0 |
| Small-Cap Equities | 0 | 8068.8 |
| Emerging Equities | 23986 | 0 |

The final plot uses the plotFrontier function to display the efficient frontier and the initial portfolio for the fully specified portfolio optimization problem. It also adds the location of the moderate-risk or final portfolio on the efficient frontier.

```
plotFrontier(q, 40);
hold on
scatter(estimatePortRisk(q, qwgt), estimatePortReturn(q, qwgt), 'filled', 'r');
h = legend('Initial Portfolio', 'Efficient Frontier', 'Final Portfolio', 'location',
set(h, 'Fontsize', 8);
hold off
```



## Utility Functions

function displayReturns(pret0, pret, qret)
fprintf('Annualized Portfolio Returns ... \n');
fprintf(' $\quad$ \%6s $\% 6 s \backslash n^{\prime},{ }^{\prime}$ Gross', 'Net');
fprintf('Initial Portfolio Return $\quad \% 6.2 f \% \% \% 6.2 f$ \% $\%$ n', 100*pret0, 100*pret0)
fprintf('Minimum Efficient Portfolio Return \%6.2f \%\% \%6.2f \%\%\n',100*pret(1), 100*qret fprintf('Maximum Efficient Portfolio Return \%6.2f \%\% \%6.2f \%\%\n',100*pret(2),100*qret end
function displayReturnLevel(Level, qret, qrsk)
fprintf('Portfolio at \%g\%\% return level on efficient frontier ... ${ }^{\prime} n^{\prime}, 100 *$ Level); fprintf('\%10s \%10s\n','Return','Risk');
fprintf('\%10.2f \%10.2f\n',100*qret,100*qrsk); end

## See Also

Portfolio| addGroups |estimateAssetMoments |estimateBounds | estimateFrontierByRisk|estimateFrontierLimits|estimatePortRisk| plotFrontier|setAssetMoments|setBounds

## Related Examples

- "Creating the Portfolio Object" on page 4-28
- "Working with Portfolio Constraints Using Defaults" on page 4-68
- "Validate the Portfolio Problem for Portfolio Object" on page 4-110
- "Estimate Efficient Portfolios for Entire Efficient Frontier for Portfolio Object" on page 4-115
- "Estimate Efficient Frontiers for Portfolio Object" on page 4-140
- "Postprocessing Results to Set Up Tradable Portfolios" on page 4-149


## More About

- "Portfolio Object" on page 4-23
- "Portfolio Optimization Theory" on page 4-3
- "Portfolio Object Workflow" on page 4-21


## External Websites

- Using MATLAB to Optimize Portfolios with Financial Toolbox (33 min 24 sec )
- MATLAB for Advanced Portfolio Construction and Stock Selection Models ( 30 min 28 $\mathrm{sec})$


## Portfolio Optimization with Semicontinuous and Cardinality Constraints

This example shows how to use a Portfolio object to directly handle semicontinuous and cardinality constraints when performing portfolio optimization. Portfolio optimization finds the asset allocation that maximizes the return or minimizes the risk, subject to a set of investment constraints. The Portfolio class in Financial Toolbox ${ }^{\mathrm{TM}}$ is designed and implemented based on the Markowitz Mean-Variance Optimization framework. The MeanVariance Optimization framework handles problems where the return is the expected portfolio return, and the risk is the variance of portfolio returns. Using the Portfolio class, you can minimize the risk on the efficient frontier (EF), maximize the return on the EF, maximize the return for a given risk, and minimize the risk for a given return. Such optimization problems integrate with constraints such as group, linear inequality, turnover, and tracking error constraints. These constraints are formulated as nonlinear programming (NLP) problems with continuous variables represented as the asset weights $x_{i}$.

Semicontinuous and cardinality constraints are two other common categories of portfolio constraints that are formulated mathematically by adding the binary variables $v_{i}$.

- A semicontinuous constraint confines the allocation of an asset. For example, you can use this constraint to confine the allocated weight of an allocated asset to between 5\% and $50 \%$. By using this constraint, you can avoid very small or large positions to minimize the churns and operational costs. To mathematically formulate this type of constraint, a binary variable ${ }^{v_{i}}$ is needed, where $v_{i}$ is 0 or 1 . The value 0 indicates that asset $i$ is not allocated and the value 1 indicates that asset $i$ is allocated. The mathematical form is $\mathrm{lb} * v_{i} \leq x_{i} \leq \mathrm{ub} * v_{i}$, where $v_{i}$ is 0 or 1 . Specify this type of constraint as a 'Conditional' BoundType in the Portfolio class using the setBounds function.
- A cardinality constraint limits the number of assets in the optimal allocation, For example, for a portfolio with a universe of 100 assets, you can specify an optimal portfolio allocation between 20 and 40 assets. This capability helps limit the number of positions, and thus reduce operational costs. To mathematically formulate this type of
constraint, binary variables represented as $v_{i}$ are needed, where $v_{i}$ is 0 or 1 . The value 0 indicates that asset $i$ is not allocated and the value 1 indicates that asset $i$ is


#### Abstract

allocated. The mathematical form is MinNumAssets $\leq \sum_{1}^{\text {NumAssels }} v_{i} \leq$ MaxNumAssets where ${ }^{v_{i}}$ is 0 or 1 . Specify this type of constraint by setting the 'MinNumAssets ' and 'MaxNumAssets ' constraints in the Portfolio class using the setMinMaxNumAssets function.


For more information on semicontinuous and cardinality constraints, see "Algorithms" on page 19-1612.

When semicontinuous and cardinality constraints are used for portfolio optimization, this leads to mixed integer nonlinear programming problems (MINLP). The Portfolio class allows you to configure these two constraints, specifically, semicontinuous constraints using setBounds with 'Conditional' BoundType, and cardinality constraints using setMinMaxNumAssets. The Portfolio class automatically formulates the mathematical problems and validates the specified constraints. The Portfolio class also provides built-in MINLP solvers and flexible solver options for you to tune the solver performance using the setSolverMINLP function.

This example demonstrates a Portfolio object with semicontinuous and cardinality constraints and uses the BlueChipStockMoments dataset, which has a universe of 30 assets.
load BlueChipStockMoments
numAssets $=$ numel(AssetList)
numAssets $=30$

## Limit the Minimum Weight for Each Allocated Asset

Create a fully invested portfolio with only long positions: $x_{i} \geq 0$ and $\operatorname{sum}\left(x_{i}\right)=1$. These are configured with setDefaultConstraints.

```
p = Portfolio('AssetList', AssetList,'AssetCovar', AssetCovar, 'AssetMean', AssetMean)
p = setDefaultConstraints(p);
```

Suppose that you want to avoid very small positions to minimize the churn and operational costs. Add another constraint to confine the allocated positions to be no less than $5 \%$, by setting the constraints $x_{i}=0$ or $x_{i} \geq 0.05$ using setBounds with a 'Conditional' BoundType.
pWithMinWeight = setBounds(p, 0.05, 'BoundType', 'Conditional');

Plot the efficient frontiers for both Portfolio objects.
wgt = estimateFrontier(p);
wgtWithMinWeight = estimateFrontier(pWithMinWeight);
figure(1);
plotFrontier(p, wgt); hold on;
plotFrontier(pWithMinWeight, wgtWithMinWeight); hold off;
legend('Baseline portfolio', 'With minWeight constraint', 'location', 'best');


The figure shows that the two Portfolio objects have almost identical efficient frontiers. However, the one with the minimum weight requirement is more practical, since it prevents the close-to-zero positions.

Check the optimal weights for the portfolio with default constraints to see how many assets are below the 5\% limit for each optimal allocation.

```
toler = eps;
sum(wgt>toler & wgt<0.05)
ans = 1\times10
```

    \(\begin{array}{llllllllll}5 & 7 & 5 & 4 & 2 & 3 & 4 & 2 & 0 & 0\end{array}\)
    Use estimateFrontierByReturn to investigate the portfolio compositions for a target return on the frontier for both cases.

```
targetRetn = 0.011;
pwgt = estimateFrontierByReturn(p, targetRetn);
pwgtWithMinWeight = estimateFrontierByReturn(pWithMinWeight, targetRetn);
```

Plot the composition of the two Portfolio objects for the universe of 30 assets.

```
figure(2);
barh([pwgt, pwgtWithMinWeight]);
grid on
xlabel('Proportion of Investment')
yticks(1:p.NumAssets);
yticklabels(p.AssetList);
title('Asset Allocation');
legend('Without min weight limit', 'With min weight limit', 'location', 'best');
```



Show only the allocated assets.

```
idx = (pwgt>toler) | (pwgtWithMinWeight>toler);
barh([pwgt(idx), pwgtWithMinWeight(idx)]);
grid on
xlabel('Proportion of Investment')
yticks(1:sum(idx));
yticklabels(p.AssetList(idx));
title('Asset Allocation');
legend('Without min weight limit', 'With min weight limit', 'location', 'best');
```



## Limit the Maximum Number of Assets to Allocate

Use setMinMaxNumAssets to set the maximum number of allocated assets for the Portfolio object. Suppose that you want no more than eight assets invested in the optimal portfolio. To do this with a Portfolio object, use setMinMaxNumAssets.

```
pWithMaxNumAssets = setMinMaxNumAssets(p, [], 8);
```

wgt = estimateFrontier(p);
wgtWithMaxNumAssets = estimateFrontier(pWithMaxNumAssets);
plotFrontier(p, wgt) ; hold on;
plotFrontier(pWithMaxNumAssets, wgtWithMaxNumAssets); hold off;
legend('Baseline portfolio', 'With MaxNumAssets constraint', 'location', 'best');


Use estimateFrontierByReturn to find the allocation that minimizes the risk on the frontier for the given target return.

```
pwgtWithMaxNum = estimateFrontierByReturn(pWithMaxNumAssets, targetRetn);
```

Plot the composition of the two Portfolio objects for the universe of 30 assets.

```
idx = (pwgt>toler) | (pwgtWithMaxNum>toler);
barh([pwgt(idx), pwgtWithMaxNum(idx)]);
grid on
xlabel('Proportion of Investment')
yticks(1:sum(idx));
yticklabels(p.AssetList(idx));
```

```
title('Asset Allocation');
legend('Baseline portfolio', 'With MaxNumAssets constraint', 'location', 'best');
```



Count the total number of allocated assets to verify that only eight assets at most are allocated.

```
sum(abs(pwgtWithMaxNum)>toler)
ans = 8
```


## Limit the Minimum and Maximum Number of Assets to Allocate

Suppose that you want to set both the lower and upper bounds for the number of assets to allocate in a portfolio, given the universe of assets. Use setBounds to specify the allowed number of assets to allocate as from 5 through 10, and the allocated weight as no less than $5 \%$.

```
p1 = setMinMaxNumAssets(p, 5, 10);
p1 = setBounds(p1, 0.05, 'BoundType', 'conditional');
```

If an asset is allocated, it is necessary to clearly define the minimum weight requirement for that asset. This is done using setBounds with a 'Conditional' BoundType. Otherwise, the optimizer cannot evaluate which assets are allocated and cannot formulate the MinNumAssets constraint. For more details, see "Conditional Bounds with LowerBound Defined as Empty or Zero" on page 4-156.

Plot the efficient frontier to compare this portfolio to the baseline portfolio, which has only default constraints.

```
wgt = estimateFrontier(p);
wgt1 = estimateFrontier(pl);
plotFrontier(p, wgt); hold on;
plotFrontier(p1, wgt1); hold off;
legend('Baseline portfolio', 'With MaxNumAssets constraint', 'location', 'best');
```



## Asset Allocation for an Equal-Weighted Portfolio

Create an equal-weighted portfolio using both setBounds and setMinMaxNumAssets functions.

```
numAssetsAllocated = 8;
weight= 1/numAssetsAllocated;
p2 = setBounds(p, weight, weight, 'BoundType', 'conditional');
p2 = setMinMaxNumAssets(p2, numAssetsAllocated, numAssetsAllocated);
```

When any one, or any combination of 'Conditional ' BoundType, MinNumAssets, or MaxNumAssets are active, the optimization problem is formulated as a mixed integer nonlinear programming (MINLP) problem. The Portfolio class automatically constructs the MINLP problem based on the specified constraints.

When working with a Portfolio object, you can select one of three solvers using the setSolverMINLP function. In this example, instead of using default MINLP solver options, customize the solver options to help with a convergence issue. Use a large number (50) for 'MaxIterationsInactiveCut' with setSolverMINLP, instead of the default value of 30 for 'MaxIterationsInactiveCut'. The value 50 works well in finding the efficient frontier of optimal asset allocation.

```
p2 = setSolverMINLP(p2, 'OuterApproximation', 'MaxIterationsInactiveCut', 50);
```

Plot the efficient frontiers for the baseline and equal-weighted portfolios.

```
wgt = estimateFrontier(p);
wgt2 = estimateFrontier(p2);
plotFrontier(p, wgt); hold on;
plotFrontier(p2, wgt2); hold off;
legend('Baseline portfolio', 'Equal Weighted portfolio', 'location', 'best');
```



Use estimateFrontierByRisk to optimize for a specific risk level, in this case . 05 , to determine what allocation maximizes the portfolio return.

```
targetRisk = 0.05;
pwgt = estimateFrontierByRisk(p, targetRisk);
pwgt2 = estimateFrontierByRisk(p2, targetRisk);
idx = (pwgt>toler) | (pwgt2>toler);
barh([pwgt(idx), pwgt2(idx)]);
grid on
xlabel('Proportion of investment')
yticks(1:sum(idx));
yticklabels(p.AssetList(idx));
```

```
title('Asset Allocation');
legend('Baseline portfolio', 'Equal weighted portfolio', 'location', 'best');
```



## Use 'Conditional' BoundType, MinNumAssets, and MaxNumAssets Constraints with Other Constraints

You can define other constraints for a Portfolio object using the set functions. These other constraints for a Portfolio object, such as group, linear inequality, turnover, and tracking error can be used together with the 'Conditional' BoundType, 'MinNumAssets', and 'MaxNumAssets' constraints. For example, specify a tracking error constraint using setTrackingError.

```
ii = [15, 16, 20, 21, 23, 25, 27, 29, 30]; % indexes of assets to include in tracki|
trackingPort(ii) = 1/numel(ii);
q = setTrackingError(p, 0.5, trackingPort);
```

Then use setMinMaxNumAssets to add a constraint to limit maximum number of assets to invest.

```
q = setMinMaxNumAssets(q, [], 8);
```

On top of these previously specified constraints, use setBounds to add a constraint to limit the weight for the allocated assets. You can use constraints with mixed BoundType values, where 'Simple' means $\mathrm{lb} \leq x_{i} \leq \mathrm{ub}$ and 'Conditional' means $x_{i}=0$ or $\mathrm{lb} \leq x_{i} \leq \mathrm{ub}$.

Allow the assets in trackingPort to have the BoundType value 'Conditional' in the optimum allocation.

```
lb = zeros(q.NumAssets, 1);
ub = zeros(q.NumAssets, 1)*0.5;
lb(ii) = 0.1;
ub(ii) = 0.3;
boundType = repmat("simple",q.NumAssets,1);
boundType(ii) = "conditional";
q = setBounds(q, lb, ub, 'BoundType',boundType);
```

Plot the efficient frontier:

```
plotFrontier(q);
```



Use estimateFrontierByReturn to find the allocation that minimizes the risk for a given return at 0.125.

```
targetRetn = 0.0125;
```

pwgt = estimateFrontierByReturn(q, targetRetn);

Show the allocation of assets by weight.

```
idx = abs(pwgt)>eps;
assetnames = q.AssetList';
Asset = assetnames(idx);
Weight = pwgt(idx);
resultAlloc = table(Asset, Weight)
```

```
resultAlloc=7\times2 table
    Asset Weight
    'JNJ' 0.1
    'MMM' 0.19503
    'MO' 0.1485
    'MSFT' 0.1
    'PG' 0.1
    'WMT' 0.2212
    'XOM' 0.13527
```


## See Also

Portfolio| setBounds | setMinMaxNumAssets | setSolverMINLP

## Related Examples

- "Creating the Portfolio Object" on page 4-28
- "Working with Portfolio Constraints Using Defaults" on page 4-68
- "Asset Allocation Case Study" on page 4-196
- "Portfolio Optimization Examples" on page 4-167


## More About

- "Portfolio Object" on page 4-23
- "Portfolio Optimization Theory" on page 4-3
- "Working with 'Conditional' BoundType, MinNumAssets, and MaxNumAssets Constraints Using Portfolio Object" on page 4-94
- "Portfolio Object Workflow" on page 4-21


## External Websites

- Using MATLAB to Optimize Portfolios with Financial Toolbox (33 min 24 sec )
- MATLAB for Advanced Portfolio Construction and Stock Selection Models (30 min 28 sec)


## Portfolio Optimization Against a Benchmark

This example shows how to perform portfolio optimization using the Portfolio object in Financial Toolbox ${ }^{\mathrm{TM}}$.

This example, in particular, demonstrates optimizing a portfolio to maximize the information ratio relative to a market benchmark. Specifically, financial data contained in a table is read into MATLAB® and visualizations (at both daily and annual levels) are performed. A Portfolio object is created with the market data using an active daily return for each asset. Using functions supporting a Portfolio object, the efficient frontier is calculated directly. Then, a customized optimization problem is solved to find the asset allocation with the maximized information ratio.


## Import Historical Data Using MATLAB®

Import historical prices for the asset universe and the Dow Jones Industrial Average (DJI) market benchmark. The data is imported into a table from a Microsoft ${ }^{\circledR}$ Excel ${ }^{\circledR}$ spreadsheet using the MATLAB® readtable function.

```
data = readtable('dowPortfolio.xlsx');
head(data, 10)
```

| Dates | DJI | AA | AIG | AXP | BA | C | CAT | DD |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 03-Jan-2006 | 10847 | 28.72 | 68.41 | 51.53 | 68.63 | 45.26 | 55.86 | 40.68 |
| 04-Jan-2006 | 10880 | 28.89 | 68.51 | 51.03 | 69.34 | 44.42 | 57.29 | 40.4 |
| 05-Jan-2006 | 10882 | 29.12 | 68.6 | 51.57 | 68.53 | 44.65 | 57.29 | 40.3 |
| 06-Jan-2006 | 10959 | 29.02 | 68.89 | 51.75 | 67.57 | 44.65 | 58.43 | 40.5 |
| 09-Jan-2006 | 11012 | 29.37 | 68.57 | 53.04 | 67.01 | 44.43 | 59.49 | 40.3 |
| 10-Jan-2006 | 11012 | 28.44 | 69.18 | 52.88 | 67.33 | 44.57 | 59.25 | 40. |
| 11-Jan-2006 | 11043 | 28.05 | 69.6 | 52.59 | 68.3 | 44.98 | 59.28 | 38.8 |
| 12-Jan-2006 | 10962 | 27.68 | 69.04 | 52.6 | 67.9 | 45.02 | 60.13 | 38.0 |
| 13-Jan-2006 | 10960 | 27.81 | 68.84 | 52.5 | 67.7 | 44.92 | 60.24 | 37.8 |
| 17-Jan-2006 | 10896 | 27.97 | 67.84 | 52.03 | 66.93 | 44.47 | 60.85 | 37.7 |

Separate the asset names, asset prices, and DJI benchmark prices from the table. The visualization shows the evolution of all the asset prices normalized to start at unity, that is accumulative returns.

```
dates = datenum(data.Dates);
benchPrice = data.DJI;
assetNames = data.Properties.VariableNames(3:2:end); % using half of the assets for di
assetPrice = data(:,assetNames).Variables;
assetP = assetPrice./assetPrice(1, :);
benchmarkP = benchPrice / benchPrice(1);
figure;
plot(dates,assetP);
hold on;
plot(dates,benchmarkP,'LineWidth',3,'Color','k');
hold off;
xlabel('Date');
ylabel('Normalized Price');
title('Normalized Asset Prices and Benchmark');
datetick('x');
grid on;
```



The bold line indicates the DJIA market benchmark.

## Compute Returns and Risk-Adjusted Returns

Calculate the return series from the price series and compute the asset moments (historical returns and standard deviations). The visualization shows a scatter plot of the risk-return characteristics of all the assets and the DJI market benchmark.

```
benchReturn = tick2ret(benchPrice);
assetReturn = tick2ret(assetPrice);
benchRetn = mean(benchReturn);
benchRisk = std(benchReturn);
```

```
assetRetn = mean(assetReturn);
assetRisk = std(assetReturn);
```

Calculate historical statistics and plot the annual risk-return. Note that the plot is at the annual level, therefore scaling is performed on the daily returns.

```
scale = 252;
assetRiskR = sqrt(scale) * assetRisk;
benchRiskR = sqrt(scale) * benchRisk;
assetReturnR = scale * assetRetn;
benchReturnR = scale * benchRetn;
figure;
scatter(assetRiskR, assetReturnR, 6, 'm', 'Filled');
hold on
scatter(benchRiskR, benchReturnR, 6, 'g', 'Filled');
for k = 1:length(assetNames)
    text(assetRiskR(k) + 0.005, assetReturnR(k), assetNames{k}, 'FontSize', 8);
end
text(benchRiskR + 0.005, benchReturnR, 'Benchmark', 'Fontsize', 8);
hold off;
xlabel('Risk (Std Dev of Return)');
ylabel('Expected Annual Return');
grid on;
```



## Set Up a Portfolio Optimization

Set up a portfolio optimization problem by populating the object using Portfolio. Because the goal is to optimize portfolio allocation against a benchmark, the active return of each asset is computed and used in the Portfolio object. In this example, the expected returns and covariances of the assets in the portfolio are set to their historical values.
p = Portfolio('AssetList',assetNames);
Set up default portfolio constraints (all weights sum to 1, no shorting, and 100\% investment in risky assets).
$\mathrm{p}=$ setDefaultConstraints(p);

```
Add asset returns and covariance to the Portfolio object.
activReturn = assetReturn - benchReturn;
pAct = estimateAssetMoments(p,activReturn,'missingdata',false)
pAct =
    Portfolio with properties:
            BuyCost: []
            SellCost: []
        RiskFreeRate: []
            AssetMean: [15x1 double]
            AssetCovar: [15\times15 double]
        TrackingError: []
            TrackingPort: []
            Turnover: []
            BuyTurnover: []
            SellTurnover: []
            Name: []
            NumAssets: 15
            AssetList: {1x15 cell}
            InitPort: []
            AInequality: []
            bInequality: []
            AEquality: []
            bEquality: []
            LowerBound: [15x1 double]
            UpperBound: []
            LowerBudget: 1
            UpperBudget: 1
            GroupMatrix: []
            LowerGroup: []
            UpperGroup: []
                            GroupA: []
                            GroupB: []
            LowerRatio: []
            UpperRatio: []
            MinNumAssets: []
            MaxNumAssets: []
            BoundType: [15x1 categorical]
```


## Compute the Efficient Frontier Using the Portfolio Object

Compute the mean-variance efficient frontier of 20 optimal portfolios. Visualize the frontier over the risk-return characteristics of the individual assets. Furthermore, calculate and visualize the information ratio for each portfolio along the frontier.
pwgtAct = estimateFrontier(pAct, 20); \% Estimate weights [portRiskAct, portRetnAct] = estimatePortMoments(pAct, pwgtAct); \% Get risk and return

```
% Extract asset moments & names
[assetActRetnDaily, assetActCovarDaily] = getAssetMoments(pAct);
assetActRiskDaily = sqrt(diag(assetActCovarDaily));
assetNames = pAct.AssetList;
```

\% Rescale
assetActRiskAnnual = sqrt(scale) * assetActRiskDaily;
portRiskAnnual $=\operatorname{sqrt}(s c a l e)$ * portRiskAct;
assetActRetnAnnual $=$ scale $*$ assetActRetnDaily;
portRetnAnnual $=$ scale * portRetnAct;
figure;
subplot(2,1,1);
plot(portRiskAnnual, portRetnAnnual, 'bo-', 'MarkerFaceColor', 'b');
hold on;
scatter(assetActRiskAnnual, assetActRetnAnnual, 12, 'm', 'Filled');
hold on;
for $k=1: l e n g t h(a s s e t N a m e s)$
text(assetActRiskAnnual(k) + 0.005, assetActRetnAnnual(k), assetNames\{k\}, 'FontSiz
end
hold off;
xlabel('Risk (Std Dev of Active Return)');
ylabel('Expected Active Return');
grid on;
subplot(2,1,2);
plot(portRiskAnnual, portRetnAnnual./portRiskAnnual, 'bo-', 'MarkerFaceColor', 'b');
xlabel('Risk (Std Dev of Active Return)');
ylabel('Information Ratio');
grid on;


## Perform Information Ratio Maximization Using Optimization Toolbox ${ }^{\text {Tm }}$

Run a hybrid optimization to find the portfolio along the frontier with the maximum information ratio. The information ratio is the ratio of relative return to relative risk (known as "tracking error"). Whereas the Sharpe ratio looks at returns relative to a riskless asset, the information ratio is based on returns relative to a risky benchmark, in this case the DJI benchmark. This is done by running an optimization that finds the optimal return constraint for which the portfolio optimization problem returns the maximum information ratio portfolio. The portfolio optimization functions are called from an objective function infoRatioTargetReturn that is optimized by the Optimization Toolbox ${ }^{\mathrm{TM}}$ function fminbnd. The local function infoRatioTargetReturn calculates a minimum (active) risk portfolio given a target active return.

The infoRatioTargetReturn local function is called as an objective function in an optimization routine (fminbnd) that seeks to find the target return that maximizes the information ratio and minimizes a negative information ratio.

```
objFun = @(targetReturn) -infoRatioTargetReturn(targetReturn,pAct);
options = optimset('TolX',1.0e-8);
[optPortRetn, ~, exitflag] = fminbnd(objFun,0,max(portRetnAct),options);
```

Get weights, information ratio, and risk return for the optimal portfolio.

```
[optInfoRatio,optWts] = infoRatioTargetReturn(optPortRetn,pAct);
optPortRisk = estimatePortRisk(pAct,optWts)
optPortRisk = 0.0040
```


## Plot the Optimal Portfolio

Verify that the portfolio found is indeed the maximum information-ratio portfolio.

```
% Rescale
optPortRiskAnnual = sqrt(scale) * optPortRisk;
optPortReturnAnnual = scale * optPortRetn;
figure;
subplot(2,1,1);
scatter(assetActRiskAnnual, assetActRetnAnnual, 6, 'm', 'Filled');
hold on
for k = 1:length(assetNames)
    text(assetActRiskAnnual(k) + 0.005,assetActRetnAnnual(k),assetNames{k},'FontSize',
end
plot(portRiskAnnual,portRetnAnnual,'bo-','MarkerSize',4,'MarkerFaceColor','b');
plot(optPortRiskAnnual,optPortReturnAnnual,'ro-','MarkerFaceColor','r');
hold off;
xlabel('Risk (Std Dev of Active Return)');
ylabel('Expected Active Return');
grid on;
subplot(2,1,2);
plot(portRiskAnnual,portRetnAnnual./portRiskAnnual, 'bo- ','MarkerSize' ,4,'MarkerFaceCold
hold on
plot(optPortRiskAnnual,optPortReturnAnnual./optPortRiskAnnual, 'ro-' ','MarkerFaceColor',
hold off;
```

```
xlabel('Risk (Std Dev of Active Return)');
ylabel('Information Ratio');
title('Information Ratio with Optimal Portfolio');
grid on;
```



## Display the Portfolio Optimization Solution

Display the portfolio optimization solution.
assetIndx = optWts > .001;
results = table(assetNames(assetIndx)', optWts(assetIndx)*100, 'VariableNames',\{'Asset disp('Maximum Information Ratio Portfolio:');

Maximum Information Ratio Portfolio:

```
disp(results);
    Asset Weight
    'AA' 1.539
    'AXP' 0.3555
    'C' 9.6533
    'DD' 4.0684
    'HPQ' 17.698
    'JPM' 21.565
    'MCD' 26.736
    'MO' 13.648
    'MSFT' 2.6858
    'UTX' 2.0509
```

fprintf('Max. Info Ratio portfolio has expected active return \%0.2f\%\%)n', optPortRetn*
Max. Info Ratio portfolio has expected active return 12.14\%
fprintf('Max. Info Ratio portfolio has expected tracking error of \%0.2f\%\%)n', optPortR
Max. Info Ratio portfolio has expected tracking error of 6.32\%

## Local Function

```
function [infoRatio,wts] = infoRatioTargetReturn(targetReturn,portObj)
% Calculate information ratio for a target-return portfolio along the
% efficient frontier
wts = estimateFrontierByReturn(portObj,targetReturn);
portRiskAct = estimatePortRisk(port0bj,wts);
infoRatio = targetReturn/portRiskAct;
end
```


## See Also

Portfolio| fminbnd |inforatio

## Related Examples

- "Creating the Portfolio Object" on page 4-28
- "Working with Portfolio Constraints Using Defaults" on page 4-68
- "Validate the Portfolio Problem for Portfolio Object" on page 4-110
- "Estimate Efficient Portfolios for Entire Efficient Frontier for Portfolio Object" on page 4-115
- "Estimate Efficient Frontiers for Portfolio Object" on page 4-140
- "Postprocessing Results to Set Up Tradable Portfolios" on page 4-149
- "Portfolio Optimization Examples" on page 4-167
- "Information Ratio" on page 7-9


## More About

- "Performance Metrics Overview" on page 7-2
- "Portfolio Object" on page 4-23
- "Portfolio Optimization Theory" on page 4-3
- "Portfolio Object Workflow" on page 4-21


## External Websites

- Using MATLAB to Optimize Portfolios with Financial Toolbox (33 min 24 sec )
- MATLAB for Advanced Portfolio Construction and Stock Selection Models ( 30 min 28 sec)


## CVaR Portfolio Optimization Tools

- "Portfolio Optimization Theory" on page 5-3
- "Portfolio Set for Optimization Using PortfolioCVaR Object" on page 5-10
- "Default Portfolio Problem" on page 5-18
- "PortfolioCVaR Object Workflow" on page 5-20
- "PortfolioCVaR Object" on page 5-22
- "Creating the PortfolioCVaR Object" on page 5-27
- "Common Operations on the PortfolioCVaR Object" on page 5-36
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## Portfolio Optimization Theory

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```


## Portfolio Optimization Problems

Portfolio optimization problems involve identifying portfolios that satisfy three criteria:

- Minimize a proxy for risk.
- Match or exceed a proxy for return.
- Satisfy basic feasibility requirements.

Portfolios are points from a feasible set of assets that constitute an asset universe. A portfolio specifies either holdings or weights in each individual asset in the asset universe. The convention is to specify portfolios in terms of weights, although the portfolio optimization tools work with holdings as well.

The set of feasible portfolios is necessarily a nonempty, closed, and bounded set. The proxy for risk is a function that characterizes either the variability or losses associated with portfolio choices. The proxy for return is a function that characterizes either the gross or net benefits associated with portfolio choices. The terms "risk" and "risk proxy" and "return" and "return proxy" are interchangeable. The fundamental insight of Markowitz (see "Portfolio Optimization" on page A-7) is that the goal of the portfolio choice problem is to seek minimum risk for a given level of return and to seek maximum return for a given level of risk. Portfolios satisfying these criteria are efficient portfolios and the graph of the risks and returns of these portfolios forms a curve called the efficient frontier.

## Portfolio Problem Specification

To specify a portfolio optimization problem, you need the following:

- Proxy for portfolio return ( $\mu$ )
- Proxy for portfolio risk ( $\Sigma$ )
- Set of feasible portfolios ( $X$ ), called a portfolio set

Financial Toolbox has three objects to solve specific types of portfolio optimization problems:

- The Portfolio object (Portfolio) supports mean-variance portfolio optimization (see Markowitz [46], [47] at "Portfolio Optimization" on page A-7). This object has either gross or net portfolio returns as the return proxy, the variance of portfolio returns as the risk proxy, and a portfolio set that is any combination of the specified constraints to form a portfolio set.
- The PortfolioCVaR object (PortfolioCVaR) implements what is known as conditional value-at-risk portfolio optimization (see Rockafellar and Uryasev [48], [49] at "Portfolio Optimization" on page A-7), which is generally referred to as CVaR portfolio optimization. CVaR portfolio optimization works with the same return proxies and portfolio sets as mean-variance portfolio optimization but uses conditional value-at-risk of portfolio returns as the risk proxy.
- The PortfolioMAD object (PortfolioMAD) implements what is known as meanabsolute deviation portfolio optimization (see Konno and Yamazaki [50] at "Portfolio Optimization" on page A-7), which is referred to as MAD portfolio optimization. MAD portfolio optimization works with the same return proxies and portfolio sets as mean-variance portfolio optimization but uses mean-absolute deviation portfolio returns as the risk proxy.


## Return Proxy

The proxy for portfolio return is a function $\mu: X \rightarrow R$ on a portfolio set $X \subset R^{n}$ that characterizes the rewards associated with portfolio choices. Usually, the proxy for portfolio return has two general forms, gross and net portfolio returns. Both portfolio return forms separate the risk-free rate $r_{0}$ so that the portfolio $x \in X$ contains only risky assets.

Regardless of the underlying distribution of asset returns, a collection of $S$ asset returns $y_{1}, \ldots, y_{S}$ has a mean of asset returns

$$
m=\frac{1}{S} \sum_{s=1}^{S} y_{s}
$$

and (sample) covariance of asset returns

$$
C=\frac{1}{S-1} \sum_{s=1}^{S}\left(y_{s}-m\right)\left(y_{s}-m\right)^{T} .
$$

These moments (or alternative estimators that characterize these moments) are used directly in mean-variance portfolio optimization to form proxies for portfolio risk and return.

## Gross Portfolio Returns

The gross portfolio return for a portfolio $x \in X$ is

$$
\mu(x)=r_{0}+\left(m-r_{0} 1\right)^{T} x,
$$

where:
$r_{0}$ is the risk-free rate (scalar).
$m$ is the mean of asset returns ( $n$ vector).
If the portfolio weights sum to 1 , the risk-free rate is irrelevant. The properties in the Portfolio object to specify gross portfolio returns are:

- RiskFreeRate for $r_{0}$
- AssetMean for $m$


## Net Portfolio Returns

The net portfolio return for a portfolio $x \in X$ is

$$
\mu(x)=r_{0}+\left(m-r_{0} 1\right)^{T} x-b^{T} \max \left\{0, x-x_{0}\right\}-s^{T} \max \left\{0, x_{0}-x\right\},
$$

where:
$r_{0}$ is the risk-free rate (scalar).
$m$ is the mean of asset returns ( $n$ vector).
$b$ is the proportional cost to purchase assets ( $n$ vector).
$s$ is the proportional cost to sell assets ( $n$ vector).
You can incorporate fixed transaction costs in this model also. Though in this case, it is necessary to incorporate prices into such costs. The properties in the Portfolio object to specify net portfolio returns are:

- RiskFreeRate for $r_{0}$
- AssetMean for $m$
- InitPort for $x_{0}$
- BuyCost for $b$
- SellCost for s


## Risk Proxy

The proxy for portfolio risk is a function $\Sigma: X \rightarrow R$ on a portfolio set $X \subset R^{n}$ that characterizes the risks associated with portfolio choices.

## Variance

The variance of portfolio returns for a portfolio $x \in X$ is

$$
\Sigma(x)=x^{T} C x
$$

where $C$ is the covariance of asset returns ( $n$-by-n positive-semidefinite matrix).
The property in the Portfolio object to specify the variance of portfolio returns is AssetCovar for $C$.

Although the risk proxy in mean-variance portfolio optimization is the variance of portfolio returns, the square root, which is the standard deviation of portfolio returns, is often reported and displayed. Moreover, this quantity is often called the "risk" of the portfolio. For details, see Markowitz ("Portfolio Optimization" on page A-7).

## Conditional Value-at-Risk

The conditional value-at-risk for a portfolio $x \in X$, which is also known as expected shortfall, is defined as

$$
C \operatorname{VaR}_{\alpha}(x)=\frac{1}{1-\alpha} \int_{f(x, y) \geq V a R_{\alpha}(x)} f(x, y) p(y) d y
$$

where:
$\alpha$ is the probability level such that $0<\alpha<1$.
$f(x, y)$ is the loss function for a portfolio $x$ and asset return $y$.
$p(y)$ is the probability density function for asset return $y$.
$V a R_{\alpha}$ is the value-at-risk of portfolio $x$ at probability level $\alpha$.
The value-at-risk is defined as

$$
\operatorname{VaR}_{\alpha}(x)=\min \{\gamma: \operatorname{Pr}[f(x, Y) \leq \gamma] \geq \alpha\} .
$$

An alternative formulation for CVaR has the form:

$$
\operatorname{CVa}_{\alpha}(x)=\operatorname{VaR}_{\alpha}(x)+\frac{1}{1-\alpha} \int_{R^{n}} \max \left\{0,\left(f(x, y)-\operatorname{Va}_{\alpha}(x)\right)\right\} p(y) d y
$$

The choice for the probability level $\alpha$ is typically 0.9 or 0.95 . Choosing $\alpha$ implies that the value-at-risk $\operatorname{VaR}_{\alpha}(x)$ for portfolio $x$ is the portfolio return such that the probability of portfolio returns falling below this level is $(1-\alpha)$. Given $\operatorname{VaR}_{\alpha}(x)$ for a portfolio $x$, the conditional value-at-risk of the portfolio is the expected loss of portfolio returns above the value-at-risk return.

Note Value-at-risk is a positive value for losses so that the probability level $\alpha$ indicates the probability that portfolio returns are below the negative of the value-at-risk.

To describe the probability distribution of returns, the PortfolioCVaR object takes a finite sample of return scenarios $y_{s}$, with $s=1, \ldots, S$. Each $y_{s}$ is an $n$ vector that contains the returns for each of the $n$ assets under the scenario $s$. This sample of $S$ scenarios is stored as a scenario matrix of size $S$-by-n. Then, the risk proxy for CVaR portfolio optimization, for a given portfolio $x \in X$ and $\alpha \in(0,1)$, is computed as

$$
\sum(x)=V a R_{\alpha}(x)+\frac{1}{(1-\alpha) S} \sum_{s=1}^{S} \max \left\{0,-y_{s}^{T} x-V a R_{\alpha}(x)\right\}
$$

The value-at-risk, $\operatorname{Va}_{\alpha}(x)$, is estimated whenever the CVaR is estimated. The loss function is $f\left(x, y_{s}\right)=-y_{s}^{T} x$, which is the portfolio loss under scenario $s$.

Under this definition, VaR and CVaR are sample estimators for VaR and CVaR based on the given scenarios. Better scenario samples yield more reliable estimates of VaR and CVaR.

For more information, see Rockafellar and Uryasev [48], [49], and Cornuejols and Tütüncü, [51], at "Portfolio Optimization" on page A-7.

## Mean Absolute-Deviation

The mean-absolute deviation (MAD) for a portfolio $x \in X$ is defined as

$$
\sum(x)=\frac{1}{S} \sum_{s=1}^{S}\left|\left(y_{s}-m\right)^{T} x\right|
$$

where:
$y_{s}$ are asset returns with scenarios $s=1, \ldots S$ ( $S$ collection of $n$ vectors).
$f(x, y)$ is the loss function for a portfolio $x$ and asset return $y$.
$m$ is the mean of asset returns ( $n$ vector).
such that

$$
m=\frac{1}{S} \sum_{s=1}^{S} y_{s}
$$

For more information, see Konno and Yamazaki [50] at "Portfolio Optimization" on page A7.

## See Also

PortfolioCVaR

## Related Examples

- "Creating the PortfolioCVaR Object" on page 5-27
- "Working with CVaR Portfolio Constraints Using Defaults" on page 5-63


## More About

- "PortfolioCVaR Object" on page 5-22
- "Portfolio Set for Optimization Using PortfolioCVaR Object" on page 5-10
- "Default Portfolio Problem" on page 5-18
- "PortfolioCVaR Object Workflow" on page 5-20


## External Websites

- Analyzing Investment Strategies with CVaR Portfolio Optimization in MATLAB (50 $\min 42 \mathrm{sec}$ )


## Portfolio Set for Optimization Using PortfolioCVaR Object

The final element for a complete specification of a portfolio optimization problem is the set of feasible portfolios, which is called a portfolio set. A portfolio set $X \subset R^{n}$ is specified by construction as the intersection of sets formed by a collection of constraints on portfolio weights. A portfolio set necessarily and sufficiently must be a nonempty, closed, and bounded set.

When setting up your portfolio set, ensure that the portfolio set satisfies these conditions. The most basic or "default" portfolio set requires portfolio weights to be nonnegative (using the lower-bound constraint) and to sum to 1 (using the budget constraint). The most general portfolio set handled by the portfolio optimization tools can have any of these constraints:

- Linear inequality constraints
- Linear equality constraints
- Bound constraints
- Budget constraints
- Group constraints
- Group ratio constraints
- Average turnover constraints
- One-way turnover constraints


## Linear Inequality Constraints

Linear inequality constraints are general linear constraints that model relationships among portfolio weights that satisfy a system of inequalities. Linear inequality constraints take the form

$$
A_{I} x \leq b_{I}
$$

where:
$x$ is the portfolio ( $n$ vector).
$A_{I}$ is the linear inequality constraint matrix ( $n_{I}$-by- $n$ matrix).
$b_{I}$ is the linear inequality constraint vector ( $n_{I}$ vector).
$n$ is the number of assets in the universe and $n_{I}$ is the number of constraints.
Portfolio object properties to specify linear inequality constraints are:

- AInequality for $A_{I}$
- bInequality for $b_{I}$
- NumAssets for $n$

The default is to ignore these constraints.

## Linear Equality Constraints

Linear equality constraints are general linear constraints that model relationships among portfolio weights that satisfy a system of equalities. Linear equality constraints take the form

$$
A_{E} x=b_{E}
$$

where:
$x$ is the portfolio ( $n$ vector).
$A_{E}$ is the linear equality constraint matrix ( $n_{E}$-by- $n$ matrix).
$b_{E}$ is the linear equality constraint vector ( $n_{E}$ vector).
$n$ is the number of assets in the universe and $n_{E}$ is the number of constraints.
Portfolio object properties to specify linear equality constraints are:

- AEquality for $A_{E}$
- bEquality for $b_{E}$
- NumAssets for $n$

The default is to ignore these constraints.

## Bound Constraints

Bound constraints are specialized linear constraints that confine portfolio weights to fall either above or below specific bounds. Since every portfolio set must be bounded, it is often a good practice, albeit not necessary, to set explicit bounds for the portfolio problem. To obtain explicit bounds for a given portfolio set, use the estimateBounds function. Bound constraints take the form

$$
l_{B} \leq x \leq u_{B}
$$

where:
$x$ is the portfolio ( $n$ vector).
$l_{B}$ is the lower-bound constraint ( $n$ vector).
$u_{B}$ is the upper-bound constraint ( $n$ vector).
$n$ is the number of assets in the universe.
Portfolio object properties to specify bound constraints are:

- LowerBound for $l_{B}$
- UpperBound for $u_{B}$
- NumAssets for $n$

The default is to ignore these constraints.
The default portfolio optimization problem (see "Default Portfolio Problem" on page 518) has $l_{B}=0$ with $u_{B}$ set implicitly through a budget constraint.

## Budget Constraints

Budget constraints are specialized linear constraints that confine the sum of portfolio weights to fall either above or below specific bounds. The constraints take the form

$$
l_{S} \leq 1^{T} x \leq u_{S}
$$

where:
$x$ is the portfolio ( $n$ vector).
1 is the vector of ones ( $n$ vector).
$l_{S}$ is the lower-bound budget constraint (scalar).
$u_{S}$ is the upper-bound budget constraint (scalar).
$n$ is the number of assets in the universe.
Portfolio object properties to specify budget constraints are:

- LowerBudget for $l_{S}$
- UpperBudget for $u_{S}$
- NumAssets for $n$

The default is to ignore this constraint.
The default portfolio optimization problem (see "Default Portfolio Problem" on page 518) has $l_{S}=u_{S}=1$, which means that the portfolio weights sum to 1 . If the portfolio optimization problem includes possible movements in and out of cash, the budget constraint specifies how far portfolios can go into cash. For example, if $l_{S}=0$ and $u_{S}=1$, then the portfolio can have 0-100\% invested in cash. If cash is to be a portfolio choice, set RiskFreeRate ( $r_{0}$ ) to a suitable value (see "Portfolio Problem Specification" on page 5-3 and "Working with a Riskless Asset" on page 5-56).

## Group Constraints

Group constraints are specialized linear constraints that enforce "membership" among groups of assets. The constraints take the form

$$
l_{G} \leq G x \leq u_{G}
$$

where:
$x$ is the portfolio ( $n$ vector).
$l_{G}$ is the lower-bound group constraint ( $n_{G}$ vector).
$u_{G}$ is the upper-bound group constraint ( $n_{G}$ vector).
$G$ is the matrix of group membership indexes ( $n_{G}$-by- $n$ matrix).

Each row of $G$ identifies which assets belong to a group associated with that row. Each row contains either 0 s or 1 s with 1 indicating that an asset is part of the group or 0 indicating that the asset is not part of the group.

Portfolio object properties to specify group constraints are:

- GroupMatrix for $G$
- LowerGroup for $l_{G}$
- UpperGroup for $u_{G}$
- NumAssets for $n$

The default is to ignore these constraints.

## Group Ratio Constraints

Group ratio constraints are specialized linear constraints that enforce relationships among groups of assets. The constraints take the form

$$
l_{R i}\left(G_{B} x\right)_{i} \leq\left(G_{A} x\right)_{i} \leq u_{R i}\left(G_{B} x\right)_{i}
$$

for $i=1, \ldots, n_{R}$ where:
$x$ is the portfolio ( $n$ vector).
$l_{R}$ is the vector of lower-bound group ratio constraints ( $n_{R}$ vector).
$u_{R}$ is the vector matrix of upper-bound group ratio constraints ( $n_{R}$ vector).
$G_{A}$ is the matrix of base group membership indexes ( $n_{R}$-by- $n$ matrix).
$G_{B}$ is the matrix of comparison group membership indexes ( $n_{R}$-by- $n$ matrix).
$n$ is the number of assets in the universe and $n_{R}$ is the number of constraints.
Each row of $G_{A}$ and $G_{B}$ identifies which assets belong to a base and comparison group associated with that row.

Each row contains either 0 s or 1 s with 1 indicating that an asset is part of the group or 0 indicating that the asset is not part of the group.

Portfolio object properties to specify group ratio constraints are:

- GroupA for $G_{A}$
- GroupB for $G_{B}$
- LowerRatio for $l_{R}$
- UpperRatio for $u_{R}$
- NumAssets for $n$

The default is to ignore these constraints.

## Average Turnover Constraints

Turnover constraint is a linear absolute value constraint that ensures estimated optimal portfolios differ from an initial portfolio by no more than a specified amount. Although portfolio turnover is defined in many ways, the turnover constraints implemented in Financial Toolbox computes portfolio turnover as the average of purchases and sales. Average turnover constraints take the form

$$
\frac{1}{2} 1^{T}\left|x-x_{0}\right| \leq \tau
$$

where:
$x$ is the portfolio ( $n$ vector).
1 is the vector of ones ( $n$ vector).
$x_{0}$ is the initial portfolio ( $n$ vector).
$\tau$ is the upper bound for turnover (scalar).
$n$ is the number of assets in the universe.
Portfolio object properties to specify the average turnover constraint are:

- Turnover for $\tau$
- InitPort for $x_{0}$
- NumAssets for $n$

The default is to ignore this constraint.

## One-way Turnover Constraints

One-way turnover constraints ensure that estimated optimal portfolios differ from an initial portfolio by no more than specified amounts according to whether the differences are purchases or sales. The constraints take the forms

$$
\begin{aligned}
& 1^{T} \times \max \left\{0, x-x_{0}\right\} \leq \tau_{B} \\
& 1^{T} \times \max \left\{0, x_{0}-x\right\} \leq \tau_{S}
\end{aligned}
$$

where:
$x$ is the portfolio ( $n$ vector)
1 is the vector of ones ( $n$ vector).
$x_{0}$ is the Initial portfolio ( $n$ vector).
$\tau_{B}$ is the upper bound for turnover constraint on purchases (scalar).
$\tau_{S}$ is the upper bound for turnover constraint on sales (scalar).
To specify one-way turnover constraints, use the following properties in the Portfolio or PortfolioCVaR object:

- BuyTurnover for $\tau_{B}$
- SellTurnover for $\tau_{S}$
- InitPort for $x_{0}$

The default is to ignore this constraint.

Note The average turnover constraint (see "Average Turnover Constraints" on page 5-15) with $\tau$ is not a combination of the one-way turnover constraints with $\tau=\tau_{B}=\tau_{s}$.

## See Also

PortfolioCVaR

## Related Examples

- "Creating the PortfolioCVaR Object" on page 5-27
- "Working with CVaR Portfolio Constraints Using Defaults" on page 5-63


## More About

- "PortfolioCVaR Object" on page 5-22
- "Default Portfolio Problem" on page 5-18
- "PortfolioCVaR Object Workflow" on page 5-20


## External Websites

- Analyzing Investment Strategies with CVaR Portfolio Optimization in MATLAB (50 $\min 42 \mathrm{sec}$ )


## Default Portfolio Problem

The default portfolio optimization problem has a risk and return proxy associated with a given problem, and a portfolio set that specifies portfolio weights to be nonnegative and to sum to 1 . The lower bound combined with the budget constraint is sufficient to ensure that the portfolio set is nonempty, closed, and bounded. The default portfolio optimization problem characterizes a long-only investor who is fully invested in a collection of assets.

- For mean-variance portfolio optimization, it is sufficient to set up the default problem. After setting up the problem, data in the form of a mean and covariance of asset returns are then used to solve portfolio optimization problems.
- For conditional value-at-risk portfolio optimization, the default problem requires the additional specification of a probability level that must be set explicitly. Generally, "typical" values for this level are 0.90 or 0.95 . After setting up the problem, data in the form of scenarios of asset returns are then used to solve portfolio optimization problems.
- For MAD portfolio optimization, it is sufficient to set up the default problem. After setting up the problem, data in the form of scenarios of asset returns are then used to solve portfolio optimization problems.


## See Also

PortfolioCVaR

## Related Examples

- "Creating the PortfolioCVaR Object" on page 5-27
- "Working with CVaR Portfolio Constraints Using Defaults" on page 5-63


## More About

- "PortfolioCVaR Object" on page 5-22
- "Portfolio Set for Optimization Using PortfolioCVaR Object" on page 5-10
- "PortfolioCVaR Object Workflow" on page 5-20


## External Websites

- Analyzing Investment Strategies with CVaR Portfolio Optimization in MATLAB (50 $\min 42 \mathrm{sec}$ )


## PortfolioCVaR Object Workflow

The PortfolioCVaR object workflow for creating and modeling a CVaR portfolio is:
1 Create a CVaR Portfolio.
Create a PortfolioCVaR object for conditional value-at-risk (CVaR) portfolio optimization. For more information, see "Creating the PortfolioCVaR Object" on page 5-27.

2 Define asset returns and scenarios.
Evaluate scenarios for portfolio asset returns, including assets with missing data and financial time series data. For more information, see "Asset Returns and Scenarios Using PortfolioCVaR Object" on page 5-44.
3 Specify the CVaR Portfolio Constraints.
Define the constraints for portfolio assets such as linear equality and inequality, bound, budget, group, group ratio, and turnover constraints. For more information, see "Working with CVaR Portfolio Constraints Using Defaults" on page 5-63.
4 Validate the CVaR Portfolio.
Identify errors for the portfolio specification. For more information, see "Validate the CVaR Portfolio Problem" on page 5-96.
5 Estimate the efficient portfolios and frontiers.
Analyze the efficient portfolios and efficient frontiers for a CVaR portfolio. For more information, see "Estimate Efficient Portfolios for Entire Frontier for PortfolioCVaR Object" on page 5-101 and "Estimate Efficient Frontiers for PortfolioCVaR Object" on page 5-118.
6 Postprocess the results.
Use the efficient portfolios and efficient frontiers results to set up trades. For more information, see "Postprocessing Results to Set Up Tradable Portfolios" on page 5129.

## See Also

## More About

- "Portfolio Optimization Theory" on page 5-3


## External Websites

- Analyzing Investment Strategies with CVaR Portfolio Optimization in MATLAB (50 $\min 42 \mathrm{sec}$ )


## PortfolioCVaR Object

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"Working with PortfolioCVaR Objects" on page 5-22
"Setting and Getting Properties" on page 5-23
"Displaying PortfolioCVaR Objects" on page 5-24
"Saving and Loading PortfolioCVaR Objects" on page 5-24
"Estimating Efficient Portfolios and Frontiers" on page 5-24
"Arrays of PortfolioCVaR Objects" on page 5-24
"Subclassing PortfolioCVaR Objects" on page 5-25
"Conventions for Representation of Data" on page 5-25
```


## PortfolioCVaR Object Properties and Functions

The PortfolioCVaR object implements conditional value-at-risk (CVaR) portfolio optimization. Every property and function of the PortfolioCVaR object is public, although some properties and functions are hidden. SeePortfolioCVaR for the properties and functions of a PortfolioCVaR object. The PortfolioCVaR object is a value object where every instance of the object is a distinct version of the object. Since the PortfolioCVaR object is also a MATLAB object, it inherits the default functions associated with MATLAB objects.

## Working with PortfolioCVaR Objects

The PortfolioCVaR object and its functions are an interface for conditional value-at-risk portfolio optimization. So, almost everything you do with the PortfolioCVaR object can be done using the functions. The basic workflow is:

1 Design your portfolio problem.
2 Use PortfolioCVaR to create the PortfolioCVaR object or use the various set functions to set up your portfolio problem.
3 Use estimate functions to solve your portfolio problem.
In addition, functions are available to help you view intermediate results and to diagnose your computations. Since MATLAB features are part of a PortfolioCVaR object, you can
save and load objects from your workspace and create and manipulate arrays of objects. After settling on a problem, which, in the case of CVaR portfolio optimization, means that you have either scenarios, data, or moments for asset returns, a probability level, and a collection of constraints on your portfolios, use PortfolioCVaR to set the properties for the PortfolioCVaR object.

PortfolioCVaR lets you create an object from scratch or update an existing object. Since the PortfolioCVaR object is a value object, it is easy to create a basic object, then use functions to build upon the basic object to create new versions of the basic object. This is useful to compare a basic problem with alternatives derived from the basic problem. For details, see "Creating the PortfolioCVaR Object" on page 5-27.

## Setting and Getting Properties

You can set properties of a PortfolioCVaR object using either the PortfolioCVaR object or various set functions.

Note Although you can also set properties directly, it is not recommended since errorchecking is not performed when you set a property directly.

The PortfolioCVaR object supports setting properties with name-value pair arguments such that each argument name is a property and each value is the value to assign to that property. For example, to set the LowerBound, Budget, and ProbabilityLevel properties in an existing PortfolioCVaR object $p$, use the syntax:
p = PortfolioCVaR(p,'LowerBound', 0, 'Budget', 'ProbabilityLevel', 0.95);
In addition to the PortfolioCVaR object, which lets you set individual properties one at a time, groups of properties are set in a PortfolioCVaR object with various "set" and "add" functions. For example, to set up an average turnover constraint, use the setTurnover function to specify the bound on portfolio turnover and the initial portfolio. To get individual properties from a PortfolioCVaR object, obtain properties directly or use an assortment of "get" functions that obtain groups of properties from a PortfolioCVaR object. The PortfolioCVaR object and set functions have several useful features:

- The PortfolioCVaR object and set functions try to determine the dimensions of your problem with either explicit or implicit inputs.
- The PortfolioCVaR object and set functions try to resolve ambiguities with default choices.
- The PortfolioCVaR object and set functions perform scalar expansion on arrays when possible.
- The CVaR functions try to diagnose and warn about problems.


## Displaying PortfolioCVaR Objects

The PortfolioCVaR object uses the default display functions provided by MATLAB, where display and disp display a PortfolioCVaR object and its properties with or without the object variable name.

## Saving and Loading PortfolioCVaR Objects

Save and load PortfolioCVaR objects using the MATLAB save and load commands.

## Estimating Efficient Portfolios and Frontiers

Estimating efficient portfolios and efficient frontiers is the primary purpose of the CVaR portfolio optimization tools. A collection of "estimate" and "plot" functions provide ways to explore the efficient frontier. The "estimate" functions obtain either efficient portfolios or risk and return proxies to form efficient frontiers. At the portfolio level, a collection of functions estimates efficient portfolios on the efficient frontier with functions to obtain efficient portfolios:

- At the endpoints of the efficient frontier
- That attain targeted values for return proxies
- That attain targeted values for risk proxies
- Along the entire efficient frontier

These functions also provide purchases and sales needed to shift from an initial or current portfolio to each efficient portfolio. At the efficient frontier level, a collection of functions plot the efficient frontier and estimate either risk or return proxies for efficient portfolios on the efficient frontier. You can use the resultant efficient portfolios or risk and return proxies in subsequent analyses.

## Arrays of PortfolioCVaR Objects

Although all functions associated with a PortfolioCVaR object are designed to work on a scalar PortfolioCVaR object, the array capabilities of MATLAB enables you to set up and
work with arrays of PortfolioCVaR objects. The easiest way to do this is with the repmat function. For example, to create a 3-by-2 array of PortfolioCVaR objects:

```
p = repmat(PortfolioCVaR, 3, 2);
disp(p)
```

After setting up an array of PortfolioCVaR objects, you can work on individual PortfolioCVaR objects in the array by indexing. For example:

```
p(i,j) = PortfolioCVaR(p(i,j), ... );
```

This example calls PortfolioCVaR for the ( $i, j$ ) element of a matrix of PortfolioCVaR objects in the variable $p$.

If you set up an array of PortfolioCVaR objects, you can access properties of a particular PortfolioCVaR object in the array by indexing so that you can set the lower and upper bounds lb and ub for the ( $\mathrm{i}, \mathrm{j}, \mathrm{k}$ ) element of a 3-D array of PortfolioCVaR objects with

```
p(i,j,k) = setBounds(p(i,j,k), lb, ub);
```

and, once set, you can access these bounds with

```
[lb, ub] = getBounds(p(i,j,k));
```

PortfolioCVaR object functions work on only one PortfolioCVaR object at a time.

## Subclassing PortfolioCVaR Objects

You can subclass the PortfolioCVaR object to override existing functions or to add new properties or functions. To do so, create a derived class from the PortfolioCVaR class. This gives you all the properties and functions of thePortfolioCVaR class along with any new features that you choose to add to your subclassed object. ThePortfolioCVaR class is derived from an abstract class called AbstractPortfolio. Because of this, you can also create a derived class from AbstractPortfolio that implements an entirely different form of portfolio optimization using properties and functions of theAbstractPortfolio class.

## Conventions for Representation of Data

The CVaR portfolio optimization tools follow these conventions regarding the representation of different quantities associated with portfolio optimization:

- Asset returns or prices for scenarios are in matrix form with samples for a given asset going down the rows and assets going across the columns. In the case of prices, the earliest dates must be at the top of the matrix, with increasing dates going down.
- Portfolios are in vector or matrix form with weights for a given portfolio going down the rows and distinct portfolios going across the columns.
- Constraints on portfolios are formed in such a way that a portfolio is a column vector.
- Portfolio risks and returns are either scalars or column vectors (for multiple portfolio risks and returns).


## See Also

PortfolioCVaR

## Related Examples

- "Creating the PortfolioCVaR Object" on page 5-27
- "Working with CVaR Portfolio Constraints Using Defaults" on page 5-63


## More About

- "Portfolio Optimization Theory" on page 5-3
- "PortfolioCVaR Object Workflow" on page 5-20


## External Websites

- Analyzing Investment Strategies with CVaR Portfolio Optimization in MATLAB (50 $\min 42 \mathrm{sec}$ )


## Creating the PortfolioCVaR Object

## In this section...

"Syntax" on page 5-27
"PortfolioCVaR Problem Sufficiency" on page 5-28
"PortfolioCVaR Function Examples" on page 5-28
To create a fully specified CVaR portfolio optimization problem, instantiate the PortfolioCVaR object using PortfolioCVaR. For information on the workflow when using PortfolioCVaR objects, see "PortfolioCVaR Object Workflow" on page 5-20.

## Syntax

Use PortfolioCVaR to create an instance of an object of the PortfolioCVaR class. You can use PortfolioCVaR object in several ways. To set up a portfolio optimization problem in a PortfolioCVaR object, the simplest syntax is:

```
p = PortfolioCVaR;
```

This syntax creates a PortfolioCVaR object, p, such that all object properties are empty.
The PortfolioCVaR object also accepts collections of argument name-value pair arguments for properties and their values. The PortfolioCVaR object accepts inputs for public properties with the general syntax:

```
p = PortfolioCVaR('property1', value1, 'property2', value2, ... );
```

If a PortfolioCVaR object already exists, the syntax permits the first (and only the first argument) of PortfolioCVaR to be an existing object with subsequent argument namevalue pair arguments for properties to be added or modified. For example, given an existing PortfolioCVaR object in $p$, the general syntax is:

```
p = PortfolioCVaR(p, 'property1', value1, 'property2', value2, ... );
```

Input argument names are not case-sensitive, but must be completely specified. In addition, several properties can be specified with alternative argument names (see "Shortcuts for Property Names" on page 5-32). The PortfolioCVaR object tries to detect problem dimensions from the inputs and, once set, subsequent inputs can undergo various scalar or matrix expansion operations that simplify the overall process to
formulate a problem. In addition, a PortfolioCVaR object is a value object so that, given portfolio $p$, the following code creates two objects, $p$ and $q$, that are distinct:

```
q = PortfolioCVaR(p, ...)
```


## PortfolioCVaR Problem Sufficiency

A CVaR portfolio optimization problem is completely specified with the PortfolioCVaR object if the following three conditions are met:

- You must specify a collection of asset returns or prices known as scenarios such that all scenarios are finite asset returns or prices. These scenarios are meant to be samples from the underlying probability distribution of asset returns. This condition can be satisfied by the setScenarios function or with several canned scenario simulation functions.
- The set of feasible portfolios must be a nonempty compact set, where a compact set is closed and bounded. You can satisfy this condition using an extensive collection of properties that define different types of constraints to form a set of feasible portfolios. Since such sets must be bounded, either explicit or implicit constraints can be imposed and several tools, such as the estimateBounds function, provide ways to ensure that your problem is properly formulated.
- You must specify a probability level to locate the level of tail loss above which the conditional value-at-risk is to be minimized. This condition can be satisfied by the setProbabilityLevel function.

Although the general sufficient conditions for CVaR portfolio optimization go beyond the first three conditions, the PortfolioCVaR object handles all these additional conditions.

## PortfolioCVaR Function Examples

If you create a PortfolioCVaR object, p , with no input arguments, you can display it using disp:
p = PortfolioCVaR;
disp(p);
PortfolioCVaR with properties:
BuyCost: []

```
            SellCost: []
        RiskFreeRate: []
ProbabilityLevel: []
            Turnover: []
        BuyTurnover: []
SellTurnover: []
NumScenarios: []
            Name: []
        NumAssets: []
        AssetList: []
        InitPort: []
AInequality: []
bInequality: []
    AEquality: []
        bEquality: []
    LowerBound: []
    UpperBound: []
LowerBudget: []
UpperBudget: []
GroupMatrix: []
    LowerGroup: []
    UpperGroup: []
        GroupA: []
        GroupB: []
    LowerRatio: []
    UpperRatio: []
```

The approaches listed provide a way to set up a portfolio optimization problem with the PortfolioCVaR object. The custom set functions offer additional ways to set and modify collections of properties in the PortfolioCVaR object.

## Using the PortfolioCVaR Function for a Single-Step Setup

You can use the PortfolioCVaR object to directly set up a "standard" portfolio optimization problem. Given scenarios of asset returns in the variable AssetScenarios, this problem is completely specified as follows:

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;
AssetScenarios = mvnrnd(m, C, 20000);
p = PortfolioCVaR('Scenarios', AssetScenarios, ...
```

```
'LowerBound', 0, 'LowerBudget', 1, 'UpperBudget', 1, ...
```

'ProbabilityLevel', 0.95);
The LowerBound property value undergoes scalar expansion since AssetScenarios provides the dimensions of the problem.

You can use dot notation with the function plotFrontier.
p.plotFrontier;


## Using the PortfolioCVaR Function with a Sequence of Steps

An alternative way to accomplish the same task of setting up a "standard" CVaR portfolio optimization problem, given AssetScenarios variable is:

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
\(C=[0.00640 .004080 .001920\);
0.004080 .02890 .0204 0.0119;
0.001920 .02040 .05760 .0336 ;
00.01190 .03360 .1225 ];
m = m/12;
```

$$
C=C / 12 ;
$$

AssetScenarios $=$ mvnrnd(m, C, 20000);
p = PortfolioCVaR;
$p$ = setScenarios(p, AssetScenarios);
$p=$ PortfolioCVaR(p, 'LowerBound', 0);
p = PortfolioCVaR(p, 'LowerBudget', 1, 'UpperBudget', 1);
p = setProbabilityLevel(p, 0.95);
plotFrontier(p);


This way works because the calls to the are in this particular order. In this case, the call to initialize AssetScenarios provides the dimensions for the problem. If you were to do this step last, you would have to explicitly dimension the LowerBound property as follows:

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
0.00408 0.0289 0.0204 0.0119;
```

```
0.00192 0.0204 0.0576 0.0336;
0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;
AssetScenarios = mvnrnd(m, C, 20000);
p = PortfolioCVaR;
p = PortfolioCVaR(p, 'LowerBound', zeros(size(m)));
p = PortfolioCVaR(p, 'LowerBudget', 1, 'UpperBudget', 1);
p = setProbabilityLevel(p, 0.95);
p = setScenarios(p, AssetScenarios);
```

Note If you did not specify the size of Lowe rBound but, instead, input a scalar argument, the PortfolioCVaR object assumes that you are defining a single-asset problem and produces an error at the call to set asset scenarios with four assets.

## Shortcuts for Property Names

The PortfolioCVaR object has shorter argument names that replace longer argument names associated with specific properties of the PortfolioCVaR object. For example, rather than enter 'ProbabilityLevel', the PortfolioCVaR object accepts the caseinsensitive name 'plevel' to set the ProbabilityLevel property in a PortfolioCVaR object. Every shorter argument name corresponds with a single property in the PortfolioCVaR object. The one exception is the alternative argument name 'budget', which signifies both the LowerBudget and UpperBudget properties. When 'budget ' is used, then the LowerBudget and UpperBudget properties are set to the same value to form an equality budget constraint.

## Shortcuts for Property Names

| Shortcut Argument Name | Equivalent Argument / Property Name |
| :--- | :--- |
| ae | AEquality |
| ai | AInequality |
| assetnames or assets | AssetList |
| be | bEquality |
| bi | bInequality |
| budget | UpperBudget and LowerBudget |
| group | GroupMatrix |
| lb | LowerBound |
| n or num | NumAssets |
| level, problevel, or plevel | ProbabilityLevel |
| rfr | RiskFreeRate |
| scenario or <br> assetscenarios | Scenarios |
| ub | UpperBound |

For example, this call to the PortfolioCVaR object uses these shortcuts for properties:

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C=[ 0.0064 0.00408 0.00192 0;
0.00408 0.0289 0.0204 0.0119;
0.00192 0.0204 0.0576 0.0336;
0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;
AssetScenarios = mvnrnd(m, C, 20000);
p = PortfolioCVaR('scenario', AssetScenarios, 'lb', 0, 'budget', 1, 'plevel', 0.95);
plotFrontier(p);
```


## Direct Setting of Portfolio Object Properties

Although not recommended, you can set properties directly using dot notation, however no error-checking is done on your inputs:

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
```

```
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;
AssetScenarios = mvnrnd(m, C, 20000);
p = PortfolioCVaR;
p = setScenarios(p, AssetScenarios);
p.ProbabilityLevel = 0.95;
p.LowerBudget = 1;
p.UpperBudget = 1;
p.LowerBound = zeros(size(m));
plotFrontier(p);
```

Note Scenarios cannot be assigned directly using dot notation to a PortfolioCVaR object. Scenarios must always be set through either the PortfolioCVaR object, the setScenarios function, or any of the scenario simulation functions.

## See Also

PortfolioCVaR|estimateBounds

## Related Examples

- "Common Operations on the PortfolioCVaR Object" on page 5-36
- "Working with CVaR Portfolio Constraints Using Defaults" on page 5-63


## More About

- "PortfolioCVaR Object" on page 5-22
- "Portfolio Optimization Theory" on page 5-3
- "PortfolioCVaR Object Workflow" on page 5-20


## External Websites

- Analyzing Investment Strategies with CVaR Portfolio Optimization in MATLAB (50 $\min 42 \mathrm{sec}$ )


# Common Operations on the PortfolioCVaR Object 

```
In this section...
"Naming a PortfolioCVaR Object" on page 5-36
"Configuring the Assets in the Asset Universe" on page 5-36
"Setting Up a List of Asset Identifiers" on page 5-37
"Truncating and Padding Asset Lists" on page 5-38
```


## Naming a PortfolioCVaR Object

To name a PortfolioCVaR object, use the Name property. Name is informational and has no effect on any portfolio calculations. If the Name property is nonempty, Name is the title for the efficient frontier plot generated by plotFrontier. For example, if you set up an asset allocation fund, you could name the PortfolioCVaR object Asset Allocation Fund:

```
p = PortfolioCVaR('Name','Asset Allocation Fund');
disp(p.Name);
Asset Allocation Fund
```


## Configuring the Assets in the Asset Universe

The fundamental quantity in the PortfolioCVaR object is the number of assets in the asset universe. This quantity is maintained in the NumAssets property. Although you can set this property directly, it is usually derived from other properties such as the number of assets in the scenarios or the initial portfolio. In some instances, the number of assets may need to be set directly. This example shows how to set up a PortfolioCVaR object that has four assets:

```
p = PortfolioCVaR('NumAssets', 4);
disp(p.NumAssets);
```

4
After setting the NumAssets property, you cannot modify it (unless no other properties are set that depend on NumAssets). The only way to change the number of assets in an existing PortfolioCVaR object with a known number of assets is to create a new PortfolioCVaR object.

## Setting Up a List of Asset Identifiers

When working with portfolios, you must specify a universe of assets. Although you can perform a complete analysis without naming the assets in your universe, it is helpful to have an identifier associated with each asset as you create and work with portfolios. You can create a list of asset identifiers as a cell vector of character vectors in the property AssetList. You can set up the list using the next two methods.

## Setting Up Asset Lists Using the PortfolioCVaR Function

Suppose that you have a PortfolioCVaR object, p , with assets with symbols 'AA' ', 'BA', 'CAT', 'DD', and 'ETR'. You can create a list of these asset symbols in the object using the PortfolioCVaR object:

```
p = PortfolioCVaR('assetlist', { 'AA', 'BA', 'CAT', 'DD', 'ETR' });
disp(p.AssetList);
'AA' 'BA' 'CAT' 'DD' 'ETR'
```

Notice that the property AssetList is maintained as a cell array that contains character vectors, and that it is necessary to pass a cell array into the PortfolioCVaR object to set AssetList. In addition, notice that the property NumAssets is set to 5 based on the number of symbols used to create the asset list:

```
disp(p.NumAssets);
```

5

## Setting Up Asset Lists Using the setAssetList Function

You can also specify a list of assets using the setAssetList function. Given the list of asset symbols 'AA', 'BA', 'CAT', 'DD', and 'ETR', you can use setAssetList with:

```
p = PortfolioCVaR;
p = setAssetList(p, { 'AA', 'BA', 'CAT', 'DD', 'ETR' });
disp(p.AssetList);
    'AA' 'BA' 'CAT' 'DD' 'ETR'
```

setAssetList also enables you to enter symbols directly as a comma-separated list without creating a cell array of character vectors. For example, given the list of assets symbols 'AA', 'BA', 'CAT','DD', and 'ETR', use setAssetList:

```
p = PortfolioCVaR;
p = setAssetList(p, 'AA', 'BA', 'CAT', 'DD', 'ETR');
disp(p.AssetList);
'AA' 'BA' 'CAT' 'DD' 'ETR'
```

setAssetList has many additional features to create lists of asset identifiers. If you use setAssetList with just a PortfolioCVaR object, it creates a default asset list according to the name specified in the hidden public property defaultforAssetList (which is 'Asset' by default). The number of asset names created depends on the number of assets in the property NumAssets. If NumAssets is not set, then NumAssets is assumed to be 1 .

For example, if a PortfolioCVaR object $p$ is created with NumAssets $=5$, then this code fragment shows the default naming behavior:

```
p = PortfolioCVaR('numassets',5);
p = setAssetList(p);
disp(p.AssetList);
'Asset1' 'Asset2' 'Asset3' 'Asset4' 'Asset5'
```

Suppose that your assets are, for example, ETFs and you change the hidden property defaultforAssetList to 'ETF', you can then create a default list for ETFs:

```
p = PortfolioCVaR('numassets',5);
p.defaultforAssetList = 'ETF';
p = setAssetList(p);
disp(p.AssetList);
'ETF1' 'ETF2' 'ETF3' 'ETF4' 'ETF5'
```


## Truncating and Padding Asset Lists

If the NumAssets property is already set and you pass in too many or too few identifiers, the PortfolioCVaR object, and the setAssetList function truncate or pad the list with numbered default asset names that use the name specified in the hidden public property defaultforAssetList. If the list is truncated or padded, a warning message indicates the discrepancy. For example, assume that you have a PortfolioCVaR object with five ETFs and you only know the first three CUSIPs '921937835', '922908769' , and ' 922042775 '. Use this syntax to create an asset list that pads the remaining asset identifiers with numbered 'UnknownCUSIP' placeholders:

```
p = PortfolioCVaR('numassets',5);
p.defaultforAssetList = 'UnknownCUSIP';
```

```
p = setAssetList(p, '921937835', '922908769', '922042775');
disp(p.AssetList);
Warning: Input list of assets has 2 too few identifiers. Padding with numbered assets.
> In PortfolioCVaR.setAssetList at 118
    921937835' '922908769' '922042775' 'UnknownCUSIP4' 'UnknownCUSIP5'
```

Alternatively, suppose that you have too many identifiers and need only the first four assets. This example illustrates truncation of the asset list using the PortfolioCVaR object:

```
p = PortfolioCVaR('numassets',4);
p = PortfolioCVaR(p, 'assetlist', { 'AGG', 'EEM', 'MDY', 'SPY', 'VEU' });
disp(p.AssetList);
Warning: AssetList has 1 too many identifiers. Using first 4 assets.
> In PortfolioCVaR.checkarguments at 399
    In PortfolioCVaR.PortfolioCVaR>PortfolioCVaR.PortfolioCVaR at 195
        'AGG' 'EEM' 'MDY' 'SPY'
```

The hidden public property uppercaseAssetList is a Boolean flag to specify whether to convert asset names to uppercase letters. The default value for uppercaseAssetList is false. This example shows how to use the uppercaseAssetList flag to force identifiers to be uppercase letters:

```
p = PortfolioCVaR;
p.uppercaseAssetList = true;
p = setAssetList(p, { 'aa', 'ba', 'cat', 'dd', 'etr' });
disp(p.AssetList);
'AA' 'BA' 'CAT' 'DD' 'ETR'
```


## See Also

PortfolioCVaR|checkFeasibility|estimateBounds|setAssetList| setInitPort

## Related Examples

- "Setting Up an Initial or Current Portfolio" on page 5-41
- "Working with CVaR Portfolio Constraints Using Defaults" on page 5-63
- "Asset Returns and Scenarios Using PortfolioCVaR Object" on page 5-44


## More About

- "PortfolioCVaR Object" on page 5-22
- "Portfolio Optimization Theory" on page 5-3
- "PortfolioCVaR Object Workflow" on page 5-20


## External Websites

- Analyzing Investment Strategies with CVaR Portfolio Optimization in MATLAB (50 $\min 42 \mathrm{sec}$ )


## Setting Up an Initial or Current Portfolio

In many applications, creating a new optimal portfolio requires comparing the new portfolio with an initial or current portfolio to form lists of purchases and sales. The PortfolioCVaR object property InitPort lets you identify an initial or current portfolio. The initial portfolio also plays an essential role if you have either transaction costs or turnover constraints. The initial portfolio need not be feasible within the constraints of the problem. This can happen if the weights in a portfolio have shifted such that some constraints become violated. To check if your initial portfolio is feasible, use the checkFeasibility function described in "Validating CVaR Portfolios" on page 5-98. Suppose that you have an initial portfolio in x0, then use the PortfolioCVaR object to set up an initial portfolio:

```
x0 = [ 0.3; 0.2; 0.2; 0.0 ];
p = PortfolioCVaR('InitPort', x0);
disp(p.InitPort);
```

0.3000
0.2000
0.2000

0

As with all array properties, you can set InitPort with scalar expansion. This is helpful to set up an equally weighted initial portfolio of, for example, 10 assets:

```
p = PortfolioCVaR('NumAssets', 10, 'InitPort', 1/10);
disp(p.InitPort);
```

0.1000
0.1000
0.1000
0.1000
0.1000
0.1000
0.1000
0.1000
0.1000
0.1000

To clear an initial portfolio from your PortfolioCVaR object, use either the PortfolioCVaR object or the setInitPort function with an empty input for the InitPort property. If transaction costs or turnover constraints are set, it is not possible to clear the InitPort property in this way. In this case, to clear InitPort, first clear the dependent properties and then clear theInitPort property.

The InitPort property can also be set with setInitPort which lets you specify the number of assets if you want to use scalar expansion. For example, given an initial portfolio in $\times 0$, use setInitPort to set the InitPort property:

```
p = PortfolioCVaR;
x0 = [ 0.3; 0.2; 0.2; 0.0 ];
p = setInitPort(p, x0);
disp(p.InitPort);
0.3000
0.2000
0.2000
0
```

To create an equally weighted portfolio of four assets, use setInitPort:

```
p = PortfolioCVaR;
p = setInitPort(p, 1/4, 4);
disp(p.InitPort);
    0.2500
    0.2500
    0.2500
    0.2500
```

PortfolioCVaR object functions that work with either transaction costs or turnover constraints also depend on the InitPort property. So, the set functions for transaction costs or turnover constraints permit the assignment of a value for the InitPort property as part of their implementation. For details, see "Working with Average Turnover Constraints Using PortfolioCVaR Object" on page 5-88, "Working with One-Way Turnover Constraints Using PortfolioCVaR Object" on page 5-92, and "Working with Transaction Costs" on page 5-58. If either transaction costs or turnover constraints are used, then the InitPort property must have a nonempty value. Absent a specific value assigned through the PortfolioCVaR object or various set functions, the PortfolioCVaR object sets InitPort to 0 and warns if BuyCost, SellCost, or Turnover properties are set. This example shows what happens if you specify an average turnover constraint with an initial portfolio:

```
p = PortfolioCVaR('Turnover', 0.3, 'InitPort', [ 0.3; 0.2; 0.2; 0.0 ]);
disp(p.InitPort);
```

0.3000
0.2000
0.2000

In contrast, this example shows what happens if an average turnover constraint is specified without an initial portfolio:

```
p = PortfolioCVaR('Turnover', 0.3);
disp(p.InitPort);
```


## Warning: InitPort and NumAssets are empty and either transaction costs or turnover constraints specified.

Will set NumAssets $=1$ and InitPort $=0$.
> In PortfolioCVaR.checkarguments at 322
In PortfolioCVaR.PortfolioCVaR>PortfolioCVaR.PortfolioCVaR at 195
0

## See Also

PortfolioCVaR|checkFeasibility|estimateBounds|setAssetList| setInitPort

## Related Examples

- "Creating the PortfolioCVaR Object" on page 5-27
- "Common Operations on the PortfolioCVaR Object" on page 5-36
- "Working with CVaR Portfolio Constraints Using Defaults" on page 5-63
- "Asset Returns and Scenarios Using PortfolioCVaR Object" on page 5-44


## More About

- "PortfolioCVaR Object" on page 5-22
- "Portfolio Optimization Theory" on page 5-3
- "PortfolioCVaR Object Workflow" on page 5-20


## External Websites

- Analyzing Investment Strategies with CVaR Portfolio Optimization in MATLAB (50 $\min 42 \mathrm{sec}$ )


# Asset Returns and Scenarios Using PortfolioCVaR Object 

In this section...<br>"How Stochastic Optimization Works" on page 5-44<br>"What Are Scenarios?" on page 5-45<br>"Setting Scenarios Using the PortfolioCVaR Function" on page 5-45<br>"Setting Scenarios Using the setScenarios Function" on page 5-46<br>"Estimating the Mean and Covariance of Scenarios" on page 5-47<br>"Simulating Normal Scenarios" on page 5-48<br>"Simulating Normal Scenarios from Returns or Prices" on page 5-48<br>"Simulating Normal Scenarios with Missing Data" on page 5-50<br>"Simulating Normal Scenarios from Time Series Data" on page 5-51<br>"Simulating Normal Scenarios with Mean and Covariance" on page 5-53

## How Stochastic Optimization Works

The CVaR of a portfolio is a conditional expectation. (For the definition of the CVaR function, see "Risk Proxy" on page 5-6.) Therefore, the CVaR portfolio optimization problem is a stochastic optimization problem. Given a sample of scenarios, the conditional expectation that defines the sample CVaR of the portfolio can be expressed as a finite sum, a weighted average of losses. The weights of the losses depend on their relative magnitude; for a confidence level $\alpha$, only the worst $(1-\alpha)$ x $100 \%$ losses get a positive weight. As a function of the portfolio weights, the CVaR of the portfolio is a convex function (see [48], [49] Rockafellar \& Uryasev at "Portfolio Optimization" on page A-7). It is also a nonsmooth function, but its edges are less sharp as the sample size increases.

There are reformulations of the CVaR portfolio optimization problem (see [48], [49] at Rockafellar \& Uryasev) that result in a linear programming problem, which can be solved either with standard linear programming techniques or with stochastic programming solvers. The PortfolioCVaR object, however, does not reformulate the problem in such a manner. The PortfolioCVaR object computes the CVaR as a nonlinear function. The convexity of the CVaR, as a function of the portfolio weights and the dull edges when the number of scenarios is large, make the CVaR portfolio optimization problem tractable, in practice, for certain nonlinear programming solvers, such as fmincon from Optimization Toolbox. The problem can also be solved using a cutting-plane method (see Kelley [45] at "Portfolio Optimization" on page A-7). For more information, see Algorithms section of
setSolver. To learn more about the workflow when using PortfolioCVaR objects, see "PortfolioCVaR Object Workflow" on page 5-20.

## What Are Scenarios?

Since conditional value-at-risk portfolio optimization works with scenarios of asset returns to perform the optimization, several ways exist to specify and simulate scenarios. In many applications with CVaR portfolio optimization, asset returns may have distinctly nonnormal probability distributions with either multiple modes, binning of returns, truncation of distributions, and so forth. In other applications, asset returns are modeled as the result of various simulation methods that might include Monte-Carlo simulation, quasi-random simulation, and so forth. Often, the underlying probability distribution for risk factors may be multivariate normal but the resultant transformations are sufficiently nonlinear to result in distinctively nonnormal asset returns.

For example, this occurs with bonds and derivatives. In the case of bonds with a nonzero probability of default, such scenarios would likely include asset returns that are $-100 \%$ to indicate default and some values slightly greater than $-100 \%$ to indicate recovery rates.

Although the PortfolioCVaR object has functions to simulate multivariate normal scenarios from either data or moments (simulateNormalScenariosByData and simulateNormalScenariosByMoments), the usual approach is to specify scenarios directly from your own simulation functions. These scenarios are entered directly as a matrix with a sample for all assets across each row of the matrix and with samples for an asset down each column of the matrix. The architecture of the CVaR portfolio optimization tools references the scenarios through a function handle so scenarios that have been set cannot be accessed directly as a property of the PortfolioCVaR object.

## Setting Scenarios Using the PortfolioCVaR Function

Suppose that you have a matrix of scenarios in the AssetScenarios variable. The scenarios are set through the PortfolioCVaR object with:

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
0.00408 0.0289 0.0204 0.0119;
0.00192 0.0204 0.0576 0.0336;
0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;
```

```
AssetScenarios = mvnrnd(m, C, 20000);
p = PortfolioCVaR('Scenarios', AssetScenarios);
disp(p.NumAssets);
disp(p.NumScenarios);
4
```

20000

Notice that the PortfolioCVaR object determines and fixes the number of assets in NumAssets and the number of scenarios in NumScenarios based on the scenario's matrix. You can change the number of scenarios by calling the PortfolioCVaR object with a different scenario matrix. However, once the NumAssets property has been set in the object, you cannot enter a scenario matrix with a different number of assets. The getScenarios function lets you recover scenarios from a PortfolioCVaR object. You can also obtain the mean and covariance of your scenarios using estimateScenarioMoments.

Although not recommended for the casual user, an alternative way exists to recover scenarios by working with the function handle that points to scenarios in the PortfolioCVaR object. To access some or all the scenarios from a PortfolioCVaR object, the hidden property localScenarioHandle is a function handle that points to a function to obtain scenarios that have already been set. To get scenarios directly from a PortfolioCVaR object p, use

```
scenarios = p.localScenarioHandle([], []);
```

and to obtain a subset of scenarios from rows startrow to endrow, use
scenarios = p.localScenarioHandle(startrow, endrow);
where $1 \leq$ startrow $\leq$ endrow $\leq$ numScenarios.

## Setting Scenarios Using the setScenarios Function

You can also set scenarios using setScenarios. For example, given the mean and covariance of asset returns in the variables $m$ and $C$, the asset moment properties can be set:
m = [ 0.05; 0.1; 0.12; 0.18 ];
$C=[0.00640 .004080 .001920$;

```
0.00408 0.0289 0.0204 0.0119;
0.00192 0.0204 0.0576 0.0336;
0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;
AssetScenarios = mvnrnd(m, C, 20000);
p = PortfolioCVaR;
p = setScenarios(p, AssetScenarios);
disp(p.NumAssets);
disp(p.NumScenarios);
```

4
20000

## Estimating the Mean and Covariance of Scenarios

The estimateScenarioMoments function obtains estimates for the mean and covariance of scenarios in a PortfolioCVaR object.

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
0.00408 0.0289 0.0204 0.0119;
0.00192 0.0204 0.0576 0.0336;
0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;
```

AssetScenarios = mvnrnd(m, C, 20000);
p = PortfolioCVaR;
p = setScenarios(p, AssetScenarios);
[mean, covar] = estimateScenarioMoments(p)
mean =
0.0043
0.0085
0.0098

### 0.0153

covar =

| 0.0005 | 0.0003 | 0.0002 | 0.0000 |
| :--- | :--- | :--- | :--- |
| 0.0003 | 0.0024 | 0.0017 | 0.0010 |
| 0.0002 | 0.0017 | 0.0049 | 0.0029 |
| 0.0000 | 0.0010 | 0.0029 | 0.0102 |

## Simulating Normal Scenarios

As a convenience, the two functions (simulateNormalScenariosByData and simulateNormalScenariosByMoments) exist to simulate scenarios from data or moments under an assumption that they are distributed as multivariate normal random asset returns.

## Simulating Normal Scenarios from Returns or Prices

Given either return or price data, use the function simulateNormalScenariosByData to simulate multivariate normal scenarios. Either returns or prices are stored as matrices with samples going down the rows and assets going across the columns. In addition, returns or prices can be stored in a table or timetable (see "Simulating Normal Scenarios from Time Series Data" on page 5-51). To illustrate using simulateNormalScenariosByData, generate random samples of 120 observations of asset returns for four assets from the mean and covariance of asset returns in the variables m and C with portsim. The default behavior of portsim creates simulated data with estimated mean and covariance identical to the input moments $m$ and $C$. In addition to a return series created by portsim in the variable $X$, a price series is created in the variable $Y$ :

```
m = [ 0.0042; 0.0083; 0.01; 0.15 ];
C = [ 0.005333 0.00034 0.00016 0;
0.00034 0.002408 0.0017 0.000992;
0.00016 0.0017 0.0048 0.0028;
0 0.000992 0.0028 0.010208 ];
X = portsim(m', C, 120);
Y = ret2tick(X);
```

Note Portfolio optimization requires that you use total returns and not just price returns. So, "returns" should be total returns and "prices" should be total return prices.

Given asset returns and prices in variables $X$ and $Y$ from above, this sequence of examples demonstrates equivalent ways to simulate multivariate normal scenarios for the PortfolioCVaR object. Assume a PortfolioCVaR object created in p that uses the asset returns in $X$ uses simulateNormalScenariosByData:

```
p = PortfolioCVaR;
```

p = simulateNormalScenariosByData(p, X, 20000);
[passetmean, passetcovar] = estimateScenarioMoments(p)
passetmean =
0.0043
0.0083
0.0102
0.1507
passetcovar =

| 0.0053 | 0.0003 | 0.0002 | 0.0000 |
| :--- | :--- | :--- | :--- |
| 0.0003 | 0.0024 | 0.0017 | 0.0010 |
| 0.0002 | 0.0017 | 0.0049 | 0.0028 |
| 0.0000 | 0.0010 | 0.0028 | 0.0101 |

The moments that you obtain from this simulation will likely differ from the moments listed here because the scenarios are random samples from the estimated multivariate normal probability distribution of the input returns $X$.

The default behavior of simulateNormalScenariosByData is to work with asset returns. If, instead, you have asset prices as in the variable $Y$, simulateNormalScenariosByData accepts a name-value pair argument name 'DataFormat' with a corresponding value set to 'prices' to indicate that the input to the function is in the form of asset prices and not returns (the default value for the 'DataFormat' argument is 'returns'). This example simulates scenarios with the asset price data in $Y$ for the PortfolioCVaR object q:

```
p = PortfolioCVaR;
p = simulateNormalScenariosByData(p, Y, 20000, 'dataformat', 'prices');
[passetmean, passetcovar] = estimateScenarioMoments(p)
```


## passetmean =

0.0043
0.0084
0.0094
0.1490

## passetcovar =

| 0.0054 | 0.0004 | 0.0001 | -0.0000 |
| ---: | ---: | ---: | ---: |
| 0.0004 | 0.0024 | 0.0016 | 0.0009 |
| 0.0001 | 0.0016 | 0.0048 | 0.0028 |
| -0.0000 | 0.0009 | 0.0028 | 0.0100 |

## Simulating Normal Scenarios with Missing Data

Often when working with multiple assets, you have missing data indicated by NaN values in your return or price data. Although "Multivariate Normal Regression" on page 9-2 goes into detail about regression with missing data, the simulateNormalScenariosByData function has a name-value pair argument name 'MissingData' that indicates with a Boolean value whether to use the missing data capabilities of Financial Toolbox. The default value for 'MissingData' is false which removes all samples with NaN values. If, however, 'MissingData' is set to true, simulateNormalScenariosByData uses the ECM algorithm to estimate asset moments. This example shows how this works on price data with missing values:

```
m = [ 0.0042; 0.0083; 0.01; 0.15 ];
C = [ 0.005333 0.00034 0.00016 0;
0.00034 0.002408 0.0017 0.000992;
0.00016 0.0017 0.0048 0.0028;
0 0.000992 0.0028 0.010208 ];
X = portsim(m', C, 120);
Y = ret2tick(X);
Y(1:20,1) = NaN;
Y(1:12,4) = NaN;
```

Notice that the prices above in Y have missing values in the first and fourth series.

```
p = PortfolioCVaR;
p = simulateNormalScenariosByData(p, Y, 20000, 'dataformat', 'prices');
q = PortfolioCVaR;
q = simulateNormalScenariosByData(q, Y, 20000, 'dataformat', 'prices', 'missingdata', true);
```

```
[passetmean, passetcovar] = estimateScenarioMoments(p)
[qassetmean, qassetcovar] = estimateScenarioMoments(q)
passetmean =
    0.0020
    0.0074
    0.0078
    0.1476
passetcovar =
\begin{tabular}{rrrr}
0.0055 & 0.0003 & -0.0001 & -0.0003 \\
0.0003 & 0.0024 & 0.0019 & 0.0012 \\
-0.0001 & 0.0019 & 0.0050 & 0.0028 \\
-0.0003 & 0.0012 & 0.0028 & 0.0101
\end{tabular}
qassetmean =
    0.0024
    0.0085
    0.0106
    0.1482
qassetcovar =
\begin{tabular}{rrrr}
0.0071 & 0.0004 & -0.0001 & -0.0004 \\
0.0004 & 0.0032 & 0.0022 & 0.0012 \\
-0.0001 & 0.0022 & 0.0063 & 0.0034 \\
-0.0004 & 0.0012 & 0.0034 & 0.0127
\end{tabular}
```

The first PortfolioCVaR object, $p$, contains scenarios obtained from price data in $Y$ where NaN values are discarded and the second PortfolioCVaR object, q, contains scenarios obtained from price data in $Y$ that accommodate missing values. Each time you run this example, you get different estimates for the moments in p and q .

## Simulating Normal Scenarios from Time Series Data

The simulateNormalScenariosByData function implicitly works with matrices of data or data in a table or timetable object using the same rules for whether the data are returns or prices. To illustrate, use array2timetable to create a timetable for 14 assets from CAPMuniverse and the use the timetable to simulate scenarios for PortfolioCVaR.

[^0]```
ans =
struct with fields:
    Description: '
            UserData: []
    DimensionNames: {'Time' 'Variables'}
        VariableNames: {'AAPL' 'AMZN' 'CSCO' 'DELL' 'EBAY' 'GOOG' 'HPQ' 'IBM
    VariableDescriptions: {}
        VariableUnits: {}
VariableContinuity: []
    RowTimes: [1471\times1 datetime]
```

Use the 'MissingData' option offered by PortfolioCVaR to account for the missing data.

```
p = PortfolioCVaR;
passetmean =
    0.0012
    0.0007
    -0.0005
    -0.0000
        0.0016
        0.0043
        -0.0001
        0.0000
        0.0001
        -0.0002
        0.0000
        0.0004
        0.0001
        0.0001
```

p = simulateNormalScenariosByData(p, stockTT, 20000 ,'missingdata',true);
[passetmean, passetcovar] = estimateScenarioMoments(p)
passetcovar =

| 0.0013 | 0.0005 | 0.0006 | 0.0005 | 0.0006 | 0.0003 | 0.0005 | 0.0003 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.0005 | 0.0024 | 0.0007 | 0.0005 | 0.0010 | 0.0005 | 0.0005 | 0.0003 |
| 0.0006 | 0.0007 | 0.0013 | 0.0006 | 0.0007 | 0.0004 | 0.0006 | 0.0004 |
| 0.0005 | 0.0005 | 0.0006 | 0.0009 | 0.0006 | 0.0002 | 0.0005 | 0.0003 |
| 0.0006 | 0.0010 | 0.0007 | 0.0006 | 0.0018 | 0.0007 | 0.0005 | 0.0003 |
| 0.0003 | 0.0005 | 0.0004 | 0.0002 | 0.0007 | 0.0013 | 0.0002 | 0.0002 |
| 0.0005 | 0.0005 | 0.0006 | 0.0005 | 0.0005 | 0.0002 | 0.0010 | 0.0003 |


| 0.0003 | 0.0003 | 0.0004 | 0.0003 | 0.0003 | 0.0002 | 0.0003 | 0.0005 | $0 .(1)$ |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 0.0006 | 0.0006 | 0.0008 | 0.0006 | 0.0006 | 0.0002 | 0.0005 | 0.0004 | $0 .(1)$ |
| 0.0004 | 0.0004 | 0.0005 | 0.0004 | 0.0005 | 0.0002 | 0.0003 | 0.0002 | $0 .(1)$ |
| 0.0005 | 0.0006 | 0.0008 | 0.0005 | 0.0007 | 0.0003 | 0.0005 | 0.0004 | 0.0 |
| 0.0006 | 0.0011 | 0.0008 | 0.0006 | 0.0011 | 0.0011 | 0.0006 | 0.0004 | 0.0 |
| 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0001 | 0.0002 | 0.0002 | $0 .(1)$ |
| -0.0000 | -0.0000 | -0.0000 | -0.0000 | -0.0000 | -0.0000 | -0.0000 | 0.0000 | -0.0 |

Use the name-value input 'DataFormat ' to handle return or price data and 'MissingData' to ignore or use samples with missing values. In addition, simulateNormalScenariosByData extracts asset names or identifiers from a table or timetable if the argument 'GetAssetList' is set to true (the default value is false). If the 'GetAssetList' value is true, the identifiers are used to set the AssetList property of the PortfolioCVaR object. Thus, repeating the formation of the PortfolioCVaR object $p$ from the previous example with the 'GetAssetList' flag set to true extracts the column names from the timetable object:

```
p = simulateNormalScenariosByData(p, stockTT, 20000 ,'missingdata',true, 'GetAssetList', true);
disp(p.AssetList)
    'AAPL' 'AMZN' 'CSCO' 'DELL' 'EBAY' 'GOOG' 'HPQ' 'IBM' 'INTC'
```

If you set the 'GetAssetList' flag set to true and your input data is in a matrix, simulateNormalScenariosByData uses the default labeling scheme from setAssetList as described in "Setting Up a List of Asset Identifiers" on page 5-37.

## Simulating Normal Scenarios with Mean and Covariance

Given the mean and covariance of asset returns, use the simulateNormalScenariosByMoments function to simulate multivariate normal scenarios. The mean can be either a row or column vector and the covariance matrix must be a symmetric positive-semidefinite matrix. Various rules for scalar expansion apply. To illustrate using simulateNormalScenariosByMoments, start with moments in $m$ and $C$ and generate 20,000 scenarios:

```
m = [ 0.0042; 0.0083; 0.01; 0.15 ];
C = [ 0.005333 0.00034 0.00016 0;
0.00034 0.002408 0.0017 0.000992;
0.00016 0.0017 0.0048 0.0028;
0 0.000992 0.0028 0.010208 ];
p = PortfolioCVaR;
p = simulateNormalScenariosByMoments(p, m, C, 20000);
[passetmean, passetcovar] = estimateScenarioMoments(p)
```


## passetmean =

0.0049
0.0083
0.0101
0.1503
passetcovar =

| 0.0053 | 0.0003 | 0.0002 | -0.0000 |
| ---: | ---: | ---: | ---: |
| 0.0003 | 0.0024 | 0.0017 | 0.0010 |
| 0.0002 | 0.0017 | 0.0047 | 0.0028 |
| -0.0000 | 0.0010 | 0.0028 | 0.0101 |

simulateNormalScenariosByMoments performs scalar expansion on arguments for the moments of asset returns. If NumAssets has not already been set, a scalar argument is interpreted as a scalar with NumAssets set to 1.
simulateNormalScenariosByMoments provides an additional optional argument to specify the number of assets so that scalar expansion works with the correct number of assets. In addition, if either a scalar or vector is input for the covariance of asset returns, a diagonal matrix is formed such that a scalar expands along the diagonal and a vector becomes the diagonal.

## See Also

PortfolioCVaR|estimatePortVaR|setCosts | setProbabilityLevel| setScenarios|simulateNormalScenariosByData|
simulateNormalScenariosByMoments

## Related Examples

- "Working with a Riskless Asset" on page 5-56
- "Working with Transaction Costs" on page 5-58
- "Creating the PortfolioCVaR Object" on page 5-27
- "Working with CVaR Portfolio Constraints Using Defaults" on page 5-63
- "Validate the CVaR Portfolio Problem" on page 5-96
- "Estimate Efficient Portfolios for Entire Frontier for PortfolioCVaR Object" on page 5-101
- "Estimate Efficient Frontiers for PortfolioCVaR Object" on page 5-118
- "Asset Returns and Scenarios Using PortfolioCVaR Object" on page 5-44


## More About

- "PortfolioCVaR Object" on page 5-22
- "Portfolio Optimization Theory" on page 5-3
- "PortfolioCVaR Object Workflow" on page 5-20


## External Websites

- Analyzing Investment Strategies with CVaR Portfolio Optimization in MATLAB (50 $\min 42 \mathrm{sec}$ )


## Working with a Riskless Asset

The PortfolioCVaR object has a separate RiskFreeRate property that stores the rate of return of a riskless asset. Thus, you can separate your universe into a riskless asset and a collection of risky assets. For example, assume that your riskless asset has a return in the scalar variable r0, then the property for the RiskFreeRate is set using the PortfolioCVaR object:

```
r0 = 0.01/12;
p = PortfolioCVaR;
p = PortfolioCVaR('RiskFreeRate', r0);
disp(p.RiskFreeRate);
```

8.3333e-04

Note If your portfolio problem has a budget constraint such that your portfolio weights must sum to 1 , then the riskless asset is irrelevant.

## See Also

PortfolioCVaR|estimatePortVaR|setCosts | setProbabilityLevel| setScenarios|simulateNormalScenariosByData| simulateNormalScenariosByMoments

## Related Examples

- "Asset Returns and Scenarios Using PortfolioCVaR Object" on page 5-44
- "Working with Transaction Costs" on page 5-58
- "Creating the PortfolioCVaR Object" on page 5-27
- "Working with CVaR Portfolio Constraints Using Defaults" on page 5-63
- "Validate the CVaR Portfolio Problem" on page 5-96
- "Estimate Efficient Portfolios for Entire Frontier for PortfolioCVaR Object" on page 5-101
- "Estimate Efficient Frontiers for PortfolioCVaR Object" on page 5-118


## More About

- "PortfolioCVaR Object" on page 5-22
- "Portfolio Optimization Theory" on page 5-3
- "PortfolioCVaR Object Workflow" on page 5-20


## External Websites

- Analyzing Investment Strategies with CVaR Portfolio Optimization in MATLAB (50 $\min 42 \mathrm{sec}$ )


## Working with Transaction Costs

The difference between net and gross portfolio returns is transaction costs. The net portfolio return proxy has distinct proportional costs to purchase and to sell assets which are maintained in the PortfolioCVaR object properties BuyCost and SellCost.
Transaction costs are in units of total return and, as such, are proportional to the price of an asset so that they enter the model for net portfolio returns in return form. For example, suppose that you have a stock currently priced $\$ 40$ and your usual transaction costs are 5 cents per share. Then the transaction cost for the stock is $0.05 / 40=0.00125$ (as defined in "Net Portfolio Returns" on page 5-5). Costs are entered as positive values and credits are entered as negative values.

## Setting Transaction Costs Using the PortfolioCVaR Function

To set up transaction costs, you must specify an initial or current portfolio in the InitPort property. If the initial portfolio is not set when you set up the transaction cost properties, InitPort is 0 . The properties for transaction costs can be set using the PortfolioCVaR object. For example, assume that purchase and sale transaction costs are in the variables bc and sc and an initial portfolio is in the variable $\times 0$, then transaction costs are set:

```
bc = [ 0.00125; 0.00125; 0.00125; 0.00125; 0.00125 ];
Sc = [ 0.00125; 0.007; 0.00125; 0.00125; 0.0024 ];
x0 = [ 0.4; 0.2; 0.2; 0.1; 0.1 ];
p = PortfolioCVaR('BuyCost', bc, 'SellCost', sc, 'InitPort', x0);
disp(p.NumAssets);
disp(p.BuyCost);
disp(p.SellCost);
disp(p.InitPort);
```

    5
    0.0013
0.0013
0.0013
0.0013
0.0013
0.0013
0.0070
0.0013
0.0013
0.0024
0.4000
0.2000
0.2000
0.1000
0.1000

## Setting Transaction Costs Using the setCosts Function

You can also set the properties for transaction costs using setCosts. Assume that you have the same costs and initial portfolio as in the previous example. Given a PortfolioCVaR object $p$ with an initial portfolio already set, use setCosts to set up transaction costs:

```
bc = [ 0.00125; 0.00125; 0.00125; 0.00125; 0.00125 ];
sc = [ 0.00125; 0.007; 0.00125; 0.00125; 0.0024 ];
x0 = [ 0.4; 0.2; 0.2; 0.1; 0.1 ];
p = PortfolioCVaR('InitPort', x0);
p = setCosts(p, bc, sc);
disp(p.NumAssets);
disp(p.BuyCost);
disp(p.SellCost);
disp(p.InitPort);
```

5
0.0013
0.0013
0.0013
0.0013
0.0013
0.0013
0.0070
0.0013
0.0013
0.0024
0.4000
0.2000
0.2000
0.1000
0.1000

You can also set up the initial portfolio's InitPort value as an optional argument to setCosts so that the following is an equivalent way to set up transaction costs:
bc = [ 0.00125; 0.00125; 0.00125; 0.00125; 0.00125 ];
sc $=$ [ $0.00125 ; 0.007 ; 0.00125 ; 0.00125 ; 0.0024$ ];
$\mathrm{x} 0=[0.4 ; 0.2 ; 0.2 ; 0.1 ; 0.1$ ];
p = PortfolioCVaR;
p = setCosts(p, bc, sc, x0);
disp(p.NumAssets);
disp(p.BuyCost);
disp(p.SellCost);
disp(p.InitPort);
5
0.0013
0.0013
0.0013
0.0013
0.0013
0.0013
0.0070
0.0013
0.0013
0.0024
0.4000
0.2000
0.2000
0.1000
0.1000

## Setting Transaction Costs with Scalar Expansion

Both the PortfolioCVaR object and setCosts function implement scalar expansion on the arguments for transaction costs and the initial portfolio. If the NumAssets property is already set in the PortfolioCVaR object, scalar arguments for these properties are expanded to have the same value across all dimensions. In addition, setCosts lets you
specify NumAssets as an optional final argument. For example, assume that you have an initial portfolio $x 0$ and you want to set common transaction costs on all assets in your universe. You can set these costs in any of these equivalent ways:

```
x0 = [ 0.4; 0.2; 0.2; 0.1; 0.1 ];
or
x0 = [ 0.4; 0.2; 0.2; 0.1; 0.1 ];
p = PortfolioCVaR('InitPort', x0);
p = setCosts(p, 0.002, 0.002);
or
x0 = [ 0.4; 0.2; 0.2; 0.1; 0.1 ];
p = PortfolioCVaR;
p = setCosts(p, 0.002, 0.002, x0);
```

$\mathrm{p}=$ PortfolioCVaR('InitPort', $\times 0$, 'BuyCost', 0.002, 'SellCost', 0.002);

To clear costs from your PortfolioCVaR object, use either the PortfolioCVaR object or setCosts with empty inputs for the properties to be cleared. For example, you can clear sales costs from the PortfolioCVaR object $p$ in the previous example:

```
p = PortfolioCVaR(p, 'SellCost', []);
```


## See Also

PortfolioCVaR|estimatePortVaR|setCosts | setProbabilityLevel| setScenarios|simulateNormalScenariosByData|
simulateNormalScenariosByMoments

## Related Examples

- "Asset Returns and Scenarios Using PortfolioCVaR Object" on page 5-44
- "Working with a Riskless Asset" on page 5-56
- "Creating the PortfolioCVaR Object" on page 5-27
- "Working with CVaR Portfolio Constraints Using Defaults" on page 5-63
- "Validate the CVaR Portfolio Problem" on page 5-96
- "Estimate Efficient Portfolios for Entire Frontier for PortfolioCVaR Object" on page 5-101
- "Estimate Efficient Frontiers for PortfolioCVaR Object" on page 5-118


## More About

- "PortfolioCVaR Object" on page 5-22
- "Portfolio Optimization Theory" on page 5-3
- "PortfolioCVaR Object Workflow" on page 5-20


## External Websites

- Analyzing Investment Strategies with CVaR Portfolio Optimization in MATLAB (50 $\min 42 \mathrm{sec}$ )


## Working with CVaR Portfolio Constraints Using Defaults

The final element for a complete specification of a portfolio optimization problem is the set of feasible portfolios, which is called a portfolio set. A portfolio set $X \subset R^{n}$ is specified by construction as the intersection of sets formed by a collection of constraints on portfolio weights. A portfolio set necessarily and sufficiently must be a nonempty, closed, and bounded set.

When setting up your portfolio set, ensure that the portfolio set satisfies these conditions. The most basic or "default" portfolio set requires portfolio weights to be nonnegative (using the lower-bound constraint) and to sum to 1 (using the budget constraint). For information on the workflow when using PortfolioCVaR objects, see "PortfolioCVaR Object Workflow" on page 5-20.

## Setting Default Constraints for Portfolio Weights Using PortfolioCVaR Object

The "default" CVaR portfolio problem has two constraints on portfolio weights:

- Portfolio weights must be nonnegative.
- Portfolio weights must sum to 1 .

Implicitly, these constraints imply that portfolio weights are no greater than 1, although this is a superfluous constraint to impose on the problem.

## Setting Default Constraints Using the PortfolioCVaR Function

Given a portfolio optimization problem with NumAssets $=20$ assets, use the PortfolioCVaR object to set up a default problem and explicitly set bounds and budget constraints:

```
p = PortfolioCVaR('NumAssets', 20, 'LowerBound', 0, 'Budget', 1);
disp(p);
    PortfolioCVaR with properties:
```

        BuyCost: []
            SellCost: []
        RiskFreeRate: []
    ProbabilityLevel: []
    ```
Turnover: []
    BuyTurnover: []
SellTurnover: []
NumScenarios: []
            Name: []
    NumAssets: 20
    AssetList: []
        InitPort: []
    AInequality: []
    bInequality: []
        AEquality: []
        bEquality: []
    LowerBound: [20x1 double]
    UpperBound: []
    LowerBudget: 1
    UpperBudget: 1
    GroupMatrix: []
    LowerGroup: []
    UpperGroup: []
        GroupA: []
        GroupB: []
    LowerRatio: []
    UpperRatio: []
```


## Setting Default Constraints Using the setDefaultConstraints Function

An alternative approach is to use the setDefaultConstraints function. If the number of assets is already known in a PortfolioCVaR object, use setDefaultConstraints with no arguments to set up the necessary bound and budget constraints. Suppose that you have 20 assets to set up the portfolio set for a default problem:

```
p = PortfolioCVaR('NumAssets', 20);
p = setDefaultConstraints(p);
disp(p);
PortfolioCVaR with properties:
            BuyCost: []
            SellCost: []
        RiskFreeRate: []
    ProbabilityLevel: []
            Turnover: []
        BuyTurnover: []
        SellTurnover: []
        NumScenarios: []
```

Name: []
NumAssets: 20
AssetList: []
InitPort: []
AInequality: []
bInequality: []
AEquality: []
bEquality: []
LowerBound: [20x1 double]
UpperBound: []
LowerBudget: 1
UpperBudget: 1
GroupMatrix: []
LowerGroup: []
UpperGroup: []
GroupA: []
GroupB: []
LowerRatio: []
UpperRatio: []
If the number of assets is unknown, setDefaultConstraints accepts NumAssets as an optional argument to form a portfolio set for a default problem. Suppose that you have 20 assets:

```
p = PortfolioCVaR;
p = setDefaultConstraints(p, 20);
disp(p);
```

PortfolioCVaR with properties:
BuyCost: []
SellCost: []
RiskFreeRate: []
ProbabilityLevel: []
Turnover: []
BuyTurnover: []
SellTurnover: []
NumScenarios: []
Name: []
NumAssets: 20
AssetList: []
InitPort: []
AInequality: []
bInequality: []
AEquality: []

```
        bEquality: []
        LowerBound: [20x1 double]
        UpperBound: []
        LowerBudget: 1
        UpperBudget: 1
        GroupMatrix: []
        LowerGroup: []
        UpperGroup: []
        GroupA: []
        GroupB: []
    LowerRatio: []
    UpperRatio: []
```


## See Also

PortfolioCVaR | setBounds | setBudget | setDefaultConstraints | setEquality | setGroupRatio| setGroups| setInequality| setOneWayTurnover|setTurnover

## Related Examples

- "Working with Bound Constraints Using PortfolioCVaR Object" on page 5-68
- "Working with Budget Constraints Using PortfolioCVaR Object" on page 5-71
- "Working with Group Constraints Using PortfolioCVaR Object" on page 5-74
- "Working with Group Ratio Constraints Using PortfolioCVaR Object" on page 5-78
- "Working with Linear Equality Constraints Using PortfolioCVaR Object" on page 582
- "Working with Linear Inequality Constraints Using PortfolioCVaR Object" on page 585
- "Working with Average Turnover Constraints Using PortfolioCVaR Object" on page 5-88
- "Working with One-Way Turnover Constraints Using PortfolioCVaR Object" on page 5-92
- "Creating the PortfolioCVaR Object" on page 5-27
- "Validate the CVaR Portfolio Problem" on page 5-96
- "Estimate Efficient Portfolios for Entire Frontier for PortfolioCVaR Object" on page 5-101
- "Estimate Efficient Frontiers for PortfolioCVaR Object" on page 5-118
- "Asset Returns and Scenarios Using PortfolioCVaR Object" on page 5-44


## More About

- "PortfolioCVaR Object" on page 5-22
- "Portfolio Optimization Theory" on page 5-3
- "PortfolioCVaR Object Workflow" on page 5-20


## External Websites

- Analyzing Investment Strategies with CVaR Portfolio Optimization in MATLAB (50 $\min 42 \mathrm{sec}$ )


## Working with Bound Constraints Using PortfolioCVaR Object

Bound constraints are optional linear constraints that maintain upper and lower bounds on portfolio weights (see "Bound Constraints" on page 5-12). Although every CVaR portfolio set must be bounded, it is not necessary to specify a CVaR portfolio set with explicit bound constraints. For example, you can create a CVaR portfolio set with an implicit upper bound constraint or a CVaR portfolio set with average turnover constraints. The bound constraints have properties LowerBound for the lower-bound constraint and UpperBound for the upper-bound constraint. Set default values for these constraints using the setDefaultConstraints function (see "Setting Default Constraints for Portfolio Weights Using PortfolioCVaR Object" on page 5-63).

## Setting Bounds Using the PortfolioCVaR Function

The properties for bound constraints are set through the PortfolioCVaR object. Suppose that you have a balanced fund with stocks that can range from $50 \%$ to $75 \%$ of your portfolio and bonds that can range from $25 \%$ to $50 \%$ of your portfolio. The bound constraints for a balanced fund are set with:

```
lb = [ 0.5; 0.25 ];
ub = [ 0.75; 0.5 ];
p = PortfolioCVaR('LowerBound', lb, 'UpperBound', ub);
disp(p.NumAssets);
disp(p.LowerBound);
disp(p.UpperBound);
```

2
0.5000
0.2500
0.7500
0.5000

To continue with this example, you must set up a budget constraint. For details, see "Working with Budget Constraints Using PortfolioCVaR Object" on page 5-71.

## Setting Bounds Using the setBounds Function

You can also set the properties for bound constraints using setBounds. Suppose that you have a balanced fund with stocks that can range from $50 \%$ to $75 \%$ of your portfolio and bonds that can range from $25 \%$ to $50 \%$ of your portfolio. Given a PortfolioCVaR object $p$, use setBounds to set the bound constraints:

```
lb = [ 0.5; 0.25 ];
ub = [ 0.75; 0.5 ];
p = PortfolioCVaR;
p = setBounds(p, lb, ub);
disp(p.NumAssets);
disp(p.LowerBound);
disp(p.UpperBound);
```

    2
    0.5000
0.2500
0.7500
0.5000

## Setting Bounds Using the PortfolioCVaR Function or setBounds Function

Both the PortfolioCVaR object and setBounds function implement scalar expansion on either the LowerBound or UpperBound properties. If the NumAssets property is already set in the PortfolioCVaR object, scalar arguments for either property expand to have the same value across all dimensions. In addition, setBounds lets you specify NumAssets as an optional argument. Suppose that you have a universe of 500 assets and you want to set common bound constraints on all assets in your universe. Specifically, you are a long-only investor and want to hold no more than $5 \%$ of your portfolio in any single asset. You can set these bound constraints in any of these equivalent ways:

```
p = PortfolioCVaR('NumAssets', 500, 'LowerBound', 0, 'UpperBound', 0.05);
or
p = PortfolioCVaR('NumAssets', 500);
p = setBounds(p, 0, 0.05);
```

or

```
p = PortfolioCVaR;
p = setBounds(p, 0, 0.05, 500);
```

To clear bound constraints from your PortfolioCVaR object, use either the PortfolioCVaR object or setBounds with empty inputs for the properties to be cleared. For example, to clear the upper-bound constraint from the PortfolioCVaR object $p$ in the previous example:
p = PortfolioCVaR(p, 'UpperBound', []);

## See Also

PortfolioCVaR|setBounds | setBudget | setDefaultConstraints | setEquality | setGroupRatio| setGroups | setInequality | setOneWayTurnover| setTurnover

## Related Examples

- "Creating the PortfolioCVaR Object" on page 5-27
- "Working with CVaR Portfolio Constraints Using Defaults" on page 5-63
- "Validate the CVaR Portfolio Problem" on page 5-96
- "Estimate Efficient Portfolios for Entire Frontier for PortfolioCVaR Object" on page 5-101
- "Estimate Efficient Frontiers for PortfolioCVaR Object" on page 5-118
- "Asset Returns and Scenarios Using PortfolioCVaR Object" on page 5-44


## More About

- "PortfolioCVaR Object" on page 5-22
- "Portfolio Optimization Theory" on page 5-3
- "PortfolioCVaR Object Workflow" on page 5-20


## External Websites

- Analyzing Investment Strategies with CVaR Portfolio Optimization in MATLAB (50 $\min 42 \mathrm{sec}$ )


## Working with Budget Constraints Using PortfolioCVaR Object

The budget constraint is an optional linear constraint that maintains upper and lower bounds on the sum of portfolio weights (see "Budget Constraints" on page 5-12). Budget constraints have properties LowerBudget for the lower budget constraint and UpperBudget for the upper budget constraint. If you set up a CVaR portfolio optimization problem that requires portfolios to be fully invested in your universe of assets, you can set LowerBudget to be equal to UpperBudget. These budget constraints can be set with default values equal to 1 using setDefaultConstraints (see "Setting Default Constraints Using the PortfolioCVaR Function" on page 5-63).

## Setting Budget Constraints Using the PortfolioCVaR Function

The properties for the budget constraint can also be set using the PortfolioCVaR object. Suppose that you have an asset universe with many risky assets and a riskless asset and you want to ensure that your portfolio never holds more than $1 \%$ cash, that is, you want to ensure that you are 99-100\% invested in risky assets. The budget constraint for this portfolio can be set with:

```
p = PortfolioCVaR('LowerBudget', 0.99, 'UpperBudget', 1);
disp(p.LowerBudget);
disp(p.UpperBudget);
```

0.9900

1

## Setting Budget Constraints Using the setBudget Function

You can also set the properties for a budget constraint using setBudget. Suppose that you have a fund that permits up to $10 \%$ leverage which means that your portfolio can be from $100 \%$ to $110 \%$ invested in risky assets. Given a PortfolioCVaR object p, use setBudget to set the budget constraints:

```
p = PortfolioCVaR;
p = setBudget(p, 1, 1.1);
disp(p.LowerBudget);
disp(p.UpperBudget);
```

If you were to continue with this example, then set the RiskFreeRate property to the borrowing rate to finance possible leveraged positions. For details on the RiskFreeRate property, see "Working with a Riskless Asset" on page 5-56. To clear either bound for the budget constraint from your PortfolioCVaR object, use either the PortfolioCVaR object or setBudget with empty inputs for the properties to be cleared. For example, clear the upper-budget constraint from the PortfolioCVaR object $p$ in the previous example with:

```
p = PortfolioCVaR(p, 'UpperBudget', []);
```


## See Also

PortfolioCVaR|setBounds | setBudget | setDefaultConstraints | setEquality|setGroupRatio|setGroups|setInequality| setOneWayTurnover| setTurnover

## Related Examples

- "Creating the PortfolioCVaR Object" on page 5-27
- "Working with CVaR Portfolio Constraints Using Defaults" on page 5-63
- "Validate the CVaR Portfolio Problem" on page 5-96
- "Estimate Efficient Portfolios for Entire Frontier for PortfolioCVaR Object" on page 5-101
- "Estimate Efficient Frontiers for PortfolioCVaR Object" on page 5-118
- "Asset Returns and Scenarios Using PortfolioCVaR Object" on page 5-44


## More About

- "PortfolioCVaR Object" on page 5-22
- "Portfolio Optimization Theory" on page 5-3
- "PortfolioCVaR Object Workflow" on page 5-20


## External Websites

- Analyzing Investment Strategies with CVaR Portfolio Optimization in MATLAB (50 $\min 42 \mathrm{sec}$ )


## Working with Group Constraints Using PortfolioCVaR Object

Group constraints are optional linear constraints that group assets together and enforce bounds on the group weights (see "Group Constraints" on page 5-13). Although the constraints are implemented as general constraints, the usual convention is to form a group matrix that identifies membership of each asset within a specific group with Boolean indicators (either true or false or with 1 or 0) for each element in the group matrix. Group constraints have properties GroupMatrix for the group membership matrix, LowerGroup for the lower-bound constraint on groups, and UpperGroup for the upper-bound constraint on groups.

## Setting Group Constraints Using the PortfolioCVaR Function

The properties for group constraints are set through the PortfolioCVaR object. Suppose that you have a portfolio of five assets and want to ensure that the first three assets constitute no more than $30 \%$ of your portfolio, then you can set group constraints:

```
G = [ 1 1 1 1 0 0 ];
p = PortfolioCVaR('GroupMatrix', G, 'UpperGroup', 0.3);
disp(p.NumAssets);
disp(p.GroupMatrix);
disp(p.UpperGroup);
```

5
$\begin{array}{lllll}1 & 1 & 1 & 0 & 0\end{array}$

### 0.3000

The group matrix $G$ can also be a logical matrix so that the following code achieves the same result.

```
G = [ true true true false false ];
p = PortfolioCVaR('GroupMatrix', G, 'UpperGroup', 0.3);
disp(p.NumAssets);
disp(p.GroupMatrix);
disp(p.UpperGroup);
```

5
$\begin{array}{lllll}1 & 1 & 1 & 0 & 0\end{array}$

## Setting Group Constraints Using the setGroups and addGroups Functions

You can also set the properties for group constraints using setGroups. Suppose that you have a portfolio of five assets and want to ensure that the first three assets constitute no more than $30 \%$ of your portfolio. Given a PortfolioCVaR object p, use setGroups to set the group constraints:

```
G = [ true true true false false ];
p = PortfolioCVaR;
p = setGroups(p, G, [], 0.3);
disp(p.NumAssets);
disp(p.GroupMatrix);
disp(p.UpperGroup);
```

5
$\begin{array}{lllll}1 & 1 & 1 & 0 & 0\end{array}$

### 0.3000

In this example, you would set the LowerGroup property to be empty ([ ]).
Suppose that you want to add another group constraint to make odd-numbered assets constitute at least $20 \%$ of your portfolio. Set up an augmented group matrix and introduce infinite bounds for unconstrained group bounds or use the addGroups function to build up group constraints. For this example, create another group matrix for the second group constraint:

```
p = PortfolioCVaR;
G = [ true true true false false ];
% group matrix for first group constraint
p = setGroups(p, G, [], 0.3)
G = [ true false true false true ]; % group matrix for second group constraint
p = addGroups(p, G, 0.2);
disp(p.NumAssets);
disp(p.GroupMatrix);
disp(p.LowerGroup);
disp(p.UpperGroup);
```


## 5

$\begin{array}{lllll}1 & 1 & 1 & 0 & 0\end{array}$
$\begin{array}{lllll}1 & 0 & 1 & 0 & 1\end{array}$
-Inf 0.2000

### 0.3000

Inf
addGroups determines which bounds are unbounded so you only need to focus on the constraints that you want to set.

The PortfolioCVaR object, setGroups, and addGroups implement scalar expansion on either the LowerGroup or UpperGroup properties based on the dimension of the group matrix in the property GroupMatrix. Suppose that you have a universe of 30 assets with 6 asset classes such that assets $1-5$, assets $6-12$, assets $13-18$, assets 19-22, assets 2327, and assets 28-30 constitute each of your asset classes and you want each asset class to fall from $0 \%$ to $25 \%$ of your portfolio. Let the following group matrix define your groups and scalar expansion define the common bounds on each group:

```
p = PortfolioCVaR;
G = blkdiag(true(1,5), true(1,7), true(1,6), true(1,4), true(1,5), true(1,3));
p = setGroups(p, G, 0, 0.25);
disp(p.NumAssets);
disp(p.GroupMatrix);
disp(p.LowerGroup);
disp(p.UpperGroup);
3 0
```

Columns 1 through 16

| 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Columns 17 through 30

| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  |
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  |
| 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |  |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |  |
| 0 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 0 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 0 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 0 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 0 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 0 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 0.2500 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 0.2500 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 0 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

## See Also

PortfolioCVaR|setBounds | setBudget | setDefaultConstraints | setEquality|setGroupRatio|setGroups|setInequality| setOneWayTurnover | setTurnover

## Related Examples

- "Creating the PortfolioCVaR Object" on page 5-27
- "Working with CVaR Portfolio Constraints Using Defaults" on page 5-63
- "Validate the CVaR Portfolio Problem" on page 5-96
- "Estimate Efficient Portfolios for Entire Frontier for PortfolioCVaR Object" on page 5-101
- "Estimate Efficient Frontiers for PortfolioCVaR Object" on page 5-118
- "Asset Returns and Scenarios Using PortfolioCVaR Object" on page 5-44


## More About

- "PortfolioCVaR Object" on page 5-22
- "Portfolio Optimization Theory" on page 5-3
- "PortfolioCVaR Object Workflow" on page 5-20


## External Websites

- Analyzing Investment Strategies with CVaR Portfolio Optimization in MATLAB (50 $\min 42 \mathrm{sec}$ )


## Working with Group Ratio Constraints Using PortfolioCVaR Object

Group ratio constraints are optional linear constraints that maintain bounds on proportional relationships among groups of assets (see "Group Ratio Constraints" on page 5-14). Although the constraints are implemented as general constraints, the usual convention is to specify a pair of group matrices that identify membership of each asset within specific groups with Boolean indicators (either true or false or with 1 or 0 ) for each element in each of the group matrices. The goal is to ensure that the ratio of a base group compared to a comparison group fall within specified bounds. Group ratio constraints have properties:

- GroupA for the base membership matrix
- GroupB for the comparison membership matrix
- LowerRatio for the lower-bound constraint on the ratio of groups
- UpperRatio for the upper-bound constraint on the ratio of groups


## Setting Group Ratio Constraints Using the PortfolioCVaR Function

The properties for group ratio constraints are set using PortfolioCVaR object. For example, assume that you want the ratio of financial to nonfinancial companies in your portfolios to never go above 50\%. Suppose that you have six assets with three financial companies (assets 1-3) and three nonfinancial companies (assets 4-6). To set group ratio constraints:

```
GA = [ 1 1 1 0 0 0 ]; % financial companies
GB = [ 0000 1 1 1 ]; % nonfinancial companies
p = PortfolioCVaR('GroupA', GA, 'GroupB', GB, 'UpperRatio', 0.5);
disp(p.NumAssets);
disp(p.GroupA);
disp(p.GroupB);
disp(p.UpperRatio);
6
1 1 1 
0
0.5000
```

Group matrices GA and GB in this example can be logical matrices with true and false elements that yield the same result:

```
GA = [ true true true false false false ]; % financial companies
GB = [ false false false true true true ]; % nonfinancial companies
p = PortfolioCVaR('GroupA', GA, 'GroupB', GB, 'UpperRatio', 0.5);
disp(p.NumAssets);
disp(p.GroupA);
disp(p.GroupB);
disp(p.UpperRatio);
```

6

| 1 | 1 | 1 | 0 | 0 | 0 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 1 | 1 | 1 |

0.5000

## Setting Group Ratio Constraints Using the setGroupRatio and addGroupRatio Functions

You can also set the properties for group ratio constraints using setGroupRatio. For example, assume that you want the ratio of financial to nonfinancial companies in your portfolios to never go above 50\%. Suppose that you have six assets with three financial companies (assets 1-3) and three nonfinancial companies (assets 4-6). Given a PortfolioCVaR object $p$, use setGroupRatio to set the group constraints:

```
GA = [ true true true false false false ]; % financial companies
GB = [ false false false true true true ]; % nonfinancial companies
p = PortfolioCVaR;
p = setGroupRatio(p, GA, GB, [], 0.5);
disp(p.NumAssets);
disp(p.GroupA);
disp(p.GroupB);
disp(p.UpperRatio);
6
1 1 1 1 1 0
0
0.5000
```

In this example, you would set the LowerRatio property to be empty ([ ]).

Suppose that you want to add another group ratio constraint to ensure that the weights in odd-numbered assets constitute at least $20 \%$ of the weights in nonfinancial assets your portfolio. You can set up augmented group ratio matrices and introduce infinite bounds for unconstrained group ratio bounds, or you can use the addGroupRatio function to build up group ratio constraints. For this example, create another group matrix for the second group constraint:

```
p = PortfolioCVaR;
GA = [ true true true false false false ]; % financial companies
GB = [ false false false true true true ]; % nonfinancial companies
p = setGroupRatio(p, GA, GB, [], 0.5);
GA = [ true false true false true false ]; % odd-numbered companies
GB = [ false false false true true true ]; % nonfinancial companies
p = addGroupRatio(p, GA, GB, 0.2);
disp(p.NumAssets);
disp(p.GroupA);
disp(p.GroupB);
disp(p.LowerRatio);
disp(p.UpperRatio);
6
\begin{tabular}{llllll}
1 & 1 & 1 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 & 1 & 0 \\
& & & & & \\
0 & 0 & 0 & 1 & 1 & 1 \\
0 & 0 & 0 & 1 & 1 & 1
\end{tabular}
    - Inf
0.2000
0.5000
Inf
```

Notice that addGroupRatio determines which bounds are unbounded so you only need to focus on the constraints you want to set.

The PortfolioCVaR object, setGroupRatio, and addGroupRatio implement scalar expansion on either the LowerRatio or UpperRatio properties based on the dimension of the group matrices in GroupA and GroupB properties.

## See Also

PortfolioCVaR|setBounds | setBudget| setDefaultConstraints | setEquality|setGroupRatio|setGroups|setInequality| setOneWayTurnover| setTurnover

## Related Examples

- "Creating the PortfolioCVaR Object" on page 5-27
- "Working with CVaR Portfolio Constraints Using Defaults" on page 5-63
- "Validate the CVaR Portfolio Problem" on page 5-96
- "Estimate Efficient Portfolios for Entire Frontier for PortfolioCVaR Object" on page 5-101
- "Estimate Efficient Frontiers for PortfolioCVaR Object" on page 5-118
- "Asset Returns and Scenarios Using PortfolioCVaR Object" on page 5-44


## More About

- "PortfolioCVaR Object" on page 5-22
- "Portfolio Optimization Theory" on page 5-3
- "PortfolioCVaR Object Workflow" on page 5-20


## External Websites

- Analyzing Investment Strategies with CVaR Portfolio Optimization in MATLAB (50 $\min 42 \mathrm{sec}$ )


## Working with Linear Equality Constraints Using PortfolioCVaR Object

Linear equality constraints are optional linear constraints that impose systems of equalities on portfolio weights (see "Linear Equality Constraints" on page 5-11). Linear equality constraints have properties AEquality, for the equality constraint matrix, and bEquality, for the equality constraint vector.

## Setting Linear Equality Constraints Using the PortfolioCVaR Function

The properties for linear equality constraints are set using the PortfolioCVaR object. Suppose that you have a portfolio of five assets and want to ensure that the first three assets are $50 \%$ of your portfolio. To set this constraint:

```
A = [ 1 1 1 0 0 ];
b = 0.5;
p = PortfolioCVaR('AEquality', A, 'bEquality', b);
disp(p.NumAssets);
disp(p.AEquality);
disp(p.bEquality);
5
1 
0.5000
```


## Setting Linear Equality Constraints Using the setEquality and addEquality Functions

You can also set the properties for linear equality constraints using setEquality. Suppose that you have a portfolio of five assets and want to ensure that the first three assets are $50 \%$ of your portfolio. Given a PortfolioCVaR object p, use setEquality to set the linear equality constraints:
$\mathrm{A}=\left[\begin{array}{lllll}1 & 1 & 1 & 0 & 0\end{array}\right] ;$
b $=0.5$;
p = PortfolioCVaR;
$\mathrm{p}=\operatorname{setEquality}(\mathrm{p}, \mathrm{A}, \mathrm{b})$;

```
disp(p.NumAssets);
disp(p.AEquality);
disp(p.bEquality);
5
1 1 1 1 0
0.5000
```

Suppose that you want to add another linear equality constraint to ensure that the last three assets also constitute $50 \%$ of your portfolio. You can set up an augmented system of linear equalities or use addEquality to build up linear equality constraints. For this example, create another system of equalities:

```
p = PortfolioCVaR;
A = [ 1 1 1 0 0 ]; % first equality constraint
b = 0.5;
p = setEquality(p, A, b);
A = [ 0 0 1 1 1 ]; % second equality constraint
b = 0.5;
p = addEquality(p, A, b);
disp(p.NumAssets);
disp(p.AEquality);
disp(p.bEquality);
5
\begin{tabular}{lllll}
1 & 1 & 1 & 0 & 0
\end{tabular}
0.5000
0.5000
```

The PortfolioCVaR object, setEquality, and addEquality implement scalar expansion on the bEquality property based on the dimension of the matrix in the AEquality property.

## See Also

PortfolioCVaR|setBounds | setBudget| setDefaultConstraints | setEquality|setGroupRatio|setGroups|setInequality| setOneWayTurnover|setTurnover

## Related Examples

- "Creating the PortfolioCVaR Object" on page 5-27
- "Working with CVaR Portfolio Constraints Using Defaults" on page 5-63
- "Validate the CVaR Portfolio Problem" on page 5-96
- "Estimate Efficient Portfolios for Entire Frontier for PortfolioCVaR Object" on page 5-101
- "Estimate Efficient Frontiers for PortfolioCVaR Object" on page 5-118
- "Asset Returns and Scenarios Using PortfolioCVaR Object" on page 5-44


## More About

- "PortfolioCVaR Object" on page 5-22
- "Portfolio Optimization Theory" on page 5-3
- "PortfolioCVaR Object Workflow" on page 5-20


## External Websites

- Analyzing Investment Strategies with CVaR Portfolio Optimization in MATLAB (50 $\min 42 \mathrm{sec}$ )


## Working with Linear Inequality Constraints Using PortfolioCVaR Object

Linear inequality constraints are optional linear constraints that impose systems of inequalities on portfolio weights (see "Linear Inequality Constraints" on page 5-10). Linear inequality constraints have properties AInequality for the inequality constraint matrix, and bInequality for the inequality constraint vector.

## Setting Linear Inequality Constraints Using the PortfolioCVaR Function

The properties for linear inequality constraints are set using the PortfolioCVaR object. Suppose that you have a portfolio of five assets and you want to ensure that the first three assets are no more than $50 \%$ of your portfolio. To set up these constraints:

```
A = [ 1 1 1 0 0 ];
b = 0.5;
p = PortfolioCVaR('AInequality', A, 'bInequality', b);
disp(p.NumAssets);
disp(p.AInequality);
disp(p.bInequality);
5
1 1 1 1 1 0 0
0.5000
```


## Setting Linear Inequality Constraints Using the setInequality and addInequality Functions

You can also set the properties for linear inequality constraints using setInequality. Suppose that you have a portfolio of five assets and you want to ensure that the first three assets constitute no more than $50 \%$ of your portfolio. Given a PortfolioCVaR object p, use setInequality to set the linear inequality constraints:
$\mathrm{A}=\left[\begin{array}{lllll}1 & 1 & 1 & 0\end{array}\right] ;$
b $=0.5$;
p = PortfolioCVaR;
p = setInequality(p, A, b);

```
disp(p.NumAssets);
disp(p.AInequality);
disp(p.bInequality);
5
1 1 1 1 0
0.5000
```

Suppose that you want to add another linear inequality constraint to ensure that the last three assets constitute at least 50\% of your portfolio. You can set up an augmented system of linear inequalities or use the addInequality function to build up linear inequality constraints. For this example, create another system of inequalities:

```
p = PortfolioCVaR;
A = [ 1 1 1 0 0 ]; % first inequality constraint
b = 0.5;
p = setInequality(p, A, b);
A = [ 0 0 -1 -1 -1 ]; % second inequality constraint
b = -0.5;
p = addInequality(p, A, b);
disp(p.NumAssets);
disp(p.AInequality);
disp(p.bInequality);
5
\begin{tabular}{rrrrr}
1 & 1 & 1 & 0 & 0 \\
0 & 0 & -1 & -1 & -1
\end{tabular}
0.5000
-0.5000
```

The PortfolioCVaR object, setInequality, and addInequality implement scalar expansion on the bInequality property based on the dimension of the matrix in the AInequality property.

## See Also

PortfolioCVaR|setBounds | setBudget| setDefaultConstraints | setEquality|setGroupRatio|setGroups|setInequality| setOneWayTurnover| setTurnover

## Related Examples

- "Creating the PortfolioCVaR Object" on page 5-27
- "Working with CVaR Portfolio Constraints Using Defaults" on page 5-63
- "Validate the CVaR Portfolio Problem" on page 5-96
- "Estimate Efficient Portfolios for Entire Frontier for PortfolioCVaR Object" on page 5-101
- "Estimate Efficient Frontiers for PortfolioCVaR Object" on page 5-118
- "Asset Returns and Scenarios Using PortfolioCVaR Object" on page 5-44


## More About

- "PortfolioCVaR Object" on page 5-22
- "Portfolio Optimization Theory" on page 5-3
- "PortfolioCVaR Object Workflow" on page 5-20


## External Websites

- Analyzing Investment Strategies with CVaR Portfolio Optimization in MATLAB (50 $\min 42 \mathrm{sec}$ )


## Working with Average Turnover Constraints Using PortfolioCVaR Object

The turnover constraint is an optional linear absolute value constraint (see "Average Turnover Constraints" on page 5-15) that enforces an upper bound on the average of purchases and sales. The turnover constraint can be set using the PortfolioCVaR object or the setTurnover function. The turnover constraint depends on an initial or current portfolio, which is assumed to be zero if not set when the turnover constraint is set. The turnover constraint has properties Turnover, for the upper bound on average turnover, and InitPort, for the portfolio against which turnover is computed.

## Setting Average Turnover Constraints Using the PortfolioCVaR Function

The properties for the turnover constraints are set using the PortfolioCVaR object. Suppose that you have an initial portfolio of 10 assets in a variable $x 0$ and you want to ensure that average turnover is no more than $30 \%$. To set this turnover constraint:

```
x0 = [ 0.12; 0.09; 0.08; 0.07; 0.1; 0.1; 0.15; 0.11; 0.08; 0.1 ];
p = PortfolioCVaR('Turnover', 0.3, 'InitPort', x0);
disp(p.NumAssets);
disp(p.Turnover);
disp(p.InitPort);
```

10
0.3000
0.1200
0.0900
0.0800
0.0700
0.1000
0.1000
0.1500
0.1100
0.0800
0. 1000

Note if the NumAssets or InitPort properties are not set before or when the turnover constraint is set, various rules are applied to assign default values to these properties (see "Setting Up an Initial or Current Portfolio" on page 5-41).

## Setting Average Turnover Constraints Using the setTurnover Function

You can also set properties for portfolio turnover using setTurnover to specify both the upper bound for average turnover and an initial portfolio. Suppose that you have an initial portfolio of 10 assets in a variable $x 0$ and want to ensure that average turnover is no more than $30 \%$. Given a PortfolioCVaR object p, use setTurnover to set the turnover constraint with and without the initial portfolio being set previously:

```
x0 = [ 0.12; 0.09; 0.08; 0.07; 0.1; 0.1; 0.15; 0.11; 0.08; 0.1 ];
p = PortfolioCVaR('InitPort', x0);
p = setTurnover(p, 0.3);
disp(p.NumAssets);
disp(p.Turnover);
disp(p.InitPort);
1 0
0.3000
0.1200
0.0900
0.0800
0.0700
0.1000
0.1000
0.1500
0.1100
0.0800
0.1000
```

or
$\mathrm{x} 0=[0.12 ; 0.09 ; 0.08 ; 0.07 ; 0.1 ; 0.1 ; 0.15 ; 0.11 ; 0.08 ; 0.1]$;
p = PortfolioCVaR;
$p=\operatorname{set}$ Turnover $(p, 0.3, x 0)$;
disp(p.NumAssets);
disp(p.Turnover);
disp(p.InitPort);
10
0.3000
0.1200
0.0900
0.0800
0.0700
0.1000
0.1000
0.1500
0.1100
0.0800
0.1000
setTurnover implements scalar expansion on the argument for the initial portfolio. If the NumAssets property is already set in the PortfolioCVaR object, a scalar argument for InitPort expands to have the same value across all dimensions. In addition, setTurnover lets you specify NumAssets as an optional argument. To clear turnover from your PortfolioCVaR object, use the PortfolioCVaR object or setTurnover with empty inputs for the properties to be cleared.

## See Also

PortfolioCVaR|setBounds | setBudget | setDefaultConstraints | setEquality | setGroupRatio| setGroups|setInequality| setOneWayTurnover| setTurnover

## Related Examples

- "Creating the PortfolioCVaR Object" on page 5-27
- "Working with CVaR Portfolio Constraints Using Defaults" on page 5-63
- "Validate the CVaR Portfolio Problem" on page 5-96
- "Estimate Efficient Portfolios for Entire Frontier for PortfolioCVaR Object" on page 5-101
- "Estimate Efficient Frontiers for PortfolioCVaR Object" on page 5-118
- "Asset Returns and Scenarios Using PortfolioCVaR Object" on page 5-44


## More About

- "PortfolioCVaR Object" on page 5-22
- "Portfolio Optimization Theory" on page 5-3
- "PortfolioCVaR Object Workflow" on page 5-20


## External Websites

- Analyzing Investment Strategies with CVaR Portfolio Optimization in MATLAB (50 $\min 42 \mathrm{sec}$ )


## Working with One-Way Turnover Constraints Using PortfolioCVaR Object

One-way turnover constraints are optional constraints (see "One-way Turnover Constraints" on page 5-16) that enforce upper bounds on net purchases or net sales. Oneway turnover constraints can be set using the PortfolioCVaR object or the setOneWayTurnover function. One-way turnover constraints depend upon an initial or current portfolio, which is assumed to be zero if not set when the turnover constraints are set. One-way turnover constraints have properties BuyTurnover, for the upper bound on net purchases, SellTurnover, for the upper bound on net sales, and InitPort, for the portfolio against which turnover is computed.

## Setting One-Way Turnover Constraints Using the PortfolioCVaR Function

The Properties for the one-way turnover constraints are set using the PortfolioCVaR object. Suppose that you have an initial portfolio with 10 assets in a variable $x 0$ and you want to ensure that turnover on purchases is no more than $30 \%$ and turnover on sales is no more than $20 \%$ of the initial portfolio. To set these turnover constraints:

```
x0 = [ 0.12; 0.09; 0.08; 0.07; 0.1; 0.1; 0.15; 0.11; 0.08; 0.1 ];
p = PortfolioCVaR('BuyTurnover', 0.3, 'SellTurnover', 0.2, 'InitPort', x0);
disp(p.NumAssets);
disp(p.BuyTurnover);
disp(p.SellTurnover);
disp(p.InitPort);
```

    10
    0.3000
    0.2000
    0.1200
    0.0900
    0.0800
    0.0700
    0.1000
    0.1000
    0.1500
    0.1100
    0.0800
    0.1000
    If the NumAssets or InitPort properties are not set before or when the turnover constraint is set, various rules are applied to assign default values to these properties (see "Setting Up an Initial or Current Portfolio" on page 5-41).

## Setting Turnover Constraints Using the setOneWayTurnover Function

You can also set properties for portfolio turnover using setOneWayTurnover to specify to the upper bounds for turnover on purchases (BuyTurnover) and sales
(SellTurnover) and an initial portfolio. Suppose that you have an initial portfolio of 10 assets in a variable $x 0$ and want to ensure that turnover on purchases is no more than $30 \%$ and that turnover on sales is no more than $20 \%$ of the initial portfolio. Given a PortfolioCVaR object $p$, use setOneWayTurnover to set the turnover constraints with and without the initial portfolio being set previously:

```
x0 = [ 0.12; 0.09; 0.08; 0.07; 0.1; 0.1; 0.15; 0.11; 0.08; 0.1 ];
p = PortfolioCVaR('InitPort', x0);
p = setOneWayTurnover(p, 0.3, 0.2);
disp(p.NumAssets);
disp(p.BuyTurnover);
disp(p.SellTurnover);
disp(p.InitPort);
    1 0
    0.3000
    0.2000
    0.1200
    0.0900
    0.0800
    0.0700
    0.1000
    0.1000
    0.1500
    0.1100
    0.0800
    0.1000
Or
x0 = [ 0.12; 0.09; 0.08; 0.07; 0.1; 0.1; 0.15; 0.11; 0.08; 0.1 ];
p = PortfolioCVaR;
```

```
p = setOneWayTurnover(p, 0.3, 0.2, x0);
disp(p.NumAssets);
disp(p.BuyTurnover);
disp(p.SellTurnover);
disp(p.InitPort);
    10
    0.3000
    0.2000
    0.1200
    0.0900
    0.0800
    0.0700
    0.1000
    0.1000
    0.1500
    0.1100
    0.0800
    0.1000
```

setOneWayTurnover implements scalar expansion on the argument for the initial portfolio. If the NumAssets property is already set in the PortfolioCVaR object, a scalar argument for InitPort expands to have the same value across all dimensions. In addition, setOneWayTurnover lets you specify NumAssets as an optional argument. To remove one-way turnover from your PortfolioCVaR object, use thePortfolioCVaR object or setOneWayTurnover with empty inputs for the properties to be cleared.

## See Also

PortfolioCVaR|setBounds | setBudget|setDefaultConstraints | setEquality | setGroupRatio| setGroups | setInequality | setOneWayTurnover| setTurnover

## Related Examples

- "Creating the PortfolioCVaR Object" on page 5-27
- "Working with CVaR Portfolio Constraints Using Defaults" on page 5-63
- "Validate the CVaR Portfolio Problem" on page 5-96
- "Estimate Efficient Portfolios for Entire Frontier for PortfolioCVaR Object" on page 5-101
- "Estimate Efficient Frontiers for PortfolioCVaR Object" on page 5-118
- "Asset Returns and Scenarios Using PortfolioCVaR Object" on page 5-44


## More About

- "PortfolioCVaR Object" on page 5-22
- "Portfolio Optimization Theory" on page 5-3
- "PortfolioCVaR Object Workflow" on page 5-20


## External Websites

- Analyzing Investment Strategies with CVaR Portfolio Optimization in MATLAB (50 $\min 42 \mathrm{sec}$ )


## Validate the CVaR Portfolio Problem

In this section...<br>"Validating a CVaR Portfolio Set" on page 5-96<br>"Validating CVaR Portfolios" on page 5-98

Sometimes, you may want to validate either your inputs to, or outputs from, a portfolio optimization problem. Although most error checking that occurs during the problem setup phase catches most difficulties with a portfolio optimization problem, the processes to validate CVaR portfolio sets and portfolios are time consuming and are best done offline. So, the portfolio optimization tools have specialized functions to validate CVaR portfolio sets and portfolios. For information on the workflow when using PortfolioCVaR objects, see "PortfolioCVaR Object Workflow" on page 5-20.

## Validating a CVaR Portfolio Set

Since it is necessary and sufficient that your CVaR portfolio set must be a nonempty, closed, and bounded set to have a valid portfolio optimization problem, the estimateBounds function lets you examine your portfolio set to determine if it is nonempty and, if nonempty, whether it is bounded. Suppose that you have the following CVaR portfolio set which is an empty set because the initial portfolio at 0 is too far from a portfolio that satisfies the budget and turnover constraint:

```
p = PortfolioCVaR('NumAssets', 3, 'Budget', 1);
p = setTurnover(p, 0.3, 0);
```

If a CVaR portfolio set is empty, estimateBounds returns NaN bounds and sets the isbounded flag to []:
[lb, ub, isbounded] = estimateBounds(p)
lb $=$
NaN
NaN
NaN
ub $=$
NaN
NaN

NaN
isbounded =
[]
Suppose that you create an unbounded CVaR portfolio set as follows:

```
p = PortfolioCVaR('AInequality', [1 -1; 1 1 ], 'bInequality', 0);
[lb, ub, isbounded] = estimateBounds(p)
lb =
    -Inf
    -Inf
ub =
    1.0e-008
    -0.3712
        Inf
isbounded =
```

0

In this case, estimateBounds returns (possibly infinite) bounds and sets the isbounded flag to false. The result shows which assets are unbounded so that you can apply bound constraints as necessary.

Finally, suppose that you created a CVaR portfolio set that is both nonempty and bounded. estimateBounds not only validates the set, but also obtains tighter bounds which are useful if you are concerned with the actual range of portfolio choices for individual assets in your portfolio set:

```
p = PortfolioCVaR;
p = setBudget(p, 1,1);
p = setBounds(p, [ -0.1; 0.2; 0.3; 0.2 ], [ 0.5; 0.3; 0.9; 0.8 ]);
[lb, ub, isbounded] = estimateBounds(p)
lb =
    -0.1000
    0.2000
```

```
    0.3000
    0.2000
ub =
    0.3000
    0.3000
    0.7000
    0.6000
isbounded =
    1
```

In this example, all but the second asset has tighter upper bounds than the input upper bound implies.

## Validating CVaR Portfolios

Given a CVaR portfolio set specified in a PortfolioCVaR object, you often want to check if specific portfolios are feasible with respect to the portfolio set. This can occur with, for example, initial portfolios and with portfolios obtained from other procedures. The checkFeasibility function determines whether a collection of portfolios is feasible. Suppose that you perform the following portfolio optimization and want to determine if the resultant efficient portfolios are feasible relative to a modified problem.

First, set up a problem in the PortfolioCVaR object p, estimate efficient portfolios in pwgt, and then confirm that these portfolios are feasible relative to the initial problem:

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;
AssetScenarios = mvnrnd(m, C, 20000);
p = PortfolioCVaR;
p = setScenarios(p, AssetScenarios);
p = setDefaultConstraints(p);
p = setProbabilityLevel(p, 0.95);
```

```
pwgt = estimateFrontier(p);
checkFeasibility(p, pwgt)
ans =
    1 1rllllllllll
```

Next, set up a different portfolio problem that starts with the initial problem with an additional a turnover constraint and an equally weighted initial portfolio:

```
q = setTurnover(p, 0.3, 0.25);
checkFeasibility(q, pwgt)
ans =
```

$\begin{array}{llllllllll}0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0\end{array}$
In this case, only two of the 10 efficient portfolios from the initial problem are feasible relative to the new problem in PortfolioCVaR object q. Solving the second problem using checkFeasibility demonstrates that the efficient portfolio for PortfolioCVaR object q is feasible relative to the initial problem:

```
qwgt = estimateFrontier(q);
checkFeasibility(p, qwgt)
ans =
```

| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

## See Also

PortfolioCVaR|checkFeasibility|estimateBounds

## Related Examples

- "Creating the PortfolioCVaR Object" on page 5-27
- "Working with CVaR Portfolio Constraints Using Defaults" on page 5-63
- "Estimate Efficient Portfolios for Entire Frontier for PortfolioCVaR Object" on page 5-101
- "Estimate Efficient Frontiers for PortfolioCVaR Object" on page 5-118
- "Asset Returns and Scenarios Using PortfolioCVaR Object" on page 5-44


## More About

- "PortfolioCVaR Object" on page 5-22
- "Portfolio Optimization Theory" on page 5-3
- "PortfolioCVaR Object Workflow" on page 5-20


## External Websites

- Analyzing Investment Strategies with CVaR Portfolio Optimization in MATLAB (50 $\min 42 \mathrm{sec}$ )


## Estimate Efficient Portfolios for Entire Frontier for PortfolioCVaR Object

There are two ways to look at a portfolio optimization problem that depends on what you are trying to do. One goal is to estimate efficient portfolios and the other is to estimate efficient frontiers. This section focuses on the former goal and "Estimate Efficient Frontiers for PortfolioCVaR Object" on page 5-118 focuses on the latter goal. For information on the workflow when using PortfolioCVaR objects, see "PortfolioCVaR Object Workflow" on page 5-20.

## Obtaining Portfolios Along the Entire Efficient Frontier

The most basic way to obtain optimal portfolios is to obtain points over the entire range of the efficient frontier. Given a portfolio optimization problem in a PortfolioCVaR object, the estimateFrontier function computes efficient portfolios spaced evenly according to the return proxy from the minimum to maximum return efficient portfolios. The number of portfolios estimated is controlled by the hidden property defaultNumPorts which is set to 10. A different value for the number of portfolios estimated is specified as input to estimateFrontier. This example shows the default number of efficient portfolios over the entire range of the efficient frontier:

```
m}=[ 0.05; 0.1; 0.12; 0.18 ]; 
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;
AssetScenarios = mvnrnd(m, C, 20000);
p = PortfolioCVaR;
p = setScenarios(p, AssetScenarios);
p = setDefaultConstraints(p);
p = setProbabilityLevel(p, 0.95);
pwgt = estimateFrontier(p);
disp(pwgt);
Columns 1 through 8
\begin{tabular}{llllllll}
0.8670 & 0.7046 & 0.5421 & 0.3825 & 0.2236 & 0.0570 & 0.0000 & 0.0000 \\
0.0413 & 0.1193 & 0.1963 & 0.2667 & 0.3392 & 0.4159 & 0.3392 & 0.1753 \\
0.0488 & 0.0640 & 0.0811 & 0.1012 & 0.1169 & 0.1427 & 0.1568 & 0.1754 \\
0.0429 & 0.1120 & 0.1806 & 0.2496 & 0.3203 & 0.3844 & 0.5040 & 0.6493
\end{tabular}
```

Columns 9 through 10
$\begin{array}{ll}0.0000 & 0.0000 \\ 0.0230 & 0.0000\end{array}$

| 0.1777 | 0.0000 |
| :--- | :--- |
| 0.7993 | 1.0000 |

If you want only four portfolios in the previous example:

```
pwgt = estimateFrontier(p, 4);
disp(pwgt);
\begin{tabular}{llll}
0.8670 & 0.3825 & 0.0000 & 0.0000 \\
0.0413 & 0.2667 & 0.3392 & 0.0000 \\
0.0488 & 0.1012 & 0.1568 & 0.0000 \\
0.0429 & 0.2496 & 0.5040 & 1.0000
\end{tabular}
```

Starting from the initial portfolio, estimateFrontier also returns purchases and sales to get from your initial portfolio to each efficient portfolio on the efficient frontier. For example, given an initial portfolio in pwgt0, you can obtain purchases and sales:

```
pwgt0 = [ 0.3; 0.3; 0.2; 0.1 ];
p = setInitPort(p, pwgt0);
[pwgt, pbuy, psell] = estimateFrontier(p);
display(pwgt);
display(pbuy);
display(psell);
pwgt =
    Columns 1 through 8
\begin{tabular}{llllllll}
0.8670 & 0.7046 & 0.5421 & 0.3825 & 0.2236 & 0.0570 & 0.0000 & 0.0000 \\
0.0413 & 0.1193 & 0.1963 & 0.2667 & 0.3392 & 0.4159 & 0.3392 & 0.1753 \\
0.0488 & 0.0640 & 0.0811 & 0.1012 & 0.1169 & 0.1427 & 0.1568 & 0.1754 \\
0.0429 & 0.1120 & 0.1806 & 0.2496 & 0.3203 & 0.3844 & 0.5040 & 0.6493
\end{tabular}
```

Columns 9 through 10

| 0.0000 | 0.0000 |
| :--- | :--- |
| 0.0230 | 0.0000 |
| 0.1777 | 0.0000 |
| 0.7993 | 1.0000 |

pbuy =
Columns 1 through 8

| 0.5670 | 0.4046 | 0.2421 | 0.0825 | 0 | 0 | 0 | 0 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 0 | 0 | 0 | 0 | 0.0392 | 0.1159 | 0.0392 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0.0120 | 0.0806 | 0.1496 | 0.2203 | 0.2844 | 0.4040 | 0.5493 |

Columns 9 through 10

| 0 | 0 |
| ---: | ---: |
| 0 | 0 |
| 0 | 0 |
| 0.6993 | 0.9000 |

```
psell =
    Columns 1 through 8
\begin{tabular}{rrrrrrrr}
0 & 0 & 0 & 0 & 0.0764 & 0.2430 & 0.3000 & 0.3000 \\
0.2587 & 0.1807 & 0.1037 & 0.0333 & 0 & 0 & 0 & 0.1247 \\
0.1512 & 0.1360 & 0.1189 & 0.0988 & 0.0831 & 0.0573 & 0.0432 & 0.0246 \\
0.0571 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{tabular}
Columns 9 through 10
```

$0.3000 \quad 0.3000$
$0.2770 \quad 0.3000$
$0.0223 \quad 0.2000$

If you do not specify an initial portfolio, the purchase and sale weights assume that your initial portfolio is 0 .

## See Also

PortfolioCVaR|estimateFrontier|estimateFrontierByReturn| estimateFrontierByRisk|estimateFrontierByRisk| estimateFrontierLimits|estimatePortReturn|estimatePortRisk| setSolver

## Related Examples

- "Obtaining Endpoints of the Efficient Frontier" on page 5-105
- "Obtaining Efficient Portfolios for Target Returns" on page 5-108
- "Obtaining Efficient Portfolios for Target Risks" on page 5-111
- "Obtaining CVaR Portfolio Risks and Returns" on page 5-118
- "Obtaining Portfolio Standard Deviation and VaR" on page 5-120
- "Plotting the Efficient Frontier for a PortfolioCVaR Object" on page 5-122
- "Creating the PortfolioCVaR Object" on page 5-27
- "Working with CVaR Portfolio Constraints Using Defaults" on page 5-63
- "Estimate Efficient Frontiers for PortfolioCVaR Object" on page 5-118
- "Asset Returns and Scenarios Using PortfolioCVaR Object" on page 5-44
- "Troubleshooting CVaR Portfolio Optimization Results" on page 5-136


## More About

- "PortfolioCVaR Object" on page 5-22
- "Portfolio Optimization Theory" on page 5-3
- "PortfolioCVaR Object Workflow" on page 5-20


## External Websites

- Analyzing Investment Strategies with CVaR Portfolio Optimization in MATLAB (50 $\min 42 \mathrm{sec}$ )


## Obtaining Endpoints of the Efficient Frontier

Often, you might be interested in the endpoint portfolios for the efficient frontier. Suppose that you want to determine the range of returns from minimum to maximum to refine a search for a portfolio with a specific target return. Use the estimateFrontierLimits function to obtain the endpoint portfolios:

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;
AssetScenarios = mvnrnd(m, C, 20000);
p = PortfolioCVaR;
p = setScenarios(p, AssetScenarios);
p = setDefaultConstraints(p);
p = setProbabilityLevel(p, 0.95);
pwgt = estimateFrontierLimits(p);
disp(pwgt);
0.8646 0.0000
0.0470 0.0000
0.0414 0.0000
0.0470 1.0000
```

Note The endpoints of the efficient frontier depend upon the Scenarios in the PortfolioCVaR object. If you change the Scenarios, you are likely to obtain different endpoints.

Starting from an initial portfolio, estimateFrontierLimits also returns purchases and sales to get from the initial portfolio to the endpoint portfolios on the efficient frontier. For example, given an initial portfolio in pwgt0, you can obtain purchases and sales:

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
```

```
    0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;
AssetScenarios = mvnrnd(m, C, 20000);
p = PortfolioCVaR;
p = setScenarios(p, AssetScenarios);
p = setDefaultConstraints(p);
p = setProbabilityLevel(p, 0.95);
pwgt0 = [ 0.3; 0.3; 0.2; 0.1 ];
p = setInitPort(p, pwgt0);
[pwgt, pbuy, psell] = estimateFrontierLimits(p);
display(pwgt);
display(pbuy);
display(psell);
pwgt =
\begin{tabular}{ll}
0.8624 & 0.0000 \\
0.0513 & 0.0000 \\
0.0452 & 0.0000 \\
0.0411 & 1.0000
\end{tabular}
pbuy =
\begin{tabular}{rr}
0.5624 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0.9000
\end{tabular}
psell =
\begin{tabular}{rr}
0 & 0.3000 \\
0.2487 & 0.3000 \\
0.1548 & 0.2000 \\
0.0589 & 0
\end{tabular}
```

If you do not specify an initial portfolio, the purchase and sale weights assume that your initial portfolio is 0 .

## See Also

PortfolioCVaR|estimateFrontier|estimateFrontierByReturn| estimateFrontierByRisk|estimateFrontierByRisk|
estimateFrontierLimits|estimatePortReturn|estimatePortRisk| setSolver

## Related Examples

- "Estimate Efficient Portfolios for Entire Frontier for PortfolioCVaR Object" on page 5-101
- "Creating the PortfolioCVaR Object" on page 5-27
- "Working with CVaR Portfolio Constraints Using Defaults" on page 5-63
- "Estimate Efficient Frontiers for PortfolioCVaR Object" on page 5-118
- "Asset Returns and Scenarios Using PortfolioCVaR Object" on page 5-44
- "Troubleshooting CVaR Portfolio Optimization Results" on page 5-136


## More About

- "PortfolioCVaR Object" on page 5-22
- "Portfolio Optimization Theory" on page 5-3
- "PortfolioCVaR Object Workflow" on page 5-20


## External Websites

- Analyzing Investment Strategies with CVaR Portfolio Optimization in MATLAB (50 $\min 42 \mathrm{sec}$ )


## Obtaining Efficient Portfolios for Target Returns

To obtain efficient portfolios that have targeted portfolio returns, the estimateFrontierByReturn function accepts one or more target portfolio returns and obtains efficient portfolios with the specified returns. For example, assume that you have a universe of four assets where you want to obtain efficient portfolios with target portfolio returns of $7 \%, 10 \%$, and $12 \%$ :

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
AssetScenarios = mvnrnd(m, C, 20000);
p = PortfolioCVaR;
p = setScenarios(p, AssetScenarios);
p = setDefaultConstraints(p);
p = setProbabilityLevel(p, 0.95);
pwgt = estimateFrontierByReturn(p, [0.07, 0.10, .12]);
display(pwgt);
pwgt =
    0.7526 0.3773 0.1306
    0.1047 0.3079 0.4348
    0.0662 0.1097 0.1426
    0.0765 0.2051 0.2920
```

Sometimes, you can request a return for which no efficient portfolio exists. Based on the previous example, suppose that you want a portfolio with a $4 \%$ return (which is the return of the first asset). A portfolio that is fully invested in the first asset, however, is inefficient. estimateFrontierByReturn warns if your target returns are outside the range of efficient portfolio returns and replaces it with the endpoint portfolio of the efficient frontier closest to your target return:

```
pwgt = estimateFrontierByReturn(p, [0.04]);
Warning: One or more target return values are outside the feasible range [
0.066388, 0.178834 ].
    Will return portfolios associated with endpoints of the range for these values.
> In PortfolioCVaR.estimateFrontierByReturn at 93
```

The best way to avoid this situation is to bracket your target portfolio returns with estimateFrontierLimits and estimatePortReturn (see "Obtaining Endpoints of the Efficient Frontier" on page 5-105 and "Obtaining CVaR Portfolio Risks and Returns" on page 5-118).

```
pret = estimatePortReturn(p, p.estimateFrontierLimits);
display(pret);
pret =
    0.0664
    0.1788
```

This result indicates that efficient portfolios have returns that range from 6.5\% to $17.8 \%$. Note, your results for these examples may be different due to the random generation of scenarios.

If you have an initial portfolio, estimateFrontierByReturn also returns purchases and sales to get from your initial portfolio to the target portfolios on the efficient frontier. For example, given an initial portfolio in pwgt0, to obtain purchases and sales with target returns of $7 \%, 10 \%$, and $12 \%$ :

```
pwgt0 = [ 0.3; 0.3; 0.2; 0.1 ];
p = setInitPort(p, pwgt0);
[pwgt, pbuy, psell] = estimateFrontierByReturn(p, [0.07, 0.10, .12]);
display(pwgt);
display(pbuy);
display(psell);
pwgt =
\begin{tabular}{lll}
0.7526 & 0.3773 & 0.1306 \\
0.1047 & 0.3079 & 0.4348 \\
0.0662 & 0.1097 & 0.1426 \\
0.0765 & 0.2051 & 0.2920
\end{tabular}
pbuy =
    0.4526 0.0773 0
        0 0.0079 0.1348
        0 0 0
        0.1051 0.1920
```


## psell =

| 0 | 0 | 0.1694 |
| ---: | ---: | ---: |
| 0.1953 | 0 | 0 |
| 0.1338 | 0.0903 | 0.0574 |
| 0.0235 | 0 | 0 |

If you do not have an initial portfolio, the purchase and sale weights assume that your initial portfolio is 0 .

## See Also

PortfolioCVaR|estimateFrontier|estimateFrontierByReturn| estimateFrontierByRisk|estimateFrontierByRisk| estimateFrontierLimits|estimatePortReturn|estimatePortRisk| setSolver

## Related Examples

- "Estimate Efficient Portfolios for Entire Frontier for PortfolioCVaR Object" on page 5-101
- "Creating the PortfolioCVaR Object" on page 5-27
- "Working with CVaR Portfolio Constraints Using Defaults" on page 5-63
- "Estimate Efficient Frontiers for PortfolioCVaR Object" on page 5-118
- "Asset Returns and Scenarios Using PortfolioCVaR Object" on page 5-44
- "Troubleshooting CVaR Portfolio Optimization Results" on page 5-136


## More About

- "PortfolioCVaR Object" on page 5-22
- "Portfolio Optimization Theory" on page 5-3
- "PortfolioCVaR Object Workflow" on page 5-20


## External Websites

- Analyzing Investment Strategies with CVaR Portfolio Optimization in MATLAB (50 $\min 42 \mathrm{sec}$ )


## Obtaining Efficient Portfolios for Target Risks

To obtain efficient portfolios that have targeted portfolio risks, the estimateFrontierByRisk function accepts one or more target portfolio risks and obtains efficient portfolios with the specified risks. Suppose that you have a universe of four assets where you want to obtain efficient portfolios with target portfolio risks of 12\%, $14 \%$, and $16 \%$.

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
AssetScenarios = mvnrnd(m, C, 20000);
p = PortfolioCVaR;
p = setScenarios(p, AssetScenarios);
p = setDefaultConstraints(p);
p = setProbabilityLevel(p, 0.9);
pwgt = estimateFrontierByRisk(p, [0.12, 0.14, 0.16]);
display(pwgt);
pwgt =
\begin{tabular}{lll}
0.3594 & 0.2524 & 0.1543 \\
0.3164 & 0.3721 & 0.4248 \\
0.1044 & 0.1193 & 0.1298 \\
0.2199 & 0.2563 & 0.2910
\end{tabular}
```

Sometimes, you can request a risk for which no efficient portfolio exists. Based on the previous example, suppose that you want a portfolio with $6 \%$ risk (individual assets in this universe have risks ranging from $7 \%$ to $42.5 \%$ ). It turns out that a portfolio with $6 \%$ risk cannot be formed with these four assets. estimateFrontierByRisk warns if your target risks are outside the range of efficient portfolio risks and replaces it with the endpoint of the efficient frontier closest to your target risk:

```
pwgt = estimateFrontierByRisk(p, 0.06)
```

```
Warning: One or more target risk values are outside the feasible range [
0.0735749, 0.436667 ].
    Will return portfolios associated with endpoints of the range for these values.
```

```
> In PortfolioCVaR.estimateFrontierByRisk at 80
pwgt =
    0.7899
    0.0856
    0.0545
    0.0700
```

The best way to avoid this situation is to bracket your target portfolio risks with estimateFrontierLimits and estimatePortRisk (see "Obtaining Endpoints of the Efficient Frontier" on page 5-105 and "Obtaining CVaR Portfolio Risks and Returns" on page 5-118).

```
prsk = estimatePortRisk(p, p.estimateFrontierLimits);
display(prsk);
prsk =
    0.0736
    0.4367
```

This result indicates that efficient portfolios have risks that range from 7\% to 42.5\%. Note, your results for these examples may be different due to the random generation of scenarios.

Starting with an initial portfolio, estimateFrontierByRisk also returns purchases and sales to get from your initial portfolio to the target portfolios on the efficient frontier. For example, given an initial portfolio in pwgt0, you can obtain purchases and sales from the example with target risks of $12 \%, 14 \%$, and $16 \%$ :

```
pwgt0 = [ 0.3; 0.3; 0.2; 0.1 ];
p = setInitPort(p, pwgt0);
[pwgt, pbuy, psell] = estimateFrontierByRisk(p, [0.12, 0.14, 0.16]);
display(pwgt);
display(pbuy);
display(psell);
pwgt =
    0.3594 0.2524 0.1543
    0.3164 0.3721 0.4248
    0.1044 0.1193 0.1298
    0.2199 0.2563 0.2910
```

pbuy =

| 0.0594 | 0 | 0 |
| ---: | ---: | ---: |
| 0.0164 | 0.0721 | 0.1248 |
| 0 | 0 | 0 |
| 0.1199 | 0.1563 | 0.1910 |

psell =

| 0 | 0.0476 | 0.1457 |
| ---: | ---: | ---: |
| 0 | 0 | 0 |
| 0.0956 | 0.0807 | 0.0702 |
| 0 | 0 | 0 |

If you do not specify an initial portfolio, the purchase and sale weights assume that your initial portfolio is 0 .

## See Also

PortfolioCVaR|estimateFrontier|estimateFrontierByReturn| estimateFrontierByRisk|estimateFrontierByRisk| estimateFrontierLimits|estimatePortReturn|estimatePortRisk| setSolver

## Related Examples

- "Estimate Efficient Portfolios for Entire Frontier for PortfolioCVaR Object" on page 5-101
- "Creating the PortfolioCVaR Object" on page 5-27
- "Working with CVaR Portfolio Constraints Using Defaults" on page 5-63
- "Estimate Efficient Frontiers for PortfolioCVaR Object" on page 5-118
- "Asset Returns and Scenarios Using PortfolioCVaR Object" on page 5-44
- "Troubleshooting CVaR Portfolio Optimization Results" on page 5-136


## More About

- "PortfolioCVaR Object" on page 5-22
- "Portfolio Optimization Theory" on page 5-3
- "PortfolioCVaR Object Workflow" on page 5-20


## External Websites

- Analyzing Investment Strategies with CVaR Portfolio Optimization in MATLAB (50 $\min 42 \mathrm{sec}$ )


## Choosing and Controlling the Solver

When solving portfolio optimizations for a PortfolioCVaR object, all variations of fmincon from Optimization Toolbox are supported. Alternatively, you can use ' cuttingplane' , a solver that implements Kelley's cutting plane method (see Kelley [45] at "Portfolio Optimization" on page A-7).

Unlike Optimization Toolbox which uses the interior-point algorithm as the default algorithm for fmincon, the portfolio optimization for a PortfolioCVaR object uses the sqp algorithm. For details about fmincon and constrained nonlinear optimization algorithms and options, see "Constrained Nonlinear Optimization Algorithms" (Optimization Toolbox).

To modify fmincon options for CVaR portfolio optimizations, use setSolver to set the hidden properties solverType and solverOptions to specify and control the solver. (Note that you can see the default options by creating a dummyPortfolioCVaR object, using $p=$ PortfolioCVaR and then type $p$.solverOptions.) Since these solver properties are hidden, you cannot set them using the PortfolioCVaR object. The default solver is fmincon with the sqp algorithm objective function, gradients turned on, and no displayed output, so you do not need to use setSolver to specify this.

If you want to specify additional options associated with the fmincon solver, setSolver accepts these options as name-value pair arguments. For example, if you want to use fmincon with the trust-region-reflective algorithm and with no displayed output, use setSolver with:

```
p = PortfolioCVaR;
p = setSolver(p, 'fmincon', 'Algorithm', 'trust-region-reflective', 'Display', 'on');
display(p.solverOptions.Algorithm);
display(p.solverOptions.Display);
trust-region-reflective
on
```

Alternatively, setSolver accepts an optimoptions object as the second argument. For example, you can change the algorithm to trust-region-reflective with no displayed output as follows:

```
p = PortfolioCVaR;
options = optimoptions('fmincon','Algorithm', 'trust-region-reflective', 'Display', 'off');
p = setSolver(p, 'fmincon', options);
display(p.solverOptions.Algorithm);
display(p.solverOptions.Display);
trust-region-reflective
off
```

The 'cuttingplane' solver has options to control the number iterations and stopping tolerances. Moreover, this solver uses linprog as the master solver, and all linprog options are supported using optimoptions structures. All these options are set using setSolver.

For example, you can use setSolver to increase the number of iterations for 'cuttingplane':
p = PortfolioCVaR;
p = setSolver(p, 'cuttingplane', 'MaxIter', 2000);
display(p.solverType);
display(p.solverOptions);

## cuttingplane

MaxIter: 2000
AbsTol: 1.0000e-06
RelTol: 1.0000e-05
MasterSolverOptions: [1x1 optim.options.Linprog]
To change the master solver algorithm to 'interior-point', with no display, use setSolver to modify 'MasterSolverOptions':

```
p = PortfolioCVaR;
options = optimoptions('linprog','Algorithm','interior-point','Display','off');
p = setSolver(p,'cuttingplane','MasterSolverOptions',options);
display(p.solverType)
display(p.solverOptions)
display(p.solverOptions.MasterSolverOptions.Algorithm)
display(p.solverOptions.MasterSolverOptions.Display)
```

cuttingplane
MaxIter: 1000
AbsTol: 1.0000e-06
RelTol: 1.0000e-05
MasterSolverOptions: [1x1 optim.options.Linprog]
interior-point
off

## See Also

PortfolioCVaR|estimateFrontier|estimateFrontierByReturn| estimateFrontierByRisk|estimateFrontierByRisk| estimateFrontierLimits|estimatePortReturn|estimatePortRisk| setSolver

## Related Examples

- "Estimate Efficient Portfolios for Entire Frontier for PortfolioCVaR Object" on page 5-101
- "Creating the PortfolioCVaR Object" on page 5-27
- "Working with CVaR Portfolio Constraints Using Defaults" on page 5-63
- "Estimate Efficient Frontiers for PortfolioCVaR Object" on page 5-118
- "Asset Returns and Scenarios Using PortfolioCVaR Object" on page 5-44
- "Troubleshooting CVaR Portfolio Optimization Results" on page 5-136


## More About

- "PortfolioCVaR Object" on page 5-22
- "Portfolio Optimization Theory" on page 5-3
- "PortfolioCVaR Object Workflow" on page 5-20


## External Websites

- Analyzing Investment Strategies with CVaR Portfolio Optimization in MATLAB (50 $\min 42 \mathrm{sec}$ )


# Estimate Efficient Frontiers for PortfolioCVaR Object 

In this section...<br>"Obtaining CVaR Portfolio Risks and Returns" on page 5-118<br>"Obtaining Portfolio Standard Deviation and VaR" on page 5-120

Whereas "Estimate Efficient Portfolios for Entire Frontier for PortfolioCVaR Object" on page 5-101 focused on estimation of efficient portfolios, this section focuses on the estimation of efficient frontiers. For information on the workflow when using PortfolioCVaR objects, see "PortfolioCVaR Object Workflow" on page 5-20.

## Obtaining CVaR Portfolio Risks and Returns

Given any portfolio and, in particular, efficient portfolios, the functions estimatePortReturn and estimatePortRisk provide estimates for the return (or return proxy), risk (or the risk proxy). Each function has the same input syntax but with different combinations of outputs. Suppose that you have this following portfolio optimization problem that gave you a collection of portfolios along the efficient frontier in pwgt:

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
0.00408 0.0289 0.0204 0.0119;
0.00192 0.0204 0.0576 0.0336;
0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;
AssetScenarios = mvnrnd(m, C, 20000);
p = PortfolioCVaR;
p = setScenarios(p, AssetScenarios);
p = setDefaultConstraints(p);
p = setProbabilityLevel(p, 0.95);
pwgt0 = [ 0.3; 0.3; 0.2; 0.1 ];
p = setInitPort(p, pwgt0);
pwgt = estimateFrontier(p);
```

Note Remember that the risk proxy for CVaR portfolio optimization is CVaR.

Given pwgt0 and pwgt, use the portfolio risk and return estimation functions to obtain risks and returns for your initial portfolio and the portfolios on the efficient frontier:

```
prsk0 = estimatePortRisk(p, pwgt0);
pret0 = estimatePortReturn(p, pwgt0);
prsk = estimatePortRisk(p, pwgt);
pret = estimatePortReturn(p, pwgt);
```

You obtain these risks and returns:

```
display(prsk0);
```

display(pret0);
display(prsk);
display(pret);
prsk0 =
0.0591
pret0 =
0.0067
prsk =
0.0414
0.0453
0.0553
0.0689
0.0843
0. 1006
0.1193
0.1426
0.1689
0.1969
pret =
0.0050
0.0060
0.0070
0.0080
0.0089
0.0099
0.0109
0.0119
0.0129
0.0139

## Obtaining Portfolio Standard Deviation and VaR

The PortfolioCVaR object has functions to compute standard deviations of portfolio returns and the value-at-risk of portfolios with the functions estimatePortStd and estimatePortVaR. These functions work with any portfolios, not necessarily efficient portfolios. For example, the following example obtains five portfolios (pwgt) on the efficient frontier and also has an initial portfolio in pwgt0. Various portfolio statistics are computed that include the return, risk, standard deviation, and value-at-risk. The listed estimates are for the initial portfolio in the first row followed by estimates for each of the five efficient portfolios in subsequent rows.

```
m = [ 0.0042; 0.0083; 0.01; 0.15 ];
C = [ 0.005333 0.00034 0.00016 0;
0.00034 0.002408 0.0017 0.000992;
0.00016 0.0017 0.0048 0.0028;
0 0.000992 0.0028 0.010208 ];
pwgt0 = [ 0.3; 0.3; 0.2; 0.1 ];
p = PortfolioCVaR('initport', pwgt0);
p = simulateNormalScenariosByMoments(p, m, C, 20000);
p = setDefaultConstraints(p);
p = setProbabilityLevel(p, 0.9);
pwgt = estimateFrontier(p, 5);
pret = estimatePortReturn(p, [pwgt0, pwgt]);
prsk = estimatePortRisk(p, [pwgt0, pwgt]);
pstd = estimatePortStd(p, [pwgt0, pwgt]);
pvar = estimatePortVaR(p, [pwgt0, pwgt]);
[pret, prsk, pstd, pvar]
```

ans =

| 0.0207 | 0.0464 | 0.0381 | 0.0283 |
| :--- | ---: | ---: | ---: |
| 0.1009 | 0.0214 | 0.0699 | -0.0109 |
| 0.1133 | 0.0217 | 0.0772 | -0.0137 |
| 0.1256 | 0.0226 | 0.0849 | -0.0164 |
| 0.1380 | 0.0240 | 0.0928 | -0.0182 |
| 0.1503 | 0.0262 | 0.1011 | -0.0197 |

## See Also

PortfolioCVaR|estimatePortReturn|estimatePortStd|estimatePortVaR| plotFrontier

## Related Examples

- "Plotting the Efficient Frontier for a PortfolioCVaR Object" on page 5-122
- "Creating the PortfolioCVaR Object" on page 5-27
- "Working with CVaR Portfolio Constraints Using Defaults" on page 5-63
- "Asset Returns and Scenarios Using PortfolioCVaR Object" on page 5-44
- "Estimate Efficient Portfolios for Entire Frontier for PortfolioCVaR Object" on page 5-101
- "Postprocessing Results to Set Up Tradable Portfolios" on page 5-129


## More About

- "PortfolioCVaR Object" on page 5-22
- "Portfolio Optimization Theory" on page 5-3
- "PortfolioCVaR Object Workflow" on page 5-20


## External Websites

- Analyzing Investment Strategies with CVaR Portfolio Optimization in MATLAB (50 $\min 42 \mathrm{sec}$ )


## Plotting the Efficient Frontier for a PortfolioCVaR Object

The plotFrontier function creates a plot of the efficient frontier for a given portfolio optimization problem. This function accepts several types of inputs and generates a plot with an optional possibility to output the estimates for portfolio risks and returns along the efficient frontier. plotFrontier has four different ways that it can be used. In addition to a plot of the efficient frontier, if you have an initial portfolio in the InitPort property, plotFrontier also displays the return versus risk of the initial portfolio on the same plot. If you have a well-posed portfolio optimization problem set up in a PortfolioCVaR object and you use plotFrontier, you get a plot of the efficient frontier with the default number of portfolios on the frontier (the default number is currently 10 and is maintained in the hidden property defaultNumPorts). This example illustrates a typical use of plotFrontier to create a new plot:

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;
AssetScenarios = mvnrnd(m, C, 20000);
p = PortfolioCVaR;
p = setScenarios(p, AssetScenarios);
p = setDefaultConstraints(p);
p = setProbabilityLevel(p, 0.95);
plotFrontier(p);
```



The Name property appears as the title of the efficient frontier plot if you set it in the PortfolioCVaR object. Without an explicit name, the title on the plot would be "Efficient Frontier." If you want to obtain a specific number of portfolios along the efficient frontier, use plotFrontier with the number of portfolios that you want. Suppose that you have the PortfolioCVaR object from the previous example and you want to plot 20 portfolios along the efficient frontier and to obtain 20 risk and return values for each portfolio:

```
[prsk, pret] = plotFrontier(p, 20);
display([pret, prsk]);
ans =
\begin{tabular}{ll}
0.0051 & 0.0406 \\
0.0056 & 0.0414 \\
0.0061 & 0.0437 \\
0.0066 & 0.0471 \\
0.0071 & 0.0515 \\
0.0076 & 0.0567 \\
0.0082 & 0.0624 \\
0.0087 & 0.0687
\end{tabular}
```

| 0.0092 | 0.0753 |
| :--- | :--- |
| 0.0097 | 0.0821 |
| 0.0102 | 0.0891 |
| 0.0107 | 0.0962 |
| 0.0112 | 0.1044 |
| 0.0117 | 0.1142 |
| 0.0122 | 0.1251 |
| 0.0127 | 0.1369 |
| 0.0133 | 0.1496 |
| 0.0138 | 0.1628 |
| 0.0143 | 0.1766 |
| 0.0148 | 0.1907 |



## Plotting Existing Efficient Portfolios

If you already have efficient portfolios from any of the "estimateFrontier" functions (see "Estimate Efficient Portfolios for Entire Frontier for PortfolioCVaR Object" on page 5101), pass them into plotFrontier directly to plot the efficient frontier:

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;
```

AssetScenarios $=$ mvnrnd(m, C, 20000);
pwgt0 = [ 0.3; 0.3; 0.2; 0.1 ];
p = PortfolioCVaR('Name', 'Asset Allocation Portfolio', 'InitPort', pwgt0);
p = setScenarios(p, AssetScenarios);
p = setDefaultConstraints(p);
p = setProbabilityLevel(p, 0.95);
pwgt = estimateFrontier(p, 20);
plotFrontier(p, pwgt);


## Plotting Existing Efficient Portfolio Risks and Returns

If you already have efficient portfolio risks and returns, you can use the interface to plotFrontier to pass them into plotFrontier to obtain a plot of the efficient frontier:

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
AssetScenarios = mvnrnd(m, C, 20000);
pwgt0 = [ 0.3; 0.3; 0.2; 0.1 ];
p = PortfolioCVaR('Name', 'Asset Allocation Portfolio', 'InitPort', pwgt0);
p = setScenarios(p, AssetScenarios);
p = setDefaultConstraints(p);
p = setProbabilityLevel(p, 0.95);
pwgt = estimateFrontier(p);
pret= estimatePortReturn(p, pwgt);
prsk = estimatePortRisk(p, pwgt);
plotFrontier(p, prsk, pret);
```



## See Also

PortfolioCVaR|estimatePortReturn|estimatePortStd|estimatePortVaR| plotFrontier

## Related Examples

- "Estimate Efficient Frontiers for PortfolioCVaR Object" on page 5-118
- "Creating the PortfolioCVaR Object" on page 5-27
- "Working with CVaR Portfolio Constraints Using Defaults" on page 5-63
- "Asset Returns and Scenarios Using PortfolioCVaR Object" on page 5-44
- "Estimate Efficient Portfolios for Entire Frontier for PortfolioCVaR Object" on page 5-101
- "Postprocessing Results to Set Up Tradable Portfolios" on page 5-129


## More About

- "PortfolioCVaR Object" on page 5-22
- "Portfolio Optimization Theory" on page 5-3
- "PortfolioCVaR Object Workflow" on page 5-20


## External Websites

- Analyzing Investment Strategies with CVaR Portfolio Optimization in MATLAB (50 $\min 42 \mathrm{sec}$ )


## Postprocessing Results to Set Up Tradable Portfolios

After obtaining efficient portfolios or estimates for expected portfolio risks and returns, use your results to set up trades to move toward an efficient portfolio. For information on the workflow when using PortfolioCVaR objects, see "PortfolioCVaR Object Workflow" on page 5-20.

## Setting Up Tradable Portfolios

Suppose that you set up a portfolio optimization problem and obtained portfolios on the efficient frontier. Use the dataset object from Statistics and Machine Learning Toolbox to form a blotter that lists your portfolios with the names for each asset. For example, suppose that you want to obtain five portfolios along the efficient frontier. You can set up a blotter with weights multiplied by 100 to view the allocations for each portfolio:

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
0.00408 0.0289 0.0204 0.0119;
0.00192 0.0204 0.0576 0.0336;
0 0.0119 0.0336 0.1225 ];
pwgt0 = [ 0.3; 0.3; 0.2; 0.1 ];
p = PortfolioCVaR;
p = setAssetList(p, 'Bonds','Large-Cap Equities','Small-Cap Equities','Emerging Equities');
p = setInitPort(p, pwgt0);
p = simulateNormalScenariosByMoments(p, m, C, 20000);
p = setDefaultConstraints(p);
p = setProbabilityLevel(p, 0.9);
pwgt = estimateFrontier(p, 5);
pnames = cell(1,5);
for i = 1:5
pnames{i} = sprintf('Port%d',i);
end
Blotter = dataset([{100*pwgt},pnames],'obsnames',p.AssetList);
display(Blotter);
Blotter =
\begin{tabular}{llllrr} 
& Port1 & Port2 & Port3 & Port4 & Port5 \\
Bonds & 78.84 & 43.688 & 8.3448 & 0 & \(1.2501 \mathrm{e}-12\) \\
Large-Cap Equities & 9.3338 & 29.131 & 48.467 & 23.602 & \(9.4219 \mathrm{e}-13\) \\
Small-Cap Equities & 4.8843 & 8.1284 & 12.419 & 16.357 & \(8.281 \mathrm{e}-14\) \\
Emerging Equities & 6.9419 & 19.053 & 30.769 & 60.041 & 100
\end{tabular}
```

Note Your results may differ from this result due to the simulation of scenarios.

This result indicates that you would invest primarily in bonds at the minimum-risk/ minimum-return end of the efficient frontier (Port1), and that you would invest completely in emerging equity at the maximum-risk/maximum-return end of the efficient frontier (Port5). You can also select a particular efficient portfolio, for example, suppose that you want a portfolio with $15 \%$ risk and you add purchase and sale weights outputs obtained from the "estimateFrontier" functions to set up a trade blotter:

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
0.00408 0.0289 0.0204 0.0119;
0.00192 0.0204 0.0576 0.0336;
0 0.0119 0.0336 0.1225 ];
pwgt0 = [ 0.3; 0.3; 0.2; 0.1 ];
p = PortfolioCVaR;
p = setAssetList(p, 'Bonds','Large-Cap Equities','Small-Cap Equities','Emerging Equities');
p = setInitPort(p, pwgt0);
p = simulateNormalScenariosByMoments(p, m, C, 20000);
p = setDefaultConstraints(p);
p = setProbabilityLevel(p, 0.9);
[pwgt, pbuy, psell] = estimateFrontierByRisk(p, 0.15);
Blotter = dataset([{100*[pwgt0, pwgt, pbuy, psell]}, ...
{'Initial','Weight', 'Purchases','Sales'}],'obsnames',p.AssetList);
display(Blotter);
Blotter =
\begin{tabular}{lllcr} 
& Initial & Weight & Purchases & Sales \\
Bonds & 30 & 15.036 & 0 & 14.964 \\
Large-Cap Equities & 30 & 45.357 & 15.357 & 0 \\
Small-Cap Equities & 20 & 12.102 & 0 & 7.8982 \\
Emerging Equities & 10 & 27.505 & 17.505 & 0
\end{tabular}
```

If you have prices for each asset (in this example, they can be ETFs), add them to your blotter and then use the tools of the dataset object to obtain shares and shares to be traded.

## See Also

PortfolioCVaR|checkFeasibility|estimateScenarioMoments

## Related Examples

- "Troubleshooting CVaR Portfolio Optimization Results" on page 5-136
- "Creating the PortfolioCVaR Object" on page 5-27
- "Working with CVaR Portfolio Constraints Using Defaults" on page 5-63
- "Asset Returns and Scenarios Using PortfolioCVaR Object" on page 5-44
- "Estimate Efficient Portfolios for Entire Frontier for PortfolioCVaR Object" on page 5-101
- "Estimate Efficient Frontiers for PortfolioCVaR Object" on page 5-118


## More About

- "PortfolioCVaR Object" on page 5-22
- "Portfolio Optimization Theory" on page 5-3
- "PortfolioCVaR Object Workflow" on page 5-20


## External Websites

- Analyzing Investment Strategies with CVaR Portfolio Optimization in MATLAB (50 $\min 42 \mathrm{sec}$ )


## Working with Other Portfolio Objects

The PortfolioCVaR object is for CVaR portfolio optimization. The Portfolio object is for mean-variance portfolio optimization. Sometimes, you might want to examine portfolio optimization problems according to different combinations of return and risk proxies. A common example is that you want to do a CVaR portfolio optimization and then want to work primarily with moments of portfolio returns. Suppose that you set up a CVaR portfolio optimization problem with:

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
0.00408 0.0289 0.0204 0.0119;
0.00192 0.0204 0.0576 0.0336;
0 0.0119 0.0336 0.1225 ];
pwgt0 = [ 0.3; 0.3; 0.2; 0.1 ];
p = PortfolioCVaR;
p = setAssetList(p, 'Bonds','Large-Cap Equities','Small-Cap Equities','Emerging Equities');
p = setInitPort(p, pwgt0);
p = simulateNormalScenariosByMoments(p, m, C, 20000);
p = setDefaultConstraints(p);
p = setProbabilityLevel(p, 0.9);
```

To work with the same problem in a mean-variance framework, you can use the scenarios from the PortfolioCVaR object to set up a Portfolio object so that p contains a CVaR optimization problem and q contains a mean-variance optimization problem based on the same data.

```
q = Portfolio('AssetList', p.AssetList);
q = estimateAssetMoments(q, p.getScenarios);
q = setDefaultConstraints(q);
pwgt = estimateFrontier(p);
qwgt = estimateFrontier(q);
```

Since each object has a different risk proxy, it is not possible to compare results side by side. To obtain means and standard deviations of portfolio returns, you can use the functions associated with each object to obtain:

```
pret = estimatePortReturn(p, pwgt);
pstd = estimatePortStd(p, pwgt);
qret = estimatePortReturn(q, qwgt);
qstd = estimatePortStd(q, qwgt);
[pret, qret]
[pstd, qstd]
```

```
ans =
\begin{tabular}{ll}
0.0665 & 0.0585 \\
0.0787 & 0.0716 \\
0.0910 & 0.0848 \\
0.1033 & 0.0979 \\
0.1155 & 0.1111 \\
0.1278 & 0.1243 \\
0.1401 & 0.1374 \\
0.1523 & 0.1506 \\
0.1646 & 0.1637 \\
0.1769 & 0.1769
\end{tabular}
ans =
    0.0797 0.0774
    0.0912 0.0835
    0.1095 0.0995
    0.1317 0.1217
    0.1563 0.1472
    0.1823 0.1746
    0.2135 0.2059
    0.2534 0.2472
    0.2985 0.2951
    0.3499 0.3499
```

To produce comparable results, you can use the returns or risks from one portfolio optimization as target returns or risks for the other portfolio optimization.

```
qwgt = estimateFrontierByReturn(q, pret);
qret = estimatePortReturn(q, qwgt);
qstd = estimatePortStd(q, qwgt);
[pret, qret]
[pstd, qstd]
ans =
\begin{tabular}{ll}
0.0665 & 0.0665 \\
0.0787 & 0.0787 \\
0.0910 & 0.0910 \\
0.1033 & 0.1033 \\
0.1155 & 0.1155 \\
0.1278 & 0.1278
\end{tabular}
```

| 0.1401 | 0.1401 |
| :---: | :---: |
| 0.1523 | 0.1523 |
| 0.1646 | 0.1646 |
| 0.1769 | 0.1769 |
|  |  |
| ans $=$ |  |
|  |  |
| 0.0797 | 0.0797 |
| 0.0912 | 0.0912 |
| 0.1095 | 0.1095 |
| 0.1317 | 0.1317 |
| 0.1563 | 0.1563 |
| 0.1823 | 0.1823 |
| 0.2135 | 0.2135 |
| 0.2534 | 0.2534 |
| 0.2985 | 0.2985 |
| 0.3499 | 0.3499 |

Now it is possible to compare standard deviations of portfolio returns from either type of portfolio optimization.

## See Also

Portfolio| PortfolioCVaR

## Related Examples

- "Creating the Portfolio Object" on page 4-28
- "Creating the PortfolioCVaR Object" on page 5-27
- "Working with CVaR Portfolio Constraints Using Defaults" on page 5-63
- "Asset Returns and Scenarios Using PortfolioCVaR Object" on page 5-44
- "Estimate Efficient Portfolios for Entire Frontier for PortfolioCVaR Object" on page 5-101
- "Estimate Efficient Frontiers for PortfolioCVaR Object" on page 5-118


## More About

- "PortfolioCVaR Object" on page 5-22
- "Portfolio Optimization Theory" on page 5-3
- "PortfolioCVaR Object Workflow" on page 5-20
- "Portfolio Object Workflow" on page 4-21


## External Websites

- Analyzing Investment Strategies with CVaR Portfolio Optimization in MATLAB (50 $\min 42 \mathrm{sec}$ )


# Troubleshooting CVaR Portfolio Optimization Results 

## PortfolioCVaR Object Destroyed When Modifying

If a PortfolioCVaR object is destroyed when modifying, remember to pass an existing object into the PortfolioCVaR object if you want to modify it, otherwise it creates a new object. See "Creating the PortfolioCVaR Object" on page 5-27 for details.

## Matrix Incompatibility and "Non-Conformable" Errors

If you get matrix incompatibility or "non-conformable" errors, the representation of data in the tools follows a specific set of basic rules described in "Conventions for Representation of Data" on page 5-25.

## CVaR Portfolio Optimization Warns About "Max Iterations"

If the ' cuttingplane' solver displays the following warning:

```
Warning: Max iterations reached. Consider modifying the solver options, or using fmincon.
```

> In @PortfolioCVaR\private\cvar_cuttingplane_solver at 255
In @PortfolioCVaR\private\cvar_optim_min_risk at 85
In PortfolioCVaR.estimateFrontier at 69
this warning indicates that some of the reported efficient portfolios may not be accurate enough.

This warning is usually related to portfolios in the lower-left end of the efficient frontier. The cutting plane solver may have gotten very close to the solution, but there may be too many portfolios with very similar risks and returns in that neighborhood, and the solver runs out of iterations before reaching the desired accuracy.

To correct this problem, you can use setSolver to make any of these changes:

- Increase the maximum number of iterations ('MaxIter').
- Relax the stopping tolerances ('AbsTol' and/or 'RelTol').
- Use a different master solver algorithm ('MasterSolverOptions').
- Alternatively, you can try the 'fmincon' solver.

When the default maximum number of iterations of the 'cuttingplane' solver is reached, the solver usually needs many more iterations to reach the accuracy required by
the default stopping tolerances. You may want to combine increasing the number of iterations (e.g., multiply by 5 ) with relaxing the stopping tolerances (e.g., multiply by 10 or 100). Since the CVaR is a stochastic optimization problem, the accuracy of the solution is relative to the scenario sample, so a looser stopping tolerance may be acceptable. Keep in mind that the solution time may increase significantly when you increase the number of iterations. For example, doubling the number of iterations more than doubles the solution time. Sometimes using a different master solver (e.g., switching to 'interior-point ' if you are using the default 'simplex') can get the 'cuttingplane' solver to converge without changing the maximum number of iterations.

Alternatively, the 'fmincon' solver may be faster than the 'cuttingplane' solver for problems where cutting plane reaches the maximum number of iterations.

## CVaR Portfolio Optimization Errors with "Could Not Solve" Message

If the 'cuttingplane' solver generates the following error:

```
Error using cvar_cuttingplane_solver (line 251)
Could not solve the problem. Consider modifying the solver options, or using fmincon.
Error in cvar optim by return (line 100)
    [x,~,~, exit\overline{flag}] = cvar_cuttingplane_solver(...
Error in PortfolioCVaR/estimateFrontier (line 80)
    pwgt = cvar_optim_by_return(obj, r(2:end-1), obj.NumAssets, ...
```

this error means that the master solver failed to solve one of the master problems. The error may be due to numerical instability or other problem-specific situation.

To correct this problem, you can use setSolver to make any of these changes:

- Modify the master solver options ('MasterSolverOptions'), for example, change the algorithm ('Algorithm') or the termination tolerance ('TolFun').
- Alternatively, you can try the 'fmincon' solver.


## Missing Data Estimation Fails

If asset return data has missing or NaN values, the simulateNormalScenariosByData function with the 'missingdata' flag set to true may fail with either too many iterations or a singular covariance. To correct this problem, consider this:

- If you have asset return data with no missing or NaN values, you can compute a covariance matrix that may be singular without difficulties. If you have missing or NaN
values in your data, the supported missing data feature requires that your covariance matrix must be positive-definite, that is, nonsingular.
- simulateNormalScenariosByData uses default settings for the missing data estimation procedure that might not be appropriate for all problems.

In either case, you might want to estimate the moments of asset returns separately with either the ECM estimation functions such as ecmnmle or with your own functions.

## cvar_optim_transform Errors

If you obtain optimization errors such as:

```
Error using cvar_optim_transform (line 276)
Portfolio set appears to be either empty or unbounded. Check constraints.
Error in PortfolioCVaR/estimateFrontier (line 64)
    [AI, bI, AE, bE, lB, uB, f0, f, x0] = cvar_optim_transform(obj);
Or
Error using cvar_optim_transform (line 281)
Cannot obtain fin̄ite lōwer bounds for specified portfolio set.
Error in PortfolioCVaR/estimateFrontier (line 64)
    [AI, bI, AE, bE, lB, uB, f0, f, x0] = cvar_optim_transform(obj);
```

Since the portfolio optimization tools require a bounded portfolio set, these errors (and similar errors) can occur if your portfolio set is either empty and, if nonempty, unbounded. Specifically, the portfolio optimization algorithm requires that your portfolio set have at least a finite lower bound. The best way to deal with these problems is to use the validation functions in "Validate the CVaR Portfolio Problem" on page 5-96.
Specifically, use estimateBounds to examine your portfolio set, and use checkFeasibility to ensure that your initial portfolio is either feasible and, if infeasible, that you have sufficient turnover to get from your initial portfolio to the portfolio set.

Tip To correct this problem, try solving your problem with larger values for turnover and gradually reduce to the value that you want.

## Efficient Portfolios Do Not Make Sense

If you obtain efficient portfolios that, do not seem to make sense, this can happen if you forget to set specific constraints or you set incorrect constraints. For example, if you
allow portfolio weights to fall between 0 and 1 and do not set a budget constraint, you can get portfolios that are $100 \%$ invested in every asset. Although it may be hard to detect, the best thing to do is to review the constraints you have set with display of the PortfolioCVaR object. If you get portfolios with $100 \%$ invested in each asset, you can review the display of your object and quickly see that no budget constraint is set. Also, you can use estimateBounds and checkFeasibility to determine if the bounds for your portfolio set make sense and to determine if the portfolios you obtained are feasible relative to an independent formulation of your portfolio set.

## See Also

PortfolioCVaR|checkFeasibility|estimateScenarioMoments

## Related Examples

- "Postprocessing Results to Set Up Tradable Portfolios" on page 5-129
- "Creating the PortfolioCVaR Object" on page 5-27
- "Working with CVaR Portfolio Constraints Using Defaults" on page 5-63
- "Asset Returns and Scenarios Using PortfolioCVaR Object" on page 5-44
- "Estimate Efficient Portfolios for Entire Frontier for PortfolioCVaR Object" on page 5-101
- "Estimate Efficient Frontiers for PortfolioCVaR Object" on page 5-118


## More About

- "PortfolioCVaR Object" on page 5-22
- "Portfolio Optimization Theory" on page 5-3
- "PortfolioCVaR Object Workflow" on page 5-20


## External Websites

- Analyzing Investment Strategies with CVaR Portfolio Optimization in MATLAB (50 $\min 42 \mathrm{sec}$ )


## MAD Portfolio Optimization Tools

- "Portfolio Optimization Theory" on page 6-3
- "Portfolio Set for Optimization Using PortfolioMAD Object" on page 6-10
- "Default Portfolio Problem" on page 6-18
- "PortfolioMAD Object Workflow" on page 6-19
- "PortfolioMAD Object" on page 6-21
- "Creating the PortfolioMAD Object" on page 6-26
- "Common Operations on the PortfolioMAD Object" on page 6-34
- "Setting Up an Initial or Current Portfolio" on page 6-39
- "Asset Returns and Scenarios Using PortfolioMAD Object" on page 6-42
- "Working with a Riskless Asset" on page 6-54
- "Working with Transaction Costs" on page 6-56
- "Working with MAD Portfolio Constraints Using Defaults" on page 6-61
- "Working with Bound Constraints Using PortfolioMAD Object" on page 6-66
- "Working with Budget Constraints Using PortfolioMAD Object" on page 6-69
- "Working with Group Constraints Using PortfolioMAD Object" on page 6-71
- "Working with Group Ratio Constraints Using PortfolioMAD Object" on page 6-75
- "Working with Linear Equality Constraints Using PortfolioMAD Object" on page 6-79
- "Working with Linear Inequality Constraints Using PortfolioMAD Object" on page 6-82
- "Working with Average Turnover Constraints Using PortfolioMAD Object" on page 6-85
- "Working with One-Way Turnover Constraints Using PortfolioMAD Object" on page 6-88
- "Validate the MAD Portfolio Problem" on page 6-91
- "Estimate Efficient Portfolios Along the Entire Frontier for PortfolioMAD Object" on page 6-96
- "Obtaining Endpoints of the Efficient Frontier" on page 6-100
- "Obtaining Efficient Portfolios for Target Returns" on page 6-103
- "Obtaining Efficient Portfolios for Target Risks" on page 6-106
- "Choosing and Controlling the Solver" on page 6-109
- "Estimate Efficient Frontiers for PortfolioMAD Object" on page 6-111
- "Plotting the Efficient Frontier for a PortfolioMAD Object" on page 6-115
- "Postprocessing Results to Set Up Tradable Portfolios" on page 6-122
- "Working with Other Portfolio Objects" on page 6-125
- "Troubleshooting MAD Portfolio Optimization Results" on page 6-129


## Portfolio Optimization Theory

```
In this section...
"Portfolio Optimization Problems" on page 6-3
"Portfolio Problem Specification" on page 6-3
"Return Proxy" on page 6-4
"Risk Proxy" on page 6-6
```


## Portfolio Optimization Problems

Portfolio optimization problems involve identifying portfolios that satisfy three criteria:

- Minimize a proxy for risk.
- Match or exceed a proxy for return.
- Satisfy basic feasibility requirements.

Portfolios are points from a feasible set of assets that constitute an asset universe. A portfolio specifies either holdings or weights in each individual asset in the asset universe. The convention is to specify portfolios in terms of weights, although the portfolio optimization tools work with holdings as well.

The set of feasible portfolios is necessarily a nonempty, closed, and bounded set. The proxy for risk is a function that characterizes either the variability or losses associated with portfolio choices. The proxy for return is a function that characterizes either the gross or net benefits associated with portfolio choices. The terms "risk" and "risk proxy" and "return" and "return proxy" are interchangeable. The fundamental insight of Markowitz (see "Portfolio Optimization" on page A-7) is that the goal of the portfolio choice problem is to seek minimum risk for a given level of return and to seek maximum return for a given level of risk. Portfolios satisfying these criteria are efficient portfolios and the graph of the risks and returns of these portfolios forms a curve called the efficient frontier.

## Portfolio Problem Specification

To specify a portfolio optimization problem, you need the following:

- Proxy for portfolio return ( $\mu$ )
- Proxy for portfolio risk ( $\Sigma$ )
- Set of feasible portfolios (X), called a portfolio set

Financial Toolbox has three objects to solve specific types of portfolio optimization problems:

- The Portfolio object (Portfolio) supports mean-variance portfolio optimization (see Markowitz [46], [47] at "Portfolio Optimization" on page A-7). This object has either gross or net portfolio returns as the return proxy, the variance of portfolio returns as the risk proxy, and a portfolio set that is any combination of the specified constraints to form a portfolio set.
- The PortfolioCVaR object (PortfolioCVaR) implements what is known as conditional value-at-risk portfolio optimization (see Rockafellar and Uryasev [48], [49] at "Portfolio Optimization" on page A-7), which is generally referred to as CVaR portfolio optimization. CVaR portfolio optimization works with the same return proxies and portfolio sets as mean-variance portfolio optimization but uses conditional value-at-risk of portfolio returns as the risk proxy.
- The PortfolioMAD object (PortfolioMAD) implements what is known as meanabsolute deviation portfolio optimization (see Konno and Yamazaki [50] at "Portfolio Optimization" on page A-7), which is generally referred to as MAD portfolio optimization. MAD portfolio optimization works with the same return proxies and portfolio sets as mean-variance portfolio optimization but uses mean-absolute deviation portfolio returns as the risk proxy.


## Return Proxy

The proxy for portfolio return is a function $\mu: X \rightarrow R$ on a portfolio set $X \subset R^{n}$ that characterizes the rewards associated with portfolio choices. Usually, the proxy for portfolio return has two general forms, gross and net portfolio returns. Both portfolio return forms separate the risk-free rate $r_{0}$ so that the portfolio $x \in X$ contains only risky assets.

Regardless of the underlying distribution of asset returns, a collection of $S$ asset returns $y_{1}, \ldots, y_{S}$ has a mean of asset returns

$$
m=\frac{1}{S} \sum_{s=1}^{S} y_{s}
$$

and (sample) covariance of asset returns

$$
C=\frac{1}{S-1} \sum_{s=1}^{S}\left(y_{s}-m\right)\left(y_{s}-m\right)^{T} .
$$

These moments (or alternative estimators that characterize these moments) are used directly in mean-variance portfolio optimization to form proxies for portfolio risk and return.

## Gross Portfolio Returns

The gross portfolio return for a portfolio $x \in X$ is

$$
\mu(x)=r_{0}+\left(m-r_{0} 1\right)^{T} x,
$$

where:
$r_{0}$ is the risk-free rate (scalar).
$m$ is the mean of asset returns ( $n$ vector).
If the portfolio weights sum to 1 , the risk-free rate is irrelevant. The properties in the Portfolio object to specify gross portfolio returns are:

- RiskFreeRate for $r_{0}$
- AssetMean for $m$


## Net Portfolio Returns

The net portfolio return for a portfolio $x \in X$ is

$$
\mu(x)=r_{0}+\left(m-r_{0} 1\right)^{T} x-b^{T} \max \left\{0, x-x_{0}\right\}-s^{T} \max \left\{0, x_{0}-x\right\},
$$

where:
$r_{0}$ is the risk-free rate (scalar).
$m$ is the mean of asset returns ( $n$ vector).
$b$ is the proportional cost to purchase assets ( $n$ vector).
$s$ is the proportional cost to sell assets ( $n$ vector).
You can incorporate fixed transaction costs in this model also. Though in this case, it is necessary to incorporate prices into such costs. The properties in the Portfolio object to specify net portfolio returns are:

- RiskFreeRate for $r_{0}$
- AssetMean for $m$
- InitPort for $x_{0}$
- BuyCost for $b$
- SellCost for $s$


## Risk Proxy

The proxy for portfolio risk is a function $\Sigma: X \rightarrow R$ on a portfolio set $X \subset R^{n}$ that characterizes the risks associated with portfolio choices.

## Variance

The variance of portfolio returns for a portfolio $x \in X$ is

$$
\Sigma(x)=x^{T} C x
$$

where $C$ is the covariance of asset returns ( $n$-by-n positive-semidefinite matrix).
The property in the Portfolio object to specify the variance of portfolio returns is AssetCovar for $C$.

Although the risk proxy in mean-variance portfolio optimization is the variance of portfolio returns, the square root, which is the standard deviation of portfolio returns, is often reported and displayed. Moreover, this quantity is often called the "risk" of the portfolio. For details, see Markowitz [46], [47] at ("Portfolio Optimization" on page A7).

## Conditional Value-at-Risk

The conditional value-at-risk for a portfolio $x \in X$, which is also known as expected shortfall, is defined as

$$
C \operatorname{VaR}_{\alpha}(x)=\frac{1}{1-\alpha} \int_{f(x, y) \geq V a R_{\alpha}(x)} f(x, y) p(y) d y
$$

where:
$\alpha$ is the probability level such that $0<\alpha<1$.
$f(x, y)$ is the loss function for a portfolio $x$ and asset return $y$.
$p(y)$ is the probability density function for asset return $y$.
$V a R_{\alpha}$ is the value-at-risk of portfolio $x$ at probability level $\alpha$.
The value-at-risk is defined as

$$
\operatorname{VaR}_{\alpha}(x)=\min \{\gamma: \operatorname{Pr}[f(x, Y) \leq \gamma] \geq \alpha\} .
$$

An alternative formulation for CVaR has the form:

$$
\operatorname{CVa}_{\alpha}(x)=\operatorname{VaR}_{\alpha}(x)+\frac{1}{1-\alpha} \int_{R^{n}} \max \left\{0,\left(f(x, y)-\operatorname{Va}_{\alpha}(x)\right)\right\} p(y) d y
$$

The choice for the probability level $\alpha$ is typically 0.9 or 0.95 . Choosing $\alpha$ implies that the value-at-risk $\operatorname{VaR}_{\alpha}(x)$ for portfolio $x$ is the portfolio return such that the probability of portfolio returns falling below this level is $(1-\alpha)$. Given $\operatorname{VaR}_{\alpha}(x)$ for a portfolio $x$, the conditional value-at-risk of the portfolio is the expected loss of portfolio returns above the value-at-risk return.

Note Value-at-risk is a positive value for losses so that the probability level $\alpha$ indicates the probability that portfolio returns are below the negative of the value-at-risk.

To describe the probability distribution of returns, the PortfolioCVaR object takes a finite sample of return scenarios $y_{s}$, with $s=1, \ldots, S$. Each $y_{s}$ is an $n$ vector that contains the returns for each of the $n$ assets under the scenario $s$. This sample of $S$ scenarios is stored as a scenario matrix of size $S$-by-n. Then, the risk proxy for CVaR portfolio optimization, for a given portfolio $x \in X$ and $\alpha \in(0,1)$, is computed as

$$
\sum(x)=V a R_{\alpha}(x)+\frac{1}{(1-\alpha) S} \sum_{s=1}^{S} \max \left\{0,-y_{s}^{T} x-V a R_{\alpha}(x)\right\}
$$

The value-at-risk, $\operatorname{Va}_{\alpha}(x)$, is estimated whenever the CVaR is estimated. The loss function is $f\left(x, y_{s}\right)=-y_{s}^{T} x$, which is the portfolio loss under scenario $s$.

Under this definition, VaR and CVaR are sample estimators for VaR and CVaR based on the given scenarios. Better scenario samples yield more reliable estimates of VaR and CVaR.

For more information, see Rockafellar and Uryasev [48], [49], and Cornuejols and Tütüncü, [51], at "Portfolio Optimization" on page A-7.

## Mean Absolute-Deviation

The mean-absolute deviation (MAD) for a portfolio $x \in X$ is defined as

$$
\sum(x)=\frac{1}{S} \sum_{s=1}^{S}\left|\left(y_{s}-m\right)^{T} x\right|
$$

where:
$y_{s}$ are asset returns with scenarios $s=1, \ldots S$ ( $S$ collection of $n$ vectors).
$f(x, y)$ is the loss function for a portfolio $x$ and asset return $y$.
$m$ is the mean of asset returns ( $n$ vector).
such that

$$
m=\frac{1}{S} \sum_{s=1}^{S} y_{s}
$$

For more information, see Konno and Yamazaki [50] at "Portfolio Optimization" on page A7.

## See Also

Portfolio| PortfolioCVaR|PortfolioMAD

## Related Examples

- "Creating the PortfolioMAD Object" on page 6-26
- "Working with MAD Portfolio Constraints Using Defaults" on page 6-61


## More About

- "PortfolioMAD Object" on page 6-21
- "Portfolio Set for Optimization Using PortfolioMAD Object" on page 6-10
- "Default Portfolio Problem" on page 6-18
- "PortfolioMAD Object Workflow" on page 6-19


## Portfolio Set for Optimization Using PortfolioMAD Object

The final element for a complete specification of a portfolio optimization problem is the set of feasible portfolios, which is called a portfolio set. A portfolio set $X \subset R^{n}$ is specified by construction as the intersection of sets formed by a collection of constraints on portfolio weights. A portfolio set necessarily and sufficiently must be a nonempty, closed, and bounded set.

When setting up your portfolio set, ensure that the portfolio set satisfies these conditions. The most basic or "default" portfolio set requires portfolio weights to be nonnegative (using the lower-bound constraint) and to sum to 1 (using the budget constraint). The most general portfolio set handled by the portfolio optimization tools can have any of these constraints:

- Linear inequality constraints
- Linear equality constraints
- Bound constraints
- Budget constraints
- Group constraints
- Group ratio constraints
- Average turnover constraints
- One-way turnover constraints


## Linear Inequality Constraints

Linear inequality constraints are general linear constraints that model relationships among portfolio weights that satisfy a system of inequalities. Linear inequality constraints take the form

$$
A_{I} x \leq b_{I}
$$

where:
$x$ is the portfolio ( $n$ vector).
$A_{I}$ is the linear inequality constraint matrix ( $n_{I}$-by- $n$ matrix).
$b_{I}$ is the linear inequality constraint vector ( $n_{I}$ vector).
$n$ is the number of assets in the universe and $n_{I}$ is the number of constraints.
Portfolio object properties to specify linear inequality constraints are:

- AInequality for $A_{I}$
- bInequality for $b_{I}$
- NumAssets for $n$

The default is to ignore these constraints.

## Linear Equality Constraints

Linear equality constraints are general linear constraints that model relationships among portfolio weights that satisfy a system of equalities. Linear equality constraints take the form

$$
A_{E} x=b_{E}
$$

where:
$x$ is the portfolio ( $n$ vector).
$A_{E}$ is the linear equality constraint matrix ( $n_{E}$-by- $n$ matrix).
$b_{E}$ is the linear equality constraint vector ( $n_{E}$ vector).
$n$ is the number of assets in the universe and $n_{E}$ is the number of constraints.
Portfolio object properties to specify linear equality constraints are:

- AEquality for $A_{E}$
- bEquality for $b_{E}$
- NumAssets for $n$

The default is to ignore these constraints.

## Bound Constraints

Bound constraints are specialized linear constraints that confine portfolio weights to fall either above or below specific bounds. Since every portfolio set must be bounded, it is often a good practice, albeit not necessary, to set explicit bounds for the portfolio problem. To obtain explicit bounds for a given portfolio set, use the estimateBounds function. Bound constraints take the form

$$
l_{B} \leq x \leq u_{B}
$$

where:
$x$ is the portfolio ( $n$ vector).
$l_{B}$ is the lower-bound constraint ( $n$ vector).
$u_{B}$ is the upper-bound constraint ( $n$ vector).
$n$ is the number of assets in the universe.
Portfolio object properties to specify bound constraints are:

- LowerBound for $l_{B}$
- UpperBound for $u_{B}$
- NumAssets for $n$

The default is to ignore these constraints.
The default portfolio optimization problem (see "Default Portfolio Problem" on page 618) has $l_{B}=0$ with $u_{B}$ set implicitly through a budget constraint.

## Budget Constraints

Budget constraints are specialized linear constraints that confine the sum of portfolio weights to fall either above or below specific bounds. The constraints take the form

$$
l_{S} \leq 1^{T} x \leq u_{S}
$$

where:
$x$ is the portfolio ( $n$ vector).
1 is the vector of ones ( $n$ vector).
$l_{S}$ is the lower-bound budget constraint (scalar).
$u_{S}$ is the upper-bound budget constraint (scalar).
$n$ is the number of assets in the universe.
Portfolio object properties to specify budget constraints are:

- LowerBudget for $l_{S}$
- UpperBudget for $u_{S}$
- NumAssets for $n$

The default is to ignore this constraint.
The default portfolio optimization problem (see "Default Portfolio Problem" on page 618) has $l_{S}=u_{S}=1$, which means that the portfolio weights sum to 1 . If the portfolio optimization problem includes possible movements in and out of cash, the budget constraint specifies how far portfolios can go into cash. For example, if $l_{S}=0$ and $u_{S}=1$, then the portfolio can have 0-100\% invested in cash. If cash is to be a portfolio choice, set RiskFreeRate ( $r_{0}$ ) to a suitable value (see "Return Proxy" on page 6-4 and "Working with a Riskless Asset" on page 6-54).

## Group Constraints

Group constraints are specialized linear constraints that enforce "membership" among groups of assets. The constraints take the form

$$
l_{G} \leq G x \leq u_{G}
$$

where:
$x$ is the portfolio ( $n$ vector).
$l_{G}$ is the lower-bound group constraint ( $n_{G}$ vector).
$u_{G}$ is the upper-bound group constraint ( $n_{G}$ vector).
$G$ is the matrix of group membership indexes ( $n_{G}$-by- $n$ matrix).

Each row of $G$ identifies which assets belong to a group associated with that row. Each row contains either 0 s or 1 s with 1 indicating that an asset is part of the group or 0 indicating that the asset is not part of the group.

Portfolio object properties to specify group constraints are:

- GroupMatrix for $G$
- LowerGroup for $l_{G}$
- UpperGroup for $u_{G}$
- NumAssets for $n$

The default is to ignore these constraints.

## Group Ratio Constraints

Group ratio constraints are specialized linear constraints that enforce relationships among groups of assets. The constraints take the form

$$
l_{R i}\left(G_{B} x\right)_{i} \leq\left(G_{A} x\right)_{i} \leq u_{R i}\left(G_{B} x\right)_{i}
$$

for $i=1, \ldots, n_{R}$ where:
$x$ is the portfolio ( $n$ vector).
$l_{R}$ is the vector of lower-bound group ratio constraints ( $n_{R}$ vector).
$u_{R}$ is the vector matrix of upper-bound group ratio constraints ( $n_{R}$ vector).
$G_{A}$ is the matrix of base group membership indexes ( $n_{R}$-by- $n$ matrix).
$G_{B}$ is the matrix of comparison group membership indexes ( $n_{R}$-by- $n$ matrix).
$n$ is the number of assets in the universe and $n_{R}$ is the number of constraints.
Each row of $G_{A}$ and $G_{B}$ identifies which assets belong to a base and comparison group associated with that row.

Each row contains either 0 s or 1 s with 1 indicating that an asset is part of the group or 0 indicating that the asset is not part of the group.

Portfolio object properties to specify group ratio constraints are:

- GroupA for $G_{A}$
- GroupB for $G_{B}$
- LowerRatio for $l_{R}$
- UpperRatio for $u_{R}$
- NumAssets for $n$

The default is to ignore these constraints.

## Average Turnover Constraints

Turnover constraint is a linear absolute value constraint that ensures estimated optimal portfolios differ from an initial portfolio by no more than a specified amount. Although portfolio turnover is defined in many ways, the turnover constraints implemented in Financial Toolbox compute portfolio turnover as the average of purchases and sales. Average turnover constraints take the form

$$
\frac{1}{2} 1^{T}\left|x-x_{0}\right| \leq \tau
$$

where:
$x$ is the portfolio ( $n$ vector).
1 is the vector of ones ( $n$ vector).
$x_{0}$ is the initial portfolio ( $n$ vector).
$\tau$ is the upper bound for turnover (scalar).
$n$ is the number of assets in the universe.
Portfolio object properties to specify the average turnover constraint are:

- Turnover for $\tau$
- InitPort for $x_{0}$
- NumAssets for $n$

The default is to ignore this constraint.

## One-way Turnover Constraints

One-way turnover constraints ensure that estimated optimal portfolios differ from an initial portfolio by no more than specified amounts according to whether the differences are purchases or sales. The constraints take the forms

$$
\begin{aligned}
& 1^{T} \times \max \left\{0, x-x_{0}\right\} \leq \tau_{B} \\
& 1^{T} \times \max \left\{0, x_{0}-x\right\} \leq \tau_{S}
\end{aligned}
$$

where:
$x$ is the portfolio ( $n$ vector)
1 is the vector of ones ( $n$ vector).
$x_{0}$ is the Initial portfolio ( $n$ vector).
$\tau_{B}$ is the upper bound for turnover constraint on purchases (scalar).
$\tau_{S}$ is the upper bound for turnover constraint on sales (scalar).
To specify one-way turnover constraints, use the following properties in the Portfolio, PortfolioCVaR, or PotfolioMAD object:

- BuyTurnover for $\tau_{B}$
- SellTurnover for $\tau_{S}$
- InitPort for $x_{0}$

The default is to ignore this constraint.

Note The average turnover constraint (see "Working with Average Turnover Constraints Using PortfolioMAD Object" on page 6-85) with $\tau$ is not a combination of the one-way turnover constraints with $\tau=\tau_{B}=\tau_{S}$.

## See Also

Portfolio|PortfolioCVaR|PortfolioMAD

## Related Examples

- "Creating the PortfolioMAD Object" on page 6-26
- "Working with MAD Portfolio Constraints Using Defaults" on page 6-61


## More About

- "PortfolioMAD Object" on page 6-21
- "Default Portfolio Problem" on page 6-18
- "PortfolioMAD Object Workflow" on page 6-19


## Default Portfolio Problem

The default portfolio optimization problem has a risk and return proxy associated with a given problem, and a portfolio set that specifies portfolio weights to be nonnegative and to sum to 1 . The lower bound combined with the budget constraint is sufficient to ensure that the portfolio set is nonempty, closed, and bounded. The default portfolio optimization problem characterizes a long-only investor who is fully invested in a collection of assets.

- For mean-variance portfolio optimization, it is sufficient to set up the default problem. After setting up the problem, data in the form of a mean and covariance of asset returns are then used to solve portfolio optimization problems.
- For conditional value-at-risk portfolio optimization, the default problem requires the additional specification of a probability level that must be set explicitly. Generally, "typical" values for this level are 0.90 or 0.95 . After setting up the problem, data in the form of scenarios of asset returns are then used to solve portfolio optimization problems.
- For MAD portfolio optimization, it is sufficient to set up the default problem. After setting up the problem, data in the form of scenarios of asset returns are then used to solve portfolio optimization problems.


## See Also

Portfolio| PortfolioCVaR| PortfolioMAD

## Related Examples

- "Creating the PortfolioMAD Object" on page 6-26
- "Working with MAD Portfolio Constraints Using Defaults" on page 6-61


## More About

- "PortfolioMAD Object" on page 6-21
- "Portfolio Set for Optimization Using PortfolioMAD Object" on page 6-10
- "PortfolioMAD Object Workflow" on page 6-19


## PortfolioMAD Object Workflow

The PortfolioMAD object workflow for creating and modeling a MAD portfolio is:
1 Create a MAD Portfolio.
Create a PortfolioMAD object for mean-absolute deviation (MAD) portfolio optimization. "Creating the PortfolioMAD Object" on page 6-26.
2 Define asset returns and scenarios.
Evaluate scenarios for portfolio asset returns, including assets with missing data and financial time series data. For more information, see "Asset Returns and Scenarios Using PortfolioMAD Object" on page 6-42.

3 Specify the MAD Portfolio Constraints.
Define the constraints for portfolio assets such as linear equality and inequality, bound, budget, group, group ratio, and turnover constraints. For more information, see "Working with MAD Portfolio Constraints Using Defaults" on page 6-61.
4 Validate the MAD Portfolio.
Identify errors for the portfolio specification. For more information, see "Validate the MAD Portfolio Problem" on page 6-91.
5 Estimate the efficient portfolios and frontiers.
Analyze the efficient portfolios and efficient frontiers for a portfolio. For more information, see "Estimate Efficient Portfolios Along the Entire Frontier for PortfolioMAD Object" on page 6-96 and "Estimate Efficient Frontiers for PortfolioMAD Object" on page 6-111.
6 Postprocess the results.
Use the efficient portfolios and efficient frontiers results to set up trades. For more information, see "Postprocessing Results to Set Up Tradable Portfolios" on page 6122.

## See Also

## More About

- "Portfolio Optimization Theory" on page 6-3


## PortfolioMAD Object

```
In this section...
"PortfolioMAD Object Properties and Functions" on page 6-21
"Working with PortfolioMAD Objects" on page 6-21
"Setting and Getting Properties" on page 6-22
"Displaying PortfolioMAD Objects" on page 6-23
"Saving and Loading PortfolioMAD Objects" on page 6-23
"Estimating Efficient Portfolios and Frontiers" on page 6-23
"Arrays of PortfolioMAD Objects" on page 6-23
"Subclassing PortfolioMAD Objects" on page 6-24
"Conventions for Representation of Data" on page 6-24
```


## PortfolioMAD Object Properties and Functions

The PortfolioMAD object implements mean absolute-deviation (MAD) portfolio optimization and is derived from the abstract class AbstractPortfolio. Every property and function of the PortfolioMAD object is public, although some properties and functions are hidden. The PortfolioMAD object is a value object where every instance of the object is a distinct version of the object. Since the PortfolioMAD object is also a MATLAB object, it inherits the default functions associated with MATLAB objects.

## Working with PortfolioMAD Objects

The PortfolioMAD object and its functions are an interface for mean absolute-deviation portfolio optimization. So, almost everything you do with the PortfolioMAD object can be done using the functions. The basic workflow is:

1 Design your portfolio problem.
2 Use PortfolioMAD to create the PortfolioMAD object or use the various set functions to set up your portfolio problem.
3 Use estimate functions to solve your portfolio problem.
In addition, functions are available to help you view intermediate results and to diagnose your computations. Since MATLAB features are part of a PortfolioMAD object, you can
save and load objects from your workspace and create and manipulate arrays of objects. After settling on a problem, which, in the case of MAD portfolio optimization, means that you have either scenarios, data, or moments for asset returns, and a collection of constraints on your portfolios, use PortfolioMAD to set the properties for the PortfolioMAD object.

PortfolioMAD lets you create an object from scratch or update an existing object. Since the PortfolioMAD object is a value object, it is easy to create a basic object, then use functions to build upon the basic object to create new versions of the basic object. This is useful to compare a basic problem with alternatives derived from the basic problem. For details, see "Creating the PortfolioMAD Object" on page 6-26.

## Setting and Getting Properties

You can set properties of a PortfolioMAD object using either PortfolioMAD or various set functions.

Note Although you can also set properties directly, it is not recommended since errorchecking is not performed when you set a property directly.

The PortfolioMAD object supports setting properties with name-value pair arguments such that each argument name is a property and each value is the value to assign to that property. For example, to set the LowerBound and Budget properties in an existing PortfolioMAD object $p$, use the syntax:
p = PortfolioMAD(p,'LowerBound', 0,'Budget',1);
In addition to the PortfolioMAD object, which lets you set individual properties one at a time, groups of properties are set in a PortfolioMAD object with various "set" and "add" functions. For example, to set up an average turnover constraint, use the setTurnover function to specify the bound on portfolio turnover and the initial portfolio. To get individual properties from a PortfolioMAD object, obtain properties directly or use an assortment of "get" functions that obtain groups of properties from a PortfolioMAD object. The PortfolioMAD object and set functions have several useful features:

- The PortfolioMAD object and set functions try to determine the dimensions of your problem with either explicit or implicit inputs.
- The PortfolioMAD object and set functions try to resolve ambiguities with default choices.
- The PortfolioMAD object and set functions perform scalar expansion on arrays when possible.
- The PortfolioMAD functions try to diagnose and warn about problems.


## Displaying PortfolioMAD Objects

The PortfolioMAD object uses the default display function provided by MATLAB, where display and disp display a PortfolioMAD object and its properties with or without the object variable name.

## Saving and Loading PortfolioMAD Objects

Save and load PortfolioMAD objects using the MATLAB save and load commands.

## Estimating Efficient Portfolios and Frontiers

Estimating efficient portfolios and efficient frontiers is the primary purpose of the MAD portfolio optimization tools. A collection of "estimate" and "plot" functions provide ways to explore the efficient frontier. The "estimate" functions obtain either efficient portfolios or risk and return proxies to form efficient frontiers. At the portfolio level, a collection of functions estimates efficient portfolios on the efficient frontier with functions to obtain efficient portfolios:

- At the endpoints of the efficient frontier
- That attain targeted values for return proxies
- That attain targeted values for risk proxies
- Along the entire efficient frontier

These functions also provide purchases and sales needed to shift from an initial or current portfolio to each efficient portfolio. At the efficient frontier level, a collection of functions plot the efficient frontier and estimate either risk or return proxies for efficient portfolios on the efficient frontier. You can use the resultant efficient portfolios or risk and return proxies in subsequent analyses.

## Arrays of PortfolioMAD Objects

Although all functions associated with a PortfolioMAD object are designed to work on a scalar PortfolioMAD object, the array capabilities of MATLAB enable you to set up and
work with arrays of PortfolioMAD objects. The easiest way to do this is with the repmat function. For example, to create a 3-by-2 array of PortfolioMAD objects:

```
p = repmat(PortfolioMAD, 3, 2);
disp(p)
```

After setting up an array of PortfolioMAD objects, you can work on individual PortfolioMAD objects in the array by indexing. For example:
p(i,j) = PortfolioMAD(p(i,j), ... );
This example calls PortfolioMAD for the ( $\mathbf{i}, \mathrm{j}$ ) element of a matrix of PortfolioMAD objects in the variable p .

If you set up an array of PortfolioMAD objects, you can access properties of a particular PortfolioMAD object in the array by indexing so that you can set the lower and upper bounds lb and ub for the ( $\mathrm{i}, \mathrm{j}, \mathrm{k}$ ) element of a 3-D array of PortfolioMAD objects with
$p(i, j, k)=\operatorname{setBounds}(p(i, j, k), l b, u b)$;
and, once set, you can access these bounds with

```
[lb, ub] = getBounds(p(i,j,k));
```

PortfolioMAD object functions work on only one PortfolioMAD object at a time.

## Subclassing PortfolioMAD Objects

You can subclass the PortfolioMAD object to override existing functions or to add new properties or functions. To do so, create a derived class from the PortfolioMAD class. This gives you all the properties and functions of the PortfolioMAD class along with any new features that you choose to add to your subclassed object. ThePortfolioMAD class is derived from an abstract class called AbstractPortfolio. Because of this, you can also create a derived class from AbstractPortfolio that implements an entirely different form of portfolio optimization using properties and functions of theAbstractPortfolio class.

## Conventions for Representation of Data

The MAD portfolio optimization tools follow these conventions regarding the representation of different quantities associated with portfolio optimization:

- Asset returns or prices for scenarios are in matrix form with samples for a given asset going down the rows and assets going across the columns. In the case of prices, the earliest dates must be at the top of the matrix, with increasing dates going down.
- Portfolios are in vector or matrix form with weights for a given portfolio going down the rows and distinct portfolios going across the columns.
- Constraints on portfolios are formed in such a way that a portfolio is a column vector.
- Portfolio risks and returns are either scalars or column vectors (for multiple portfolio risks and returns).


## See Also

PortfolioMAD

## Related Examples

- "Creating the PortfolioMAD Object" on page 6-26
- "Working with MAD Portfolio Constraints Using Defaults" on page 6-61


## More About

- "Portfolio Optimization Theory" on page 6-3
- "PortfolioMAD Object Workflow" on page 6-19


## Creating the PortfolioMAD Object

## In this section...

"Syntax" on page 6-26
"PortfolioMAD Problem Sufficiency" on page 6-27
"PortfolioMAD Function Examples" on page 6-27
To create a fully specified MAD portfolio optimization problem, instantiate the PortfolioMAD object using the PortfolioMAD object. For information on the workflow when using PortfolioMAD objects, see "PortfolioMAD Object Workflow" on page 6-19.

## Syntax

Use PortfolioMAD to create an instance of an object of the PortfolioMAD class. You can use the PortfolioMAD object in several ways. To set up a portfolio optimization problem in a PortfolioMAD object, the simplest syntax is:

```
p = PortfolioMAD;
```

This syntax creates a PortfolioMAD object, p, such that all object properties are empty.
The PortfolioMAD object also accepts collections of argument name-value pair arguments for properties and their values. The PortfolioMAD object accepts inputs for public properties with the general syntax:

```
p = PortfolioMAD('property1', value1, 'property2', value2, ... );
```

If a PortfolioMAD object already exists, the syntax permits the first (and only the first argument) of PortfolioMAD to be an existing object with subsequent argument namevalue pair arguments for properties to be added or modified. For example, given an existing PortfolioMAD object in $p$, the general syntax is:

```
p = PortfolioMAD(p, 'property1', value1, 'property2', value2, ... );
```

Input argument names are not case-sensitive, but must be completely specified. In addition, several properties can be specified with alternative argument names (see "Shortcuts for Property Names" on page 6-31). The PortfolioMAD object tries to detect problem dimensions from the inputs and, once set, subsequent inputs can undergo various scalar or matrix expansion operations that simplify the overall process to
formulate a problem. In addition, a PortfolioMAD object is a value object so that, given portfolio $p$, the following code creates two objects, $p$ and $q$, that are distinct:
$q=\operatorname{PortfolioMAD(p,~...)~}$

## PortfolioMAD Problem Sufficiency

A MAD portfolio optimization problem is completely specified with the PortfolioMAD object if the following three conditions are met:

- You must specify a collection of asset returns or prices known as scenarios such that all scenarios are finite asset returns or prices. These scenarios are meant to be samples from the underlying probability distribution of asset returns. This condition can be satisfied by the setScenarios function or with several canned scenario simulation functions.
- The set of feasible portfolios must be a nonempty compact set, where a compact set is closed and bounded. You can satisfy this condition using an extensive collection of properties that define different types of constraints to form a set of feasible portfolios. Since such sets must be bounded, either explicit or implicit constraints can be imposed and several tools, such as the estimateBounds function, provide ways to ensure that your problem is properly formulated.

Although the general sufficient conditions for MAD portfolio optimization go beyond these conditions, the PortfolioMAD object handles all these additional conditions.

## PortfolioMAD Function Examples

If you create a PortfolioMAD object, $p$, with no input arguments, you can display it using disp:
$\mathrm{p}=$ PortfolioMAD;
disp(p);
PortfolioMAD with properties:
BuyCost: []
SellCost: []
RiskFreeRate: []
Turnover: []
BuyTurnover: []
SellTurnover: []

```
NumScenarios: []
    Name: []
    NumAssets: []
    AssetList: []
        InitPort: []
AInequality: []
bInequality: []
    AEquality: []
    bEquality: []
    LowerBound: []
    UpperBound: []
LowerBudget: []
UpperBudget: []
GroupMatrix: []
    LowerGroup: []
    UpperGroup: []
        GroupA: []
        GroupB: []
    LowerRatio: []
    UpperRatio: []
```

The approaches listed provide a way to set up a portfolio optimization problem with the PortfolioMAD object. The custom set functions offer additional ways to set and modify collections of properties in the PortfolioMAD object.

## Using the PortfolioMAD Function for a Single-Step Setup

You can use the PortfolioMAD object to directly set up a "standard" portfolio optimization problem. Given scenarios of asset returns in the variable AssetScenarios, this problem is completely specified as follows:

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ]:
m = m/12;
C = C/12;
AssetScenarios = mvnrnd(m, C, 20000);
p = PortfolioMAD('Scenarios', AssetScenarios, ...
'LowerBound', 0, 'LowerBudget', 1, 'UpperBudget', 1);
```

The LowerBound property value undergoes scalar expansion since AssetScenarios provides the dimensions of the problem.

You can use dot notation with the function plotFrontier.
p.plotFrontier;


## Using the PortfolioMAD Function with a Sequence of Steps

An alternative way to accomplish the same task of setting up a "standard" MAD portfolio optimization problem, given AssetScenarios variable is:

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
0.00408 0.0289 0.0204 0.0119;
0.00192 0.0204 0.0576 0.0336;
0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;
```

AssetScenarios $=$ mvnrnd(m, C, 20000);
p = PortfolioMAD;
$p=$ setScenarios(p, AssetScenarios);
p = PortfolioMAD(p, 'LowerBound', 0);

```
p = PortfolioMAD(p, 'LowerBudget', 1, 'UpperBudget', 1);
plotFrontier(p);
```



This way works because the calls to the PortfolioMAD object are in this particular order. In this case, the call to initialize AssetScenarios provides the dimensions for the problem. If you were to do this step last, you would have to explicitly dimension the LowerBound property as follows:
m = [ 0.05; 0.1; 0.12; 0.18 ];
$C=[0.00640 .00408$ 0.00192 0;
0.004080 .02890 .0204 0.0119;
0.001920 .02040 .05760 .0336 ;
00.01190 .03360 .1225 ];
$\mathrm{m}=\mathrm{m} / 12$;
$C=C / 12 ;$
AssetScenarios = mvnrnd(m, C, 20000);

```
p = PortfolioMAD;
p = PortfolioMAD(p, 'LowerBound', zeros(size(m)));
p = PortfolioMAD(p, 'LowerBudget', 1, 'UpperBudget', 1);
p = setScenarios(p, AssetScenarios);
```

Note If you did not specify the size of LowerBound but, instead, input a scalar argument, the PortfolioMAD object assumes that you are defining a single-asset problem and produces an error at the call to set asset scenarios with four assets.

## Shortcuts for Property Names

The PortfolioMAD object has shorter argument names that replace longer argument names associated with specific properties of the PortfolioMAD object. For example, rather than enter 'AInequality', the PortfolioMAD object accepts the case-insensitive name 'ai' to set the AInequality property in a PortfolioMAD object. Every shorter argument name corresponds with a single property in the PortfolioMAD object. The one exception is the alternative argument name 'budget ', which signifies both the LowerBudget and UpperBudget properties. When 'budget' is used, then the LowerBudget and UpperBudget properties are set to the same value to form an equality budget constraint.

Shortcuts for Property Names

| Shortcut Argument Name | Equivalent Argument / Property Name |
| :--- | :--- |
| ae | AEquality |
| ai | AInequality |
| assetnames or assets | AssetList |
| be | bEquality |
| bi | bInequality |
| budget | UpperBudget and LowerBudget |
| group | GroupMatrix |
| lb | LowerBound |
| n or num | NumAssets |
| rfr | RiskFreeRate |
| scenario or <br> assetscenarios | Scenarios |
| ub | UpperBound |

For example, this call to PortfolioMAD uses these shortcuts for properties:

```
m=[0.05; 0.1; 0.12; 0.18];
C = [ 0.0064 0.00408 0.00192 0;
0.00408 0.0289 0.0204 0.0119;
0.00192 0.0204 0.0576 0.0336;
0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;
AssetScenarios = mvnrnd(m, C, 20000);
p = PortfolioMAD('scenario', AssetScenarios, 'lb', 0, 'budget', 1);
plotFrontier(p);
```


## Direct Setting of Portfolio Object Properties

Although not recommended, you can set properties directly using dot notation, however no error-checking is done on your inputs:

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
```

```
    0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;
AssetScenarios = mvnrnd(m, C, 20000);
p = PortfolioMAD;
p = setScenarios(p, AssetScenarios);
p.LowerBudget = 1;
p.UpperBudget = 1;
p.LowerBound = zeros(size(m));
plotFrontier(p);
```

Note Scenarios cannot be assigned directly using dot notation to a PortfolioMAD object. Scenarios must always be set through either the PortfolioMAD object, the setScenarios function, or any of the scenario simulation functions.

## See Also

PortfolioMAD|estimateBounds

## Related Examples

- "Common Operations on the PortfolioMAD Object" on page 6-34
- "Working with MAD Portfolio Constraints Using Defaults" on page 6-61


## More About

- "PortfolioMAD Object" on page 6-21
- "Portfolio Optimization Theory" on page 6-3
- "PortfolioMAD Object Workflow" on page 6-19


## Common Operations on the PortfolioMAD Object

```
In this section...
"Naming a PortfolioMAD Object" on page 6-34
"Configuring the Assets in the Asset Universe" on page 6-34
"Setting Up a List of Asset Identifiers" on page 6-35
"Truncating and Padding Asset Lists" on page 6-36
```


## Naming a PortfolioMAD Object

To name a PortfolioMAD object, use the Name property. Name is informational and has no effect on any portfolio calculations. If the Name property is nonempty, Name is the title for the efficient frontier plot generated by plotFrontier. For example, if you set up an asset allocation fund, you could name the PortfolioMAD object Asset Allocation Fund:

```
p = PortfolioMAD('Name','Asset Allocation Fund');
disp(p.Name);
Asset Allocation Fund
```


## Configuring the Assets in the Asset Universe

The fundamental quantity in the PortfolioMAD object is the number of assets in the asset universe. This quantity is maintained in the NumAssets property. Although you can set this property directly, it is usually derived from other properties such as the number of assets in the scenarios or the initial portfolio. In some instances, the number of assets may need to be set directly. This example shows how to set up a PortfolioMAD object that has four assets:

```
p = PortfolioMAD('NumAssets', 4);
disp(p.NumAssets);
4
```

After setting the NumAssets property, you cannot modify it (unless no other properties are set that depend on NumAssets). The only way to change the number of assets in an existing PortfolioMAD object with a known number of assets is to create a new PortfolioMAD object.

## Setting Up a List of Asset Identifiers

When working with portfolios, you must specify a universe of assets. Although you can perform a complete analysis without naming the assets in your universe, it is helpful to have an identifier associated with each asset as you create and work with portfolios. You can create a list of asset identifiers as a cell vector of character vectors in the property AssetList. You can set up the list using the next two methods.

## Setting Up Asset Lists Using the PortfolioMAD Function

Suppose that you have a PortfolioMAD object, $p$, with assets with symbols 'AA' ', 'BA', 'CAT', 'DD', and 'ETR'. You can create a list of these asset symbols in the object using PortfolioMAD:

```
p = PortfolioMAD('assetlist', { 'AA', 'BA', 'CAT', 'DD', 'ETR' });
disp(p.AssetList);
'AA' 'BA' 'CAT' 'DD' 'ETR'
```

Notice that the property AssetList is maintained as a cell array that contains character vectors, and that it is necessary to pass a cell array into PortfolioMAD to set AssetList. In addition, notice that the property NumAssets is set to 5 based on the number of symbols used to create the asset list:

```
disp(p.NumAssets);
```

5

## Setting Up Asset Lists Using the setAssetList Function

You can also specify a list of assets using the setAssetList function. Given the list of asset symbols 'AA', 'BA', 'CAT', 'DD', and 'ETR', you can use setAssetList with:

```
p = PortfolioMAD;
p = setAssetList(p, { 'AA', 'BA', 'CAT', 'DD', 'ETR' });
disp(p.AssetList);
    'AA' 'BA' 'CAT' 'DD' 'ETR'
```

setAssetList also enables you to enter symbols directly as a comma-separated list without creating a cell array of character vectors. For example, given the list of assets symbols 'AA', 'BA', 'CAT', 'DD', and 'ETR', use setAssetList:

```
p = PortfolioMAD;
p = setAssetList(p, 'AA', 'BA', 'CAT', 'DD', 'ETR');
disp(p.AssetList);
    'AA' 'BA' 'CAT' 'DD' 'ETR'
```

setAssetList has many additional features to create lists of asset identifiers. If you use setAssetList with just a PortfolioMAD object, it creates a default asset list according to the name specified in the hidden public property defaultforAssetList (which is 'Asset' by default). The number of asset names created depends on the number of assets in the property NumAssets. If NumAssets is not set, then NumAssets is assumed to be 1 .

For example, if a PortfolioMAD object $p$ is created with NumAssets $=5$, then this code fragment shows the default naming behavior:

```
p = PortfolioMAD('numassets',5);
p = setAssetList(p);
disp(p.AssetList);
'Asset1' 'Asset2' 'Asset3' 'Asset4' 'Asset5'
```

Suppose that your assets are, for example, ETFs and you change the hidden property defaultforAssetList to 'ETF', you can then create a default list for ETFs:

```
p = PortfolioMAD('numassets',5);
p.defaultforAssetList = 'ETF';
p = setAssetList(p);
disp(p.AssetList);
```

'ETF1' 'ETF2' 'ETF3' 'ETF4' 'ETF5'

## Truncating and Padding Asset Lists

If the NumAssets property is already set and you pass in too many or too few identifiers, the PortfolioMAD object, and the setAssetList function truncate or pad the list with numbered default asset names that use the name specified in the hidden public property defaultforAssetList. If the list is truncated or padded, a warning message indicates the discrepancy. For example, assume that you have a PortfolioMAD object with five ETFs and you only know the first three CUSIPs '921937835', '922908769' , and ' $922042775^{\prime}$. Use this syntax to create an asset list that pads the remaining asset identifiers with numbered 'UnknownCUSIP' placeholders:

```
p = PortfolioMAD('numassets',5);
p.defaultforAssetList = 'UnknownCUSIP';
```

```
p = setAssetList(p, '921937835', '922908769', '922042775');
disp(p.AssetList);
Warning: Input list of assets has 2 too few identifiers. Padding with numbered
assets.
> In PortfolioMAD.setAssetList at 121
    Columns 1 through 4
    '921937835' '922908769' '922042775' 'UnknownCUSIP4'
    Column 5
```

    'UnknownCUSIP5
    Alternatively, suppose that you have too many identifiers and need only the first four assets. This example illustrates truncation of the asset list using the PortfolioMAD object:

```
p = PortfolioMAD('numassets',4);
p = PortfolioMAD(p, 'assetlist', { 'AGG', 'EEM', 'MDY', 'SPY', 'VEU' });
disp(p.AssetList);
Warning: AssetList has 1 too many identifiers. Using first 4 assets.
> In PortfolioMAD.checkarguments at 410
    In PortfolioMAD.PortfolioMAD>PortfolioMAD.PortfolioMAD at 187
        'AGG' 'EEM' 'MDY' 'SPY'
```

The hidden public property uppercaseAssetList is a Boolean flag to specify whether to convert asset names to uppercase letters. The default value for uppercaseAssetList is false. This example shows how to use the uppercaseAssetList flag to force identifiers to be uppercase letters:

```
p = PortfolioMAD;
p.uppercaseAssetList = true;
p = setAssetList(p, { 'aa', 'ba', 'cat', 'dd', 'etr' });
disp(p.AssetList);
'AA' 'BA' 'CAT' 'DD' 'ETR'
```


## See Also

PortfolioMAD|checkFeasibility|estimateBounds| setAssetList| setInitPort

## Related Examples

- "Setting Up an Initial or Current Portfolio" on page 6-39
- "Working with MAD Portfolio Constraints Using Defaults" on page 6-61
- "Asset Returns and Scenarios Using PortfolioMAD Object" on page 6-42


## More About

- "PortfolioMAD Object" on page 6-21
- "Portfolio Optimization Theory" on page 6-3
- "PortfolioMAD Object Workflow" on page 6-19


## Setting Up an Initial or Current Portfolio

In many applications, creating a new optimal portfolio requires comparing the new portfolio with an initial or current portfolio to form lists of purchases and sales. The PortfolioMAD object property InitPort lets you identify an initial or current portfolio. The initial portfolio also plays an essential role if you have either transaction costs or turnover constraints. The initial portfolio need not be feasible within the constraints of the problem. This can happen if the weights in a portfolio have shifted such that some constraints become violated. To check if your initial portfolio is feasible, use thecheckFeasibility function described in "Validating MAD Portfolios" on page 6-93. Suppose that you have an initial portfolio in x0, then use the PortfolioMAD object to set up an initial portfolio:

```
x0 = [ 0.3; 0.2; 0.2; 0.0 ];
p = PortfolioMAD('InitPort', x0);
disp(p.InitPort);
    0.3000
0.2000
0.2000
0
```

As with all array properties, you can set InitPort with scalar expansion. This is helpful to set up an equally weighted initial portfolio of, for example, 10 assets:

```
p = PortfolioMAD('NumAssets', 10, 'InitPort', 1/10);
disp(p.InitPort);
```

0.1000
0.1000
0.1000
0.1000
0.1000
0.1000
0.1000
0.1000
0.1000
0.1000

To clear an initial portfolio from your PortfolioMAD object, use either the PortfolioMAD object or the setInitPort function with an empty input for the InitPort property. If transaction costs or turnover constraints are set, it is not possible to clear the InitPort property in this way. In this case, to clear InitPort, first clear the dependent properties and then clear theInitPort property.

The InitPort property can also be set with setInitPort which lets you specify the number of assets if you want to use scalar expansion. For example, given an initial portfolio in $\times 0$, use setInitPort to set the InitPort property:

```
p = PortfolioMAD;
x0 = [ 0.3; 0.2; 0.2; 0.0 ];
p = setInitPort(p, x0);
disp(p.InitPort);
0.3000
0.2000
0.2000
0
```

To create an equally weighted portfolio of four assets, use setInitPort:

```
p = PortfolioMAD;
p = setInitPort(p, 1/4, 4);
disp(p.InitPort);
    0.2500
    0.2500
    0.2500
    0.2500
```

PortfolioMAD object functions that work with either transaction costs or turnover constraints also depend on the InitPort property. So, the set functions for transaction costs or turnover constraints permit the assignment of a value for the InitPort property as part of their implementation. For details, see "Working with Average Turnover Constraints Using PortfolioMAD Object" on page 6-85, "Working with One-Way Turnover Constraints Using PortfolioMAD Object" on page 6-88, and "Working with Transaction Costs" on page 6-56. If either transaction costs or turnover constraints are used, then the InitPort property must have a nonempty value. Absent a specific value assigned through the PortfolioMAD object or various set functions, the PortfolioMAD object sets InitPort to 0 and warns if BuyCost, SellCost, or Turnover properties are set. This example shows what happens if you specify an average turnover constraint with an initial portfolio:

```
p = PortfolioMAD('Turnover', 0.3, 'InitPort', [ 0.3; 0.2; 0.2; 0.0 ]);
disp(p.InitPort);
```

0.3000
0.2000
0.2000

In contrast, this example shows what happens if an average turnover constraint is specified without an initial portfolio:

## p = PortfolioMAD('Turnover', 0.3);

disp(p.InitPort);
Warning: InitPort and NumAssets are empty and either transaction costs or
turnover constraints specified. Will set NumAssets $=1$ and InitPort $=0$.
> In PortfolioMAD.checkarguments at 446
In PortfolioMAD. PortfolioMAD>PortfolioMAD. PortfolioMAD at 190
0

## See Also

PortfolioMAD|checkFeasibility|estimateBounds|setAssetList| setInitPort

## Related Examples

- "Common Operations on the PortfolioMAD Object" on page 6-34
- "Working with MAD Portfolio Constraints Using Defaults" on page 6-61
- "Asset Returns and Scenarios Using PortfolioMAD Object" on page 6-42


## More About

- "PortfolioMAD Object" on page 6-21
- "Portfolio Optimization Theory" on page 6-3
- "PortfolioMAD Object Workflow" on page 6-19


# Asset Returns and Scenarios Using PortfolioMAD Object 

In this section...<br>"How Stochastic Optimization Works" on page 6-42<br>"What Are Scenarios?" on page 6-43<br>"Setting Scenarios Using the PortfolioMAD Function" on page 6-43<br>"Setting Scenarios Using the setScenarios Function" on page 6-45<br>"Estimating the Mean and Covariance of Scenarios" on page 6-45<br>"Simulating Normal Scenarios" on page 6-46<br>"Simulating Normal Scenarios from Returns or Prices" on page 6-46<br>"Simulating Normal Scenarios with Missing Data" on page 6-48<br>"Simulating Normal Scenarios from Time Series Data" on page 6-50<br>"Simulating Normal Scenarios for Mean and Covariance" on page 6-52

## How Stochastic Optimization Works

The MAD of a portfolio is mean-absolute deviation. For the definition of the MAD function, see "Risk Proxy" on page 6-6. Although analytic solutions for MAD exist for a few probability distributions, an alternative is to compute the expectation for MAD with samples from the probability distribution of asset returns. These samples are called scenarios and, given a collection of scenarios, the portfolio optimization problem becomes a stochastic optimization problem.

As a function of the portfolio weights, the MAD of the portfolio is a convex non-smooth function (see Konno and Yamazaki [50] at "Portfolio Optimization" on page A-7). The PortfolioMAD object computes MAD as this nonlinear function which can be handled by the solver fmincon Optimization Toolbox. The nonlinear programming solver fmincon has several algorithms that can be selected with the setSolver function, the two algorithms that work best in practice are 'sqp' and 'active-set'.

There are reformulations of the MAD portfolio optimization problem (see Konno and Yamazaki [50] at "Portfolio Optimization" on page A-7) that result in a linear programming problem, which can be solved either with standard linear programming techniques or with stochastic programming solvers. The PortfolioMAD object, however, does not reformulate the problem in such a manner. The PortfolioMAD object computes the MAD as a nonlinear function. The convexity of the MAD, as a function of the portfolio
weights and the dull edges when the number of scenarios is large, make the MAD portfolio optimization problem tractable, in practice, for certain nonlinear programming solvers, such as fmincon from Optimization Toolbox. To learn more about the workflow when using PortfolioMAD objects, see "PortfolioMAD Object Workflow" on page 6-19.

## What Are Scenarios?

Since mean absolute deviation portfolio optimization works with scenarios of asset returns to perform the optimization, several ways exist to specify and simulate scenarios. In many applications with MAD portfolio optimization, asset returns may have distinctly nonnormal probability distributions with either multiple modes, binning of returns, truncation of distributions, and so forth. In other applications, asset returns are modeled as the result of various simulation methods that might include Monte-Carlo simulation, quasi-random simulation, and so forth. Often, the underlying probability distribution for risk factors may be multivariate normal but the resultant transformations are sufficiently nonlinear to result in distinctively nonnormal asset returns.

For example, this occurs with bonds and derivatives. In the case of bonds with a nonzero probability of default, such scenarios would likely include asset returns that are $-100 \%$ to indicate default and some values slightly greater than $-100 \%$ to indicate recovery rates.

Although the PortfolioMAD object has functions to simulate multivariate normal scenarios from either data or moments (simulateNormalScenariosByData and simulateNormalScenariosByMoments), the usual approach is to specify scenarios directly from your own simulation functions. These scenarios are entered directly as a matrix with a sample for all assets across each row of the matrix and with samples for an asset down each column of the matrix. The architecture of the MAD portfolio optimization tools references the scenarios through a function handle so scenarios that have been set cannot be accessed directly as a property of the PortfolioMAD object.

## Setting Scenarios Using the PortfolioMAD Function

Suppose that you have a matrix of scenarios in the AssetScenarios variable. The scenarios are set through the PortfolioMAD object with:

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
0.00408 0.0289 0.0204 0.0119;
0.00192 0.0204 0.0576 0.0336;
0 0.0119 0.0336 0.1225 ];
```

```
m = m/12;
C = C/12;
AssetScenarios = mvnrnd(m, C, 20000);
p = PortfolioMAD('Scenarios', AssetScenarios);
disp(p.NumAssets);
disp(p.NumScenarios);
4
20000
```

Notice that the PortfolioMAD object determines and fixes the number of assets in NumAssets and the number of scenarios in NumScenarios based on the scenario's matrix. You can change the number of scenarios by calling the PortfolioMAD object with a different scenario matrix. However, once the NumAssets property has been set in the object, you cannot enter a scenario matrix with a different number of assets. The getScenarios function lets you recover scenarios from a PortfolioMAD object. You can also obtain the mean and covariance of your scenarios using estimateScenarioMoments.

Although not recommended for the casual user, an alternative way exists to recover scenarios by working with the function handle that points to scenarios in the PortfolioMAD object. To access some or all the scenarios from a PortfolioMAD object, the hidden property localScenarioHandle is a function handle that points to a function to obtain scenarios that have already been set. To get scenarios directly from a PortfolioMAD object p, use

```
scenarios = p.localScenarioHandle([], []);
```

and to obtain a subset of scenarios from rows startrow to endrow, use
scenarios = p.localScenarioHandle(startrow, endrow);
where $1 \leq$ startrow $\leq$ endrow $\leq$ numScenarios.

## Setting Scenarios Using the setScenarios Function

You can also set scenarios using setScenarios. For example, given the mean and covariance of asset returns in the variables $m$ and $C$, the asset moment properties can be set:

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
0.00408 0.0289 0.0204 0.0119;
0.00192 0.0204 0.0576 0.0336;
0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;
AssetScenarios = mvnrnd(m, C, 20000);
p = PortfolioMAD;
p = setScenarios(p, AssetScenarios);
disp(p.NumAssets);
disp(p.NumScenarios);
```

4
20000

## Estimating the Mean and Covariance of Scenarios

The estimateScenarioMoments function obtains estimates for the mean and covariance of scenarios in a PortfolioMAD object.

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
0.00408 0.0289 0.0204 0.0119;
0.00192 0.0204 0.0576 0.0336;
0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;
```

AssetScenarios $=$ mvnrnd(m, C, 20000);

```
p = PortfolioMAD;
p = setScenarios(p, AssetScenarios);
[mean, covar] = estimateScenarioMoments(p)
mean =
    0.0044
    0.0084
    0.0108
    0.0155
covar =
\begin{tabular}{rrrr}
0.0005 & 0.0003 & 0.0002 & -0.0000 \\
0.0003 & 0.0024 & 0.0017 & 0.0010 \\
0.0002 & 0.0017 & 0.0047 & 0.0028 \\
-0.0000 & 0.0010 & 0.0028 & 0.0103
\end{tabular}
```


## Simulating Normal Scenarios

As a convenience, the two functions (simulateNormalScenariosByData and simulateNormalScenariosByMoments) exist to simulate scenarios from data or moments under an assumption that they are distributed as multivariate normal random asset returns.

## Simulating Normal Scenarios from Returns or Prices

Given either return or price data, use the simulateNormalScenariosByData function to simulate multivariate normal scenarios. Either returns or prices are stored as matrices with samples going down the rows and assets going across the columns. In addition, returns or prices can be stored in a table or timetable (see "Simulating Normal Scenarios from Time Series Data" on page 6-50). To illustrate using simulateNormalScenariosByData, generate random samples of 120 observations of asset returns for four assets from the mean and covariance of asset returns in the variables m and C with portsim. The default behavior of portsim creates simulated data with estimated mean and covariance identical to the input moments $m$ and $C$. In addition to a return series created by portsim in the variable $X$, a price series is created in the variable $Y$ :
m = [ 0.0042; 0.0083; 0.01; 0.15 ];
$C=[0.0053330 .000340 .000160$;

```
0.00034 0.002408 0.0017 0.000992;
0.00016 0.0017 0.0048 0.0028;
0 0.000992 0.0028 0.010208 ];
X = portsim(m', C, 120);
Y = ret2tick(X);
```

Note Portfolio optimization requires that you use total returns and not just price returns. So, "returns" should be total returns and "prices" should be total return prices.

Given asset returns and prices in variables $X$ and $Y$ from above, this sequence of examples demonstrates equivalent ways to simulate multivariate normal scenarios for the PortfolioMAD object. Assume a PortfolioMAD object created in p that uses the asset returns in $X$ uses simulateNormalScenariosByData:

```
p = PortfolioMAD;
p = simulateNormalScenariosByData(p, X, 20000);
```

[passetmean, passetcovar] = estimateScenarioMoments(p)
passetmean =
0.0033
0.0085
0.0095
0.1503
passetcovar =

| 0.0055 | 0.0004 | 0.0002 | 0.0001 |
| :--- | :--- | :--- | :--- |
| 0.0004 | 0.0024 | 0.0017 | 0.0010 |
| 0.0002 | 0.0017 | 0.0049 | 0.0028 |
| 0.0001 | 0.0010 | 0.0028 | 0.0102 |

The moments that you obtain from this simulation will likely differ from the moments listed here because the scenarios are random samples from the estimated multivariate normal probability distribution of the input returns $X$.

The default behavior of simulateNormalScenariosByData is to work with asset returns. If, instead, you have asset prices as in the variable $Y$, simulateNormalScenariosByData accepts a name-value pair argument name
'DataFormat ' with a corresponding value set to 'prices' to indicate that the input to the function is in the form of asset prices and not returns (the default value for the 'DataFormat' argument is 'returns'). This example simulates scenarios with the asset price data in $Y$ for the PortfolioMAD object q:

```
p = PortfolioMAD;
p = simulateNormalScenariosByData(p, Y, 20000, 'dataformat', 'prices');
[passetmean, passetcovar] = estimateScenarioMoments(p)
passetmean =
0.0043
0.0083
0.0099
0.1500
passetcovar =
\begin{tabular}{llll}
0.0053 & 0.0003 & 0.0001 & 0.0002 \\
0.0003 & 0.0024 & 0.0017 & 0.0010 \\
0.0001 & 0.0017 & 0.0047 & 0.0027 \\
0.0002 & 0.0010 & 0.0027 & 0.0100
\end{tabular}
```


## Simulating Normal Scenarios with Missing Data

Often when working with multiple assets, you have missing data indicated by NaN values in your return or price data. Although "Multivariate Normal Regression" on page 9-2 goes into detail about regression with missing data, the simulateNormalScenariosByData function has a name-value pair argument name 'MissingData' that indicates with a Boolean value whether to use the missing data capabilities of Financial Toolbox. The default value for 'MissingData' is false which removes all samples with NaN values. If, however, 'MissingData' is set to true, simulateNormalScenariosByData uses the ECM algorithm to estimate asset moments. This example shows how this works on price data with missing values:

```
m = [ 0.0042; 0.0083; 0.01; 0.15 ];
C = [ 0.005333 0.00034 0.00016 0;
0.00034 0.002408 0.0017 0.000992;
0.00016 0.0017 0.0048 0.0028;
0 0.000992 0.0028 0.010208 ];
X = portsim(m', C, 120);
```

```
Y = ret2tick(X);
Y(1:20,1) = NaN;
Y(1:12,4) = NaN;
```

Notice that the prices above in $Y$ have missing values in the first and fourth series.

```
p = PortfolioMAD
p = simulateNormalScenariosByData(p, Y, 20000, 'dataformat', 'prices');
q = PortfolioMAD;
q = simulateNormalScenariosByData(q, Y, 20000, 'dataformat', 'prices', 'missingdata', true);
[passetmean, passetcovar] = estimateScenarioMoments(p)
[qassetmean, qassetcovar] = estimateScenarioMoments(q)
passetmean =
    0.0095
    0.0103
    0.0124
    0.1505
passetcovar =
\begin{tabular}{rrrr}
0.0054 & 0.0000 & -0.0005 & -0.0006 \\
0.0000 & 0.0021 & 0.0015 & 0.0010 \\
-0.0005 & 0.0015 & 0.0046 & 0.0026 \\
-0.0006 & 0.0010 & 0.0026 & 0.0100
\end{tabular}
qassetmean =
    0.0092
    0.0082
    0.0094
    0.1463
qassetcovar =
\begin{tabular}{rrrr}
0.0071 & -0.0000 & -0.0006 & -0.0006 \\
-0.0000 & 0.0032 & 0.0023 & 0.0015 \\
-0.0006 & 0.0023 & 0.0064 & 0.0036 \\
-0.0006 & 0.0015 & 0.0036 & 0.0133
\end{tabular}
```

The first PortfolioMAD object, $p$, contains scenarios obtained from price data in $Y$ where NaN values are discarded and the second PortfolioMAD object, q, contains scenarios
obtained from price data in $Y$ that accommodate missing values. Each time you run this example, you get different estimates for the moments in p and q .

## Simulating Normal Scenarios from Time Series Data

The simulateNormalScenariosByData function accepts asset returns or prices stored in table or timetable. The simulateNormalScenariosByData function implicitly works with matrices of data or data in a table or timetable object using the same rules for whether the data are returns or prices. To illustrate, use array2timetable to create a timetable for 14 assets from CAPMuniverse and the use the timetable to simulate scenarios for PortfolioCVaR.

```
load CAPMuniverse
time = datetime(Dates,'ConvertFrom','datenum');
stockTT = array2timetable(Data,'RowTimes',time, 'VariableNames', Assets);
stockTT.Properties
% Notice that GOOG has missing data, because it was not listed before Aug 2004
head(stockTT, 5);
ans =
    struct with fields:
```

```
            Description: '
```

            Description: '
                    UserData: []
                    UserData: []
        DimensionNames: {'Time' 'Variables'}
        DimensionNames: {'Time' 'Variables'}
            VariableNames: {'AAPL' 'AMZN' 'CSCO' 'DELL' 'EBAY' 'GOOG' 'HPQ' 'IBM
            VariableNames: {'AAPL' 'AMZN' 'CSCO' 'DELL' 'EBAY' 'GOOG' 'HPQ' 'IBM
    VariableDescriptions: {}
    VariableDescriptions: {}
            VariableUnits: {}
            VariableUnits: {}
    VariableContinuity: []
    VariableContinuity: []
                            RowTimes: [1471×1 datetime]
    ```
                            RowTimes: [1471×1 datetime]
```

Use the 'MissingData' option offered by PortfolioMAD to account for the missing data.

```
p = PortfolioMAD;
p = simulateNormalScenariosByData(p, stockTT, 20000 ,'missingdata',true);
[passetmean, passetcovar] = estimateScenarioMoments(p)
```

passetmean =
0.0017
0.0013
0.0005
0.0001
0.0019
0.0049
0.0003

$$
\begin{array}{r}
0.0003 \\
0.0006 \\
-0.0001 \\
0.0005 \\
0.0011 \\
0.0002 \\
0.0001
\end{array}
$$

```
passetcovar =
```

| 0.0014 | 0.0005 | 0.0006 | 0.0006 | 0.0006 | 0.0003 | 0.0005 | 0.0003 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.0005 | 0.0025 | 0.0007 | 0.0005 | 0.0010 | 0.0005 | 0.0005 | 0.0003 |
| 0.0006 | 0.0007 | 0.0013 | 0.0006 | 0.0007 | 0.0004 | 0.0006 | 0.0004 |
| 0.0006 | 0.0005 | 0.0006 | 0.0009 | 0.0006 | 0.0002 | 0.0005 | 0.0003 |
| 0.0006 | 0.0010 | 0.0007 | 0.0006 | 0.0018 | 0.0007 | 0.0005 | 0.0003 |
| 0.0003 | 0.0005 | 0.0004 | 0.0002 | 0.0007 | 0.0013 | 0.0002 | 0.0002 |
| 0.0005 | 0.0005 | 0.0006 | 0.0005 | 0.0005 | 0.0002 | 0.0010 | 0.0003 |
| 0.0003 | 0.0003 | 0.0004 | 0.0003 | 0.0003 | 0.0002 | 0.0003 | 0.0005 |
| 0.0006 | 0.0006 | 0.0008 | 0.0006 | 0.0006 | 0.0002 | 0.0005 | 0.0004 |
| 0.0004 | 0.0004 | 0.0005 | 0.0004 | 0.0005 | 0.0002 | 0.0003 | 0.0002 |
| 0.0005 | 0.0006 | 0.0008 | 0.0005 | 0.0007 | 0.0003 | 0.0005 | 0.0004 |
| 0.0007 | 0.0012 | 0.0008 | 0.0006 | 0.0011 | 0.0011 | 0.0006 | 0.0004 |
| 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0001 | 0.0002 | 0.0002 |
| -0.0000 | -0.0000 | -0.0000 | -0.0000 | 0.0000 | -0.0000 | -0.0000 | -0.0000 |

Use the name-value input 'DataFormat' to handle return or price data and 'MissingData' to ignore or use samples with missing values. In addition, simulateNormalScenariosByData extracts asset names or identifiers from a table or timetable if the argument 'GetAssetList' is set to true (the default value is false). If the 'GetAssetList' value is true, the identifiers are used to set the AssetList property of the PortfolioMAD object. Thus, repeating the formation of the PortfolioMAD object $p$ from the previous example with the 'GetAssetList' flag set to true extracts the column names from the timetable object:

```
p = simulateNormalScenariosByData(p, stockTT, 20000 ,'missingdata',true, 'GetAssetList', true);
disp(p.AssetList)
    'AAPL' 'AMZN' 'CSCO' 'DELL' 'EBAY' 'GOOG' 'HPQ' 'IBM' 'INTC'
```

If you set the 'GetAssetList ' flag set to true and your input data is in a matrix, simulateNormalScenariosByData uses the default labeling scheme from setAssetList as described in "Setting Up a List of Asset Identifiers" on page 5-37.

## Simulating Normal Scenarios for Mean and Covariance

Given the mean and covariance of asset returns, use the simulateNormalScenariosByMoments function to simulate multivariate normal scenarios. The mean can be either a row or column vector and the covariance matrix must be a symmetric positive-semidefinite matrix. Various rules for scalar expansion apply. To illustrate using simulateNormalScenariosByMoments, start with moments in $m$ and $C$ and generate 20,000 scenarios:

```
m = [ 0.0042; 0.0083; 0.01; 0.15 ];
C = [ 0.005333 0.00034 0.00016 0;
0.00034 0.002408 0.0017 0.000992;
0.00016 0.0017 0.0048 0.0028;
0 0.000992 0.0028 0.010208 ];
p = PortfolioMAD;
p = simulateNormalScenariosByMoments(p, m, C, 20000);
[passetmean, passetcovar] = estimateScenarioMoments(p)
passetmean =
    0.0040
    0.0084
    0.0105
    0.1513
passetcovar =
\begin{tabular}{llll}
0.0053 & 0.0003 & 0.0002 & 0.0001 \\
0.0003 & 0.0024 & 0.0017 & 0.0009 \\
0.0002 & 0.0017 & 0.0048 & 0.0028 \\
0.0001 & 0.0009 & 0.0028 & 0.0102
\end{tabular}
```

simulateNormalScenariosByMoments performs scalar expansion on arguments for the moments of asset returns. If NumAssets has not already been set, a scalar argument is interpreted as a scalar with NumAssets set to 1 .
simulateNormalScenariosByMoments provides an additional optional argument to specify the number of assets so that scalar expansion works with the correct number of assets. In addition, if either a scalar or vector is input for the covariance of asset returns, a diagonal matrix is formed such that a scalar expands along the diagonal and a vector becomes the diagonal.

## See Also

PortfolioMAD|setCosts|setScenarios|simulateNormalScenariosByData| simulateNormalScenariosByMoments

## Related Examples

- "Working with a Riskless Asset" on page 6-54
- "Working with Transaction Costs" on page 6-56
- "Creating the PortfolioMAD Object" on page 6-26
- "Working with MAD Portfolio Constraints Using Defaults" on page 6-61
- "Validate the MAD Portfolio Problem" on page 6-91
- "Estimate Efficient Portfolios Along the Entire Frontier for PortfolioMAD Object" on page 6-96
- "Estimate Efficient Frontiers for PortfolioMAD Object" on page 6-111
- "Asset Returns and Scenarios Using PortfolioMAD Object" on page 6-42


## More About

- "PortfolioMAD Object" on page 6-21
- "Portfolio Optimization Theory" on page 6-3
- "PortfolioMAD Object Workflow" on page 6-19


## Working with a Riskless Asset

The PortfolioMAD object has a separate RiskFreeRate property that stores the rate of return of a riskless asset. Thus, you can separate your universe into a riskless asset and a collection of risky assets. For example, assume that your riskless asset has a return in the scalar variable r0, then the property for the RiskFreeRate is set using the PortfolioMAD object:

```
r0 = 0.01/12;
p = PortfolioMAD;
p = PortfolioMAD('RiskFreeRate', r0);
disp(p.RiskFreeRate);
```

8.3333e-04

Note If your portfolio problem has a budget constraint such that your portfolio weights must sum to 1 , then the riskless asset is irrelevant.

## See Also

PortfolioMAD|setCosts| setScenarios|simulateNormalScenariosByData| simulateNormalScenariosByMoments

## Related Examples

- "Working with Transaction Costs" on page 6-56
- "Asset Returns and Scenarios Using PortfolioMAD Object" on page 6-42
- "Creating the PortfolioMAD Object" on page 6-26
- "Working with MAD Portfolio Constraints Using Defaults" on page 6-61
- "Validate the MAD Portfolio Problem" on page 6-91
- "Estimate Efficient Portfolios Along the Entire Frontier for PortfolioMAD Object" on page 6-96
- "Estimate Efficient Frontiers for PortfolioMAD Object" on page 6-111
- "Asset Returns and Scenarios Using PortfolioMAD Object" on page 6-42


## More About

- "PortfolioMAD Object" on page 6-21
- "Portfolio Optimization Theory" on page 6-3
- "PortfolioMAD Object Workflow" on page 6-19


## Working with Transaction Costs

The difference between net and gross portfolio returns is transaction costs. The net portfolio return proxy has distinct proportional costs to purchase and to sell assets which are maintained in the PortfolioMAD object properties BuyCost and SellCost.
Transaction costs are in units of total return and, as such, are proportional to the price of an asset so that they enter the model for net portfolio returns in return form. For example, suppose that you have a stock currently priced $\$ 40$ and your usual transaction costs are 5 cents per share. Then the transaction cost for the stock is $0.05 / 40=0.00125$ (as defined in "Net Portfolio Returns" on page 6-5). Costs are entered as positive values and credits are entered as negative values.

## Setting Transaction Costs Using the PortfolioMAD Function

To set up transaction costs, you must specify an initial or current portfolio in the InitPort property. If the initial portfolio is not set when you set up the transaction cost properties, InitPort is 0 . The properties for transaction costs can be set using the PortfolioMAD object. For example, assume that purchase and sale transaction costs are in the variables bc and sc and an initial portfolio is in the variable $\times 0$, then transaction costs are set:

```
bc = [ 0.00125; 0.00125; 0.00125; 0.00125; 0.00125 ];
sc = [ 0.00125; 0.007; 0.00125; 0.00125; 0.0024 ];
x0 = [ 0.4; 0.2; 0.2; 0.1; 0.1 ];
p = PortfolioMAD('BuyCost', bc, 'SellCost', sc, 'InitPort', x0);
disp(p.NumAssets);
disp(p.BuyCost);
disp(p.SellCost);
disp(p.InitPort);
```

5
0.0013
0.0013
0.0013
0.0013
0.0013
0.0013
0.0070
0.0013
0.0013
0.0024
0.4000
0.2000
0.2000
0.1000
0.1000

## Setting Transaction Costs Using the setCosts Function

You can also set the properties for transaction costs using setCosts. Assume that you have the same costs and initial portfolio as in the previous example. Given a PortfolioMAD object $p$ with an initial portfolio already set, use setCosts to set up transaction costs:

```
bc = [ 0.00125; 0.00125; 0.00125; 0.00125; 0.00125 ];
sc = [ 0.00125; 0.007; 0.00125; 0.00125; 0.0024 ];
x0 = [ 0.4; 0.2; 0.2; 0.1; 0.1 ];
p = PortfolioMAD('InitPort', x0);
p = setCosts(p, bc, sc);
disp(p.NumAssets);
disp(p.BuyCost);
disp(p.SellCost);
disp(p.InitPort);
```


## 5

```
0.0013
0.0013
0.0013
0.0013
0.0013
0.0013
0.0070
0.0013
0.0013
0.0024
0.4000
0.2000
0.2000
```

0.1000
0.1000

You can also set up the initial portfolio's InitPort value as an optional argument to setCosts so that the following is an equivalent way to set up transaction costs:
bc = [ 0.00125; 0.00125; 0.00125; 0.00125; 0.00125 ];
sc = [ 0.00125; 0.007; 0.00125; 0.00125; 0.0024 ];
$\mathrm{x} 0=[0.4 ; 0.2 ; 0.2 ; 0.1 ; 0.1$ ];
p = PortfolioMAD;
$\mathrm{p}=$ setCosts(p, bc, sc, x0);
disp(p.NumAssets);
disp(p.BuyCost);
disp(p.SellCost);
disp(p.InitPort);
5
0.0013
0.0013
0.0013
0.0013
0.0013
0.0013
0.0070
0.0013
0.0013
0.0024
0.4000
0.2000
0.2000
0.1000
0.1000

## Setting Transaction Costs with Scalar Expansion

Both the PortfolioMAD object and setCosts function implement scalar expansion on the arguments for transaction costs and the initial portfolio. If the NumAssets property is already set in the PortfolioMAD object, scalar arguments for these properties are expanded to have the same value across all dimensions. In addition, setCosts lets you
specify NumAssets as an optional final argument. For example, assume that you have an initial portfolio $x 0$ and you want to set common transaction costs on all assets in your universe. You can set these costs in any of these equivalent ways:

```
x0 = [ 0.4; 0.2; 0.2; 0.1; 0.1 ];
p = PortfolioMAD('InitPort', x0, 'BuyCost', 0.002, 'SellCost', 0.002);
Or
x0 = [ 0.4; 0.2; 0.2; 0.1; 0.1 ];
p = PortfolioMAD('InitPort', x0);
p = setCosts(p, 0.002, 0.002);
or
x0 = [ 0.4; 0.2; 0.2; 0.1; 0.1 ];
p = PortfolioMAD;
p = setCosts(p, 0.002, 0.002, x0);
```

To clear costs from your PortfolioMAD object, use either thePortfolioMAD object or setCosts with empty inputs for the properties to be cleared. For example, you can clear sales costs from the PortfolioMAD object $p$ in the previous example:

```
p = PortfolioMAD(p, 'SellCost', []);
```


## See Also

PortfolioMAD|setCosts| setScenarios|simulateNormalScenariosByData| simulateNormalScenariosByMoments

## Related Examples

- "Working with a Riskless Asset" on page 6-54
- "Asset Returns and Scenarios Using PortfolioMAD Object" on page 6-42
- "Creating the PortfolioMAD Object" on page 6-26
- "Working with MAD Portfolio Constraints Using Defaults" on page 6-61
- "Validate the MAD Portfolio Problem" on page 6-91
- "Estimate Efficient Portfolios Along the Entire Frontier for PortfolioMAD Object" on page 6-96
- "Estimate Efficient Frontiers for PortfolioMAD Object" on page 6-111
- "Asset Returns and Scenarios Using PortfolioMAD Object" on page 6-42


## More About

- "PortfolioMAD Object" on page 6-21
- "Portfolio Optimization Theory" on page 6-3
- "PortfolioMAD Object Workflow" on page 6-19


## Working with MAD Portfolio Constraints Using Defaults

The final element for a complete specification of a portfolio optimization problem is the set of feasible portfolios, which is called a portfolio set. A portfolio set $X \subset R^{n}$ is specified by construction as the intersection of sets formed by a collection of constraints on portfolio weights. A portfolio set necessarily and sufficiently must be a nonempty, closed, and bounded set.

When setting up your portfolio set, ensure that the portfolio set satisfies these conditions. The most basic or "default" portfolio set requires portfolio weights to be nonnegative (using the lower-bound constraint) and to sum to 1 (using the budget constraint). For information on the workflow when using PortfolioMAD objects, see "PortfolioMAD Object Workflow" on page 6-19.

## Setting Default Constraints for Portfolio Weights Using PortfolioMAD Object

The "default" MAD portfolio problem has two constraints on portfolio weights:

- Portfolio weights must be nonnegative.
- Portfolio weights must sum to 1 .

Implicitly, these constraints imply that portfolio weights are no greater than 1, although this is a superfluous constraint to impose on the problem.

## Setting Default Constraints Using the PortfolioMAD Function

Given a portfolio optimization problem with NumAssets $=20$ assets, use the PortfolioMAD object to set up a default problem and explicitly set bounds and budget constraints:

```
p = PortfolioMAD('NumAssets', 20, 'LowerBound', 0, 'Budget', 1);
disp(p);
PortfolioMAD with properties:
```

    BuyCost: []
    SellCost: []
    RiskFreeRate: []
    Turnover: []
    ```
BuyTurnover: []
SellTurnover: []
NumScenarios: []
    Name: []
    NumAssets: 20
    AssetList: []
    InitPort: []
AInequality: []
bInequality: []
    AEquality: []
    bEquality: []
    LowerBound: [20x1 double]
    UpperBound: []
LowerBudget: 1
UpperBudget: 1
GroupMatrix: []
    LowerGroup: []
    UpperGroup: []
    GroupA: []
    GroupB: []
LowerRatio: []
UpperRatio: []
```


## Setting Default Constraints Using the setDefaultConstraints Function

An alternative approach is to use the setDefaultConstraints function. If the number of assets is already known in a PortfolioMAD object, use setDefaultConstraints with no arguments to set up the necessary bound and budget constraints. Suppose that you have 20 assets to set up the portfolio set for a default problem:

```
p = PortfolioMAD('NumAssets', 20);
p = setDefaultConstraints(p);
disp(p);
PortfolioMAD with properties:
    BuyCost: []
        SellCost: []
    RiskFreeRate: []
        Turnover: []
    BuyTurnover: []
    SellTurnover: []
    NumScenarios: []
        Name: []
    NumAssets: 20
```

```
    AssetList: []
        InitPort: []
AInequality: []
bInequality: []
    AEquality: []
    bEquality: []
    LowerBound: [20x1 double]
    UpperBound: []
LowerBudget: 1
UpperBudget: 1
GroupMatrix: []
    LowerGroup: []
    UpperGroup: []
        GroupA: []
        GroupB: []
    LowerRatio: []
    UpperRatio: []
```

If the number of assets is unknown, setDefaultConstraints accepts NumAssets as an optional argument to form a portfolio set for a default problem. Suppose that you have 20 assets:
p = PortfolioMAD;
p = setDefaultConstraints(p, 20);
disp(p);
PortfolioMAD with properties:
BuyCost: []
SellCost: []
RiskFreeRate: []
Turnover: []
BuyTurnover: []
SellTurnover: []
NumScenarios: []
Name: []
NumAssets: 20
AssetList: []
InitPort: []
AInequality: []
bInequality: []
AEquality: []
bEquality: []
LowerBound: [20x1 double]
UpperBound: []

```
LowerBudget: 1
UpperBudget: 1
GroupMatrix: []
    LowerGroup: []
    UpperGroup: []
    GroupA: []
    GroupB: []
    LowerRatio: []
    UpperRatio: []
```


## See Also

PortfolioMAD| setBounds| setBudget| setDefaultConstraints| setEquality | setGroupRatio| setGroups | setInequality | setOneWayTurnover | setTurnover

## Related Examples

- "Working with Bound Constraints Using PortfolioMAD Object" on page 6-66
- "Working with Budget Constraints Using PortfolioMAD Object" on page 6-69
- "Working with Group Constraints Using PortfolioMAD Object" on page 6-71
- "Working with Group Ratio Constraints Using PortfolioMAD Object" on page 6-75
- "Working with Linear Equality Constraints Using PortfolioMAD Object" on page 679
- "Working with Linear Inequality Constraints Using PortfolioMAD Object" on page 682
- "Working with Average Turnover Constraints Using PortfolioMAD Object" on page 685
- "Working with One-Way Turnover Constraints Using PortfolioMAD Object" on page 6-88
- "Creating the PortfolioMAD Object" on page 6-26
- "Validate the MAD Portfolio Problem" on page 6-91
- "Estimate Efficient Portfolios Along the Entire Frontier for PortfolioMAD Object" on page 6-96
- "Estimate Efficient Frontiers for PortfolioMAD Object" on page 6-111
- "Asset Returns and Scenarios Using PortfolioMAD Object" on page 6-42


## More About

- "PortfolioMAD Object" on page 6-21
- "Portfolio Optimization Theory" on page 6-3
- "PortfolioMAD Object Workflow" on page 6-19


## Working with Bound Constraints Using PortfolioMAD Object

Bound constraints are optional linear constraints that maintain upper and lower bounds on portfolio weights (see "Bound Constraints" on page 6-12). Although every MAD portfolio set must be bounded, it is not necessary to specify a MAD portfolio set with explicit bound constraints. For example, you can create a MAD portfolio set with an implicit upper bound constraint or a MAD portfolio set with average turnover constraints. The bound constraints have properties LowerBound for the lower-bound constraint and UpperBound for the upper-bound constraint. Set default values for these constraints using the setDefaultConstraints function (see "Setting Default Constraints for Portfolio Weights Using PortfolioMAD Object" on page 6-61).

## Setting Bounds Using the PortfolioMAD Function

The properties for bound constraints are set through the PortfolioMAD object. Suppose that you have a balanced fund with stocks that can range from $50 \%$ to $75 \%$ of your portfolio and bonds that can range from $25 \%$ to $50 \%$ of your portfolio. The bound constraints for a balanced fund are set with:

```
lb = [ 0.5; 0.25 ];
ub = [ 0.75; 0.5 ];
p = PortfolioMAD('LowerBound', lb, 'UpperBound', ub);
disp(p.NumAssets);
disp(p.LowerBound);
disp(p.UpperBound);
    2
    0.5000
    0.2500
    0.7500
    0.5000
```

To continue with this example, you must set up a budget constraint. For details, see "Budget Constraints" on page 6-12.

## Setting Bounds Using the setBounds Function

You can also set the properties for bound constraints using setBounds. Suppose that you have a balanced fund with stocks that can range from $50 \%$ to $75 \%$ of your portfolio and bonds that can range from $25 \%$ to $50 \%$ of your portfolio. Given a PortfolioMAD object $p$, use setBounds to set the bound constraints:

```
lb = [ 0.5; 0.25 ];
ub = [ 0.75; 0.5 ];
p = PortfolioMAD;
p = setBounds(p, lb, ub);
disp(p.NumAssets);
disp(p.LowerBound);
disp(p.UpperBound);
```

    2
    0.5000
    0.2500
    0.7500
    0.5000
    
## Setting Bounds Using the PortfolioMAD Function or setBounds Function

Both the PortfolioMAD object and setBounds function implement scalar expansion on either the LowerBound or UpperBound properties. If the NumAssets property is already set in the PortfolioMAD object, scalar arguments for either property expand to have the same value across all dimensions. In addition, setBounds lets you specify NumAssets as an optional argument. Suppose that you have a universe of 500 assets and you want to set common bound constraints on all assets in your universe. Specifically, you are a long-only investor and want to hold no more than $5 \%$ of your portfolio in any single asset. You can set these bound constraints in any of these equivalent ways:

```
p = PortfolioMAD('NumAssets', 500, 'LowerBound', 0, 'UpperBound', 0.05);
or
p = PortfolioMAD('NumAssets', 500);
p = setBounds(p, 0, 0.05);
or
```

```
p = PortfolioMAD;
p = setBounds(p, 0, 0.05, 500);
```

To clear bound constraints from your PortfolioMAD object, use either the PortfolioMAD object or setBounds with empty inputs for the properties to be cleared. For example, to clear the upper-bound constraint from the PortfolioMAD object p in the previous example:

```
p = PortfolioMAD(p, 'UpperBound', []);
```


## See Also

PortfolioMAD | setBounds | setBudget | setDefaultConstraints| setEquality | setGroupRatio| setGroups | setInequality| setOneWayTurnover| setTurnover

## Related Examples

- "Setting Default Constraints for Portfolio Weights Using PortfolioMAD Object" on page 6-61
- "Creating the PortfolioMAD Object" on page 6-26
- "Validate the MAD Portfolio Problem" on page 6-91
- "Estimate Efficient Portfolios Along the Entire Frontier for PortfolioMAD Object" on page 6-96
- "Estimate Efficient Frontiers for PortfolioMAD Object" on page 6-111
- "Asset Returns and Scenarios Using PortfolioMAD Object" on page 6-42


## More About

- "PortfolioMAD Object" on page 6-21
- "Portfolio Optimization Theory" on page 6-3
- "PortfolioMAD Object Workflow" on page 6-19


## Working with Budget Constraints Using PortfolioMAD Object

The budget constraint is an optional linear constraint that maintains upper and lower bounds on the sum of portfolio weights (see "Budget Constraints" on page 5-12). Budget constraints have properties LowerBudget for the lower budget constraint and UpperBudget for the upper budget constraint. If you set up a MAD portfolio optimization problem that requires portfolios to be fully invested in your universe of assets, you can set LowerBudget to be equal to UpperBudget. These budget constraints can be set with default values equal to 1 using setDefaultConstraints (see "Setting Default Constraints for Portfolio Weights Using PortfolioMAD Object" on page 6-61).

## Setting Budget Constraints Using the PortfolioMAD Function

The properties for the budget constraint can also be set using the PortfolioMAD object. Suppose that you have an asset universe with many risky assets and a riskless asset and you want to ensure that your portfolio never holds more than $1 \%$ cash, that is, you want to ensure that you are 99-100\% invested in risky assets. The budget constraint for this portfolio can be set with:

```
p = PortfolioMAD('LowerBudget', 0.99, 'UpperBudget', 1);
disp(p.LowerBudget);
disp(p.UpperBudget);
```

0.9900

1

## Setting Budget Constraints Using the setBudget Function

You can also set the properties for a budget constraint using setBudget. Suppose that you have a fund that permits up to $10 \%$ leverage which means that your portfolio can be from $100 \%$ to $110 \%$ invested in risky assets. Given a PortfolioMAD object p, use setBudget to set the budget constraints:

```
p = PortfolioMAD;
p = setBudget(p, 1, 1.1);
disp(p.LowerBudget);
disp(p.UpperBudget);
```

1
1.1000

If you were to continue with this example, then set the RiskFreeRate property to the borrowing rate to finance possible leveraged positions. For details on the RiskFreeRate property, see "Working with a Riskless Asset" on page 6-54. To clear either bound for the budget constraint from your PortfolioMAD object, use either the PortfolioMAD object or setBudget with empty inputs for the properties to be cleared. For example, clear the upper-budget constraint from the PortfolioMAD object $p$ in the previous example with:
p = PortfolioMAD(p, 'UpperBudget', []);

## See Also

PortfolioMAD | setBounds | setBudget | setDefaultConstraints| setEquality | setGroupRatio| setGroups|setInequality| setOneWayTurnover| setTurnover

## Related Examples

- "Setting Default Constraints for Portfolio Weights Using PortfolioMAD Object" on page 6-61
- "Creating the PortfolioMAD Object" on page 6-26
- "Validate the MAD Portfolio Problem" on page 6-91
- "Estimate Efficient Portfolios Along the Entire Frontier for PortfolioMAD Object" on page 6-96
- "Estimate Efficient Frontiers for PortfolioMAD Object" on page 6-111
- "Asset Returns and Scenarios Using PortfolioMAD Object" on page 6-42


## More About

- "PortfolioMAD Object" on page 6-21
- "Portfolio Optimization Theory" on page 6-3
- "PortfolioMAD Object Workflow" on page 6-19


## Working with Group Constraints Using PortfolioMAD Object

Group constraints are optional linear constraints that group assets together and enforce bounds on the group weights (see "Group Constraints" on page 6-13). Although the constraints are implemented as general constraints, the usual convention is to form a group matrix that identifies membership of each asset within a specific group with Boolean indicators (either true or false or with 1 or 0) for each element in the group matrix. Group constraints have properties GroupMatrix for the group membership matrix, LowerGroup for the lower-bound constraint on groups, and UpperGroup for the upper-bound constraint on groups.

## Setting Group Constraints Using the PortfolioMAD Function

The properties for group constraints are set through the PortfolioMAD object. Suppose that you have a portfolio of five assets and want to ensure that the first three assets constitute no more than $30 \%$ of your portfolio, then you can set group constraints:

```
G = [ [\begin{array}{llllll}{1}&{1}&{1}&{0}&{0}\end{array}];
p = PortfolioMAD('GroupMatrix', G, 'UpperGroup', 0.3);
disp(p.NumAssets);
disp(p.GroupMatrix);
disp(p.UpperGroup);
```

5
$\begin{array}{lllll}1 & 1 & 1 & 0 & 0\end{array}$

### 0.3000

The group matrix $G$ can also be a logical matrix so that the following code achieves the same result.

```
G = [ true true true false false ];
p = PortfolioMAD('GroupMatrix', G, 'UpperGroup', 0.3);
disp(p.NumAssets);
disp(p.GroupMatrix);
disp(p.UpperGroup);
```

5
$\begin{array}{lllll}1 & 1 & 1 & 0 & 0\end{array}$

## Setting Group Constraints Using the setGroups and addGroups Functions

You can also set the properties for group constraints using setGroups. Suppose that you have a portfolio of five assets and want to ensure that the first three assets constitute no more than $30 \%$ of your portfolio. Given a PortfolioMAD object p, use setGroups to set the group constraints:

```
G = [ true true true false false ];
p = PortfolioMAD;
p = setGroups(p, G, [], 0.3);
disp(p.NumAssets);
disp(p.GroupMatrix);
disp(p.UpperGroup);
```

5
$\begin{array}{lllll}1 & 1 & 1 & 0 & 0\end{array}$

### 0.3000

In this example, you would set the LowerGroup property to be empty ([ ]).
Suppose that you want to add another group constraint to make odd-numbered assets constitute at least $20 \%$ of your portfolio. Set up an augmented group matrix and introduce infinite bounds for unconstrained group bounds or use the addGroups function to build up group constraints. For this example, create another group matrix for the second group constraint:

```
p = PortfolioMAD;
G = [ true true true false false ];
% group matrix for first group constraint
p = setGroups(p, G, [], 0.3)
G = [ true false true false true ]; % group matrix for second group constraint
p = addGroups(p, G, 0.2);
disp(p.NumAssets);
disp(p.GroupMatrix)
disp(p.LowerGroup);
disp(p.UpperGroup);
```

5
$\begin{array}{lllll}1 & 1 & 1 & 0 & 0\end{array}$
$\begin{array}{lllll}1 & 0 & 1 & 0 & 1\end{array}$

- Inf 0.2000


### 0.3000

Inf
addGroups determines which bounds are unbounded so you only need to focus on the constraints that you want to set.

The PortfolioMAD object, setGroups, and addGroups implement scalar expansion on either the LowerGroup or UpperGroup properties based on the dimension of the group matrix in the property GroupMatrix. Suppose that you have a universe of 30 assets with 6 asset classes such that assets $1-5$, assets 6-12, assets 13-18, assets 19-22, assets 2327, and assets 28-30 constitute each of your asset classes and you want each asset class to fall from $0 \%$ to $25 \%$ of your portfolio. Let the following group matrix define your groups and scalar expansion define the common bounds on each group:

```
p = PortfolioMAD;
G = blkdiag(true(1,5), true(1,7), true(1,6), true(1,4), true(1,5), true(1,3));
p = setGroups(p, G, 0, 0.25);
disp(p.NumAssets);
disp(p.GroupMatrix)
disp(p.LowerGroup);
disp(p.UpperGroup);
30
```

Columns 1 through 13

| 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Columns 14 through 26

| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Columns 27 through 30

| 0 | 0 | 0 | 0 |
| :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 |

## See Also

PortfolioMAD | setBounds | setBudget | setDefaultConstraints| setEquality | setGroupRatio| setGroups | setInequality| setOneWayTurnover| setTurnover

## Related Examples

- "Setting Default Constraints for Portfolio Weights Using PortfolioMAD Object" on page 6-61
- "Creating the PortfolioMAD Object" on page 6-26
- "Validate the MAD Portfolio Problem" on page 6-91
- "Estimate Efficient Portfolios Along the Entire Frontier for PortfolioMAD Object" on page 6-96
- "Estimate Efficient Frontiers for PortfolioMAD Object" on page 6-111
- "Asset Returns and Scenarios Using PortfolioMAD Object" on page 6-42


## More About

- "PortfolioMAD Object" on page 6-21
- "Portfolio Optimization Theory" on page 6-3
- "PortfolioMAD Object Workflow" on page 6-19


## Working with Group Ratio Constraints Using PortfolioMAD Object

Group ratio constraints are optional linear constraints that maintain bounds on proportional relationships among groups of assets (see "Group Ratio Constraints" on page 6-14). Although the constraints are implemented as general constraints, the usual convention is to specify a pair of group matrices that identify membership of each asset within specific groups with Boolean indicators (either true or false or with 1 or 0 ) for each element in each of the group matrices. The goal is to ensure that the ratio of a base group compared to a comparison group fall within specified bounds. Group ratio constraints have properties:

- GroupA for the base membership matrix
- GroupB for the comparison membership matrix
- LowerRatio for the lower-bound constraint on the ratio of groups
- UpperRatio for the upper-bound constraint on the ratio of groups


## Setting Group Ratio Constraints Using the PortfolioMAD Function

The properties for group ratio constraints are set using PortfolioMAD object. For example, assume that you want the ratio of financial to nonfinancial companies in your portfolios to never go above 50\%. Suppose that you have six assets with three financial companies (assets 1-3) and three nonfinancial companies (assets 4-6). To set group ratio constraints:

```
GA = [ 1 1 1 0 0 0 ]; % financial companies
GB = [ 0 0 0 1 1 1 ]; % nonfinancial companies
p = PortfolioMAD('GroupA', GA, 'GroupB', GB, 'UpperRatio', 0.5);
disp(p.NumAssets);
disp(p.GroupA);
disp(p.GroupB);
disp(p.UpperRatio);
6
1 1 1 1 1 0
0
0.5000
```

Group matrices GA and GB in this example can be logical matrices with true and false elements that yield the same result:

```
GA = [ true true true false false false ]; % financial companies
GB = [ false false false true true true ]; % nonfinancial companies
p = PortfolioMAD('GroupA', GA, 'GroupB', GB, 'UpperRatio', 0.5);
disp(p.NumAssets);
disp(p.GroupA);
disp(p.GroupB)
disp(p.UpperRatio);
```

6

| 1 | 1 | 1 | 0 | 0 | 0 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 1 | 1 | 1 |

0.5000

## Setting Group Ratio Constraints Using the setGroupRatio and addGroupRatio Functions

You can also set the properties for group ratio constraints using setGroupRatio. For example, assume that you want the ratio of financial to nonfinancial companies in your portfolios to never go above 50\%. Suppose that you have six assets with three financial companies (assets 1-3) and three nonfinancial companies (assets 4-6). Given a PortfolioMAD object p, use setGroupRatio to set the group constraints:

```
GA = [ true true true false false false ]; % financial companies
GB = [ false false false true true true ]; % nonfinancial companies
p = PortfolioMAD;
p = setGroupRatio(p, GA, GB, [], 0.5);
disp(p.NumAssets);
disp(p.GroupA);
disp(p.GroupB);
disp(p.UpperRatio);
6
1 1 1 1 1 0
0
0.5000
```

In this example, you would set the LowerRatio property to be empty ([ ]).

Suppose that you want to add another group ratio constraint to ensure that the weights in odd-numbered assets constitute at least $20 \%$ of the weights in nonfinancial assets your portfolio. You can set up augmented group ratio matrices and introduce infinite bounds for unconstrained group ratio bounds, or you can use the addGroupRatio function to build up group ratio constraints. For this example, create another group matrix for the second group constraint:

```
p = PortfolioMAD;
GA = [ true true true false false false ]; % financial companies
GB = [ false false false true true true ]; % nonfinancial companies
p = setGroupRatio(p, GA, GB, [], 0.5);
GA = [ true false true false true false ]; % odd-numbered companies
GB = [ false false false true true true ]; % nonfinancial companies
p = addGroupRatio(p, GA, GB, 0.2);
disp(p.NumAssets);
disp(p.GroupA);
disp(p.GroupB);
disp(p.LowerRatio);
disp(p.UpperRatio);
6
\begin{tabular}{llllll}
1 & 1 & 1 & 0 & 0 & 0
\end{tabular}
\begin{tabular}{llllll}
1 & 0 & 1 & 0 & 1 & 0
\end{tabular}
0
0
    - Inf
0.2000
0.5000
Inf
```

Notice that addGroupRatio determines which bounds are unbounded so you only need to focus on the constraints you want to set.

ThePortfolioMAD object, setGroupRatio, and addGroupRatio implement scalar expansion on either the LowerRatio or UpperRatio properties based on the dimension of the group matrices in GroupA and GroupB properties.

## See Also

PortfolioMAD | setBounds | setBudget | setDefaultConstraints| setEquality | setGroupRatio| setGroups | setInequality| setOneWayTurnover| setTurnover

## Related Examples

- "Setting Default Constraints for Portfolio Weights Using PortfolioMAD Object" on page 6-61
- "Creating the PortfolioMAD Object" on page 6-26
- "Validate the MAD Portfolio Problem" on page 6-91
- "Estimate Efficient Portfolios Along the Entire Frontier for PortfolioMAD Object" on page 6-96
- "Estimate Efficient Frontiers for PortfolioMAD Object" on page 6-111
- "Asset Returns and Scenarios Using PortfolioMAD Object" on page 6-42


## More About

- "PortfolioMAD Object" on page 6-21
- "Portfolio Optimization Theory" on page 6-3
- "PortfolioMAD Object Workflow" on page 6-19


## Working with Linear Equality Constraints Using PortfolioMAD Object

Linear equality constraints are optional linear constraints that impose systems of equalities on portfolio weights (see "Linear Equality Constraints" on page 6-11). Linear equality constraints have properties AEquality, for the equality constraint matrix, and bEquality, for the equality constraint vector.

## Setting Linear Equality Constraints Using the PortfolioMAD Function

The properties for linear equality constraints are set using the PortfolioMAD object. Suppose that you have a portfolio of five assets and want to ensure that the first three assets are $50 \%$ of your portfolio. To set this constraint:

```
A = [ 1 1 1 0 0 ];
b = 0.5;
p = PortfolioMAD('AEquality', A, 'bEquality', b);
disp(p.NumAssets);
disp(p.AEquality);
disp(p.bEquality);
5
1 1 1 1 1 0 0
0.5000
```


## Setting Linear Equality Constraints Using the setEquality and addEquality Functions

You can also set the properties for linear equality constraints using setEquality. Suppose that you have a portfolio of five assets and want to ensure that the first three assets are $50 \%$ of your portfolio. Given a PortfolioMAD object $p$, use setEquality to set the linear equality constraints:
$\mathrm{A}=\left[\begin{array}{lllll}1 & 1 & 1 & 0\end{array}\right] ;$
b $=0.5$;
$\mathrm{p}=$ PortfolioMAD;
p = setEquality(p, A, b);

```
disp(p.NumAssets);
disp(p.AEquality);
disp(p.bEquality);
5
1 1 1 1 0
0.5000
```

Suppose that you want to add another linear equality constraint to ensure that the last three assets also constitute $50 \%$ of your portfolio. You can set up an augmented system of linear equalities or use addEquality to build up linear equality constraints. For this example, create another system of equalities:

```
p = PortfolioMAD;
A = [ 1 1 1 0 0 ]; % first equality constraint
b = 0.5;
p = setEquality(p, A, b);
A = [ 0 0 1 1 1 ]; % second equality constraint
b = 0.5;
p = addEquality(p, A, b);
disp(p.NumAssets);
disp(p.AEquality);
disp(p.bEquality);
5
\begin{tabular}{lllll}
1 & 1 & 1 & 0 & 0 \\
0 & 0 & 1 & 1 & 1
\end{tabular}
0.5000
0.5000
```

The PortfolioMAD object, setEquality, and addEquality implement scalar expansion on the bEquality property based on the dimension of the matrix in the AEquality property.

## See Also

PortfolioMAD|setBounds | setBudget| setDefaultConstraints| setEquality | setGroupRatio| setGroups | setInequality| setOneWayTurnover| setTurnover

## Related Examples

- "Setting Default Constraints for Portfolio Weights Using PortfolioMAD Object" on page 6-61
- "Creating the PortfolioMAD Object" on page 6-26
- "Validate the MAD Portfolio Problem" on page 6-91
- "Estimate Efficient Portfolios Along the Entire Frontier for PortfolioMAD Object" on page 6-96
- "Estimate Efficient Frontiers for PortfolioMAD Object" on page 6-111
- "Asset Returns and Scenarios Using PortfolioMAD Object" on page 6-42


## More About

- "PortfolioMAD Object" on page 6-21
- "Portfolio Optimization Theory" on page 6-3
- "PortfolioMAD Object Workflow" on page 6-19


## Working with Linear Inequality Constraints Using PortfolioMAD Object

Linear inequality constraints are optional linear constraints that impose systems of inequalities on portfolio weights (see "Linear Inequality Constraints" on page 6-10). Linear inequality constraints have properties AInequality for the inequality constraint matrix, and bInequality for the inequality constraint vector.

## Setting Linear Inequality Constraints Using the PortfolioMAD Function

The properties for linear inequality constraints are set using the PortfolioMAD object. Suppose that you have a portfolio of five assets and you want to ensure that the first three assets are no more than $50 \%$ of your portfolio. To set up these constraints:

```
A = [ 1 1 1 0 0 ];
b = 0.5;
p = PortfolioMAD('AInequality', A, 'bInequality', b);
disp(p.NumAssets);
disp(p.AInequality);
disp(p.bInequality);
5
1 
0.5000
```


## Setting Linear Inequality Constraints Using the setInequality and addInequality Functions

You can also set the properties for linear inequality constraints using setInequality. Suppose that you have a portfolio of five assets and you want to ensure that the first three assets constitute no more than $50 \%$ of your portfolio. Given a PortfolioMAD object $p$, use setInequality to set the linear inequality constraints:
$\mathrm{A}=\left[\begin{array}{lllll}1 & 1 & 1 & 0\end{array}\right] ;$
b $=0.5$;
$\mathrm{p}=$ PortfolioMAD;
$\mathrm{p}=\operatorname{setInequality}(\mathrm{p}, \mathrm{A}, \mathrm{b})$;

```
disp(p.NumAssets);
disp(p.AInequality);
disp(p.bInequality);
5
1 1 1 1 0
0.5000
```

Suppose that you want to add another linear inequality constraint to ensure that the last three assets constitute at least 50\% of your portfolio. You can set up an augmented system of linear inequalities or use the addInequality function to build up linear inequality constraints. For this example, create another system of inequalities:

```
p = PortfolioMAD;
A = [ 1 1 1 0 0 ]; % first inequality constraint
b = 0.5;
p = setInequality(p, A, b);
A = [ 0 0 -1 -1 -1 ]; % second inequality constraint
b = -0.5;
p = addInequality(p, A, b);
disp(p.NumAssets);
disp(p.AInequality);
disp(p.bInequality);
5
\begin{tabular}{rrrrr}
1 & 1 & 1 & 0 & 0 \\
0 & 0 & -1 & -1 & -1
\end{tabular}
0.5000
-0.5000
```

The PortfolioMAD object, setInequality, and addInequality implement scalar expansion on the bInequality property based on the dimension of the matrix in the AInequality property.

## See Also

PortfolioMAD | setBounds | setBudget | setDefaultConstraints| setEquality | setGroupRatio| setGroups | setInequality| setOneWayTurnover| setTurnover

## Related Examples

- "Setting Default Constraints for Portfolio Weights Using PortfolioMAD Object" on page 6-61
- "Creating the PortfolioMAD Object" on page 6-26
- "Validate the MAD Portfolio Problem" on page 6-91
- "Estimate Efficient Portfolios Along the Entire Frontier for PortfolioMAD Object" on page 6-96
- "Estimate Efficient Frontiers for PortfolioMAD Object" on page 6-111
- "Asset Returns and Scenarios Using PortfolioMAD Object" on page 6-42


## More About

- "PortfolioMAD Object" on page 6-21
- "Portfolio Optimization Theory" on page 6-3
- "PortfolioMAD Object Workflow" on page 6-19


## Working with Average Turnover Constraints Using PortfolioMAD Object

The turnover constraint is an optional linear absolute value constraint (see "Average Turnover Constraints" on page 6-15) that enforces an upper bound on the average of purchases and sales. The turnover constraint can be set using the PortfolioMAD object or the setTurnover function. The turnover constraint depends on an initial or current portfolio, which is assumed to be zero if not set when the turnover constraint is set. The turnover constraint has properties Turnover, for the upper bound on average turnover, and InitPort, for the portfolio against which turnover is computed.

## Setting Average Turnover Constraints Using the PortfolioMAD Function

The properties for the turnover constraints are set using the PortfolioMAD object. Suppose that you have an initial portfolio of 10 assets in a variable $x 0$ and you want to ensure that average turnover is no more than $30 \%$. To set this turnover constraint:

```
x0 = [ 0.12; 0.09; 0.08; 0.07; 0.1; 0.1; 0.15; 0.11; 0.08; 0.1 ];
p = PortfolioMAD('Turnover', 0.3, 'InitPort', x0);
disp(p.NumAssets);
disp(p.Turnover);
disp(p.InitPort);
```

10
0.3000
0.1200
0.0900
0.0800
0.0700
0.1000
0.1000
0.1500
0.1100
0.0800
0.1000

Note if the NumAssets or InitPort properties are not set before or when the turnover constraint is set, various rules are applied to assign default values to these properties (see "Setting Up an Initial or Current Portfolio" on page 6-39).

## Setting Average Turnover Constraints Using the setTurnover Function

You can also set properties for portfolio turnover using setTurnover to specify both the upper bound for average turnover and an initial portfolio. Suppose that you have an initial portfolio of 10 assets in a variable $x 0$ and want to ensure that average turnover is no more than $30 \%$. Given a PortfolioMAD object p, use setTurnover to set the turnover constraint with and without the initial portfolio being set previously:

```
x0 = [ 0.12; 0.09; 0.08; 0.07; 0.1; 0.1; 0.15; 0.11; 0.08; 0.1 ];
p = PortfolioMAD('InitPort', x0);
p = setTurnover(p, 0.3);
disp(p.NumAssets);
disp(p.Turnover);
disp(p.InitPort);
1 0
0.3000
0.1200
0.0900
0.0800
0.0700
0.1000
0.1000
0.1500
0.1100
0.0800
0.1000
Or
x0 = [ 0.12; 0.09; 0.08; 0.07; 0.1; 0.1; 0.15; 0.11; 0.08; 0.1 ];
p = PortfolioMAD;
p = setTurnover(p, 0.3, x0);
disp(p.NumAssets);
disp(p.Turnover);
disp(p.InitPort);
1 0
0.3000
0.1200
0.0900
```

0.0800
0.0700
0.1000
0.1000
0.1500
0.1100
0.0800
0.1000
setTurnover implements scalar expansion on the argument for the initial portfolio. If the NumAssets property is already set in the PortfolioMAD object, a scalar argument for InitPort expands to have the same value across all dimensions. In addition, setTurnover lets you specify NumAssets as an optional argument. To clear turnover from your PortfolioMAD object, use the PortfolioMAD object or setTurnover with empty inputs for the properties to be cleared.

## See Also

PortfolioMAD | setBounds | setBudget | setDefaultConstraints| setEquality | setGroupRatio| setGroups| setInequality| setOneWayTurnover| setTurnover

## Related Examples

- "Setting Default Constraints for Portfolio Weights Using PortfolioMAD Object" on page 6-61
- "Creating the PortfolioMAD Object" on page 6-26
- "Validate the MAD Portfolio Problem" on page 6-91
- "Estimate Efficient Portfolios Along the Entire Frontier for PortfolioMAD Object" on page 6-96
- "Estimate Efficient Frontiers for PortfolioMAD Object" on page 6-111
- "Asset Returns and Scenarios Using PortfolioMAD Object" on page 6-42


## More About

- "PortfolioMAD Object" on page 6-21
- "Portfolio Optimization Theory" on page 6-3
- "PortfolioMAD Object Workflow" on page 6-19


## Working with One-Way Turnover Constraints Using PortfolioMAD Object

One-way turnover constraints are optional constraints (see "One-way Turnover Constraints" on page 6-16) that enforce upper bounds on net purchases or net sales. Oneway turnover constraints can be set using the PortfolioMAD object or the setOneWayTurnover function. One-way turnover constraints depend upon an initial or current portfolio, which is assumed to be zero if not set when the turnover constraints are set. One-way turnover constraints have properties BuyTurnover, for the upper bound on net purchases, SellTurnover, for the upper bound on net sales, and InitPort, for the portfolio against which turnover is computed.

## Setting One-Way Turnover Constraints Using the PortfolioMAD Function

The Properties for the one-way turnover constraints are set using the PortfolioMAD object. Suppose that you have an initial portfolio with 10 assets in a variable x0 and you want to ensure that turnover on purchases is no more than $30 \%$ and turnover on sales is no more than $20 \%$ of the initial portfolio. To set these turnover constraints:

```
x0 = [ 0.12; 0.09; 0.08; 0.07; 0.1; 0.1; 0.15; 0.11; 0.08; 0.1 ];
p = PortfolioMAD('BuyTurnover', 0.3, 'SellTurnover', 0.2, 'InitPort', x0);
disp(p.NumAssets);
disp(p.BuyTurnover);
disp(p.SellTurnover);
disp(p.InitPort);
```

10
0.3000
0.2000
0.1200
0.0900
0.0800
0.0700
0.1000
0.1000
0.1500
0.1100
0.0800
0.1000

If the NumAssets or InitPort properties are not set before or when the turnover constraint is set, various rules are applied to assign default values to these properties (see "Setting Up an Initial or Current Portfolio" on page 6-39).

## Setting Turnover Constraints Using the setOneWayTurnover Function

You can also set properties for portfolio turnover using setOneWayTurnover to specify to the upper bounds for turnover on purchases (BuyTurnover) and sales (SellTurnover) and an initial portfolio. Suppose that you have an initial portfolio of 10 assets in a variable $\times 0$ and want to ensure that turnover on purchases is no more than $30 \%$ and that turnover on sales is no more than $20 \%$ of the initial portfolio. Given a PortfolioMAD object $p$, use setOneWayTurnover to set the turnover constraints with and without the initial portfolio being set previously:

```
x0 = [ 0.12; 0.09; 0.08; 0.07; 0.1; 0.1; 0.15; 0.11; 0.08; 0.1 ];
p = PortfolioMAD('InitPort', x0);
p = setOneWayTurnover(p, 0.3, 0.2);
disp(p.NumAssets);
disp(p.BuyTurnover);
disp(p.SellTurnover);
disp(p.InitPort);
or
x0 = [ 0.12; 0.09; 0.08; 0.07; 0.1; 0.1; 0.15; 0.11; 0.08; 0.1 ];
p = PortfolioMAD;
p = setOneWayTurnover(p, 0.3, 0.2, x0);
disp(p.NumAssets);
disp(p.BuyTurnover);
disp(p.SellTurnover);
disp(p.InitPort);
```

setOneWayTurnover implements scalar expansion on the argument for the initial portfolio. If the NumAssets property is already set in the PortfolioMAD object, a scalar argument for InitPort expands to have the same value across all dimensions. In addition, setOneWayTurnover lets you specify NumAssets as an optional argument. To remove one-way turnover from your PortfolioMAD object, use the PortfolioMAD object or setOneWayTurnover with empty inputs for the properties to be cleared.

## See Also

PortfolioMAD | setBounds | setBudget | setDefaultConstraints| setEquality | setGroupRatio| setGroups|setInequality|setOneWayTurnover| setTurnover

## Related Examples

- "Setting Default Constraints for Portfolio Weights Using PortfolioMAD Object" on page 6-61
- "Creating the PortfolioMAD Object" on page 6-26
- "Validate the MAD Portfolio Problem" on page 6-91
- "Estimate Efficient Portfolios Along the Entire Frontier for PortfolioMAD Object" on page 6-96
- "Estimate Efficient Frontiers for PortfolioMAD Object" on page 6-111
- "Asset Returns and Scenarios Using PortfolioMAD Object" on page 6-42


## More About

- "PortfolioMAD Object" on page 6-21
- "Portfolio Optimization Theory" on page 6-3
- "PortfolioMAD Object Workflow" on page 6-19


## Validate the MAD Portfolio Problem

In this section...<br>"Validating a MAD Portfolio Set" on page 6-91<br>"Validating MAD Portfolios" on page 6-93

Sometimes, you may want to validate either your inputs to, or outputs from, a portfolio optimization problem. Although most error checking that occurs during the problem setup phase catches most difficulties with a portfolio optimization problem, the processes to validate MAD portfolio sets and portfolios are time consuming and are best done offline. So, the portfolio optimization tools have specialized functions to validate MAD portfolio sets and portfolios. For information on the workflow when using PortfolioMAD objects, see "PortfolioMAD Object Workflow" on page 6-19.

## Validating a MAD Portfolio Set

Since it is necessary and sufficient that your MAD portfolio set must be a nonempty, closed, and bounded set to have a valid portfolio optimization problem, the estimateBounds function lets you examine your portfolio set to determine if it is nonempty and, if nonempty, whether it is bounded. Suppose that you have the following MAD portfolio set which is an empty set because the initial portfolio at 0 is too far from a portfolio that satisfies the budget and turnover constraint:

```
p = PortfolioMAD('NumAssets', 3, 'Budget', 1);
p = setTurnover(p, 0.3, 0);
```

If a MAD portfolio set is empty, estimateBounds returns NaN bounds and sets the isbounded flag to []:
[lb, ub, isbounded] = estimateBounds(p)
lb =
NaN
NaN
NaN
$u b=$
NaN
NaN

NaN
isbounded =
[]
Suppose that you create an unbounded MAD portfolio set as follows:

```
p = PortfolioMAD('AInequality', [1 -1; 1 1 ], 'bInequality', 0);
[lb, ub, isbounded] = estimateBounds(p)
lb =
    -Inf
    -Inf
ub =
    1.0e-008
    -0.3712
        Inf
isbounded =
```

0
In this case, estimateBounds returns (possibly infinite) bounds and sets the isbounded flag to false. The result shows which assets are unbounded so that you can apply bound constraints as necessary.

Finally, suppose that you created a MAD object that is both nonempty and bounded. estimateBounds not only validates the set, but also obtains tighter bounds which are useful if you are concerned with the actual range of portfolio choices for individual assets in your portfolio:

```
p = PortfolioMAD;
p = setBudget(p, 1,1);
p = setBounds(p, [ -0.1; 0.2; 0.3; 0.2 ], [ 0.5; 0.3; 0.9; 0.8 ]);
[lb, ub, isbounded] = estimateBounds(p)
lb =
    0.1000
    0.2000
```

```
    0.3000
    0.2000
ub =
    0.3000
    0.3000
    0.7000
    0.6000
isbounded =
    1
```

In this example, all but the second asset has tighter upper bounds than the input upper bound implies.

## Validating MAD Portfolios

Given a MAD portfolio set specified in a PortfolioMAD object, you often want to check if specific portfolios are feasible with respect to the portfolio set. This can occur with, for example, initial portfolios and with portfolios obtained from other procedures. The checkFeasibility function determines whether a collection of portfolios is feasible. Suppose that you perform the following portfolio optimization and want to determine if the resultant efficient portfolios are feasible relative to a modified problem.

First, set up a problem in the PortfolioMAD object p, estimate efficient portfolios in pwgt, and then confirm that these portfolios are feasible relative to the initial problem:

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;
AssetScenarios = mvnrnd(m, C, 20000);
p = PortfolioMAD;
p = setScenarios(p, AssetScenarios);
p = setDefaultConstraints(p);
```

```
pwgt = estimateFrontier(p);
checkFeasibility(p, pwgt)
ans =
    1 1 1 1 1 
```

Next, set up a different portfolio problem that starts with the initial problem with an additional a turnover constraint and an equally weighted initial portfolio:

```
q = setTurnover(p, 0.3, 0.25);
```

checkFeasibility(q, pwgt)
ans =

| 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

In this case, only two of the 10 efficient portfolios from the initial problem are feasible relative to the new problem in PortfolioMAD object q. Solving the second problem using checkFeasibility demonstrates that the efficient portfolio for PortfolioMAD object q is feasible relative to the initial problem:

```
qwgt = estimateFrontier(q);
checkFeasibility(p, qwgt)
ans =
\begin{tabular}{llllllllll}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1
\end{tabular}
```


## See Also

PortfolioMAD|checkFeasibility|estimateBounds

## Related Examples

- "Creating the PortfolioMAD Object" on page 6-26
- "Working with MAD Portfolio Constraints Using Defaults" on page 6-61
- "Estimate Efficient Portfolios Along the Entire Frontier for PortfolioMAD Object" on page 6-96
- "Estimate Efficient Frontiers for PortfolioMAD Object" on page 6-111
- "Asset Returns and Scenarios Using PortfolioMAD Object" on page 6-42


## More About

- "PortfolioMAD Object" on page 6-21
- "Portfolio Optimization Theory" on page 6-3
- "PortfolioMAD Object Workflow" on page 6-19


## Estimate Efficient Portfolios Along the Entire Frontier for PortfolioMAD Object

There are two ways to look at a portfolio optimization problem that depends on what you are trying to do. One goal is to estimate efficient portfolios and the other is to estimate efficient frontiers. This section focuses on the former goal and "Estimate Efficient Frontiers for PortfolioMAD Object" on page 6-111 focuses on the latter goal. For information on the workflow when using PortfolioMAD objects, see "PortfolioMAD Object Workflow" on page 6-19.

## Obtaining Portfolios Along the Entire Efficient Frontier

The most basic way to obtain optimal portfolios is to obtain points over the entire range of the efficient frontier. Given a portfolio optimization problem in a PortfolioMAD object, the estimateFrontier function computes efficient portfolios spaced evenly according to the return proxy from the minimum to maximum return efficient portfolios. The number of portfolios estimated is controlled by the hidden property defaultNumPorts which is set to 10. A different value for the number of portfolios estimated is specified as input to estimateFrontier. This example shows the default number of efficient portfolios over the entire range of the efficient frontier:

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;
AssetScenarios = mvnrnd(m, C, 20000);
p = PortfolioMAD;
p = setScenarios(p, AssetScenarios);
p = setDefaultConstraints(p);
pwgt = estimateFrontier(p);
disp(pwgt);
Columns 1 through }
\begin{tabular}{lrrrrrrr}
0.8907 & 0.7289 & 0.5614 & 0.3946 & 0.2257 & 0.0612 & 0 & 0.0000 \\
0.0330 & 0.1163 & 0.2119 & 0.3042 & 0.3998 & 0.4876 & 0.4400 & 0.3125 \\
0.0420 & 0.0469 & 0.0472 & 0.0505 & 0.0534 & 0.0580 & 0.0374 & 0.0018 \\
0.0343 & 0.1079 & 0.1794 & 0.2507 & 0.3211 & 0.3933 & 0.5226 & 0.6857
\end{tabular}
```

Columns 9 through 10

| 0.0000 | 0.0000 |
| :--- | :--- |
| 0.1570 | 0.0000 |
| 0.0000 | 0.0000 |
| 0.8430 | 1.0000 |

If you want only four portfolios in the previous example:

```
pwgt = estimateFrontier(p, 4);
disp(pwgt);
\begin{tabular}{lrrr}
0.8907 & 0.3946 & 0 & 0.0000 \\
0.0330 & 0.3042 & 0.4401 & 0.0000 \\
0.0420 & 0.0505 & 0.0373 & 0.0000 \\
0.0343 & 0.2507 & 0.5227 & 1.0000
\end{tabular}
```

Starting from the initial portfolio, estimateFrontier also returns purchases and sales to get from your initial portfolio to each efficient portfolio on the efficient frontier. For example, given an initial portfolio in pwgt0, you can obtain purchases and sales:

```
pwgt0 = [ 0.3; 0.3; 0.2; 0.1 ];
p = setInitPort(p, pwgt0);
[pwgt, pbuy, psell] = estimateFrontier(p);
display(pwgt);
display(pbuy);
display(psell);
pwgt =
```

    Columns 1 through 8
    | 0.8907 | 0.7289 | 0.5614 | 0.3946 | 0.2257 | 0.0612 | 0 | 0.0000 |
| :--- | :--- | :--- | :--- | :--- | :--- | ---: | ---: |
| 0.0330 | 0.1163 | 0.2119 | 0.3042 | 0.3998 | 0.4876 | 0.4400 | 0.3125 |
| 0.0420 | 0.0469 | 0.0472 | 0.0505 | 0.0534 | 0.0580 | 0.0374 | 0.0018 |
| 0.0343 | 0.1079 | 0.1794 | 0.2507 | 0.3211 | 0.3933 | 0.5226 | 0.6857 |

Columns 9 through 10
$0.0000 \quad 0.0000$
$0.1570 \quad 0.0000$
$0.0000 \quad 0.0000$
$0.8430 \quad 1.0000$
pbuy $=$
Columns 1 through 8

| 0.5907 | 0.4289 | 0.2614 | 0.0946 | 0 | 0 | 0 | 0 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 0 | 0 | 0 | 0.0042 | 0.0998 | 0.1876 | 0.1400 | 0.0125 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0.0079 | 0.0794 | 0.1507 | 0.2211 | 0.2933 | 0.4226 | 0.5857 |

Columns 9 through 10

| 0 | 0 |
| ---: | ---: |
| 0 | 0 |
| 0 | 0 |
| 0.7430 | 0.9000 |

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```
psell =
    Columns 1 through 8
\begin{tabular}{rrrrrrrr}
0 & 0 & 0 & 0 & 0.0743 & 0.2388 & 0.3000 & 0.3000 \\
0.2670 & 0.1837 & 0.0881 & 0 & 0 & 0 & 0 & 0 \\
0.1580 & 0.1531 & 0.1528 & 0.1495 & 0.1466 & 0.1420 & 0.1626 & 0.1982 \\
0.0657 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{tabular}
Columns 9 through 10
```

$0.3000 \quad 0.3000$
$0.1430 \quad 0.3000$
$0.2000 \quad 0.2000$

If you do not specify an initial portfolio, the purchase and sale weights assume that your initial portfolio is 0 .

## See Also

PortfolioMAD |estimateFrontier|estimateFrontierByReturn | estimateFrontierByRisk|estimateFrontierByRisk| estimateFrontierLimits|estimatePortReturn|estimatePortRisk| setSolver

## Related Examples

- "Obtaining Endpoints of the Efficient Frontier" on page 6-100
- "Obtaining Efficient Portfolios for Target Returns" on page 6-103
- "Obtaining Efficient Portfolios for Target Risks" on page 6-106
- "Obtaining MAD Portfolio Risks and Returns" on page 6-111
- "Obtaining the PortfolioMAD Standard Deviation" on page 6-113
- "Plotting the Efficient Frontier for a PortfolioMAD Object" on page 6-115
- "Creating the PortfolioMAD Object" on page 6-26
- "Working with MAD Portfolio Constraints Using Defaults" on page 6-61
- "Estimate Efficient Frontiers for PortfolioMAD Object" on page 6-111
- "Asset Returns and Scenarios Using PortfolioMAD Object" on page 6-42


## More About

- "PortfolioMAD Object" on page 6-21
- "Portfolio Optimization Theory" on page 6-3
- "PortfolioMAD Object Workflow" on page 6-19


## Obtaining Endpoints of the Efficient Frontier

Often, you might be interested in the endpoint portfolios for the efficient frontier. Suppose that you want to determine the range of returns from minimum to maximum to refine a search for a portfolio with a specific target return. Use the estimateFrontierLimits function to obtain the endpoint portfolios:

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;
AssetScenarios = mvnrnd(m, C, 20000);
p = PortfolioMAD;
p = setScenarios(p, AssetScenarios);
p = setDefaultConstraints(p);
pwgt = estimateFrontierLimits(p);
disp(pwgt);
    0.8875 0.0000
    0.0373 0.0000
    0.0386 0.0000
    0.0366 1.0000
```

Note The endpoints of the efficient frontier depend upon the Scenarios in the PortfolioMAD object. If you change the Scenarios, you are likely to obtain different endpoints.

Starting from an initial portfolio, estimateFrontierLimits also returns purchases and sales to get from the initial portfolio to the endpoint portfolios on the efficient frontier. For example, given an initial portfolio in pwgt0, you can obtain purchases and sales:

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
```

```
m = m/12
C = C/12;
```

AssetScenarios = mvnrnd(m, C, 20000);
p = PortfolioMAD;
p = setScenarios(p, AssetScenarios);
p = setDefaultConstraints(p);
pwgt0 = [ 0.3; 0.3; 0.2; 0.1 ];
p = setInitPort(p, pwgt0);
[pwgt, pbuy, psell] = estimateFrontierLimits(p);
display(pwgt);
display(pbuy);
display(psell);
pwgt =

| 0.8927 | 0.0000 |
| :--- | :--- |
| 0.0334 | 0.0000 |
| 0.0422 | 0.0000 |
| 0.0317 | 1.0000 |

pbuy =
0.59270
$0 \quad 0$
$0 \quad 0$
$0 \quad 0.9000$
psell =

| 0 | 0.3000 |
| ---: | ---: |
| 0.2666 | 0.3000 |
| 0.1578 | 0.2000 |
| 0.0683 | 0 |

If you do not specify an initial portfolio, the purchase and sale weights assume that your initial portfolio is 0 .

## See Also

PortfolioMAD |estimateFrontier|estimateFrontierByReturn| estimateFrontierByRisk|estimateFrontierByRisk|
estimateFrontierLimits|estimatePortReturn|estimatePortRisk| setSolver

## Related Examples

- "Estimate Efficient Portfolios Along the Entire Frontier for PortfolioMAD Object" on page 6-96
- "Creating the PortfolioMAD Object" on page 6-26
- "Working with MAD Portfolio Constraints Using Defaults" on page 6-61
- "Estimate Efficient Frontiers for PortfolioMAD Object" on page 6-111
- "Asset Returns and Scenarios Using PortfolioMAD Object" on page 6-42


## More About

- "PortfolioMAD Object" on page 6-21
- "Portfolio Optimization Theory" on page 6-3
- "PortfolioMAD Object Workflow" on page 6-19


## Obtaining Efficient Portfolios for Target Returns

To obtain efficient portfolios that have targeted portfolio returns, the estimateFrontierByReturn function accepts one or more target portfolio returns and obtains efficient portfolios with the specified returns. For example, assume that you have a universe of four assets where you want to obtain efficient portfolios with target portfolio returns of $7 \%, 10 \%$, and $12 \%$ :

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
AssetScenarios = mvnrnd(m, C, 20000);
p = PortfolioMAD;
p = setScenarios(p, AssetScenarios);
p = setDefaultConstraints(p);
pwgt = estimateFrontierByReturn(p, [0.07, 0.10, .12]);
display(pwgt);
pwgt =
\begin{tabular}{lll}
0.7537 & 0.3899 & 0.1478 \\
0.1113 & 0.2934 & 0.4136 \\
0.0545 & 0.1006 & 0.1319 \\
0.0805 & 0.2161 & 0.3066
\end{tabular}
```

Sometimes, you can request a return for which no efficient portfolio exists. Based on the previous example, suppose that you want a portfolio with a $4 \%$ return (which is the return of the first asset). A portfolio that is fully invested in the first asset, however, is inefficient. estimateFrontierByReturn warns if your target returns are outside the range of efficient portfolio returns and replaces it with the endpoint portfolio of the efficient frontier closest to your target return:

```
pwgt = estimateFrontierByReturn(p, [0.04]);
Warning: One or more target return values are outside the feasible range [
0.0591121, 0.182542 ].
    Will return portfolios associated with endpoints of the range for these values.
> In PortfolioMAD.estimateFrontierByReturn at 90
```

The best way to avoid this situation is to bracket your target portfolio returns with estimateFrontierLimits and estimatePortReturn (see "Obtaining Endpoints of
the Efficient Frontier" on page 6-100 and "Obtaining MAD Portfolio Risks and Returns" on page 6-111).

```
pret = estimatePortReturn(p, p.estimateFrontierLimits);
display(pret);
pret =
    0.0591
    0.1825
```

This result indicates that efficient portfolios have returns that range from $6.5 \%$ to $17.8 \%$. Note, your results for these examples may be different due to the random generation of scenarios.

If you have an initial portfolio, estimateFrontierByReturn also returns purchases and sales to get from your initial portfolio to the target portfolios on the efficient frontier. For example, given an initial portfolio in pwgt0, to obtain purchases and sales with target returns of $7 \%, 10 \%$, and $12 \%$ :

```
pwgt0 = [ 0.3; 0.3; 0.2; 0.1 ];
p = setInitPort(p, pwgt0);
[pwgt, pbuy, psell] = estimateFrontierByReturn(p, [0.07, 0.10, .12]);
display(pwgt);
display(pbuy);
display(psell);
pwgt =
\begin{tabular}{lll}
0.7537 & 0.3899 & 0.1478 \\
0.1113 & 0.2934 & 0.4136 \\
0.0545 & 0.1006 & 0.1319 \\
0.0805 & 0.2161 & 0.3066
\end{tabular}
pbuy =
    0.4537 0.0899 0
        0 0 0.1136
        0 0 0
        0 0.1161 0.2066
```

psell =

| 0 | 0 | 0.1522 |
| ---: | ---: | ---: |
| 0.1887 | 0.0066 | 0 |
| 0.1455 | 0.0994 | 0.0681 |
| 0.0195 | 0 | 0 |

If you do not have an initial portfolio, the purchase and sale weights assume that your initial portfolio is 0 .

## See Also

PortfolioMAD|estimateFrontier|estimateFrontierByReturn| estimateFrontierByRisk|estimateFrontierByRisk|
estimateFrontierLimits|estimatePortReturn|estimatePortRisk| setSolver

## Related Examples

- "Estimate Efficient Portfolios Along the Entire Frontier for PortfolioMAD Object" on page 6-96
- "Creating the PortfolioMAD Object" on page 6-26
- "Working with MAD Portfolio Constraints Using Defaults" on page 6-61
- "Estimate Efficient Frontiers for PortfolioMAD Object" on page 6-111
- "Asset Returns and Scenarios Using PortfolioMAD Object" on page 6-42


## More About

- "PortfolioMAD Object" on page 6-21
- "Portfolio Optimization Theory" on page 6-3
- "PortfolioMAD Object Workflow" on page 6-19


## Obtaining Efficient Portfolios for Target Risks

To obtain efficient portfolios that have targeted portfolio risks, the estimateFrontierByRisk function accepts one or more target portfolio risks and obtains efficient portfolios with the specified risks. Suppose that you have a universe of four assets where you want to obtain efficient portfolios with target portfolio risks of 12\%, $14 \%$, and $16 \%$.

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
AssetScenarios = mvnrnd(m, C, 20000);
p = PortfolioMAD;
p = setScenarios(p, AssetScenarios);
p = setDefaultConstraints(p);
pwgt = estimateFrontierByRisk(p, [0.12, 0.14, 0.16]);
display(pwgt);
pwgt =
\begin{tabular}{lrr}
0.2102 & 0.0621 & 0 \\
0.3957 & 0.4723 & 0.4305 \\
0.1051 & 0.1204 & 0.1291 \\
0.2889 & 0.3452 & 0.4404
\end{tabular}
```

Sometimes, you can request a risk for which no efficient portfolio exists. Based on the previous example, suppose that you want a portfolio with $6 \%$ risk (individual assets in this universe have risks ranging from 7\% to 42.5\%). It turns out that a portfolio with $6 \%$ risk cannot be formed with these four assets. estimateFrontierByRisk warns if your target risks are outside the range of efficient portfolio risks and replaces it with the endpoint of the efficient frontier closest to your target risk:

```
pwgt = estimateFrontierByRisk(p, 0.06)
Warning: One or more target risk values are outside the feasible range [
0.0610574, 0.278711 ].
    Will return portfolios associated with endpoints of the range for these values.
> In PortfolioMAD.estimateFrontierByRisk at 82
```

The best way to avoid this situation is to bracket your target portfolio risks with estimateFrontierLimits and estimatePortRisk (see "Obtaining Endpoints of the Efficient Frontier" on page 6-100 and "Estimate Efficient Frontiers for PortfolioMAD Object" on page 6-111).

```
prsk = estimatePortRisk(p, p.estimateFrontierLimits);
display(prsk);
prsk =
```

    0.0611
    0.2787
    This result indicates that efficient portfolios have risks that range from $7 \%$ to $42.5 \%$. Note, your results for these examples may be different due to the random generation of scenarios.

Starting with an initial portfolio, estimateFrontierByRisk also returns purchases and sales to get from your initial portfolio to the target portfolios on the efficient frontier. For example, given an initial portfolio in pwgt0, you can obtain purchases and sales from the example with target risks of $12 \%, 14 \%$, and $16 \%$ :

```
pwgt0 = [ 0.3; 0.3; 0.2; 0.1 ];
p = setInitPort(p, pwgt0);
[pwgt, pbuy, psell] = estimateFrontierByRisk(p, [0.12, 0.14, 0.16]);
display(pwgt);
display(pbuy);
display(psell);
pwgt =
\begin{tabular}{lrr}
0.2102 & 0.0621 & 0 \\
0.3957 & 0.4723 & 0.4305 \\
0.1051 & 0.1204 & 0.1291 \\
0.2889 & 0.3452 & 0.4404
\end{tabular}
pbuy =
```

| 0 | 0 | 0 |
| ---: | ---: | ---: |
| 0.0957 | 0.1723 | 0.1305 |
| 0 | 0 | 0 |
| 0.1889 | 0.2452 | 0.3404 |

psell =

| 0.0898 | 0.2379 | 0.3000 |
| ---: | ---: | ---: |
| 0 | 0 | 0 |
| 0.0949 | 0.0796 | 0.0709 |
| 0 | 0 | 0 |

If you do not specify an initial portfolio, the purchase and sale weights assume that your initial portfolio is 0 .

## See Also

PortfolioMAD|estimateFrontier|estimateFrontierByReturn| estimateFrontierByRisk |estimateFrontierByRisk| estimateFrontierLimits|estimatePortReturn|estimatePortRisk| setSolver

## Related Examples

- "Estimate Efficient Portfolios Along the Entire Frontier for PortfolioMAD Object" on page 6-96
- "Creating the PortfolioMAD Object" on page 6-26
- "Working with MAD Portfolio Constraints Using Defaults" on page 6-61
- "Estimate Efficient Frontiers for PortfolioMAD Object" on page 6-111
- "Asset Returns and Scenarios Using PortfolioMAD Object" on page 6-42


## More About

- "PortfolioMAD Object" on page 6-21
- "Portfolio Optimization Theory" on page 6-3
- "PortfolioMAD Object Workflow" on page 6-19


## Choosing and Controlling the Solver

When solving portfolio optimizations for a PortfolioMAD object, while all variations of fmincon from Optimization Toolbox are supported, using 'sqp' and 'active-set' algorithms for fmincon is recommended and the use of 'interior-point' algorithm is not recommended for MAD portfolio optimization.

Unlike Optimization Toolbox which uses the 'trust-region-reflective' algorithm as the default algorithm for fmincon, the portfolio optimization for a PortfolioMAD object uses the 'active-set ' algorithm. For details about fmincon and constrained nonlinear optimization algorithms and options, see "Constrained Nonlinear Optimization Algorithms" (Optimization Toolbox).

To modify fmincon options for MAD portfolio optimizations, use setSolver to set the hidden properties solverType and solverOptions to specify and control the solver. Since these solver properties are hidden, you cannot set them using the PortfolioMAD object. The default solver is fmincon with the 'sqb' algorithm and no displayed output, so you do not need to use setSolver to specify this.

If you want to specify additional options associated with the fmincon solver, setSolver accepts these options as name-value pair arguments. For example, if you want to use fmincon with the sqp algorithm and with displayed output, use setSolver with:

```
p = PortfolioMAD
p = setSolver(p, 'fmincon', 'Algorithm', 'sqp', 'Display', 'final');
display(p.solverOptions.Algorithm);
display(p.solverOptions.Display);
sqp
final
```

Alternatively, the setSolver function accepts an optimoptions object as the second argument. For example, you can change the algorithm to trust-region-reflective with no displayed output as follows:

```
p = PortfolioMAD;
options = optimoptions('fmincon', 'Algorithm', 'trust-region-reflective', 'Display', 'off');
p = setSolver(p, 'fmincon', options);
display(p.solverOptions.Algorithm);
display(p.solverOptions.Display);
trust-region-reflective
off
```


## See Also

PortfolioMAD |estimateFrontier|estimateFrontierByReturn| estimateFrontierByRisk|estimateFrontierByRisk|
estimateFrontierLimits|estimatePortReturn|estimatePortRisk| setSolver

## Related Examples

- "Estimate Efficient Portfolios Along the Entire Frontier for PortfolioMAD Object" on page 6-96
- "Creating the PortfolioMAD Object" on page 6-26
- "Working with MAD Portfolio Constraints Using Defaults" on page 6-61
- "Estimate Efficient Frontiers for PortfolioMAD Object" on page 6-111
- "Asset Returns and Scenarios Using PortfolioMAD Object" on page 6-42


## More About

- "PortfolioMAD Object" on page 6-21
- "Portfolio Optimization Theory" on page 6-3
- "PortfolioMAD Object Workflow" on page 6-19


## Estimate Efficient Frontiers for PortfolioMAD Object

In this section...<br>"Obtaining MAD Portfolio Risks and Returns" on page 6-111<br>"Obtaining the PortfolioMAD Standard Deviation" on page 6-113

Whereas "Estimate Efficient Portfolios Along the Entire Frontier for PortfolioMAD Object" on page 6-96 focused on estimation of efficient portfolios, this section focuses on the estimation of efficient frontiers. For information on the workflow when using PortfolioMAD objects, see "PortfolioMAD Object Workflow" on page 6-19.

## Obtaining MAD Portfolio Risks and Returns

Given any portfolio and, in particular, efficient portfolios, the functions estimatePortReturn and estimatePortRisk provide estimates for the return (or return proxy), risk (or the risk proxy). Each function has the same input syntax but with different combinations of outputs. Suppose that you have this following portfolio optimization problem that gave you a collection of portfolios along the efficient frontier in pwgt:

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
0.00408 0.0289 0.0204 0.0119;
0.00192 0.0204 0.0576 0.0336;
0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;
AssetScenarios = mvnrnd(m, C, 20000);
p = PortfolioMAD;
p = setScenarios(p, AssetScenarios);
p = setDefaultConstraints(p);
pwgt0 = [ 0.3; 0.3; 0.2; 0.1 ];
p = setInitPort(p, pwgt0);
pwgt = estimateFrontier(p)
```

pwgt =

| 0.8954 | 0.7264 | 0.5573 | 0.3877 | 0.2176 | 0.0495 | 0.0000 | 0 |
| :--- | ---: | :--- | :--- | :--- | :--- | ---: | ---: |
| 0.0310 | 0.1239 | 0.2154 | 0.3081 | 0.4028 | 0.4924 | 0.4069 | 0.2386 |
| 0.0409 | 0.0524 | 0.0660 | 0.0792 | 0.0907 | 0.1047 | 0.1054 | 0.1132 |
| 0.0328 | 0.0973 | 0.1613 | 0.2250 | 0.2890 | 0.3534 | 0.4877 | 0.6482 |

Columns 9 through 10

| 0 | 0.0000 |
| ---: | ---: |
| 0.0694 | 0.0000 |
| 0.1221 | 0.0000 |
| 0.8084 | 1.0000 |

Note Remember that the risk proxy for MAD portfolio optimization is mean-absolute deviation.

Given pwgt0 and pwgt, use the portfolio risk and return estimation functions to obtain risks and returns for your initial portfolio and the portfolios on the efficient frontier:

```
prsk0 = estimatePortRisk(p, pwgt0);
pret0 = estimatePortReturn(p, pwgt0);
prsk = estimatePortRisk(p, pwgt);
pret = estimatePortReturn(p, pwgt);
display(prsk0);
display(pret0);
display(prsk);
display(pret);
You obtain these risks and returns:
prsk0 =
    0.0256
pret0 =
    0.0072
prsk =
    0.0178
    0.0193
    0.0233
    0.0286
```

0.0348
0.0414
0.0489
0.0584
0.0692
0.0809

## pret =

0.0047
0.0059
0.0072
0.0084
0.0096
0.0108
0.0120
0.0133
0.0145
0.0157

## Obtaining the PortfolioMAD Standard Deviation

The PortfolioMAD object has a function to compute standard deviations of portfolio returns, estimatePortStd. This function works with any portfolios, not necessarily efficient portfolios. For example, the following example obtains five portfolios (pwgt) on the efficient frontier and also has an initial portfolio in pwgt0. Various portfolio statistics are computed that include the return, risk, and standard deviation. The listed estimates are for the initial portfolio in the first row followed by estimates for each of the five efficient portfolios in subsequent rows.

```
m = [ 0.0042; 0.0083; 0.01; 0.15 ];
C = [ 0.005333 0.00034 0.00016 0;
0.00034 0.002408 0.0017 0.000992;
0.00016 0.0017 0.0048 0.0028;
0 0.000992 0.0028 0.010208 ];
pwgt0 = [ 0.3; 0.3; 0.2; 0.1 ];
p = PortfolioMAD('initport', pwgt0);
p = simulateNormalScenariosByMoments(p, m, C, 20000);
p = setDefaultConstraints(p);
```

```
pwgt = estimateFrontier(p, 5);
pret = estimatePortReturn(p, [pwgt0, pwgt]);
prsk = estimatePortRisk(p, [pwgt0, pwgt]);
pstd = estimatePortStd(p, [pwgt0, pwgt]);
[pret, prsk, pstd]
ans =
\begin{tabular}{lll}
0.0212 & 0.0305 & 0.0381 \\
0.0187 & 0.0326 & 0.0407 \\
0.0514 & 0.0369 & 0.0462 \\
0.0841 & 0.0484 & 0.0607 \\
0.1168 & 0.0637 & 0.0796 \\
0.1495 & 0.0807 & 0.1009
\end{tabular}
```


## See Also

PortfolioMAD|estimatePortReturn|estimatePortStd|plotFrontier

## Related Examples

- "Plotting the Efficient Frontier for a PortfolioMAD Object" on page 6-115
- "Creating the PortfolioMAD Object" on page 6-26
- "Working with MAD Portfolio Constraints Using Defaults" on page 6-61
- "Estimate Efficient Portfolios Along the Entire Frontier for PortfolioMAD Object" on page 6-96
- "Asset Returns and Scenarios Using PortfolioMAD Object" on page 6-42
- "Postprocessing Results to Set Up Tradable Portfolios" on page 6-122


## More About

- "PortfolioMAD Object" on page 6-21
- "Portfolio Optimization Theory" on page 6-3
- "PortfolioMAD Object Workflow" on page 6-19


## Plotting the Efficient Frontier for a PortfolioMAD Object

The plotFrontier function creates a plot of the efficient frontier for a given portfolio optimization problem. This function accepts several types of inputs and generates a plot with an optional possibility to output the estimates for portfolio risks and returns along the efficient frontier. plotFrontier has four different ways that it can be used. In addition to a plot of the efficient frontier, if you have an initial portfolio in the InitPort property, plotFrontier also displays the return versus risk of the initial portfolio on the same plot. If you have a well-posed portfolio optimization problem set up in a PortfolioMAD object and you use plotFrontier, you get a plot of the efficient frontier with the default number of portfolios on the frontier (the default number is 10 and is maintained in the hidden property defaultNumPorts). This example illustrates a typical use of plotFrontier to create a new plot:

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;
AssetScenarios = mvnrnd(m, C, 20000);
p = PortfolioMAD;
p = setScenarios(p, AssetScenarios);
p = setDefaultConstraints(p);
plotFrontier(p);
```



The Name property appears as the title of the efficient frontier plot if you set it in the PortfolioMAD object. Without an explicit name, the title on the plot would be "Efficient Frontier." If you want to obtain a specific number of portfolios along the efficient frontier, use plotFrontier with the number of portfolios that you want. Suppose that you have the PortfolioMAD object from the previous example and you want to plot 20 portfolios along the efficient frontier and to obtain 20 risk and return values for each portfolio:

```
[prsk, pret] = plotFrontier(p, 20);
display([pret, prsk]);
ans =
\begin{tabular}{ll}
0.0049 & 0.0176 \\
0.0054 & 0.0179 \\
0.0058 & 0.0189 \\
0.0063 & 0.0205 \\
0.0068 & 0.0225 \\
0.0073 & 0.0248 \\
0.0078 & 0.0274 \\
0.0083 & 0.0302
\end{tabular}
```

| 0.0088 | 0.0331 |
| :--- | :--- |
| 0.0093 | 0.0361 |
| 0.0098 | 0.0392 |
| 0.0103 | 0.0423 |
| 0.0108 | 0.0457 |
| 0.0112 | 0.0496 |
| 0.0117 | 0.0539 |
| 0.0122 | 0.0586 |
| 0.0127 | 0.0635 |
| 0.0132 | 0.0687 |
| 0.0137 | 0.0744 |
| 0.0142 | 0.0806 |



## Plotting Existing Efficient Portfolios

If you already have efficient portfolios from any of the "estimateFrontier" functions (see "Estimate Efficient Frontiers for PortfolioMAD Object" on page 6-111), pass them into plotFrontier directly to plot the efficient frontier:

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;
```

AssetScenarios $=$ mvnrnd(m, C, 20000);
pwgt0 = [ 0.3; 0.3; 0.2; 0.1 ];
p = PortfolioMAD('Name', 'Asset Allocation Portfolio', 'InitPort', pwgt0);
$\mathrm{p}=$ setScenarios( p, AssetScenarios);
p = setDefaultConstraints(p);
pwgt = estimateFrontier(p, 20);
plotFrontier(p, pwgt)


## Plotting Existing Efficient Portfolio Risks and Returns

If you already have efficient portfolio risks and returns, you can use the interface to plotFrontier to pass them into plotFrontier to obtain a plot of the efficient frontier:

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
AssetScenarios = mvnrnd(m, C, 20000);
pwgt0 = [ 0.3; 0.3; 0.2; 0.1 ];
p = PortfolioMAD('Name', 'Asset Allocation Portfolio', 'InitPort', pwgt0);
p = setScenarios(p, AssetScenarios);
p = setDefaultConstraints(p);
pwgt = estimateFrontier(p);
pret= estimatePortReturn(p, pwgt)
prsk = estimatePortRisk(p, pwgt)
plotFrontier(p, prsk, pret)
pret =
    0.0590
    0.0723
    0.0857
    0.0991
    0.1124
    0.1258
    0.1391
    0.1525
    0.1658
    0.1792
prsk =
    0.0615
    0.0664
    0.0795
    0.0976
    0.1184
    0.1408
```

0.1663
0.1992
0.2368
0.2787


## See Also

PortfolioMAD|estimatePortReturn|estimatePortStd|plotFrontier

## Related Examples

- "Estimate Efficient Frontiers for PortfolioMAD Object" on page 6-111
- "Creating the PortfolioMAD Object" on page 6-26
- "Working with MAD Portfolio Constraints Using Defaults" on page 6-61
- "Estimate Efficient Portfolios Along the Entire Frontier for PortfolioMAD Object" on page 6-96
- "Asset Returns and Scenarios Using PortfolioMAD Object" on page 6-42
- "Postprocessing Results to Set Up Tradable Portfolios" on page 6-122


## More About

- "PortfolioMAD Object" on page 6-21
- "Portfolio Optimization Theory" on page 6-3
- "PortfolioMAD Object Workflow" on page 6-19


## Postprocessing Results to Set Up Tradable Portfolios

After obtaining efficient portfolios or estimates for expected portfolio risks and returns, use your results to set up trades to move toward an efficient portfolio. For information on the workflow when using PortfolioMAD objects, see "PortfolioMAD Object Workflow" on page 6-19.

## Setting Up Tradable Portfolios

Suppose that you set up a portfolio optimization problem and obtained portfolios on the efficient frontier. Use the dataset object from Statistics and Machine Learning Toolbox to form a blotter that lists your portfolios with the names for each asset. For example, suppose that you want to obtain five portfolios along the efficient frontier. You can set up a blotter with weights multiplied by 100 to view the allocations for each portfolio:

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
0.00408 0.0289 0.0204 0.0119;
0.00192 0.0204 0.0576 0.0336;
0 0.0119 0.0336 0.1225 ];
pwgt0 = [ 0.3; 0.3; 0.2; 0.1 ];
p = PortfolioMAD;
p = setAssetList(p, 'Bonds','Large-Cap Equities','Small-Cap Equities','Emerging Equities');
p = setInitPort(p, pwgt0);
p = simulateNormalScenariosByMoments(p, m, C, 20000);
p = setDefaultConstraints(p);
pwgt = estimateFrontier(p, 5);
pnames = cell(1,5);
for i = 1:5
pnames{i} = sprintf('Port%d',i);
end
Blotter = dataset([{100*pwgt},pnames],'obsnames',p.AssetList);
display(Blotter);
Blotter =
\begin{tabular}{llllrr} 
& Port1 & Port2 & Port3 & \multicolumn{1}{l}{ Port4 } & Port5 \\
Bonds & 88.154 & 50.867 & 13.611 & 0 & \(1.0609 \mathrm{e}-12\) \\
Large-Cap Equities & 4.0454 & 22.571 & 41.276 & 23.38 & \(7.9362 \mathrm{e}-13\) \\
Small-Cap Equities & 4.2804 & 9.3108 & 14.028 & 17.878 & \(6.4823 \mathrm{e}-14\) \\
Emerging Equities & 3.5202 & 17.252 & 31.084 & 58.743 & 100
\end{tabular}
```

Note Your results may differ from this result due to the simulation of scenarios.

This result indicates that you would invest primarily in bonds at the minimum-risk/ minimum-return end of the efficient frontier (Port1), and that you would invest completely in emerging equity at the maximum-risk/maximum-return end of the efficient frontier (Port5). You can also select a particular efficient portfolio, for example, suppose that you want a portfolio with $15 \%$ risk and you add purchase and sale weights outputs obtained from the "estimateFrontier" functions to set up a trade blotter:

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
0.00408 0.0289 0.0204 0.0119;
0.00192 0.0204 0.0576 0.0336;
0 0.0119 0.0336 0.1225 ];
pwgt0 = [ 0.3; 0.3; 0.2; 0.1 ];
p = PortfolioMAD;
p = setAssetList(p, 'Bonds','Large-Cap Equities','Small-Cap Equities','Emerging Equities');
p = setInitPort(p, pwgt0);
p = simulateNormalScenariosByMoments(p, m, C, 20000);
p = p.setDefaultConstraints;
[pwgt, pbuy, psell] = estimateFrontierByRisk(p, 0.15);
Blotter = dataset([{100*[pwgt0, pwgt, pbuy, psell]},
{'Initial','Weight', 'Purchases','Sales'}],'obsnames',p.AssetList);
display(Blotter);
Blotter =
\begin{tabular}{lllrr} 
& Initial & Weight & Purchases & Sales \\
Bonds & 30 & \(6.0364 \mathrm{e}-18\) & 0 & 30 \\
Large-Cap Equities & 30 & 50.179 & 20.179 & 0 \\
Small-Cap Equities & 20 & 13.43 & 0 & 6.5696 \\
Emerging Equities & 10 & 36.391 & 26.391 & 0
\end{tabular}
```

If you have prices for each asset (in this example, they can be ETFs), add them to your blotter and then use the tools of the dataset object to obtain shares and shares to be traded.

## See Also

PortfolioMAD|checkFeasibility|estimateScenarioMoments

## Related Examples

- "Creating the PortfolioMAD Object" on page 6-26
- "Working with MAD Portfolio Constraints Using Defaults" on page 6-61
- "Estimate Efficient Portfolios Along the Entire Frontier for PortfolioMAD Object" on page 6-96
- "Estimate Efficient Frontiers for PortfolioMAD Object" on page 6-111
- "Asset Returns and Scenarios Using PortfolioMAD Object" on page 6-42


## More About

- "PortfolioMAD Object" on page 6-21
- "Portfolio Optimization Theory" on page 6-3
- "PortfolioMAD Object Workflow" on page 6-19


## Working with Other Portfolio Objects

The PortfolioMAD object is for MAD portfolio optimization. The PortfolioCVaR object is for CVaR portfolio optimization. The Portfolio object is for mean-variance portfolio optimization. Sometimes, you might want to examine portfolio optimization problems according to different combinations of return and risk proxies. A common example is that you want to do a MAD portfolio optimization and then want to work primarily with moments of portfolio returns. Suppose that you set up a MAD portfolio optimization problem with:

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
0.00408 0.0289 0.0204 0.0119;
0.00192 0.0204 0.0576 0.0336;
0 0.0119 0.0336 0.1225 ];
pwgt0 = [ 0.3; 0.3; 0.2; 0.1 ];
p = PortfolioMAD;
p = setAssetList(p, 'Bonds','Large-Cap Equities','Small-Cap Equities','Emerging Equities');
p = setInitPort(p, pwgt0);
p = simulateNormalScenariosByMoments(p, m, C, 20000);
p = setDefaultConstraints(p);
```

To work with the same problem in a mean-variance framework, you can use the scenarios from the PortfolioMAD object to set up a Portfolio object so that p contains a MAD optimization problem and q contains a mean-variance optimization problem based on the same data.

```
q = Portfolio('AssetList', p.AssetList);
q = estimateAssetMoments(q, p.getScenarios);
q = setDefaultConstraints(q);
pwgt = estimateFrontier(p);
qwgt = estimateFrontier(q);
```

Since each object has a different risk proxy, it is not possible to compare results side by side. To obtain means and standard deviations of portfolio returns, you can use the functions associated with each object to obtain:

```
pret = estimatePortReturn(p, pwgt);
pstd = estimatePortStd(p, pwgt);
qret = estimatePortReturn(q, qwgt);
qstd = estimatePortStd(q, qwgt);
[pret, qret]
[pstd, qstd]
```

```
ans =
\begin{tabular}{ll}
0.0592 & 0.0590 \\
0.0730 & 0.0728 \\
0.0868 & 0.0867 \\
0.1006 & 0.1005 \\
0.1145 & 0.1143 \\
0.1283 & 0.1282 \\
0.1421 & 0.1420 \\
0.1559 & 0.1558 \\
0.1697 & 0.1697 \\
0.1835 & 0.1835
\end{tabular}
ans =
\begin{tabular}{ll}
0.0767 & 0.0767 \\
0.0829 & 0.0828 \\
0.0989 & 0.0987 \\
0.1208 & 0.1206 \\
0.1461 & 0.1459 \\
0.1732 & 0.1730 \\
0.2042 & 0.2040 \\
0.2453 & 0.2452 \\
0.2929 & 0.2928 \\
0.3458 & 0.3458
\end{tabular}
```

To produce comparable results, you can use the returns or risks from one portfolio optimization as target returns or risks for the other portfolio optimization.

```
qwgt = estimateFrontierByReturn(q, pret);
qret = estimatePortReturn(q, qwgt);
qstd = estimatePortStd(q, qwgt);
[pret, qret]
[pstd, qstd]
ans =
\begin{tabular}{ll}
0.0592 & 0.0592 \\
0.0730 & 0.0730 \\
0.0868 & 0.0868 \\
0.1006 & 0.1006 \\
0.1145 & 0.1145 \\
0.1283 & 0.1283
\end{tabular}
```

| 0.1421 | 0.1421 |
| :---: | :---: |
| 0.1559 | 0.1559 |
| 0.1697 | 0.1697 |
| 0.1835 | 0.1835 |
|  |  |
| ans $=$ |  |
|  |  |
| 0.0767 | 0.0767 |
| 0.0829 | 0.0829 |
| 0.0989 | 0.0989 |
| 0.1208 | 0.1208 |
| 0.1461 | 0.1461 |
| 0.1732 | 0.1732 |
| 0.2042 | 0.2042 |
| 0.2453 | 0.2453 |
| 0.2929 | 0.2929 |
| 0.3458 | 0.3458 |

Now it is possible to compare standard deviations of portfolio returns from either type of portfolio optimization.

## See Also

Portfolio| PortfolioMAD

## Related Examples

- "Creating the Portfolio Object" on page 4-28
- "Creating the PortfolioMAD Object" on page 6-26
- "Working with MAD Portfolio Constraints Using Defaults" on page 6-61
- "Estimate Efficient Portfolios Along the Entire Frontier for PortfolioMAD Object" on page 6-96
- "Estimate Efficient Frontiers for PortfolioMAD Object" on page 6-111
- "Asset Returns and Scenarios Using PortfolioMAD Object" on page 6-42


## More About

- "PortfolioMAD Object" on page 6-21

6 MAD Portfolio Optimization Tools

- "Portfolio Optimization Theory" on page 6-3
- "PortfolioMAD Object Workflow" on page 6-19


# Troubleshooting MAD Portfolio Optimization Results 

## PortfolioMAD Object Destroyed When Modifying

If a PortfolioMAD object is destroyed when modifying, remember to pass an existing object into the PortfolioMAD object if you want to modify it, otherwise it creates a new object. See "Creating the PortfolioMAD Object" on page 6-26 for details.

## Matrix Incompatibility and "Non-Conformable" Errors

If you get matrix incompatibility or "non-conformable" errors, the representation of data in the tools follows a specific set of basic rules described in "Conventions for Representation of Data" on page 6-24.

## Missing Data Estimation Fails

If asset return data has missing or NaN values, the simulateNormalScenariosByData function with the 'missingdata' flag set to true may fail with either too many iterations or a singular covariance. To correct this problem, consider this:

- If you have asset return data with no missing or NaN values, you can compute a covariance matrix that may be singular without difficulties. If you have missing or NaN values in your data, the supported missing data feature requires that your covariance matrix must be positive-definite, that is, nonsingular.
- simulateNormalScenariosByData uses default settings for the missing data estimation procedure that might not be appropriate for all problems.

In either case, you might want to estimate the moments of asset returns separately with either the ECM estimation functions such as ecmnmle or with your own functions.

## mad_optim_transform Errors

If you obtain optimization errors such as:

```
Error using mad_optim_transform (line 276)
Portfolio set appears to be either empty or unbounded. Check constraints.
Error in PortfolioMAD/estimateFrontier (line 64)
    [AI, bI, AE, bE, lB, uB, f0, f, x0] = mad_optim_transform(obj);
or
```

Error using mad_optim_transform (line 281)
Cannot obtain finite lower bounds for specified portfolio set.
Error in PortfolioMAD/estimateFrontier (line 64)
[AI, bI, AE, bE, lB, uB, f0, f, x0] = mad optim transform(obj);
Since the portfolio optimization tools require a bounded portfolio set, these errors (and similar errors) can occur if your portfolio set is either empty and, if nonempty, unbounded. Specifically, the portfolio optimization algorithm requires that your portfolio set have at least a finite lower bound. The best way to deal with these problems is to use the validation methods in "Validate the MAD Portfolio Problem" on page 6-91. Specifically, use estimateBounds to examine your portfolio set, and use checkFeasibility to ensure that your initial portfolio is either feasible and, if infeasible, that you have sufficient turnover to get from your initial portfolio to the portfolio set.

Tip To correct this problem, try solving your problem with larger values for turnover and gradually reduce to the value that you want.

## Efficient Portfolios Do Not Make Sense

If you obtain efficient portfolios that, do not seem to make sense, this can happen if you forget to set specific constraints or you set incorrect constraints. For example, if you allow portfolio weights to fall between 0 and 1 and do not set a budget constraint, you can get portfolios that are $100 \%$ invested in every asset. Although it may be hard to detect, the best thing to do is to review the constraints you have set with display of the PortfolioMAD object. If you get portfolios with $100 \%$ invested in each asset, you can review the display of your object and quickly see that no budget constraint is set. Also, you can use estimateBounds and checkFeasibility to determine if the bounds for your portfolio set make sense and to determine if the portfolios you obtained are feasible relative to an independent formulation of your portfolio set.

## See Also

PortfolioMAD|checkFeasibility|estimateScenarioMoments

## Related Examples

- "Postprocessing Results to Set Up Tradable Portfolios" on page 6-122
- "Creating the PortfolioMAD Object" on page 6-26
- "Working with MAD Portfolio Constraints Using Defaults" on page 6-61
- "Estimate Efficient Portfolios Along the Entire Frontier for PortfolioMAD Object" on page 6-96
- "Estimate Efficient Frontiers for PortfolioMAD Object" on page 6-111
- "Asset Returns and Scenarios Using PortfolioMAD Object" on page 6-42


## More About

- "PortfolioMAD Object" on page 6-21
- "Portfolio Optimization Theory" on page 6-3
- "PortfolioMAD Object Workflow" on page 6-19


## Investment Performance Metrics

- "Performance Metrics Overview" on page 7-2
- "Performance Metrics Illustration" on page 7-4
- "Using the Sharpe Ratio" on page 7-7
- "Using the Information Ratio" on page 7-9
- "Using Tracking Error" on page 7-11
- "Using Risk-Adjusted Return" on page 7-13
- "Using Sample and Expected Lower Partial Moments" on page 7-16
- "Using Maximum and Expected Maximum Drawdown" on page 7-19


## Performance Metrics Overview

## Performance Metrics Types

Sharpe first proposed a ratio of excess return to total risk as an investment performance metric. Subsequent work by Sharpe, Lintner, and Mossin extended these ideas to entire asset markets in what is called the Capital Asset Pricing Model (CAPM). Since the development of the CAPM, various investment performance metrics has evolved.

This section presents four types of investment performance metrics:

- The first type of metrics is absolute investment performance metrics that are called "classic" metrics since they are based on the CAPM. They include the Sharpe ratio, the information ratio, and tracking error. To compute the Sharpe ratio from data, use sharpe to calculate the ratio for one or more asset return series. To compute the information ratio and associated tracking error, use inforatio to calculate these quantities for one or more asset return series.
- The second type of metrics is relative investment performance metrics to compute risk-adjusted returns. These metrics are also based on the CAPM and include Beta, Jensen's Alpha, the Security Market Line (SML), Modigliani and Modigliani RiskAdjusted Return, and the Graham-Harvey measures. To calculate risk-adjusted alpha and return, use portalpha.
- The third type of metrics is alternative investment performance metrics based on lower partial moments. To calculate lower partial moments, use lpm for sample lower partial moments and elpm for expected lower partial moments.
- The fourth type of metrics is performance metrics based on maximum drawdown and expected maximum drawdown. To calculate maximum or expected maximum drawdowns, use maxdrawdown and emaxdrawdown.


## See Also

elpm | emaxdrawdown | inforatio|lpm|maxdrawdown | portalpha|ret2tick| sharpe|tick2ret

## Related Examples

- "Using the Sharpe Ratio" on page 7-7
- "Using the Information Ratio" on page 7-9
- "Using Tracking Error" on page 7-11
- "Using Risk-Adjusted Return" on page 7-13
- "Using Sample and Expected Lower Partial Moments" on page 7-16
- "Using Maximum and Expected Maximum Drawdown" on page 7-19


## Performance Metrics Illustration

To illustrate the functions for investment performance metrics, you work with three financial time series objects using performance data for:

- An actively managed, large-cap value mutual fund
- A large-cap market index
- 90-day Treasury bills

The data is monthly total return prices that cover a span of five years.
The following plot illustrates the performance of each series in terms of total returns to an initial $\$ 1$ invested at the start of this 5 -year period:

```
load FundMarketCash
plot(TestData)
hold on
title('\bfFive-Year Total Return Performance');
legend('Fund','Market','Cash','Location','SouthEast');
hold off
```



The mean (Mean) and standard deviation (Sigma) of returns for each series are
Returns = tick2ret(TestData);

## Assets

Mean = mean(Returns)
Sigma = std(Returns, 1)
which gives the following result:
Assets =
'Fund ${ }^{\prime}$
'Market'
'Cash'

Mean =
$0.0038 \quad 0.0030$
Sigma =
$0.0229 \quad 0.0389$
0.0017
0.0389
0.0009

Note Functions for investment performance metrics use total return price and total returns. To convert between total return price and total returns, use ret2tick and tick2ret.

## See Also

elpm | emaxdrawdown |inforatio | lpm| maxdrawdown | portalpha|ret2tick| sharpe|tick2ret

## Related Examples

- "Using the Sharpe Ratio" on page 7-7
- "Using the Information Ratio" on page 7-9
- "Using Tracking Error" on page 7-11
- "Using Risk-Adjusted Return" on page 7-13
- "Using Sample and Expected Lower Partial Moments" on page 7-16
- "Using Maximum and Expected Maximum Drawdown" on page 7-19


## Using the Sharpe Ratio

```
In this section...
"Introduction" on page 7-7
"Sharpe Ratio" on page 7-7
```


## Introduction

The Sharpe ratio is the ratio of the excess return of an asset divided by the asset's standard deviation of returns. The Sharpe ratio has the form:

```
(Mean - Riskless) / Sigma
```

Here Mean is the mean of asset returns, Riskless is the return of a riskless asset, and Sigma is the standard deviation of asset returns. A higher Sharpe ratio is better than a lower Sharpe ratio. A negative Sharpe ratio indicates "anti-skill" since the performance of the riskless asset is superior. For more information, see sharpe.

## Sharpe Ratio

To compute the Sharpe ratio, the mean return of the cash asset is used as the return for the riskless asset. Thus, given asset return data and the riskless asset return, the Sharpe ratio is calculated with

```
load FundMarketCash
Returns = tick2ret(TestData);
Riskless = mean(Returns(:,3))
Sharpe = sharpe(Returns, Riskless)
which gives the following result:
```


## Riskless =

0.0017

Sharpe =
$0.0886 \quad 0.0315 \quad 0$
The Sharpe ratio of the example fund is significantly higher than the Sharpe ratio of the market. As is demonstrated with portalpha, this translates into a strong risk-adjusted return. Since the Cash asset is the same as Riskless, it makes sense that its Sharpe
ratio is 0 . The Sharpe ratio was calculated with the mean of cash returns. It can also be calculated with the cash return series as input for the riskless asset

Sharpe $=$ sharpe(Returns, Returns(:,3))
which gives the following result:

## Sharpe =

$0.0886 \quad 0.0315 \quad 0$
When using the Portfolio object, you can use the estimateMaxSharpeRatio function to estimate an efficient portfolio that maximizes the Sharpe ratio. For more information, see "Efficient Portfolio That Maximizes Sharpe Ratio" on page 4-128.

## See Also

Portfolio|elpm|emaxdrawdown | inforatio|lpm|maxdrawdown | portalpha| ret2tick|sharpe|tick2ret

## Related Examples

- "Performance Metrics Overview" on page 7-2
- "Using the Information Ratio" on page 7-9
- "Using Tracking Error" on page 7-11
- "Using Risk-Adjusted Return" on page 7-13
- "Using Sample and Expected Lower Partial Moments" on page 7-16
- "Using Maximum and Expected Maximum Drawdown" on page 7-19


## Using the Information Ratio

```
In this section...
"Introduction" on page 7-9
"Information Ratio" on page 7-9
```


## Introduction

Although originally called the "appraisal ratio" by Treynor and Black, the information ratio is the ratio of relative return to relative risk (known as "tracking error"). Whereas the Sharpe ratio looks at returns relative to a riskless asset, the information ratio is based on returns relative to a risky benchmark which is known colloquially as a "bogey." Given an asset or portfolio of assets with random returns designated by Asset and a benchmark with random returns designated by Benchmark, the information ratio has the form:

Mean(Asset - Benchmark) / Sigma (Asset - Benchmark)
Here Mean(Asset - Benchmark) is the mean of Asset minus Benchmark returns, and Sigma(Asset - Benchmark) is the standard deviation of Asset minus Benchmark returns. A higher information ratio is considered better than a lower information ratio. For more information, see inforatio.

## Information Ratio

To calculate the information ratio using the example data, the mean return of the market series is used as the return of the benchmark. Thus, given asset return data and the riskless asset return, compute the information ratio with

```
load FundMarketCash
Returns = tick2ret(TestData);
Benchmark = Returns(:,2);
InfoRatio = inforatio(Returns, Benchmark)
```

which gives the following result:
0.0432 NaN -0.0315

Since the market series has no risk relative to itself, the information ratio for the second series is undefined (which is represented as NaN in MATLAB software). Its standard deviation of relative returns in the denominator is 0 .

## See Also

elpm | emaxdrawdown |inforatio | lpm| maxdrawdown | portalpha|ret2tick| sharpe|tick2ret

## Related Examples

- "Performance Metrics Overview" on page 7-2
- "Using the Sharpe Ratio" on page 7-7
- "Using Tracking Error" on page 7-11
- "Using Risk-Adjusted Return" on page 7-13
- "Using Sample and Expected Lower Partial Moments" on page 7-16
- "Using Maximum and Expected Maximum Drawdown" on page 7-19


## Using Tracking Error

In this section...<br>"Introduction" on page 7-11<br>"Tracking Error" on page 7-11

## Introduction

Given an asset or portfolio of assets and a benchmark, the relative standard deviation of returns between the asset or portfolio of assets and the benchmark is called tracking error.

## Tracking Error

The function inforatio computes tracking error and returns it as a second argument
load FundMarketCash
Returns = tick2ret(TestData);
Benchmark = Returns(:,2);
[InfoRatio, TrackingError] = inforatio(Returns, Benchmark)
which gives the following results:

```
InfoRatio =
    0.0432 NaN -0.0315
TrackingError =
    0.0187 0 0.0390
```

Tracking error is a useful measure of performance relative to a benchmark since it is in units of asset returns. For example, the tracking error of $1.87 \%$ for the fund relative to the market in this example is reasonable for an actively managed, large-cap value fund.

## See Also

elpm | emaxdrawdown |inforatio|lpm|maxdrawdown | portalpha|ret2tick| sharpe|tick2ret

## Related Examples

- "Performance Metrics Overview" on page 7-2
- "Using the Sharpe Ratio" on page 7-7
- "Using the Information Ratio" on page 7-9
- "Using Risk-Adjusted Return" on page 7-13
- "Using Sample and Expected Lower Partial Moments" on page 7-16
- "Using Maximum and Expected Maximum Drawdown" on page 7-19


## Using Risk-Adjusted Return

## In this section...

"Introduction" on page 7-13
"Risk-Adjusted Return" on page 7-13

## Introduction

Risk-adjusted return either shifts the risk (which is the standard deviation of returns) of a portfolio to match the risk of a market portfolio or shifts the risk of a market portfolio to match the risk of a fund. According to the Capital Asset Pricing Model (CAPM), the market portfolio and a riskless asset are points on a Security Market Line (SML). The return of the resultant shifted portfolio, levered or unlevered, to match the risk of the market portfolio, is the risk-adjusted return. The SML provides another measure of riskadjusted return, since the difference in return between the fund and the SML, return at the same level of risk.

## Risk-Adjusted Return

Given our example data with a fund, a market, and a cash series, you can calculate the risk-adjusted return and compare it with the fund and market's mean returns

```
load FundMarketCash
Returns = tick2ret(TestData);
Fund = Returns(:,1);
Market = Returns(:,2);
Cash = Returns(:,3);
MeanFund = mean(Fund)
MeanMarket = mean(Market)
[MM, aMM] = portalpha(Fund, Market, Cash, 'MM')
[GH1, aGH1] = portalpha(Fund, Market, Cash, 'gh1')
[GH2, aGH2] = portalpha(Fund, Market, Cash, 'gh2')
[SML, aSML] = portalpha(Fund, Market, Cash, 'sml')
```

which gives the following results:
MeanFund =
0.0038

```
MeanMarket =
    0.0030
MM =
    0.0022
aMM =
    0.0052
GH1 =
    0.0013
aGH1 =
    0.0025
GH2 =
    0.0022
aGH2 =
    0.0052
SML =
    0.0013
aSML =
    0.0025
```

Since the fund's risk is much less than the market's risk, the risk-adjusted return of the fund is much higher than both the nominal fund and market returns.

## See Also

elpm | emaxdrawdown | inforatio|lpm|maxdrawdown | portalpha|ret2tick| sharpe|tick2ret

## Related Examples

- "Performance Metrics Overview" on page 7-2
- "Using the Sharpe Ratio" on page 7-7
- "Using the Information Ratio" on page 7-9
- "Using Tracking Error" on page 7-11
- "Using Sample and Expected Lower Partial Moments" on page 7-16
- "Using Maximum and Expected Maximum Drawdown" on page 7-19


# Using Sample and Expected Lower Partial Moments 

In this section...<br>"Introduction" on page 7-16<br>"Sample Lower Partial Moments" on page 7-16<br>"Expected Lower Partial Moments" on page 7-17

## Introduction

Use lower partial moments to examine what is colloquially known as "downside risk." The main idea of the lower partial moment framework is to model moments of asset returns that fall below a minimum acceptable level of return. To compute lower partial moments from data, use lpm to calculate lower partial moments for multiple asset return series and for multiple moment orders. To compute expected values for lower partial moments under several assumptions about the distribution of asset returns, use elpm to calculate lower partial moments for multiple assets and for multiple orders.

## Sample Lower Partial Moments

The following example demonstrates $l \mathrm{pm}$ to compute the zero-order, first-order, and second-order lower partial moments for the three time series, where the mean of the third time series is used to compute MAR (minimum acceptable return) with the so-called risk-free rate.

```
load FundMarketCash
Returns = tick2ret(TestData);
Assets
MAR = mean(Returns(:,3))
LPM = lpm(Returns, MAR, [0 1 2])
```

which gives the following results:

```
Assets =
    'Fund' 'Market' 'Cash'
MAR =
    0.0017
LPM =
    0.4333 0.4167 0.6167
    0.0075 0.0140 0.0004
    0.0003 0.0008 0.0000
```

The first row of LPM contains zero-order lower partial moments of the three series. The fund and market index fall below MAR about $40 \%$ of the time and cash returns fall below its own mean about $60 \%$ of the time.

The second row contains first-order lower partial moments of the three series. The fund and market have large average shortfall returns relative to MAR by 75 and 140 basis points per month. On the other hand, cash underperforms MAR by about only four basis points per month on the downside.

The third row contains second-order lower partial moments of the three series. The square root of these quantities provides an idea of the dispersion of returns that fall below the MAR. The market index has a much larger variation on the downside when compared to the fund.

## Expected Lower Partial Moments

To compare realized values with expected values, use elpm to compute expected lower partial moments based on the mean and standard deviations of normally distributed asset returns. The elpm function works with the mean and standard deviations for multiple assets and multiple orders.
load FundMarketCash
Returns = tick2ret(TestData);
MAR $=$ mean(Returns(:, 3$)$ )
Mean = mean(Returns)
Sigma $=$ std(Returns, 1)
Assets
ELPM $=$ elpm(Mean, Sigma, MAR, $\left.\left[\begin{array}{lll}0 & 1 & 2\end{array}\right]\right)$
which gives the following results:
Assets =
'Fund' 'Market' 'Cash'
ELPM =

| 0.4647 | 0.4874 | 0.5000 |
| :--- | :--- | :--- |
| 0.0082 | 0.0149 | 0.0004 |
| 0.0002 | 0.0007 | 0.0000 |

Based on the moments of each asset, the expected values for lower partial moments imply better than expected performance for the fund and market and worse than expected performance for cash. This function works with either degenerate or nondegenerate normal random variables. For example, if cash were truly riskless, its standard deviation would be 0 . You can examine the difference in average shortfall.

```
RisklessCash = elpm(Mean(3), 0, MAR, 1)
```

which gives the following result:

```
RisklessCash =
    0
```


## See Also

elpm | emaxdrawdown |inforatio | lpm|maxdrawdown | portalpha|ret2tick| sharpe|tick2ret

## Related Examples

- "Performance Metrics Overview" on page 7-2
- "Using the Sharpe Ratio" on page 7-7
- "Using the Information Ratio" on page 7-9
- "Using Tracking Error" on page 7-11
- "Using Risk-Adjusted Return" on page 7-13
- "Using Maximum and Expected Maximum Drawdown" on page 7-19


# Using Maximum and Expected Maximum Drawdown 

In this section...<br>"Introduction" on page 7-19<br>"Maximum Drawdown" on page 7-19<br>"Expected Maximum Drawdown" on page 7-22

## Introduction

Maximum drawdown is the maximum decline of a series, measured as return, from a peak to a nadir over a period of time. Although additional metrics exist that are used in the hedge fund and commodity trading communities (see Pederson and Rudholm-Alfvin [20] in "Bibliography" on page A-2), the original definition and subsequent implementation of these metrics is not yet standardized.

It is possible to compute analytically the expected maximum drawdown for a Brownian motion with drift (see Magdon-Ismail, Atiya, Pratap, and Abu-Mostafa [16] "Bibliography" on page A-2). These results are used to estimate the expected maximum drawdown for a series that approximately follows a geometric Brownian motion.

Use maxdrawdown and emaxdrawdown to calculate the maximum and expected maximum drawdowns.

## Maximum Drawdown

This example demonstrates how to compute the maximum drawdown (MaxDD) using our example data with a fund, a market, and a cash series:

```
load FundMarketCash
MaxDD = maxdrawdown(TestData)
```

which gives the following results:

## MaxDD =

$$
\begin{array}{lll}
0.1658 & 0.3381 & 0
\end{array}
$$

The maximum drop in the given time period was of $16.58 \%$ for the fund series, and $33.81 \%$ for the market. There was no decline in the cash series, as expected, because the cash account never loses value.
maxdrawdown can also return the indices (MaxDDIndex) of the maximum drawdown intervals for each series in an optional output argument:
[MaxDD, MaxDDIndex] = maxdrawdown(TestData)
which gives the following results:
MaxDD $=$
$0.1658 \quad 0.3381 \quad 0$

MaxDDIndex =
22 NaN
1818 NaN
The first two series experience their maximum drawdowns from the second to the 18th month in the data. The indices for the third series are NaNs because it never has a drawdown.

The $16.58 \%$ value loss from month 2 to month 18 for the fund series is verified using the reported indices:

```
Start = MaxDDIndex(1,:);
End = MaxDDIndex(2,:);
(TestData(Start(1),1) - TestData(End(1),1))/TestData(Start(1),1)
ans =
    0.1658
```

Although the maximum drawdown is measured in terms of returns, maxdrawdown can measure the drawdown in terms of absolute drop in value, or in terms of log-returns. To contrast these alternatives more clearly, we work with the fund series assuming, an initial investment of 50 dollars:

```
Fund50 = 50*TestData(:,1);
plot(Fund50);
title('\bfFive-Year Fund Performance, Initial Investment 50 usd');
xlabel('Months');
ylabel('Value of Investment');
```



First, we compute the standard maximum drawdown, which coincides with the results above because returns are independent of the initial amounts invested:

MaxDD50Ret $=$ maxdrawdown(Fund50)
MaxDD50Ret =
0.1658

Next, we compute the maximum drop in value, using the arithmetic argument:
[MaxDD50Arith, Ind50Arith] = maxdrawdown(Fund50,'arithmetic')
MaxDD50Arith =
8.4285

Ind50Arith =

The value of this investment was $\$ 50.84$ in month 2 , but by month 18 the value was down to $\$ 42.41$, a drop of $\$ 8.43$. This is the largest loss in dollar value from a previous high in the given time period. In this case, the maximum drawdown period, 2nd to 18th month, is the same independently of whether drawdown is measured as return or as dollar value loss.

Last, we compute the maximum decline based on log-returns using the geometric argument. In this example, the log-returns result in a maximum drop of 18.13\%, again from the second to the 18th month, not far from the $16.58 \%$ obtained using standard returns.
[MaxDD50LogRet, Ind50LogRet] = maxdrawdown(Fund50,'geometric')
MaxDD50LogRet =
0.1813

Ind50LogRet $=$
2
18
Note, the last measure is equivalent to finding the arithmetic maximum drawdown for the log of the series:

```
MaxDD50LogRet2 = maxdrawdown(log(Fund50),'arithmetic')
```

MaxDD50LogRet2 =
0.1813

## Expected Maximum Drawdown

This example demonstrates using the log-return moments of the fund to compute the expected maximum drawdown (EMaxDD) and then compare it with the realized maximum drawdown (MaxDD).
load FundMarketCash
logReturns $=\log ($ TestData(2:end,:) ./ TestData(1:end - 1,:));

```
Mu = mean(logReturns(:,1));
Sigma = std(logReturns(:,1),1);
T = size(logReturns,1);
MaxDD = maxdrawdown(TestData(:,1),'geometric')
EMaxDD = emaxdrawdown(Mu-0.5*Sigma^2, Sigma, T)
which gives the following results:
```


## MaxDD =

```
0.1813
EMaxDD =
0.1588
```

The drawdown observed in this time period is above the expected maximum drawdown. There is no contradiction here. The expected maximum drawdown is not an upper bound on the maximum losses from a peak, but an estimate of their average, based on a geometric Brownian motion assumption.

## See Also

elpm | emaxdrawdown | inforatio | lpm|maxdrawdown | portalpha|ret2tick| sharpe|tick2ret

## Related Examples

- "Performance Metrics Overview" on page 7-2
- "Using the Sharpe Ratio" on page 7-7
- "Using the Information Ratio" on page 7-9
- "Using Tracking Error" on page 7-11
- "Using Risk-Adjusted Return" on page 7-13
- "Using Sample and Expected Lower Partial Moments" on page 7-16


## Credit Risk Analysis

- "Estimation of Transition Probabilities" on page 8-2
- "Forecasting Corporate Default Rates" on page 8-25
- "Credit Quality Thresholds" on page 8-57
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## Estimation of Transition Probabilities

In this section...<br>"Introduction" on page 8-2<br>"Estimate Transition Probabilities" on page 8-3<br>"Estimate Transition Probabilities for Different Rating Scales" on page 8-5<br>"Working with a Transition Matrix Containing NR Rating" on page 8-6<br>"Estimate Point-in-Time and Through-the-Cycle Probabilities" on page 8-11<br>"Estimate t-Year Default Probabilities" on page 8-14<br>"Estimate Bootstrap Confidence Intervals" on page 8-15<br>"Group Credit Ratings" on page 8-16<br>"Work with Nonsquare Matrices" on page 8-18<br>"Remove Outliers" on page 8-20<br>"Estimate Probabilities for Different Segments" on page 8-21<br>"Work with Large Datasets" on page 8-22

## Introduction

Credit ratings rank borrowers according to their credit worthiness. Though this ranking is, in itself, useful, institutions are also interested in knowing how likely it is that borrowers in a particular rating category will be upgraded or downgraded to a different rating, and especially, how likely it is that they will default.

Transition probabilities offer one way to characterize the past changes in credit quality of obligors (typically firms), and are cardinal inputs to many risk management applications. Financial Toolbox software supports the estimation of transition probabilities using both cohort and duration (also known as hazard rate or intensity) approaches using transprob and related functions.

Note The sample dataset used throughout this section is simulated using a single transition matrix. No attempt is made to match historical trends in transition rates.

## Estimate Transition Probabilities

The Data_TransProb.mat file contains sample credit ratings data.
load Data TransProb
data(1:10,:)
ans =

| ID | Date | Rating |
| :---: | :---: | :---: |
| '00010283' | '10-Nov-1984' | ' CCC' |
| '00010283' | '12-May-1986' | 'B' |
| '00010283' | ' 29-Jun-1988' | ' CCC' |
| '00010283' | '12-Dec-1991' | 'D' |
| '00013326' | ' 09-Feb-1985' | 'A' |
| '00013326' | ' 24-Feb-1994' | ' $\mathrm{AA}^{\prime}$ |
| '00013326' | ' 10-Nov-2000' | ' ${ }^{\prime}{ }^{\prime}{ }^{\prime}$ |
| '00014413' | '23-Dec-1982' | 'B' |
| '00014413' | ' 20-Apr-1988' | 'BB' |
| '00014413' | '16-Jan-1998' | 'B' |

The sample data is formatted as a cell array with three columns. Each row contains an ID (column 1), a date (column 2), and a credit rating (column 3). The assigned credit rating corresponds to the associated ID on the associated date. All information corresponding to the same ID must be stored in contiguous rows. In this example, IDs, dates, and ratings are stored in character vector format, but you also can enter them in numeric format.

In this example, the simplest calling syntax for transprob passes the nRecords-by-3 cell array as the only input argument. The default startDate and endDate are the earliest and latest dates in the data. The default estimation algorithm is the duration method and one-year transition probabilities are estimated:


Provide explicit start and end dates, otherwise, the estimation window for two different datasets can differ, and the estimates might not be comparable. From this point, assume that the time window of interest is the five-year period from the end of 1995 to the end of 2000. For comparisons, compute the estimates for this time window. First use the duration algorithm (default option), and then the cohort algorithm explicitly set.

```
startDate = '31-Dec-1995';
endDate = '31-Dec-2000';
transMat1 = transprob(data,'startDate',startDate,'endDate',endDate)
transMat2 = transprob(data,'startDate',startDate,'endDate',endDate,...
'algorithm','cohort')
transMat1 =
\begin{tabular}{rrrrrrrr}
90.6236 & 7.9051 & 1.0314 & 0.4123 & 0.0210 & 0.0020 & 0.0003 & 0.0043 \\
4.4780 & 89.5558 & 4.5298 & 1.1225 & 0.2284 & 0.0094 & 0.0009 & 0.0754 \\
0.3983 & 6.1164 & 87.0641 & 5.4801 & 0.7637 & 0.0892 & 0.0050 & 0.0832 \\
0.1029 & 0.8572 & 10.7918 & 83.0204 & 3.9971 & 0.7001 & 0.1313 & 0.3992 \\
0.1043 & 0.3745 & 2.2962 & 14.0954 & 78.9840 & 3.0013 & 0.0463 & 1.0980 \\
0.0113 & 0.0544 & 0.7055 & 3.2925 & 15.4350 & 75.5988 & 1.8166 & 3.0860 \\
0.0044 & 0.0189 & 0.1903 & 1.9743 & 6.2320 & 10.2334 & 75.9983 & 5.3484 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 100.0000
\end{tabular}
transMat2 =
\begin{tabular}{rrrrrrrr}
90.1554 & 8.5492 & 0.9067 & 0.3886 & 0 & 0 & 0 & 0 \\
4.9512 & 88.5221 & 5.1763 & 1.0503 & 0.2251 & 0 & 0 & 0.0750 \\
0.2770 & 6.6482 & 86.2188 & 6.0942 & 0.6233 & 0.0693 & 0 & 0.0693 \\
0.0794 & 0.8737 & 11.6759 & 81.6521 & 4.3685 & 0.7943 & 0.1589 & 0.3971 \\
0.1002 & 0.4008 & 1.9038 & 15.4309 & 77.8557 & 3.4068 & 0 & 0.9018 \\
0 & 0 & 0.2262 & 2.4887 & 17.4208 & 74.2081 & 2.2624 & 3.3937 \\
0 & 0 & 0.7576 & 1.5152 & 6.0606 & 10.6061 & 75.0000 & 6.0606 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 100.0000
\end{tabular}
```

By default, the cohort algorithm internally gets yearly snapshots of the credit ratings, but the number of snapshots per year is definable using the parameter/value pair snapsPerYear. To get the estimates using quarterly snapshots:

| transMat3 = |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 90.4765 | 8.0881 | 1.0072 | 0.4069 | 0.0164 | 0.0015 | 0.0002 | 0.0032 |
| 4.5949 | 89.3216 | 4.6489 | 1.1239 | 0.2276 | 0.0074 | 0.0007 | 0.0751 |
| 0.3747 | 6.3158 | 86.7380 | 5.6344 | 0.7675 | 0.0856 | 0.0040 | 0.0800 |
| 0.0958 | 0.7967 | 11.0441 | 82.6138 | 4.1906 | 0.7230 | 0.1372 | 0.3987 |
| 0.1028 | 0.3571 | 2.3312 | 14.4954 | 78.4276 | 3.1489 | 0.0383 | 1.0987 |
| 0.0084 | 0.0399 | 0.6465 | 3.0962 | 16.0789 | 75.1300 | 1.9044 | 3.0956 |
| 0.0031 | 0.0125 | 0.1445 | 1.8759 | 6.2613 | 10.7022 | 75.6300 | 5.3705 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 100.0000 |

Both duration and cohort compute one-year transition probabilities by default, but the time interval for the transitions is definable using the parameter/value pair transInterval. For example, to get the two-year transition probabilities using the cohort algorithm with the same snapshot periodicity and estimation window:

| transMat4 = |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 82.2358 | 14.6092 | 2.2062 | 0.8543 | 0.0711 | 0.0074 | 0.0011 | 0.0149 |
| 8.2803 | 80.4584 | 8.3606 | 2.2462 | 0.4665 | 0.0316 | 0.0030 | 0.1533 |
| 0.9604 | 11.1975 | 76.1729 | 9.7284 | 1.5322 | 0.2044 | 0.0162 | 0.1879 |
| 0.2483 | 2.0903 | 18.8440 | 69.5145 | 6.9601 | 1.2966 | 0.2329 | 0.8133 |
| 0.2129 | 0.8713 | 5.4893 | 23.5776 | 62.6438 | 4.9464 | 0.1390 | 2.1198 |
| 0.0378 | 0.1895 | 1.7679 | 7.2875 | 24.9444 | 57.1783 | 2.8816 | 5.7132 |
| 0.0154 | 0.0716 | 0.6576 | 4.2157 | 11.4465 | 16.3455 | 57.4078 | 9.8399 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 100.0000 |

## Estimate Transition Probabilities for Different Rating Scales

The dataset data from Data_TransProb. mat contains sample credit ratings using the default rating scale \{'AAA', 'AA','A', 'BBB', 'BB', 'B', 'CCC', 'D'\}. It also contains the dataset dataIGSG with ratings investment grade (' IG'), speculative grade ('SG'), and default ('D'). To estimate the transition matrix for this dataset, use the labels argument.
load Data TransProb
startDate = '31-Dec-1995';
endDate = '31-Dec-2000';
dataIGSG(1:10,:)
transMatIGSG = transprob(dataIGSG,'labels',\{'IG','SG','D'\},...
'startDate', startDate, 'endDate', endDate)

```
ans =
```

| 00011253' | '04-Apr-1983' |
| :---: | :---: |
| 00012751' | '17-Feb-1985' |
| '00012751' | '19-May-1986' |
| '00014690' | '17-Jan-1983' |
| '00012144' | '21-Nov-1984' |
| '00012144' | '25-Mar-1992' |
| '00012144' | '07-May-1994' |
| 00012144' | '23-Jan-2000' |
| '00012144' | '20-Aug-2001' |
| 00012937 ' | 07-Feb-1984' |

## transMatIGSG =

| 98.1986 | 1.5179 | 0.2835 |
| ---: | ---: | ---: |
| 8.5396 | 89.4891 | 1.9713 |
| 0 | 0 | 100.0000 |

There is another dataset, dataIGSGnum, with the same information as dataIGSG, except the ratings are mapped to a numeric scale where ' $I G^{\prime}=1$, ' $\mathrm{SG}^{\prime}=2$, and ' $\mathrm{D}^{\prime}=3$. To estimate the transition matrix, use the labels optional argument specifying the numeric scale as a cell array.

```
dataIGSGnum(1:10,:)
% Note {1,2,3} and num2cell(1:3) are equivalent; num2cell is convenient
% when the number of ratings is larger
transMatIGSGnum = transprob(dataIGSGnum,'labels',{1,2,3},....
'startDate',startDate,'endDate',endDate)
ans =
    '00011253' '04-Apr-1983' [1]
    '00012751' '17-Feb-1985' [2]
    '00012751' '19-May-1986' [3]
    '00014690' '17-Jan-1983' [1]
    '00012144' '21-Nov-1984' [1]
    '00012144' '25-Mar-1992' [2]
    '00012144' '07-May-1994' [1]
    '00012144' '23-Jan-2000' [2]
    '00012144' '20-Aug-2001' [1]
    '00012937' '07-Feb-1984' [1]
transMatIGSGnum =
\begin{tabular}{rrr}
98.1986 & 1.5179 & 0.2835 \\
8.5396 & 89.4891 & 1.9713 \\
0 & 0 & 100.0000
\end{tabular}
```

Any time the input dataset contains ratings not included in the default rating scale \{'AAA', 'AA', 'A', 'BBB', 'BB', 'B', 'CCC', 'D'\}, the full rating scale must be specified using the labels optional argument. For example, if the dataset contains ratings 'AAA', ..., 'CCC, 'D', and 'NR' (not rated), use labels with this cell array \{'AAA', 'AA', 'A','BBB','BB','B','CCC','D','NR'\}.

## Working with a Transition Matrix Containing NR Rating

This example demonstrates how 'NR' (not rated) ratings are handled by transprob, and how to get transition matrix that use the 'NR ' rating information for the estimation, but that do not show the 'NR ' rating in the final transition probabilities.

The dataset data from Data_TransProb.mat contains sample credit ratings using the default rating scale \{'AAA', 'AA','A', 'BBB', 'BB', 'B', 'CCC', 'D'\}.

```
load Data TransProb
head(data,12)
ans =
    12\times3 table
```

ID
'00010283'
'00010283'
' 00010283
'00010283'
'00013326'
'00013326'
'00013326'
'00014413'
'00014413'
'00014413'
'00014413'
'00012126'

Date
' 10-Nov-1984'
' 12 -May-1986'
' 29-Jun-1988'
' 12-Dec-1991'
' 09-Feb-1985'
' 24 - Feb-1994 '
' 10-Nov-2000'
'23-Dec-1982'
' 20-Apr-1988'
' 16-Jan-1998'
'25-Nov-1999'
' 17 -Feb-1985'

Rating
' CCC'
'B'
'CCC'
'D'
' A '
' $A A^{\prime}$
' BBB '
'B'
'BB'
'B'
'BB'
'CCC'

Replace a transition to ' B ' with a transition to ' NR ' for the first company. Note that there is a subsequent transition from 'NR' to 'CCC'.

```
dataNR = data;
dataNR.Rating{2} = 'NR';
dataNR.Rating{7} = 'NR';
head(dataNR,12)
ans =
```

$12 \times 3$ table

| ID | Date | Rating |
| :---: | :---: | :---: |
| '00010283' | ' 10-Nov-1984' | ' CCC' |
| '00010283' | '12-May-1986' | 'NR' |
| '00010283' | ' 29-Jun-1988' | ' CCC' |
| '00010283' | '12-Dec-1991' | 'D' |
| '00013326' | ' 09-Feb-1985 ' | 'A' |
| '00013326' | ' 24-Feb-1994' | ' AA ' |


| '00013326' | ' 10-Nov-2000' | ' NR' |
| :---: | :---: | :---: |
| '00014413' | '23-Dec-1982' | 'B' |
| '00014413' | ' 20-Apr-1988' | 'BB' |
| '00014413' | ' 16-Jan-1998' | 'B' |
| '00014413' | '25-Nov-1999' | 'BB' |
| '00012126' | ' 17-Feb-1985' | ' $\mathrm{CCC}{ }^{\prime}$ |

' NR ' is treated as another rating. The transition matrix shows the estimated probability of transitioning into and out of ' NR'. In this example, the transprob function uses the ' cohort ' algorithm, and the 'NR' rating is treated as another rating. The same behavior exists when using the transprob function with the 'duration' algorithm.

```
RatingsLabelsNR = {'AAA','AA','A','BBB','BB','B','CCC','D','NR'};
[MatrixNRCohort,TotalsNRCohort] = transprob(dataNR,...
    'Labels',RatingsLabelsNR,...
    'Algorithm','cohort');
fprintf('Transition probability, cohort, including NR:\n')
disp(array2table(MatrixNRCohort,'VariableNames',RatingsLabelsNR,...
    'RowNames',RatingsLabelsNR))
fprintf('Total transitions out of given rating, including 6 out of NR (5 NR->NR, 1 NR->
disp(array2table(TotalsNRCohort.totalsVec,'VariableNames',RatingsLabelsNR))
Transition probability, cohort, including NR:
\begin{tabular}{|c|c|c|c|c|c|c|c|}
\hline & AAA & AA & A & BBB & BB & B & CCC \\
\hline AAA & 93.135 & 5.9335 & 0.74557 & 0.15533 & 0.031066 & 0 & \\
\hline AA & 1.7359 & 92.92 & 4.5446 & 0.58514 & 0.15604 & 0 & \\
\hline A & 0.12683 & 2.9716 & 91.991 & 4.3124 & 0.4711 & 0.054358 & \\
\hline BBB & 0.021048 & 0.37887 & 5.0726 & 89.771 & 4.0413 & 0.46306 & 0.042 \\
\hline BB & 0.022099 & 0.1105 & 0.68508 & 6.232 & 88.376 & 3.6464 & 0.28 \\
\hline B & 0 & 0 & 0.076161 & 0.72353 & 7.997 & 86.215 & 2.7 \\
\hline CCC & 0 & 0 & 0 & 0.30936 & 1.8561 & 4.4857 & 80. \\
\hline D & 0 & 0 & 0 & 0 & 0 & 0 & \\
\hline NR & 0 & 0 & 0 & 0 & 0 & 0 & 16 \\
\hline
\end{tabular}
```



To remove transitions to 'NR' from transition matrix, you need to work with the sampleTotals optional output from transprob. The sampleTotals structure has informations of transition counts between ratings (totalsMat field), and total transitions out of each rating (totalsVec).

To remove the 'NR' from the transition matrix, you need to remove the transitions into and out of the 'NR' rating. This means, remove the row and the column in totalsMat that correspond to 'NR'.

Because removing these transitions changes the total number of transitions out of other ratings (any rating transitioning into 'NR' is affected), the totalsVec vector must be updated with the new total counts that excludes transitions in and out of 'NR'. The adjusted count per rating is totalsVec computed as the sum by rows in the new totalsMat. Then use transprobbytotals to obtain the transition matrix.

```
TotalsCohort = TotalsNRCohort;
TotalsCohort.totalsMat = TotalsCohort.totalsMat(1:end-1,1:end-1);
TotalsCohort.totalsVec = sum(TotalsCohort.totalsMat,2)';
TotalsCohort.algorithm = 'cohort';
RatingsLabels = {'AAA','AA','A','BBB','BB','B','CCC','D'};
MatrixCohort = transprobbytotals(TotalsCohort);
fprintf('Transition probability, cohort, after postprocessing to remove NR:\n')
disp(array2table(MatrixCohort,'VariableNames',RatingsLabels,...
    'RowNames',RatingsLabels))
fprintf('Total transitions out of given rating, AA and CCC have one less than before:\\
disp(array2table(TotalsCohort.totalsVec,'VariableNames',RatingsLabels))
```




This procedure is different from removing the 'NR' rows from the data itself.
For example, if you remove the ' NR ' rows in this example, the first company seems to stay in its initial rating of ' CCC' all the way from the initial date in 1984 to the default event in 1991. With the previous approach, the estimation knows that the company transitioned out of ' CCC' at some point, it knows it was staying at 'CCC' all the time.

The second company seems to have stayed in the sample as an ' $A$ ' ' company until the end of the sample. With the previous approach, the estimation knows that this company stopped being an 'AA' earlier.

```
dataNR2 = dataNR;
dataNR2([2 7],:) = [];
head(dataNR2,12)
ans =
    12\times3 table
\begin{tabular}{|c|c|c|}
\hline ID & Date & Rating \\
\hline '00010283' & '10-Nov-1984' & ' CCC' \\
\hline '00010283' & '29-Jun-1988' & CCC \({ }^{\prime}\) \\
\hline '00010283' & '12-Dec-1991' & D' \\
\hline '00013326' & '09-Feb-1985' & 'A' \\
\hline '00013326' & '24-Feb-1994' & 'AA' \\
\hline '00014413' & '23-Dec-1982' & B' \\
\hline '00014413' & '20-Apr-1988' & BB' \\
\hline '00014413' & '16-Jan-1998' & B' \\
\hline '00014413' & '25-Nov-1999' & BB' \\
\hline '00012126' & '17-Feb-1985' & 'CCC' \\
\hline '00012126' & '08-Mar-1989' & 'D' \\
\hline '00011692' & '11-May-1984' & 'BB' \\
\hline
\end{tabular}
```

The transition matrices will be different. The probability of staying at ' CCC ' goes slightly up, and so does the probability of staying at 'AA'.

```
[MatrixCohort2,TotalsCohort2] = transprob(dataNR2,..
    'Labels',RatingsLabels,...
    'Algorithm','cohort');
fprintf('Transition probability, cohort, if NR rows are removed from data:\n')
disp(array2table(MatrixCohort2,'VariableNames',RatingsLabels,...
    'RowNames',RatingsLabels))
fprintf('Total transitions out of given rating, many more out of CCC and AA:\n')
disp(array2table(TotalsCohort2.totalsVec,'VariableNames',RatingsLabels))
```




## Estimate Point-in-Time and Through-the-Cycle Probabilities

Transition probability estimates are sensitive to the length of the estimation window. When the estimation window is small, the estimates only capture recent credit events, and these can change significantly from one year to the next. These are called point-intime (PIT) estimates. In contrast, a large time window yields fairly stable estimates that average transition rates over a longer period of time. These are called through-the-cycle (TTC) estimates.

The estimation of PIT probabilities requires repeated calls to transprob with a rolling estimation window. Use transprobprep every time repeated calls to transprob are required. transprobprep performs a preprocessing step on the raw dataset that is independent of the estimation window. The benefits of transprobprep are greater as
the number of repeated calls to transprob increases. Also, the performance gains from transprobprep are more significant for the cohort algorithm.

```
load Data TransProb
prepData = transprobprep(data);
Years = 1991:2000;
nYears = length(Years);
nRatings = length(prepData.ratingsLabels);
transMatPIT = zeros(nRatings,nRatings,nYears);
algorithm = 'duration';
sampleTotals(nYears,1) = struct('totalsVec',[],'totalsMat',[],...
'algorithm',algorithm);
for t = 1:nYears
    startDate = ['31-Dec-' num2str(Years(t)-1)];
    endDate = ['31-Dec-' num2str(Years(t))];
    [transMatPIT(:,:,t),sampleTotals(t)] = transprob(prepData,...
        'startDate',startDate,'endDate',endDate, 'algorithm',algorithm);
end
```

Here is the PIT transition matrix for 1993. Recall that the sample dataset contains simulated credit migrations so the PIT estimates in this example do not match actual historical transition rates.

| transMatPIT(:, : , Years==1993) |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ans = |  |  |  |  |  |  |  |
| 95.3193 | 4.5999 | 0.0802 | 0.0004 | 0.0002 | 0.0000 | 0.0000 | 0.0000 |
| 2.0631 | 94.5931 | 3.3057 | 0.0254 | 0.0126 | 0.0002 | 0.0000 | 0.0000 |
| 0.0237 | 2.1748 | 95.5901 | 1.4700 | 0.7284 | 0.0131 | 0.0000 | 0.0000 |
| 0.0003 | 0.0372 | 3.2585 | 95.2914 | 1.3876 | 0.0250 | 0.0001 | 0.0000 |
| 0.0000 | 0.0005 | 0.0657 | 3.8292 | 92.7474 | 3.3459 | 0.0111 | 0.0001 |
| 0.0000 | 0.0001 | 0.0128 | 0.7977 | 8.0926 | 90.4897 | 0.5958 | 0.0113 |
| 0.0000 | 0.0000 | 0.0005 | 0.0459 | 0.5026 | 11.1621 | 84.9315 | 3.3574 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 100.0000 |

A structure array stores the sampleTotals optional output from transprob. The sampleTotals structure contains summary information on the total time spent on each rating, and the number of transitions out of each rating, for each year under consideration. For more information on the sampleTotals structure, see transprob.

As an example, the sampleTotals structure for 1993 is used here. The total time spent on each rating is stored in the totalsVec field of the structure. The total transitions out of each rating are stored in the totalsMat field. A third field, algorithm, indicates the algorithm used to generate the structure.

[^1]```
    144.4411 230.0356 262.2438 204.9671 246.1315 147.0767 
```

ans $=$

| 0 | 7 | 0 | 0 | 0 | 0 | 0 | 0 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 5 | 0 | 8 | 0 | 0 | 0 | 0 | 0 |
| 0 | 6 | 0 | 4 | 2 | 0 | 0 | 0 |
| 0 | 0 | 7 | 0 | 3 | 0 | 0 | 0 |
| 0 | 0 | 0 | 10 | 0 | 9 | 0 | 0 |
| 0 | 0 | 0 | 1 | 13 | 0 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 7 | 0 | 2 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

ans =
duration
To get the TTC transition matrix, pass the sampleTotals structure array to transprobbytotals. Internally, transprobbytotals aggregates the information in the sampleTotals structures to get the total time spent on each rating over the 10 years considered in this example, and the total number of transitions out of each rating during the same period. transprobbytotals uses the aggregated information to get the TTC matrix, or average one-year transition matrix.

| transMatTTC $=$ transprobbytotals(sampleTotals) |  |  |  |  |  |  |  |
| ---: | :--- | ---: | :--- | ---: | ---: | ---: | ---: |
| transMatTTC $=$ |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |
| 92.8544 | 6.1068 | 0.7463 | 0.2761 | 0.0123 | 0.0009 | 0.0001 | 0.0032 |
| 2.9399 | 92.2329 | 3.8394 | 0.7349 | 0.1676 | 0.0050 | 0.0004 | 0.0799 |
| 0.2410 | 4.5963 | 90.3468 | 3.9572 | 0.6909 | 0.0521 | 0.0025 | 0.1133 |
| 0.0530 | 0.4729 | 7.9221 | 87.2751 | 3.5075 | 0.4650 | 0.0791 | 0.2254 |
| 0.0460 | 0.1636 | 1.1873 | 9.3442 | 85.4305 | 2.9520 | 0.1150 | 0.7615 |
| 0.0031 | 0.0152 | 0.2608 | 1.5563 | 10.4468 | 83.8525 | 1.9771 | 1.8882 |
| 0.0009 | 0.0041 | 0.0542 | 0.8378 | 2.9996 | 7.3614 | 82.4758 | 6.2662 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 100.0000 |

The same TTC matrix could be obtained with a direct call to transprob, setting the estimation window to the 10 years under consideration. But it is much more efficient to use the sampleTotals structures, whenever they are available. (Note, for the duration algorithm, these alternative workflows can result in small numerical differences in the estimates whenever leap years are part of the sample.)

In "Estimate Transition Probabilities" on page 8-3, a 1-year transition matrix is estimated using the 5 -year time window from 1996 through 2000. This is another example of a TTC matrix and this can also be computed using the sampleTotals structure array.

```
transprobbytotals(sampleTotals(Years>=1996&Years<=2000))
\begin{tabular}{rrrrrrrr} 
ans \(=\) \\
& & & & & & \\
90.6239 & 7.9048 & 1.0313 & 0.4123 & 0.0210 & 0.0020 & 0.0003 & 0.0043 \\
4.4776 & 89.5565 & 4.5294 & 1.1224 & 0.2283 & 0.0094 & 0.0009 & 0.0754
\end{tabular}
```

| 0.3982 | 6.1159 | 87.0651 | 5.4797 | 0.7636 | 0.0892 | 0.0050 | 0.0832 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 0.1029 | 0.8571 | 10.7909 | 83.0218 | 3.9968 | 0.7001 | 0.1313 | 0.3991 |
| 0.1043 | 0.3744 | 2.2960 | 14.0947 | 78.9851 | 3.0012 | 0.0463 | 1.0980 |
| 0.0113 | 0.0544 | 0.7054 | 3.2922 | 15.4341 | 75.6004 | 1.8165 | 3.0858 |
| 0.0044 | 0.0189 | 0.1903 | 1.9742 | 6.2318 | 10.2332 | 75.9990 | 5.3482 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 100.0000 |

## Estimate t-Year Default Probabilities

By varying the start and end dates, the amount of data considered for the estimation is changed, but the output still contains, by default, one-year transition probabilities. You can change the default behavior by specifying the transInterval argument, as illustrated in "Estimate Transition Probabilities" on page 8-3.

However, when $t$-year transition probabilities are required for a whole range of values of $t$, for example, 1-year, 2 -year, 3 -year, 4 -year, and 5 -year transition probabilities, it is more efficient to call transprob once to get the optional output sampleTotals. You can use the same sampleTotals structure can be used to get the $t$-year transition matrix for any transition interval $t$. Given a sampleTotals structure and a transition interval, you can get the corresponding transition matrix by using transprobbytotals.

```
load Data_TransProb
startDate = '31-Dec-1995';
endDate = '31-Dec-2000';
[~,sampleTotals] = transprob(data,'startDate', ...
startDate, 'endDate',endDate);
DefProb = zeros(7,5);
for t = 1:5
    transMatTemp = transprobbytotals(sampleTotals,'transInterval',t);
    DefProb(:,t) = transMatTemp(1:7,8);
end
DefProb
DefProb =
\begin{tabular}{lrrrr}
0.0043 & 0.0169 & 0.0377 & 0.0666 & 0.1033 \\
0.0754 & 0.1542 & 0.2377 & 0.3265 & 0.4213 \\
0.0832 & 0.1936 & 0.3276 & 0.4819 & 0.6536 \\
0.3992 & 0.8127 & 1.2336 & 1.6566 & 2.0779 \\
1.0980 & 2.1189 & 3.0668 & 3.9468 & 4.7644 \\
3.0860 & 5.6994 & 7.9281 & 9.8418 & 11.4963 \\
5.3484 & 9.8053 & 13.5320 & 16.6599 & 19.2964
\end{tabular}
```


## Estimate Bootstrap Confidence Intervals

transprob also returns the idTotals structure array which contains, for each ID, or company, the total time spent on each rating, and the total transitions out of each rating. For more information on the idTotals structure, see transprob. The idTotals structure is similar to the sampleTotals structures (see "Estimate Point-in-Time and Through-the-Cycle Probabilities" on page 8-11), but idTotals has the information at an ID level. Because most companies only migrate between few ratings, the numeric arrays in idTotals are stored as sparse arrays to reduce memory requirements.

You can use the idTotals structure array to estimate confidence intervals for the transition probabilities using a bootstrapping procedure, as the following example demonstrates. To do this, call transprob and keep the third output argument, idTotals. The idTotals fields are displayed for the last company in the sample. Within the estimation window, this company spends almost a year as 'AA' and it is then upgraded to 'AAA'.

```
load Data TransProb
startDate = '31-Dec-1995'
endDate = '31-Dec-2000';
[transMat,~,idTotals] = transprob(data,...
    startDate',startDate,'endDate',endDate);
% Total time spent on each rating
full(idTotals(end).totalsVec)
% Total transitions out of each rating
full(idTotals(end).totalsMat)
% Algorithm
idTotals(end).algorithm
ans =
    4.0820 0.9180 0
ans =
\begin{tabular}{llllllll}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{tabular}
ans =
duration
```

Next, use bootstrp from Statistics and Machine Learning Toolbox with transprobbytotals as the bootstrap function and idTotals as the data to sample from. Each bootstrap sample corresponds to a dataset made of companies sampled with replacement from the original data. However, you do not have to draw companies from the original data, because a bootstrap idTotals sample contains all the information required to compute the transition probabilities. transprobbytotals aggregates all structures in each bootstrap idTotals sample and finds the corresponding transition matrix.

To estimate $95 \%$ confidence intervals for the transition matrix and display the probabilities of default together with its upper and lower confidence bounds:

```
PD = transMat(1:7,8);
bootstat = bootstrp(100,@(totals)transprobbytotals(totals),idTotals);
ci = prctile(bootstat,[2.5 97.5]); % 95% confidence
CIlower = reshape(ci(1,:),8,8);
CIupper = reshape(ci(2,:),8,8);
PD_LB = CIlower(1:7,8);
PD_UB = CIupper(1:7,8);
[PD_LB PD PD_UB]
ans =
\begin{tabular}{lll}
0.0004 & 0.0043 & 0.0106 \\
0.0028 & 0.0754 & 0.2192 \\
0.0126 & 0.0832 & 0.2180 \\
0.1659 & 0.3992 & 0.6617 \\
0.5703 & 1.0980 & 1.7260 \\
1.7264 & 3.0860 & 4.7602 \\
1.7678 & 5.3484 & 9.5055
\end{tabular}
```


## Group Credit Ratings

Credit rating scales can be more or less granular. For example, there are ratings with qualifiers (such as, 'AA+', 'BB-' , and so on), whole ratings ('AA', 'BB' , and so on), and investment or speculative grade ('IG', 'SG') categories. Given a dataset with credit ratings at a more granular level, transition probabilities for less granular categories can be of interest. For example, you might be interested in a transition matrix for investment and speculative grades given a dataset with whole ratings. Use transprobgrouptotals for this evaluation, as illustrated in the following examples. The sample dataset data has whole credit ratings:

```
load Data TransProb
startDate = '31-Dec-1995';
endDate = '31-Dec-2000';
data(1:5,:)
ans =
\begin{tabular}{lll} 
'00010283' & '10-Nov-1984' & 'CCC' \\
'00010283' & '12-May-1986' & 'B' \\
'00010283' & '29-Jun-1988' & 'CCC' \\
'00010283' & '12-Dec-1991' & 'D ' \\
'00013326' & '09-Feb-1985' & 'A
\end{tabular}
```

A call to transprob returns the transition matrix and totals structures for the eight ('AAA' to 'D') whole credit ratings. The array with number of transitions out of each credit rating is displayed after the call to transprob:

```
[transMat,sampleTotals,idTotals] = transprob(data,'startDate',startDate,...
'endDate',endDate);
sampleTotals.totalsMat
ans =
\begin{tabular}{rrrrrrrr}
0 & 67 & 7 & 3 & 0 & 0 & 0 & 0 \\
67 & 0 & 68 & 15 & 3 & 0 & 0 & 1 \\
4 & 101 & 0 & 93 & 11 & 1 & 0 & 1 \\
1 & 7 & 163 & 0 & 62 & 10 & 2 & 5 \\
1 & 3 & 16 & 168 & 0 & 37 & 0 & 11 \\
0 & 0 & 2 & 10 & 83 & 0 & 10 & 14 \\
0 & 0 & 0 & 2 & 8 & 16 & 0 & 7 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{tabular}
```

Next, use transprobgrouptotals to group whole ratings into investment and speculative grades. This function takes a totals structure as the first argument. The second argument indicates the edges between rating categories. In this case, ratings 1 through 4 ('AAA' through 'BBB') correspond to the first category ('IG'), ratings 5 through 7 ('BB' through 'CCC') to the second category ('SG'), and rating 8 ('D') is a category of its own. transprobgrouptotals adds up the total time spent on ratings that belong to the same category. For example, total times spent on 'AAA' through 'BBB' are added up as the total time spent on 'IG'. transprobgrouptotals also adds up the total number of transitions between any 'IG' rating and any 'SG' rating, for example, a credit migration from ' BBB ' to ' BB '.

The grouped totals can then be passed to transprobbytotals to obtain the transition matrix for investment and speculative grades. Both totalsMat and the new transition matrix are both 3-by-3, corresponding to the grouped categories 'IG', 'SG', and 'D'.
sampleTotalsIGSG = transprobgrouptotals(sampleTotals,[4 7 8]) transMatIGSG = transprobbytotals(sampleTotalsIGSG)
sampleTotalsIGSG =
totalsVec: [4.8591e+003 1.5034e+003 1.1621e+003]
totalsMat: [3x3 double]
algorithm: 'duration'
transMatIGSG =

| 98.1591 | 1.6798 | 0.1611 |
| ---: | ---: | ---: |
| 12.3228 | 85.6961 | 1.9811 |
| 0 | 0 | 100.0000 |

When a totals structure array is passed to transprobgrouptotals, this function groups each structure in the array individually and preserves sparsity, if the fields in the input structures are sparse. One way to exploit this feature is to compute confidence intervals for the investment grade default rate and the speculative grade default rate (see also "Estimate Bootstrap Confidence Intervals" on page 8-15).

```
PDIGSG = transMatIGSG(1:2,3);
idTotalsIGSG = transprobgrouptotals(idTotals,[4 7 8]);
bootstat = bootstrp(100,@(totals)transprobbytotals(totals),idTotalsIGSG);
ci = prctile(bootstat,[2.5 97.5]); % 95% confidence
CIlower = reshape(ci(1,:),3,3);
CIupper = reshape(ci(2,:),3,3);
PDIGSG_LB = CIlower(1:2,3);
PDIGSG_UB = CIupper(1:2,3);
[PDIGSG_LB PDIGSG PDIGSG_UB]
ans =
    0.0603 0.1611 0.2538
    1.3470 1.9811 2.6195
```


## Work with Nonsquare Matrices

Transition probabilities and the number of transitions between ratings are usually reported without the 'D' ('Default') row. For example, a credit report can contain the
following table, indicating the number of issuers starting in each rating (first column), and the number of transitions between ratings (remaining columns):

| Initial | AAA | AA | A | BBB | BB | B | CCC | D |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| AAA | 98 | 88 | 9 | 1 | 0 | 0 | 0 | 0 | 0 |
| AA | 389 | 0 | 368 | 19 | 2 | 0 | 0 | 0 | 0 |
| A | 1165 | 1 | 21 | 1087 | 56 | 0 | 0 | 0 | 0 |
| BBB | 1435 | 0 | 2 | 89 | 1289 | 45 | 8 | 0 | 2 |
| BB | 915 | 0 | 0 | 1 | 60 | 776 | 73 | 2 | 3 |
| B | 867 | 0 | 0 | 1 | 7 | 88 | 715 | 39 | 17 |
| CCC | 112 | 0 | 0 | 0 | 1 | 3 | 34 | 61 | 13 |

You can store the information in this table in a totals structure compatible with the cohort algorithm. For more information on the cohort algorithm and the totals structure, see transprob. The totalsMat field is a nonsquare array in this case.

```
% Define totals structure
totals.totalsVec = [98 389 1165 1435 915 867 112];
totals.totalsMat = [
    88
        0
        1 21 1087 56 0 0 0 0 0
        0
        0
        0
        0 0 0 0 1 3 34 61 13];
totals.algorithm = 'cohort';
```

transprobbytotals and transprobgrouptotals accept totals inputs with nonsquare totalsMat fields. To get the transition matrix corresponding to the previous table, and to group ratings into investment and speculative grade with the corresponding matrix:

```
transMat = transprobbytotals(totals)
% Group into IG/SG and get IG/SG transition matrix
totalsIGSG = transprobgrouptotals(totals,[4 7]);
transMatIGSG = transprobbytotals(totalsIGSG)
transMat =
\begin{tabular}{rrrrrrrr}
89.7959 & 9.1837 & 1.0204 & 0 & 0 & 0 & 0 & 0 \\
0 & 94.6015 & 4.8843 & 0.5141 & 0 & 0 & 0 & 0 \\
0.0858 & 1.8026 & 93.3047 & 4.8069 & 0 & 0 & 0 & 0 \\
0 & 0.1394 & 6.2021 & 89.8258 & 3.1359 & 0.5575 & 0 & 0.1394 \\
0 & 0 & 0.1093 & 6.5574 & 84.8087 & 7.9781 & 0.2186 & 0.3279 \\
0 & 0 & 0.1153 & 0.8074 & 10.1499 & 82.4683 & 4.4983 & 1.9608 \\
0 & 0 & 0 & 0.8929 & 2.6786 & 30.3571 & 54.4643 & 11.6071
\end{tabular}
```

| 98.2183 | 1.7169 | 0.0648 |
| ---: | ---: | ---: |
| 3.6959 | 94.5618 | 1.7423 |

## Remove Outliers

The idTotals output from transprob can also be exploited to update the transition probability estimates after removing some outlier information. For more information on idTotals, see transprob. For example, if you know that the credit rating migration information for the 4th and 27th companies in the data have problems, you can remove those companies and efficiently update the transition probabilities as follows:

```
load Data TransProb
startDate = '31-Dec-1995';
endDate = '31-Dec-2000';
[transMat,~,idTotals] = transprob(data,'startDate', ...
startDate, 'endDate',endDate);
transMat
transMat =
\begin{tabular}{rrrrrrrr}
90.6236 & 7.9051 & 1.0314 & 0.4123 & 0.0210 & 0.0020 & 0.0003 & 0.0043 \\
4.4780 & 89.5558 & 4.5298 & 1.1225 & 0.2284 & 0.0094 & 0.0009 & 0.0754 \\
0.3983 & 6.1164 & 87.0641 & 5.4801 & 0.7637 & 0.0892 & 0.0050 & 0.0832 \\
0.1029 & 0.8572 & 10.7918 & 83.0204 & 3.9971 & 0.7001 & 0.1313 & 0.3992 \\
0.1043 & 0.3745 & 2.2962 & 14.0954 & 78.9840 & 3.0013 & 0.0463 & 1.0980 \\
0.0113 & 0.0544 & 0.7055 & 3.2925 & 15.4350 & 75.5988 & 1.8166 & 3.0860 \\
0.0044 & 0.0189 & 0.1903 & 1.9743 & 6.2320 & 10.2334 & 75.9983 & 5.3484 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 100.0000
\end{tabular}
nIDs = length(idTotals)
keepInd = setdiff(1:nIDs,[4 27]);
transMatNoOutlier = transprobbytotals(idTotals(keepInd))
transMatNoOutlier \(=\)
\begin{tabular}{rrrrrrrr}
90.6241 & 7.9067 & 1.0290 & 0.4124 & 0.0211 & 0.0020 & 0.0003 & 0.0043 \\
4.4917 & 89.5918 & 4.4779 & 1.1240 & 0.2288 & 0.0094 & 0.0009 & 0.0756 \\
0.3990 & 6.1220 & 87.0530 & 5.4841 & 0.7643 & 0.0893 & 0.0050 & 0.0833 \\
0.1030 & 0.8576 & 10.7909 & 83.0207 & 3.9971 & 0.7001 & 0.1313 & 0.3992 \\
0.1043 & 0.3746 & 2.2960 & 14.0955 & 78.9840 & 3.0013 & 0.0463 & 1.0980 \\
0.0113 & 0.0544 & 0.7054 & 3.2925 & 15.4350 & 75.5988 & 1.8166 & 3.0860 \\
0.0044 & 0.0189 & 0.1903 & 1.9743 & 6.2320 & 10.2334 & 75.9983 & 5.3484 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 100.0000
\end{tabular}
```

Deciding which companies to remove is a case-by-case situation. Reasons to remove a company can include a typo in one of the ratings histories, or an unusual migration between ratings whose impact on the transition probability estimates must be measured. transprob does not reorder the companies in any way. The ordering of companies in the input data is the same as the ordering in the idTotals array.

## Estimate Probabilities for Different Segments

You can use idTotals efficiently to get estimates over different segments of the sample. For more information on idTotals, see transprob. For example, assume that the companies in the example are grouped into three geographic regions and that the companies were grouped by geographic regions previously, so that the first 340 companies correspond to the first region, the next 572 companies to the second region, and the rest to the third region. You can efficiently get transition probabilities for each region as follows:

```
load Data TransProb
startDate-= '31-Dec-1995';
endDate = '31-Dec-2000';
[~,~,idTotals] = transprob(data,'startDate', ...
startDate, 'endDate',endDate);
n1 = 340;
n2 = 572;
transMatG1 = transprobbytotals(idTotals(1:n1))
transMatG2 = transprobbytotals(idTotals(n1+1:n1+n2))
transMatG3 = transprobbytotals(idTotals(n1+n2+1:end))
```

transMatG1 =

| 90.8299 | 7.6501 | 0.3178 | 1.1700 | 0.0255 | 0.0044 | 0.0021 | 0.0002 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 4.3572 | 89.0262 | 5.7838 | 0.8039 | 0.0245 | 0.0029 | 0.0013 | 0.0001 |
| 0.7066 | 6.7567 | 86.6320 | 5.4950 | 0.3721 | 0.0252 | 0.0101 | 0.0023 |
| 0.0626 | 1.3688 | 10.3895 | 83.5022 | 3.6823 | 0.6466 | 0.3084 | 0.0396 |
| 0.0256 | 0.7884 | 2.6970 | 13.7857 | 78.8321 | 2.8310 | 0.0561 | 0.9842 |
| 0.0026 | 0.1095 | 0.4280 | 3.5204 | 21.1437 | 72.9230 | 1.6456 | 0.2273 |
| 0.0005 | 0.0216 | 0.0730 | 0.4574 | 4.9586 | 4.2821 | 80.3062 | 9.9006 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 100.0000 |
|  |  |  |  |  |  |  |  |
| transMatG2 |  |  |  |  |  |  |  |
| 90.5798 | 8.4877 | 0.8202 | 0.0884 | 0.0132 | 0.0011 | 0.0000 | 0.0096 |
| 4.1999 | 90.0371 | 3.8657 | 1.4744 | 0.2144 | 0.0128 | 0.0001 | 0.1956 |
| 0.3022 | 5.9869 | 86.7128 | 5.5526 | 1.0411 | 0.1902 | 0.0015 | 0.2127 |
| 0.0204 | 0.5606 | 10.9342 | 82.9195 | 4.0123 | 0.7398 | 0.0059 | 0.8073 |
| 0.0089 | 0.3338 | 2.1185 | 16.6496 | 76.2395 | 3.1241 | 0.0261 | 1.4995 |
| 0.0013 | 0.0465 | 0.6710 | 2.4731 | 14.7281 | 76.7378 | 1.2993 | 4.0428 |
| 0.0002 | 0.0080 | 0.0681 | 0.4598 | 4.1324 | 8.4380 | 80.9092 | 5.9843 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 100.0000 |
|  |  |  |  |  |  |  |  |
| transMatG3 | $=$ |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |
| 90.5655 | 7.5408 | 1.5288 | 0.3369 | 0.0258 | 0.0015 | 0.0003 | 0.0004 |
| 4.8073 | 89.3842 | 4.4865 | 0.9582 | 0.3509 | 0.0095 | 0.0009 | 0.0025 |
| 0.3153 | 5.8771 | 87.6353 | 5.4101 | 0.7160 | 0.0322 | 0.0052 | 0.0088 |
| 0.1995 | 0.8625 | 10.8682 | 82.8717 | 4.1423 | 0.6903 | 0.1565 | 0.2090 |
| 0.2465 | 0.1091 | 2.1558 | 12.0289 | 81.5803 | 3.0057 | 0.0616 | 0.8122 |
| 0.0227 | 0.0400 | 0.9380 | 4.3175 | 12.3632 | 75.9429 | 2.5766 | 3.7991 |
| 0.0149 | 0.0180 | 0.3414 | 3.6918 | 8.1414 | 13.6010 | 70.7254 | 3.4661 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 100.0000 |

## Work with Large Datasets

This example shows how to aggregate estimates from two (or more) datasets. It is possible that two datasets, coming from two different databases, must be considered for the estimation of the transition probabilities. Also, if a dataset is too large and cannot be loaded into memory, the dataset can be split into two (or more) datasets. In these cases, it is simple to apply transprob to each individual dataset, and then get the final estimates corresponding to the aggregated data with a call to transprobbytotals at the end.

For example, the dataset data is artificially split into two sections in this example. In practice the two datasets would come from different files or databases. When aggregating multiple datasets, the history of a company cannot be split across datasets. You can analyze that this condition is satisfied for the arbitrarily chosen cut-off point.

```
load Data_TransProb
```

cutoff = 2099;
data(cutoff-5:cutoff,:)
data(cutoff+1:cutoff+6,:)

```
ans =
```

| '000011166' | '24-Aug-1995' | 'BBB' |
| :--- | :--- | :--- |
| '00011166' | '25-Jan-1997' | 'A' |
| '00011166' | '01-Feb-1998' | 'AA' |
| '00014878' | '15-Mar-1983' | 'B' |
| '00014878' | '21-Sep-1986' | 'BB' |
| '00014878' | '17-Jan-1998' | 'BBB' |

ans $=$

| '00012043' | '09-Feb-1985' | 'BBB' |
| :--- | :--- | :--- |
| '00012043' | '03-Jan-1988' | 'A' |
| '00012043' | '15-Jan-1994' | 'AAA' |
| '00011157' | '24-Jun-1984' | 'A' |
| '00011157' | '09-Dec-1999' | 'BBB' |
| '00011157' | '28-Mar-2001' | 'A' |

When working with multiple datasets, it is important to set the start and end dates explicitly. Otherwise, the estimation window differs for each dataset because the default start and end dates used by transprob are the earliest and latest dates found in the input data.

```
startDate = '31-Dec-1995';
endDate = '31-Dec-2000';
```

In practice, this is the point where you can read in the first dataset. Now, the dataset is already obtained. Call transprob with the first dataset and the explicit start and end dates. Keep only the sampleTotals output. For details on sampleTotals, see transprob.

```
[~,sampleTotals(1)] = transprob(data(1:cutoff,:),...
    'startDate',startDate,'endDate',endDate);
```

Repeat for the remaining datasets. Note the different sampleTotals structures are stored in a structured array.

```
[~,sampleTotals(2)] = transprob(data(cutoff+1:end,:),...
    'startDate',startDate,'endDate',endDate);
```

To get the transition matrix corresponding to the aggregated dataset, use transprobbytotals. When the totals input is a structure array, transprobbytotals aggregates the information over all structures, and returns a single transition matrix.

| transMatAggr = |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 90.6236 | 7.9051 | 1.0314 | 0.4123 | 0.0210 | 0.0020 | 0.0003 | 0.0043 |
| 4.4780 | 89.5558 | 4.5298 | 1.1225 | 0.2284 | 0.0094 | 0.0009 | 0.0754 |
| 0.3983 | 6.1164 | 87.0641 | 5.4801 | 0.7637 | 0.0892 | 0.0050 | 0.0832 |
| 0.1029 | 0.8572 | 10.7918 | 83.0204 | 3.9971 | 0.7001 | 0.1313 | 0.3992 |
| 0.1043 | 0.3745 | 2.2962 | 14.0954 | 78.9840 | 3.0013 | 0.0463 | 1.0980 |
| 0.0113 | 0.0544 | 0.7055 | 3.2925 | 15.4350 | 75.5988 | 1.8166 | 3.0860 |
| 0.0044 | 0.0189 | 0.1903 | 1.9743 | 6.2320 | 10.2334 | 75.9983 | 5.3484 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 100.0000 |

As a sanity check, for this example you can analyze that the aggregation procedure yields the same estimates (up to numerical differences) as estimating the probabilities directly over the entire sample:

| transMatWhole = |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 90.6236 | 7.9051 | 1.0314 | 0.4123 | 0.0210 | 0.0020 | 0.0003 | 0.0043 |
| 4.4780 | 89.5558 | 4.5298 | 1.1225 | 0.2284 | 0.0094 | 0.0009 | 0.0754 |
| 0.3983 | 6.1164 | 87.0641 | 5.4801 | 0.7637 | 0.0892 | 0.0050 | 0.0832 |
| 0.1029 | 0.8572 | 10.7918 | 83.0204 | 3.9971 | 0.7001 | 0.1313 | 0.3992 |
| 0.1043 | 0.3745 | 2.2962 | 14.0954 | 78.9840 | 3.0013 | 0.0463 | 1.0980 |
| 0.0113 | 0.0544 | 0.7055 | 3.2925 | 15.4350 | 75.5988 | 1.8166 | 3.0860 |
| 0.0044 | 0.0189 | 0.1903 | 1.9743 | 6.2320 | 10.2334 | 75.9983 | 5.3484 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 100.0000 |

## See Also

bootstrp|transprob|transprobbytotals|transprobfromthresholds | transprobgrouptotals|transprobprep|transprobtothresholds

## Related Examples

- "Credit Quality Thresholds" on page 8-57
- "Credit Rating by Bagging Decision Trees" (Statistics and Machine Learning Toolbox)
- "Forecasting Corporate Default Rates" on page 8-25


## External Websites

- Credit Risk Modeling with MATLAB (53 min 09 sec )
- Forecasting Corporate Default Rates with MATLAB (54 min 36 sec )


## Forecasting Corporate Default Rates

This example shows how to build a forecasting model for corporate default rates.
Risk parameters are dynamic in nature, and understanding how these parameters change in time is a fundamental task for risk management.

In the first part of this example, we work with historical credit migrations data to construct some time series of interest, and to visualize default rates dynamics. In the second part of this example, we use some of the series constructed in the first part, and some additional data, to fit a forecasting model for corporate default rates, and to show some backtesting and stress testing concepts. A linear regression model for corporate default rates is presented, but the tools and concepts described can be used with other forecasting methodologies. The appendix at the end references the handling of models for full transition matrices.

People interested in forecasting, backtesting, and stress testing can go directly to the second part of this example. The first part of this example is more relevant for people who work with credit migration data.

## Part I: Working with Credit Migrations Data

We work with historical transition probabilities for corporate issuers (variable TransMat). This is yearly data for the period 1981-2005, from [10]. The data includes, for each year, the number of issuers per rating at the beginning of the year (variable nIssuers), and the number of new issuers per rating per year (variable nNewIssuers). There is also a corporate profits forecast, from [9], and a corporate spread, from [4] (variables CPF and SPR). A variable indicating recession years (Recession), consistent with recession dates from [7], is used mainly for visualizations.

Example_LoadData

## Getting Default Rates for Different Ratings Categories

We start by performing some aggregations to get corporate default rates for Investment Grade (IG) and Speculative Grade (SG) issuers, and the overall corporate default rate.

Aggregation and segmentation are relative terms. IG is an aggregate with respect to credit ratings, but a segment from the perspective of the overall corporate portfolio. Other segments are of interest in practice, for example, economic sectors, industries, or geographic regions. The data we use, however, is aggregated by credit ratings, so further
segmentation is not possible. Nonetheless, the tools and workflow discussed here can be useful to work with other segment-specific models.

Use functionality in Financial Toolbox ${ }^{\mathrm{TM}}$, specifically, the functions transprobgrouptotals and transprobbytotals, to perform the aggregation. These functions take as inputs structures with credit migration information in a particular format. We set up the inputs here and visualize them below to understand their information and format.

```
% Pre-allocate the struct array
totalsByRtg(nYears,1) = struct('totalsVec',[],'totalsMat',[],...
    'algorithm','cohort');
for t = 1:nYears
    % Number of issuers per rating at the beginning of the year
    totalsByRtg(t).totalsVec = nIssuers(t,:);
    % Number of transitions between ratings during the year
    totalsByRtg(t).totalsMat = round(diag(nIssuers(t,:))*...
        (0.01*TransMat(:,:,t)));
    % Algorithm
    totalsByRtg(t).algorithm = 'cohort';
end
```

It is useful to see both the original data and the data stored in these totals structures side to side. The original data contains number of issuers and transition probabilities for each year. For example, for 2005:

```
fprintf('\nTransition matrix for 2005:\n\n')
Transition matrix for 2005:
Example_DisplayTransitions(squeeze(TransMat(:,:,end)),nIssuers(end,:),...
    {'AAA','AA','A','BBB','BB','B','CCC'},...
    {'AAA','AA','A','BBB','BB','B','CCC','D','NR'})
\begin{tabular}{rrrrrrrrrrr} 
\\
& Init & AAA & AA & A & BBB & BB & B & CCC & D & NR \\
AAA & 98 & 88.78 & 9.18 & 1.02 & 0 & 0 & 0 & 0 & 0 & 1.02 \\
AA & 407 & 0 & 90.66 & 4.91 & 0.49 & 0 & 0 & 0 & 0 & 3.93 \\
A & 1224 & 0.08 & 1.63 & 88.89 & 4.41 & 0 & 0 & 0 & 0 & 4.98 \\
BBB & 1535 & 0 & 0.2 & 5.93 & 84.04 & 3.06 & 0.46 & 0 & 0.07 & 6.25 \\
BB & 1015 & 0 & 0 & 0 & 5.71 & 76.75 & 6.9 & 0.2 & 0.2 & 10.25 \\
B & 1010 & 0 & 0 & 0.1 & 0.59 & 8.51 & 70.59 & 3.76 & 1.58 & 14.85 \\
CCC & 126 & 0 & 0 & 0 & 0.79 & 0.79 & 25.4 & 46.83 & 8.73 & 17.46
\end{tabular}
```

The totals structure stores the total number of issuers per rating at the beginning of the year in the totalsVec field, and the total number of migrations between ratings (instead of transition probabilities) in the totalsMat field. Here is the information for 2005:

| Transition counts (totals struct) for 2005: |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ```Example_DisplayTransitions(totalsByRtg(end).totalsMat,... totalsByRtg(end).totalsVec,... {'AAA','AA','A','BBB','BB','B','CCC'},... {'AAA','AA','A','BBB','BB','B','CCC','D','NR'})``` |  |  |  |  |  |  |  |  |  |  |
|  | Init | AAA | AA | A | BBB | BB | B | CCC | D | NR |
| AAA | 98 | 87 | 9 | 1 | 0 | 0 | 0 | 0 | 0 | 1 |
| AA | 407 | 0 | 369 | 20 | 2 | 0 | 0 | 0 | 0 | 16 |
| A | 1224 | 1 | 20 | 1088 | 54 | 0 | 0 | 0 | 0 | 61 |
| BBB | 1535 | 0 | 3 | 91 | 1290 | 47 | 7 | 0 | 1 | 96 |
| BB | 1015 | 0 | 0 | 0 | 58 | 779 | 70 | 2 | 2 | 104 |
| B | 1010 | 0 | 0 | 1 | 6 | 86 | 713 | 38 | 16 | 150 |
| CCC | 126 | 0 | 0 | 0 | 1 | 1 | 32 | 59 | 11 | 22 |

The third field in the totals structure, algorithm, indicates that we are working with the cohort method (duration is also supported, although the information in totalsVec and totalsMat would be different). These structures are obtained as optional outputs from transprob, but this example shows how you can define these structures directly.

Use transprobgrouptotals to group the ratings 'AAA' to 'BBB' (ratings 1 to 4) into the IG category and ratings 'BB' to 'CCC' (ratings 5 to 7 ) into the SG category. The edges argument tells the function which ratings are to be grouped together ( 1 to 4 , and 5 to 7). We also group all non-default ratings into one category. These are preliminary steps to get the IG, SG, and overall default rates for each year.

```
edgesIGSG = [4 7];
totalsIGSG = transprobgrouptotals(totalsByRtg,edgesIGSG);
edgesAll = 7; % could also use edgesAll = 2 with totalsIGSG
totalsAll = transprobgrouptotals(totalsByRtg,edgesAll);
```

Here are the 2005 totals grouped at IG/SG level, and the corresponding transition matrix, recovered using transprobbytotals.
fprintf('\nTransition counts for 2005 at IG/SG level:\n\n')
Transition counts for 2005 at IG/SG level:

```
Example_DisplayTransitions(totalsIGSG(end).totalsMat,...
    totalsIGSG(end).totalsVec,...
    {'IG','SG'},...
    {'IG','SG','D','NR'})
\begin{tabular}{lrrrrr} 
& Init & IG & SG & D & NR \\
IG & 3264 & 3035 & 54 & 1 & 174 \\
SG & 2151 & 66 & 1780 & 29 & 276
\end{tabular}
fprintf('\nTransition matrix for 2005 at IG/SG level:\n\n')
Transition matrix for 2005 at IG/SG level:
Example_DisplayTransitions(transprobbytotals(totalsIGSG(end)),[],...
    {'IG','SG'},...
    {'IG','SG','D','NR'})
\begin{tabular}{lrrrr} 
& IG & SG & D & NR \\
IG & 92.98 & 1.65 & 0.03 & 5.33 \\
SG & 3.07 & 82.75 & 1.35 & 12.83
\end{tabular}
```

Now get transition matrices for every year both at IG/SG and non-default/default levels and store the default rates only (we do not use the rest of the transition probabilities).

```
DefRateIG = zeros(nYears,1);
DefRateSG = zeros(nYears,1);
DefRate = zeros(nYears,1);
for t=1:nYears
    % Get transition matrix at IG/SG level and extract IG default rate and
    % SG default rate for year t
    tmIGSG = transprobbytotals(totalsIGSG(t));
    DefRateIG(t) = tmIGSG(1,3);
    DefRateSG(t) = tmIGSG(2,3);
    % Get transition matrix at most aggregate level and extract overall
    % corporate default rate for year t
    tmAll = transprobbytotals(totalsAll(t));
    DefRate(t) = tmAll(1,2);
end
```

Here is a visualization of the dynamics of IG, SG, and overall corporate default rates together. To emphasize their patterns, rather than their magnitudes, a log scale is used. The shaded bands indicate recession years. The patterns of SG and IG are slightly different. For example, the IG rate is higher in 1994 than in 1995, but the opposite is true for SG. More noticeably, the IG default rate peaked after the 2001 recession, in 2002,
whereas the peak for SG is in 2001. This suggests that models for the dynamics of the IG and SG default rates could have important differences, a common situation when working with different segments. The overall corporate default rate is by construction a combination of the other two, and its pattern is closer to SG, most likely due to the relative magnitude of SG versus IG.

```
minIG = min(DefRateIG(DefRateIG~=0));
figure
plot(Years,log(DefRateSG),'m-*')
hold on
plot(Years,log(DefRate),'b-o')
plot(Years,log(max(DefRateIG,minIG-0.001)),'r-+')
Example_RecessionBands
hold off
grid on
title('{\bf Default Rates (log scale)}')
ylabel('log %')
legend({'SG','Overall','IG'},'location','NW')
```



## Getting Default Rates for Different Time Periods

The default rates obtained are examples of point-in-time (PIT) rates, only the most recent information is used to estimate them. On the other extreme, we can use all the migrations observed in the 25 years spanned by the dataset to estimate long-term, or through-thecycle (TTC) default rates. Other rates of interest are the average default rates over recession or expansion years.

All of these are easy to estimate with the data we have and the same tools. For example, to estimate the average transition probabilities over recession years, pass to transprobbytotals the totals structures corresponding to the recession years only. We use logical indexing below, taking advantage of the Recession variable.
transprobbytotals aggregates the information over time and returns the corresponding transition matrix.

```
tmAllRec = transprobbytotals(totalsAll(Recession));
DefRateRec = tmAllRec(1,2);
tmAllExp = transprobbytotals(totalsAll(~Recession));
DefRateExp = tmAllExp(1,2);
tmAllTTC = transprobbytotals(totalsAll);
DefRateTTC = tmAllTTC(1,2);
```

The following figure shows the estimated PIT rates, TTC rates, and recession and expansion rates.

DefRateTwoValues = DefRateExp*ones(nYears,1);
DefRateTwoValues(Recession) = DefRateRec;

```
figure
plot(Years,DefRate,'bo:','LineWidth',1.2)
hold on
stairs(Years-0.5,DefRateTwoValues,'m-',''LineWidth',1.5)
plot(Years,DefRateTTC*ones(nYears,1),'r-.','LineWidth',1.5)
Example RecessionBands
hold off
grid on
title('{\bf Default Rate}')
ylabel('%')
legend({'Point-in-time (PIT)','Recession/Expansion Avg',...
    'Through-the-cycle (TTC)'},'location','NW')
```



Some analyses (see, for example, [11]) use simulations where the default rate is conditional on the general state of the economy, for example, recession v. expansion. The recession and expansion estimates obtained can be useful in such a framework. These are all historical averages, however, and may not work well if used as predictions for the actual default rates expected on any particular year. In the second part of this example, we revisit the use of these types of historical averages as forecasting tools in a backtesting exercise.

## Building Predictors Using Credit Ratings Data

Using the credit data, you can build new time series of interest. We start with an age proxy that is used as predictor in the forecasting model in the second part of this example.

Age is known to be an important factor in predicting default rates; see, e.g., [1] and [5]. Age here means the number of years since a bond was issued. By extension, the age of a portfolio is the average age of its bonds. Certain patterns have been observed historically. Many low-quality borrowers default just a few years after issuing a bond. When troubled companies issue bonds, the amount borrowed helps them make payments for a year or two. Beyond that point, their only source of money is their cash flows, and if they are insufficient, default occurs.

We cannot calculate the exact age of the portfolio, because there is no information at issuer level in the dataset. We follow [6], however, and use the number of new issuers in year $t-3$ divided by the total number of issuers at the end of year $t$ as an age proxy. Because of the lag, the age proxy starts in 1984. For the numerator, we have explicit information on the number of new issuers. For the denominator, the number of issuers at the end of a year equals the number of issuers at the beginning of next year. This is known for all years but the last one, which is set to the total transitions into a non-default rating plus the number of new issuers on that year.

```
% Total number of issuers at the end of the year
nEOY = zeros(nYears,1);
% nIssuers is number of issuers per ratings at the beginning of the year
% nEOY ( 1981 ) = sum nIssuers ( 1982 ), etc until 2004
nEOY(1:end-1) = sum(nIssuers(2:end,:),2);
% nEOY ( 2005 ) = issuers in non-default state at end of 2005 plus
% new issuers in 2005
nEOY(end) = totalsAll(end).totalsMat(1,1) + sum(nNewIssuers(end,:));
% Age proxy
AGE = 100*[nan(3,1); sum(nNewIssuers(1:end-3,:),2)./nEOY(4:end)];
```

Examples of other time series of interest are the proportion of SG issuers at the end of each year, or an age proxy for SG.

```
% nSGEOY: Number of SG issuers at the end of the year
% nSGEOY is similar to nEOY, but for SG only, from 5 ('BB') to 7 ('CCC')
indSG = 5:7;
nSGEOY = zeros(nYears,1);
nSGEOY(1:end-1) = sum(nIssuers(2:end,indSG),2);
nSGEOY(end) = sum(totalsIGSG(end).totalsMat(:,2)) +...
    sum(nNewIssuers(end,indSG));
% Proportion of SG issuers
SG = 100*nSGEOY./nEOY;
% SG age proxy: new SG issuers in t-3 / total issuers at the end of year t
AGESG = 100*[nan(3,1); sum(nNewIssuers(1:end-3,indSG),2)./nEOY(4:end)];
```


## Part II: A Forecasting Model for Default Rates

We work with the following linear regression model for corporate default rates

$$
\text { DefRate }=\beta_{0}+\beta_{a g e} A G E+\beta_{c p j} C P F+\beta_{s p r} S P R
$$

where

- AGE: Age proxy defined above
- CPF: Corporate profits forecast
- SPR: Corporate spread over treasuries

This is the same model as in [6], except the model in [6] is for IG only.
As previously discussed, age is known to be an important factor regarding default rates. The corporate profits provide information on the economic environment. The corporate spread is a proxy for credit quality. Age, environment, and quality are three dimensions frequently found in credit analysis models.

```
inSample = 4:nYears-1;
T = length(inSample);
varNames = {'AGE','CPF','SPR'};
X = [AGE CPF SPR];
X = X(inSample,:);
y = DefRate(inSample+1); % DefaultRate, year t+1
stats = regstats(y,X);
fprintf('\nConst AGE CPF SPR adjR^2\n')
Const AGE CPF SPR adjR^2
fprintf('%1.2f %1.2f %1.2f %1.2f %1.4f\n',...
    [stats.beta;stats.adjrsquare])
-1.19 0.15 -0.10 0.71 0.7424
```

The coefficients have the expected sign: default rates tend to increase with a higher proportion of 3-year issuers, decrease with good corporate profits, and increase when the corporate yields are higher. The adjusted R square shows a good fit.

The in-sample fit, or how close the model predictions are from the sample points used to fit the model, is shown in the following figure.

```
bHat = stats.beta;
yHat = [ones(T,1),X]*bHat;
figure
plot(Years(inSample+1),DefRate(inSample+1),'ko','LineWidth',1.5,...
    'MarkerSize',10,'MarkerFaceColor','g')
hold on
plot(Years(inSample+1),yHat,'b-s','LineWidth',1.2,'MarkerSize',10)
hold off
grid on
legend({'Actual','Model'},'location','NW')
title('{\bf Corporate Default Rate Models: In-Sample Fit}')
xlabel('Year')
ylabel('Percent')
```



It can be shown that there is no strong statistical evidence to conclude that the linear regression assumptions are violated. It is apparent that default rates are not normally distributed. The model, however, does not make that assumption. The only normality assumption in the model is that, given the predictors values, the error between the predicted and the observed default rates is normally distributed. By looking at the insample fit, this does not seem unreasonable. The magnitude of the errors certainly seems independent of whether the default rates are high or low. Year 2001 has a high default rate and a high error, but years 1991 or 2002 also have high rates and yet very small errors. Likewise, low default rate years like 1996 and 1997 show considerable errors, but years 2004 or 2005 have similarly low rates and tiny errors.

A thorough statistical analysis of the model is out of scope here, but there are several detailed examples in Statistics and Machine Learning Toolbox ${ }^{\mathrm{TM}}$ and Econometrics Toolbox ${ }^{\mathrm{TM}}$.

## Backtesting

To evaluate how this model performs out-of-sample, we set up a backtesting exercise. Starting at the end of 1995, we fit the linear regression model with the information available up to that date, and compare the model prediction to the actual default rate observed the following year. We repeat the same for all subsequent years until the end of the sample.

For backtesting, relative performance of a model, when compared to alternatives, is easier to assess than the performance of a model in isolation. Here we include two alternatives to determine next year's default rate, both likely candidates in practice. One is the TTC default rate, estimated with data from the beginning of the sample to the current year, a very stable default rate estimate. The other is the PIT rate, estimated using data from the most recent year only, much more sensitive to recent events.

```
XBT = [AGE,CPF,SPR];
yBT = DefRate;
iYear0 = find(Years==1984); % index of first year in sample, 1984
T = find(Years==1995); % ind "current" year, start at 1995, updated in loop
YearsBT = 1996:2005; % years predicted in BT exercise
iYearsBT = find(Years==1996):find(Years==2005); % corresponding indices
nYearsBT = length(YearsBT); % number of years in BT exercise
MethodTags = {'Model','PIT','TTC'};
nMethods = length(MethodTags);
PredDefRate = zeros(nYearsBT,nMethods);
```

```
ErrorBT = zeros(nYearsBT,nMethods);
alpha = 0.05;
PredDefLoBnd = zeros(nYearsBT,1);
PredDefUpBnd = zeros(nYearsBT,1);
for k=1:nYearsBT
    % In sample years for predictors, from 1984 to "last" year (T-1)
    inSampleBT = iYear0:T-1;
    % Method 1: Linear regression model
    % Fit regression model with data up to "current" year (T)
    s = regstats(yBT(inSampleBT+1),XBT(inSampleBT,:));
        Predict default rate for "next" year (T+1)
    PredDefRate(k,1) = [1 XBT(T,:)]*s.beta;
    % Compute prediction intervals
    tCrit = tinv(1-alpha/2,s.tstat.dfe);
    PredStd = sqrt([1 XBT(T,:)]*s.covb*[1 XBT(T,:)]'+s.mse);
    PredDefLoBnd(k) = max(0,PredDefRate(k,1) - tCrit*PredStd);
    PredDefUpBnd(k) = PredDefRate(k,1) + tCrit*PredStd;
    % Method 2: Point-in-time (PIT) default rate
    PredDefRate(k,2) = DefRate(T);
    % Method 3: Through-the-cycle (TTC) default rate
    tmAll = transprobbytotals(totalsAll(iYear0:T));
    PredDefRate(k,3) = tmAll(1,2);
    % Update error
    ErrorBT(k,:) = PredDefRate(k,:) - DefRate(T+1);
    % Move to next year
    T = T + 1;
end
```

Here are the predictions of the three alternative approaches, compared to the actual default rates observed. Unsurprisingly, TTC shows a very poor predictive power. However, it is not obvious whether PIT or the linear regression model makes better predictions in this 10 -year time span.

Example_BacktestPlot(YearsBT,DefRate(iYearsBT), PredDefRate,'Year','\%',... '\{\b̄ Default Rate Estimation Methods: Backtesting\}',...
['Actual' MethodTags],'NW')


The following plot keeps track of cumulative square error, a measure often used for comparisons in backtesting exercises. This confirms TTC as a poor alternative. PIT shows lower cumulative error than the linear regression model in the late nineties, but after the 2001 recession the situation is reversed. Cumulative square error, however, is not an intuitive measure, it is hard to get a sense of what the difference between these alternatives means in practical terms.

```
CumSqError = cumsum(ErrorBT.^2);
Example BacktestPlot(YearsBT,[],CumSqError,'Year','Cum Sq Error',...
    '{\b\overline{f}}\mathrm{ Cumulative Square Error in Backtesting Exercise}',...
    MethodTags,'NW')
```



It makes sense to translate the prediction errors into a monetary measure. Here we measure the impact of the prediction error on a simplified framework for generating loss reserves in an institution.

We assume a homogeneous portfolio, where all credits have the same probability of default, the same loss given default (LGD), and the same exposure at default (EAD). Both LGD and EAD are assumed to be known. For simplicity, we keep these values constant for the 10 years of the exercise. We set LGD at $45 \%$, and EAD per bond at 100 million. The portfolio is assumed to have a thousand bonds, so the total value of the portfolio, the total EAD , is 100 billion.

The predicted default rate for year $t$, determined at the end of year $t-1$, is used to calculate the expected loss for year $t$

$$
E L_{t}=E A D_{t} \times L G D_{t} \times \text { PredictedDefaultRate }_{t}
$$

This is the amount added to the loss reserves at the start of year $t$. At the end of the year, the actual losses are known

$$
A L_{t}=E A D_{t} \times L G D_{t} \times \text { ObservedDefaultRate }{ }_{t}
$$

We assume that unused loss reserves remain in the reserves fund. The starting balance in reserves at the beginning of the exercise is set to zero. If the actual losses surpass the expected loss, unused reserves accumulated over the years are used first, and only if these run out, capital is used to cover a shortfall. All this translates into the following formula

$$
\text { Reserves }_{t}=\text { Reserves }_{t-1}+\left(E L_{t}-A L_{t}\right)
$$

or equivalently

$$
\text { Reserves }_{t}=\sum_{s=1}^{t}\left(E L_{t}-A L_{t}\right)
$$

The following figure shows the loss reserves balance for each of the three alternatives in the backtesting exercise.

```
EAD = 100*ones(nYearsBT,1); % in billions
LGD = 0.45*ones(nYearsBT,1); % Loss given default, 45%
% Reserves excess or shortfall for each year, in billions
ReservesExcessShortfall = bsxfun(@times,EAD.*LGD,ErrorBT/100);
% Cumulative reserve balance for each year, in billions
ReservesBalanceEOY = cumsum(ReservesExcessShortfall);
Example_BacktestPlot(YearsBT,[],ReservesBalanceEOY,'Year',...
    'Bil\ions of Dollars',...
    '{\bf Reserves Balance (EOY): Backtesting}',...
    MethodTags,'SW')
grid on
```



Using the linear regression model we only observe a deficit in reserves in two out of ten years, and the maximum deficit, in 2001, is 0.09 billion, only nine basis points of the portfolio value.

In contrast, both TTC and PIT reach a deficit of 1.2 billion by 2001. Things get worse for TTC in the next two years, reaching a deficit of 2.1 billion by 2003. PIT does make a correction quickly after 2001, and by 2004 the reserves have a surplus. Yet, both TTC and PIT lead to more deficit years than surplus years in this exercise.

The linear regression model shows more of a counter-cyclical effect than the alternatives in this exercise. The money set aside using the linear regression model reaches close to a billion in 1997 and 1998. High levels of unused reserves translate into a slower pace of lending (not reflected in the exercise, because we exogenously impose the portfolio
value). Moreover, capital is only slightly impacted during the 2001 recession thanks to the reserves accumulated over the previous expansion. This translates into more capital available to back up further lending, if desired, during the economic recovery.

The last backtesting tool we discuss is the use of prediction intervals. Linear regression models provide standard formulas to compute confidence intervals for the values of new observations. These intervals are shown in the next figure for the 10 years spanned in the backtesting exercise.

```
figure
plot(YearsBT,DefRate(iYearsBT),'ko','LineWidth',1.5,'MarkerSize',10,...
    'MarkerFaceColor','g')
hold on
plot(YearsBT,PredDefRate(:,1),'b-s','LineWidth',1.2,'MarkerSize',10)
plot(YearsBT,[PredDefLoBnd PredDefUpBnd],'b:','LineWidth',1.2)
hold off
strConf = num2str((1-alpha)*100);
title(['{\bf Backtesting Results with ' strConf '% Prediction Intervals}'])
xlabel('Year');
ylabel('%');
legend({'Actual','Predicted','Conf Bounds'},'location','NW');
```

Backtesting Results with 95\% Prediction Intervals


The observed default rates fall outside the prediction intervals for two years, 1996 and 1997, where very low default rates are observed. For a $95 \%$ confidence level, two out of 10 seems high. Yet, the observed values in these cases fall barely outside the prediction interval, which is a positive sign for the model. It is also positive that the prediction intervals contain the observed values around the 2001 recession.

## Stress Testing

Stress testing is a broad area that reaches far beyond computational tools; see, for example, [3]. We show some tools that can be incorporated into a comprehensive stress testing framework. We build on the linear regression model presented above, but the concepts and tools are compatible with other forecasting methodologies.

The first tool is the use of prediction intervals to define a worst-case scenario forecasts. This is to account for uncertainty in the model only, not in the value of the predictors.

We take a baseline scenario of predictors, in our case, the latest known values of our age proxy AGE, corporate profits forecast, CPF, and corporate spread, SPR. We then use the linear regression model to compute a $95 \%$ confidence upper bound for the predicted default rate. The motivation for this is illustrated in the last plot of the backtesting section, where the $95 \%$ confidence upper limit acts as a conservative bound when the prediction underestimates the actual default rates.

```
tCrit = tinv(1-alpha/2,stats.tstat.dfe);
XLast = [AGE(end), CPF(end),SPR(end)];
yPred = [1 XLast]*stats.beta;
PredStd = sqrt([1 XLast]*stats.covb*[1 XLast]'+stats.mse);
yPredUB = yPred + tCrit*PredStd;
fprintf('\nPredicted default rate:\n');
Predicted default rate:
fprintf(' Baseline: %4.2f%%\n',yPred);
    Baseline: 1.18%
fprintf(' %g%% Upper Bound: %4.2f%%\n',(1-alpha)*100,yPredUB);
    95% Upper Bound: 2.31%
```

The next step is to incorporate stressed scenarios of the predictors in the analysis. CPF and SPR can change in the short term, whereas AGE cannot. This is important. The corporate profits forecast and the corporate spread are influenced by world events, including, for example, natural disasters. These predictors can significantly change overnight. On the other hand, AGE depends on managerial decisions that can alter the proportion of old and new loans in time, but these decisions take months, if not years, to reflect in the AGE time series. Scenarios for AGE are compatible with longer term analyses. Here we look at one year ahead only, and keep AGE fixed for the remainder of this section.

It is convenient to define the predicted default rate and the confidence bounds as functions of CPF and SPR to simplify the scenario analysis.

```
yPredFn = @(cpf,spr) [1 AGE(end) cpf spr]*stats.beta;
PredStdFn = @(cpf,spr) sqrt([1 AGE(end) cpf spr]*stats.covb*...
```

```
    [1 AGE(end) cpf spr]'+stats.mse);
yPredUBFn = @(cpf,spr) (yPredFn(cpf,spr) + tCrit*PredStdFn(cpf,spr));
yPredLBFn = @(cpf,spr) (yPredFn(cpf,spr) - tCrit*PredStdFn(cpf,spr));
```

Two extreme scenarios of interest can be a drop in the corporate profits forecast of 4\% relative to the baseline, and an increase in the corporate spread of 100 basis points over the baseline.

Moving one predictor at a time is not unreasonable in this case, because the correlation between CPF and SPR is very low. Moderate correlation levels may require perturbing predictors together to get more reliable results. Highly correlated predictors usually do not coexist in the same model, since they offer redundant information.

```
fprintf('\n\n What-if Analysis\n');
    What-if Analysis
fprintf('Scenario LB Pred UB\n');
Scenario LB Pred UB
cpf = CPF(end)-4;
spr = SPR(end);
yPredRange = [yPredLBFn(cpf,spr),yPredFn(cpf,spr),yPredUBFn(cpf,spr)];
fprintf('CPF drops 4%% %4.2f%% %4.2f%% %4.2f%%\n',yPredRange);
CPF drops 4% 0.42% 1.57% 2.71%
cpf = CPF(end);
spr = SPR(end)+1;
yPredRange = [yPredLBFn(cpf,spr),yPredFn(cpf,spr),yPredUBFn(cpf,spr)];
fprintf('SPR rises 1%% %4.2f%% %4.2f%% %4.2f%%\n',yPredRange);
```

```
SPR rises 1% 0.71% 1.88% 3.05%
```

SPR rises 1% 0.71% 1.88% 3.05%
cpf = CPF(end);
spr = SPR(end);
yPredRange = [yPredLBFn(cpf,spr),yPredFn(cpf,spr),yPredUBFn(cpf,spr)];
fprintf(' Baseline %4.2f%% %4.2f%% %4.2f%%\n',yPredRange);
Baseline 0.04% 1.18% 2.31%
fprintf('\nCorrelation between CPF and SPR: %4.3f\n',corr(CPF,SPR));
Correlation between CPF and SPR: 0.012

```

We now take a more global view of the scenario analysis. Instead of analyzing one scenario at a time, we visualize the default rate forecasts as a function of CPF and SPR. More precisely, we plot default rate contours over a whole grid of CPF and SPR values. We use the conservative \(95 \%\) upper bound.

If we assumed a particular bivariate distribution for the values of CPF and SPR, we could plot the contours of their distribution in the same figure. That would give visual information on the probability of falling on each region. Lacking such a distribution, we simply add to the plot the CPF - SPR pairs observed in our sample, as a historical, empirical distribution. The last observation in the sample, the baseline scenario, is marked in red.
```

gridCPF = 2*min(CPF):0.1:max(CPF);
gridSPR = min(SPR):0.1:2*max(SPR);
nGridCPF = length(gridCPF);
nGridSPR = length(gridSPR);
DefRateUB = zeros(nGridCPF,nGridSPR);
for i=1:nGridCPF
for j=1:nGridSPR
DefRateUB(i,j) = yPredUBFn(gridCPF(i),gridSPR(j));
end
end
Example_StressTestPlot(gridCPF,gridSPR,DefRateUB,CPF,SPR,...
'Corporate Profits Forecast (%)','Corporate Spread (%)',...
['{\bf ' strConf '% UB Default Rate Regions (in %)}'])

```


Very different predictor values result in similar default rate levels. For example, consider a profits forecast around \(10 \%\) with a spread of \(3.5 \%\), and a profits forecast of \(-2.5 \%\) with a spread of \(2 \%\), they both result in a default rate slightly above \(3 \%\). Also, only one point in the available history yields a default rate higher than \(4 \%\).

Monetary terms, once again, may be more meaningful. We use the Basel II capital requirements formula (see [2]) to translate the default rates into a monetary measure. The Basel II formula is convenient because it is analytic (there is no need to simulate to estimate the capital requirements), but also because it depends only on the probabilities of default. We define the Basel II capital requirements as a function K.
```

% Correlation as a function of PD
w = @(pd) (1-exp(-50*pd))/(1-\operatorname{exp}(-50)); % weight

```
```

R = @(pd) (0.12*W(pd)+0.24*(1-w(pd))); % correlation
% Vasicek formula
V = @(pd) normcdf(norminv(pd)+R(pd).*norminv(0.999)./sqrt(1-R(pd)));
% Parameter b for maturity adjustment
b = @(pd) (0.11852-0.05478*log(pd)).^2;
% Basel II capital requirement with LGD=45% and maturity M=2.5 (numerator
% in maturity adjustment term becomes 1)
K = @(pd) 0.45*(V(pd)-pd).*(1./(1-1.5*b(pd)));

```

Worst-case default rates for a whole grid of CPF - SPR pairs are stored in DefRateUB. By applying the function \(K\) to DefRateUB, we can visualize the capital requirements over the same grid.
```

CapReq = 100*K(DefRateUB/100);
Example_StressTestPlot(gridCPF,gridSPR,CapReq,CPF,SPR,...
'Corporate Profits Forecast (%)','Corporate Spread (%)',...
{'{\bf Capital Requirement Regions (% of value)}';...
['{\bf using ' strConf '% UB Default Rate}']})

```


The contour levels now indicate capital requirements as a percentage of portfolio value. The two scenarios above, profits of \(10 \%\) with spread of \(3.5 \%\), and profits of \(-2.5 \%\) and spread of \(2 \%\), result in capital requirements near \(2.75 \%\). The worst-case point from the historical data yields a capital requirement of about \(3 \%\).

This visualization can also be used, for example, as part of a reverse stress test analysis. Critical levels of capital can be determined first, and the figure can be used to determine regions of risk factor values (in this case CPF and SPR) that lead to those critical levels.

Instead of historical observations of CPF and SPR, an empirical distribution for the risk factors can be simulated using, for example, a vector autoregressive (VAR) model from Econometrics Toolbox \({ }^{\mathrm{TM}}\). The capital requirements corresponding to each default probability level can be found by simulation if a closed form formula is not available, and
the same plots can be generated. For large simulations, a distributed computing implementation using Parallel Computing Toolbox \({ }^{\mathrm{TM}}\) or MATLAB Distributed Computing Server \({ }^{\mathrm{TM}}\) can make the process more efficient.

\section*{Appendix: Modeling Full Transition Matrices}

Transition matrices change in time, and a full description of their dynamics requires working with multi-dimensional time series. There are, however, techniques that exploit the particular structure of transition matrices to reduce the dimensionality of the problem. In [8], for example, a single parameter related to the proportion of downgrades is used, and both [6] and [8] describe a method to shift transition probabilities using a single parameter. The latter approach is shown in this appendix.

The method takes the TTC transition matrix as a baseline.
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|}
\hline \multicolumn{10}{|l|}{tmTTC \(=\) transprobbytotals(totalsByRtg);} \\
\hline \multicolumn{10}{|l|}{Example DisplayTransitions(tmTTC, [],...} \\
\hline \multicolumn{10}{|l|}{} \\
\hline \multicolumn{10}{|l|}{} \\
\hline & AAA & AA & A & BBB & BB & B & CCC & D & NR \\
\hline AAA & 88.2 & 7.67 & 0.49 & 0.09 & 0.06 & 0 & 0 & 0 & 3.49 \\
\hline AA & 0.58 & 87.16 & 7.63 & 0.58 & 0.06 & 0.11 & 0.02 & 0.01 & 3.85 \\
\hline A & 0.05 & 1.9 & 87.24 & 5.59 & 0.42 & 0.15 & 0.03 & 0.04 & 4.58 \\
\hline BBB & 0.02 & 0.16 & 3.85 & 84.13 & 4.27 & 0.76 & 0.17 & 0.27 & 6.37 \\
\hline BB & 0.03 & 0.04 & 0.25 & 5.26 & 75.74 & 7.36 & 0.9 & 1.12 & 9.29 \\
\hline B & 0 & 0.05 & 0.19 & 0.31 & 5.52 & 72.67 & 4.21 & 5.38 & 11.67 \\
\hline CCC & 0 & 0 & 0.28 & 0.41 & 1.24 & 10.92 & 47.06 & 27.02 & 13.06 \\
\hline
\end{tabular}

An equivalent way to represent this matrix is by transforming it into credit quality thresholds, that is, critical values of a standard normal distribution that yield the same transition probabilities (row by row).
```

thresholdMat = transprobtothresholds(tmTTC);
Example_DisplayTransitions(thresholdMat,[],...
{'AAA}\mp@subsup{\overline{A}}{}{\prime},'AA','A','BBB','BB','B','CCC'},...
{'AAA','AA','A','BBB','BB','B','CCC','D','NR'})

```
\begin{tabular}{rlrrrrrrrr} 
& AAA & AA & A & BBB & BB & B & CCC & D & NR \\
AAA & Inf & -1.19 & -1.74 & -1.8 & -1.81 & -1.81 & -1.81 & -1.81 & -1.81 \\
AA & Inf & 2.52 & -1.16 & -1.68 & -1.75 & -1.75 & -1.76 & -1.77 & -1.77 \\
A & Inf & 3.31 & 2.07 & -1.24 & -1.62 & -1.66 & -1.68 & -1.68 & -1.69 \\
BBB & Inf & 3.57 & 2.91 & 1.75 & -1.18 & -1.43 & -1.49 & -1.5 & -1.52 \\
BB & Inf & 3.39 & 3.16 & 2.72 & 1.59 & -0.89 & -1.21 & -1.26 & -1.32
\end{tabular}
\begin{tabular}{rllrrrrrrr} 
B & Inf & Inf & 3.28 & 2.82 & 2.54 & 1.55 & -0.8 & -0.95 & -1.19 \\
CCC & Inf & Inf & Inf & 2.77 & 2.46 & 2.07 & 1.13 & -0.25 & -1.12
\end{tabular}

Credit quality thresholds are illustrated in the following figure. The segments in the vertical axis represent transition probabilities, and the boundaries between them determine the critical values in the horizontal axis, via the standard normal distribution. Each row in the transition matrix determines a set of thresholds. The figure shows the thresholds for the 'CCC' rating.
```

xliml = -5;
xlimr = 5;
step = 0.1;
x=xliml:step:xlimr;
thresCCC = thresholdMat(7,:);
centersY = (normcdf([thresCCC(2:end) xliml])+...
normcdf([xlimr thresCCC(2:end)]))/2;
labels = {'AAA','AA','A','BBB','BB','B','CCC','D','NR'};
figure
plot(x,normcdf(x),'m','LineWidth',1.5)
for i=2:length(labels)
val = thresCCC(i);
line([val val],[0 normcdf(val)],'LineStyle',':');
line([x(1) val],[normcdf(val) normcdf(val)],'LineStyle',':');
if (centersY(i-1)-centersY(i))>0.05
text(-4.5,centersY(i),labels{i});
end
end
xlabel('Credit Quality Thresholds')
ylabel('Cumulative Probability')
title('{\bf Visualization of Credit Quality Thresholds}')
legend('Std Normal CDF','Location','E')

```


Shifting the critical values to the right or left changes the transition probabilities. For example, here is the transition matrix obtained by shifting the TTC thresholds by 0.5 to the right. Note that default probabilities increase.
```

shiftedThresholds = thresholdMat+0.5;
Example_DisplayTransitions(transprobfromthresholds(shiftedThresholds),...
[],{'AAA','AA','A','BBB','BB','B','CCC'},...
{'AAA','AA','A','BBB','BB','B','CCC','D','NR'})

|  | AAA | AA | A | BBB | BB | B | CCC | D | NR |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| AAA | 75.34 | 13.84 | 1.05 | 0.19 | 0.13 | 0 | 0 | 0 | 9.45 |
| AA | 0.13 | 74.49 | 13.53 | 1.21 | 0.12 | 0.22 | 0.04 | 0.02 | 10.24 |
| A | 0.01 | 0.51 | 76.4 | 10.02 | 0.83 | 0.31 | 0.06 | 0.08 | 11.77 |
| BBB | 0 | 0.03 | 1.2 | 74.03 | 7.22 | 1.39 | 0.32 | 0.51 | 15.29 |

```
\begin{tabular}{rrrrrrrrrr} 
BB & 0 & 0.01 & 0.05 & 1.77 & 63.35 & 10.94 & 1.47 & 1.88 & 20.52 \\
B & 0 & 0.01 & 0.04 & 0.07 & 1.91 & 59.67 & 5.74 & 8.1 & 24.46 \\
CCC & 0 & 0 & 0.05 & 0.1 & 0.36 & 4.61 & 35.06 & 33.18 & 26.65
\end{tabular}

Given a particular PIT matrix, the idea in [6] and [8] is to vary the shifting parameter applied to the TTC thresholds so that the resulting transition matrix is as close as possible to the PIT matrix. Closeness is measured as the sum of squares of differences between corresponding transition probabilities. The optimal shifting value is called credit index. A credit index is determined for every PIT transition matrix in the sample.

Here we use fminunc from Optimization Toolbox \({ }^{\mathrm{TM}}\) to find the credit indices.
```

CreditIndex = zeros(nYears,1);
ExitFlag = zeros(nYears,1);
options = optimset('LargeScale','Off','Display','Off');
for i=1:nYears
errorfun = @(z)norm(squeeze(TransMat(:,:,i))-...
transprobfromthresholds(...
transprobtothresholds(tmTTC)+z),'fro');
[CreditIndex(i),~,ExitFlag(i)] = fminunc(errorfun,0,options);
end

```

In general, one expects that higher credit indices correspond to riskier years. The series of credit indices found does not entirely match this pattern. There may be different reasons for this. First, transition probabilities may deviate from their long-term averages in different ways that may lead to confounding effects in the single parameter trying to capture these differences, the credit index. Having separate credit indices for IG and SG, for example, may help separate confounding effects. Second, a difference of five basis points may be very significant for the 'BBB' default rate, but not as important for the ' CCC ' default rate, yet the norm used weights them equally. Other norms can be considered. Also, it is always a good idea to check the exit flags of optimization solvers, in case the algorithm could not find a solution. Here we get valid solutions for each year (all exit flags are 1).
```

figure
plot(Years,CreditIndex,'-d')
hold on
Example_RecessionBands
hold of\overline{f}
grid on
xlabel('Year')
ylabel('Shift')
title('{\bf Credit Index}')

```


The workflow above can be adapted to work with the series of credit indices instead of the series of corporate default rates. A model can be fit to predict a credit index for the following year, and a predicted transition matrix can be inferred and used for risk analyses.

\section*{References}
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[11] Wilson, T. C., "Portfolio Credit Risk," FRBNY Economic Policy Review, October 1998.

\section*{See Also}
bootstrp|transprob|transprobbytotals|transprobfromthresholds | transprobgrouptotals|transprobprep|transprobtothresholds

\section*{Related Examples}
- "Credit Quality Thresholds" on page 8-57
- "Credit Rating by Bagging Decision Trees" (Statistics and Machine Learning Toolbox)

\section*{External Websites}
- Credit Risk Modeling with MATLAB (53 min 09 sec )
- Forecasting Corporate Default Rates with MATLAB (54 min 36 sec )

\section*{Credit Quality Thresholds}
```

In this section...
"Introduction" on page 8-57
"Compute Credit Quality Thresholds" on page 8-57
"Visualize Credit Quality Thresholds" on page 8-58

```

\section*{Introduction}

An equivalent way to represent transition probabilities is by transforming them into credit quality thresholds. These are critical values of a standard normal distribution that yield the same transition probabilities.

An M-by-N matrix of transition probabilities TRANS and the corresponding M-by-N matrix of credit quality thresholds THRESH are related as follows. The thresholds THRESH \((i, j)\) are critical values of a standard normal distribution \(z\), such that
```

TRANS(i,N) = P[z < THRESH(i,N)],
TRANS(i,j) = P[z < THRESH(i,j)] - P[z < THRESH(i,j+1)], for 1<=j<N

```

Financial Toolbox supports the transformation between transition probabilities and credit quality thresholds with the functions transprobtothresholds and transprobfromthresholds.

\section*{Compute Credit Quality Thresholds}

To compute credit quality thresholds, transition probabilities are required as input. Here is a transition matrix estimated from credit ratings data:
```

load Data_TransProb
trans = transprob(data)
trans =

| 93.1170 | 5.8428 | 0.8232 | 0.1763 | 0.0376 | 0.0012 | 0.0001 | 0.0017 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 1.6166 | 93.1518 | 4.3632 | 0.6602 | 0.1626 | 0.0055 | 0.0004 | 0.0396 |
| 0.1237 | 2.9003 | 92.2197 | 4.0756 | 0.5365 | 0.0661 | 0.0028 | 0.0753 |
| 0.0236 | 0.2312 | 5.0059 | 90.1846 | 3.7979 | 0.4733 | 0.0642 | 0.2193 |
| 0.0216 | 0.1134 | 0.6357 | 5.7960 | 88.9866 | 3.4497 | 0.2919 | 0.7050 |
| 0.0010 | 0.0062 | 0.1081 | 0.8697 | 7.3366 | 86.7215 | 2.5169 | 2.4399 |
| 0.0002 | 0.0011 | 0.0120 | 0.2582 | 1.4294 | 4.2898 | 81.2927 | 12.7167 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 100.0000 |

```

Convert the transition matrix to credit quality thresholds using transprobtothresholds:
thresh \(=\) transprobtothresholds(trans)
thresh
\begin{tabular}{rrrrrrrr} 
Inf & -1.4846 & -2.3115 & -2.8523 & -3.3480 & -4.0083 & -4.1276 & -4.1413 \\
Inf & 2.1403 & -1.6228 & -2.3788 & -2.8655 & -3.3166 & -3.3523 & -3.3554 \\
Inf & 3.0264 & 1.8773 & -1.6690 & -2.4673 & -2.9800 & -3.1631 & -3.1736 \\
Inf & 3.4963 & 2.8009 & 1.6201 & -1.6897 & -2.4291 & -2.7663 & -2.8490 \\
Inf & 3.5195 & 2.9999 & 2.4225 & 1.5089 & -1.7010 & -2.3275 & -2.4547 \\
Inf & 4.2696 & 3.8015 & 3.0477 & 2.3320 & 1.3838 & -1.6491 & -1.9703 \\
Inf & 4.6241 & 4.2097 & 3.6472 & 2.7803 & 2.1199 & 1.5556 & -1.1399 \\
Inf & Inf & Inf & Inf & Inf & Inf & Inf & Inf
\end{tabular}

Conversely, given a matrix of thresholds, you can compute transition probabilities using transprobfromthresholds. For example, take the thresholds computed previously as input to recover the original transition probabilities:
trans1 = transprobfromthresholds(thresh)
trans1 \(=\)
\begin{tabular}{rrrrrrrr}
93.1170 & 5.8428 & 0.8232 & 0.1763 & 0.0376 & 0.0012 & 0.0001 & 0.0017 \\
1.6166 & 93.1518 & 4.3632 & 0.6602 & 0.1626 & 0.0055 & 0.0004 & 0.0396 \\
0.1237 & 2.9003 & 92.2197 & 4.0756 & 0.5365 & 0.0661 & 0.0028 & 0.0753 \\
0.0236 & 0.2312 & 5.0059 & 90.1846 & 3.7979 & 0.4733 & 0.0642 & 0.2193 \\
0.0216 & 0.1134 & 0.6357 & 5.7960 & 88.9866 & 3.4497 & 0.2919 & 0.7050 \\
0.0010 & 0.0062 & 0.1081 & 0.8697 & 7.3366 & 86.7215 & 2.5169 & 2.4399 \\
0.0002 & 0.0011 & 0.0120 & 0.2582 & 1.4294 & 4.2898 & 81.2927 & 12.7167 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 100.0000
\end{tabular}

\section*{Visualize Credit Quality Thresholds}

You can graphically represent the relationship between credit quality thresholds and transition probabilities. Here, this example shows the relationship for the 'CCC' credit rating. In the plot, the thresholds are marked by the vertical lines and the transition probabilities are the area below the standard normal density curve:
```

load Data_TransProb
trans = transprob(data);
thresh = transprobtothresholds(trans);
xliml = -5;
xlimr = 5;
step = 0.1;
x=xliml:step:xlimr;
thresCCC = thresh(7,:);
labels = {'AAA','AA','A','BBB','BB','B','CCC','D'};

```
```

centersX = ([5 thresCCC(2:end)]+[thresCCC(2:end) -5])*0.5;
omag = round(log10(trans(7,:)));
omag (omag>0)=omag (omag>0).^2;
fs = 14+2*omag;
figure
plot(x,normpdf(x),'LineWidth',1.5)
text(centersX(1),0.2,labels{1},'FontSize',fs(1),...
'HorizontalAlignment','center')
for i=2:length(labels)
val = thresCCC(i);
line([val val],[0 0.4],'LineStyle',':')
text(centersX(i),0.2,labels{i},'FontSize',fs(i),...
'HorizontalAlignment','center')
end
xlabel('Credit Quality Thresholds')
ylabel('Probability Density Function')
title('{\bf Visualization of Credit Quality Thresholds}')
legend('Std Normal PDF','Location','S')

```


The second plot uses the cumulative density function instead. The thresholds are represented by vertical lines. The transition probabilities are given by the distance between horizontal lines.
```

figure
plot(x,normcdf(x),'m','LineWidth',1.5)
text(centersX(1),0.2,labels{1},'FontSize',fs(1),...
'HorizontalAlignment','center')
for i=2:length(labels)
val = thresCCC(i);
line([val val],[0 normcdf(val)],'LineStyle',':');
line([x(1) val],[normcdf(val) normcdf(val)],'LineStyle',':');
text(centersX(i),0.2,labels{i},'FontSize',fs(i),...
'HorizontalAlignment','center')
end
xlabel('Credit Quality Thresholds')
ylabel('Cumulative Probability')
title('{\bf Visualization of Credit Quality Thresholds}')
legend('Std Normal CDF','Location','W')

```


\section*{See Also}
bootstrp|transprob|transprobbytotals|transprobfromthresholds | transprobgrouptotals|transprobprep|transprobtothresholds

\section*{Related Examples}
- "Estimation of Transition Probabilities" on page 8-2
- "Credit Rating by Bagging Decision Trees" (Statistics and Machine Learning Toolbox)
- "Forecasting Corporate Default Rates" on page 8-25

\section*{External Websites}
- Credit Risk Modeling with MATLAB (53 min 09 sec )
- Forecasting Corporate Default Rates with MATLAB (54 min 36 sec )

\section*{About Credit Scorecards}

\author{
In this section... \\ "What Is a Credit Scorecard?" on page 8-62 \\ "Credit Scorecard Development Process" on page 8-65
}

\section*{What Is a Credit Scorecard?}

Credit scoring is one of the most widely used credit risk analysis tools. The goal of credit scoring is ranking borrowers by their credit worthiness. In the context of retail credit (credit cards, mortgages, car loans, and so on), credit scoring is performed using a credit scorecard. Credit scorecards represent different characteristics of a customer (age, residential status, time at current address, time at current job, and so on) translated into points and the total number of points becomes the credit score. The credit worthiness of customers is summarized by their credit score; high scores usually correspond to low-risk customers, and conversely. Scores are also used for corporate credit analysis of small and medium enterprises, and, large corporations.

A credit scorecard is a lookup table that maps specific characteristics of a borrower into points. The total number of points becomes the credit score. Credit scorecards are a widely used type of credit scoring model. As such, the goal of a credit scorecard is to distinguish between customers who repay their loans ("good" customers), and customers who will not ("bad" customers). Like other credit scoring models, credit scorecards quantify the risk that a borrower will not repay a loan in the form of a score and a probability of default.

For example, a credit scorecard can give individual borrowers points for their age and income according to the following table. Other characteristics such as residential status, employment status, might also be included, although, for brevity, they are not shown in this table.
Age Points
Up to 25 ..... 10
26 to 40 ..... 25
41 to 65 ..... 38
66 and up ..... 43
Income
Up to 40 k ..... 16
40k to 70k ..... 28
Total score ..... (Sum of Points)

Using the credit scorecard in this example, a particular customer who is 31 and has an income of \(\$ 52,000\) a year, is placed into the second age group (26-40) and receives 25 points for their age, and similarly, receives 28 points for their income. Other characteristics (not shown here) might contribute additional points to their score. The total score is the sum of all points, which in this example is assumed to give the customer a total of 238 points (this is a fictitious example on an arbitrary scoring scale).


\section*{John:}
- 31 years old
- 52k a year
- Single

Score 238

Technically, to determine the credit scorecard points, start out by selecting a set of potential predictors (column 1 in the next figure). Then, bin data into groups (for example, ages 'Up to 25 ', '25 to 40 ' (column 2 in the figure). This grouping helps to distinguish between "good" and "bad" customers. The Weight of Evidence (WOE) is a way to measure how well the distribution of "good" and "bad" are separated across bins or groups for each individual predictor (column 3 in the figure). By fitting a logistic regression model, you can identify which predictors, when put together, do a better job distinguishing between "good" and "bad" customers. The model is summarized by its coefficients (column 4 in the figure). Finally, the combination of WOE's and model coefficients (commonly scaled, shifted, and rounded) make up the scorecard points (column 5 in the figure).
\begin{tabular}{|c|c|c|c|c|}
\hline Predictor & Bin & WOE & Model & Points* \\
\hline \multirow[t]{5}{*}{Age} & & & \(\beta_{\text {age }}\) & \\
\hline & 'Up to \(25{ }^{\prime}\) & WOE \(\mathrm{E}_{\text {ago }}\) ('Up to \(25{ }^{\text {a }}\) ) & &  \\
\hline & ' 26 to \(40^{\prime}\) & WOE \(\mathrm{Eggo}^{( }\)(26 to 40') & & \(\beta_{\text {ago }}{ }^{*}\) WOE \(_{\text {ago }}\) ( 26 to 40') \\
\hline & '41 to 65 ' & WOE \(\mathrm{E}_{\text {spe }}\left(41\right.\) to \(65{ }^{\prime}\) ) & & \(\beta_{\text {spe }}{ }^{*}\) WOE \(_{\text {age }}\) ( 41 to 65') \\
\hline & '66 and up' & WOE \(\mathrm{esge}^{\text {( }} 66\) and up') & & \(\beta_{\text {age }}{ }^{*} W^{\text {P }}\) ( \(E_{\text {ape }}\) ('66 and up') \\
\hline \multirow[t]{4}{*}{Income} & & & \(\beta_{\text {ncore }}\) & \\
\hline & 'Up to 40k' & WOE \({ }_{\text {income }}(\) 'Up to 40 K ) & &  \\
\hline & '40k to \(70{ }^{\prime}\) & WOE \(\mathrm{Emcomo}(40 \mathrm{k}\) to 70 k ) & &  \\
\hline & ... & & & \\
\hline
\end{tabular}
- Points may include a constant and may be scaled and rounded.

\section*{Credit Scorecard Development Process}

1 Data gathering and preparation phase
This includes data gathering and integration, such as querying, merging, aligning. It also includes treatment of missing information and outliers. There is a prescreening step based on reports of association measures between the predictors and the response variable. Finally, there is a sampling step, to produce a training set, sometimes called the modeling view, and usually a validation set, too. The training set, in the form of a table, is the required data input to the creditscorecard object, and this training set table must be prepared before creating a creditscorecard object in the Modeling phase.
2 Modeling phase
Use the creditscorecard object and associated object functions to develop a credit scorecard model. You can bin the data, apply the Weight of Evidence (WOE) transformation, and compute other statistics, such as the Information Value. You can fit a logistic regression model and also review the resulting scorecard points and format their scaling and rounding. For details on using the creditscorecard object, see creditscorecard.

3 Deployment phase

Deployment entails integrating a credit scorecard model into an IT production environment and keeping tracking logs, performance reports, and so on.

The creditscorecard object is designed for the Modeling phase of the credit scorecard workflow. Support for all three phases requires other MathWorks \({ }^{\circledR}\) products.

\section*{See Also}
autobinning|bindata|bininfo|creditscorecard|displaypoints|fitmodel | formatpoints | modifybins | modifypredictor|plotbins | predictorinfo| probdefault |score|setmodel|validatemodel

\section*{Related Examples}
- "Troubleshooting Credit Scorecard Results" on page 8-91
- "Case Study for a Credit Scorecard Analysis" on page 8-101

\section*{More About}
- "Credit Scorecard Modeling Workflow" on page 8-67
- "Credit Scorecard Modeling Using Observation Weights" on page 8-70

\section*{Credit Scorecard Modeling Workflow}

Create, model, and analyze credit scorecards as follows.
1 Create a creditscorecard object.
Create a creditscorecard object for credit scorecard analysis by specifying "training" data in table format. The training data, sometimes called the modeling view, is the result of multiple data preparation tasks (see "About Credit Scorecards" on page 8-62) that must be performed before creating a creditscorecard object.

You can use optional input arguments for creditscorecard to specify scorecard properties such as the response variable and the GoodLabel. Perform some initial data exploration when the creditscorecard object is created, although data analysis is usually done in combination with data binning (see step 2). For more information and examples, see creditscorecard and step 1 in "Case Study for a Credit Scorecard Analysis" on page 8-101.

2 Bin the data.
Perform manual or automatic binning of the data loaded into the creditscorecard object.

A common starting point is to apply automatic binning to all or selected variables using autobinning, report using bininfo, and visualize bin information with respect to bin counts and statistics or association measures such as Weight of Evidence (WOE) using plotbins. The bins can be modified or fine-tuned either manually using modifybins or with a different automatic binning algorithm using autobinning. Bins that show a close-to-linear trend in the WOE are frequently desired in the credit scorecard context.

Alternatively, with Risk Management Toolbox \({ }^{\mathrm{TM}}\), you can use the Binning Explorer app to interactively bin. The Binning Explorer enables you to interactively apply a binning algorithm and modify bins. For more information, see Binning Explorer.

For more information and examples, see autobinning, modifybins, bininfo, and plotbins and step 2 in "Case Study for a Credit Scorecard Analysis" on page 8-101.

3 Fit a logistic regression model.
Fit a logistic regression model to the WOE data from the creditscorecard object. The fitmodel function internally bins the training data, transforms it into WOE
values, maps the response variable so that 'Good ' is 1 , and fits a linear logistic regression model.

By default, fitmodel uses a stepwise procedure to determine which predictors should be in the model, but optional input arguments can also be used, for example, to fit a full model. For more information and examples, see fitmodel and step 3 in "Case Study for a Credit Scorecard Analysis" on page 8-101.
4 Review and format credit scorecard points.
After fitting the logistic model, use displaypoints to summarize the scorecard points. By default, the points are unscaled and come directly from the combination of Weight of Evidence (WOE) values and model coefficients.

The formatpoints function lets you control scaling and rounding of scorecard points. For more information and examples, see displaypoints and formatpoints and step 4 in "Case Study for a Credit Scorecard Analysis" on page 8-101.

5 Score the data.
The score function computes the scores for the training data.
An optional data input can also be passed to score, for example, validation data. The points per predictor for each customer are also provided as an optional output. For more information and examples, see score and step 5 in "Case Study for a Credit Scorecard Analysis" on page 8-101.
6 Calculate the probability of default for credit scorecard scores.
The probdefault function to calculate the probability of default for training data.
In addition, you can compute likelihood of default for a different dataset (for example, a validation data set) using the probdefault function. For more information and examples, see probdefault and step 6 in "Case Study for a Credit Scorecard Analysis" on page 8-101.
7 Validate the credit scorecard model.
Use the validatemodel function to validate the quality of the credit scorecard model.

You can obtain the Cumulative Accuracy Profile (CAP), Receiver Operating Characteristic (ROC), and Kolmogorov-Smirnov (KS) plots and statistics for a given
dataset using the validatemodel function. For more information and examples, see validatemodel and step 7 in "Case Study for a Credit Scorecard Analysis" on page 8-101.

For an example of this workflow, see "Case Study for a Credit Scorecard Analysis" on page 8-101.

\section*{See Also}
autobinning|bindata|bininfo|creditscorecard|displaypoints|fitmodel |formatpoints|modifybins |modifypredictor|plotbins|predictorinfo| probdefault|score|setmodel|validatemodel

\section*{Related Examples}
- "Troubleshooting Credit Scorecard Results" on page 8-91
- "Case Study for a Credit Scorecard Analysis" on page 8-101

\section*{More About}
- "About Credit Scorecards" on page 8-62
- "Credit Scorecard Modeling Using Observation Weights" on page 8-70
- "Credit Scorecard Modeling with Missing Values" on page 8-73

\section*{Credit Scorecard Modeling Using Observation Weights}

When creating a creditscorecard object, the table used for the input data argument either defines or does not define observational weights. If the data does not use weights, then the "counts" for Good, Bad, and Odds are used by credit score card functions. However, if the optional WeightsVar argument is specified when creating a creditscorecard object, then the "counts" for Good, Bad, and Odds are the sum of weights.

For example, here is a snippet of an input table that does not define observational weights:
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline CustID & CustAge & TmAtAddress & ResStatus & EmpStatus & CustIncome & status \\
\hline 11 & 52 & 24 & 'Tenant' & 'Unknown' & 34000 & 1 \\
\hline 12 & 48 & 91 & 'Other' & 'Unknown' & 44000 & 0 \\
\hline 13 & 65 & 63 & 'Home Owner' & 'Unknown' & 48000 & 0 \\
\hline 14 & 44 & 75 & 'Other' & 'Unknown' & 41000 & 0 \\
\hline 15 & 46 & 82 & 'Other' & 'Employed' & 46000 & 0 \\
\hline 16 & 33 & 47 & 'Home Owner' & 'Employed' & 36000 & 0 \\
\hline 17 & 39 & 10 & 'Tenant' & 'Employed' & 34000 & 1 \\
\hline 18 & 24 & 7 & 'Home Owner' & 'Employed' & 22000 & 0 \\
\hline 19 & 53 & 14 & 'Home Owner' & 'Employed' & 51000 & 1 \\
\hline 20 & 52 & 55 & 'Other' & 'Unknown' & 42000 & 0 \\
\hline
\end{tabular}

If you bin the customer age predictor data, with customers up to 45 years old in one bin, and 46 and up in another bin, you get these statistics:
\begin{tabular}{|c|c|c|c}
\hline Bin & Good & Bad & Odds \\
\hline '[-Inf,46)' & 381 & 241 & 1.581 \\
\hline '[46,Inf]' & 422 & 156 & 2.705 \\
\hline 'Totals' & 803 & 397 & 2.023 \\
\hline
\end{tabular}

Good means the total number of rows with a 0 value in the status response variable. Bad the number of 1's in the status column. Odds is the ratio of Good to Bad. The Good, Bad, and Odds is reported for each bin. This means that there are 381 people in the sample who are 45 and under who paid their loans, 241 in the same age range who defaulted, and therefore, the odds of being good for that age range is 1.581 .

Suppose that the modeler thinks that people 45 and younger are underrepresented in this sample. The modeler wants to give all rows with ages up to 45 a higher weight. Assume
that the modeler thinks the up to 45 age group should have \(50 \%\) more weight than rows with ages 46 and up. The table data is expanded to include the observation weights. A Weight column is added to the table, where all rows with ages 45 and under have a weight of 1.5 , and all other rows a weight of 1 . There are other reasons to use weights, for example, recent data points may be given higher weights than older data points.
\begin{tabular}{|c|c|c|c|c|c|c|c|}
\hline CustID & CustAge & TmAtAddress & ResStatus & EmpStatus & CustIncome & status & Weight \\
\hline 11 & 52 & 24 & 'Tenant' & 'Unknown' & 34000 & 1 & 1.0 \\
\hline 12 & 48 & 91 & 'Other' & 'Unknown' & 44000 & 0 & 1.0 \\
\hline 13 & 65 & 63 & 'Home Owner' & 'Unknown' & 48000 & 0 & 1.0 \\
\hline 14 & 44 & 75 & 'Other' & 'Unknown' & 41000 & 0 & 1.5 \\
\hline 15 & 46 & 82 & 'Other' & 'Employed' & 46000 & 0 & 1.0 \\
\hline 16 & 33 & 47 & 'Home Owner' & 'Employed' & 36000 & 0 & 1.5 \\
\hline 17 & 39 & 10 & 'Tenant' & 'Employed' & 34000 & 1 & 1.5 \\
\hline 18 & 24 & 7 & 'Home Owner' & 'Employed' & 22000 & 0 & 1.5 \\
\hline 19 & 53 & 14 & 'Home Owner' & 'Employed' & 51000 & 1 & 1.0 \\
\hline 20 & 52 & 55 & 'Other' & 'Unknown' & 42000 & 0 & 1.0 \\
\hline
\end{tabular}

If you bin the weighted data based on age ( 45 and under, versus 46 and up) the expectation is that each row with age 45 and under must count as 1.5 observations, and therefore the Good and Bad "counts" are increased by 50\%:
\begin{tabular}{|c|c|c|c}
\hline Bin & Good & Bad & Odds \\
\hline '[-Inf,46)' & 571.5 & 361.5 & 1.581 \\
\hline '[46,Inf]' & 422 & 156 & 2.705 \\
\hline 'Totals' & 993.5 & 517.5 & 1.920 \\
\hline
\end{tabular}

The "counts" are now "weighted frequencies" and are no longer integer values. The 0dds do not change for the first bin. The particular weights given in this example have the effect of scaling the total Good and Bad counts in the first bin by the same scaling factor, therefore their ratio does not change. However, the Odds value of the total sample does change; the first bin now carries a higher weight, and because the odds in that bin are lower, the total Odds are now lower, too. Other credit scorecard statistics not shown here, such as WOE and Information Value are affected in a similar way.

In general, the effect of weights is not simply to scale frequencies in a particular bin, because members of that bin will have different weights. The goal of this example is to demonstrate the concept of switching from counts to the sum of weights.

\section*{See Also \\ autobinning|bininfo|creditscorecard|fitmodel|validatemodel \\ More About}
- "About Credit Scorecards" on page 8-62
- "Credit Scorecard Modeling Workflow" on page 8-67

\section*{Credit Scorecard Modeling with Missing Values}

This example shows how to handle missing values when working with creditscorecard objects. First, the example shows how to use the creditscorecard functionality to create an explicit bin for missing data with corresponding points. Then, the example shows how to "treat" the missing data to get a final scorecard with no explicit bins for missing values. To develop a scorecard without explicit bins for missing data, data must be treated and treatments for old and new data sets prior to scoring must be consistent.

\section*{Develop a Credit Scorecard with Explicit Bins for Missing Values}

When creating a creditscorecard object, the data may contain missing values. When using creditscorecard to create a creditscorecard object, you can specify the name-value pair argument for 'BinMissingData' set to true. In this case, the missing data for numeric predictors ( NaN values) and for categorical predictors (<undefined> values) is binned in a separate bin labeled <missing> that appears at the end of the bins. Predictors with no missing values in the training data have no <missing> bin. If you do not specify the 'BinMissingData' argument or if you set 'BinMissingData' to false, the creditscorecard function discards missing observations when computing frequencies of Good and Bad, and neither the bininfo nor plotbins functions reports such observations.

The <missing> bin remains in place throughout the scorecard modeling process. The final scorecard explicitly indicates the points to be assigned to missing values for predictors that have a <missing> bin. These points are determined from the Weight-ofEvidence (WOE) of the <missing> bin and the predictor's coefficient in the logistic model. For predictors without an explicit <missing> bin, you can assign points to missing values.

The dataMissing table in the CreditCardData.mat file has two predictors, CustAge and ResStatus, with missing values.
load CreditCardData.mat
head(dataMissing,5)
\begin{tabular}{|c|c|c|c|c|c|}
\hline CustID & CustAge & TmAtAddress & ResStatus & EmpStatus & CustIncome \\
\hline 1 & 53 & 62 & <undefined> & Unknown & 50000 \\
\hline 2 & 61 & 22 & Home Owner & Employed & 52000 \\
\hline 3 & 47 & 30 & Tenant & Employed & 37000 \\
\hline
\end{tabular}
\begin{tabular}{rrrrll}
4 & NaN & 75 & Home Owner & Employed & 53000 \\
5 & 68 & 56 & Home Owner & Employed & 53000
\end{tabular}

Create a creditscorecard object using the CreditCardData.mat file to load the dataMissing table with missing values. Set the 'BinMissingData' argument to true. Apply automatic binning.
```

sc = creditscorecard(dataMissing,'IDVar','CustID','BinMissingData',true);
sc = autobinning(sc);

```

The bin information and bin plots for the predictors that have missing data both show a <missing> bin at the end.
```

bi = bininfo(sc,'CustAge');
disp(bi)

```
\begin{tabular}{|c|c|c|c|c|c|}
\hline Bin & Good & Bad & 0dds & WOE & InfoValue \\
\hline '[-Inf,33)' & 69 & 52 & 1.3269 & -0.42156 & 0.018993 \\
\hline ' \([33,37\) )' & 63 & 45 & 1.4 & -0.36795 & 0.012839 \\
\hline '[37,40)' & 72 & 47 & 1.5319 & -0.2779 & 0.0079824 \\
\hline '[40,46)' & 172 & 89 & 1.9326 & -0.04556 & 0.0004549 \\
\hline ' \([46,48\) )' & 59 & 25 & 2.36 & 0.15424 & 0.0016199 \\
\hline '[48,51)' & 99 & 41 & 2.4146 & 0.17713 & 0.0035449 \\
\hline ' \([51,58\) )' & 157 & 62 & 2.5323 & 0.22469 & 0.0088407 \\
\hline '[58,Inf]' & 93 & 25 & 3.72 & 0.60931 & 0.032198 \\
\hline '<missing>' & 19 & 11 & 1.7273 & -0.15787 & 0.00063885 \\
\hline 'Totals' & 803 & 397 & 2.0227 & NaN & 0.087112 \\
\hline
\end{tabular}
plotbins(sc,'CustAge')

```

bi = bininfo(sc,'ResStatus');
disp(bi)

```
\begin{tabular}{lrrrrrr}
\multicolumn{1}{c}{ Bin } & Good & Bad & & \multicolumn{1}{c}{ Odds } & & \multicolumn{1}{c}{ WOE }
\end{tabular}
plotbins(sc,'ResStatus')


The training data for the 'CustAge' and 'ResStatus ' predictors has missing data (NaNs and <undefined>). The binning process estimates WOE values of -0.15787 and 0.026469 , respectively, for the missing data in these predictors.

The training data for EmpStatus and CustIncome has no explicit bin for <missing> values because there are no missing values for these predictors.
```

bi = bininfo(sc,'EmpStatus');
disp(bi)

| Bin | Good | Bad | Odds | WOE | InfoValue |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 'Unknown ' | 396 | 239 | 1.6569 | -0.19947 | 0.0 |

```


Use fitmodel to fit a logistic regression model using Weight of Evidence (WOE) values. fitmodel internally transforms all the predictor variables into WOE values, using the bins found during the automatic binning process. By default, fitmodel then fits a logistic regression model using a stepwise method. For predictors that have missing data, there is an explicit <missing> bin with a corresponding WOE value computed from the data. When using fitmodel, the corresponding WOE value for the <missing> bin is applied when performing the WOE transformation.
```

[sc,mdl] = fitmodel(sc);

```
1. Adding CustIncome, Deviance = 1490.8527, Chi2Stat = 32.588614, PValue = 1.1387992e-(
2. Adding TmWBank, Deviance \(=1467.1415\), Chi2Stat \(=23.711203\), PValue \(=1.1192909 \mathrm{e}-06\)
3. Adding AMBalance, Deviance \(=1455.5715\), Chi2Stat \(=11.569967\), PValue \(=0.0006702560\) 4. Adding EmpStatus, Deviance \(=1447.3451\), Chi2Stat \(=8.2264038\), PValue \(=0.0041285257\)
5. Adding CustAge, Deviance \(=1442.8477\), Chi2Stat \(=4.4974731\), PValue \(=0.033944979\)
6. Adding ResStatus, Deviance \(=1438.9783\), Chi2Stat \(=3.86941\), PValue \(=0.049173805\)
7. Adding OtherCC, Deviance \(=1434.9751\), Chi2Stat \(=4.0031966\), PValue \(=0.045414057\)

Generalized linear regression model:
status ~ [Linear formula with 8 terms in 7 predictors]
Distribution = Binomial

Estimated Coefficients:

\begin{tabular}{lrrrr} 
CustAge & 0.57421 & 0.25708 & 2.2335 & 0.025513 \\
ResStatus & 1.3629 & 0.66952 & 2.0356 & 0.04179 \\
EmpStatus & 0.88373 & 0.2929 & 3.0172 & 0.002551 \\
CustIncome & 0.73535 & 0.2159 & 3.406 & 0.00065929 \\
TmWBank & 1.1065 & 0.23267 & 4.7556 & \(1.9783 e-06\) \\
OtherCC & 1.0648 & 0.52826 & 2.0156 & 0.043841 \\
AMBalance & 1.0446 & 0.32197 & 3.2443 & 0.0011775
\end{tabular}

1200 observations, 1192 error degrees of freedom Dispersion: 1
Chi^2-statistic vs. constant model: 88.5, p -value \(=2.55 \mathrm{e}-16\)
Scale the scorecard points by the points-to-double-the-odds (PDO) method using the ' Points0ddsAndPDO ' argument of formatpoints. Suppose that you want a score of 500 points to have odds of 2 (twice as likely to be good than to be bad) and that the odds double every 50 points (so that 550 points would have odds of 4).

Display the scorecard showing the scaled points for predictors retained in the fitting model.
```

sc = formatpoints(sc,'Points0ddsAndPDO',[500 2 50]);
PointsInfo = displaypoints(sc)

```
PointsInfo=33×3 table
Predictors
'CustAge'
'CustAge'
'CustAge'
'CustAge'
'CustAge'
'CustAge'
'CustAge'
'CustAge'
'CustAge'
'ResStatus'
'ResStatus'
'ResStatus'
'ResStatus'
'EmpStatus'
'EmpStatus'
'CustIncome'

Bin
'[-Inf,33)'
54.062
' \([33,37\) )
' \([37,40\) ) '
' \([40,46\) )'
\([46,48)\)\(\quad 77.912\)
\begin{tabular}{ll} 
\\
\hline
\end{tabular}\([48,51)\) ' 78.86
\(\begin{array}{ll} \\ \\ {[51,58)} & 80.83\end{array}\)
'[58,Inf]' 96.76
'<missing>' 64.984
'Tenant' 62.138
'Home Owner' 73.248
'Other' 90.828
'<missing>' 74.125
'Unknown' 58.807
'Employed' 86.937
'[-Inf,29000)' 29.375

Notice that points for the <missing> bin for CustAge and ResStatus are explicitly shown (as 64.9836 and 74.1250 , respectively). These points are computed from the WOE value for the <missing> bin and the logistic model coefficients.

Predictors that have no missing data in the training set have no explicit <missing> bin. By default, the points are set to NaN for missing data and they lead to a score of NaN when running score. For predictors that have no explicit <missing> bin, use the namevalue argument 'Missing' in formatpoints to indicate how missing data should be treated for scoring purposes.

The scorecard is ready for scoring new data sets. You can also use the scorecard to compute probabilities of default or perform model validation. For details, see score, probdefault, and validatemodel. To further explore the handling of missing data, take a few rows from the original data as test data and introduce some missing data.
```

tdata = dataMissing(11:14,mdl.PredictorNames); % Keep only the predictors retained in
% Set some missing values
tdata.CustAge(1) = NaN;
tdata.ResStatus(2) = '<undefined>';
tdata.EmpStatus(3) = '<undefined>';
tdata.CustIncome(4) = NaN;
disp(tdata)

| CustAge | ResStatus | EmpStatus | CustIncome | TmWBank | OtherCC | AMBal |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| NaN | Tenant | Unknown | 34000 | 44 | Yes | 119 |
| 48 | <undefined> | Unknown | 44000 | 14 | Yes | 403. |
| 65 | Home Owner | <undefined> | 48000 | 6 | No | 111 |
| 44 | Other | Unknown | NaN | 35 | No | 436 |

```

Score the new data and see how points for missing data are differently assigned for CustAge and ResStatus and for EmpStatus and CustIncome. CustAge and ResStatus have an explicit <missing> bin for missing data. However, for EmpStatus and CustIncome the score function sets the points to NaN.
```

[Scores,Points] = score(sc,tdata);
disp(Scores)

```
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline \multicolumn{7}{|l|}{NaN NaN} \\
\hline \multicolumn{7}{|l|}{disp(Points)} \\
\hline CustAge & ResStatus & EmpStatus & CustIncome & TmWBank & OtherCC & AMBalance \\
\hline 64.984 & 62.138 & 58.807 & 67.893 & 61.858 & 75.622 & 89.922 \\
\hline 78.86 & 74.125 & 58.807 & 82.439 & 61.061 & 75.622 & 89.922 \\
\hline 96.76 & 73.248 & NaN & 96.969 & 51.132 & 50.914 & 89.922 \\
\hline 69.636 & 90.828 & 58.807 & NaN & 61.858 & 50.914 & 89.922 \\
\hline
\end{tabular}

Use the name-value argument 'Missing' in formatpoints to choose how to assign points to missing values for predictors that do not have an explicit <missing> bin. For this example, use the 'MinPoints ' option for the 'Missing ' argument. For EmpStatus and CustIncome, the minimum numbers of points in the scorecard are 58.8072 and 29.3753, respectively.
```

sc = formatpoints(sc,'Missing','MinPoints');
[Scores,Points] = score(sc,tdata);
disp(Scores)
481.2231
520.8353
517.7532
451.3405

```
disp(Points)
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline CustAge & ResStatus & EmpStatus & CustIncome & TmWBank & OtherCC & AMBalance \\
\hline 64.984 & 62.138 & 58.807 & 67.893 & 61.858 & 75.622 & 89.922 \\
\hline 78.86 & 74.125 & 58.807 & 82.439 & 61.061 & 75.622 & 89.922 \\
\hline 96.76 & 73.248 & 58.807 & 96.969 & 51.132 & 50.914 & 89.922 \\
\hline 69.636 & 90.828 & 58.807 & 29.375 & 61.858 & 50.914 & 89.922 \\
\hline
\end{tabular}

\section*{Treat Missing Data and Develop a New Credit Scorecard Without Bins for Missing Values}

You can use one of two alternative workflows to develop a scorecard without explicit bins for missing data.

The first alternative is to discard the missing data during the analysis. If the creditscorecard is created with the 'BinMissingData' argument set to false (by
default, it is set to false if not specified), the missing observations are discarded when computing frequencies of Good and Bad and are not reported by bininfo or plotbins. For the fitting of the logistic model, rows with missing values are also discarded. With this approach, the missing data indirectly influences the results because the total number of observations used to compute bin statistics such as Weight-of-Evidence (WOE), or the total number of rows used to fit a logistic model, is reduced by the number of missing observations. For more information on this workflow, see "Credit Scorecard Modeling Workflow" on page 8-67.

The second alternative is to first gather information about the missing values, then treat or replace the missing values so that the training data has no missing observations, and then create a creditscorecard object with the treated data set. This approach modifies the training data, allowing the reporting of missing observations in the bin counts and the inclusion of missing observations for fitting the logistic model. However, in this approach, the treatment of the training data and the treatment of any new data set that requires scoring must be the same.

The following example explains the second alternative workflow, which gathers missing data, treats the training data, develops a new creditscorecard, and treats new data before scoring.

The dataMissing table in the CreditCardData.mat file has two predictors, CustAge and ResStatus, with missing values.
```

load CreditCardData.mat
head(dataMissing,5)

| ans $=5 \times 11$ table <br> CustID | CustAge |  | TmAtAddress |  | ResStatus |  | EmpStatus |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | CustIncome

```

First, use the untreated training data to analyze the missing data information.
Create a creditscorecard object using the CreditCardData.mat file to load the dataMissing with missing values and set the 'BinMissingData' argument to true to explicitly report information on missing values. Apply automatic binning.
```

sc = creditscorecard(dataMissing,'IDVar','CustID','BinMissingData',true);
sc = autobinning(sc);

```

The bin information and bin plots for predictors that have missing data both show a <missing> bin at the end. The two predictors with missing values in this data set are CustAge and ResStatus.
```

bi = bininfo(sc,'CustAge');
disp(bi)

```
\begin{tabular}{|c|c|c|c|c|c|}
\hline Bin & Good & Bad & 0dds & WOE & InfoValue \\
\hline '[-Inf, 33) ' & 69 & 52 & 1.3269 & -0.42156 & 0.018993 \\
\hline '[33,37)' & 63 & 45 & 1.4 & -0.36795 & 0.012839 \\
\hline ' \([37,40\) )' & 72 & 47 & 1.5319 & -0.2779 & 0.0079824 \\
\hline ' \([40,46\) )' & 172 & 89 & 1.9326 & -0.04556 & 0.0004549 \\
\hline ' \([46,48\) )' & 59 & 25 & 2.36 & 0.15424 & 0.0016199 \\
\hline ' \([48,51\) )' & 99 & 41 & 2.4146 & 0.17713 & 0.0035449 \\
\hline ' \([51,58\) ) ' & 157 & 62 & 2.5323 & 0.22469 & 0.0088407 \\
\hline '[58,Inf]' & 93 & 25 & 3.72 & 0.60931 & 0.032198 \\
\hline '<missing>' & 19 & 11 & 1.7273 & -0.15787 & 0.00063885 \\
\hline 'Totals' & 803 & 397 & 2.0227 & NaN & 0.087112 \\
\hline
\end{tabular}

```

bi = bininfo(sc,'ResStatus');
disp(bi)

```
\begin{tabular}{lrrrrrr}
\multicolumn{1}{c}{ Bin } & Good & Bad & & \multicolumn{1}{c}{ Odds } & & \multicolumn{1}{c}{ WOE }
\end{tabular}
plotbins(sc,'ResStatus')


To treat missing values, you can apply different criteria. This example follows a straightforward approach to replace missing observations with the most common or typical value in the data distribution, which is the value of mode for the data. For this example, the mode happens to have a similar WOE value as the original <missing> bin. The similarity in values is favorable because similar WOE values means similar points in a scorecard.

For CustAge, bin 4 is the bin with the most observations and the mode value of the original data is 43 .
```

modeCustAge = mode(dataMissing.CustAge);
disp(modeCustAge)

```

The WOE value of the <missing> bin is similar to the WOE value of bin 4. Therefore, replacing the missing values in CustAge with the value of mode is reasonable.

To treat the data, create a copy of the data and fill the missing values.
```

dataTreated = dataMissing;
dataTreated.CustAge = fillmissing(dataTreated.CustAge,'constant',modeCustAge);

```

For ResStatus, the value of 'Home Owner' is the value of the mode of the data, and the WOE value of the <missing> bin is closest to that of the 'Home Owner' bin.
```

modeResStatus = mode(dataMissing.ResStatus);
disp(modeResStatus)

```

Home Owner
Replace the missing data with 'Home Owner'. Replacing the missing values preserves both the observed WOE values and the typical characteristics observed in the data set.
```

dataTreated.ResStatus = fillmissing(dataTreated.ResStatus,'constant',string(modeResSta

```

The treated data set now has no missing values.
```

disp(any(any(ismissing(dataTreated))))

```
    0

Using the treated data set, apply the typical creditscorecard workflow. Create a creditscorecard object with the treated data and applying automatic binning.
```

scTreated = creditscorecard(dataTreated,'IDVar','CustID');
scTreated = autobinning(scTreated);

```

Compare the bin information of the untreated data for CustAge with the bin information of the treated data for CustAge.
```

bi = bininfo(sc,'CustAge');
disp(bi)

| Bin | Good | Bad | Odds | WOE | InfoValue |
| :---: | :---: | :---: | :---: | :---: | :---: |
| '[-Inf, 33) ' | 69 | 52 | 1.3269 | -0.42156 | 0.018993 |
| ' $[33,37)^{\prime}$ | 63 | 45 | 1.4 | -0.36795 | 0.012839 |
| ' $[37,40$ )' | 72 | 47 | 1.5319 | -0.2779 | 0.0079824 |

```
\begin{tabular}{lrrrrr} 
'[40,46)' & 172 & 89 & 1.9326 & -0.04556 & 0.0004549 \\
'[46,48)' & 59 & 25 & 2.36 & 0.15424 & 0.0016199 \\
'[48,51)' & 99 & 41 & 2.4146 & 0.17713 & 0.0035449 \\
'[51,58)' & 157 & 62 & 2.5323 & 0.22469 & 0.0088407 \\
'[58,Inf]' & 93 & 25 & 3.72 & 0.60931 & 0.032198 \\
'<missing>' & 19 & 11 & 1.7273 & -0.15787 & 0.00063885 \\
'Totals' & 803 & 397 & 2.0227 & NaN & 0.087112
\end{tabular}
```

biTreated = bininfo(scTreated,'CustAge');
disp(biTreated)

```
\begin{tabular}{|c|c|c|c|c|c|}
\hline Bin & Good & Bad & Odds & WOE & InfoValue \\
\hline '[-Inf,33)' & 69 & 52 & 1.3269 & -0.42156 & 0.018993 \\
\hline '[33,37)' & 63 & 45 & 1.4 & -0.36795 & 0.012839 \\
\hline ' \([37,40\) )' & 72 & 47 & 1.5319 & -0.2779 & 0.0079824 \\
\hline '[40,45)' & 156 & 86 & 1.814 & -0.10891 & 0.0024345 \\
\hline ' \([45,48)^{\prime}\) & 94 & 39 & 2.4103 & 0.17531 & 0.0033002 \\
\hline ' \([48,58)^{\prime}\) & 256 & 103 & 2.4854 & 0.20603 & 0.01223 \\
\hline '[58,Inf]' & 93 & 25 & 3.72 & 0.60931 & 0.032198 \\
\hline 'Totals' & 803 & 397 & 2.0227 & NaN & 0.089977 \\
\hline
\end{tabular}

The first few bins are the same, but the treatment of missing values influences the binning results, starting with the bin where the missing data is placed. You can further explore your binning results using autobinning with a different algorithm or you can manually modify the bins using modifybins.

For ResStatus, the results for the treated data look similar to the initial results, except for the higher counts in the 'Home Owner' bin due to the treatment. For a categorical variable with more categories (or levels), an automatic algorithm may find category groups and the results may show more differences for before and after the treatment. You can further explore your binning results using autobinning with a different algorithm or you can manually modify the bins using modifybins.
```

bi = bininfo(sc,'ResStatus');
disp(bi)

| Bin | Good | Bad | 0dds | WOE | InfoValue |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 'Tenant' | 296 | 161 | 1.8385 | -0.095463 | 0.0035249 |
| 'Home Owner' | 352 | 171 | 2.0585 | 0.017549 | 0.00013382 |
| 'Other' | 128 | 52 | 2.4615 | 0.19637 | 0.0055808 |

```
\begin{tabular}{lrrrrr} 
'<missing>' & 27 & 13 & 2.0769 & 0.026469 & \(2.3248 \mathrm{e}-05\) \\
'Totals' & 803 & 397 & 2.0227 & NaN & 0.0092627
\end{tabular}
```

biTreated = bininfo(scTreated,'ResStatus');
disp(biTreated)

```
\begin{tabular}{|c|c|c|c|c|c|}
\hline Bin & Good & Bad & Odds & WOE & InfoValue \\
\hline 'Tenant' & 296 & 161 & 1.8385 & -0.095463 & 0.0035249 \\
\hline 'Home Owner' & 379 & 184 & 2.0598 & 0.018182 & 0.00015462 \\
\hline 'Other' & 128 & 52 & 2.4615 & 0.19637 & 0.0055808 \\
\hline 'Totals' & 803 & 397 & 2.0227 & NaN & 0.0092603 \\
\hline
\end{tabular}

Fit the logistic model, scale the points, and display the final scorecard.
```

scTreated = fitmodel(scTreated,'Display','off');
scTreated = formatpoints(scTreated,'PointsOddsAndPDO',[500 2 50]);
ScPoints = displaypoints(scTreated);
disp(ScPoints)

```
\begin{tabular}{|c|c|c|}
\hline Predictors & Bin & Points \\
\hline 'CustAge' & '[-Inf,33)' & 53.507 \\
\hline 'CustAge' & ' \([33,37)\) ' & 55.798 \\
\hline 'CustAge' & ' \([37,40\) )' & 59.646 \\
\hline 'CustAge' & ' \([40,45\) )' & 66.868 \\
\hline 'CustAge' & ' \([45,48\) ) ' & 79.013 \\
\hline 'CustAge' & ' \([48,58\) )' & 80.326 \\
\hline 'CustAge' & '[58,Inf]' & 97.559 \\
\hline 'ResStatus' & 'Tenant' & 62.161 \\
\hline 'ResStatus' & 'Home Owner' & 73.305 \\
\hline 'ResStatus' & 'Other' & 90.777 \\
\hline 'EmpStatus' & 'Unknown' & 58.846 \\
\hline 'EmpStatus' & 'Employed' & 86.887 \\
\hline 'CustIncome' & '[-Inf, 29000)' & 29.906 \\
\hline 'CustIncome' & ' [29000, 33000) & 56.219 \\
\hline 'CustIncome' & ' \([33000,35000)^{\prime}\) & 67.938 \\
\hline 'CustIncome' & '[35000, 40000) ' & 70.123 \\
\hline 'CustIncome' & '[40000, 42000) ' & 70.931 \\
\hline 'CustIncome' & '[42000, 47000) ' & 82.3 \\
\hline 'CustIncome' & '[47000,Inf]' & 96.647 \\
\hline 'TmWBank' & '[-Inf,12)' & 51.05 \\
\hline 'TmWBank' & ' \([12,23\) ) & 61.018 \\
\hline
\end{tabular}
\begin{tabular}{lll} 
'TmWBank' & '[23,45)' & 61.818 \\
'TmWBank' & '[45,71)' & 92.921 \\
'TmWBank' & '[71,Inf]' & 133.14 \\
'OtherCC' & 'No' & 50.806 \\
'OtherCC' & 'Yes' & 75.642 \\
'AMBalance' & '[-Inf,558.88)' & 89.788 \\
'AMBalance' & '[558.88,1254.28)' & 63.088 \\
'AMBalance' & '[1254.28,1597.44)' & 59.711 \\
'AMBalance' & '[1597.44,Inf]' & 49.157
\end{tabular}

There are no explicit <missing> bins in the final scorecard. If you need to score a new data set and it contains missing data, by default the score function sets the points to NaN . To further explore the handling of missing data, take a few rows from the original data as test data and introduce some missing data.
```

tdata = dataTreated(11:14,mdl.PredictorNames); % Keep only the predictors retained in
% Set some missing values
tdata.CustAge(1) = NaN;
tdata.ResStatus(2) = '<undefined>';
tdata.EmpStatus(3) = '<undefined>';
tdata.CustIncome(4) = NaN;
disp(tdata)

| CustAge | ResStatus | EmpStatus | CustIncome | TmWBank | OtherCC | AMBal |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| NaN | Tenant | Unknown | 34000 | 44 | Yes | 119 |
| 48 | <undefined> | Unknown | 44000 | 14 | Yes | 403. |
| 65 | Home Owner | <undefined> | 48000 | 6 | No | 111. |
| 44 | Other | Unknown | NaN | 35 | No | 436. |

```

Score the new data and see how points are set to NaN, which leads to NaN scores.
```

[Scores,Points] = score(scTreated,tdata);
disp(Scores)

```
    NaN
    NaN
    NaN
    NaN
disp(Points)
CustAge ResStatus EmpStatus CustIncome TmWBank OtherCC AMBalance
\begin{tabular}{rrrrrrr}
NaN & 62.161 & 58.846 & 67.938 & 61.818 & 75.642 & 89.788 \\
80.326 & NaN & 58.846 & 82.3 & 61.018 & 75.642 & 89.788 \\
97.559 & 73.305 & NaN & 96.647 & 51.05 & 50.806 & 89.788 \\
66.868 & 90.777 & 58.846 & NaN & 61.818 & 50.806 & 89.788
\end{tabular}

For untreated predictors, such as EmpStatus or CustIncome, you can use the namevalue argument 'Missing ' in formatpoints to choose how to assign points to missing values.

Use the 'MinPoints' option for the 'Missing' argument. This assigns the minimum number of possible points in the scorecard to the missing data. In this example, the minimum number of possible points for CustIncome is 29.906 , so the last row in the table gets 29.906 points for the missing Cust Income value.
```

scTreated = formatpoints(scTreated,'Missing','MinPoints');
[Scores,Points] = score(scTreated,tdata);
disp(Scores)

```
    469.7003
    510.0812
    518.0013
    448.8099
disp(Points)
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline CustAge & ResStatus & EmpStatus & CustIncome & TmWBank & OtherCC & AMBalance \\
\hline 53.507 & 62.161 & 58.846 & 67.938 & 61.818 & 75.642 & 89.788 \\
\hline 80.326 & 62.161 & 58.846 & 82.3 & 61.018 & 75.642 & 89.788 \\
\hline 97.559 & 73.305 & 58.846 & 96.647 & 51.05 & 50.806 & 89.788 \\
\hline 66.868 & 90.777 & 58.846 & 29.906 & 61.818 & 50.806 & 89.788 \\
\hline
\end{tabular}

However, for predictors that were treated in the training data, such as CustAge, the effect of the 'Missing ' argument is inconsistent with the treatment of the training data. For example, for CustAge, the first observation gets 53.507 points for the missing value, yet if the new data were "treated," and the missing value for CustAge were replaced with the mode of the training data (age of 43), this observation falls in the [40,45) bin and receives 66.868 points.

Therefore, before scoring, data sets must be treated the same way the training data was treated. The use of the 'Missing' argument is still important to assign points for
untreated predictors and the treated predictors receive points in a way that is consistent with the way the model was developed.
```

tdataTreated = tdata;
tdataTreated.CustAge = fillmissing(tdataTreated.CustAge,'constant',modeCustAge);
tdataTreated.ResStatus = fillmissing(tdataTreated.ResStatus,'constant',string(modeResS
disp(tdataTreated)

```
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline CustAge & ResStatus & EmpStatus & CustIncome & TmWBank & OtherCC & AMBala \\
\hline 43 & Tenant & Unknown & 34000 & 44 & Yes & 119. \\
\hline 48 & Home Owner & Unknown & 44000 & 14 & Yes & 403.6 \\
\hline 65 & Home Owner & <undefined> & 48000 & 6 & No & 111.8 \\
\hline 44 & Other & Unknown & NaN & 35 & No & 436.4 \\
\hline
\end{tabular}
```

[Scores,Points] = score(scTreated,tdataTreated);
disp(Scores)

```
    483.0606
    521.2249
    518.0013
    448.8099
disp(Points)
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline CustAge & ResStatus & EmpStatus & CustIncome & TmWBank & OtherCC & AMBalance \\
\hline 66.868 & 62.161 & 58.846 & 67.938 & 61.818 & 75.642 & 89.788 \\
\hline 80.326 & 73.305 & 58.846 & 82.3 & 61.018 & 75.642 & 89.788 \\
\hline 97.559 & 73.305 & 58.846 & 96.647 & 51.05 & 50.806 & 89.788 \\
\hline 66.868 & 90.777 & 58.846 & 29.906 & 61.818 & 50.806 & 89.788 \\
\hline
\end{tabular}

\section*{See Also}
bininfo|creditscorecard|plotbins

\section*{More About}
- "About Credit Scorecards" on page 8-62
- "Credit Scorecard Modeling Workflow" on page 8-67

\section*{Troubleshooting Credit Scorecard Results}
```

In this section...
"Predictor Name Is Unspecified and the Parser Returns an Error" on page 8-91
"Using bininfo or plotbins Before Binning" on page 8-91
"If Categorical Data Is Given as Numeric" on page 8-94
"NaNs Returned When Scoring a "Test" Dataset" on page 8-97

```

This topic shows some of the results when using credit scorecards that need troubleshooting. These examples cover the full range of the credit score card workflow. For details on the overall process of creating and developing credit scorecards, see "Credit Scorecard Modeling Workflow" on page 8-67.

\section*{Predictor Name Is Unspecified and the Parser Returns an Error}

If you attempt to use modifybins, bininfo, or plotbins and omit the predictor's name, the parser returns an error.
```

load CreditCardData
sc = creditscorecard(data,'IDVar','CustID','GoodLabel',0);
modifybins(sc,'CutPoints',[20 30 50 65])

```
Error using creditscorecard/modifybins (line 79)
Expected a string for the parameter name, instead the input type was 'double'.

Solution: Make sure to include the predictor's name when using these functions. Use this syntax to specify the PredictorName when using modifybins.
load CreditCardData
sc = creditscorecard(data,'IDVar','CustID','GoodLabel',0);
modifybins(sc,'CustIncome','CutPoints',[20 3050 65]);

\section*{Using bininfo or plotbins Before Binning}

If you use bininfo or plotbins before binning, the results might be unusable.
load CreditCardData
sc = creditscorecard(data,'IDVar','CustID','GoodLabel',0);

\section*{bininfo(sc,'CustAge') plotbins(sc,'CustAge')}
ans \(=\)
\begin{tabular}{|c|c|c|c|c|c|}
\hline Bin & Good & Bad & Odds & WOE & InfoValue \\
\hline '21' & 2 & 1 & 2 & -0.011271 & 3.1821e-07 \\
\hline '22' & 3 & 1 & 3 & 0.39419 & 0.00047977 \\
\hline '23' & 1 & 2 & 0.5 & -1.3976 & 0.0053002 \\
\hline '24' & 3 & 4 & 0.75 & -0.9921 & 0.0062895 \\
\hline '25' & 3 & 1 & 3 & 0.39419 & 0.00047977 \\
\hline '26' & 4 & 2 & 2 & -0.011271 & 6.3641e-07 \\
\hline '27' & 6 & 5 & 1.2 & -0.5221 & 0.0026744 \\
\hline '28' & 10 & 2 & 5 & 0.90502 & 0.0067112 \\
\hline '29' & 8 & 6 & 1.3333 & -0.41674 & 0.0021465 \\
\hline '30' & 9 & 10 & 0.9 & -0.80978 & 0.011321 \\
\hline '31' & 8 & 6 & 1.3333 & -0.41674 & 0.0021465 \\
\hline '32' & 13 & 13 & 1 & -0.70442 & 0.011663 \\
\hline '33' & 9 & 11 & 0.81818 & -0.90509 & 0.014934 \\
\hline '34' & 14 & 12 & 1.1667 & -0. 05027 & 0.0070391 \\
\hline '35' & 18 & 10 & 1.8 & -0.11663 & 0.00032342 \\
\hline '36' & 23 & 14 & 1.6429 & -0.20798 & 0.0013772 \\
\hline '37' & 28 & 19 & 1.4737 & -0.31665 & 0.0041132 \\
\hline '38' & 24 & 14 & 1.7143 & -0.16542 & 0.0008894 \\
\hline '39' & 21 & 14 & 1.5 & -0.29895 & 0.0027242 \\
\hline '40' & 31 & 12 & 2.5833 & 0.24466 & 0.0020499 \\
\hline '41' & 21 & 18 & 1. 1667 & -0.55027 & 0.010559 \\
\hline '42' & 29 & 9 & 3.2222 & 0.46565 & 0.0062605 \\
\hline '43' & 29 & 23 & 1.2609 & -0.47262 & 0.010312 \\
\hline '44' & 28 & 16 & 1.75 & -0.1448 & 0.00078672 \\
\hline '45' & 36 & 16 & 2.25 & 0.10651 & 0.00048246 \\
\hline '46' & 33 & 19 & 1.7368 & -0.15235 & 0.0010303 \\
\hline '47' & 28 & 6 & 4.6667 & 0.83603 & 0.016516 \\
\hline '48' & 32 & 17 & 1.8824 & -0.071896 & 0.00021357 \\
\hline '49' & 38 & 10 & 3.8 & 0.63058 & 0.013957 \\
\hline '50' & 33 & 14 & 2.3571 & 0.15303 & 0.00089239 \\
\hline '51' & 28 & 9 & 3.1111 & 0.43056 & 0.0052525 \\
\hline '52' & 35 & 8 & 4.375 & 0.77149 & 0.01808 \\
\hline '53' & 14 & 8 & 1.75 & -0.1448 & 0.00039336 \\
\hline '54' & 27 & 12 & 2.25 & 0.10651 & 0.00036184 \\
\hline '55' & 20 & 9 & 2.2222 & 0.094089 & 0.00021044 \\
\hline '56' & 20 & 11 & 1.8182 & -0.10658 & 0.00029856 \\
\hline '57' & 16 & 7 & 2.2857 & 0.12226 & 0.00028035 \\
\hline '58' & 11 & 7 & 1.5714 & -0.25243 & 0.00099297 \\
\hline '59' & 11 & 6 & 1.8333 & -0.098283 & 0.00013904 \\
\hline '60' & 9 & 4 & 2.25 & 0.10651 & 0.00012061 \\
\hline '61' & 11 & 2 & 5.5 & 1.0003 & 0.0086637 \\
\hline '62' & 8 & 0 & Inf & Inf & Inf \\
\hline '63' & 7 & 1 & 7 & 1.2415 & 0.0076953 \\
\hline '64' & 10 & 0 & Inf & Inf & Inf \\
\hline '65' & 4 & 1 & 4 & 0.68188 & 0.0016791 \\
\hline '66' & 6 & 1 & 6 & 1.0873 & 0.0053857 \\
\hline '67' & 2 & 3 & 0.66667 & -1.1099 & 0.0056227 \\
\hline '68' & 6 & 1 & 6 & 1.0873 & 0.0053857 \\
\hline '69' & 6 & 0 & Inf & Inf & Inf \\
\hline '70' & 1 & 0 & Inf & Inf & Inf \\
\hline '71' & 1 & 0 & Inf & Inf & Inf \\
\hline '72' & 1 & 0 & Inf & Inf & Inf \\
\hline '73' & 3 & 0 & Inf & Inf & Inf \\
\hline
\end{tabular}


The plot for CustAge is not readable because it has too many bins. Also, bininfo returns data that have Inf values for the WOE due to zero observations for either Good or Bad.

Solution: Bin the data using autobinning or modifybins before plotting or inquiring about the bin statistics, to avoid having too many bins or having NaNs and Infs. For example, you can use the name-value pair argument for AlgoOptions with the autobinning function to define the number of bins.
```

load CreditCardData
sc = creditscorecard(data,'IDVar','CustID','GoodLabel',0);
AlgoOptions = {'NumBins',4};
sc = autobinning(sc,'CustAge','Algorithm','EqualFrequency',...
'AlgorithmOptions',AlgoOptions);
bininfo(sc,'CustAge','Totals','off')
plotbins(sc,'CustAge')
ans =

```



\section*{If Categorical Data Is Given as Numeric}

Categorical data is often recorded using numeric values, and can be stored in a numeric array. Although you know that the data should be interpreted as categorical information, for creditscorecard this predictor looks like a numeric array.

To show the case where categorical data is given as numeric data, the data for the variable ResStatus is intentionally converted to numeric values.
```

load CreditCardData
data.ResStatus = double(data.ResStatus);
sc = creditscorecard(data,'IDVar','CustID')
sc =

```
    creditscorecard with properties:
```

    GoodLabel: 0
    ResponseVar: 'status'
        VarNames: {1x11 cell}
    NumericPredictors: {1x7 cell}
    CategoricalPredictors: {'EmpStatus' 'OtherCC'}
IDVar: 'CustID'
PredictorVars: {1x9 cell}

```

Note that 'ResStatus' appears as part of the NumericPredictors property. If we applied automatic binning, the resulting bin information raises flags regarding the predictor type.
```

sc = autobinning(sc,'ResStatus');
[bi,cg] = bininfo(sc,'ResStatus')

```
bi \(=\)
\begin{tabular}{|c|c|c|c|c|c|}
\hline Bin & Good & Bad & Odds & WOE & InfoValue \\
\hline '[-Inf,2)' & 365 & 177 & 2.0621 & 0.019329 & 0.0001682 \\
\hline '[2,Inf]' & 438 & 220 & 1.9909 & -0.015827 & 0.00013772 \\
\hline 'Totals' & 803 & 397 & 2.0227 & NaN & 0.00030592 \\
\hline
\end{tabular}
\(\mathrm{Cg}=\)

2

The numeric ranges in the bin labels show that 'ResStatus' is being treated as a numeric variable. This is also confirmed by the fact that the optional output from bininfo is a numeric array of cut points, as opposed to a table with category groupings. Moreover, the output from predictorinfo confirms that the credit scorecard is treating the data as numeric.
```

[T,Stats] = predictorinfo(sc,'ResStatus')

```
\(\mathrm{T}=\)

```

Stats =
Value
Min

```
```

Mean 1.7017
Std 0.71863

```

Solution: For creditscorecard, 'Categorical' means a MATLAB categorical data type. For more information, see categorical. To treat'ResStatus ' as categorical, change the 'PredictorType' of the PredictorName 'ResStatus' from 'Numeric' to 'Categorical' using modifypredictor.
```

sc = modifypredictor(sc,'ResStatus','PredictorType','Categorical')
[T,Stats] = predictorinfo(sc,'ResStatus')

```
SC \(=\)
creditscorecard with properties:
GoodLabel: 0
ResponseVar: 'status'
VarNames: \{1x11 cell\}
NumericPredictors: \{1x6 cell\}
CategoricalPredictors: \{'ResStatus' 'EmpStatus' 'OtherCC'\}
IDVar: 'CustID'
PredictorVars: \{1x9 cell\}
\(T=\)
ResStatus \begin{tabular}{llll} 
'Categorical' & \begin{tabular}{l} 
PredictorType
\end{tabular} & \(\left.\begin{array}{l}\text { Ordinal } \\
\text { 'Original Data' }\end{array}\right)\) LatestBinning \\
\hline
\end{tabular}

Stats =
\begin{tabular}{ll} 
& Count \\
& C1 \\
C2 & 542 \\
C3 & 474 \\
& 184
\end{tabular}

Note that 'ResStatus ' now appears as part of the Categorical predictors. Also, predictorinfo now describes 'ResStatus' as categorical and displays the category counts.

If you apply autobinning, the categories are now reordered, as shown by calling bininfo, which also shows the category labels, as opposed to numeric ranges. The optional output of bininfo is now a category grouping table.
```

sc = autobinning(sc,'ResStatus');
[bi,cg] = bininfo(sc,'ResStatus')

```
\begin{tabular}{|c|c|c|c|c|c|}
\hline Bin & Good & Bad & Odds & WOE & InfoValue \\
\hline 'C2' & 307 & 167 & 1.8383 & -0.095564 & 0.0036638 \\
\hline 'C1' & 365 & 177 & 2.0621 & 0.019329 & 0.0001682 \\
\hline 'C3' & 131 & 53 & 2.4717 & 0.20049 & 0.0059418 \\
\hline 'Totals' & 803 & 397 & 2.0227 & NaN & 0.0097738 \\
\hline
\end{tabular}
```

Cg =

| Category |  | BinNumber |  |
| :--- | :--- | :--- | :---: |
| 'C2' |  | 1 |  |
| 'C1' $^{\prime}$ | 2 |  |  |
| 'C3' $^{\prime}$ | 3 |  |  |

```

\section*{NaNs Returned When Scoring a "Test" Dataset}

When applying a creditscorecard model to a "test" dataset using the score function, if an observation in the "test" dataset has a NaN or <undefined> value, a NaN total score is returned for each of these observations. For example, a creditscorecard object is created using "training" data.
```

load CreditCardData
sc = creditscorecard(data,'IDVar','CustID');
sc = autobinning(sc);
sc = fitmodel(sc);

1. Adding CustIncome, Deviance = 1490.8527, Chi2Stat = 32.588614, PValue = 1.1387992e-08
. Adding TmWBank, Deviance = 1467.1415, Chi2Stat = 23.711203, PValue = 1.1192909e-06
. Adding AMBalance, Deviance = 1455.5715, Chi2Stat = 11.569967, PValue = 0.00067025601
2. Adding EmpStatus, Deviance = 1447.3451, Chi2Stat = 8.2264038, PValue = 0.0041285257
3. Adding CustAge, Deviance = 1441.994, Chi2Stat = 5.3511754, PValue = 0.020708306
4. Adding ResStatus, Deviance = 1437.8756, Chi2Stat = 4.118404, PValue = 0.042419078
5. Adding OtherCC, Deviance = 1433.707, Chi2Stat = 4.1686018, PValue = 0.041179769
```
Generalized Linear regression model:
    logit(status) ~ \(1+\) CustAge + ResStatus + EmpStatus + CustIncome + TmWBank + OtherCC + AMBalance
    Distribution = Binomial
Estimated Coefficients:
\begin{tabular}{|c|c|c|c|c|}
\hline & Estimate & SE & tStat & pValue \\
\hline (Intercept) & 0.70239 & 0.064001 & 10.975 & \(5.0538 \mathrm{e}-28\) \\
\hline CustAge & 0.60833 & 0.24932 & 2.44 & 0.014687 \\
\hline ResStatus & 1.377 & 0.65272 & 2.1097 & 0.034888 \\
\hline EmpStatus & 0.88565 & 0.293 & 3.0227 & 0.0025055 \\
\hline CustIncome & 0.70164 & 0.21844 & 3.2121 & 0.0013179 \\
\hline TmWBank & 1.1074 & 0.23271 & 4.7589 & \(1.9464 \mathrm{e}-06\) \\
\hline OtherCC & 1.0883 & 0.52912 & 2.0569 & 0.039696 \\
\hline AMBalance & 1.045 & 0.32214 & 3.2439 & 0.0011792 \\
\hline
\end{tabular}

1200 observations, 1192 error degrees of freedom
Dispersion: 1
Chi^2-statistic vs. constant model: 89.7, p -value \(=1.4 \mathrm{e}-16\)
Suppose that a missing observation ( Nan ) is added to the data and then newdata is scored using the score function. By default, the points and score assigned to the missing value is NaN .
```

newdata = data(1:10,:);
newdata.CustAge(1) = NaN;
[Scores,Points] = score(sc,newdata)

```
Scores =
        NaN
        1.4646
    0.7662
    1.5779
    1.4535
    1.8944
    \(-0.0872\)
    0.9207
    1.0399
    0.8252
Points \(=\)
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline CustAge & ResStatus & EmpStatus & CustIncome & TmWBank & OtherCC & AMBalance \\
\hline NaN & -0.031252 & -0.076317 & 0.43693 & 0.39607 & 0.15842 & -0.017472 \\
\hline 0.479 & 0.12696 & 0.31449 & 0.43693 & -0.033752 & 0.15842 & -0.017472 \\
\hline 0.21445 & -0.031252 & 0.31449 & 0.081611 & 0.39607 & -0.19168 & -0.017472 \\
\hline 0.23039 & 0.12696 & 0.31449 & 0.43693 & -0.044811 & 0.15842 & 0.35551 \\
\hline 0.479 & 0.12696 & 0.31449 & 0.43693 & -0.044811 & 0.15842 & -0.017472 \\
\hline 0.479 & 0.12696 & 0.31449 & 0.43693 & 0.39607 & 0.15842 & -0.017472 \\
\hline -0.14036 & 0.12696 & -0.076317 & -0.10466 & -0.033752 & 0.15842 & -0.017472 \\
\hline 0.23039 & 0.37641 & 0.31449 & 0.43693 & -0.033752 & -0.19168 & -0.21206 \\
\hline 0.23039 & -0.031252 & -0.076317 & 0.43693 & -0.033752 & 0.15842 & 0.35551 \\
\hline 0.23039 & 0.12696 & -0.076317 & 0.43693 & -0.033752 & 0.15842 & -0.017472 \\
\hline
\end{tabular}

Also, notice that because the CustAge predictor for the first observation is NaN , the corresponding Scores output is NaN also.

Solution: To resolve this issue, use the formatpoints function with the name-value pair argument Missing. When using Missing, you can replace a predictor's NaN value according to three alternative criteria ('ZeroWoe', 'MinPoints', or 'MaxPoints').

For example, use Missing to replace the missing value with the 'MinPoints ' option. The row with the missing data now has a score corresponding to assigning it the minimum possible points for CustAge.
```

sc = formatpoints(sc,'Missing','MinPoints');
[Scores,Points] = score(sc,newdata)
PointsTable = displaypoints(sc);
PointsTable(1:7,:)

```

\section*{Scores =}
0.7074
1.4646
0.7662
1.5779
1.4535
1.8944
0.0872
0.9207
1.0399
0.8252

Points =
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline CustAge & ResStatus & EmpStatus & CustIncome & TmWBank & OtherCC & AMBalance \\
\hline -0.15894 & -0.031252 & -0.076317 & 0.43693 & 0.39607 & 0.15842 & -0.017472 \\
\hline 0.479 & 0.12696 & 0.31449 & 0.43693 & -0.033752 & 0.15842 & -0.017472 \\
\hline 0.21445 & -0.031252 & 0.31449 & 0.081611 & 0.39607 & -0.19168 & -0.017472 \\
\hline 0.23039 & 0.12696 & 0.31449 & 0.43693 & -0.044811 & 0.15842 & 0.35551 \\
\hline 0.479 & 0.12696 & 0.31449 & 0.43693 & -0.044811 & 0.15842 & -0.017472 \\
\hline 0.479 & 0.12696 & 0.31449 & 0.43693 & 0.39607 & 0.15842 & -0.017472 \\
\hline -0.14036 & 0.12696 & -0.076317 & -0.10466 & -0.033752 & 0.15842 & -0.017472 \\
\hline 0.23039 & 0.37641 & 0.31449 & 0.43693 & -0.033752 & -0.19168 & -0.21206 \\
\hline 0.23039 & -0.031252 & -0.076317 & 0.43693 & -0.033752 & 0.15842 & 0.35551 \\
\hline 0.23039 & 0.12696 & -0.076317 & 0.43693 & -0.033752 & 0.15842 & -0.017472 \\
\hline
\end{tabular}
\begin{tabular}{|c|c|c|}
\hline Predictors & Bin & Points \\
\hline 'CustAge ' & '[-Inf,33) ' & -0.15894 \\
\hline 'CustAge' & ' \([33,37)^{\prime}\) & -0.14036 \\
\hline 'CustAge' & ' \([37,40\) )' & -0.060323 \\
\hline 'CustAge' & ' \([40,46\) )' & 0.046408 \\
\hline 'CustAge' & ' \([46,48)^{\prime}\) & 0.21445 \\
\hline 'CustAge' & ' \([48,58)^{\prime}\) & 0.23039 \\
\hline 'CustAge' & '[58,Inf]' & 0.479 \\
\hline
\end{tabular}

Notice that the Scores output has a value for the first customer record because CustAge now has a value and the score can be calculated for the first customer record.

\section*{See Also}
autobinning|bindata|bininfo|creditscorecard|displaypoints|fitmodel | formatpoints |modifybins |modifypredictor|plotbins|predictorinfo| probdefault|score|setmodel|validatemodel

\section*{Related Examples}
- "Case Study for a Credit Scorecard Analysis" on page 8-101

\section*{More About}
- "About Credit Scorecards" on page 8-62
- "Credit Scorecard Modeling Workflow" on page 8-67

\section*{Case Study for a Credit Scorecard Analysis}

This example shows how to create a creditscorecard object, bin data, display, and plot binned data information. This example also shows how to fit a logistic regression model, obtain a score for the scorecard model, and determine the probabilities of default and validate the credit scorecard model using three different metrics.

\section*{Step 1. Create a creditscorecard object.}

Use the CreditCardData.mat file to load the data (using a dataset from Refaat 2011). By default, 'ResponseVar' is set to the last column in the data ('status' in this example) and the 'GoodLabel' to the response value with the highest count ( 0 in this example). The syntax for creditscorecard indicates that 'CustID' is the 'IDVar' to remove from the list of predictors. Also, while not demonstrated in this example, when creating a creditscorecard object using creditscorecard, you can use the optional name-value pair argument 'WeightsVar' to specify observation (sample) weights or 'BinMissingData' to bin missing data.
```

load CreditCardData
sc = creditscorecard(data,'IDVar','CustID')
sc =
creditscorecard with properties:

```
                        GoodLabel: 0
            ResponseVar: 'status'
                        WeightsVar:
                            VarNames: \{1x11 cell\}
            NumericPredictors: \{1x6 cell\}
        CategoricalPredictors: \{'ResStatus' 'EmpStatus' 'OtherCC'\}
            BinMissingData: 0
                            IDVar: 'CustID'
                            PredictorVars: \{1x9 cell\}
                            Data: [1200x11 table]

Perform some initial data exploration. Inquire about predictor statistics for the categorical variable 'ResStatus' and plot the bin information for 'ResStatus'.
```

bininfo(sc,'ResStatus')
ans=4\times6 table
Bin Good Bad Odds WOE InfoValue

```
\begin{tabular}{lr} 
'Home Owner' & 365 \\
'Tenant' & 307 \\
'Other' & 131 \\
'Totals' & 803 \\
& \\
& \\
&
\end{tabular}
\begin{tabular}{rrrr}
177 & 2.0621 & 0.019329 & 0.0001682 \\
167 & 1.8383 & -0.095564 & 0.0036638 \\
53 & 2.4717 & 0.20049 & 0.0059418 \\
397 & 2.0227 & \(N a N\) & 0.0097738
\end{tabular}
plotbins(sc,'ResStatus')

ResStatus

\(\square\) Good \(\square \mathrm{Bad}\)

This bin information contains the frequencies of "Good" and "Bad," and bin statistics. Avoid having bins with frequencies of zero because they lead to infinite or undefined ( NaN ) statistics. Use the modifybins or autobinning functions to bin the data accordingly.

For numeric data, a common first step is "fine classing." This means binning the data into several bins, defined with a regular grid. To illustrate this point, use the predictor 'CustIncome'.
```

cp = 20000:5000:60000;

```
sc = modifybins(sc,'CustIncome','CutPoints',cp);
bininfo(sc,'CustIncome')
ans=11×6 table
\begin{tabular}{|c|c|c|c|c|c|}
\hline Bin & Good & Bad & 0dds & WOE & InfoValue \\
\hline ' [-Inf, 20000) & 3 & 5 & 0.6 & -1.2152 & 0.010765 \\
\hline '[20000, 25000)' & 23 & 16 & 1.4375 & -0.34151 & 0.0039819 \\
\hline '[25000, 30000) ' & 38 & 47 & 0.80851 & -0.91698 & 0.065166 \\
\hline '[30000, 35000)' & 131 & 75 & 1.7467 & -0.14671 & 0.003782 \\
\hline '[35000, 40000) ' & 193 & 98 & 1.9694 & -0.026696 & 0.00017359 \\
\hline '[40000, 45000) ' & 173 & 76 & 2.2763 & 0.11814 & 0.0028361 \\
\hline ' [45000, 50000) ' & 131 & 47 & 2.7872 & 0.32063 & 0.014348 \\
\hline '[50000, 55000) ' & 82 & 24 & 3.4167 & 0.52425 & 0.021842 \\
\hline '[55000,60000)' & 21 & 8 & 2.625 & 0.26066 & 0.0015642 \\
\hline '[60000,Inf]' & 8 & 1 & 8 & 1.375 & 0.010235 \\
\hline 'Totals' & 803 & 397 & 2.0227 & NaN & 0.13469 \\
\hline
\end{tabular}
```

plotbins(sc,'CustIncome')

```


\section*{Step 2a. Automatically bin the data.}

Use the autobinning function to perform automatic binning for every predictor variable, using the default 'Monotone' algorithm with default algorithm options.
sc = autobinning(sc);
After the automatic binning step, every predictor bin must be reviewed using the bininfo and plotbins functions and fine-tuned. A monotonic, ideally linear trend in the Weight of Evidence (WOE) is desirable for credit scorecards because this translates into linear points for a given predictor. The WOE trends can be visualized using plotbins.
plotbins(sc,sc.PredictorVars)










Unlike the initial plot of 'ResStatus ' when the scorecard was created, the new plot for 'ResStatus ' shows an increasing WOE trend. This is because the autobinning function, by default, sorts the order of the categories by increasing odds.

These plots show that the 'Monotone' algorithm does a good job finding monotone WOE trends for this dataset. To complete the binning process, it is necessary to make only a few manual adjustments for some predictors using the modifybins function.

\section*{Step 2b. Fine-tune the bins using manual binning.}

Common steps to manually modify bins are:
- Use the bininfo function with two output arguments where the second argument contains binning rules.
- Manually modify the binning rules using the second output argument from bininfo.
- Set the updated binning rules with modifybins and then use plotbins or bininfo to review the updated bins.

For example, based on the plot for 'CustAge' in Step 2a, bins number 1 and 2 have similar WOE's as do bins number 5 and 6 . To merge these bins using the steps outlined above:
```

[bi,cp] = bininfo(sc,'CustAge');
cp([1 5]) = []; % To merge bins 1 and 2, and bins 5 and 6
sc = modifybins(sc,'CustAge','CutPoints',cp);
plotbins(sc,'CustAge')

```


For 'CustIncome' , based on the plot above, it is best to merge bins 3, 4 and 5 because they have similar WOE's. To merge these bins:
```

[bi,cp] = bininfo(sc,'CustIncome');
cp([3 4]) = [];
sc = modifybins(sc,'CustIncome','CutPoints',cp);
plotbins(sc,'CustIncome')

```


For 'TmWBank' , based on the plot above, it is best to merge bins 2 and 3 because they have similar WOE's. To merge these bins:
```

[bi,cp] = bininfo(sc,'TmWBank');

```
\(\mathrm{cp}(2)=[] ;\)
```

sc = modifybins(sc,'TmWBank','CutPoints',cp);
plotbins(sc,'TmWBank')

```

TmWBank


Good Bad

For 'AMBalance', based on the plot above, it is best to merge bins 2 and 3 because they have similar WOE's. To merge these bins:
```

[bi,cp] = bininfo(sc,'AMBalance');
cp(2) = [];
sc = modifybins(sc,'AMBalance','CutPoints',cp);
plotbins(sc,'AMBalance')

```


Now that the binning fine-tuning is completed, the bins for all predictors have close-tolinear WOE trends.

\section*{Step 3. Fit a logistic regression model.}

The fitmodel function fits a logistic regression model to the WOE data. fitmodel internally bins the training data, transforms it into WOE values, maps the response variable so that 'Good ' is 1 , and fits a linear logistic regression model. By default, fitmodel uses a stepwise procedure to determine which predictors should be in the model.
sc = fitmodel(sc);
1. Adding CustIncome, Deviance \(=1490.8954\), Chi2Stat \(=32.545914\), PValue \(=1.1640961 \mathrm{e}-(\)
2. Adding TmWBank, Deviance \(=1467.3249\), Chi2Stat \(=23.570535\), PValue \(=1.2041739 \mathrm{e}-06\)
3. Adding AMBalance, Deviance \(=1455.858\), Chi2Stat \(=11.466846\), PValue \(=0.00070848829\)
4. Adding EmpStatus, Deviance \(=1447.6148\), Chi2Stat \(=8.2432677\), PValue \(=0.0040903428\)
5. Adding CustAge, Deviance \(=1442.06\), Chi2Stat \(=5.5547849\), PValue \(=0.018430237\)
6. Adding ResStatus, Deviance \(=1437.9435\), Chi2Stat \(=4.1164321\), PValue \(=0.042468555\)
7. Adding OtherCC, Deviance \(=1433.7372\), Chi2Stat \(=4.2063597\), PValue \(=0.040272676\)

Generalized linear regression model:
status ~ [Linear formula with 8 terms in 7 predictors] Distribution = Binomial

Estimated Coefficients:
\begin{tabular}{|c|c|c|c|}
\hline Estimate & SE & tStat & pValue \\
\hline 0.7024 & 0.064 & 10.975 & 5.0407e-28 \\
\hline 0.61562 & 0.24783 & 2.4841 & 0.012988 \\
\hline 1.3776 & 0.65266 & 2.1107 & 0.034799 \\
\hline 0.88592 & 0.29296 & 3.024 & 0.0024946 \\
\hline 0.69836 & 0.21715 & 3.216 & 0.0013001 \\
\hline 1.106 & 0.23266 & 4.7538 & \(1.9958 \mathrm{e}-06\) \\
\hline 1.0933 & 0.52911 & 2.0662 & 0.038806 \\
\hline 1.0437 & 0.32292 & 3.2322 & 0.0012285 \\
\hline
\end{tabular}

1200 observations, 1192 error degrees of freedom
Dispersion: 1
Chi^2-statistic vs. constant model: 89.7, p-value \(=1.42 \mathrm{e}-16\)

\section*{Step 4. Review and format scorecard points.}

After fitting the logistic model, by default the points are unscaled and come directly from the combination of WOE values and model coefficients. The displaypoints function summarizes the scorecard points.
```

p1 = displaypoints(sc);
disp(p1)

| Predictors | Bin | Points |
| :---: | :---: | :---: |
| 'CustAge' | '[-Inf,37)' | -0.15314 |
| 'CustAge' | '[37,40)' | -0.062247 |
| 'CustAge' | ' $[40,46$ ) | 0.045763 |

```
\begin{tabular}{|c|c|c|}
\hline CustAge ' & ' \([46,58\) ) ' & 0.22888 \\
\hline 'CustAge' & '[58,Inf]' & 0.48354 \\
\hline 'ResStatus' & 'Tenant' & -0.031302 \\
\hline 'ResStatus' & 'Home Owner' & 0.12697 \\
\hline 'ResStatus' & 'Other' & 0.37652 \\
\hline 'EmpStatus' & 'Unknown' & -0.076369 \\
\hline 'EmpStatus' & 'Employed' & 0.31456 \\
\hline 'CustIncome' & '[-Inf, 29000)' & -0.45455 \\
\hline 'CustIncome' & ' \([29000,33000)\) ' & -0.1037 \\
\hline 'CustIncome' & '[33000, 42000)' & 0.077768 \\
\hline 'CustIncome' & '[42000, 47000)' & 0.24406 \\
\hline 'CustIncome' & '[47000,Inf]' & 0.43536 \\
\hline 'TmWBank' & '[-Inf,12)' & -0.18221 \\
\hline 'TmWBank' & ' \([12,45\) )' & -0.038279 \\
\hline 'TmWBank' & ' \([45,71\) )' & 0.39569 \\
\hline 'TmWBank' & '[71,Inf]' & 0.95074 \\
\hline 'OtherCC' & 'No' & -0.193 \\
\hline 'OtherCC' & 'Yes' & 0.15868 \\
\hline 'AMBalance' & '[-Inf, 558.88)' & 0.3552 \\
\hline 'AMBalance' & '[558.88, 1597.44)' & -0.026797 \\
\hline 'AMBalance ' & '[1597.44,Inf] & -0.21168 \\
\hline
\end{tabular}

This is a good time to modify the bin labels, if this is something of interest for cosmetic reasons. To do so, use modifybins to change the bin labels.
```

sc = modifybins(sc,'CustAge','BinLabels',...
{'Up to 36' '37 to 39' '40 to 45' '46 to 57' '58 and up'});
sc = modifybins(sc,'CustIncome','BinLabels',...
{'Up to 28999' '29000 to 32999' '33000 to 41999' '42000 to 46999' '47000 and up'});
sc = modifybins(sc,'TmWBank','BinLabels',...
{'Up to 11' '12 to 44' '45 to 70' '71 and up'});
sc = modifybins(sc,'AMBalance','BinLabels',...
{'Up to 558.87' '558.88 to 1597.43' '1597.44 and up'});
p1 = displaypoints(sc);
disp(p1)

| Predictors | Bin | Points |
| :---: | :---: | :---: |
| 'CustAge' | 'Up to 36' | -0.15314 |
| 'CustAge' | '37 to 39' | -0.062247 |

```

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\begin{tabular}{|c|c|c|}
\hline 'CustAge' & '40 to 45' & 0.045763 \\
\hline 'CustAge' & '46 to 57' & 0.22888 \\
\hline 'CustAge' & '58 and up' & 0.48354 \\
\hline 'ResStatus' & 'Tenant' & -0.031302 \\
\hline 'ResStatus' & 'Home Owner' & 0.12697 \\
\hline 'ResStatus' & 'Other' & 0.37652 \\
\hline 'EmpStatus' & 'Unknown' & -0.076369 \\
\hline 'EmpStatus' & 'Employed' & 0.31456 \\
\hline 'CustIncome' & 'Up to 28999' & -0.45455 \\
\hline 'CustIncome' & '29000 to 32999' & -0.1037 \\
\hline 'CustIncome' & '33000 to 41999' & 0.077768 \\
\hline 'CustIncome' & '42000 to 46999' & 0.24406 \\
\hline 'CustIncome' & '47000 and up' & 0.43536 \\
\hline 'TmWBank' & 'Up to 11' & -0.18221 \\
\hline 'TmWBank' & '12 to 44' & -0.038279 \\
\hline 'TmWBank' & '45 to 70' & 0.39569 \\
\hline 'TmWBank' & '71 and up' & 0.95074 \\
\hline 'OtherCC' & 'No' & -0.193 \\
\hline 'OtherCC' & 'Yes' & 0.15868 \\
\hline 'AMBalance' & 'Up to 558.87' & 0.3552 \\
\hline 'AMBalance' & '558.88 to 1597.43' & -0.026797 \\
\hline 'AMBalance' & '1597.44 and up' & -0.21168 \\
\hline
\end{tabular}

Points are usually scaled and also often rounded. To do this, use the formatpoints function. For example, you can set a target level of points corresponding to a target odds level and also set the required points-to-double-the-odds (PDO).
```

TargetPoints = 500;
TargetOdds = 2;
PDO = 50; % Points to double the odds

```
```

sc = formatpoints(sc,'PointsOddsAndPDO',[TargetPoints TargetOdds PDO]);

```
sc = formatpoints(sc,'PointsOddsAndPDO',[TargetPoints TargetOdds PDO]);
p2 = displaypoints(sc);
p2 = displaypoints(sc);
disp(p2)
```

disp(p2)

```
\begin{tabular}{|c|c|c|}
\hline Predictors & Bin & Points \\
\hline 'CustAge' & 'Up to 36' & 53.239 \\
\hline 'CustAge' & '37 to 39' & 59.796 \\
\hline 'CustAge' & '40 to 45' & 67.587 \\
\hline 'CustAge' & '46 to 57' & 80.796 \\
\hline 'CustAge' & '58 and up' & 99.166 \\
\hline 'ResStatus' & 'Tenant' & 62.028 \\
\hline 'ResStatus' & 'Home Owner' & 73. \\
\hline
\end{tabular}
```

'ResStatus'
'EmpStatus'
'EmpStatus'
'CustIncome'
'CustIncome'
'CustIncome'
'CustIncome'
'CustIncome'
'TmWBank'
'TmWBank'
'TmWBank'
'TmWBank'
'OtherCC'
'OtherCC'
'AMBalance'
'AMBalance'
'AMBalance'

```
'Other'
91.446
'Unknown
58.777
'Employed ' 86.976
'Up to 28999' 31.497
'29000 to 32999' 56.805
'33000 to 41999' 69.896
'42000 to 46999' 81.891
'47000 and up' 95.69
'Up to 11' 51.142
'12 to 44 ' 61.524
\begin{tabular}{ll} 
\\
\hline
\end{tabular} \(\mathbf{4 5}\) to 70 ' 92.829
'71 and up' 132.87
'No' 50.364
'Yes' 75.732
'Up to 558.87' 89.908
'558.88 to 1597.43' 62.353
'1597.44 and up' 49.016

\section*{Step 5. Score the data.}

The score function computes the scores for the training data. An optional data input can also be passed to score, for example, validation data. The points per predictor for each customer are provided as an optional output.
```

[Scores,Points] = score(sc);
disp(Scores(1:10))

```
    528.2044
554.8861
505.2406
564.0717
554.8861
586.1904
441.8755
515.8125
524.4553
508.3169
disp(Points(1:10,:))
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline CustAge & ResStatus & EmpStatus & CustIncome & TmWBank & OtherCC & AMBalance \\
\hline 80.796 & 62.028 & 58.777 & 95.69 & 92.829 & 75.732 & 62.353 \\
\hline 99.166 & 73.445 & 86.976 & 95.69 & 61.524 & 75.732 & 62.353 \\
\hline
\end{tabular}

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\begin{tabular}{lrrrrrr}
80.796 & 62.028 & 86.976 & 69.896 & 92.829 & 50.364 & 62.353 \\
80.796 & 73.445 & 86.976 & 95.69 & 61.524 & 75.732 & 89.908 \\
99.166 & 73.445 & 86.976 & 95.69 & 61.524 & 75.732 & 62.353 \\
99.166 & 73.445 & 86.976 & 95.69 & 92.829 & 75.732 & 62.353 \\
53.239 & 73.445 & 58.777 & 56.805 & 61.524 & 75.732 & 62.353 \\
80.796 & 91.446 & 86.976 & 95.69 & 61.524 & 50.364 & 49.016 \\
80.796 & 62.028 & 58.777 & 95.69 & 61.524 & 75.732 & 89.908 \\
80.796 & 73.445 & 58.777 & 95.69 & 61.524 & 75.732 & 62.353
\end{tabular}

\section*{Step 6. Calculate the probability of default.}

To calculate the probability of default, use the probdefault function.
```

pd = probdefault(sc);

```

Define the probability of being "Good" and plot the predicted odds versus the formatted scores. Visually analyze that the target points and target odds match and that the points-to-double-the-odds (PDO) relationship holds.
```

ProbGood = 1-pd;
PredictedOdds = ProbGood./pd;
figure
scatter(Scores,Predicted0dds)
title('Predicted Odds vs. Score')
xlabel('Score')
ylabel('Predicted Odds')
hold on
xLimits = xlim;
yLimits = ylim;
% Target points and odds
plot([TargetPoints TargetPoints],[yLimits(1) TargetOdds],'k:')
plot([xLimits(1) TargetPoints],[TargetOdds TargetOdds],'k:')
% Target points plus PDO
plot([TargetPoints+PDO TargetPoints+PDO],[yLimits(1) 2*TargetOdds],'k:')
plot([xLimits(1) TargetPoints+PD0],[2*Target0dds 2*Target0dds],'k:')
% Target points minus PDO
plot([TargetPoints-PDO TargetPoints-PDO],[yLimits(1) Target0dds/2],'k:')
plot([xLimits(1) TargetPoints-PDO],[TargetOdds/2 TargetOdds/2],'k:')

```
hold off


\section*{Step 7. Validate the credit scorecard model using the CAP, ROC, and KolmogorovSmirnov statistic}

The creditscorecard class supports three validation methods, the Cumulative Accuracy Profile (CAP), the Receiver Operating Characteristic (ROC), and the Kolmogorov-Smirnov (K-S) statistic. For more information on CAP, ROC, and KS, see "Cumulative Accuracy Profile (CAP)" on page 19-2240, "Receiver Operating Characteristic (ROC)" on page 19-2241, and "Kolmogorov-Smirnov statistic (KS)" on page 19-2241.
[Stats,T] = validatemodel(sc,'Plot',\{'CAP','ROC','KS'\});



\begin{abstract}
K-S Plot

disp(Stats)
\begin{tabular}{|c|c|}
\hline Measure & Value \\
\hline 'Accuracy Ratio' & 0.32225 \\
\hline 'Area under ROC curve' & 0.66113 \\
\hline 'KS statistic' & 0.22324 \\
\hline 'KS score' & 499.18 \\
\hline
\end{tabular}
```

disp(T(1:15,:))

```

Scores ProbDefault
TrueBads
FalseBads
TrueGoods
FalseGoods
\end{abstract}
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline 369.4 & 0.7535 & 0 & 1 & 802 & 397 & \\
\hline 377.86 & 0.73107 & 1 & 1 & 802 & 396 & 0.00 \\
\hline 379.78 & 0.7258 & 2 & 1 & 802 & 395 & 0.00 \\
\hline 391.81 & 0.69139 & 3 & 1 & 802 & 394 & 0.00 \\
\hline 394.77 & 0.68259 & 3 & 2 & 801 & 394 & 0.00 \\
\hline 395.78 & 0.67954 & 4 & 2 & 801 & 393 & 0.0 \\
\hline 396.95 & 0.67598 & 5 & 2 & 801 & 392 & 0.0 \\
\hline 398.37 & 0.67167 & 6 & 2 & 801 & 391 & 0.0 \\
\hline 401.26 & 0.66276 & 7 & 2 & 801 & 390 & 0.0 \\
\hline 403.23 & 0.65664 & 8 & 2 & 801 & 389 & 0.0 \\
\hline 405.09 & 0.65081 & 8 & 3 & 800 & 389 & 0.0 \\
\hline 405.15 & 0.65062 & 11 & 5 & 798 & 386 & 0.0 \\
\hline 405.37 & 0.64991 & 11 & 6 & 797 & 386 & 0.0 \\
\hline 406.18 & 0.64735 & 12 & 6 & 797 & 385 & 0.0 \\
\hline 407.14 & 0.64433 & 13 & 6 & 797 & 384 & 0.0 \\
\hline
\end{tabular}

\section*{See Also}
autobinning|bindata|bininfo|creditscorecard|displaypoints|fitmodel | formatpoints | modifybins | modifypredictor|plotbins | predictorinfo|
probdefault|score|setmodel|validatemodel

\section*{Related Examples}
- "Troubleshooting Credit Scorecard Results" on page 8-91
- "Credit Rating by Bagging Decision Trees" (Statistics and Machine Learning Toolbox)

\section*{More About}
- "About Credit Scorecards" on page 8-62
- "Credit Scorecard Modeling Workflow" on page 8-67
- "Credit Scorecard Modeling Using Observation Weights" on page 8-70
- Monotone Adjacent Pooling Algorithm (MAPA) on page 19-2190

\section*{External Websites}
- Credit Risk Modeling with MATLAB (53 min 10 sec )

\section*{Credit Default Swap (CDS)}

A credit default swap (CDS) is a contract that protects against losses resulting from credit defaults. The transaction involves two parties, the protection buyer and the protection seller, and also a reference entity, usually a bond. The protection buyer pays a stream of premiums to the protection seller, who in exchange offers to compensate the buyer for the loss in the bond's value if a credit event occurs. The stream of premiums is called the premium leg, and the compensation when a credit event occurs is called the protection leg. Credit events usually include situations in which the bond issuer goes bankrupt, misses coupon payments, or enters a restructuring process. Financial Instruments Toolbox \({ }^{\text {TM }}\) software supports:

\section*{CDS Functions}
\begin{tabular}{|l|l|}
\hline Function & Purpose \\
\hline cdsbootstrap & \begin{tabular}{l} 
Compute default probability parameters from CDS market \\
quotes.
\end{tabular} \\
\hline cdsspread & Compute breakeven spreads for the CDS contracts. \\
\hline cdsprice & Compute the price for the CDS contracts. \\
\hline
\end{tabular}

\section*{See Also}
cdsbootstrap|cdsprice|cdsrpv01|cdsspread

\section*{Related Examples}
- "First-to-Default Swaps" (Financial Instruments Toolbox)
- "Credit Default Swap Option" (Financial Instruments Toolbox)
- "Counterparty Credit Risk and CVA" (Financial Instruments Toolbox)

\section*{External Websites}
- Pricing and Valuation of Credit Default Swaps (4 min 22 sec)

\section*{Bootstrapping a Default Probability Curve}

This example shows how to bootstrap default probabilities from CDS market quotes. To bootstrap default probabilities from bond market data, see bondDefaultBootstrap. In a typical workflow, pricing a new CDS contract involves first estimating a default probability term structure using cdsbootstrap. This requires market quotes of existing CDS contracts, or quotes of CDS indices (e.g., iTraxx). The estimated default probability curve is then used as input to cdsspread or cdsprice. If the default probability information is already known, cdsbootstrap can be bypassed and cdsspread or cdsprice can be called directly.

The market information in this example is provided in the form of running spreads of CDS contracts maturing on the CDS standard payment dates closest to \(1,2,3,5\), and 7 years from the valuation date.
```

Settle = '17-Jul-2009'; % valuation date for the CDS
MarketDates = datenum({'20-Sep-10','20-Sep-11','20-Sep-12','20-Sep-14',....
'20-Sep-16'});
MarketSpreads = [140 175 210 265 310]';
MarketData = [MarketDates MarketSpreads];
ZeroDates = datenum({'17-Jan-10','17-Jul-10','17-Jul-11','17-Jul-12', ...
'17-Jul-13','17-Jul-14'});
ZeroRates = [1.35 1.43 1.9 2.47 2.936 3.311]'/100;
ZeroData = [ZeroDates ZeroRates];
[ProbData,HazData] = cdsbootstrap(ZeroData,MarketData,Settle);

```

The bootstrapped default probability curve is plotted against time, in years, from the valuation date.
```

ProbTimes = yearfrac(Settle,ProbData(:,1));
figure
plot([0; ProbTimes],[0; ProbData(:,2)])
grid on
axis([0 ProbTimes(end,1) 0 ProbData(end,2)])
xlabel('Time (years)')
ylabel('Cumulative Default Probability')
title('Bootstrapped Default Probability Curve')

```


The associated hazard rates are returned as an optional output. The convention is that the first hazard rate applies from the settlement date to the first market date, the second hazard rate from the first to the second market date, and so on, and the last hazard rate applies from the second-to-last market date onwards. The following plot displays the bootstrapped hazard rates, plotted against time, in years, from the valuation date:
```

HazTimes = yearfrac(Settle,HazData(:,1));
figure
stairs([0; HazTimes(1:end-1,1); HazTimes(end,1)+1],...
[HazData(:,2);HazData(end,2)])
grid on
axis([0 HazTimes(end,1)+1 0.9*HazData(1,2) 1.1*HazData(end,2)])
xlabel('Time (years)')
ylabel('Hazard Rate')
title('Bootstrapped Hazard Rates')

```


\section*{See Also}
bondDefaultBootstrap|cdsbootstrap|cdsprice|cdsrpv01|cdsspread

\section*{Related Examples}
- "First-to-Default Swaps" (Financial Instruments Toolbox)
- "Credit Default Swap Option" (Financial Instruments Toolbox)
- "Counterparty Credit Risk and CVA" (Financial Instruments Toolbox)

\section*{External Websites}
- Pricing and Valuation of Credit Default Swaps (4 min 22 sec )

\section*{Finding Breakeven Spread for New CDS Contract}

The breakeven, or running, spread is the premium a protection buyer must pay, with no upfront payments involved, to receive protection for credit events associated to a given reference entity. Spreads are expressed in basis points (bp). There is a notional amount associated to the CDS contract to determine the monetary amounts of the premium payments.

New quotes for CDS contracts can be obtained with cdsspread. After obtaining a default probability curve using cdsbootstrap, you get quotes that are consistent with current market conditions.

In this example, instead of standard CDS payment dates, define new maturity dates. Using the period from three to five years (CDS standard dates), maturities are defined within this range spaced at quarterly intervals (measuring time from the valuation date):
```

Settle = '17-Jul-2009'; % valuation date for the CDS
MarketDates = datenum({'20-Sep-10','20-Sep-11','20-Sep-12','20-Sep-14',...
'20-Sep-16'});
MarketSpreads = [140 175 210 265 310]';
MarketData = [MarketDates MarketSpreads];
ZeroDates = datenum({'17-Jan-10','17-Jul-10','17-Jul-11','17-Jul-12' ,...
'17-Jul-13','17-Jul-14'});
ZeroRates = [1.35 1.43 1.9 2.47 2.936 3.311]'/100;
ZeroData = [ZeroDates ZeroRates];
[ProbData,HazData] = cdsbootstrap(ZeroData,MarketData,Settle);
Maturity1 = datestr(daysadd('17-Jul-09',360*(3.25:0.25:5),1));
Spread1 = cdsspread(ZeroData,ProbData,Settle,Maturity1);
figure
scatter(yearfrac(Settle,Maturity1),Spread1,'*')
hold on
scatter(yearfrac(Settle,MarketData(3:4,1)),MarketData(3:4,2))
hold off
grid on
xlabel('Time (years)')
ylabel('Spread (bp)')
title('CDS Spreads')
legend('New Quotes','Market','location','SouthEast')

```

This plot displays the resulting spreads:


To evaluate the effect of the recovery rate parameter, instead of \(40 \%\) (default value), use a recovery rate of \(35 \%\) :
```

Spread1Rec35 = cdsspread(ZeroData,ProbData,Settle,Maturity1,...
'RecoveryRate',0.35);
figure
plot(yearfrac(Settle,Maturity1),Spread1,...
yearfrac(Settle,Maturity1),Spread1Rec35,' --')
grid on
xlabel('Time (years)')
ylabel('Spread (bp)')
title('CDS Spreads with Different Recovery Rates')
legend('40%','35%','location','SouthEast')

```

The resulting plot shows that smaller recovery rates produce higher premia, as expected, since in the event of default, the protection payments are higher:


\section*{See Also}
cdsbootstrap|cdsprice|cdsrpv01|cdsspread

\section*{Related Examples}
- "First-to-Default Swaps" (Financial Instruments Toolbox)
- "Credit Default Swap Option" (Financial Instruments Toolbox)
- "Counterparty Credit Risk and CVA" (Financial Instruments Toolbox)

\section*{External Websites}
- \(\quad\) Pricing and Valuation of Credit Default Swaps (4 min 22 sec )

\section*{Valuing an Existing CDS Contract}

The current value, or mark-to-market, of an existing CDS contract is the amount of money the contract holder would receive (if positive) or pay (if negative) to unwind this contract. The upfront of the contract is the current value expressed as a fraction of the notional amount of the contract, and it is commonly used to quote market values.

The value of existing CDS contracts is obtained with cdsprice. By default, cdsprice treats contracts as long positions. Whether a contract position is long or short is determined from a protection standpoint, that is, long means that protection was bought, and short means protection was sold. In the following example, an existing CDS contract pays a premium that is lower than current market conditions. The price is positive, as expected, since it would be more costly to buy the same type of protection today.
```

Settle = '17-Jul-2009'; % valuation date for the CDS
MarketDates = datenum({'20-Sep-10','20-Sep-11','20-Sep-12','20-Sep-14',...
'20-Sep-16'});
MarketSpreads = [140 175 210 265 310]';
MarketData = [MarketDates MarketSpreads];
ZeroDates = datenum({'17-Jan-10','17-Jul-10','17-Jul-11','17-Jul-12',...
'17-Jul-13','17-Jul-14'});
ZeroRates = [1.35 1.43 1.9 2.47 2.936 3.311]'/100;
ZeroData = [ZeroDates ZeroRates];
[ProbData,HazData] = cdsbootstrap(ZeroData,MarketData,Settle);
Maturity2 = '20-Sep-2012';
Spread2 = 196;
[Price,AccPrem,PaymentDates,PaymentTimes,PaymentCF] = cdsprice(ZeroData,...
ProbData,Settle,Maturity2,Spread2)
fprintf('Dirty Price: %8.2f\n',Price);
fprintf('Accrued Premium: %8.2f\n',AccPrem);
fprintf('Clean Price: %8.2f\n',Price-AccPrem);
fprintf('\nPayment Schedule:\n\n');
fprintf('Date \t\t Time Frac \t Amount\n');
for k = 1:length(PaymentDates)
fprintf('%s \t %5.4f \t %8.2f\n',datestr(PaymentDates(k)),...
PaymentTimes(k),PaymentCF(k));
end

```

This resulting payment schedule is:
```

Dirty Price: 41628.50

```
Accrued Premium: 15244.44
Clean Price: 26384.05

Payment Schedule:
\begin{tabular}{lrl} 
Date & Time Frac & Amount \\
20-Sep-2009 & 0.1806 & 35388.89 \\
20-Dec-2009 & 0.2528 & 49544.44 \\
20-Mar-2010 & 0.2500 & 49000.00 \\
20-Jun-2010 & 0.2556 & 50088.89 \\
20-Sep-2010 & 0.2556 & 50088.89 \\
20-Dec-2010 & 0.2528 & 49544.44 \\
20-Mar-2011 & 0.2500 & 49000.00 \\
20-Jun-2011 & 0.2556 & 50088.89 \\
20-Sep-2011 & 0.2556 & 50088.89 \\
20-Dec-2011 & 0.2528 & 49544.44 \\
20-Mar-2012 & 0.2528 & 49544.44 \\
20-Jun-2012 & 0.2556 & 50088.89 \\
20-Sep-2012 & 0.2556 & 50088.89
\end{tabular}

Also, you can use cdsprice to value a portfolio of CDS contracts. In the following example, a simple hedged position with two vanilla CDS contracts, one long, one short, with slightly different spreads is priced in a single call and the value of the portfolio is the sum of the returned prices:
```

[Price2,AccPrem2] = cdsprice(ZeroData,ProbData,Settle,...
repmat(datenum(Maturity2),2,1),[Spread2;Spread2+3],...
'Notional',[1e7; -1e7]);
fprintf('Contract \t Dirty Price \t Acc Premium \t Clean Price\n');
fprintf(' Long \t \$ %9.2f \t \$ %9.2f \t \$ %9.2f \t\n',...
Price2(1), AccPrem2(1), Price2(1) - AccPrem2(1));
fprintf(' Short \t \$ %8.2f \t \$ %8.2f \t \$ %8.2f \t\n',...
Price2(2), AccPrem2(2), Price2(2) - AccPrem2(2));
fprintf('Mark-to-market of hedged position: \$ %8.2f\n',sum(Price2));

```

This resulting value of the portfolio is:
\begin{tabular}{rlll} 
Contract & Dirty Price & Acc Premium & Clean Price \\
Long & \(\$ 41628.50\) & \(\$ 15244.44\) & \(\$ 26384.05\) \\
Short & \(\$-32708.11\) & \(\$-15477.78\) & \(\$-17230.33\)
\end{tabular}

Mark-to-market of hedged position: \$ 8920.39

\section*{See Also}
cdsbootstrap|cdsprice|cdsrpv01|cdsspread

\section*{Related Examples}
- "First-to-Default Swaps" (Financial Instruments Toolbox)
- "Credit Default Swap Option" (Financial Instruments Toolbox)
- "Counterparty Credit Risk and CVA" (Financial Instruments Toolbox)

\section*{External Websites}
- \(\quad\) Pricing and Valuation of Credit Default Swaps (4 min 22 sec )

\section*{Converting from Running to Upfront}

A CDS market quote is given in terms of a standard spread (usually 100 bp or 500 bp ) and an upfront payment, or in terms of an equivalent running or breakeven spread, with no upfront payment. The functions cdsbootstrap, cdsspread, and cdsprice perform upfront to running or running to upfront conversions.

For example, to convert the market quotes to upfront quotes for a standard spread of 100 bp:
```

Settle = '17-Jul-2009'; % valuation date for the CDS
MarketDates = datenum({'20-Sep-10','20-Sep-11','20-Sep-12','20-Sep-14',...
'20-Sep-16'});
MarketSpreads = [140 175 210 265 310]';
MarketData = [MarketDates MarketSpreads];
ZeroDates = datenum({'17-Jan-10','17-Jul-10','17-Jul-11','17-Jul-12',...
'17-Jul-13', '17-Jul-14'});
ZeroRates = [1.35 1.43 1.9 2.47 2.936 3.311]'/100;
ZeroData = [ZeroDates ZeroRates];
[ProbData,HazData] = cdsbootstrap(ZeroData,MarketData,Settle);
Maturity3 = MarketData(:,1);
Spread3Run = MarketData(:,2);
Spread3Std = 100*ones(size(Maturity3));
Price3 = cdsprice(ZeroData,ProbData,Settle,Maturity3,Spread3Std);
Upfront3 = Price3/10000000; % Standard notional of 10MM
display(Upfront3);

```

This resulting value is:
```

Upfront3 =
0.0047
0.0158
0.0327
0.0737
0.1182

```

The conversion can be reversed to convert upfront quotes to market quotes:
```

ProbData3Upf = cdsbootstrap(ZeroData,[Maturity3 Upfront3 Spread3Std],Settle);
Spread3RunFromUpf = cdsspread(ZeroData,ProbData3Upf,Settle,Maturity3);

```
display([Spread3Run Spread3RunFromUpf]);

Comparing the results of this conversion to the original market spread demonstrates the reversal:
\begin{tabular}{ll} 
ans \(=\) \\
140.0000 & 140.0000 \\
175.0000 & 175.0000 \\
210.0000 & 210.0000 \\
265.0000 & 265.0000 \\
310.0000 & 310.0000
\end{tabular}

Under the flat-hazard rate (FHR) quoting convention, a single market quote is used to calibrate a probability curve. This convention yields a single point in the probability curve, and a single hazard rate value. For example, assume a four-year (standard dates) CDS contract with a current FHR-based running spread of 550 bp needs conversion to a CDS contract with a standard spread of 500 bp :
```

Maturity4 = datenum('20-Sep-13');
Spread4Run = 550;
ProbData4Run = cdsbootstrap(ZeroData,[Maturity4 Spread4Run],Settle);
Spread4Std = 500;
Price4 = cdsprice(ZeroData,ProbData4Run,Settle,Maturity4,Spread4Std);
Upfront4 = Price4/10000000;
fprintf('A running spread of %5.2f is equivalent to\n',Spread4Run);
fprintf(' a standard spread of %5.2f with an upfront of %8.7f\n',...
Spread4Std,Upfront4);
A running spread of 550.00 is equivalent to
a standard spread of 500.00 with an upfront of 0.0167576

```

To reverse the conversion:
```

ProbData4Upf = cdsbootstrap(ZeroData,[Maturity4 Upfront4 Spread4Std],Settle);
Spread4RunFromUpf = cdsspread(ZeroData,ProbData4Upf,Settle,Maturity4);
fprintf('A standard spread of %5.2f with an upfront of %8.7f\n',...
Spread4Std,Upfront4);
fprintf(' is equivalent to a running spread of %5.2f\n',Spread4RunFromUpf);
A standard spread of 500.00 with an upfront of 0.0167576
is equivalent to a running spread of 550.00

```

As discussed in Beumee et. al., 2009 (see "Credit Derivatives" on page A-7), the FHR approach is a quoting convention only, and leads to quotes inconsistent with market data. For example, calculating the upfront for the three-year (standard dates) CDS contract with a standard spread of 100 bp using the FHR approach and comparing the results to the upfront amounts previously calculated, demonstrates that the FHR-based approach yields a different upfront amount:
```

Maturity5 = MarketData(3,1);
Spread5Run = MarketData(3,2);
ProbData5Run = cdsbootstrap(ZeroData,[Maturity5 Spread5Run],Settle);

```
```

Spread5Std = 100;
Price5 = cdsprice(ZeroData,ProbData5Run,Settle,Maturity5,Spread5Std);
Upfront5 = Price5/10000000;
fprintf('Relative error of FHR-based upfront amount: %3.1f%%\n',...
((Upfront5-Upfront3(3))/Upfront3(3))*100);
Relative error of FHR-based upfront amount: -0.8%

```

\section*{See Also}
cdsbootstrap|cdsprice|cdsrpv01|cdsspread

\section*{Related Examples}
- "First-to-Default Swaps" (Financial Instruments Toolbox)
- "Credit Default Swap Option" (Financial Instruments Toolbox)
- "Counterparty Credit Risk and CVA" (Financial Instruments Toolbox)

\section*{External Websites}
- \(\quad\) Pricing and Valuation of Credit Default Swaps (4 min 22 sec )

\section*{Bootstrapping from Inverted Market Curves}

The following two examples demonstrate the behavior of bootstrapping with inverted CDS market curves, that is, market quotes with higher spreads for short-term CDS contracts. The first example is handled normally by cdsbootstrap:
```

Settle = '17-Jul-2009'; % valuation date for the CDS
MarketDates = datenum({'20-Sep-10','20-Sep-11','20-Sep-12','20-Sep-14',···..
'20-Sep-16'});
ZeroDates = datenum({'17-Jan-10','17-Jul-10','17-Jul-11','17-Jul-12', ....
'17-Jul-13','17-Jul-14'});
ZeroRates = [1.35 1.43 1.9 2.47 2.936 3.311]'/100;
ZeroData = [ZeroDates ZeroRates];
MarketSpreadsInv1 = [750 650 550 500 450]';
MarketDataInv1 = [MarketDates MarketSpreadsInv1];
[ProbDataInv1,HazDataInv1] = cdsbootstrap(ZeroData,MarketDataInv1,Settle)
ProbDataInv1 =
1.0e+05 *

| 7.3440 | 0.0000 |
| :--- | :--- |
| 7.3477 | 0.0000 |
| 7.3513 | 0.0000 |
| 7.3586 | 0.0000 |
| 7.3659 | 0.0000 |

HazDataInv1 =
$1.0 \mathrm{e}+05$ *

| 7.3440 | 0.0000 |
| :--- | :--- |
| 7.3477 | 0.0000 |
| 7.3513 | 0.0000 |
| 7.3586 | 0.0000 |
| 7.3659 | 0.0000 |

```

In the second example, cdsbootstrap generates a warning:
```

MarketSpreadsInv2 = [800 550 400 250 100]';
MarketDataInv2 = [MarketDates MarketSpreadsInv2];
[ProbDataInv2,HazDataInv2] = cdsbootstrap(ZeroData,MarketDataInv2,Settle);
Warning: Found non-monotone default probabilities (negative hazard rates)

```

A non-monotone bootstrapped probability curve implies negative default probabilities and negative hazard rates for certain time intervals. Extreme market conditions can lead to these types of situations. In these cases, you must assess the reliability and usefulness of the bootstrapped results.

The following plot illustrates these bootstrapped probability curves. The curves are concave, meaning that the marginal default probability decreases with time. This result is consistent with the market information that indicates a higher default risk in the short term. The second bootstrapped curve is non-monotone, as indicated by the warning.
```

ProbTimes = yearfrac(Settle, MarketDates);
figure
plot([0; ProbTimes],[0; ProbDataInv1(:,2)])
hold on
plot([0; ProbTimes],[0; ProbDataInv2(:,2)],'--')
hold off
grid on
axis([0 ProbTimes(end,1) 0 ProbDataInv1(end,2)])
xlabel('Time (years)')
ylabel('Cumulative Default Probability')
title('Probability Curves for Inverted Spread Curves')
legend('1st instance','2nd instance','location','SouthEast')

```

The resulting plot


Also in line with the previous plot, the hazard rates for these bootstrapped curves are decreasing because the short-term risk is higher. Some bootstrapped parameters in the second curve are negative, as indicated by the warning.
```

HazTimes = yearfrac(Settle, MarketDates);
figure
stairs([0; HazTimes(1:end-1,1); HazTimes(end,1)+1],...
[HazDataInv1(:,2);HazDataInv1(end,2)])
hold on
stairs([0; HazTimes(1:end-1,1); HazTimes(end,1)+1],...
[HazDataInv2(:,2);HazDataInv2(end,2)],'--')
hold off
grid on
xlabel('Time (years)')
ylabel('Hazard Rate')
title('Hazard Rates for Inverted Spread Curves')
legend('1st instance','2nd instance','location','NorthEast')

```

The resulting plot shows the hazard rates for both bootstrapped curves:


For further discussion on inverted curves, and their relationship to arbitrage, see O'Kane and Turnbull, 2003 ("Credit Derivatives" on page A-7).

\section*{See Also}
cdsbootstrap|cdsprice|cdsrpv01|cdsspread

\section*{Related Examples}
- "First-to-Default Swaps" (Financial Instruments Toolbox)
- "Credit Default Swap Option" (Financial Instruments Toolbox)
- "Counterparty Credit Risk and CVA" (Financial Instruments Toolbox)

\section*{External Websites}
- \(\quad\) Pricing and Valuation of Credit Default Swaps (4 min 22 sec )

\section*{Regression with Missing Data}
- "Multivariate Normal Regression" on page 9-2
- "Maximum Likelihood Estimation with Missing Data" on page 9-8
- "Multivariate Normal Regression Functions" on page 9-12
- "Multivariate Normal Regression Types" on page 9-16
- "Troubleshooting Multivariate Normal Regression" on page 9-23
- "Portfolios with Missing Data" on page 9-27
- "Valuation with Missing Data" on page 9-33

\section*{Multivariate Normal Regression}
```

In this section...
"Introduction" on page 9-2
"Multivariate Normal Linear Regression" on page 9-2
"Maximum Likelihood Estimation" on page 9-3
"Special Case of Multiple Linear Regression Model" on page 9-4
"Least-Squares Regression" on page 9-5
"Mean and Covariance Estimation" on page 9-5
"Convergence" on page 9-5
"Fisher Information" on page 9-6
"Statistical Tests" on page 9-6

```

\section*{Introduction}

This section focuses on using likelihood-based methods for multivariate normal regression. The parameters of the regression model are estimated via maximum likelihood estimation. For multiple series, this requires iteration until convergence. The complication due to the possibility of missing data is incorporated into the analysis with a variant of the EM algorithm known as the ECM algorithm.

The underlying theory of maximum likelihood estimation and the definition and significance of the Fisher information matrix can be found in Caines [1] and Cramér [2]. The underlying theory of the ECM algorithm can be found in Meng and Rubin [8] and Sexton and Swensen [9].

In addition, these two examples of maximum likelihood estimation are presented:
- "Portfolios with Missing Data" on page 9-27
- "Estimation of Some Technology Stock Betas" on page 9-35

\section*{Multivariate Normal Linear Regression}

Suppose that you have a multivariate normal linear regression model in the form
\[
\left[\begin{array}{c}
Z_{1} \\
\vdots \\
Z_{m}
\end{array}\right] \sim N\left(\left[\begin{array}{c}
H_{1} b \\
\vdots \\
H_{m} b
\end{array}\right],\left[\begin{array}{ccc}
C & & 0 \\
& \ddots & \\
0 & & C
\end{array}\right]\right)
\]
where the model has \(m\) observations of \(n\)-dimensional random variables \(\mathbf{Z}_{1}, \ldots, \mathbf{Z}_{m}\) with a linear regression model that has a \(p\)-dimensional model parameter vector \(\mathbf{b}\). In addition, the model has a sequence of \(m\) design matrices \(\mathbf{H}_{1}, \ldots, \mathbf{H}_{m}\), where each design matrix is a known \(n\)-by- \(p\) matrix.

Given a parameter vector \(\mathbf{b}\) and a collection of design matrices, the collection of \(m\) independent variables \(\mathbf{Z}_{k}\) is assumed to have independent identically distributed multivariate normal residual errors \(\mathbf{Z}_{k}-\mathbf{H}_{k} \mathbf{b}\) with \(n\)-vector mean \(\mathbf{0}\) and n-by-n covariance matrix \(\mathbf{C}\) for each \(k=1, \ldots, m\).

A concise way to write this model is
\[
Z_{k} \sim N\left(H_{k} b, C\right)
\]
for \(k=1, \ldots, m\).
The goal of multivariate normal regression is to obtain maximum likelihood estimates for \(\mathbf{b}\) and \(\mathbf{C}\) given a collection of \(m\) observations \(\mathbf{z}_{1}, \ldots, \mathbf{z}_{m}\) of the random variables \(\mathbf{Z}_{1}, \ldots, \mathbf{Z}_{m}\). The estimated parameters are the \(p\) distinct elements of \(\mathbf{b}\) and the \(n(n+1) / 2\) distinct elements of \(\mathbf{C}\) (the lower-triangular elements of \(\mathbf{C}\) ).

Note Quasi-maximum likelihood estimation works with the same models but with a relaxation of the assumption of normally distributed residuals. In this case, however, the parameter estimates are asymptotically optimal.

\section*{Maximum Likelihood Estimation}

To estimate the parameters of the multivariate normal linear regression model using maximum likelihood estimation, it is necessary to maximize the log-likelihood function over the estimation parameters given observations \(\mathbf{z}_{1}, \ldots, \mathbf{z}_{m}\).

Given the multivariate normal model to characterize residual errors in the regression model, the log-likelihood function is
\[
\begin{aligned}
L\left(z_{1}, \ldots, z_{m} ; b, C\right)= & \frac{1}{2} m n \log (2 \pi)+\frac{1}{2} m \log (\operatorname{det}(C)) \\
& +\frac{1}{2} \sum_{k=1}^{m}\left(z_{k}-H_{k} b\right)^{T} C^{-1}\left(z_{k}-H_{k} b\right) .
\end{aligned}
\]

Although the cross-sectional residuals must be independent, you can use this loglikelihood function for quasi-maximum likelihood estimation. In this case, the estimates for the parameters \(\mathbf{b}\) and \(\mathbf{C}\) provide estimates to characterize the first and second moments of the residuals. See Caines [1] for details.

Except for a special case (see "Special Case of Multiple Linear Regression Model" on page 9-4), if both the model parameters in \(\mathbf{b}\) and the covariance parameters in \(\mathbf{C}\) are to be estimated, the estimation problem is intractably nonlinear and a solution must use iterative methods. Denote estimates for the parameters \(\mathbf{b}\) and \(\mathbf{C}\) for iteration \(t=0,1, \ldots\) with the superscript notation \(\mathbf{b}^{(t)}\) and \(\mathbf{C}^{(t)}\).

Given initial estimates \(\mathbf{b}^{(0)}\) and \(\mathbf{C}^{(0)}\) for the parameters, the maximum likelihood estimates for \(\mathbf{b}\) and \(\mathbf{C}\) are obtained using a two-stage iterative process with
\[
b^{(t+1)}=\left(\sum_{k=1}^{m} H_{k}^{T}\left(C^{(t)}\right)^{-1} H_{k}\right)^{-1}\left(\sum_{k=1}^{m} H_{k}^{T}\left(C^{(t)}\right)^{-1} z_{k}\right)
\]
and
\[
C^{(t+1)}=\frac{1}{m} \sum_{k=1}^{m}\left(z_{k}-H_{k} b^{(t+1)}\right)\left(z_{k}-H_{k} b^{(t+1)}\right)^{T}
\]
for \(t=0,1, \ldots\).

\section*{Special Case of Multiple Linear Regression Model}

The special case mentioned in "Maximum Likelihood Estimation" on page 9-3 occurs if \(n\) \(=1\) so that the sequence of observations is a sequence of scalar observations. This model is known as a multiple linear regression model. In this case, the covariance matrix \(\mathbf{C}\) is a 1-by-1 matrix that drops out of the maximum likelihood iterates so that a single-step estimate for \(\mathbf{b}\) and \(\mathbf{C}\) can be obtained with converged estimates \(\mathbf{b}^{(1)}\) and \(\mathbf{C}^{(1)}\).

\section*{Least-Squares Regression}

Another simplification of the general model is called least-squares regression. If \(\mathbf{b}^{(0)}=\mathbf{0}\) and \(\mathbf{C}^{(0)}=\mathbf{I}\), then \(\mathbf{b}^{(1)}\) and \(\mathbf{C}^{(1)}\) from the two-stage iterative process are least-squares estimates for \(\mathbf{b}\) and \(\mathbf{C}\), where
\[
b^{L S}=\left(\sum_{k=1}^{m} H_{k}{ }^{T} H_{k}\right)^{-1}\left(\sum_{k=1}^{m} H_{k}{ }^{T} z_{k}\right)
\]
and
\[
C^{L S}=\frac{1}{m} \sum_{k=1}^{m}\left(z_{k}-H_{k} b^{L S}\right)\left(z_{k}-H_{k} b^{L S}\right)^{T}
\]

\section*{Mean and Covariance Estimation}

A final simplification of the general model is to estimate the mean and covariance of a sequence of \(n\)-dimensional observations \(\mathbf{z}_{1}, \ldots, \mathbf{z}_{m}\). In this case, the number of series is equal to the number of model parameters with \(n=p\) and the design matrices are identity matrices with \(\mathbf{H}_{k}=\mathbf{I}\) for \(i=1, \ldots, m\) so that \(\mathbf{b}\) is an estimate for the mean and \(\mathbf{C}\) is an estimate of the covariance of the collection of observations \(\mathbf{z}_{1}, \ldots, \mathbf{z}_{m}\).

\section*{Convergence}

If the iterative process continues until the log-likelihood function increases by no more than a specified amount, the resultant estimates are said to be maximum likelihood estimates \(\mathbf{b}^{M L}\) and \(\mathbf{C}^{M L}\).

If \(n=1\) (which implies a single data series), convergence occurs after only one iterative step, which, in turn, implies that the least-squares and maximum likelihood estimates are identical. If, however, \(n>1\), the least-squares and maximum likelihood estimates are usually distinct.

In Financial Toolbox software, both the changes in the log-likelihood function and the norm of the change in parameter estimates are monitored. Whenever both changes fall below specified tolerances (which should be something between machine precision and its square root), the toolbox functions terminate under an assumption that convergence has been achieved.

\section*{Fisher Information}

Since maximum likelihood estimates are formed from samples of random variables, their estimators are random variables; an estimate derived from such samples has an uncertainty associated with it. To characterize these uncertainties, which are called standard errors, two quantities are derived from the total log-likelihood function.

The Hessian of the total log-likelihood function is
\[
\nabla^{2} L\left(z_{1}, \ldots, z_{m} ; \theta\right)
\]
and the Fisher information matrix is
\[
I(\theta)=-E\left[\nabla^{2} L\left(z_{1}, \ldots, z_{m} ; \theta\right)\right]
\]
where the partial derivatives of the \(\nabla^{2}\) operator are taken with respect to the combined parameter vector \(\Theta\) that contains the distinct components of \(\mathbf{b}\) and \(\mathbf{C}\) with a total of \(q=p\) \(+n(n+1) / 2\) parameters.

Since maximum likelihood estimation is concerned with large-sample estimates, the central limit theorem applies to the estimates and the Fisher information matrix plays a key role in the sampling distribution of the parameter estimates. Specifically, maximum likelihood parameter estimates are asymptotically normally distributed such that
\[
\left(\theta^{(t)}-\theta\right) \sim N\left(0, I^{-1},\left(\theta^{(t)}\right)\right) \text { as } t \rightarrow \infty
\]
where \(\Theta\) is the combined parameter vector and \(\Theta^{(t)}\) is the estimate for the combined parameter vector at iteration \(t=0,1, \ldots\).

The Fisher information matrix provides a lower bound, called a Cramér-Rao lower bound, for the standard errors of estimates of the model parameters.

\section*{Statistical Tests}

Given an estimate for the combined parameter vector \(\Theta\), the squared standard errors are the diagonal elements of the inverse of the Fisher information matrix
\[
s^{2}\left(\hat{\theta}_{i}\right)=\left(I^{-1}\left(\hat{\theta}_{i}\right)\right)_{i i}
\]
for \(i=1, \ldots, q\).
Since the standard errors are estimates for the standard deviations of the parameter estimates, you can construct confidence intervals so that, for example, a \(95 \%\) interval for each parameter estimate is approximately
\[
\hat{\theta}_{i} \pm 1.96 s\left(\hat{\theta}_{i}\right)
\]
for \(i=1, \ldots, q\).
Error ellipses at a level-of-significance \(\alpha \varepsilon[0,1]\) for the parameter estimates satisfy the inequality
\[
(\theta-\hat{\theta})^{T} I(\hat{\theta})(\theta-\hat{\theta}) \leq \chi_{1-\alpha, q}^{2}
\]
and follow a \(\chi^{2}\) distribution with \(q\) degrees-of-freedom. Similar inequalities can be formed for any subcollection of the parameters.

In general, given parameter estimates, the computed Fisher information matrix, and the log-likelihood function, you can perform numerous statistical tests on the parameters, the model, and the regression.

\section*{See Also}
convert2sur|ecmlsrmle |ecmlsrobj|ecmmvnrfish|ecmmvnrfish|ecmmvnrmle | ecmmvnrobj| ecmmvnrstd |ecmmvnrstd|ecmnfish|ecmnhess |ecmninit| ecmnmle |ecmnobj|ecmnstd|mvnrfish|mvnrmle|mvnrobj|mvnrstd

\section*{Related Examples}
- "Maximum Likelihood Estimation with Missing Data" on page 9-8
- "Multivariate Normal Regression Types" on page 9-16
- "Valuation with Missing Data" on page 9-33

\title{
Maximum Likelihood Estimation with Missing Data
}

\author{
In this section... \\ "Introduction" on page 9-8 \\ "ECM Algorithm" on page 9-8 \\ "Standard Errors" on page 9-9 \\ "Data Augmentation" on page 9-9
}

\section*{Introduction}

Suppose that a portion of the sample data is missing, where missing values are represented as NaNs. If the missing values are missing-at-random and ignorable, where Little and Rubin [7] have precise definitions for these terms, it is possible to use a version of the Expectation Maximization, or EM, algorithm of Dempster, Laird, and Rubin [3] to estimate the parameters of the multivariate normal regression model. The algorithm used in Financial Toolbox software is the ECM (Expectation Conditional Maximization) algorithm of Meng and Rubin [8] with enhancements by Sexton and Swensen [9].

Each sample \(\mathbf{z}_{k}\) for \(k=1, \ldots, m\), is either complete with no missing values, empty with no observed values, or incomplete with both observed and missing values. Empty samples are ignored since they contribute no information.

To understand the missing-at-random and ignorable conditions, consider an example of stock price data before an IPO. For a counterexample, censored data, in which all values greater than some cutoff are replaced with NaNs, does not satisfy these conditions.

In sample \(k\), let \(\mathbf{x}_{k}\) represent the missing values in \(\mathbf{z}_{k}\) and \(\mathbf{y}_{k}\) represent the observed values. Define a permutation matrix \(\mathbf{P}_{\mathrm{k}}\) so that
\[
z_{k}=P_{k}\left[\begin{array}{l}
x_{k} \\
y_{k}
\end{array}\right]
\]
for \(k=1, \ldots, m\).

\section*{ECM Algorithm}

The ECM algorithm has two steps - an E, or expectation step, and a CM, or conditional maximization, step. As with maximum likelihood estimation, the parameter estimates
evolve according to an iterative process, where estimates for the parameters after \(t\) iterations are denoted as \(\mathbf{b}^{(t)}\) and \(\mathbf{C}^{(t)}\).

The E step forms conditional expectations for the elements of missing data with
\[
\begin{aligned}
& E\left[X_{k} \mid Y_{k}=y_{k} ; b^{(t)}, C^{(t)}\right] \\
& \operatorname{cov}\left[X_{k} \mid Y_{k}=y_{k} ; b^{(t)}, C^{(t)}\right]
\end{aligned}
\]
for each sample \(k \in\{1, \ldots, m\}\) that has missing data.
The CM step proceeds in the same manner as the maximum likelihood procedure without missing data. The main difference is that missing data moments are imputed from the conditional expectations obtained in the E step.

The E and CM steps are repeated until the log-likelihood function ceases to increase. One of the important properties of the ECM algorithm is that it is always guaranteed to find a maximum of the log-likelihood function and, under suitable conditions, this maximum can be a global maximum.

\section*{Standard Errors}

The negative of the expected Hessian of the log-likelihood function and the Fisher information matrix are identical if no data is missing. However, if data is missing, the Hessian, which is computed over available samples, accounts for the loss of information due to missing data. So, the Fisher information matrix provides standard errors that are a Cramér-Rao lower bound whereas the Hessian matrix provides standard errors that may be greater if there is missing data.

\section*{Data Augmentation}

The ECM functions do not "fill in" missing values as they estimate model parameters. In some cases, you may want to fill in the missing values. Although you can fill in the missing values in your data with conditional expectations, you would get optimistic and unrealistic estimates because conditional estimates are not random realizations.

Several approaches are possible, including resampling methods and multiple imputation (see Little and Rubin [7] and Shafer [10] for details). A somewhat informal sampling method for data augmentation is to form random samples for missing values based on the
conditional distribution for the missing values. Given parameter estimates for \(X \subset R^{n}\) and \(\hat{C}\), each observation has moments
\[
E\left[Z_{k}\right]=H_{k} \hat{b}
\]
and
\[
\operatorname{cov}\left(Z_{k}\right)=H_{k} \hat{C} H_{k}^{T}
\]
for \(k=1, \ldots, m\), where you have dropped the parameter dependence on the left sides for notational convenience.

For observations with missing values partitioned into missing values \(\mathbf{X}_{k}\) and observed values \(\mathbf{Y}_{k}=\mathbf{y}_{k}\), you can form conditional estimates for any subcollection of random variables within a given observation. Thus, given estimates \(\boldsymbol{E}\left[\mathbf{Z}_{k}\right]\) and \(\operatorname{cov}\left(\mathbf{Z}_{k}\right)\) based on the parameter estimates, you can create conditional estimates
\[
E\left[X_{k} \mid y_{k}\right]
\]
and
\[
\operatorname{cov}\left(X_{k} \mid y_{k}\right)
\]
using standard multivariate normal distribution theory. Given these conditional estimates, you can simulate random samples for the missing values from the conditional distribution
\[
X_{k} \sim N\left(E\left[X_{k} \mid y_{k}\right], \operatorname{cov}\left(X_{k} \mid y_{k}\right)\right) .
\]

The samples from this distribution reflect the pattern of missing and nonmissing values for observations \(k=1, \ldots, m\). You must sample from conditional distributions for each observation to preserve the correlation structure with the nonmissing values at each observation.

If you follow this procedure, the resultant filled-in values are random and generate mean and covariance estimates that are asymptotically equivalent to the ECM-derived mean and covariance estimates. Note, however, that the filled-in values are random and reflect
likely samples from the distribution estimated over all the data and may not reflect "true" values for a particular observation.

\section*{See Also}
convert2sur|ecmlsrmle|ecmlsrobj|ecmmvnrfish|ecmmvnrfish|ecmmvnrmle | ecmmvnrobj| ecmmvnrstd|ecmmvnrstd|ecmnfish|ecmnhess|ecmninit| ecmnmle |ecmnobj|ecmnstd|mvnrfish|mvnrmle|mvnrobj|mvnrstd

\section*{Related Examples}
- "Multivariate Normal Regression" on page 9-2
- "Multivariate Normal Regression Types" on page 9-16
- "Valuation with Missing Data" on page 9-33

\section*{Multivariate Normal Regression Functions}

\author{
In this section... \\ "Multivariate Normal Regression Without Missing Data" on page 9-13 \\ "Multivariate Normal Regression With Missing Data" on page 9-14 \\ "Least-Squares Regression With Missing Data" on page 9-14 \\ "Multivariate Normal Parameter Estimation With Missing Data" on page 9-14 \\ "Support Functions" on page 9-15
}

Financial Toolbox software has a number of functions for multivariate normal regression with or without missing data. The toolbox functions solve four classes of regression problems with functions to estimate parameters, standard errors, log-likelihood functions, and Fisher information matrices. The four classes of regression problems are:
- "Multivariate Normal Regression Without Missing Data" on page 9-13
- "Multivariate Normal Regression With Missing Data" on page 9-14
- "Least-Squares Regression With Missing Data" on page 9-14
- "Multivariate Normal Parameter Estimation With Missing Data" on page 9-14

Additional support functions are also provided, see "Support Functions" on page 9-15.
In all functions, the MATLAB representation for the number of observations (or samples) is NumSamples \(=m\), the number of data series is NumSeries \(=n\), and the number of model parameters is NumParams \(=p\). The moment estimation functions have NumSeries \(=\) NumParams.

The collection of observations (or samples) is stored in a MATLAB matrix Data such that
\[
\operatorname{Data}(\mathrm{k},:)=z_{k}^{T}
\]
for \(\mathrm{k}=1, \ldots\), NumSamples, where Data is a NumSamples-by-NumSeries matrix.
For the multivariate normal regression or least-squares functions, an additional required input is the collection of design matrices that is stored as either a MATLAB matrix or a vector of cell arrays denoted as Design.

If Numseries \(=1\), Design can be a NumSamples-by-NumParams matrix. This is the "standard" form for regression on a single data series.

If Numseries \(=1\), Design can be either a cell array with a single cell or a cell array with NumSamples cells. Each cell in the cell array contains a NumSeries-by-NumParams matrix such that
\[
\operatorname{Design}\{\mathrm{k}\}=H_{k}
\]
for \(\mathrm{k}=1\), ..., NumSamples. If Design has a single cell, it is assumed to be the same Design matrix for each sample such that
\[
\operatorname{Design}\{1\}=H_{1}=\ldots=H_{m}
\]

Otherwise, Design must contain individual design matrices for each sample.
The main distinction among the four classes of regression problems depends upon how missing values are handled and where missing values are represented as the MATLAB value NaN . If a sample is to be ignored given any missing values in the sample, the problem is said to be a problem "without missing data." If a sample is to be ignored if and only if every element of the sample is missing, the problem is said to be a problem "with missing data" since the estimation must account for possible NaN values in the data.

In general, Data may or may not have missing values and Design should have no missing values. In some cases, however, if an observation in Data is to be ignored, the corresponding elements in Design are also ignored. Consult the function reference pages for details.

\section*{Multivariate Normal Regression Without Missing Data}

You can use the following functions for multivariate normal regression without missing data.
\begin{tabular}{ll} 
mvnrmle & \begin{tabular}{l} 
Estimate model parameters, residuals, and the residual \\
covariance.
\end{tabular} \\
mvnrstd & \begin{tabular}{l} 
Estimate standard errors of model and covariance \\
parameters.
\end{tabular} \\
mvnrfish & Estimate the Fisher information matrix. \\
mvnrobj & Calculate the log-likelihood function.
\end{tabular}

The first two functions are the main estimation functions. The second two are supporting functions that can be used for more detailed analyses.

\section*{Multivariate Normal Regression With Missing Data}

You can use the following functions for multivariate normal regression with missing data.
\begin{tabular}{ll} 
ecmmvnrmle & \begin{tabular}{l} 
Estimate model parameters, residuals, and the residual \\
covariance.
\end{tabular} \\
ecmmvnrstd & \begin{tabular}{l} 
Estimate standard errors of model and covariance \\
parameters.
\end{tabular} \\
ecmmvnrfish & Estimate the Fisher information matrix. \\
ecmmvnrobj & Calculate the log-likelihood function.
\end{tabular}

The first two functions are the main estimation functions. The second two are supporting functions used for more detailed analyses.

\section*{Least-Squares Regression With Missing Data}

You can use the following functions for least-squares regression with missing data or for covariance-weighted least-squares regression with a fixed covariance matrix.
```

ecmlsrmle Estimate model parameters, residuals, and the residual covariance.
ecmlsrobj Calculate the least-squares objective function (pseudo log-likelihood).

```

To compute standard errors and estimates for the Fisher information matrix, the multivariate normal regression functions with missing data are used.
```

ecmmvnrstd Estimate standard errors of model and covariance parameters.
ecmmvnrfish
Estimate the Fisher information matrix.

```

\section*{Multivariate Normal Parameter Estimation With Missing Data}

You can use the following functions to estimate the mean and covariance of multivariate normal data.
ecmnmle Estimate the mean and covariance of the data.
```

ecmnstd
ecmnfish
ecmnhess
ecmnobj

```

Estimate standard errors of the mean and covariance of the data.

Estimate the Fisher information matrix.
Estimate the Fisher information matrix using the Hessian.
Calculate the log-likelihood function.

These functions behave slightly differently from the more general regression functions since they solve a specialized problem. Consult the function reference pages for details.

\section*{Support Functions}

Two support functions are included.
\begin{tabular}{ll} 
convert2sur & Convert a multivariate normal regression model into an \\
ecmninit & SUR model. \\
& Obtain initial estimates for the mean and covariance of a \\
& Data matrix.
\end{tabular}

The convert2sur function converts a multivariate normal regression model into a seemingly unrelated regression, or SUR, model. The second function ecmninit is a specialized function to obtain initial ad hoc estimates for the mean and covariance of a Data matrix with missing data. (If there are no missing values, the estimates are the maximum likelihood estimates for the mean and covariance.)

\section*{See Also}
convert2sur|ecmlsrmle |ecmlsrobj|ecmmvnrfish|ecmmvnrfish|ecmmvnrmle | ecmmvnrobj|ecmmvnrstd|ecmmvnrstd|ecmnfish|ecmnhess|ecmninit| ecmnmle |ecmnobj|ecmnstd|mvnrfish|mvnrmle|mvnrobj|mvnrstd

\section*{Related Examples}
- "Multivariate Normal Regression" on page 9-2
- "Multivariate Normal Regression Types" on page 9-16
- "Valuation with Missing Data" on page 9-33

\section*{Multivariate Normal Regression Types}
```

In this section...
"Regressions" on page 9-16
"Multivariate Normal Regression" on page 9-17
"Multivariate Normal Regression Without Missing Data" on page 9-17
"Multivariate Normal Regression With Missing Data" on page 9-17
"Least-Squares Regression" on page 9-17
"Least-Squares Regression Without Missing Data" on page 9-18
"Least-Squares Regression With Missing Data" on page 9-18
"Covariance-Weighted Least Squares" on page 9-18
"Covariance-Weighted Least Squares Without Missing Data" on page 9-19
"Covariance-Weighted Least Squares With Missing Data" on page 9-19
"Feasible Generalized Least Squares" on page 9-19
"Feasible Generalized Least Squares Without Missing Data" on page 9-19
"Feasible Generalized Least Squares With Missing Data" on page 9-20
"Seemingly Unrelated Regression" on page 9-20
"Seemingly Unrelated Regression Without Missing Data" on page 9-21
"Seemingly Unrelated Regression With Missing Data" on page 9-21
"Mean and Covariance Parameter Estimation" on page 9-22

```

\section*{Regressions}

Each regression function has a specific operation. This section shows how to use these functions to perform specific types of regressions. To illustrate use of the functions for various regressions, "typical" usage is shown with optional arguments kept to a minimum. For a typical regression, you estimate model parameters and residual covariance matrices with the mle functions and estimate the standard errors of model parameters with the std functions. The regressions "without missing data" essentially ignore samples with any missing values, and the regressions "with missing data" ignore samples with every value missing.

\section*{Multivariate Normal Regression}

Multivariate normal regression, or MVNR, is the "standard" implementation of the regression functions in Financial Toolbox software.

\section*{Multivariate Normal Regression Without Missing Data}

\section*{Estimate Parameters}
[Parameters, Covariance] = mvnrmle(Data, Design);
Estimate Standard Errors
StdParameters = mvnrstd(Data, Design, Covariance);

\section*{Multivariate Normal Regression With Missing Data}

\section*{Estimate Parameters}
[Parameters, Covariance] = ecmmvnrmle(Data, Design);
Estimate Standard Errors
StdParameters = ecmmvnrstd(Data, Design, Covariance);

\section*{Least-Squares Regression}

Least-squares regression, or LSR, sometimes called ordinary least-squares or multiple linear regression, is the simplest linear regression model. It also enjoys the property that, independent of the underlying distribution, it is a best linear unbiased estimator (BLUE).

Given \(m=\) NumSamples observations, the typical least-squares regression model seeks to minimize the objective function
\[
\sum_{k=1}^{m}\left(Z_{k}-H_{k} b\right)^{T}\left(Z_{k}-H_{k} b\right),
\]
which, within the maximum likelihood framework of the multivariate normal regression routine mvnrmle, is equivalent to a single-iteration estimation of just the parameters to obtain Parameters with the initial covariance matrix Covariance held fixed as the
identity matrix. In the case of missing data, however, the internal algorithm to handle missing data requires a separate routine ecmlsrmle to do least-squares instead of multivariate normal regression.

\section*{Least-Squares Regression Without Missing Data}

\section*{Estimate Parameters}
[Parameters, Covariance] = mvnrmle(Data, Design, 1);

\section*{Estimate Standard Errors}
```

StdParameters = mvnrstd(Data, Design, Covariance);

```

\section*{Least-Squares Regression With Missing Data}

\section*{Estimate Parameters}
[Parameters, Covariance] = ecmlsrmle(Data, Design);
Estimate Standard Errors
```

StdParameters = ecmmvnrstd(Data, Design, Covariance);

```

\section*{Covariance-Weighted Least Squares}

Given \(m=\) NUMSAMPLES observations, the typical covariance-weighted least squares, or CWLS, regression model seeks to minimize the objective function
\[
\sum_{k=1}^{m}\left(Z_{k}-H_{k} b\right)^{T} C_{0}\left(Z_{k}-H_{k} b\right)
\]
with fixed covariance \(C_{0}\).

In most cases, \(C_{0}\) is a diagonal matrix. The inverse matrix \(W=C_{0}^{-1}\) has diagonal elements that can be considered relative "weights" for each series. Thus, CWLS is a form of weighted least squares with the weights applied across series.

\section*{Covariance-Weighted Least Squares Without Missing Data}

\section*{Estimate Parameters}
```

[Parameters, Covariance] = mvnrmle(Data, Design, 1, [], [], [], Covar0) ;

```

Estimate Standard Errors
StdParameters = mvnrstd(Data, Design, Covariance);

\section*{Covariance-Weighted Least Squares With Missing Data}

\section*{Estimate Parameters}
```

[Parameters, Covariance] = ecmlsrmle(Data, Design, [], [], [], [], Covar0);

```

\section*{Estimate Standard Errors}

StdParameters = ecmmvnrstd(Data, Design, Covariance);

\section*{Feasible Generalized Least Squares}

An ad hoc form of least squares that has surprisingly good properties for misspecified or nonnormal models is known as feasible generalized least squares, or FGLS. The basic procedure is to do least-squares regression and then to do covariance-weighted leastsquares regression with the resultant residual covariance from the first regression.

\section*{Feasible Generalized Least Squares Without Missing Data}
```

Estimate Parameters
[Parameters, Covariance] = mvnrmle(Data, Design, 2, 0, 0);
or (to illustrate the FGLS process explicitly)
[Parameters, Covar0] = mvnrmle(Data, Design, 1);
[Parameters, Covariance] = mvnrmle(Data, Design, 1, [], [], [],
Covar0);

```
Estimate Standard Errors
```

StdParameters = mvnrstd(Data, Design, Covariance);

```

\title{
Feasible Generalized Least Squares With Missing Data
}

\author{
Estimate Parameters
}
```

[Parameters, Covar0] = ecmlsrmle(Data, Design);
[Parameters, Covariance] = ecmlsrmle(Data, Design, [], [], [], [], Covar0)

```

Estimate Standard Errors
StdParameters = ecmmvnrstd(Data, Design, Covariance);

\section*{Seemingly Unrelated Regression}

Given a multivariate normal regression model in standard form with a Data matrix and a Design array, it is possible to convert the problem into a seemingly unrelated regression (SUR) problem by a simple transformation of the Design array. The main idea of SUR is that instead of having a common parameter vector over all data series, you have a separate parameter vector associated with each separate series or with distinct groups of series that, nevertheless, share a common residual covariance. It is this ability to aggregate and disaggregate series and to perform comparative tests on each design that is the power of SUR.

To make the transformation, use the function convert2sur, which converts a standardform design array into an equivalent design array to do SUR with a specified mapping of the series into NUMGROUPS groups. The regression functions are used in the usual manner, but with the SUR design array instead of the original design array. Instead of having NUMPARAMS elements, the SUR output parameter vector has NUMGROUPS of stacked parameter estimates, where the first NUMPARAMS elements of Parameters contain parameter estimates associated with the first group of series, the next NUMPARAMS elements of Parameters contain parameter estimates associated with the second group of series, and so on. If the model has only one series, for example, NUMSERIES \(=1\), then the SUR design array is the same as the original design array since SUR requires two or more series to generate distinct parameter estimates.

Given NUMPARAMS parameters and NUMGROUPS groups with a parameter vector (Parameters) with NUMGROUPS * NUMPARAMS elements from any of the regression routines, the following MATLAB code fragment shows how to print a table of SUR parameter estimates with rows that correspond to each parameter and columns that correspond to each group or series:
```

fprintf(1,'Seemingly Unrelated Regression Parameter
Estimates\n');
fprintf(1,' %7s ',' ');
fprintf(1,' Group(%3d) ',1:NumGroups);
fprintf(1,'\n');
for i = 1:NumParams
fprintf(1,' %7d ',i);
ii = i;
for j = 1:NumGroups
fprintf(1,'%12g ',Param(ii));
ii = ii + NumParams;
end
fprintf(1,'\n');
end
fprintf(1,'\n');

```

\section*{Seemingly Unrelated Regression Without Missing Data}

Form an SUR Design
DesignSUR = convert2sur(Design, Group);
Estimate Parameters
[Parameters, Covariance] = mvnrmle(Data, DesignSUR);
Estimate Standard Errors
StdParameters = mvnrstd(Data, DesignSUR, Covariance);

\section*{Seemingly Unrelated Regression With Missing Data}

Form a SUR Design
DesignSUR = convert2sur(Design, Group);
Estimate Parameters
[Parameters, Covariance] = ecmmvnrmle(Data, DesignSUR);
Estimate Standard Errors

StdParameters = ecmmvnrstd(Data, DesignSUR, Covariance);

\section*{Mean and Covariance Parameter Estimation}

Without missing data, you can estimate the mean of your Data with the function mean and the covariance with the function cov. Nevertheless, the function ecmnmle does this for you if it detects an absence of missing values. Otherwise, it uses the ECM algorithm to handle missing values.

Estimate Parameters
[Mean, Covariance] = ecmnmle(Data);
Estimate Standard Errors
```

StdMean = ecmnstd(Data, Mean, Covariance);

```

\section*{See Also}
convert2sur|ecmlsrmle |ecmlsrobj|ecmmvnrfish|ecmmvnrfish|ecmmvnrmle | ecmmvnrobj| ecmmvnrstd|ecmmvnrstd|ecmnfish|ecmnhess|ecmninit| ecmnmle |ecmnobj|ecmnstd|mvnrfish|mvnrmle|mvnrobj|mvnrstd

\section*{Related Examples}
- "Multivariate Normal Regression" on page 9-2
- "Maximum Likelihood Estimation with Missing Data" on page 9-8
- "Valuation with Missing Data" on page 9-33

\section*{Troubleshooting Multivariate Normal Regression}

This section provides a few pointers to handle various technical and operational difficulties that might occur.

\section*{Biased Estimates}

If samples are ignored, the number of samples used in the estimation is less than NumSamples. Clearly the actual number of samples used must be sufficient to obtain estimates. In addition, although the model parameters Parameters (or mean estimates Mean) are unbiased maximum likelihood estimates, the residual covariance estimate Covariance is biased. To convert to an unbiased covariance estimate, multiply Covariance by
Count/(Count-1),
where Count is the actual number of samples used in the estimation with Count \(\leq\) NumSamples. None of the regression functions perform this adjustment.

\section*{Requirements}

The regression functions, particularly the estimation functions, have several requirements. First, they must have consistent values for NumSamples, NumSeries, and NumParams. As a rule, the multivariate normal regression functions require

Count \(\times\) NumSeries \(\leq \max \{\) NumParams, NumSeries \(\times(\) NumSeries +1\() / 2\}\)
and the least-squares regression functions require
Count \(\times\) NumSeries \(\leq\) NumParams,
where Count is the actual number of samples used in the estimation with
Count \(\leq\) NumSamples.

Second, they must have enough nonmissing values to converge. Third, they must have a nondegenerate covariance matrix.

Although some necessary and sufficient conditions can be found in the references, general conditions for existence and uniqueness of solutions in the missing-data case, do not exist. Nonconvergence is usually due to an ill-conditioned covariance matrix estimate, which is discussed in greater detail in "Nonconvergence" on page 9-24.

\section*{Slow Convergence}

Since worst-case convergence of the ECM algorithm is linear, it is possible to execute hundreds and even thousands of iterations before termination of the algorithm. If you are estimating with the ECM algorithm regularly with regular updates, you can use prior estimates as initial guesses for the next period's estimation. This approach often speeds up things since the default initialization in the regression functions sets the initial parameters \(\mathbf{b}\) to zero and the initial covariance \(\mathbf{C}\) to be the identity matrix.

Other improvised approaches are possible although most approaches are problemdependent. In particular, for mean and covariance estimation, the estimation function ecmnmle uses a function ecmninit to obtain an initial estimate.

\section*{Nonrandom Residuals}

Simultaneous estimates for parameters \(\mathbf{b}\) and covariances \(\mathbf{C}\) require \(\mathbf{C}\) to be positivedefinite. So, the general multivariate normal regression routines require nondegenerate residual errors. If you are faced with a model that has exact results, the least-squares routine ecmlsrmle still works, although it provides a least-squares estimate with a singular residual covariance matrix. The other regression functions fail.

\section*{Nonconvergence}

Although the regression functions are robust and work for most "typical" cases, they can fail to converge. The main failure mode is an ill-conditioned covariance matrix, where failures are either soft or hard. A soft failure wanders endlessly toward a nearly singular covariance matrix and can be spotted if the algorithm fails to converge after about 100 iterations. If MaxIterations is increased to 500 and display mode is initiated (with no output arguments), a typical soft failure looks like this.


This case, which is based on 20 observations of five assets with \(30 \%\) of data missing, shows that the log-likelihood goes linearly to infinity as the likelihood function goes to 0 . In this case, the function converges but the covariance matrix is effectively singular with a smallest eigenvalue on the order of machine precision (eps).

For the function ecmnmle, a hard error looks like this:
```

> In ecmninit at 60
In ecmnmle at 140
??? Error using ==> ecmnmle
Full covariance not positive-definite in iteration 218.

```

From a practical standpoint, if in doubt, test your residual covariance matrix from the regression routines to ensure that it is positive-definite. This is important because a soft error has a matrix that appears to be positive-definite but actually has a near-zero-valued eigenvalue to within machine precision. To do this with a covariance estimate Covariance, use cond (Covariance), where any value greater than 1/eps should be considered suspect.

If either type of failure occurs, however, note that the regression routine is indicating that something is probably wrong with the data. (Even with no missing data, two time series that are proportional to one another produce a singular covariance matrix.)

\section*{See Also}
convert2sur|ecmlsrmle|ecmlsrobj|ecmmvnrfish|ecmmvnrfish|ecmmvnrmle | ecmmvnrobj|ecmmvnrstd |ecmmvnrstd|ecmnfish|ecmnhess|ecmninit| ecmnmle |ecmnobj|ecmnstd|mvnrfish|mvnrmle|mvnrobj|mvnrstd

\section*{Related Examples}
- "Multivariate Normal Regression" on page 9-2
- "Maximum Likelihood Estimation with Missing Data" on page 9-8
- "Valuation with Missing Data" on page 9-33

\section*{Portfolios with Missing Data}

This example illustrates how to use the missing data algorithms for portfolio optimization and for valuation. This example works with five years of daily total return data for 12 computer technology stocks, with 6 hardware and 6 software companies. The example estimates the mean and covariance matrix for these stocks, forms efficient frontiers with both a naïve approach and the ECM approach, and compares results.

You can run the example directly with ecmtechdemo.m.
1 Load the following data file:
load ecmtechdemo
This file contains these three quantities:
- Assets is a cell array of the tickers for the 12 stocks in the example.
- Data is a 1254 -by- 12 matrix of 1254 daily total returns for each of the 12 stocks.
- Dates is a 1254-by- 1 column vector of the dates associated with the data.

The time period for the data extends from April 19, 2000 to April 18, 2005.
The sixth stock in Assets is Google (GOOG), which started trading on August 19, 2004. So, all returns before August 20, 2004 are missing and represented as NaNs. Also, Amazon (AMZN) had a few days with missing values scattered throughout the past five years.
2 A naïve approach to the estimation of the mean and covariance for these 12 assets is to eliminate all days that have missing values for any of the 12 assets. Use the function ecmninit with the nanskip option to do this.
[NaNMean, NaNCovar] = ecmninit(Data,'nanskip');
3 Contrast the result of this approach with using all available data and the function ecmnmle to compute the mean and covariance. First, call ecmnmle with no output arguments to establish that enough data is available to obtain meaningful estimates.
ecmnmle(Data);
The following figure shows that, even with almost \(87 \%\) of the Google data being NaN values, the algorithm converges after only four iterations.


4 Estimate the mean and covariance as computed by ecmnmle.
[ECMMean, ECMCovar] = ecmnmle(Data)

ECMMean =
0.0008
0.0008
0.0005
0.0002
0.0011
0.0038
0.0003
0.0000
\(-0.0003\)
0.0000
0.0003
0.0004

ECMCovar =
\begin{tabular}{llllll}
0.0012 & 0.0005 & 0.0006 & 0.0005 & 0.0005 & 0.0003 \\
0.0005 & 0.0024 & 0.0007 & 0.0006 & 0.0010 & 0.0004 \\
0.0006 & 0.0007 & 0.0013 & 0.0007 & 0.0007 & 0.0003 \\
0.0005 & 0.0006 & 0.0007 & 0.0009 & 0.0006 & 0.0002 \\
0.0005 & 0.0010 & 0.0007 & 0.0006 & 0.0016 & 0.0006 \\
0.0003 & 0.0004 & 0.0003 & 0.0002 & 0.0006 & 0.0022 \\
0.0005 & 0.0005 & 0.0006 & 0.0005 & 0.0005 & 0.0001 \\
0.0003 & 0.0003 & 0.0004 & 0.0003 & 0.0003 & 0.0002 \\
0.0006 & 0.0006 & 0.0008 & 0.0007 & 0.0006 & 0.0002 \\
0.0003 & 0.0004 & 0.0005 & 0.0004 & 0.0004 & 0.0001 \\
0.0005 & 0.0006 & 0.0008 & 0.0005 & 0.0007 & 0.0003 \\
0.0006 & 0.0012 & 0.0008 & 0.0007 & 0.0011 & 0.0016
\end{tabular}

ECMCovar (continued)
\begin{tabular}{llllll}
0.0005 & 0.0003 & 0.0006 & 0.0003 & 0.0005 & 0.0006 \\
0.0005 & 0.0003 & 0.0006 & 0.0004 & 0.0006 & 0.0012 \\
0.0006 & 0.0004 & 0.0008 & 0.0005 & 0.0008 & 0.0008 \\
0.0005 & 0.0003 & 0.0007 & 0.0004 & 0.0005 & 0.0007 \\
0.0005 & 0.0003 & 0.0006 & 0.0004 & 0.0007 & 0.0011 \\
0.0001 & 0.0002 & 0.0002 & 0.0001 & 0.0003 & 0.0016 \\
0.0009 & 0.0003 & 0.0005 & 0.0004 & 0.0005 & 0.0006 \\
0.0003 & 0.0005 & 0.0004 & 0.0003 & 0.0004 & 0.0004 \\
0.0005 & 0.0004 & 0.0011 & 0.0005 & 0.0007 & 0.0007 \\
0.0004 & 0.0003 & 0.0005 & 0.0006 & 0.0004 & 0.0005 \\
0.0005 & 0.0004 & 0.0007 & 0.0004 & 0.0013 & 0.0007 \\
0.0006 & 0.0004 & 0.0007 & 0.0005 & 0.0007 & 0.0020
\end{tabular}

5 Given estimates for the mean and covariance of asset returns derived from the naïve and ECM approaches, estimate portfolios, and associated expected returns and risks on the efficient frontier for both approaches.
```

[ECMRisk, ECMReturn, ECMWts] = portopt(ECMMean',ECMCovar,10);
[NaNRisk, NaNReturn, NaNWts] = portopt(NaNMean',NaNCovar,10);

```

6 Plot the results on the same graph to illustrate the differences.
```

figure(gcf)
plot(ECMRisk,ECMReturn,'-bo','MarkerFaceColor','b','MarkerSize', 3);
hold on
plot(NaNRisk,NaNReturn,'-ro','MarkerFaceColor','r','MarkerSize', 3);
title('\bfEfficient Frontiers under Various Assumptions');
legend('ECM','NaN','Location','SouthEast');
xlabel('\bfStd. Dev. of Returns');
ylabel('\bfMean of Returns');
hold off

```


7 Clearly, the naïve approach is optimistic about the risk-return trade-offs for this universe of 12 technology stocks. The proof, however, lies in the portfolio weights. To view the weights, enter

Assets
ECMWts
NaNWts
which generates
```

>> Assets
ans =
'AAPL' 'AMZN' 'CSCO' 'DELL' 'EBAY' 'GOOG'
>> ECMWts

```
ans \(=\)
\begin{tabular}{rrrrrr}
0.0358 & 0.0011 & -0.0000 & 0.0000 & 0.0000 & 0.0989 \\
0.0654 & 0.0110 & 0.0000 & 0.0000 & 0.0000 & 0.1877 \\
0.0923 & 0.0194 & 0.0000 & 0.0000 & 0.0000 & 0.2784 \\
0.1165 & 0.0264 & 0.0000 & -0.0000 & 0.0000 & 0.3712 \\
0.1407 & 0.0334 & -0.0000 & 0 & 0.0000 & 0.4639 \\
0.1648 & 0.0403 & 0.0000 & 0 & -0.0000 & 0.5566 \\
0.1755 & 0.0457 & 0.0000 & -0.0000 & -0.0000 & 0.6532 \\
0.1845 & 0.0509 & 0.0000 & 0.0000 & -0.0000 & 0.7502 \\
0.1093 & 0.0174 & -0.0000 & 0.0000 & 0 & 0.8733 \\
0 & 0 & -0.0000 & 0.0000 & 0 & 1.0000
\end{tabular}
>> NaNWts
ans \(=\)
\begin{tabular}{rr}
-0.0000 & 0.0000 \\
0.0576 & -0.0000 \\
0.1248 & -0.0000 \\
0.1969 & -0.0000 \\
0.2690 & -0.0000 \\
0.3414 & 0.0000 \\
0.4235 & 0.0000 \\
0.5245 & 0.0000 \\
0.6269 & -0.0000 \\
1.0000 & -0.0000
\end{tabular}
-0.0000
-0.0000
-0.0000
-0.0000
-0.0000
-0.0000
-0.0000
-0.0000
-0.0000
-0.0000
\begin{tabular}{rrr}
0.1185 & 0.0000 & 0.0522 \\
0.1219 & 0.0000 & 0.0854 \\
0.0952 & -0.0000 & 0.1195 \\
0.0529 & -0.0000 & 0.1551 \\
0.0105 & 0.0000 & 0.1906 \\
-0.0000 & -0.0000 & 0.2265 \\
-0.0000 & -0.0000 & 0.2639 \\
-0.0000 & -0.0000 & 0.3034 \\
-0.0000 & -0.0000 & 0.3425 \\
0.0000 & -0.0000 & 0
\end{tabular}

Assets (continued)
'HPQ' 'IBM'
' INTC
MSFT' 'ORCL
YHOO
ECMWts (continued)
\begin{tabular}{rrrrrr}
0.0535 & 0.4676 & 0.0000 & 0.3431 & -0.0000 & 0.0000 \\
0.0179 & 0.3899 & -0.0000 & 0.3282 & 0.0000 & -0.0000 \\
0 & 0.3025 & -0.0000 & 0.3074 & 0.0000 & -0.0000 \\
0.0000 & 0.2054 & -0.0000 & 0.2806 & 0.0000 & 0.0000 \\
0.0000 & 0.1083 & -0.0000 & 0.2538 & -0.0000 & 0.0000 \\
0.0000 & 0.0111 & -0.0000 & 0.2271 & -0.0000 & 0.0000 \\
0.0000 & 0.0000 & -0.0000 & 0.1255 & -0.0000 & 0.0000 \\
0.0000 & 0 & -0.0000 & 0.0143 & -0.0000 & -0.0000 \\
0.0000 & -0.0000 & -0.0000 & -0.0000 & -0.0000 & 0.0000 \\
0.0000 & -0.0000 & -0.0000 & -0.0000 & -0.0000 & 0.0000
\end{tabular}

NaNWts (continued)
\begin{tabular}{rrrrrr}
0.0824 & 0.1779 & 0.0000 & 0.5691 & -0.0000 & 0.0000 \\
0.1274 & 0.0460 & 0.0000 & 0.5617 & -0.0000 & -0.0000 \\
0.1674 & -0.0000 & 0.0000 & 0.4802 & 0.0129 & -0.0000 \\
0.2056 & -0.0000 & 0.0000 & 0.3621 & 0.0274 & -0.0000 \\
0.2438 & -0.0000 & 0.0000 & 0.2441 & 0.0419 & -0.0000 \\
0.2782 & -0.0000 & 0.0000 & 0.0988 & 0.0551 & -0.0000 \\
0.2788 & -0.0000 & 0.0000 & -0.0000 & 0.0337 & -0.0000 \\
0.1721 & -0.0000 & 0.0000 & -0.0000 & -0.0000 & -0.0000 \\
0.0306 & -0.0000 & 0.0000 & 0.0000 & 0 & -0.0000 \\
0 & 0.0000 & 0.0000 & -0.0000 & -0.0000 & -0.0000
\end{tabular}

The naïve portfolios in NaNWts tend to favor Apple Computer (AAPL), which happened to do well over the period from the Google IPO to the end of the estimation
period, while the ECM portfolios in ECMWts tend to underweight Apple Computer and to recommend increased weights in Google relative to the naïve weights.
8 To evaluate the impact of estimation error and, in particular, the effect of missing data, use ecmnstd to calculate standard errors. Although it is possible to estimate the standard errors for both the mean and covariance, the standard errors for the mean estimates alone are usually the main quantities of interest.
```

StdMeanF = ecmnstd(Data,ECMMean,ECMCovar,'fisher');

```

9 Calculate standard errors that use the data-generated Hessian matrix (which accounts for the possible loss of information due to missing data) with the option HESSIAN.
```

StdMeanH = ecmnstd(Data,ECMMean,ECMCovar,'hessian');

```

The difference in the standard errors shows the increase in uncertainty of estimation of asset expected returns due to missing data. This can be viewed by entering:

Assets
StdMeanH'
StdMeanF'
StdMeanH' - StdMeanF'
The two assets with missing data, AMZN and GOOG, are the only assets to have differences due to missing information.

\section*{See Also}
convert2sur|ecmlsrmle|ecmlsrobj|ecmmvnrfish|ecmmvnrfish|ecmmvnrmle | ecmmvnrobj|ecmmvnrstd|ecmmvnrstd|ecmnfish|ecmnhess|ecmninit| ecmnmle |ecmnobj|ecmnstd|mvnrfish|mvnrmle|mvnrobj|mvnrstd

\section*{Related Examples}
- "Multivariate Normal Regression" on page 9-2
- "Maximum Likelihood Estimation with Missing Data" on page 9-8
- "Valuation with Missing Data" on page 9-33

\section*{Valuation with Missing Data}
```

In this section...
"Introduction" on page 9-33
"Capital Asset Pricing Model" on page 9-33
"Estimation of the CAPM" on page 9-34
"Estimation with Missing Data" on page 9-35
"Estimation of Some Technology Stock Betas" on page 9-35
"Grouped Estimation of Some Technology Stock Betas" on page 9-37
"References" on page 9-40

```

\section*{Introduction}

The Capital Asset Pricing Model (CAPM) is a venerable but often maligned tool to characterize comovements between asset and market prices. Although many issues arise in CAPM implementation and interpretation, one problem that practitioners face is to estimate the coefficients of the CAPM with incomplete stock price data.

This example shows how to use the missing data regression functions to estimate the coefficients of the CAPM. You can run the example directly using CAPMdemo.m located at matlabroot/toolbox/finance/findemos.

\section*{Capital Asset Pricing Model}

Given a host of assumptions that can be found in the references (see Sharpe [11], Lintner [6], Jarrow [5], and Sharpe, et. al. [12]), the CAPM concludes that asset returns have a linear relationship with market returns. Specifically, given the return of all stocks that constitute a market denoted as \(M\) and the return of a riskless asset denoted as \(C\), the CAPM states that the return of each asset \(R_{\mathrm{i}}\) in the market has the expectational form
\[
E\left[R_{i}\right]=\alpha_{i}+C+\beta_{i}(E[M]-C)
\]
for assets \(i=1, \ldots, n\), where \(\beta_{i}\) is a parameter that specifies the degree of comovement between a given asset and the underlying market. In other words, the expected return of each asset is equal to the return on a riskless asset plus a risk-adjusted expected market return net of riskless asset returns. The collection of parameters \(\beta_{1}, \ldots, \beta_{n}\) is called asset betas.

The beta of an asset has the form
\[
\beta_{i}=\frac{\operatorname{cov}\left(R_{i}, M\right)}{\operatorname{var}(M)}
\]
which is the ratio of the covariance between asset and market returns divided by the variance of market returns. If an asset has a beta \(=1\), the asset is said to move with the market; if an asset has a beta > 1, the asset is said to be more volatile than the market. Conversely, if an asset has a beta \(<1\), the asset is said to be less volatile than the market.

\section*{Estimation of the CAPM}

The standard CAPM model is a linear model with additional parameters for each asset to characterize residual errors. For each of \(n\) assets with \(m\) samples of observed asset returns \(R_{k, i}\), market returns \(M_{k}\), and riskless asset returns \(C_{k}\), the estimation model has the form
\[
R_{k, i}=\alpha_{i}+C_{k}+\beta_{i}\left(M_{k}-C_{k}\right)+V_{k, i}
\]
for samples \(k=1, \ldots, m\) and assets \(i=1, \ldots, n\), where \(\alpha_{i}\) is a parameter that specifies the nonsystematic return of an asset, \(\beta_{i}\) is the asset beta, and \(V_{k, i}\) is the residual error for each asset with associated random variable \(V_{i}\).

The collection of parameters \(\alpha_{1}, \ldots, \alpha_{n}\) are called asset alphas. The strict form of the CAPM specifies that alphas must be zero and that deviations from zero are the result of temporary disequilibria. In practice, however, assets may have nonzero alphas, where much of active investment management is devoted to the search for assets with exploitable nonzero alphas.

To allow for the possibility of nonzero alphas, the estimation model generally seeks to estimate alphas and to perform tests to determine if the alphas are statistically equal to zero.

The residual errors \(V_{i}\) are assumed to have moments
\[
E\left[V_{i}\right]=0
\]
and
\[
E\left[V_{i} V_{j}\right]=S_{i j}
\]
for assets \(i, j=1, \ldots, n\), where the parameters \(S_{11}, \ldots, S_{n n}\) are called residual or nonsystematic variances/covariances.

The square root of the residual variance of each asset, for example, \(\operatorname{sqrt}\left(S_{i i}\right)\) for \(i=1, \ldots, n\), is said to be the residual or nonsystematic risk of the asset since it characterizes the residual variation in asset prices that are not explained by variations in market prices.

\section*{Estimation with Missing Data}

Although betas can be estimated for companies with sufficiently long histories of asset returns, it is difficult to estimate betas for recent IPOs. However, if a collection of sufficiently observable companies exists that can be expected to have some degree of correlation with the new company's stock price movements, that is, companies within the same industry as the new company, it is possible to obtain imputed estimates for new company betas with the missing-data regression routines.

\section*{Estimation of Some Technology Stock Betas}

To illustrate how to use the missing-data regression routines, estimate betas for 12 technology stocks, where a single stock (GOOG) is an IPO.

1 Load dates, total returns, and ticker symbols for the 12 stocks from the MAT-file CAPMuniverse.
load CAPMuniverse
whos Assets Data Dates
\begin{tabular}{lcrl} 
Name & Size & Bytes & Class \\
& & & \\
Assets & \(1 \times 14\) & 952 & cell array \\
Data & \(1471 \times 14\) & 164752 & double array \\
Dates & \(1471 \times 1\) & 11768 & double array
\end{tabular}

Grand total is 22135 elements using 177472 bytes
The assets in the model have the following symbols, where the last two series are proxies for the market and the riskless asset:


The data covers the period from January 1, 2000 to November 7, 2005 with daily total returns. Two stocks in this universe have missing values that are represented by NaNs. One of the two stocks had an IPO during this period and, so, has significantly less data than the other stocks.
2 Compute separate regressions for each stock, where the stocks with missing data have estimates that reflect their reduced observability.
```

[NumSamples, NumSeries] = size(Data);
NumAssets = NumSeries - 2;
StartDate = Dates(1);
EndDate = Dates(end);
fprintf(1,'Separate regressions with ');
fprintf(1,'daily total return data from %s to %s ...\n', ...
datestr(StartDate,1),datestr(EndDate,1));
fprintf(1,' %4s %-20s %-20s %-20s\n','','Alpha','Beta','Sigma');
fprintf(1,' ---- ---------------------');
fprintf(1,'------------------------------------------------- ');
for i = 1:NumAssets
% Set up separate asset data and design matrices
TestData = zeros(NumSamples,1);
TestDesign = zeros(NumSamples,2);
TestData(:) = Data(:,i) - Data(:,14);
TestDesign(:,1) = 1.0;
TestDesign(:,2) = Data(:,13) - Data(:,14);
% Estimate CAPM for each asset separately
[Param, Covar] = ecmmvnrmle(TestData, TestDesign);
% Estimate ideal standard errors for covariance parameters
[StdParam, StdCovar] = ecmmvnrstd(TestData, TestDesign, ...
Covar, 'fisher');
% Estimate sample standard errors for model parameters
StdParam = ecmmvnrstd(TestData, TestDesign, Covar, 'hessian');
% Set up results for output
Alpha = Param(1);
Beta = Param(2);
Sigma = sqrt(Covar);
StdAlpha = StdParam(1);
StdBeta = StdParam(2);

```
```

    StdSigma = sqrt(StdCovar);
    % Display estimates
fprintf(' %4s %9.4f (%8.4f) %9.4f (%8.4f) %9.4f (%8.4f)\n', ...
Assets{i},Alpha(1),abs(Alpha(1)/StdAlpha(1)), ...
Beta(1),abs(Beta(1)/StdBeta(1)),Sigma(1),StdSigma(1));
end

```

This code fragment generates the following table.
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline & Alpha & & Beta & & Sigma & \\
\hline AAPL & 0.0012 & ( 1.3882) & 1.2294 & 17.1839) & 0.0322 & \(0.0062)\) \\
\hline AMZN & 0.0006 & ( 0.5326) & 1.3661 & ( 13.6579) & 0.0449 & \(0.0086)\) \\
\hline CSCO & -0.0002 & ( 0.2878) & 1.5653 & ( 23.6085) & 0.0298 & ( 0.0057) \\
\hline DELL & -0.0000 & ( 0.0368) & 1.2594 & ( 22.2164) & 0.0255 & 0.0049) \\
\hline EBAY & 0.0014 & ( 1.4326) & 1.3441 & ( 16.0732) & 0.0376 & \(0.0072)\) \\
\hline G00G & 0.0046 & ( 3.2107) & 0.3742 & ( 1.7328) & 0.0252 & ( 0.0071) \\
\hline HPQ & 0.0001 & ( 0.1747) & 1.3745 & ( 24.2390) & 0.0255 & ( 0.0049) \\
\hline IBM & -0.0000 & ( 0.0312) & 1.0807 & ( 28.7576) & 0.0169 & ( 0.0032) \\
\hline INTC & 0.0001 & ( 0.1608) & 1.6002 & ( 27.3684) & 0.0263 & ( 0.0050) \\
\hline MSFT & -0.0002 & ( 0.4871) & 1.1765 & ( 27.4554) & 0.0193 & ( 0.0037) \\
\hline ORCL & 0.0000 & ( 0.0389) & 1.5010 & ( 21.1855) & 0.0319 & ( 0.0061) \\
\hline YHOO & 0.0001 & ( 0.1282) & 1.6543 & ( 19.3838) & 0.0384 & ( 0.0074) \\
\hline
\end{tabular}

The Alpha column contains alpha estimates for each stock that are near zero as expected. In addition, the t-statistics (which are enclosed in parentheses) generally reject the hypothesis that the alphas are nonzero at the \(99.5 \%\) level of significance.

The Beta column contains beta estimates for each stock that also have t-statistics enclosed in parentheses. For all stocks but GOOG, the hypothesis that the betas are nonzero is accepted at the \(99.5 \%\) level of significance. It seems, however, that GOOG does not have enough data to obtain a meaningful estimate for beta since its tstatistic would imply rejection of the hypothesis of a nonzero beta.

The Sigma column contains residual standard deviations, that is, estimates for nonsystematic risks. Instead of t-statistics, the associated standard errors for the residual standard deviations are enclosed in parentheses.

\section*{Grouped Estimation of Some Technology Stock Betas}

To estimate stock betas for all 12 stocks, set up a joint regression model that groups all 12 stocks within a single design. (Since each stock has the same design matrix, this model is actually an example of seemingly unrelated regression.) The routine to estimate model parameters is ecmmvnrmle, and the routine to estimate standard errors is ecmmvnrstd.

Because GOOG has a significant number of missing values, a direct use of the missing data routine ecmmvnrmle takes 482 iterations to converge. This can take a long time to compute. For the sake of brevity, the parameter and covariance estimates after the first 480 iterations are contained in a MAT-file and are used as initial estimates to compute stock betas.
load CAPMgroupparam
whos Param0 Covar0
\begin{tabular}{llrl} 
Name & Size & Bytes & Class \\
Covar0 & \(12 \times 12\) & 1152 & double array \\
Param0 & \(24 \times 1\) & 192 & double array \\
Grand total is 168 elements using 1344 & bytes
\end{tabular}

Now estimate the parameters for the collection of 12 stocks.
```

fprintf(1,'\n');
fprintf(1,'Grouped regression with ');
fprintf(1,'daily total return data from %s to %s ...\n', ...
datestr(StartDate,1), datestr(EndDate,1));
fprintf(1,' %4s %-20s %-20s %-20s\n','','Alpha','Beta','Sigma');
fprintf(1,' ---- -------------------------
fprintf(1,'------------------------------------------------}\mp@subsup{n}{}{\prime})
NumParams = 2 * NumAssets;
% Set up grouped asset data and design matrices
TestData = zeros(NumSamples, NumAssets);
TestDesign = cell(NumSamples, 1);
Design = zeros(NumAssets, NumParams);
for k = 1:NumSamples
for i = 1:NumAssets
TestData(k,i) = Data(k,i) - Data(k,14);
Design(i,2*i - 1) = 1.0;
Design(i,2*i) = Data(k,13) - Data(k,14);
end
TestDesign{k} = Design;
end
% Estimate CAPM for all assets together with initial parameter
% estimates
[Param, Covar] = ecmmvnrmle(TestData, TestDesign, [], [], [],...
Param0, Covar0);
% Estimate ideal standard errors for covariance parameters
[StdParam, StdCovar] = ecmmvnrstd(TestData, TestDesign, Covar,...
'fisher');
% Estimate sample standard errors for model parameters
StdParam = ecmmvnrstd(TestData, TestDesign, Covar, 'hessian');

```
```

% Set up results for output
Alpha = Param(1:2:end-1);
Beta = Param(2:2:end);
Sigma = sqrt(diag(Covar));
StdAlpha = StdParam(1:2:end-1);
StdBeta = StdParam(2:2:end);
StdSigma = sqrt(diag(StdCovar));
% Display estimates
for i = 1:NumAssets
fprintf(' %4s %9.4f (%8.4f) %9.4f (%8.4f) %9.4f (%8.4f)\n', ...
Assets{i},Alpha(i),abs(Alpha(i)/StdAlpha(i)), ...
Beta(i),abs(Beta(i)/StdBeta(i)),Sigma(i),StdSigma(i));
end

```

This code fragment generates the following table.
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline & Alpha & & Beta & \multicolumn{3}{|c|}{Sigma} \\
\hline AAPL & 0.0012 & 1.3882) & 1.2294 & ( 17.1839) & 0.0322 & \(0.0062)\) \\
\hline AMZN & 0.0007 & \(0.6086)\) & 1.3673 & ( 13.6427) & 0.0450 & \(0.0086)\) \\
\hline CSCO & -0.0002 & 0.2878) & 1.5653 & ( 23.6085) & 0.0298 & 0.0057) \\
\hline DELL & -0.0000 & 0.0368) & 1.2594 & ( 22.2164) & 0.0255 & 0.0049) \\
\hline EBAY & 0.0014 & 1.4326) & 1.3441 & ( 16.0732) & 0.0376 & \(0.0072)\) \\
\hline GOOG & 0.0041 & 2.8907) & 0.6173 & ( 3.1100) & 0.0337 & \(0.0065)\) \\
\hline HPQ & 0.0001 & 0.1747) & 1.3745 & ( 24.2390) & 0.0255 & \(0.0049)\) \\
\hline IBM & -0.0000 & 0.0312) & 1.0807 & ( 28.7576) & 0.0169 & \(0.0032)\) \\
\hline INTC & 0.0001 & 0.1608) & 1.6002 & ( 27.3684) & 0.0263 & 0.0050) \\
\hline MSFT & -0.0002 & 0.4871) & 1.1765 & ( 27.4554) & 0.0193 & \(0.0037)\) \\
\hline ORCL & 0.0000 & 0.0389) & 1.5010 & ( 21.1855) & 0.0319 & \(0.0061)\) \\
\hline YHOO & 0.0001 & 0.1282) & 1.6543 & ( 19.3838) & 0.0384 & 0.0074 \\
\hline
\end{tabular}

Although the results for complete-data stocks are the same, the beta estimates for AMZN and GOOG (the two stocks with missing values) are different from the estimates derived for each stock separately. Since AMZN has few missing values, the differences in the estimates are small. With GOOG, however, the differences are more pronounced.

The t-statistic for the beta estimate of GOOG is now significant at the \(99.5 \%\) level of significance. However, the t-statistics for beta estimates are based on standard errors from the sample Hessian which, in contrast to the Fisher information matrix, accounts for the increased uncertainty in an estimate due to missing values. If the t-statistic is obtained from the more optimistic Fisher information matrix, the t-statistic for GOOG is 8.25. Thus, despite the increase in uncertainty due to missing data, GOOG nonetheless has a statistically significant estimate for beta.

Finally, note that the beta estimate for GOOG is 0.62 - a value that may require some explanation. Although the market has been volatile over this period with sideways price
movements, GOOG has steadily appreciated in value. So, it is less tightly correlated with the market, implying that it is less volatile than the market (beta < 1).

\section*{References}
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[6] Lintner, J. "The Valuation of Risk Assets and the Selection of Risky Investments in Stocks," Review of Economics and Statistics, Vol. 14, 1965, pp. 13-37.
[7] Little, Roderick J. A and Donald B. Rubin. Statistical Analysis with Missing Data, 2nd ed., John Wiley \& Sons, Inc., 2002.
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[9] Sexton, Joe and Anders Rygh Swensen. "ECM Algorithms that Converge at the Rate of EM," Biometrika, Vol. 87, No. 3, 2000, pp. 651-662.
[10] Shafer, J. L. Analysis of Incomplete Multivariate Data, Chapman \& Hall/CRC, 1997.
[11] Sharpe, W. F. "Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk," Journal of Finance, Vol. 19, 1964, pp. 425-442.
[12] Sharpe, W. F., G. J. Alexander, and J. V. Bailey. Investments, 6th ed., Prentice-Hall, Inc., 1999.

\section*{See Also}
convert2sur|ecmlsrmle|ecmlsrobj|ecmmvnrfish|ecmmvnrfish|ecmmvnrmle | ecmmvnrobj| ecmmvnrstd | ecmmvnrstd | ecmnfish|ecmnhess|ecmninit|
ecmnmle |ecmnobj|ecmnstd|mvnrfish|mvnrmle|mvnrobj|mvnrstd

\section*{Related Examples}
- "Multivariate Normal Regression" on page 9-2
- "Maximum Likelihood Estimation with Missing Data" on page 9-8
- "Multivariate Normal Regression Types" on page 9-16

\section*{Solving Sample Problems}
- "Sensitivity of Bond Prices to Interest Rates" on page 10-2
- "Bond Portfolio for Hedging Duration and Convexity" on page 10-6
- "Bond Prices and Yield Curve Parallel Shifts" on page 10-10
- "Bond Prices and Yield Curve Nonparallel Shifts" on page 10-15
- "Greek-Neutral Portfolios of European Stock Options" on page 10-18
- "Term Structure Analysis and Interest-Rate Swaps" on page 10-22
- "Plotting an Efficient Frontier Using portopt" on page 10-26
- "Plotting Sensitivities of an Option" on page 10-30
- "Plotting Sensitivities of a Portfolio of Options" on page 10-33

\section*{Sensitivity of Bond Prices to Interest Rates}

Macaulay and modified duration measure the sensitivity of a bond's price to changes in the level of interest rates. Convexity measures the change in duration for small shifts in the yield curve, and thus measures the second-order price sensitivity of a bond. Both measures can gauge the vulnerability of a bond portfolio's value to changes in the level of interest rates.

Alternatively, analysts can use duration and convexity to construct a bond portfolio that is partly hedged against small shifts in the term structure. If you combine bonds in a portfolio whose duration is zero, the portfolio is insulated, to some extent, against interest rate changes. If the portfolio convexity is also zero, this insulation is even better. However, since hedging costs money or reduces expected return, you must know how much protection results from hedging duration alone compared to hedging both duration and convexity.

This example demonstrates a way to analyze the relative importance of duration and convexity for a bond portfolio using some of the SIA-compliant bond functions in Financial Toolbox software. Using duration, it constructs a first-order approximation of the change in portfolio price to a level shift in interest rates. Then, using convexity, it calculates a second-order approximation. Finally, it compares the two approximations with the true price change resulting from a change in the yield curve.

\section*{Step 1}

Define three bonds using values for the settlement date, maturity date, face value, and coupon rate. For simplicity, accept default values for the coupon payment periodicity (semiannual), end-of-month payment rule (rule in effect), and day-count basis (actual/ actual). Also, synchronize the coupon payment structure to the maturity date (no odd first or last coupon dates). Any inputs for which defaults are accepted are set to empty matrices ([]) as placeholders where appropriate.
```

Settle = '19-Aug-1999';
Maturity = ['17-Jun-2010'; '09-Jun-2015'; '14-May-2025'];
Face = [100; 100; 1000];
CouponRate = [0.07; 0.06; 0.045];

```

Also, specify the yield curve information.
```

Yields = [0.05; 0.06; 0.065];

```

\section*{Step 2}

Use Financial Toolbox functions to calculate the price, modified duration in years, and convexity in years of each bond.

The true price is quoted (clean) price plus accrued interest.
```

[CleanPrice, AccruedInterest] = bndprice(Yields, CouponRate,...
Settle, Maturity, 2, 0, [], [], [], [], [], Face);
Durations = bnddury(Yields, CouponRate, Settle, Maturity, 2, 0,···.
[], [], [], [], [], Face);
Convexities = bndconvy(Yields, CouponRate, Settle, Maturity, 2, 0,···.
[], [], [], [], [], Face);
Prices = CleanPrice + AccruedInterest;

```

\section*{Step 3}

Choose a hypothetical amount by which to shift the yield curve (here, 0.2 percentage point or 20 basis points).
```

dY = 0.002;

```

Weight the three bonds equally, and calculate the actual quantity of each bond in the portfolio, which has a total value of \(\$ 100,000\).
```

PortfolioPrice = 100000;
PortfolioWeights = ones(3,1)/3;
PortfolioAmounts = PortfolioPrice * PortfolioWeights ./ Prices;

```

\section*{Step 4}

Calculate the modified duration and convexity of the portfolio. The portfolio duration or convexity is a weighted average of the durations or convexities of the individual bonds. Calculate the first- and second-order approximations of the percent price change as a function of the change in the level of interest rates.
```

PortfolioDuration = PortfolioWeights' * Durations;
PortfolioConvexity = PortfolioWeights' * Convexities;
PercentApprox1 = -PortfolioDuration * dY * 100;
PercentApprox2 = PercentApprox1 + ...
PortfolioConvexity*dY^2*100/2.0;

```

\section*{Step 5}

Estimate the new portfolio price using the two estimates for the percent price change.
PriceApprox1 = PortfolioPrice + ...
PercentApprox1 * PortfolioPrice/100;
PriceApprox2 = PortfolioPrice + ...
PercentApprox2 * PortfolioPrice/100;

\section*{Step 6}

Calculate the true new portfolio price by shifting the yield curve.
```

[CleanPrice, AccruedInterest] = bndprice(Yields + dY,...
CouponRate, Settle, Maturity, 2, 0, [], [], [], [], [],...
Face);
NewPrice = PortfolioAmounts' * (CleanPrice + AccruedInterest);

```

\section*{Step 7}

Compare the results. The analysis results are as follows:
- The original portfolio price was \(\$ 100,000\).
- The yield curve shifted up by 0.2 percentage point or 20 basis points.
- The portfolio duration and convexity are 10.3181 and 157.6346 , respectively. These are needed for "Bond Portfolio for Hedging Duration and Convexity" on page 10-6.
- The first-order approximation, based on modified duration, predicts the new portfolio price (PriceApprox1), which is \(\$ 97,936.37\).
- The second-order approximation, based on duration and convexity, predicts the new portfolio price (PriceApprox2), which is \$97,967.90.
- The true new portfolio price (NewPrice) for this yield curve shift is \(\$ 97,967.51\).
- The estimate using duration and convexity is good (at least for this fairly small shift in the yield curve), but only slightly better than the estimate using duration alone. The importance of convexity increases as the magnitude of the yield curve shift increases. Try a larger shift (dY) to see this effect.

The approximation formulas in this example consider only parallel shifts in the term structure, because both formulas are functions of dY , the change in yield. The formulas
are not well-defined unless each yield changes by the same amount. In actual financial markets, changes in yield curve level typically explain a substantial portion of bond price movements. However, other changes in the yield curve, such as slope, may also be important and are not captured here. Also, both formulas give local approximations whose accuracy deteriorates as dY increases in size. You can demonstrate this by running the program with larger values of dY .

\section*{See Also}
blsdelta|blsgamma|blsprice|blsvega|bndconvy|bnddury|bndkrdur | bndprice|zbtprice|zero2disc|zero2fwd

\section*{Related Examples}
- "Pricing and Analyzing Equity Derivatives" on page 2-45
- on page 10-18
- "Bond Portfolio for Hedging Duration and Convexity" on page 10-6
- "Bond Prices and Yield Curve Parallel Shifts" on page 10-10
- "Bond Prices and Yield Curve Nonparallel Shifts" on page 10-15
- "Term Structure Analysis and Interest-Rate Swaps" on page 10-22
- "Plotting Sensitivities of an Option" on page 10-30
- "Plotting Sensitivities of a Portfolio of Options" on page 10-33

\section*{Bond Portfolio for Hedging Duration and Convexity}

This example constructs a bond portfolio to hedge the portfolio of "Sensitivity of Bond Prices to Interest Rates" on page 10-2. It assumes a long position in (holding) the portfolio, and that three other bonds are available for hedging. It chooses weights for these three other bonds in a new portfolio so that the duration and convexity of the new portfolio match those of the original portfolio. Taking a short position in the new portfolio, in an amount equal to the value of the first portfolio, partially hedges against parallel shifts in the yield curve.

Recall that portfolio duration or convexity is a weighted average of the durations or convexities of the individual bonds in a portfolio. As in the previous example, this example uses modified duration in years and convexity in years. The hedging problem therefore becomes one of solving a system of linear equations, which is an easy to do in MATLAB software.

\section*{Step 1}

Define three bonds available for hedging the original portfolio. Specify values for the settlement date, maturity date, face value, and coupon rate. For simplicity, accept default values for the coupon payment periodicity (semiannual), end-of-month payment rule (rule in effect), and day-count basis (actual/actual). Also, synchronize the coupon payment structure to the maturity date (that is, no odd first or last coupon dates). Set any inputs for which defaults are accepted to empty matrices ([]) as placeholders where appropriate. The intent is to hedge against duration and convexity and constrain total portfolio price.
```

Settle = '19-Aug-1999';
Maturity = ['15-Jun-2005'; '02-Oct-2010'; '01-Mar-2025'];
Face = [500; 1000; 250];
CouponRate = [0.07; 0.066; 0.08];

```

Also, specify the yield curve for each bond.
```

Yields = [0.06; 0.07; 0.075];

```

\section*{Step 2}

Use Financial Toolbox functions to calculate the price, modified duration in years, and convexity in years of each bond.

The true price is quoted (clean price plus accrued interest.
[CleanPrice, AccruedInterest] = bndprice(Yields,CouponRate,...
Settle, Maturity, 2, 0, [], [], [], [], [], Face);
Durations = bnddury(Yields, CouponRate, Settle, Maturity,...
2, 0, [], [], [], [], [], Face);
Convexities = bndconvy(Yields, CouponRate, Settle,...
Maturity, 2, 0, [], [], [], [], [], Face);
Prices = CleanPrice + AccruedInterest;

\section*{Step 3}

Set up and solve the system of linear equations whose solution is the weights of the new bonds in a new portfolio with the same duration and convexity as the original portfolio. In addition, scale the weights to sum to 1 ; that is, force them to be portfolio weights. You can then scale this unit portfolio to have the same price as the original portfolio. Recall that the original portfolio duration and convexity are 10.3181 and 157.6346, respectively. Also, note that the last row of the linear system ensures that the sum of the weights is unity.

A = [Durations'
Convexities'
11 1];
b = [ 10.3181
157.6346

1];
Weights = A\b;

\section*{Step 4}

Compute the duration and convexity of the hedge portfolio, which should now match the original portfolio.
```

PortfolioDuration = Weights' * Durations;
PortfolioConvexity = Weights' * Convexities;

```

\section*{Step 5}

Finally, scale the unit portfolio to match the value of the original portfolio and find the number of bonds required to insulate against small parallel shifts in the yield curve.
```

PortfolioValue = 100000;

```

HedgeAmounts = Weights ./ Prices * PortfolioValue;

\section*{Step 6}

Compare the results.
- As required, the duration and convexity of the new portfolio are 10.3181 and 157.6346, respectively.
- The hedge amounts for bonds 1,2 , and 3 are \(-57.37,71.70\), and 216.27, respectively.

Notice that the hedge matches the duration, convexity, and value \((\$ 100,000)\) of the original portfolio. If you are holding that first portfolio, you can hedge by taking a short position in the new portfolio.

Just as the approximations of the first example are appropriate only for small parallel shifts in the yield curve, the hedge portfolio is appropriate only for reducing the impact of small level changes in the term structure.

\section*{See Also}
blsdelta|blsgamma|blsprice|blsvega|bndconvy|bnddury|bndkrdur | bndprice | zbtprice | zero2disc|zero2fwd

\section*{Related Examples}
- "Pricing and Analyzing Equity Derivatives" on page 2-45
- on page 10-18
- "Sensitivity of Bond Prices to Interest Rates" on page 10-2
- "Bond Prices and Yield Curve Parallel Shifts" on page 10-10
- "Bond Prices and Yield Curve Nonparallel Shifts" on page 10-15
- "Term Structure Analysis and Interest-Rate Swaps" on page 10-22
- "Plotting Sensitivities of an Option" on page 10-30
- "Plotting Sensitivities of a Portfolio of Options" on page 10-33

\section*{Bond Prices and Yield Curve Parallel Shifts}

This example uses Financial Toolbox \({ }^{\mathrm{TM}}\) bond pricing functions to evaluate the impact of time-to-maturity and yield variation on the price of a bond portfolio. Also, this example shows how to visualize the price behavior of a portfolio of bonds over a wide range of yield curve scenarios, and as time progresses toward maturity.

Specify values for the settlement date, maturity date, face value, coupon rate, and coupon payment periodicity of a four-bond portfolio. For simplicity, accept default values for the end-of-month payment rule (rule in effect) and day-count basis (actual/actual). Also, synchronize the coupon payment structure to the maturity date (no odd first or last coupon dates). Any inputs for which defaults are accepted are set to empty matrices ([ ]) as placeholders where appropriate. Also, specify the points on the yield curve for each bond.
```

Settle = '15-Jan-1995';
Maturity = datenum(['03-Apr-2020'; '14-May-2025'; ...
'09-Jun-2019'; '25-Feb-2019']);
Face = [1000; 1000; 1000; 1000];
CouponRate = [0; 0.05; 0; 0.055];
Periods = [0; 2; 0; 2];
Yields = [0.078; 0.09; 0.075; 0.085];

```

Use Financial Toolbox functions to calculate the true bond prices as the sum of the quoted price plus accrued interest.
```

[CleanPrice, AccruedInterest] = bndprice(Yields,...
CouponRate,Settle, Maturity, Periods,...
[], [], [], [], [], [], Face);
Prices = CleanPrice + AccruedInterest
Prices = 4×1

```
    145.2452
    594.7757
    165.8949
    715.7584

Assume that the value of each bond is \(\$ 25,000\), and determine the quantity of each bond such that the portfolio value is \(\$ 100,000\).
```

BondAmounts = 25000 ./ Prices;

```

Compute the portfolio price for a rolling series of settlement dates over a range of yields. The evaluation dates occur annually on January 15, beginning on 15-Jan-1995 (settlement) and extending out to 15-Jan-2018. Thus, this step evaluates portfolio price on a grid of time of progression (dT) and interest rates (dY).
```

dy = -0.05:0.005:0.05; % Yield changes
D = datevec(Settle); %Get date components
dt = datenum(D(1):2018, D(2), D(3)); % Get evaluation dates
[dT, dY] = meshgrid(dt, dy); % Create grid
NumTimes = length(dt); % Number of time steps
NumYields = length(dy); % Number of yield changes
NumBonds = length(Maturity); % Number of bonds
% Preallocate vector
Prices = zeros(NumTimes*NumYields, NumBonds);

```

Now that the grid and price vectors have been created, compute the price of each bond in the portfolio on the grid one bond at a time.
```

for i = 1:NumBonds
[CleanPrice, AccruedInterest] = bndprice(Yields(i)+...
dY(:), CouponRate(i), dT(:), Maturity(i), Periods(i),...
[], [], [], [], [], [], Face(i));
Prices(:,i) = CleanPrice + AccruedInterest;
end

```

Scale the bond prices by the quantity of bonds and reshape the bond values to conform to the underlying evaluation grid.

Prices \(=\) Prices * BondAmounts;
```

Prices = reshape(Prices, NumYields, NumTimes);

```

Plot the price of the portfolio as a function of settlement date and a range of yields, and as a function of the change in yield (dY). This plot illustrates the interest-rate sensitivity of the portfolio as time progresses (dT), under a range of interest-rate scenarios. With the following graphics commands, you can visualize the three-dimensional surface relative to the current portfolio value (that is, \(\$ 100,000\) ).
```

figure % Open a new figure window
surf(dt, dy, Prices) % Draw the surface
hold on % Add the current value for reference
basemesh = mesh(dt, dy, 100000*ones(NumYields, NumTimes));
set(basemesh, 'facecolor', 'none');
set(basemesh, 'edgecolor', 'm');
set(gca, 'box', 'on');
dateaxis('x', 11);
xlabel('Evaluation Date (YY Format)');
ylabel('Change in Yield');
zlabel('Portfolio Price');
hold off
view(-25,25);

```


MATLAB® three-dimensional graphics allow you to visualize the interest-rate risk experienced by a bond portfolio over time. This example assumed parallel shifts in the term structure, but it might similarly have allowed other components to vary, such as the level and slope.

\section*{See Also}
blsdelta|blsgamma|blsprice|blsvega| bndconvy|bnddury|bndkrdur | bndprice|zbtprice|zero2disc|zero2fwd

\section*{Related Examples}
- "Pricing and Analyzing Equity Derivatives" on page 2-45
- on page 10-18
- "Sensitivity of Bond Prices to Interest Rates" on page 10-2
- "Bond Portfolio for Hedging Duration and Convexity" on page 10-6
- "Bond Prices and Yield Curve Nonparallel Shifts" on page 10-15
- "Term Structure Analysis and Interest-Rate Swaps" on page 10-22
- "Plotting Sensitivities of an Option" on page 10-30
- "Plotting Sensitivities of a Portfolio of Options" on page 10-33

\section*{Bond Prices and Yield Curve Nonparallel Shifts}

This example shows how to construct a bond portfolio to hedge the interest-rate risk of a Treasury bond maturing in 20 years. Key rate duration enables you to determine the sensitivity of the price of a bond to nonparallel shifts in the yield curve. This example uses bndkrdur to construct a portfolio to hedge the interest-rate risk of a U.S. Treasury bond maturing in 20 years.

Specify the bond.
```

Settle = datenum('2-Dec-2008');
CouponRate = 5.500/100;
Maturity = datenum('15-Aug-2028');
Price = 128.68;

```

The interest-rate risk of this bond is hedged with the following four on-the-run Treasury bonds:
```

Maturity 30 = datenum('15-May-2038'); % 30-year bond
Coupon 30 = .045;
Price_30 = 124.69;
Maturity_10 = datenum('15-Nov-2018'); %10-year note
Coupon_10 = .0375;
Price_\overline{10}0 = 109.35;
Maturity_05 = datenum('30-Nov-2013'); % 5-year note
Coupon 05 = .02;
Price_}\overline{0}5=101.67
Maturity_02 = datenum('30-Nov-2010'); % 2-year note
Coupon_02 = .01250;
Price_02 = 100.72;

```

You can get the Treasury spot or zero curve from http://www.treas.gov/offices/domestic-finance/debt-management/interest-rate/yield.shtml:
```

ZeroDates = daysadd(Settle,[30 90 180 360 360*2 360*3 360*5 ...
360*7 360*10 360*20 360*30]);
ZeroRates = ([0.09 0.07 0.44 0.81 0.90 1.16 1.71 2.13 2.72 3.51 3.22]/100)';

```

Compute the key rate durations for both the bond and the hedging portfolio.
BondKRD = bndkrdur([ZeroDates ZeroRates], CouponRate, Settle,... Maturity,'keyrates',[2 5 10 20]);
```

HedgeMaturity = [Maturity_02;Maturity_05;Maturity_10;Maturity_30];
HedgeCoupon = [Coupon 02;Coupon 05;Coupon_10;Coupon_30];
HedgeKRD = bndkrdur([ZeroDates ZeroRates], HedgeCoupon,...
Settle, HedgeMaturity, 'keyrates',[2 5 10 20])
HedgeKRD = 4×4

| 1.9675 | 0 | 0 | 0 |
| ---: | ---: | ---: | ---: |
| 0.1269 | 4.6152 | 0 | 0 |
| 0.2129 | 0.7324 | 7.4010 | 0 |
| 0.2229 | 0.7081 | 2.1487 | 14.5172 |

```

Compute the dollar durations for each of the instruments and each of the key rates (assuming holding 100 bonds).
```

PortfolioDD = 100*Price* BondKRD;
HedgeDD = bsxfun(@times, HedgeKRD,[Price_30;Price_10;Price_05;Price_02])
HedgeDD = 4×4
103 x

| 0.2453 | 0 | 0 | 0 |
| ---: | ---: | ---: | ---: |
| 0.0139 | 0.5047 | 0 | 0 |
| 0.0216 | 0.0745 | 0.7525 | 0 |
| 0.0224 | 0.0713 | 0.2164 | 1.4622 |

```

Compute the number of bonds to sell short to obtain a key rate duration that is 0 for the entire portfolio.

NumBonds = PortfolioDD/HedgeDD
NumBonds \(=1 \times 4\)
\[
\begin{array}{llll}
3.8973 & 6.1596 & 23.0282 & 80.0522
\end{array}
\]

These results indicate selling 4, 6, 23 and 80 bonds respectively of the \(2-, 5-10-\), and \(30-\) year bonds achieves a portfolio that is neutral with respect to the \(2-, 5-10\)-, and 30 -year spot rates.

\section*{See Also}
blsdelta|blsgamma|blsprice|blsvega|bndconvy|bnddury|bndkrdur | bndprice|zbtprice|zero2disc|zero2fwd

\section*{Related Examples}
- "Pricing and Analyzing Equity Derivatives" on page 2-45
- on page 10-18
- "Sensitivity of Bond Prices to Interest Rates" on page 10-2
- "Bond Portfolio for Hedging Duration and Convexity" on page 10-6
- "Bond Prices and Yield Curve Parallel Shifts" on page 10-10
- "Term Structure Analysis and Interest-Rate Swaps" on page 10-22
- "Plotting Sensitivities of an Option" on page 10-30
- "Plotting Sensitivities of a Portfolio of Options" on page 10-33

\section*{Greek-Neutral Portfolios of European Stock Options}

The option sensitivity measures familiar to most option traders are often referred to as the greeks: delta, gamma, vega, lambda, rho, and theta. Delta is the price sensitivity of an option with respect to changes in the price of the underlying asset. It represents a firstorder sensitivity measure analogous to duration in fixed income markets. Gamma is the sensitivity of an option's delta to changes in the price of the underlying asset, and represents a second-order price sensitivity analogous to convexity in fixed income markets. Vega is the price sensitivity of an option with respect to changes in the volatility of the underlying asset. See "Pricing and Analyzing Equity Derivatives" on page 2-45 or the Glossary for other definitions.

The greeks of a particular option are a function of the model used to price the option. However, given enough different options to work with, a trader can construct a portfolio with any desired values for its greeks. For example, to insulate the value of an option portfolio from small changes in the price of the underlying asset, one trader might construct an option portfolio whose delta is zero. Such a portfolio is then said to be "delta neutral." Another trader may want to protect an option portfolio from larger changes in the price of the underlying asset, and so might construct a portfolio whose delta and gamma are both zero. Such a portfolio is both delta and gamma neutral. A third trader may want to construct a portfolio insulated from small changes in the volatility of the underlying asset in addition to delta and gamma neutrality. Such a portfolio is then delta, gamma, and vega neutral.

Using the Black-Scholes model for European options, this example creates an equity option portfolio that is simultaneously delta, gamma, and vega neutral. The value of a particular greek of an option portfolio is a weighted average of the corresponding greek of each individual option. The weights are the quantity of each option in the portfolio. Hedging an option portfolio thus involves solving a system of linear equations, an easy process in MATLAB.

\section*{Step 1}

Create an input data matrix to summarize the relevant information. Each row of the matrix contains the standard inputs to Financial Toolbox Black-Scholes suite of functions: column 1 contains the current price of the underlying stock; column 2 the strike price of each option; column 3 the time to-expiry of each option in years; column 4 the annualized stock price volatility; and column 5 the annualized dividend rate of the underlying asset. Rows 1 and 3 are data related to call options, while rows 2 and 4 are data related to put options.
\begin{tabular}{rrrrll} 
DataMatrix \(=[100.000\) & 100 & 0.2 & 0.3 & 0 & \% Call \\
119.100 & 125 & 0.2 & 0.2 & 0.025 & \% Put \\
87.200 & 85 & 0.1 & 0.23 & 0 & \% Call \\
301.125 & 315 & 0.5 & 0.25 & \(0.0333]\) & \% Put
\end{tabular}

Also, assume that the annualized risk-free rate is \(10 \%\) and is constant for all maturities of interest.
```

RiskFreeRate = 0.10;

```

For clarity, assign each column of DataMatrix to a column vector whose name reflects the type of financial data in the column.
```

StockPrice = DataMatrix(:,1);
StrikePrice = DataMatrix(:,2);
ExpiryTime = DataMatrix(:,3);
Volatility = DataMatrix(:,4);
DividendRate = DataMatrix(:,5);

```

\section*{Step 2}

Based on the Black-Scholes model, compute the prices, and the delta, gamma, and vega sensitivity greeks of each of the four options. The functions blsprice and blsdelta have two outputs, while blsgamma and blsvega have only one. The price and delta of a call option differ from the price and delta of an otherwise equivalent put option, in contrast to the gamma and vega sensitivities, which are valid for both calls and puts.
```

[CallPrices, PutPrices] = blsprice(StockPrice, StrikePrice,...
RiskFreeRate, ExpiryTime, Volatility, DividendRate);
[CallDeltas, PutDeltas] = blsdelta(StockPrice,...
StrikePrice, RiskFreeRate, ExpiryTime, Volatility,...
DividendRate);
Gammas = blsgamma(StockPrice, StrikePrice, RiskFreeRate,...
ExpiryTime, Volatility , DividendRate)';
Vegas = blsvega(StockPrice, StrikePrice, RiskFreeRate,...
ExpiryTime, Volatility , DividendRate)';

```

Extract the prices and deltas of interest to account for the distinction between call and puts.

Prices \(=\) [CallPrices(1) PutPrices(2) CallPrices(3)...
PutPrices(4)];

Deltas \(=[\) CallDeltas(1) PutDeltas(2) CallDeltas(3)...
PutDeltas(4)];

\section*{Step 3}

Now, assuming an arbitrary portfolio value of \(\$ 17,000\), set up and solve the linear system of equations such that the overall option portfolio is simultaneously delta, gamma, and vega-neutral. The solution computes the value of a particular greek of a portfolio of options as a weighted average of the corresponding greek of each individual option in the portfolio. The system of equations is solved using the back slash ( \(\backslash\) ) operator discussed in "Solving Simultaneous Linear Equations" on page 1-13.
```

A = [Deltas; Gammas; Vegas; Prices];
b = [0; 0; 0; 17000];
OptionQuantities = A\b; % Quantity (number) of each option.

```

\section*{Step 4}

Finally, compute the market value, delta, gamma, and vega of the overall portfolio as a weighted average of the corresponding parameters of the component options. The weighted average is computed as an inner product of two vectors.
```

PortfolioValue = Prices * OptionQuantities;
PortfolioDelta = Deltas * OptionQuantities;
PortfolioGamma = Gammas * OptionQuantities;
PortfolioVega = Vegas * OptionQuantities;

```

The output for these computations is:
\begin{tabular}{crrrcr} 
Option & Price & Delta & Gamma & Vega & Quantity \\
1 & 6.3441 & 0.5856 & 0.0290 & 17.4293 & 22332.6131 \\
2 & 6.6035 & -0.6255 & 0.0353 & 20.0347 & 6864.0731 \\
3 & 4.2993 & 0.7003 & 0.0548 & 9.5837 & -15654.8657 \\
4 & 22.7694 & -0.4830 & 0.0074 & 83.5225 & -4510.5153
\end{tabular}

Portfolio Value: \$17000.00
Portfolio Delta: 0.00
Portfolio Gamma: -0.00
Portfolio Vega : 0.00
You can verify that the portfolio value is \(\$ 17,000\) and that the option portfolio is indeed delta, gamma, and vega neutral, as desired. Hedges based on these measures are effective only for small changes in the underlying variables.

\section*{See Also}
blsdelta|blsgamma|blsprice|blsvega|bndconvy|bnddury|bndkrdur | bndprice|zbtprice|zero2disc|zero2fwd

\section*{Related Examples}
- "Pricing and Analyzing Equity Derivatives" on page 2-45
- "Sensitivity of Bond Prices to Interest Rates" on page 10-2
- "Bond Portfolio for Hedging Duration and Convexity" on page 10-6
- "Bond Prices and Yield Curve Parallel Shifts" on page 10-10
- "Bond Prices and Yield Curve Nonparallel Shifts" on page 10-15
- "Term Structure Analysis and Interest-Rate Swaps" on page 10-22
- "Plotting Sensitivities of an Option" on page 10-30
- "Plotting Sensitivities of a Portfolio of Options" on page 10-33

\section*{Term Structure Analysis and Interest-Rate Swaps}

This example illustrates some of the term-structure analysis functions found in Financial Toolbox software. Specifically, it illustrates how to derive implied zero (spot) and forward curves from the observed market prices of coupon-bearing bonds. The zero and forward curves implied from the market data are then used to price an interest rate swap agreement.

In an interest rate swap, two parties agree to a periodic exchange of cash flows. One of the cash flows is based on a fixed interest rate held constant throughout the life of the swap. The other cash flow stream is tied to some variable index rate. Pricing a swap at inception amounts to finding the fixed rate of the swap agreement. This fixed rate, appropriately scaled by the notional principal of the swap agreement, determines the periodic sequence of fixed cash flows.

In general, interest rate swaps are priced from the forward curve such that the variable cash flows implied from the series of forward rates and the periodic sequence of fixedrate cash flows have the same current value. Thus, interest rate swap pricing and term structure analysis are intimately related.

\section*{Step 1}

Specify values for the settlement date, maturity dates, coupon rates, and market prices for 10 U.S. Treasury Bonds. This data allows you to price a five-year swap with net cash flow payments exchanged every six months. For simplicity, accept default values for the end-of-month payment rule (rule in effect) and day-count basis (actual/actual). To avoid issues of accrued interest, assume that all Treasury Bonds pay semiannual coupons and that settlement occurs on a coupon payment date.
```

Settle = datenum('15-Jan-1999');
BondData = {'15-Jul-1999' 0.06000 99.93
'15-Jan-2000' 0.06125 99.72
'15-Jul-2000' 0.06375 99.70
'15-Jan-2001' 0.06500 99.40
'15-Jul-2001' 0.06875 99.73
'15-Jan-2002' 0.07000 99.42
'15-Jul-2002' 0.07250 99.32
'15-Jan-2003' 0.07375 98.45
'15-Jul-2003' 0.07500 97.71
'15-Jan-2004' 0.08000 98.15};

```

BondData is an instance of a MATLAB cell array, indicated by the curly braces ( \(\}\) ).
Next assign the date stored in the cell array to Maturity, CouponRate, and Prices vectors for further processing.
```

Maturity = datenum(char(BondData{:,1}));
CouponRate = [BondData{:,2}]';
Prices = [BondData{:,3}]';
Period = 2; % semiannual coupons

```

\section*{Step 2}

Now that the data has been specified, use the term structure function zbtprice to bootstrap the zero curve implied from the prices of the coupon-bearing bonds. This implied zero curve represents the series of zero-coupon Treasury rates consistent with the prices of the coupon-bearing bonds such that arbitrage opportunities will not exist.
```

ZeroRates = zbtprice([Maturity CouponRate], Prices, Settle);

```

The zero curve, stored in ZeroRates, is quoted on a semiannual bond basis (the periodic, six-month, interest rate is doubled to annualize). The first element of ZeroRates is the annualized rate over the next six months, the second element is the annualized rate over the next 12 months, and so on.

\section*{Step 3}

From the implied zero curve, find the corresponding series of implied forward rates using the term structure function zero2fwd.

ForwardRates = zero2fwd(ZeroRates, Maturity, Settle);
The forward curve, stored in ForwardRates, is also quoted on a semiannual bond basis. The first element of ForwardRates is the annualized rate applied to the interval between settlement and six months after settlement, the second element is the annualized rate applied to the interval from six months to 12 months after settlement, and so on. This implied forward curve is also consistent with the observed market prices such that arbitrage activities will be unprofitable. Since the first forward rate is also a zero rate, the first element of ZeroRates and ForwardRates are the same.

\section*{Step 4}

Now that you have derived the zero curve, convert it to a sequence of discount factors with the term structure function zero2disc.
```

DiscountFactors = zero2disc(ZeroRates, Maturity, Settle);

```

\section*{Step 5}

From the discount factors, compute the present value of the variable cash flows derived from the implied forward rates. For plain interest rate swaps, the notional principal remains constant for each payment date and cancels out of each side of the present value equation. The next line assumes unit notional principal.
```

PresentValue = sum((ForwardRates/Period) .* DiscountFactors);

```

\section*{Step 6}

Compute the swap's price (the fixed rate) by equating the present value of the fixed cash flows with the present value of the cash flows derived from the implied forward rates. Again, since the notional principal cancels out of each side of the equation, it is assumed to be 1 .
```

SwapFixedRate = Period * PresentValue / sum(DiscountFactors);

```

The output for these computations is:
\begin{tabular}{cc} 
Zero Rates & Forward Rates \\
0.0614 & 0.0614 \\
0.0642 & 0.0670 \\
0.0660 & 0.0695 \\
0.0684 & 0.0758 \\
0.0702 & 0.0774 \\
0.0726 & 0.0846 \\
0.0754 & 0.0925 \\
0.0795 & 0.1077 \\
0.0827 & 0.1089 \\
0.0868 & 0.1239
\end{tabular}

Swap Price (Fixed Rate) \(=0.0845\)
All rates are in decimal format. The swap price, \(8.45 \%\), would likely be the mid-point between a market-maker's bid/ask quotes.

\section*{See Also}
blsdelta|blsgamma|blsprice|blsvega|bndconvy|bnddury|bndkrdur | bndprice|zbtprice|zero2disc|zero2fwd

\section*{Related Examples}
- "Pricing and Analyzing Equity Derivatives" on page 2-45
- on page 10-18
- "Sensitivity of Bond Prices to Interest Rates" on page 10-2
- "Bond Portfolio for Hedging Duration and Convexity" on page 10-6
- "Bond Prices and Yield Curve Parallel Shifts" on page 10-10
- "Bond Prices and Yield Curve Nonparallel Shifts" on page 10-15
- "Plotting Sensitivities of an Option" on page 10-30
- "Plotting Sensitivities of a Portfolio of Options" on page 10-33

\section*{Plotting an Efficient Frontier Using portopt}

This example plots the efficient frontier of a hypothetical portfolio of three assets. It illustrates how to specify the expected returns, standard deviations, and correlations of a portfolio of assets, how to convert standard deviations and correlations into a covariance matrix, and how to compute and plot the efficient frontier from the returns and covariance matrix. The example also illustrates how to randomly generate a set of portfolio weights, and how to add the random portfolios to an existing plot for comparison with the efficient frontier.

First, specify the expected returns, standard deviations, and correlation matrix for a hypothetical portfolio of three assets.
```

Returns = [0.1 0.15 0.12];
STDs = [0.2 0.25 0.18];
Correlations = [ lllll
0.3 1 0.3
0.4 0.3 1 ];

```

Convert the standard deviations and correlation matrix into a variance-covariance matrix with the Financial Toolbox function corr2cov.

Covariances \(=\) corr2cov(STDs, Correlations);
Evaluate and plot the efficient frontier at 20 points along the frontier, using the function portopt and the expected returns and corresponding covariance matrix. Although rather elaborate constraints can be placed on the assets in a portfolio, for simplicity accept the default constraints and scale the total value of the portfolio to 1 and constrain the weights to be positive (no short-selling).

Note portopt has been partially removed and will no longer accept ConSet or varargin arguments. Use Portfolio object instead to solve portfolio problems that are more than a long-only fully-invested portfolio. For information on the workflow when using Portfolio objects, see "Portfolio Object Workflow" on page 4-21. For more information on migrating portopt code to Portfolio, see "portopt Migration to Portfolio Object" on page 3-14.
portopt(Returns, Covariances, 20)


Now that the efficient frontier is displayed, randomly generate the asset weights for 1000 portfolios starting from the MATLAB initial state.
```

rng('default')
Weights = rand(1000, 3);

```

The previous line of code generates three columns of uniformly distributed random weights, but does not guarantee they sum to 1 . The following code segment normalizes the weights of each portfolio so that the total of the three weights represent a valid portfolio.
```

Total = sum(Weights, 2); % Add the weights
Total = Total(:,ones(3,1)); % Make size-compatible matrix
Weights = Weights./Total; % Normalize so sum = 1

```

Given the 1000 random portfolios created, compute the expected return and risk of each portfolio associated with the weights.
```

[PortRisk, PortReturn] = portstats(Returns, Covariances, ...
Weights);

```

Finally, hold the current graph, and plot the returns and risks of each portfolio on top of the existing efficient frontier for comparison. After plotting, annotate the graph with a title and return the graph to default holding status (any subsequent plots will erase the existing data). The efficient frontier appears in blue, while the 1000 random portfolios appear as a set of red dots on or below the frontier.
```

hold on
plot (PortRisk, PortReturn, '.r')
title('Mean-Variance Efficient Frontier and Random Portfolios')
hold off

```


\section*{See Also}
blsdelta|blsgamma|blsprice|blsvega|bndconvy|bnddury|bndkrdur | bndprice|corr2cov|portopt|zbtprice|zero2disc|zero2fwd

\section*{Related Examples}
- "Plotting Sensitivities of an Option" on page 10-30
- "Plotting Sensitivities of a Portfolio of Options" on page 10-33
- "Pricing and Analyzing Equity Derivatives" on page 2-45
- on page 10-18
- "Sensitivity of Bond Prices to Interest Rates" on page 10-2
- "Bond Portfolio for Hedging Duration and Convexity" on page 10-6
- "Bond Prices and Yield Curve Parallel Shifts" on page 10-10
- "Bond Prices and Yield Curve Nonparallel Shifts" on page 10-15
- "Term Structure Analysis and Interest-Rate Swaps" on page 10-22

\section*{Plotting Sensitivities of an Option}

This example creates a three-dimensional plot showing how gamma changes relative to price for a Black-Scholes option.

Recall that gamma is the second derivative of the option price relative to the underlying security price. The plot in this example shows a three-dimensional surface whose \(z\)-value is the gamma of an option as price ( \(x\)-axis) and time ( \(y\)-axis) vary. The plot adds yet a fourth dimension by showing option delta (the first derivative of option price to security price) as the color of the surface. First set the price range of the options, and set the time range to one year divided into half-months and expressed as fractions of a year.
```

Range = 10:70;
Span = length(Range);
j = 1:0.5:12;
Newj = j(ones(Span,1),:)'/12;

```

For each time period, create a vector of prices from 10 to 70 and create a matrix of all ones.
```

JSpan = ones(length(j),1);
NewRange = Range(JSpan,:);
Pad = ones(size(Newj));

```

Calculate the gamma and delta sensitivities (greeks) using the blsgamma and blsdelta functions. Gamma is the second derivative of the option price with respect to the stock price, and delta is the first derivative of the option price with respect to the stock price. The exercise price is \(\$ 40\), the risk-free interest rate is \(10 \%\), and volatility is 0.35 for all prices and periods.
```

ZVal = blsgamma(NewRange, 40*Pad, 0.1*Pad, Newj, 0.35*Pad);
Color = blsdelta(NewRange, 40*Pad, 0.1*Pad, Newj, 0.35*Pad);

```

Display the greeks as a function of price and time. Gamma is the \(z\)-axis; delta is the color.
```

mesh(Range, j, ZVal, Color);
xlabel('Stock Price (\$)');
ylabel('Time (months)');
zlabel('Gamma');
title('Call Option Price Sensitivity');
axis([10 70 1 12 -inf inf]);
view(-40, 50);
colorbar('horiz');

```

\section*{Call Option Price Sensitivity}



\section*{See Also}
blsdelta|blsgamma|blsprice|blsvega|bndconvy|bnddury|bndkrdur | bndprice | corr2cov | portopt | zbtprice | zero2disc | zero2fwd

\section*{Related Examples}
- "Plotting Sensitivities of a Portfolio of Options" on page 10-33
- "Pricing and Analyzing Equity Derivatives" on page 2-45
- on page 10-18
- "Sensitivity of Bond Prices to Interest Rates" on page 10-2
- "Bond Portfolio for Hedging Duration and Convexity" on page 10-6
- "Bond Prices and Yield Curve Parallel Shifts" on page 10-10
- "Bond Prices and Yield Curve Nonparallel Shifts" on page 10-15
- "Term Structure Analysis and Interest-Rate Swaps" on page 10-22

\section*{Plotting Sensitivities of a Portfolio of Options}

This example plots gamma as a function of price and time for a portfolio of 10 BlackScholes options.

The plot in this example shows a three-dimensional surface. For each point on the surface, the height ( \(z\)-value) represents the sum of the gammas for each option in the portfolio weighted by the amount of each option. The \(x\)-axis represents changing price, and the \(y\)-axis represents time. The plot adds a fourth dimension by showing delta as surface color. This example has applications in hedging. First set up the portfolio with arbitrary data. Current prices range from \(\$ 20\) to \(\$ 90\) for each option. Then, set the corresponding exercise prices for each option.
```

Range = 20:90;
PLen = length(Range);
ExPrice = [75 70 50 55 75 50 40 75 60 35];

```

Set all risk-free interest rates to \(10 \%\), and set times to maturity in days. Set all volatilities to 0.35 . Set the number of options of each instrument, and allocate space for matrices.
```

Rate = 0.1*ones(10,1);
Time = [[$$
\begin{array}{lllllllllllll}{36}&{36}&{36}&{27}&{18}&{18}&{18}&{9}&{9}&{9}\end{array}
$$];
Sigma = 0.35*ones(10,1);
NumOpt = 1000*[4 4 8 3 5 5 5.5 2 4.8 4.8 3 4.8 2.5];
ZVal = zeros(36, PLen);
Color = zeros(36, PLen);

```

For each instrument, create a matrix (of size Time by PLen) of prices for each period.
```

for i = 1:10
Pad = ones(Time(i),PLen);
NewR = Range(ones(Time(i),1),:);

```

Create a vector of time periods 1 to Time and a matrix of times, one column for each price.
```

T = (1:Time(i))';
NewT = T(:,ones(PLen,1));

```

Use the Black-Scholes gamma and delta sensitivity functions blsgamma and blsdelta to compute gamma and delta.
```

ZVal(36-Time(i)+1:36,:) = ZVal(36-Time(i)+1:36,:) ...
+ NumOpt(i) * blsgamma(NewR, ExPrice(i)*Pad, ...

```
```

        Rate(i)*Pad, NewT/36, Sigma(i)*Pad);
    ```
```

Color(36-Time(i)+1:36,:) = Color(36-Time(i)+1:36,:) ...

```
Color(36-Time(i)+1:36,:) = Color(36-Time(i)+1:36,:) ...
    + NumOpt(i) * blsdelta(NewR, ExPrice(i)*Pad, ...
    + NumOpt(i) * blsdelta(NewR, ExPrice(i)*Pad, ...
    Rate(i)*Pad, NewT/36, Sigma(i)*Pad);
```

    Rate(i)*Pad, NewT/36, Sigma(i)*Pad);
    ```
end
Draw the surface as a mesh, set the viewpoint, and reverse the \(x\)-axis because of the viewpoint. The axes range from 20 to 90,0 to 36 , and \(-\infty\) to \(\infty\).
```

mesh(Range, 1:36, ZVal, Color);
view(60,60);
set(gca, 'xdir','reverse', 'tag', 'mesh_axes_3');
axis([20 90 0 36 -inf inf]);

```


Add a title and axis labels and draw a box around the plot. Annotate the colors with a bar and label the color bar.
```

title('Call Option Portfolio Sensitivity');
xlabel('Stock Price (\$)');
ylabel('Time (months)');
zlabel('Gamma');
set(gca, 'box', 'on');
colorbar('horiz');

```


\section*{See Also}
blsdelta|blsgamma|blsprice|blsvega|bndconvy|bnddury|bndkrdur | bndprice|corr2cov|portopt|zbtprice|zero2disc|zero2fwd

\section*{Related Examples}
- "Plotting Sensitivities of an Option" on page 10-30
- "Pricing and Analyzing Equity Derivatives" on page 2-45
- on page 10-18
- "Sensitivity of Bond Prices to Interest Rates" on page 10-2
- "Bond Portfolio for Hedging Duration and Convexity" on page 10-6
- "Bond Prices and Yield Curve Parallel Shifts" on page 10-10
- "Bond Prices and Yield Curve Nonparallel Shifts" on page 10-15
- "Term Structure Analysis and Interest-Rate Swaps" on page 10-22

\section*{Financial Time Series Analysis}
- "Creating Financial Time Series Objects" on page 11-2
- "Visualizing Financial Time Series Objects" on page 11-31

\section*{Creating Financial Time Series Objects}
```

In this section...
"Introduction" on page 11-2
"Using the Constructor" on page 11-2
"Transforming a Text File" on page 11-28

```

\section*{Introduction}

Financial Toolbox software provides three ways to create a financial time series object:
- At the command line using the object constructor fints
- From a text data file through the function ascii2fts
- Use the Financial Time Series app, you can create a financial time series (fints) object from one or more selected variables. For more information, see "Creating a Financial Time Series Object" on page 14-11.

The structure of the object minimally consists of a description field, a frequency indicator field, the date vector field, and at least one data series vector. The names for the fields are fixed for the first three fields: desc, freq, and dates. You can specify names of your choice for any data series vectors. If you do not specify names, the object uses the default names series1, series2, series3, and so on.

If time-of-day information is incorporated in the date vector, the object contains an additional field named times.

\section*{Using the Constructor}

The object constructor function fints has five different syntaxes. These forms exist to simplify object construction. The syntaxes vary according to the types of input arguments presented to the constructor. The syntaxes are
- Single Matrix Input
- See "Time-of-Day Information Excluded" on page 11-3.
- See "Time-of-Day Information Included" on page 11-9.
- Separate Vector Input
- See "Time-of-Day Information Excluded" on page 11-12.
- See "Time-of-Day Information Included" on page 11-14.
- See "Data Name Input" on page 11-16.
- See "Frequency Indicator Input" on page 11-22.
- See "Description Field Input" on page 11-25.

\section*{Single Matrix Input}

The date information provided with this syntax must be in serial date number format. The date number may on page 11-9 or may not on page 11-3 include time-of-day information.

Note If you are unfamiliar with the concepts of date character vectors and serial date numbers, consult "Handle and Convert Dates" on page 2-2.

\section*{Time-of-Day Information Excluded}
```

fts = fints(dates and data)

```

In this simplest form of syntax, the input must be at least a two-column matrix. The first column contains the dates in serial date format; the second column is the data series. The input matrix can have more than two columns, each additional column representing a different data series or set of observations.

If the input is a two-column matrix, the output object contains four fields: desc, freq, dates, and series1. The description field, desc, defaults to blanks ' ' , and the frequency indicator field, freq, defaults to 0 . The dates field, dates, contains the serial dates from the first column of the input matrix, while the data series field, series1, has the data from the second column of the input matrix.

The first example makes two financial time series objects. The first one has only one data series, while the other has more than one. A random vector provides the values for the data series. The range of dates is arbitrarily chosen using the today function:
```

date series = (today:today+100)';
data series = exp(randn(1, 101))';
dates_and_data = [date_series data_series];
fts1 = fints(dates_and_data)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints (line 165)

```

Warning: FINTS will be removed in a future release. Use TIMETABLE instead. > In fints/display (line 66)
```

fts1 =
desc: (none)
freq: Unknown (0)
'dates: (101)' 'series1: (101)'
'03-0ct-2017' [ 0.97]
'04-0ct-2017' [ 1.20]
'05-0ct-2017' [ 0.21]
'06-Oct-2017' [ 0.92]
'07-0ct-2017' [ 4.97]
'08-0ct-2017' [ 1.10]
'09-0ct-2017' [ 1.04]
'10-0ct-2017' [ 0.48]
'11-Oct-2017' [ 0.97]
'12-0ct-2017' [ 1.26]
'13-0ct-2017' [ 1.53]
'14-0ct-2017' [ 0.69]
'15-0ct-2017' [ 0.79]
'16-0ct-2017' [ 7.57]
'17-Oct-2017' [ 0.10]
'18-0ct-2017' [ 9.29]
'19-0ct-2017' [ 1.40]
'20-0ct-2017' [ 2.72]
'21-0ct-2017' [ 0.19]
'22-0ct-2017' [ 0.55]
'23-0ct-2017' [ 0.76]
'24-0ct-2017' [ 1.53]
'25-0ct-2017' [ 0.19]
'26-0ct-2017' [ 1.60]
'27-0ct-2017' [ 0.30]
'28-0ct-2017' [ 1.07]
'29-0ct-2017' [ 1.92]
'30-0ct-2017' [ 1.39]
'31-0ct-2017' [ 2.95]
'01-Nov-2017' [ 2.73]
'02-Nov-2017' [ 0.52]
'03-Nov-2017' [ 1.29]
'04-Nov-2017' [ 0.39]
'05-Nov-2017' [ 0.27]
'06-Nov-2017' [ 2.52]

```
\begin{tabular}{|c|c|}
\hline 07-Nov-2017 ' & \(1.00]\) \\
\hline '08-Nov-2017' & \(0.95]\) \\
\hline '09-Nov-2017' & \(2.49]\) \\
\hline '10-Nov-2017' & \(1.81]\) \\
\hline '11-Nov-2017' & \(1.42]\) \\
\hline '12-Nov-2017' & 3.49] \\
\hline '13-Nov-2017' & \(2.53]\) \\
\hline '14-Nov-2017' & \(1.27]\) \\
\hline '15-Nov-2017' & \(0.50]\) \\
\hline '16-Nov-2017' & 0.52 ] \\
\hline '17-Nov-2017' & 3.29] \\
\hline '18-Nov-2017' & 0.20] \\
\hline '19-Nov-2017' & \(0.98]\) \\
\hline '20-Nov-2017' & \(0.14]\) \\
\hline '21-Nov-2017' & 2.77 ] \\
\hline '22-Nov-2017' & \(2.37]\) \\
\hline '23-Nov-2017' & 1.00] \\
\hline '24-Nov-2017' & \(0.93]\) \\
\hline '25-Nov-2017' & 0.08] \\
\hline '26-Nov-2017' & \(1.79]\) \\
\hline '27-Nov-2017' & \(0.11]\) \\
\hline '28-Nov-2017' & \(0.10]\) \\
\hline '29-Nov-2017' & \(1.08]\) \\
\hline '30-Nov-2017' & \(0.39]\) \\
\hline '01-Dec-2017' & [ 1.51] \\
\hline '02-Dec-2017' & [ 1.97] \\
\hline '03-Dec-2017' & \(2.36]\) \\
\hline '04-Dec-2017' & \(0.50]\) \\
\hline '05-Dec-2017' & 1.57] \\
\hline '06-Dec-2017' & 1.11] \\
\hline '07-Dec-2017' & \(2.28]\) \\
\hline '08-Dec-2017' & \(1.71]\) \\
\hline '09-Dec-2017' & \(2.45]\) \\
\hline '10-Dec-2017' & \(0.88]\) \\
\hline '11-Dec-2017' & \(0.86]\) \\
\hline '12-Dec-2017' & \(2.74]\) \\
\hline '13-Dec-2017' & \(0.12]\) \\
\hline '14-Dec-2017' & \(0.60]\) \\
\hline '15-Dec-2017' & \(0.28]\) \\
\hline '16-Dec-2017' & 0.68 ] \\
\hline '17-Dec-2017' & \(1.91]\) \\
\hline '18-Dec-2017' & \(2.28]\) \\
\hline '19-Dec-2017' & \(0.36]\) \\
\hline '20-Dec-2017 ' & 0.62 ] \\
\hline
\end{tabular}
\begin{tabular}{|c|c|}
\hline '21-Dec-2017' & \(1.15]\) \\
\hline '22-Dec-2017' & \(0.75]\) \\
\hline '23-Dec-2017' & \(1.35]\) \\
\hline '24-Dec-2017' & \(1.49]\) \\
\hline '25-Dec-2017' & \(0.39]\) \\
\hline '26-Dec-2017' & \(0.84]\) \\
\hline '27-Dec-2017' & \(0.12]\) \\
\hline '28-Dec-2017' & \(3.14]\) \\
\hline '29-Dec-2017' & \(0.53]\) \\
\hline 30-Dec-2017' & \(0.30]\) \\
\hline '31-Dec-2017' & 0.78 ] \\
\hline '01-Jan-2018' & \(0.24]\) \\
\hline '02-Jan-2018' & 0.98 ] \\
\hline '03-Jan-2018' & \(0.57]\) \\
\hline '04-Jan-2018' & 8.83] \\
\hline '05-Jan-2018' & \(3.12]\) \\
\hline '06-Jan-2018' & [ 0.08] \\
\hline '07-Jan-2018' & [ 1.55] \\
\hline '08-Jan-2018' & \(0.25]\) \\
\hline '09-Jan-2018' & 0.77 ] \\
\hline '10-Jan-2018' & 1.18] \\
\hline '11-Jan-2018' & 2.11] \\
\hline
\end{tabular}

Examine the contents of the object fts1. The actual date series you observe will vary according to the day when you run the example (the value of today). Also, your values in seriesl will differ from those shown, depending upon the sequence of random numbers generated:
```

fts1 =
desc: (none)
freq: Unknown (0)
'dates: (101)' 'series1: (101)'
'12-Jul-1999' [ 0.3124]
'13-Jul-1999' [ 3.2665]
'14-Jul-1999' [ 0.9847]
'15-Jul-1999' [ 1.7095]
'16-Jul-1999' [ 0.4885]
'17-Jul-1999' [ 0.5192]
'18-Jul-1999' [ 1.3694]
'19-Jul-1999' [ 1.1127]
'20-Jul-1999' [ 6.3485]
'21-Jul-1999' [ 0.7595]
'22-Jul-1999' [ 9.1390]
'23-Jul-1999' [ 4.5201]
'24-Jul-1999' [ 0.1430]
'25-Jul-1999' [ 0.1863]
'26-Jul-1999' [ 0.5635]

```
```

27-Jul-1999' [ 0.8304]
28-Jul-1999' [ 1.0090]...

```

The output is truncated for brevity. There are actually 101 data points in the object.
The desc field displays as (none) instead of ' ' , and that the contents of the object display as cell array elements. Although the object displays as such, it should be thought of as a MATLAB structure containing the default field names for a single data series object: desc, freq, dates, and series1.

Now create an object with more than one data series in it:
```

date series = (today:today+100)';
data_series1 = exp(randn(1, 101))';
data series2 = exp(randn(1, 101))';
dates and data = [date series data series1 data series2];
fts2 = fints(dates_and_data)

```

Now look at the object created (again in abbreviated form):
```

```
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
```

```
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints (line 165)
> In fints (line 165)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/display (line 66)
> In fints/display (line 66)
fts2 =
fts2 =
    desc: (none)
    desc: (none)
    freq: Unknown (0)
    freq: Unknown (0)
    dates: (101)
    dates: (101)
    04-0ct-2017
    04-0ct-2017
    05-0ct-2017
    05-0ct-2017
    06-0ct-2017
    06-0ct-2017
    07-0ct-2017
    07-0ct-2017
    08-0ct-2017'
    08-0ct-2017'
    09-0ct-2017'
    09-0ct-2017'
    10-0ct-2017'
    10-0ct-2017'
    '11-0ct-2017'
    '11-0ct-2017'
    12-0ct-2017
    12-0ct-2017
    13-0ct-2017'
    13-0ct-2017'
    13-0ct-2017'
    13-0ct-2017'
    15-0ct-2017'
    15-0ct-2017'
    '16-0ct-2017'
    '16-0ct-2017'
    '16-0ct-2017'
    '16-0ct-2017'
    18-0ct-2017'
    18-0ct-2017'
    '19-0ct-2017'
    '19-0ct-2017'
    19-0ct-2017'
    19-0ct-2017'
    21-0ct-2017'
    21-0ct-2017'
    22-0ct-2017'
    22-0ct-2017'
    23-0ct-2017'
    23-0ct-2017'
    24-0ct-2017'
    24-0ct-2017'
    25-0ct-2017'
    25-0ct-2017'
    '26-0ct-2017
```

    '26-0ct-2017
    ```
\((101)^{\prime}\)
\(0.76]\)
\(4.84]\)
\(0.62]\)
\(1.39]\)
\(1.94]\)
\(1.09]\)
\(2.41]\)
\(1.38]\)
\(0.46]\)
\(0.16]\)
\(6.41]\)
\(0.55]\)
\(1.11]\)
\(1.76]\)
\(1.12]\)
\(0.40]\)
\(0.63]\)
\(0.88]\)
\(4.39]\)
\(0.42]\)
\(2.19]\)
\(1.36]\)
\(0.79]\)
\(0.35]\)
\begin{tabular}{|c|c|}
\hline series2: & (101) \\
\hline [ & \(4.56]\) \\
\hline [ & 0.97] \\
\hline [ & 5.13] \\
\hline [ & \(0.65]\) \\
\hline [ & \(1.80]\) \\
\hline [ & \(0.94]\) \\
\hline [ & 0.13] \\
\hline [ & \(0.37]\) \\
\hline [ & \(1.85]\) \\
\hline [ & \(0.95]\) \\
\hline [ & \(0.33]\) \\
\hline [ & \(0.53]\) \\
\hline [ & 1.28] \\
\hline [ & \(0.37]\) \\
\hline [ & \(2.65]\) \\
\hline [ & \(0.53]\) \\
\hline [ & 6.10] \\
\hline [ & \(0.34]\) \\
\hline [ & 1.22] \\
\hline [ & \(0.22]\) \\
\hline [ & 0.48 ] \\
\hline [ & \(0.55]\) \\
\hline [ & 1.49] \\
\hline & 2.57 \\
\hline
\end{tabular}
```

```
'series1:
```

```
'series1:
```

| '27-0ct-2017' | [ | $0.75]$ | [ | $1.35]$ |
| :---: | :---: | :---: | :---: | :---: |
| '28-0ct-2017' | [ | $0.92]$ | [ | $0.69]$ |
| '29-0ct-2017' | [ | $0.23]$ | [ | $2.26]$ |
| '30-0ct-2017' | [ | 1.21] | [ | $2.22]$ |
| '31-0ct-2017' | [ | $0.44]$ | [ | $1.13]$ |
| '01-Nov-2017' | [ | $0.91]$ | [ | 1.77] |
| '02-Nov-2017' | [ | 1.40] | [ | $1.51]$ |
| '03-Nov-2017' | [ | $0.40]$ | [ | $0.37]$ |
| '04-Nov-2017' | [ | $0.75]$ | [ | $2.14]$ |
| '05-Nov-2017' | [ | 1.42] | [ | $0.52]$ |
| '06-Nov-2017' | [ | $0.16]$ | [ | $0.55]$ |
| '07-Nov-2017' | [ | $2.82]$ | [ | 1.19] |
| '08-Nov-2017' | [ | $11.30]$ | [ | $0.74]$ |
| '09-Nov-2017' | [ | $2.61]$ | [ | 0.88 ] |
| '10-Nov-2017' | [ | 0.73 ] | [ | 1.81] |
| '11-Nov-2017' | [ | $1.54]$ | [ | $2.85]$ |
| '12-Nov-2017' | [ | $0.35]$ | [ | $0.82]$ |
| '13-Nov-2017' | [ | 6.54] | [ | 1.39] |
| '14-Nov-2017' | [ | $2.56]$ | [ | $0.79]$ |
| '15-Nov-2017' | [ | $2.20]$ | [ | 1.26] |
| '16-Nov-2017' | [ | 0.42 ] | [ | $1.55]$ |
| '17-Nov-2017' | [ | $1.38]$ | [ | $0.54]$ |
| '18-Nov-2017' | [ | $0.57]$ | [ | $1.32]$ |
| '19-Nov-2017' | [ | $0.73]$ | [ | $1.82]$ |
| '20-Nov-2017' | [ | $0.57]$ | [ | $1.10]$ |
| '21-Nov-2017' | [ | $0.36]$ | [ | $5.64]$ |
| '22-Nov-2017' | [ | $0.40]$ | [ | $0.54]$ |
| '23-Nov-2017' | [ | $0.81]$ | [ | $0.48]$ |
| '24-Nov-2017' | [ | $0.18]$ | [ | $0.17]$ |
| '25-Nov-2017' | [ | $1.84]$ | [ | $2.49]$ |
| '26-Nov-2017' | [ | $0.89]$ | [ | $2.38]$ |
| '27-Nov-2017' | [ | $2.01]$ | [ | $0.92]$ |
| '28-Nov-2017' | [ | 1.31] | [ | $2.46]$ |
| '29-Nov-2017' | [ | $1.64]$ | [ | 1.20] |
| '30-Nov-2017' | [ | $0.23]$ | [ | $1.34]$ |
| '01-Dec-2017' | [ | $0.36]$ | [ | 1.12] |
| '02-Dec-2017' | [ | $0.64]$ | [ | $1.55]$ |
| '03-Dec-2017' | [ | 1.12] | [ | 1.11] |
| '04-Dec-2017' | [ | 3.09] | [ | 16.24] |
| '05-Dec-2017' | [ | $0.75]$ | [ | $0.31]$ |
| '06-Dec-2017' | [ | 3.53] | [ | $0.16]$ |
| '07-Dec-2017' | [ | 1.61] | [ | $0.32]$ |
| '08-Dec-2017' | [ | 3.24] | [ | $0.34]$ |
| '09-Dec-2017' | [ | 1.14] | [ | $0.65]$ |
| '10-Dec-2017' | [ | $0.52]$ | [ | $0.84]$ |
| '11-Dec-2017' | [ | $0.23]$ | [ | $0.80]$ |
| '12-Dec-2017' | [ | $1.17]$ | [ | $1.72]$ |
| '13-Dec-2017' | [ | 2.27] | [ | 1.48] |
| '14-Dec-2017' | [ | $0.75]$ | [ | $2.12]$ |
| '15-Dec-2017' | [ | $0.58]$ | [ | $5.92]$ |
| '16-Dec-2017' | [ | 0.73 ] | [ | 3.40] |
| '17-Dec-2017' | [ | $0.33]$ | [ | 0.28] |
| '18-Dec-2017' | [ | $0.61]$ | [ | $0.10]$ |
| '19-Dec-2017' | [ | $0.83]$ | [ | $2.46]$ |
| '20-Dec-2017' | [ | 1.05] | [ | $0.16]$ |
| '21-Dec-2017' | , | $0.94]$ | [ | 1.07] |
| '22-Dec-2017' | [ | $1.84]$ | [ | $1.04]$ |
| '23-Dec-2017' | [ | 1.12] | [ | 9.27] |
| '24-Dec-2017' | [ | 6.14] | [ | $0.93]$ |
| '25-Dec-2017' | [ | 1.37] | [ | $0.60]$ |
| '26-Dec-2017' | [ | 6.08] | [ | 1.27] |
| '27-Dec-2017' | [ | 0.49] | [ | 1.28] |


| '28-Dec-2017' | 1.69] | [ | $1.07]$ |
| :---: | :---: | :---: | :---: |
| '29-Dec-2017' | [ 0.77] | [ | 0.54] |
| '30-Dec-2017' | [ 1.82] | [ | 0.29] |
| '31-Dec-2017' | [ 1.81] | [ | 1.37] |
| '01-Jan-2018' | [ 0.11] | [ | 0.26] |
| '02-Jan-2018' | [ 0.27] | [ | 0.36] |
| '03-Jan-2018' | [ 0.24] | [ | $3.79]$ |
| '04-Jan-2018' | [ 1.49] | [ | 0.66] |
| '05-Jan-2018' | [ 4.35] | [ | $0.87]$ |
| '06-Jan-2018' | [ 0.72] | [ | $2.46]$ |
| '07-Jan-2018' | [ 2.25] | [ | $0.74]$ |
| '08-Jan-2018' | [ 1.73] | [ | $2.80]$ |
| '09-Jan-2018' | [ 0.35] | [ | $0.71]$ |
| '10-Jan-2018' | [ 1.49] | [ | $2.75]$ |
| '11-Jan-2018' | [ 0.47] | [ | 1.88] |

The second data series name defaults to series2, as expected.
Before you can perform any operations on the object, you must set the frequency indicator field freq to the valid frequency of the data series contained in the object. You can leave the description field desc blank.

To set the frequency indicator field to a daily frequency, enter
fts2.freq $=1$, or
fts2.freq = 'daily'.
For more information, see fints.

## Time-of-Day Information Included

The serial date number used with this form of the fints function can incorporate time-ofday information. When time-of-day information is present, the output of the function contains a field times that indicates the time of day.

If you recode the previous example on page 11-3 to include time-of-day information, you can see the additional column present in the output object:

```
time_series = (now:now+100)';
data_series = exp(randn(1, 101))';
times_and_data = [time_series data_series];
fts1 = fints(times_and_data)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints (line 165)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/display (line 66)
fts1 =
```

| 'dates: (101)' | 'times: | (101) ${ }^{\prime}$ | 'series1: | (101) ${ }^{\prime}$ |
| :---: | :---: | :---: | :---: | :---: |
| '03-0ct-2017' | '16:17' |  | , | 0.81] |
| '04-0ct-2017' | '16:17' |  | [ | 0.42 ] |
| '05-0ct-2017' | '16:17' |  | [ | $0.35]$ |
| '06-0ct-2017' | '16:17' |  | [ | $0.76]$ |
| '07-0ct-2017' | '16:17' |  | [ | $0.65]$ |
| '08-0ct-2017' | '16:17' |  | [ | $0.66]$ |
| '09-0ct-2017' | '16:17' |  | [ | $2.67]$ |
| '10-0ct-2017' | '16:17' |  | [ | 0.74 ] |
| '11-0ct-2017' | '16:17' |  | [ | $3.14]$ |
| '12-0ct-2017' | '16:17' |  | [ | $0.59]$ |
| '13-0ct-2017' | '16:17' |  | [ | $2.64]$ |
| '14-0ct-2017' | '16:17' |  | [ | $0.59]$ |
| '15-0ct-2017' | '16:17' |  | [ | 1.19] |
| '16-0ct-2017' | '16:17' |  | [ | $2.64]$ |
| '17-0ct-2017' | '16:17' |  | [ | $0.66]$ |
| '18-0ct-2017' | '16:17' |  | [ | $0.65]$ |
| '19-0ct-2017' | '16:17' |  | [ | $7.41]$ |
| '20-0ct-2017' | '16:17' |  | [ | $2.59]$ |
| '21-0ct-2017' | '16:17' |  | [ | $0.65]$ |
| '22-0ct-2017' | '16:17' |  | , | 1.91 ] |
| '23-0ct-2017' | '16:17' |  | [ | 0.70 ] |
| '24-0ct-2017' | '16:17' |  | [ | 2.031 |
| '25-0ct-2017' | '16:17' |  | [ | $4.12]$ |
| '26-0ct-2017' | '16:17' |  | [ | $0.20]$ |
| '27-0ct-2017' | '16:17' |  | - | $2.80]$ |
| '28-0ct-2017' | '16:17' |  | [ | 4.30] |
| '29-0ct-2017' | '16:17' |  | [ | $1.05]$ |
| '30-0ct-2017' | '16:17' |  | [ | $5.73]$ |
| '31-0ct-2017' | '16:17' |  | , | $1.17]$ |
| '01-Nov-2017' | '16:17' |  | [ | $0.29]$ |
| '02-Nov-2017' | '16:17' |  | [ | $0.11]$ |
| '03-Nov-2017' | '16:17' |  | [ | 0.72 ] |
| '04-Nov-2017' | '16:17' |  | , | $2.04]$ |
| '05-Nov-2017' | '16:17' |  | [ | $1.37]$ |
| '06-Nov-2017' | '16:17' |  | [ | 1.51] |
| '07-Nov-2017' | '16:17' |  | [ | $0.56]$ |
| '08-Nov-2017' | '16:17' |  | , | $1.15]$ |
| '09-Nov-2017' | '16:17' |  | [ | $0.19]$ |
| '10-Nov-2017' | '16:17' |  | [ | $0.47]$ |
| '11-Nov-2017' | '16:17' |  | [ | $0.44]$ |
| '12-Nov-2017' | '16:17' |  | [ | 1.68 ] |
| '13-Nov-2017' | '16:17' |  | [ | $0.99]$ |
| '14-Nov-2017' | '16:17' |  | [ | $0.31]$ |
| '15-Nov-2017' | '16:17' |  | [ | $0.99]$ |
| '16-Nov-2017' | '16:17' |  | , | $0.50]$ |
| '17-Nov-2017' | '16:17' |  | [ | $0.51]$ |
| '18-Nov-2017' | '16:17' |  | [ | $2.37]$ |
| '19-Nov-2017' | '16:17' |  | [ | 1.12] |
| '20-Nov-2017' | '16:17' |  | [ | $1.49]$ |
| '21-Nov-2017' | '16:17' |  | [ | 2.42 ] |


| '22-Nov-2017' | '16:17' | [ | 1.20] |
| :---: | :---: | :---: | :---: |
| '23-Nov-2017' | 16:17' | [ | 1.73] |
| '24-Nov-2017' | 16:17' | [ | 1.98] |
| '25-Nov-2017' | 16:17' | [ | 3.22] |
| '26-Nov-2017' | 16:17' | [ | 1.61] |
| '27-Nov-2017' | 16:17' | [ | $4.11]$ |
| '28-Nov-2017' | '16:17' | [ | 1.02] |
| '29-Nov-2017' | 16:17' | [ | $0.95]$ |
| '30-Nov-2017' | 16:17' | [ | 5.48] |
| '01-Dec-2017' | 16:17' | [ | 0.60] |
| '02-Dec-2017' | 16:17' | [ | 1.00] |
| '03-Dec-2017' | '16:17' | [ | $2.51]$ |
| '04-Dec-2017' | 16:17' | [ | 1.16] |
| '05-Dec-2017' | '16:17' | [ | 4.08] |
| '06-Dec-2017' | 16:17' | [ | $2.81]$ |
| '07-Dec-2017' | 16:17' | [ | $1.34]$ |
| '08-Dec-2017' | 16:17' | [ | 0.46 ] |
| '09-Dec-2017' | '16:17' | [ | $1.76]$ |
| '10-Dec-2017' | '16:17' | [ | 0.25] |
| '11-Dec-2017' | 16:17' | [ | 1.28] |
| '12-Dec-2017' | '16:17' | [ | $2.24]$ |
| '13-Dec-2017' | 16:17' | [ | 1.24] |
| '14-Dec-2017' | 16:17' | [ | 2.41 ] |
| '15-Dec-2017' | '16:17' | [ | 7.68] |
| '16-Dec-2017' | 16:17' | [ | 2.52 ] |
| '17-Dec-2017' | '16:17' | [ | 1.31] |
| '18-Dec-2017' | '16:17' | [ | 1.90] |
| '19-Dec-2017' | '16:17' | [ | 1.53] |
| '20-Dec-2017' | 16:17' | [ | 0.27] |
| '21-Dec-2017' | 16:17' | [ | $0.66]$ |
| '22-Dec-2017' | '16:17' | [ | 3.40] |
| '23-Dec-2017' | '16:17' | [ | $0.96]$ |
| '24-Dec-2017' | '16:17' | [ | 1.79] |
| '25-Dec-2017' | '16:17' | [ | $0.37]$ |
| '26-Dec-2017' | 16:17' | [ | 1.07] |
| '27-Dec-2017' | 16:17' | [ | 1.82 ] |
| '28-Dec-2017' | 16:17' | [ | 0.26] |
| '29-Dec-2017' | '16:17' | [ | 1.42] |
| '30-Dec-2017' | '16:17' | , | $0.83]$ |
| '31-Dec-2017' | 16:17' | , | $0.39]$ |
| '01-Jan-2018' | '16:17' | [ | $0.96]$ |
| '02-Jan-2018' | 16:17' | [ | $0.15]$ |
| '03-Jan-2018' | '16:17' | [ | 0.12 ] |
| '04-Jan-2018' | '16:17' | [ | 0.31 ] |
| '05-Jan-2018' | '16:17' | [ | $0.37]$ |
| '06-Jan-2018' | '16:17' | , | $0.31]$ |
| '07-Jan-2018' | 16:17' | , | 0.18 ] |
| '08-Jan-2018' | '16:17' | [ | 1.33] |
| '09-Jan-2018' | 16:17' | [ | $0.20]$ |
| '10-Jan-2018' | '16:17' | [ | 1.12] |
| '11-Jan-2018' | '16:17' | [ | 2.20] |

## Separate Vector Input

The date information provided with this syntax can be in serial date number or date character vector format. The date information may on page 11-14 or may not on page 11-12 include time-of-day information.

## Time-of-Day Information Excluded

```
fts = fints(dates, data)
```

In this second syntax the dates and data series are entered as separate vectors to fints, the financial time series object constructor function. The dates vector must be a column vector, while the data series data can be a column vector (if there is only one data series) or a column-oriented matrix (for multiple data series). A column-oriented matrix, in this context, indicates that each column is a set of observations. Different columns are different sets of data series.

Here is an example:

```
dates = (today:today+100)';
data_series1 = exp(randn(1, 101))';
data-series2 = exp(randn(1, 101))';
data = [data_series1 data_series2];
fts = fints(dates, data)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints (line 165)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/display (line 66)
fts =
    desc: (none)
    freq: Unknown (0)
\begin{tabular}{lllll} 
'dates: (101)' & 'series1: & (101)' & 'series2: & (101)' \\
'03-0ct-2017' & [ & \(1.00]\) & {\([\)} & \(0.50]\) \\
'04-0ct-2017' & {\([\)} & \(1.10]\) & {\([\)} & \(0.20]\) \\
'05-0ct-2017' & {\([\)} & \(0.69]\) & {\([\)} & \(4.31]\) \\
'06-0ct-2017' & {\([\)} & \(0.23]\) & {\([\)} & \(7.77]\) \\
'07-0ct-2017' & {\([\)} & \(0.96]\) & {\([\)} & \(1.13]\) \\
'08-0ct-2017' & {\([\)} & \(2.61]\) & {\([\)} & \(0.37]\) \\
'10-0ct-2017' & {\([\)} & \(5.69]\) & {\([\)} & \(3.31]\) \\
'11-0ct-2017' & {\([\)} & \(0.65]\) & {\([\)} & \(0.55]\) \\
'12-0ct-2017' & {\([\)} & \(0.20]\) & {\([\)} & \(0.63]\) \\
13-0ct-2017' & {\([\)} & \(1.18]\) & {\([\)} & \(2.43]\) \\
'14-0ct-2017' & {\([\)} & \(1.46]\) & {\([\)} & \(0.25]\) \\
15-0ct-2017' & {\([\)} & \(0.80]\) & {\([\)} & \(0.14]\) \\
'16-0ct-2017' & {\([\)} & \(0.32]\) & {\([\)} & \(1.52]\) \\
'17-0ct-2017' & {\([\)} & \(7.57]\) & {\([\)} & \(1.49]\) \\
'18-0ct-2017' & {\([\)} & \(0.09]\) & {\([\)} & \(1.10]\) \\
& \(0.60]\) & {\([\)} & \(1.64]\)
\end{tabular}
```

| '19-0ct-2017' | $0.27]$ | $2.95]$ |
| :---: | :---: | :---: |
| '20-0ct-2017' | [ 0.53] | $2.64]$ |
| '21-0ct-2017' | [ 1.37] | $0.57]$ |
| '22-0ct-2017' | 1.15] | $2.25]$ |
| '23-0ct-2017' | [ 0.49] | 1.19] |
| '24-0ct-2017' | 2.17] | $0.60]$ |
| '25-0ct-2017' | $1.86]$ | $0.30]$ |
| '26-0ct-2017' | 1.91] | $1.91]$ |
| '27-0ct-2017' | $0.65]$ | $0.70]$ |
| '28-0ct-2017' | $2.85]$ | $1.05]$ |
| '29-0ct-2017' | $1.94]$ | $0.45]$ |
| '30-0ct-2017' | 12.29] | $0.21]$ |
| '31-0ct-2017' | $2.90]$ | $1.19]$ |
| '01-Nov-2017' | [ 3.18] | $0.94]$ |
| '02-Nov-2017' | 1.05] | $3.32]$ |
| '03-Nov-2017' | $0.28]$ | $2.23]$ |
| '04-Nov-2017' | $0.69]$ | $2.87]$ |
| '05-Nov-2017' | [ 0.47] | $0.47]$ |
| '06-Nov-2017' | $0.57]$ | $0.39]$ |
| '07-Nov-2017' | [ 1.74] | $0.28]$ |
| '08-Nov-2017' | $0.57]$ | $1.65]$ |
| '09-Nov-2017' | $0.41]$ | $16.27]$ |
| '10-Nov-2017' | $0.66]$ | $2.07]$ |
| '11-Nov-2017' | [ 0.85] | $0.46]$ |
| '12-Nov-2017' | 1.51] | 2.31] |
| '13-Nov-2017' | $0.39]$ | $0.32]$ |
| '14-Nov-2017' | 1.37] | $0.24]$ |
| '15-Nov-2017' | $1.08]$ | $2.05]$ |
| '16-Nov-2017' | $3.76]$ | $0.46]$ |
| '17-Nov-2017' | $0.81]$ | $1.37]$ |
| '18-Nov-2017' | $0.87]$ | $4.08]$ |
| '19-Nov-2017' | $0.31]$ | 1.49] |
| '20-Nov-2017' | $0.25]$ | $2.53]$ |
| '21-Nov-2017' | [ 1.36] | $0.20]$ |
| '22-Nov-2017' | [ 0.78] | $1.94]$ |
| '23-Nov-2017' | $1.65]$ | 8.49] |
| '24-Nov-2017' | $0.41]$ | $1.72]$ |
| ' 25 -Nov-2017' | $6.74]$ | 0.21] |
| '26-Nov-2017' | 1.13] | $0.82]$ |
| '27-Nov-2017' | $2.85]$ | 0.61] |
| '28-Nov-2017' | $0.80]$ | 1.47] |
| '29-Nov-2017' | $0.85]$ | 1.51] |
| '30-Nov-2017' | 1.99] | 1.50] |
| '01-Dec-2017' | 1.74] | $0.70]$ |
| '02-Dec-2017' | $0.33]$ | $0.55]$ |
| '03-Dec-2017' | $0.22]$ | $0.55]$ |
| '04-Dec-2017' | $0.33]$ | $2.35]$ |
| '05-Dec-2017' | $0.24]$ | $0.16]$ |
| '06-Dec-2017' | $1.06]$ | $0.81]$ |
| '07-Dec-2017' | $0.66]$ | 1.31] |
| '08-Dec-2017' | $0.69]$ | $0.52]$ |
| '09-Dec-2017' | $0.26]$ | $1.61]$ |
| '10-Dec-2017' | $2.18]$ | 0.93] |
| '11-Dec-2017' | 1.55] | $0.39]$ |
| '12-Dec-2017' | [ 0.91] | 1.18] |
| '13-Dec-2017' | [ 2.78] | $0.76]$ |
| '14-Dec-2017' | [ 0.42] | $0.66]$ |
| '15-Dec-2017' | [ 1.51] | $0.49]$ |
| '16-Dec-2017' | [ 1.42] | 1.06] |
| '17-Dec-2017' | [ 1.42] | $0.16]$ |
| '18-Dec-2017' | 0.48 ] | 0.67] |
| '19-Dec-2017' | [ 1.39] | $0.58]$ |



The result is exactly the same as the first syntax. The only difference between the first and second syntax is the way the inputs are entered into the constructor function.

## Time-of-Day Information Included

With this form of the function you can enter the time-of-day information either as a serial date number or as a date character vector. If more than one serial date and time are present, the entry must be in the form of a column-oriented matrix. If more than one character vector date and time are present, the entry must be a column-oriented cell array of character vectors for dates and times.

With date character vector input, the dates and times can initially be separate columnoriented date and time series, but you must concatenate them into a single columnoriented cell array before entering them as the first input to fints.

For date character vector input the allowable formats are

- 'ddmmmyy hh:mm' or 'ddmmmyyyy hh:mm'
- 'mm/dd/yy hh:mm' or 'mm/dd/yyyy hh:mm'
- 'dd-mmm-yy hh:mm' or 'dd-mmm-yyyy hh:mm'
- 'mmm.dd,yy hh:mm' or 'mmm.dd,yyyy hh:mm'

The next example shows time-of-day information input as serial date numbers in a column-oriented matrix:

```
f = fints([now;now+1],(1:2)')
```

```
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints (line 165)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/display (line 66)
f =
    desc: (none)
    freq: Unknown (0)
\begin{tabular}{lllll} 
'dates: (2)' & 'times: & \((2) '\) & 'series1: & (2)' \\
'03-0ct-2017' & '16:19' & & {\([\)} & \(1.00]\) \\
'04-0ct-2017' & '16:19' & {\([\)} & \(2.00]\)
\end{tabular}
```

If the time-of-day information is in date character vector format, you must provide it to fints as a column-oriented cell array:

```
f = fints({'01-Jan-2001 12:00';'02-Jan-2001 12:00'},(1:2)')
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints (line 165)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/display (line 66)
f =
    desc: (none)
    freq: Unknown (0)
\begin{tabular}{lllll} 
'dates: (2)' & 'times: & (2)' & 'series1: & (2)' \\
'01-Jan-2001' & '12:00' & & {\([\)} & \(1.00]\) \\
'02-Jan-2001' & '12:00' & {\([\)} & \(2.00]\)
\end{tabular}
```

If the dates and times are in date character vector format and contained in separate matrices, you must concatenate them before using the date and time information as input to fints:
dates $=$ ['01-Jan-2001'; '02-Jan-2001'; '03-Jan-2001'];
times = ['12:00';'12:00';'12:00'];
dates time $=$ cellstr([dates, repmat(' ',size(dates,1),1),times]);
$f=$ fints(dates time,(1:3)')
Warning: FINTS will be removed in a future release. Use TIMETABLE instead. > In fints (line 165)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.

```
> In fints/display (line 66)
f =
    desc: (none)
    freq: Unknown (0)
    'dates: (3)' 'times: (3)' 'series1: (3)'
    '01-Jan-2001' '12:00' [ 1.00]
    '02-Jan-2001' '12:00' [ 2.00]
    '03-Jan-2001' '12:00' [ 3.00]
```


## Data Name Input

```
fts = fints(dates, data, datanames)
```

The third syntax lets you specify the names for the data series with the argument datanames. The datanames argument can be a MATLAB character vector for a single data series. For multiple data series names, it must be a cell array of character vectors.

Look at two examples, one with a single data series and a second with two. The first example sets the data series name to the specified name First:

```
dates = (today:today+100)';
data = exp(randn(1, 101))';
fts1 = fints(dates, data, 'First')
```

Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints (line 165)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/display (line 66)
fts1 =
desc: (none)
freq: Unknown (0)
'dates: (101)' 'First: (101)'
'03-0ct-2017' [ 1.50]
'04-0ct-2017' [ 0.24]
'05-0ct-2017' [ 0.48]
'06-0ct-2017' [ 3.15]
'07-0ct-2017' [ 1.82]
'08-0ct-2017' [ 0.28]
'09-0ct-2017' [ 0.11]

| '10-0ct-2017' | 0.56] |
| :---: | :---: |
| '11-0ct-2017' | $1.24]$ |
| '12-0ct-2017' | $2.57]$ |
| '13-0ct-2017' | 1.10] |
| '14-0ct-2017' | $0.33]$ |
| '15-0ct-2017' | $1.36]$ |
| '16-0ct-2017' | $0.31]$ |
| '17-0ct-2017' | 0.38 ] |
| '18-0ct-2017' | $0.52]$ |
| '19-0ct-2017' | $0.29]$ |
| '20-0ct-2017' | $0.76]$ |
| '21-0ct-2017' | $0.41]$ |
| '22-0ct-2017' | $0.75]$ |
| '23-0ct-2017' | $0.63]$ |
| '24-0ct-2017 ' | $0.66]$ |
| '25-0ct-2017' | $0.60]$ |
| '26-0ct-2017' | 3.43] |
| '27-0ct-2017' | $1.84]$ |
| '28-0ct-2017' | $1.06]$ |
| '29-0ct-2017' | 0.23] |
| '30-0ct-2017' | $0.20]$ |
| '31-0ct-2017' | $0.14]$ |
| '01-Nov-2017' | 13.53] |
| '02-Nov-2017' | $2.64]$ |
| '03-Nov-2017' | 1.29] |
| '04-Nov-2017' | 0.38 ] |
| '05-Nov-2017' | $0.32]$ |
| '06-Nov-2017' | 1.73 ] |
| '07-Nov-2017' | 4.78 ] |
| '08-Nov-2017' | $0.18]$ |
| '09-Nov-2017' | $0.64]$ |
| '10-Nov-2017' | 0.92 ] |
| '11-Nov-2017' | $0.14]$ |
| '12-Nov-2017' | $2.32]$ |
| '13-Nov-2017' | $0.66]$ |
| '14-Nov-2017' | $6.77]$ |
| '15-Nov-2017' | 0.68 ] |
| '16-Nov-2017' | 1.51 ] |
| '17-Nov-2017' | $0.32]$ |
| '18-Nov-2017' | 0.54 ] |
| '19-Nov-2017' | 0.31 ] |
| '20-Nov-2017' | 1.48 ] |
| '21-Nov-2017' | 3.68 ] |
| '22-Nov-2017 ' | $0.55]$ |


| '23-Nov-2017' | [ | $1.55]$ |
| :---: | :---: | :---: |
| '24-Nov-2017' | [ | 0.60] |
| '25-Nov-2017' | [ | 1.11] |
| '26-Nov-2017' | [ | 3.31] |
| '27-Nov-2017' | [ | 1.13] |
| '28-Nov-2017' | [ | $0.35]$ |
| '29-Nov-2017' | [ | 0.42] |
| '30-Nov-2017' | [ | $0.84]$ |
| '01-Dec-2017' | [ | 0.83] |
| '02-Dec-2017' | [ | 0.42] |
| '03-Dec-2017' | [ | 1.20] |
| '04-Dec-2017' | [ | $3.55]$ |
| '05-Dec-2017' | [ | 0.78] |
| '06-Dec-2017' | [ | 0.81] |
| '07-Dec-2017' | [ | 0.11] |
| '08-Dec-2017' | [ | $0.46]$ |
| '09-Dec-2017' | [ | $0.25]$ |
| '10-Dec-2017' | [ | 0.68] |
| '11-Dec-2017' | [ | 1.69] |
| '12-Dec-2017' | [ | 4.59] |
| '13-Dec-2017' | [ | $6.04]$ |
| '14-Dec-2017' | [ | 0.89] |
| '15-Dec-2017' | [ | 0.73 ] |
| '16-Dec-2017' | [ | $2.26]$ |
| '17-Dec-2017' | [ | 1.63] |
| '18-Dec-2017' | [ | $2.15]$ |
| '19-Dec-2017' | [ | $2.18]$ |
| '20-Dec-2017' | [ | 0.23] |
| '21-Dec-2017' | [ | 1.72] |
| '22-Dec-2017' | [ | 0.91] |
| '23-Dec-2017' | [ | 0.47 ] |
| '24-Dec-2017' | [ | 0.50 ] |
| '25-Dec-2017' | [ | 3.60] |
| '26-Dec-2017' | [ | 0.44] |
| '27-Dec-2017' | [ | 0.29] |
| '28-Dec-2017' | [ | 1.24] |
| '29-Dec-2017' | [ | 7.47] |
| '30-Dec-2017' | [ | 1.03] |
| '31-Dec-2017' | [ | 1.36] |
| '01-Jan-2018' | [ | 0.39] |
| '02-Jan-2018' | [ | 5.33] |
| '03-Jan-2018' | [ | 1.13] |
| '04-Jan-2018' | [ | 1.70] |
| '05-Jan-2018' | [ | 0.39] |


| '06-Jan-2018' | $[$ | $2.35]$ |
| :--- | :--- | :--- |
| '07-Jan-2018' | $[$ | $1.48]$ |
| '08-Jan-2018' | $[$ | $0.31]$ |
| '09-Jan-2018' | $[$ | $1.04]$ |
| '10-Jan-2018' | $[$ | $0.64]$ |
| '11-Jan-2018' | $[$ | $1.12]$ |

The second example provides two data series named First and Second:
dates = (today:today+100)';
data_series1 $=\exp (\operatorname{randn}(1,101))^{\prime} ;$
data_series2 $=\exp (r a n d n(1,101)) ' ;$
data = [data_series1 data_series2];
fts2 = fints(dates, data, \{'First', 'Second'\})
Warning: FINTS will be removed in a future release. Use TIMETABLE instead. > In fints (line 165)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead. > In fints/display (line 66)
fts2 =
desc: (none)
freq: Unknown (0)

| 'dates: (101)' | 'First: (101)' | 'Second: (101)' |
| :---: | :---: | :---: |
| '03-0ct-2017' | 0.78 ] | [ 3.36] |
| '04-0ct-2017' | [ 0.83] | 1.63] |
| '05-0ct-2017' | [ 0.36] | [ 2.79] |
| '06-0ct-2017' | [ 0.72] | [ 2.39] |
| '07-0ct-2017' | [ 2.15] | [ 0.68] |
| '08-0ct-2017' | 5.72] | $1.54]$ |
| '09-0ct-2017' | [ 0.31] | [ 0.74] |
| '10-0ct-2017' | 10.78] | [ 0.41] |
| '11-0ct-2017' | 4.60] | [ 1.89] |
| '12-0ct-2017' | 1.18] | [ 1.07] |
| '13-0ct-2017' | $0.74]$ | [ 0.83] |
| '14-0ct-2017' | $0.50]$ | [ 1.34] |
| '15-0ct-2017' | $2.30]$ | [ 2.69] |
| '16-0ct-2017' | $0.50]$ | [ 1.48] |
| '17-0ct-2017' | $0.63]$ | [ 1.21] |
| '18-0ct-2017' | 2.42] | [ 1.32] |
| '19-0ct-2017' | 1.55] | [ 1.05] |
| '20-0ct-2017' | $2.45]$ | [ 0.46] |
| '21-0ct-2017' | $1.66]$ | $2.20]$ |

'22-0ct-2017'
'23-0ct-2017'
'24-0ct-2017'
'25-0ct-2017'
'26-0ct-2017'
'27-0ct-2017'
'28-0ct-2017'
'29-0ct-2017'
'30-0ct-2017'
'31-0ct-2017'
' 01 -Nov-2017'
' 02 -Nov-2017'
' 03 -Nov-2017'
'04-Nov-2017'
' $05-$ Nov-2017 '
' 06 -Nov-2017'
'07-Nov-2017'
' 08 -Nov-2017'
' 09 -Nov-2017'
' 10-Nov-2017'
'11-Nov-2017'
' 12 -Nov-2017'
' 13 -Nov-2017'
' 14-Nov-2017'
' 15 -Nov-2017'
' 16 -Nov-2017'
' 17 -Nov-2017'
' 18-Nov-2017'
' 19-Nov-2017'
' 20 -Nov-2017'
' 21 -Nov-2017'
' 22 -Nov-2017'
' 23 -Nov-2017'
' 24 -Nov-2017'
' 25 -Nov-2017'
' 26 -Nov-2017 '
' 27 -Nov-2017'
' 28 -Nov-2017'
' 29 -Nov-2017 '
'30-Nov-2017'
'01-Dec-2017'
'02-Dec-2017'
'03-Dec-2017'
'04-Dec-2017'

| $0.67]$ | $[$ | $4.09]$ |
| ---: | :---: | ---: |
| $0.60]$ | $[$ | $0.59]$ |
| $2.22]$ | $[$ | $6.87]$ |
| $0.51]$ | $[$ | $0.84]$ |
| $3.28]$ | $[$ | $0.78]$ |
| $2.20]$ | $[$ | $0.41]$ |
| $1.33]$ | $[$ | $0.45]$ |
| $1.00]$ | $[$ | $0.39]$ |
| $1.44]$ | $[$ | $1.42]$ |
| $34.01]$ | $[$ | $4.94]$ |
| $0.89]$ | $[$ | $1.69]$ |
| $0.21]$ | $[$ | $2.35]$ |
| $6.79]$ | $[$ | $3.83]$ |
| $1.84]$ | $[$ | $0.08]$ |
| $0.52]$ | $[$ | $0.85]$ |
| $13.70]$ | $[$ | $1.42]$ |
| $1.73]$ | $[$ | $2.05]$ |
| $1.34]$ | $[$ | $0.27]$ |
| $0.46]$ | $[$ | $0.37]$ |
| $0.34]$ | $[$ | $2.20]$ |
| $0.17]$ | $[$ | $0.89]$ |
| $0.66]$ | $[$ | $1.74]$ |
| $0.35]$ | $[$ | $0.38]$ |
| $1.91]$ | $[$ | $0.20]$ |
| $0.73]$ | $[$ | $2.24]$ |
| $5.86]$ | $[$ | $[$ |



Note Data series names must be valid MATLAB variable names. The only allowed nonalphanumeric character is the underscore (_) character.

Because freq for fts 2 has not been explicitly indicated, the frequency indicator for fts2 is set to Unknown. Set the frequency indicator field freq before you attempt any operations on the object. You will not be able to use the object until the frequency indicator field is set to a valid indicator.

## Frequency Indicator Input

```
fts = fints(dates, data, datanames, freq)
```

With the fourth syntax you can set the frequency indicator field when you create the financial time series object. The frequency indicator field freq is set as the fourth input argument. You will not be able to use the financial time series object until freq is set to a valid indicator. Valid frequency indicators are

```
UNKNOWN, Unknown, unknown, U, u,0
DAILY, Daily, daily, D, d,1
WEEKLY, Weekly, weekly, W, w,2
MONTHLY, Monthly, monthly, M, m,3
QUARTERLY, Quarterly, quarterly, Q, q,4
SEMIANNUAL, Semiannual, semiannual, S, s,5
ANNUAL, Annual, annual, A, a,6
```

The previous example contained sets of daily data. The freq field displayed as Unknown (0) because the frequency indicator was not explicitly set. The command

```
fts = fints(dates, data, {'First', 'Second'}, 1)
```

sets the freq indicator to Daily (1) when creating the financial time series object:

```
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints (line 165)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/display (line 66)
fts =
    desc: (none)
    freq: Daily (1)
    'dates: (101)' 'First: (101)' 'Second: (101)'
    '03-0ct-2017' [ 1.27] [ 1.24]
    '04-0ct-2017' [ 0.43] [ 2.40]
    '05-Oct-2017' [ 0.28] [ 1.21]
    '06-0ct-2017' [ 1.85] [ 0.66]
```

'07-0ct-2017'
'08-0ct-2017'
'09-0ct-2017'
'10-0ct-2017'
'11-0ct-2017'
'12-0ct-2017'
'13-0ct-2017'
'14-0ct-2017'
'15-0ct-2017'
'16-0ct-2017'
'17-0ct-2017'
'18-0ct-2017'
'19-0ct-2017'
'20-0ct-2017'
'21-0ct-2017'
'22-0ct-2017'
'23-0ct-2017'
'24-0ct-2017'
'25-0ct-2017'
'26-0ct-2017'
'27-0ct-2017'
'28-0ct-2017'
'29-0ct-2017'
'30-0ct-2017'
'31-0ct-2017'
'01-Nov-2017'
'02-Nov-2017'
'03-Nov-2017'
'04-Nov-2017'
'05-Nov-2017'
'06-Nov-2017'
'07-Nov-2017'
'08-Nov-2017'
'09-Nov-2017'
'10-Nov-2017'
'11-Nov-2017'
'12-Nov-2017'
'13-Nov-2017'
'14-Nov-2017'
'15-Nov-2017'
'16-Nov-2017'
'17-Nov-2017'
'18-Nov-2017'
'19-Nov-2017'

| $1.85]$ | $[$ | $1.43]$ |
| ---: | :--- | ---: |
| $1.34]$ | $[$ | $1.04]$ |
| $1.48]$ | $[$ | $0.69]$ |
| $0.42]$ | $[$ | $5.88]$ |
| $0.61]$ | $[$ | $1.25]$ |
| $0.90]$ | $[$ | $15.34]$ |
| $0.50]$ | $[$ | $0.74]$ |
| $1.39]$ | $[$ | $1.76]$ |
| $10.65]$ | $[$ | $4.87]$ |
| $0.62]$ | $[$ | $15.32]$ |
| $1.91]$ | $[$ | $1.35]$ |
| $0.36]$ | $[$ | $0.45]$ |
| $3.82]$ | $[$ | $2.23]$ |
| $0.38]$ | $[$ | $0.27]$ |
| $1.23]$ | $[$ | $0.76]$ |
| $0.54]$ | $[$ | $1.31]$ |
| $1.67]$ | $[$ | $4.44]$ |
| $1.01]$ | $[$ | $4.21]$ |
| $0.96]$ | $[$ | $0.97]$ |
| $1.09]$ | $[$ | $2.52]$ |
| $0.53]$ | $[$ | $0.73]$ |
| $0.95]$ | $[$ | $1.94]$ |
| $14.63]$ | $[$ | $6.79]$ |
| $0.32]$ | $[$ | $1.17]$ |
| $1.74]$ | $[$ | $0.74]$ |
| $0.34]$ | $[$ | $0.61]$ |
| $2.80]$ | $[$ | $2.05]$ |
| $1.39]$ | $[$ | $3.81]$ |
| $1.92]$ | $[$ | $8.38]$ |
| $0.76]$ | $[$ | $1.06]$ |
| $1.28]$ | $[$ | $1.18]$ |
| $4.36]$ | $[$ | $0.53]$ |
| $0.10]$ | $[$ | $5.01]$ |
| $0.20]$ | $[$ | $0.93]$ |
| $1.52]$ | $[$ | $0.62]$ |
| $0.52]$ | $[$ | $8.88]$ |
| $0.74]$ | $[$ | $2.25]$ |
| $0.22]$ | $[$ | $2.05]$ |
| $0.40]$ | $[$ | $0.37]$ |
| $0.67]$ | $[$ | $1.54]$ |
| $0.48]$ | $[$ | $1.68]$ |
| $0.42]$ | $[$ | $0.34]$ |
| $0.66]$ | $[$ | $0.80]$ |
| $0.39]$ | $[$ | $0.67]$ |
|  |  |  |

' 20-Nov-2017'
' 21 -Nov-2017'
' 22 -Nov-2017'
' 23 -Nov-2017'
' 24 -Nov-2017'
'25-Nov-2017'
' 26 -Nov-2017'
' 27 -Nov-2017'
' 28 -Nov-2017 '
' 29 -Nov-2017'
' 30 -Nov-2017'
'01-Dec-2017'
'02-Dec-2017'
'03-Dec-2017'
'04-Dec-2017'
'05-Dec-2017'
'06-Dec-2017'
' 07 -Dec-2017'
' 08-Dec-2017'
' 09-Dec-2017'
' 10-Dec-2017'
'11-Dec-2017'
'12-Dec-2017'
'13-Dec-2017'
' 14-Dec-2017'
'15-Dec-2017'
'16-Dec-2017'
'17-Dec-2017'
' 18-Dec-2017'
' 19-Dec-2017'
'20-Dec-2017'
'21-Dec-2017'
'22-Dec-2017'
'23-Dec-2017'
' 24 -Dec-2017 '
' 25 -Dec-2017 '
' 26 -Dec-2017'
' 27 -Dec-2017'
' 28-Dec-2017'
' 29-Dec-2017'
' 30-Dec-2017'
'31-Dec-2017'
'01-Jan-2018'
'02-Jan-2018'

| $3.83]$ | 1.70] |
| :---: | :---: |
| $0.37]$ | $0.37]$ |
| 6.16] | $2.97]$ |
| $0.69]$ | $5.96]$ |
| $0.23]$ | $0.74]$ |
| $0.54]$ | $0.99]$ |
| $2.55]$ | 1.66] |
| $2.87]$ | $3.33]$ |
| 1.17] | $1.69]$ |
| 1.33] | 1.49] |
| 1.88] | $0.62]$ |
| 0.23] | $0.79]$ |
| 0.56] | 1.85] |
| 0.16] | 5.38] |
| 0.64] | 1.77] |
| $2.58]$ | $0.30]$ |
| $2.05]$ | $1.54]$ |
| 9.85] | $0.91]$ |
| 1.18] | 0.78 ] |
| $0.12]$ | $0.80]$ |
| 5.42] | $0.41]$ |
| 3.60] | $0.73]$ |
| 0.56] | $0.46]$ |
| 1.25] | $0.69]$ |
| 2.18] | 1.12] |
| 1.47] | 1.19] |
| $2.01]$ | $0.81]$ |
| $0.89]$ | $0.86]$ |
| 0.96] | 1.03] |
| 1.09] | 1.58] |
| $0.45]$ | $3.60]$ |
| 4.15] | $1.86]$ |
| 1.01] | $0.75]$ |
| 1.99] | $1.82]$ |
| $0.43]$ | 0.78 ] |
| $0.34]$ | $0.17]$ |
| $0.91]$ | $0.10]$ |
| 0.78] | $0.18]$ |
| 3.30] | $0.79]$ |
| 1.83] | $0.54]$ |
| 1.72] | $0.49]$ |
| $0.24]$ | $1.04]$ |
| 0.38] | $0.52]$ |
| 1.22] | $0.53]$ |


| 03-Jan-2018' |  | $0.71]$ |  | $1.84]$ |
| :---: | :---: | :---: | :---: | :---: |
| 04-Jan-2018' | [ | 3.63] | [ | $2.19]$ |
| 05-Jan-2018' | [ | 3.82] | [ | 11.43] |
| 06-Jan-2018' | [ | $0.56]$ | [ | $1.35]$ |
| 07-Jan-2018' | [ | $2.40]$ | [ | $1.06]$ |
| 08-Jan-2018' | [ | $4.04]$ |  | $0.56]$ |
| 09-Jan-2018' | [ | 1.38] |  | $0.82]$ |
| 10-Jan-2018' |  | $5.07]$ |  | $0.95]$ |
| 11-Jan-2018' |  | $2.89]$ |  | 0.17] |

When you create the object using this syntax, you can use the other valid frequency indicators for a particular frequency. For a daily data set you can use DAILY, Daily, daily, D, or d. Similarly, with the other frequencies, you can use the valid character vector indicators or their numeric counterparts.

## Description Field Input

```
fts = fints(dates, data, datanames, freq, desc)
```

With the fifth syntax, you can explicitly set the description field as the fifth input argument. The description can be anything you want. It is not used in any operations performed on the object.

This example sets the desc field to 'Test TS'.

```
dates = (today:today+100)';
data_series1 = exp(randn(1, 101))';
data series2 = exp(randn(1, 101))';
data = [data series1 data series2];
fts = fints(dates, data, {'First', 'Second'}, 1, 'Test TS')
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints (line 165)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/display (line 66)
```

fts =
desc: Test TS
freq: Daily (1)

| 'dates: (101)' | 'First: | $(101) '$ | 'Second: | (101)' |
| :--- | :--- | :--- | :--- | :--- |
| '03-0ct-2017' | $[$ | $0.77]$ | $[$ | $0.36]$ |
| '04-0ct-2017' | $[$ | $2.12]$ | $[$ | $0.79]$ |


'18-Nov-2017'
' 19-Nov-2017'
' 20 -Nov-2017'
'21-Nov-2017'
' 22 -Nov-2017'
' 23 -Nov-2017'
' 24 -Nov-2017'
' 25 -Nov-2017'
' 26 -Nov-2017 '
' 27 -Nov-2017'
' 28-Nov-2017'
' 29 -Nov-2017'
'30-Nov-2017'
'01-Dec-2017'
'02-Dec-2017'
'03-Dec-2017'
'04-Dec-2017'
'05-Dec-2017'
' 06-Dec-2017'
'07-Dec-2017'
' 08-Dec-2017'
'09-Dec-2017'
'10-Dec-2017'
'11-Dec-2017'
' 12 -Dec-2017'
'13-Dec-2017'
'14-Dec-2017'
' 15-Dec-2017'
' 16-Dec-2017'
' 17-Dec-2017'
'18-Dec-2017'
'19-Dec-2017'
' 20-Dec-2017'
'21-Dec-2017'
' 22 -Dec-2017 '
'23-Dec-2017'
'24-Dec-2017'
' 25 -Dec-2017'
' 26-Dec-2017'
' 27 -Dec-2017'
'28-Dec-2017'
' 29-Dec-2017'
' 30-Dec-2017'
'31-Dec-2017'

| $0.22]$ | $[$ | $1.51]$ |
| :--- | :--- | ---: |
| $2.79]$ | $[$ | $1.65]$ |
| $0.47]$ | $[$ | $1.09]$ |
| $7.99]$ | $[$ | $1.17]$ |
| $0.11]$ | $[$ | $0.59]$ |
| $1.57]$ | $[$ | $2.06]$ |
| $1.00]$ | $[$ | $0.43]$ |
| $0.47]$ | $[$ | $0.45]$ |
| $1.50]$ | $[$ | $2.07]$ |
| $0.45]$ | $[$ | $5.40]$ |
| $2.36]$ | $[$ | $0.68]$ |
| $1.07]$ | $[$ | $0.60]$ |
| $0.19]$ | $[$ | $1.50]$ |
| $0.09]$ | $[$ | $2.92]$ |
| $0.75]$ | $[$ | $2.63]$ |
| $3.14]$ | $[$ | $1.31]$ |
| $1.20]$ | $[$ | $0.53]$ |
| $1.06]$ | $[$ | $1.88]$ |
| $1.99]$ | $[$ | $0.92]$ |
| $0.25]$ | $[$ | $3.96]$ |
| $4.16]$ | $[$ | $0.24]$ |
| $0.41]$ | $[$ | $1.15]$ |
| $1.04]$ | $[$ | $3.63]$ |
| $0.70]$ | $[$ | $0.61]$ |
| $1.16]$ | $[$ | $0.54]$ |
| $0.14]$ | $[$ | $0.39]$ |
| $4.59]$ | $[$ | $0.76]$ |
| $1.73]$ | $[$ | $0.82]$ |
| $7.46]$ | $[$ | $1.15]$ |
| $4.12]$ | $[$ | $0.41]$ |
| $1.01]$ | $[$ | $1.09]$ |
| $0.39]$ | $[$ | $0.55]$ |
| $0.18]$ | $[$ | $0.69]$ |
| $1.02]$ | $[$ | $0.43]$ |
| $1.25]$ | $[$ | $0.75]$ |
| $2.85]$ | $[$ | $35.51]$ |
| $0.39]$ | $[$ | $30.19]$ |
| $2.21]$ | $[$ | $3.15]$ |
| $1.07]$ | $[$ | $2.20]$ |
| $0.46]$ | $[$ | $0.28]$ |
| $2.17]$ | $[$ | $0.56]$ |
| $1.30]$ | $[$ | $0.54]$ |
| $0.79]$ | $[$ | $1.71]$ |
| $6.54]$ | $[$ | $1.62]$ |
|  |  |  |


| '01-Jan-2018' | [ | $1.84]$ | $[$ | $1.11]$ |
| :--- | :--- | :--- | :--- | :--- |
| '02-Jan-2018' | [ | $0.90]$ | $[$ | $0.71]$ |
| '03-Jan-2018' | $[$ | $1.32]$ | $[$ | $0.40]$ |
| '04-Jan-2018' | $[$ | $1.09]$ | $[$ | $1.01]$ |
| '05-Jan-2018' $06-J a n-2018 '$ | $[$ | $1.19]$ | $[$ | $3.12]$ |
| '07-Jan-2018' | $[$ | $1.57]$ | $[$ | $1.53]$ |
| '08-Jan-2018' | $[$ | $0.55]$ | $[$ | $1.20]$ |
| '09-Jan-2018' | $[$ | $0.45]$ | $[$ | $1.93]$ |
| '10-Jan-2018' | $[$ | $1.43]$ | $[$ | $1.79]$ |
| '11-Jan-2018' | [ | $0.59]$ | $[$ | $0.20]$ |

Now the description field is filled with the specified character vector 'Test TS' when the constructor is called.

## Transforming a Text File

The function ascii2fts creates a financial time series object from a text (ASCII) data file if the data file conforms to a general format. The general format of the text data file is as follows:

- Can contain header text lines.
- Can contain column header information. The column header information must immediately precede the data series columns unless the skiprows argument (see below) is specified.
- Leftmost column must be the date column.
- Dates must be in a valid date character vector format:
- 'ddmmmyy' or 'ddmmmyyyy'
- 'mm/dd/yy' or 'mm/dd/yyyy'
- 'dd-mmm-yy' or 'dd-mmm-yyyy'
- 'mmm.dd, yy' or 'mmm.dd, yyyy'
- Each column must be separated either by spaces or a tab.

Several example text data files are included with the toolbox. These files are in the ftsdata subfolder within the folder matlabroot/toolbox/finance.

The syntax of the function
fts = ascii2fts(filename, descrow, colheadrow, skiprows);
takes in the data file name (filename), the row number where the text for the description field is (descrow), the row number of the column header information (colheadrow), and the row numbers of rows to be skipped (skiprows). For example, rows need to be skipped when there are intervening rows between the column head row and the start of the time series data.

Look at the beginning of the ASCII file disney.dat in the ftsdata subfolder:

| Walt Disney Company (DIS) |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Daily prices (3/29/96 to | $3 / 29 / 99)$ |  |  |  |  |
| DATE | OPEN | HIGH | LOW | CLOSE | VOLUME |
| $3 / 29 / 99$ | 33.0625 | 33.188 | 32.75 | 33.063 | 6320500 |
| $3 / 26 / 99$ | 33.3125 | 33.375 | 32.75 | 32.938 | 5552800 |
| $3 / 25 / 99$ | 33.5 | 33.625 | 32.875 | 33.375 | 7936000 |
| $3 / 24 / 99$ | 33.0625 | 33.25 | 32.625 | 33.188 | $6025400 \ldots$ |

The command-line

```
disfts = ascii2fts('disney.dat', 1, 3, 2)
```

uses disney.dat to create time series object disfts. This example

- Reads the text data file disney.dat
- Uses the first line in the file as the content of the description field
- Skips the second line
- Parses the third line in the file for column header (or data series names)
- Parses the rest of the file for the date vector and the data series values

The resulting financial time series object looks like this.

```
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In ascii2fts (line 64)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/display (line 66)
disfts =
    desc: Walt Disney Company (DIS)
    freq: Unknown (0)
    'dates: (782)' 'OPEN: (782)' 'HIGH: (782)' 'LOW: (782)'
'29-Mar-1996' [ 21.1938] [ 21.6250] [ 21.2920]
'01-Apr-1996' [ 21.1120] [ 21.6250] [ 21.4170]
'02-Apr-1996' [ 21.3165] [ 21.8750] [ 21.6670]
'03-Apr-1996' [ 21.4802] [ 21.8750] [ 21.7500]
'04-Apr-1996' [ 21.4393] [ 21.8750] [ 21.5000]
```



There are 782 data points in this object. Only the first few lines are shown here. Also, this object has two other data series, the CLOSE and VOLUME data series, that are not shown here. In creating the financial time series object, ascii2fts sorts the data into ascending chronological order.

The frequency indicator field, freq , is set to 0 for Unknown frequency. You can manually reset it to the appropriate frequency using structure syntax disfts.freq $=1$ for Daily frequency.

With a slightly different syntax, the function ascii2fts can create a financial time series object when time-of-day data is present in the ASCII file. The new syntax has the form

```
fts = ascii2fts(filename, timedata, descrow, colheadrow, skiprows);
```

Set timedata to ' T ' when time-of-day data is present and to ' NT ' when there is no time data. For an example using this function with time-of-day data, see the reference page for ascii2fts.

## See Also

ascii2fts | boxcox| convertto|datestr|diff|fillts|filter|fints| fts2mat|ftsbound|lagts|leadts|peravg|resamplets|smoothts| toannual | todaily|today|tomonthly| toquarterly|tosemi|toweekly|tsmovavg

## Related Examples

- "Working with Financial Time Series Objects" on page 13-2
- "Visualizing Financial Time Series Objects" on page 11-31
- "Using the Financial Time Series App" on page 14-11


## Visualizing Financial Time Series Objects

```
In this section...
"Introduction" on page 11-31
"Using chartfts" on page 11-31
"Zoom Tool" on page 11-34
"Combine Axes Tool" on page 11-37
```


## Introduction

Financial Toolbox software contains the function chartfts, which provides a visual representation of a financial time series object. chartfts is an interactive charting and graphing utility for financial time series objects. With this function, you can observe time series values on the entire range of dates covered by the time series.

Note Interactive charting is also available from the Graphs menu of the user interface. See "Interactive Chart" on page 15-16 for additional information.

## Using chartfts

chartfts requires a single input argument, tsobj, where tsobj is the name of the financial time series object you want to explore. Most equity financial time series objects contain four price series, such as opening, closing, highest, and lowest prices, plus an additional series containing the volume traded. However, chartfts is not limited to a time series of equity prices and volume traded. It can be used to display any time series data you may have.

To illustrate the use of chartfts, use the equity price and volume traded data for the Walt Disney Corporation (NYSE: DIS) provided in the file disney.mat:

```
load disney.mat
whos
\begin{tabular}{lcrl} 
Name & Size & Bytes & Class \\
dis & \(782 \times 5\) & 39290 & fints object \\
dis CLOSE & \(782 \times 1\) & 6256 & double array
\end{tabular}
```

| dis_HIGH | $782 \times 1$ | 6256 | double array |
| :--- | :---: | ---: | :--- |
| dis_LOW | $782 \times 1$ | 6256 | double array |
| dis_OPEN | $782 \times 1$ | 6256 | double array |
| dis_VOLUME | $782 \times 1$ | 6256 | double array |
| dis_nv | $782 \times 4$ | 32930 | fints object |
| q_dis | $13 \times 4$ | 2196 | fints object |

For charting purposes look only at the objects dis (daily equity data including volume traded) and dis_nv (daily data without volume traded). Both objects contain the series OPEN, HIGH, LOW, and CLOSE, but only dis contains the additional VOLUME series.

Use chartfts(dis) to observe the values.


The chart contains five plots, each representing one of the series in the time series object. Boxes indicate the value of each individual plot. The date box is always on the left. The number of data boxes on the right depends upon the number of data series in the time series object, five in this case. The order in which these boxes are arranged (left to right) matches the plots from top to bottom. With more than eight data series in the object, the scroll bar on the right is activated so that additional data from the other series can be brought into view.

Slide the mouse cursor over the chart. A vertical bar appears across all plots. This bar selects the set of data shown in the boxes below. Move this bar horizontally and the data changes accordingly.


Click the plot. A small information box displays the data at the point where you click the mouse button.


## Zoom Tool

The zoom feature of chartfts enables a more detailed look at the data during a selected time frame. The Zoom tool is found under the Chart Tools menu.


Note Due to the specialized nature of this feature, do not use the MATLAB zoom command or Zoom In and Zoom Out from the Tools menu.

When the feature is turned on, you will see two inactive buttons (ZOOM In and Reset ZOOM) above the boxes. The buttons become active later after certain actions have been performed.


The window title bar displays the status of the chart tool that you are using. With the Zoom tool turned on, you see Zoom ON in the title bar in addition to the name of the time series you are working with. When the tool is off, no status is displayed.


To zoom into the chart, you need to define the starting and ending dates. Define the starting date by moving the cursor over the chart until the desired date appears at the bottom-left box and click the mouse button. A blue vertical line indicates the starting date that you have selected. Next, again move the cursor over the chart until the desired ending date appears in the box and click the mouse once again. This time, a red vertical line appears and the ZOOM In button is activated.


To zoom into the chart, click the ZOOM In button.


The chart is zoomed in. The Reset ZOOM button now becomes active while the ZOOM In button becomes inactive again. To return the chart to its original state (not zoomed),
click the Reset ZOOM button. To zoom into the chart even further, repeat the steps above for zooming into the chart.

Turn off the Zoom tool by going back to the Chart Tools menu and choosing Zoom Off.


With the tool turned off, the chart stays at the last state that it was in. If you turn it off when the chart is zoomed in, the chart stays zoomed in. If you reset the zoom before turning it off, the chart becomes the original (not zoomed).

## Combine Axes Tool

The Combine Axes tool allows you to combine all axes or specific axes into one. With axes combined, you can visually spot any trends that can occur among the data series in a financial time series object.

To illustrate this tool, use dis_nv, the financial time series object that does not contain volume traded data:

```
chartfts(dis_nv)
```



To combine axes, choose the Chart Tools menu, followed by Combine Axes and On.


When the Combine Axes tool is on, check boxes appear beside each individual plot. An additional check box enables the combination of all plots.


## Combining All Axes

To combine all plots, select the Select all plots check box.


Now click the Combine Selected Graphs button to combine the chosen plots. In this case, all plots are combined.


The combined plots have a single plot axis with all data series traced. The background of each data box has changed to the color corresponding to the color of the trace that represents the data series. After the axes are combined, the tool is turned off.

## Combining Selected Axes

You can choose any combination of the available axes to combine. For example, combine the HIGH and LOW price series of the Disney time series. Click the check boxes next to the corresponding plots. The Combine Selected Graphs button appears and is active.


Click the Combine Selected Graphs button. The chart with the combined plots looks like the next figure.


The plot with the combined axes is located at the top of the chart while the remaining plots follow it. The data boxes have also been changed. The boxes that correspond to the combined axes are relocated to the beginning, and the background colors are set to the color of the respective traces. The data boxes for the remaining axes retain their original formats.

## Resetting Axes

If you have altered the chart by combining axes, you must reset the axes before you can visualize additional combinations. Reset the axes with the Reset Axes menu item under Chart Tools > Combine Axes. Now the On and Off features are turned off.


With axes reset, the interactive chart appears in its original format, and you can proceed with additional axes combinations.


## See Also

ascii2fts | boxcox|convertto|datestr|diff|fillts|filter|fints| fts2mat|ftsbound|lagts|leadts|peravg|resamplets|smoothts| toannual | todaily| today| tomonthly| toquarterly| tosemi|toweekly|tsmovavg

## Related Examples

- "Creating Financial Time Series Objects" on page 11-2
- "Working with Financial Time Series Objects" on page 13-2
- "Using the Financial Time Series App" on page 14-11


## Using Financial Timetables

## Convert Financial Time Series Objects fints to Timetables

In this section...<br>"Create Time Series" on page 12-3<br>"Index an Object" on page 12-4<br>"Transform Time Series" on page 12-4<br>"Convert Time Series" on page 12-5<br>"Merge Time Series" on page 12-7<br>"Analyze Time Series" on page 12-7<br>"Data Extraction" on page 12-8

In R2018a, financial time series (fints), and its associated methods have been replaced with a MATLAB timetable function. If you use fints or the associated methods, you receive a warning. To help you convert from the older fints to the newer timetable functionality, use the following information.

## Create Time Series

## I/O Related Operations

| Task | Old Functionality | New Functionality |
| :---: | :---: | :---: |
| Construct by passing in data and dates | fints(dates,data, datana mes) | timetable(rowTimes, var1, ..., varN, 'VariableNames' ,\{'a','b',...\}) |
| Construct by conversion of files | ascii2fts(filename,desc row, colheadrow,skiprows ) | ```T = readtable(filename,opts, Name,Value) TT = table2timetable(T,'RowTi mes',timeVarName)``` |
| Construct by using App with user interface | Using the Financial Time Series app with user interface | Using ImportData from HOME tab |
| Write files | fts2ascii(filename,tsob j, exttext) | writetable(TT, filename) |
| Convert to matrix | fts2mat(tsobj) | ```S = vartype('numeric'); TT2 = TT(:,S) TT2.Variables``` |

## Index an Object

## Indexing an Object

| Task | Old Functionality | New Functionality |
| :---: | :---: | :---: |
| Indexing with a date | myfts('05/11/99') | TT( \{'1999-05-11'\}, : ) |
| Indexing with a date range |  | ```S = timerange('1999-05-11',' 1999-05-15'); TT2 = TT(S,:)``` |
| Indexing with integers for rows | myfts.series2(1) myfts.series2([1, 3, 5]) myfts.series2(16:20) | $\begin{aligned} & \mathrm{TT}\left(1,\left\{{ }^{\prime}\right. \text { series2'\}) }\right. \\ & \mathrm{TT}([1,3,5], \\ & \{‘ \text { series2'\}) } \\ & \mathrm{TT}\left(16: 20,\left\{{ }^{\prime}\right. \text { series2'\}) }\right. \end{aligned}$ |
| Contents of a specific time field | myfts.times | timeofday(TT.Properties. RowTimes) |
| Contents for a specific field in a matrix | fts2mat(myfts.series2) | TT.series2 |

## Transform Time Series

Assume that all variables are numeric within a timetable, or the operations can be applied on TT2:

S = vartype('numeric');
$\mathrm{TT} 2=\mathrm{TT}(:, \mathrm{S})$

Filter Time Series

| Task | Old Functionality | New Functionality |
| :---: | :---: | :---: |
| Boxcox transformation | newfts = boxcox(oldfts) | TT.Variables = boxcox(TT.Variables) |
| Differencing | diff(myfts) | $\begin{aligned} & \mathrm{TT}\{2: \text { end, : }\}= \\ & \text { diff(TT.Variables) } \\ & \mathrm{TT}(1,:)=[] \end{aligned}$ |
| Indexing with integers for rows | fillts(oldfts,fill meth od) | fillmissing(TT,method) <br> (Assumes no missing dates) |
| Linear filtering | filter(B,A, myfts) | ```TT.Variables = filter(b,a,TT.Variables)``` |
| Lag or lead time series object | ```lagts(myfts,lagperiod) leadts(myfts,leadperiod )``` | $\begin{aligned} & \text { lag(TT, lagperiod) } \\ & \text { lag(TT, - leadperiod) } \end{aligned}$ <br> (Assumes a regularly spaced timetable) |
| Periodic average | peravg (myfts) | ```retime(TT,newTimes,'mean ')``` |
| Downsample data | ```resamplets(oldfts,sampl estep)``` | retime(TT, newTimeStep,me thod) |
| Smooth data | smoothts (input) | smoothdata(TT) |
| Moving average | $\begin{aligned} & \text { tsmovavg(tsobj, method,l } \\ & \text { ag) } \end{aligned}$ | $\begin{aligned} & \text { movavg(TT,type,windowSiz } \\ & \text { e) } \end{aligned}$ |

## Convert Time Series

Assume that all variables are numeric within a timetable, or the operations can be applied on TT2:

S = vartype('numeric');
$\mathrm{TT} 2=\mathrm{TT}(:, \mathrm{S})$

## Conversion Operations

| Task | Old Functionality | New Functionality |
| :--- | :--- | :--- |
| Convert to specified <br> frequency | convertto(oldfts, newfre <br> q) | retime(TT, newTimeStep, me <br> thod) |
| Convert to annual | toannual(oldfts, CalcMet <br> hod) | retime(TT, 'yearly', metho <br> d) |
| Convert to daily | todaily(oldfts, CalcMeth <br> od) | retime(TT, 'daily', method <br> ( |
| Convert to monthly | tomonthly(oldfts, CalcMe <br> thod) | retime(TT, 'monthly', meth <br> od) |
| Convert to quarterly | toquarterly(oldfts, Calc <br> Method) | retime(TT, 'quarterly', me <br> thod) |
| Convert to semiannual | tosemi(oldfts, CalcMetho <br> d) | semiAnnualDates (manually <br> generated) <br> retime(TT, <br> semiAnnualDates, method) |
| Convert to weekly | toweekly(oldfts, CalcMet <br> hod) | retime(TT, 'weekly', metho <br> d) |

## Merge Time Series

## Merge Operations

| Task | Old Functionality | New Functionality |
| :--- | :--- | :--- |
| Merge multiple time <br> series objects | merge(fts1,fts2) | $[\mathrm{TT1;TT2]} \mathrm{(requires} \mathrm{variable}$ <br> name to be the same) <br> unique(TT) |
| Concatenate financial <br> time series objects <br> horizontally | horzcat(fts1,fts2) or <br> [fts1,fts2] | horzcat[TT1, TT2] (requires <br> variable name to be the same) <br> or |
| Concatenate financial <br> time series objects <br> vertically | vertcat(fts1,fts2) or <br> [fts1;fts2] | vertcat[TT1;TT2] |

## Analyze Time Series

Due to flexibility of a timetable that can hold heterogeneous variables, a timetable does not support math operations or descriptive statistical calculations. If you would like to apply any numeric calculations on a timetable, use the following guidelines.

Assume that all variables are numeric within a timetable, or the operations can be applied on TT2:

S = vartype('numeric');
TT2 = TT(:, S)
Descriptive Statistics and Arithmetic and Math Operations

| Task | New Functionality |
| :--- | :--- |
| Extract out numerical data | TT. Variables |
| Apply some options (statistics) | cov (TT.Variables) |
| Apply some options (operations) | TT.Variables = cumsum (TT.Variables ) |

## Data Extraction

Refer to timetable documentation for data extraction methods and examples.

## See Also

diff|fillmissing|isregular|issorted|retime|rmmissing|sortrows| synchronize | timerange | timetable|unique |vartype|withtol

## Related Examples

- "Using Timetables in Finance"
- "Select Timetable Data by Row Time and Variable Type" (MATLAB)
- "Resample and Aggregate Data in Timetable" (MATLAB)
- "Combine Timetables and Synchronize Their Data" (MATLAB)
- "Retime and Synchronize Timetable Variables Using Different Methods" (MATLAB)
- "Clean Timetable with Missing, Duplicate, or Nonuniform Times" (MATLAB)


## Using Financial Time Series

- "Working with Financial Time Series Objects" on page 13-2
- "Financial Time Series Operations" on page 13-7
- "Data Transformation and Frequency Conversion" on page 13-13
- "Indexing a Financial Time Series Object" on page 13-22


# Working with Financial Time Series Objects 

In this section...<br>"Introduction" on page 13-2<br>"Financial Time Series Object Structure" on page 13-2<br>"Data Extraction" on page 13-3<br>"Object-to-Matrix Conversion" on page 13-4

## Introduction

A financial time series object is used as if it were a MATLAB structure. (See the MATLAB documentation for a description of MATLAB structures or how to use MATLAB in general.)

This part of the tutorial assumes that you know how to use MATLAB and are familiar with MATLAB structures. The terminology is similar to that of a MATLAB structure. The financial time series object term component is interchangeable with the MATLAB structure term field.

## Financial Time Series Object Structure

A financial time series object always contains three component names: desc (description field), freq (frequency indicator field), and dates (date vector). If you build the object using the constructor fints, the default value for the description field is a blank character vector (' ' ). If you build the object from a text data file using ascii2fts, the default is the name of the text data file. The default for the frequency indicator field is 0 (Unknown frequency). Objects created from operations can default the setting to 0 . For example, if you decide to pick out values selectively from an object, the frequency of the new object might not be the same as that of the object from which it came.

The date vector dates does not have a default set of values. When you create an object, you have to supply the date vector. You can change the date vector afterward but, at object creation time, you must provide a set of dates.

The final component of a financial time series object is one or more data series vectors. If you do not supply a name for the data series, the default name is series1. If you have multiple data series in an object and do not supply the names, the default is the name series followed by a number, for example, series1, series2, and series3.

## Data Extraction

Here is an exercise on how to extract data from a financial time series object. As mentioned before, you can think of the object as a MATLAB structure. Highlight each line in the exercise in the MATLAB Help browser, press the right mouse button, and select Evaluate Selection to execute it.

To begin, create a financial time series object called myfts:

```
dates = (datenum('05/11/99'):datenum('05/11/99')+100)';
data_series1 = exp(randn(1, 101))';
data_series2 = exp(randn(1, 101))';
data = [data_series1 data_series2];
myfts = fints(dates, data)
```

The myfts object looks like this:

```
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints (line 165)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/display (line 66)
myfts =
    desc: (none)
    freq: Unknown (0)
\begin{tabular}{|c|c|c|}
\hline 'dates: (101)' & seriesl: (101)' & 'series2: (101)' \\
\hline '11-May-1999' & \(2.8108]\) & [ 0.9323] \\
\hline '12-May-1999' & [ 0.2454] & [ 0.5608] \\
\hline '13-May-1999' & [ 0.3568] & [ 1.5989] \\
\hline '14-May-1999' & [ 0.5255] & [ 3.6682] \\
\hline '15-May-1999' & [ 1.1862] & [ 5.1284] \\
\hline '16-May-1999' & \(3.8376]\) & [ 0.4952] \\
\hline '17-May-1999' & \(6.9329]\) & [ 2.2417] \\
\hline ' 18-May-1999' & \(2.0987]\) & [ 0.3579] \\
\hline '19-May-1999' & [ 2.2524] & [ 3.6492] \\
\hline '20-May-1999' & \(0.8669]\) & [ 1.0150] \\
\hline '21-May-1999' & \(0.9050]\) & [ 1.2445] \\
\hline '22-May-1999' & \(0.4493]\) & [ 5.5466] \\
\hline '23-May-1999' & 1.6376] & [ 0.1251] \\
\hline '24-May-1999' & \(3.4472]\) & [ 1.1195] \\
\hline '25-May-1999' & 3.6545] & \(0.3374]\) \\
\hline
\end{tabular}
```

There are more dates in the object; only the first few lines are shown here.

Note The actual data in your series1 and series2 differs from the above because of the use of random numbers.

Now create another object with only the values for series2:

```
srs2 = myfts.series2
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/subsref (line 106)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/display (line 66)
srs2 =
    desc: (none)
    freq: Unknown (0)
    'dates: (101)' 'series2: (101)'
    '11-May-1999' [ 0.9323]
    '12-May-1999' [ 0.5608]
    '13-May-1999' [ 1.5989]
    ' 14-May-1999' [ 3.6682]
    '15-May-1999' [ 5.1284]
    '16-May-1999' [ 0.4952]
    '17-May-1999' [ 2.2417]
    '18-May-1999' [ 0.3579]
    '19-May-1999' [ 3.6492]
    '20-May-1999' [ 1.0150]
    '21-May-1999' [ 1.2445]
    '22-May-1999' [ 5.5466]
    '23-May-1999' [ 0.1251]
    '24-May-1999' [ 1.1195]
    '25-May-1999' [ 0.3374]...
```

The new object srs2 contains all the dates in myfts, but the only data series is series2. The name of the data series retains its name from the original object, myfts.

Note The output from referencing a data series field or indexing a financial time series object is always another financial time series object. The exceptions are referencing the description, frequency indicator, and dates fields, and indexing into the dates field.

## Object-to-Matrix Conversion

The function fts2mat extracts the dates and/or the data series values from an object and places them into a vector or a matrix. The default behavior extracts just the values into a vector or a matrix. Look at the next example:

```
srs2_vec = fts2mat(myfts.series2)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/subsref (line 106)
```

```
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/fts2mat (line 29)
srs2_vec =
    0.9323
    0.5608
    1.5989
    3.6682
    5.1284
    0.4952
    2.2417
    0.3579
    3.6492
    1.0150
    1.2445
    5.5466
    0.1251
    1.1195
    0.3374..
```

If you want to include the dates in the output matrix, provide a second input argument and set it to 1 . This results in a matrix whose first column is a vector of serial date numbers:

```
format long g
```

srs2_mtx = fts2mat(myfts.series2, 1)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/subsref (line 106)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/fts2mat (line 29)
srs2_mtx =
$730251 \quad 0.932251754559576$
$730252 \quad 0.560845677519876$
730253 1.59888712183914
$730254 \quad 3.6681500883527$
$730255 \quad 5.12842215360269$
$730256 \quad 0.49519254119977$
$730257 \quad 2.24174134286213$
$730258 \quad 0.357918065917634$
$730259 \quad 3.64915665824198$

| 730260 | 1.01504236943148 |
| :--- | :--- |
| 730261 | 1.24446420606078 |
| 730262 | 5.54661849025711 |
| 730263 | 0.12507959735904 |
| 730264 | 1.11953883096805 |
| 730265 | 0.337398214166607 |

The vector srs2_vec contains series2 values. The matrix srs2_mtx contains dates in the first column and the values of the series 2 data series in the second. Dates in the first column are in serial date format. Serial date format is a representation of the date character vector format (for example, serial date $=1$ is equivalent to 01-Jan-0000). (The serial date vector can include time-of-day information.)

The long g display format displays the numbers without exponentiation. (To revert to the default display format, use format short. (See the format for a description of MATLAB display formats.) Remember that both the vector and the matrix have 101 rows of data as in the original object myfts but are shown truncated here.

## See Also

ascii2fts|boxcox|convertto|datestr|diff|fillts|filter|fints| fts2mat|ftsbound|lagts|leadts|peravg| resamplets|smoothts|toannual | todaily|tomonthly|toquarterly|tosemi|toweekly|tsmovavg

## Related Examples

- "Creating Financial Time Series Objects" on page 11-2
- "Visualizing Financial Time Series Objects" on page 11-31
- "Using the Financial Time Series App" on page 14-11


## Financial Time Series Operations

Several MATLAB functions have been overloaded to work with financial time series objects. The overloaded functions include basic arithmetic functions such as addition, subtraction, multiplication, and division and other functions such as arithmetic average, filter, and difference. Also, specific methods have been designed to work with the financial time series object. For a list of functions grouped by type, enter

```
help ftseries
```

at the MATLAB command prompt.

## Basic Arithmetic

Financial time series objects permit you to do addition, subtraction, multiplication, and division, either on the entire object or on specific object fields. This is a feature that MATLAB structures do not allow. You cannot do arithmetic operations on entire MATLAB structures, only on specific fields of a structure.

You can perform arithmetic operations on two financial time series objects as long as they are compatible. (All contents are the same except for the description and the values associated with the data series.)

Note Compatible time series are not the same as equal time series. Two time series objects are equal when everything but the description fields are the same.

Here are some examples of arithmetic operations on financial time series objects.
Load a MAT-file that contains some sample financial time series objects:

```
load dji30short
```

One of the objects in dji30short is called myfts1:

```
myftsl =
desc: DJI30MAR94.dat
freq: Daily (1)
dates: (20)' 'Open: (20)' 'High: (20)' 'Low: (20)' 'Close: (20)'
'04-Mar-1994' [ 3830.90] [ 3868.04] [ 3800.50] [ 3832.30]
'07-Mar-1994' [ 3851.72] [ 3882.40] [ 3824.71] [ 3856.22]
```

| '08-Mar-1994' | $[3858.48]$ | $[3881.55]$ | $[3822.45]$ | $[3851.72]$ |
| :--- | :--- | :--- | :--- | :--- |
| '09-Mar-1994' | $[3853.97]$ | $[3874.52]$ | $[3817.95]$ | $[3853.41]$ |
| $' 10-M a r-1994 '$ | $[3852.57]$ | $[3865.51]$ | $[3801.63]$ | $[3830.62] \ldots$ |

Create another financial time series object that is identical to myfts1:

```
newfts = fints(myfts1.dates, fts2mat(myfts1)/100,...
{'Open','High','Low', 'Close'}, 1, 'New FTS')
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/subsref (line 106)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/fts2mat (line 29)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints (line 165)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/display (line 66)
newfts =
desc: New FTS
freq: Daily (1)
'dates: (20)' 'Open: (20)' 'High: (20)' 'Low: (20)' 'Close:(20)'
'04-Mar-1994' [ 38.31] [ 38.68] [ 38.01] [ 38.32]
'07-Mar-1994' [ 38.52] [ 38.82] [ 38.25] [ 38.56]
'08-Mar-1994' [ 38.58] [ 38.82] [ 38.22] [ 38.52]
'09-Mar-1994' }[\begin{array}{llll}{[38.54]}&{[38.75]}&{[38.18]}&{[38.53]}\\{'10-Mar-1994' [38.53] }&{[38.66]}&{[38.02]}&{[38.31]...}
```

Perform an addition operation on both time series objects:

```
addup = myfts1 + newfts
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In + (line 22)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/display (line 66)
addup =
desc: DJI30MAR94.dat
freq: Daily (1)
    'dates: (20)' 'Open: (20)' 'High: (20)' 'Low: (20)' 'Close: (20)'
    '04-Mar-1994' [ 3869.21] [ 3906.72] [ 3838.51] [ 3870.62]
    '07-Mar-1994' [ 3890.24] [ 3921.22] [ 3862.96] [ 3894.78]
    '08-Mar-1994' [ 3897.06] [ 3920.37] [ 3860.67] [ 3890.24]
    '09-Mar-1994' [ 3892.51] [ 3913.27] [ 3856.13] [ 3891.94]
    '10-Mar-1994' [ 3891.10] [ 3904.17] [ 3839.65] [ 3868.93]...
```

Now, perform a subtraction operation on both time series objects:

```
subout = myfts1 - newfts
```

```
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In - (line 23)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/display (line 66)
subout =
desc: DJI30MAR94.dat
freq: Daily (1)
\begin{tabular}{llllll} 
'dates: (20)' & 'Open: (20)' & 'High: (20)' & 'Low: (20)' & 'Close: (20)' \\
'04-Mar-1994' & {\([3792.59]\)} & {\([3829.36]\)} & {\([3762.49]\)} & {\([3793.98]\)} \\
'07-Mar-1994' & {\([3813.20]\)} & {\([3843.58]\)} & {\([3786.46]\)} & {\([3817.66]\)} \\
'08-Mar-1994' & {\([3819.90]\)} & {\([3842.73]\)} & {\([3784.23]\)} & {\([3813.20]\)} \\
'09-Mar-1994' & {\([3815.43]\)} & {\([3835.77]\)} & {\([3779.77]\)} & {\([3814.88]\)} \\
'10-Mar-1994' & {\([3814.04]\)} & {\([3826.85]\)} & {\([3763.61]\)} & {\([3792.31] \ldots\)}
\end{tabular}
```


## Operations with Objects and Matrices

## You can also perform operations involving a financial time series object and a matrix or

 scalar:```
addscalar = myfts1 + 10000
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In + (line 22)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/display (line 66)
addscalar =
desc: DJI30MAR94.dat
freq: Daily (1)
'dates: (20)' 'Open: (20)' 'High: (20)' 'Low: (20)' 'Close: (20)
'04-Mar-1994' [ 13830.90] [ 13868.04] [ 13800.50] [ 13832.30]
'07-Mar-1994' [ 13851.72] [ 13882.40] [ 13824.71] [ 13856.22]
08-Mar-1994' [ 13858.48] [ 13881.55] [ 13822.45] [ 13851.72]
'09-Mar-1994' [ 13853.97] [ 13874.52] [ 13817.95] [ 13853.41]
'10-Mar-1994' [ 13852.57] [ 13865.51] [ 13801.63] [ 13862.70]...
```

For operations with both an object and a matrix, the size of the matrix must match the size of the object. For example, a matrix to be subtracted from myfts 1 must be 20-by-4, since myfts 1 has 20 dates and 4 data series:

```
submtx = myfts1 - randn(20, 4)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead
> In - (line 23)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/display (line 66)
submtx =
```

| 'dates: (20)' | 'Open: (20)' | 'High: (20)' | 'Low: (20)' | 'Close: (20)' |
| :---: | :---: | :---: | :---: | :---: |
| '04-Mar-1994' | [ 3831.33] | [ 3867.75] | [ 3802.10] | [ 3832.63] |
| '07-Mar-1994' | [ 3853.39] | [ 3883.74] | [ 3824.45] | [ 3857.06] |
| '08-Mar-1994' | [ 3858.35] | [ 3880.84] | [ 3823.51] | [ 3851.22] |
| '09-Mar-1994' | [ 3853.68] | [ 3872.90] | [ 3816.53] | [ 3851.92] |
| '10-Mar-1994' | [ 3853.72] | [ 3866.20] | [ 3802.44] | [ 3831.17] |

## Arithmetic Operations with Differing Data Series Names

Arithmetic operations on two objects that have the same size but contain different data series names require the function fts2mat. This function extracts the values in an object and puts them into a matrix or vector, whichever is appropriate.

To see an example, create another financial time series object the same size as myfts 1 but with different values and data series names:

```
newfts2 = fints(myfts1.dates, fts2mat(myfts1/10000),...
{'Rat1','Rat2', 'Rat3','Rat4'}, 1, 'New FTS')
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/subsref (line 106)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In / (line 25)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/fts2mat (line 29)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints (line 165)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/display (line 66)
newfts2 =
desc: New FTS
freq: Daily (1)
\begin{tabular}{lllllllll} 
'dates: (20)' & 'Rat1: & (20)' & 'Rat2: & (20)' & 'Rat3: & (20)' & 'Rat4: (20)' \\
'04-Mar-1994' & {\([\)} & \(0.38]\) & {\([\)} & \(0.39]\) & {\([\)} & \(0.38]\) & {\([\)} & \(0.38]\) \\
'07-Mar-1994' & {\([\)} & \(0.39]\) & {\([\)} & \(0.39]\) & {\([\)} & \(0.38]\) & {\([\)} & \(0.39]\) \\
'08-Mar-1994' & {\([\)} & \(0.39]\) & {\([\)} & \(0.39]\) & {\([\)} & \(0.38]\) & {\([\)} & \(0.39]\) \\
'09-Mar-1994' & {\([\)} & \(0.39]\) & {\([\)} & \(0.39]\) & {\([\)} & \(0.38]\) & {\([\)} & \(0.39]\) \\
10-Mar-1994' & {\([\)} & \(0.39]\) & {\([\)} & \(0.39]\) & {\([\)} & \(0.38]\) & {\([\)} & \(0.38]\) \\
\hline \(11-M a r-1994 '\) & {\([\)} & \(0.38]\) & {\([\)} & \(0.39]\) & {\([\)} & \(0.38]\) & {\([\)} & \(0.39]\)
\end{tabular}
```

| ' 14-Mar-1994' |  | $0.39]$ |  | $0.39]$ | [ | $0.38]$ |  | $0.39]$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| '15-Mar-1994' | [ | 0.39] | [ | 0.39] | [ | $0.38]$ |  | $0.38]$ |
| '16-Mar-1994' | [ | $0.39]$ | [ | $0.39]$ | [ | 0.38] |  | 0.38] |
| '17-Mar-1994' | [ | $0.39]$ | [ | $0.39]$ |  | $0.38]$ |  | 0.39] |
| '18-Mar-1994' | [ | $0.39]$ | [ | $0.39]$ |  | $0.38]$ |  | $0.39]$ |
| '21-Mar-1994' |  | 0.39] |  | 0.39] |  | 0.38] |  | 0.39] |
| '22-Mar-1994' |  | $0.39]$ |  | $0.39]$ |  | 0.38] |  | 0.39] |
| '23-Mar-1994' |  | $0.39]$ |  | $0.39]$ |  | $0.38]$ |  | 0.39] |
| '24-Mar-1994' |  | 0.38] |  | 0.39] |  | 0.38] |  | 0.38] |

If you attempt to add (or subtract, and so on) this new object to myfts1, an error indicates that the objects are not identical. Although they contain the same dates, number of dates, number of data series, and frequency, the two time series objects do not have the same data series names. Use fts2mat to bypass this problem:
addother $=$ myfts1 + fts2mat(newfts2)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead. > In fints/fts2mat (line 29)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead. $>$ In + (line 22)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead. > In fints/display (line 66)
addother $=$
desc: DJI30MAR94.dat
freq: Daily (1)

| 'dates: (20)' | 'Open: (20)' | 'High: (20)' | 'Low: (20)' | 'Close: (20)' |
| :---: | :---: | :---: | :---: | :---: |
| '04-Mar-1994' | [ 3831.28] | $3868.43]$ | $3800.88]$ | $3832.68]$ |
| '07-Mar-1994' | [ 3852.11] | $3882.79]$ | [ 3825.09] | $3856.61]$ |
| '08-Mar-1994' | [ 3858.87] | [ 3881.94] | [ 3822.83] | 3852.11 ] |
| '09-Mar-1994' | [ 3854.36] | 3874.91] | [ 3818.33] | $3853.80]$ |
| ' 10-Mar-1994' | [ 3852.96] | $3865.90]$ | [ 3802.01] | $3831.00]$ |
| '11-Mar-1994' | [ 3832.96] | 3873.22] | [ 3807.07] | $3863.09]$ |
| '14-Mar-1994' | 3870.68] | $3894.60]$ | [ 3836.34] | 3863 . 37] |
| '15-Mar-1994' | $3863.80]$ | $3888.85]$ | [ 3827.23] | $3849.97]$ |
| '16-Mar-1994' | 3851.42 ] | $3879.92]$ | 3820.32] | $3848.53]$ |
| '17-Mar-1994' | 3854.01 ] | $3891.73]$ | $3822.04]$ | $3865.53]$ |
| '18-Mar-1994' | $3865.81]$ | 3912.17] | $3839.03]$ | $3896.04]$ |
| '21-Mar-1994' | 3878.77] | $3898.64]$ | $3839.03]$ | $3865.24]$ |
| '22-Mar-1994' | $3866.10]$ | $3896.62]$ | $3841.04]$ | $3862.94]$ |

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This operation adds the matrix that contains the contents of the data series in the object newfts2 to myfts1. You should carefully consider the effects on your data before deciding to combine financial time series objects in this manner.

## Other Arithmetic Operations

In addition to the basic arithmetic operations, several other mathematical functions operate directly on financial time series objects. These functions include exponential (exp), natural logarithm (log), common logarithm (log10), and many more.

## See Also

ascii2fts | boxcox| convertto|datestr|diff|fillts|filter|fints| fts2mat|ftsbound|lagts|leadts|peravg|resamplets|smoothts|toannual | todaily|tomonthly|toquarterly|tosemi|toweekly|tsmovavg

## Related Examples

- "Creating Financial Time Series Objects" on page 11-2
- "Visualizing Financial Time Series Objects" on page 11-31
- "Using the Financial Time Series App" on page 14-11


## Data Transformation and Frequency Conversion

The data transformation and the frequency conversion functions convert a data series into a different format.

## Data Transformation Functions

| Function | Purpose |
| :--- | :--- |
| boxcox | Box-Cox transformation |
| diff | Differencing |
| fillts | Fill missing values |
| filter | Filter |
| lagts | Lag time series object |
| leadts | Lead time series object |
| peravg | Periodic average |
| smoothts | Smooth data |
| tsmovavg | Moving average |
| Frequency Conversion Functions |  |
| Function | New Frequency |
| convertto | As specified |
| resamplets | As specified |
| toannual | Annual |
| todaily | Daily |
| tomonthly | Monthly |
| toquarterly | Quarterly |
| tosemi | Semiannually |
| toweekly | Weekly |

As an example look at boxcox, the Box-Cox transformation function. This function transforms the data series contained in a financial time series object into another set of data series with relatively normal distributions.

First create a financial time series object from the supplied whirlpool. dat data file.

```
whrl = ascii2fts('whirlpool.dat', 1, 2, [])
```

Warning: FINTS will be removed in a future release. Use TIMETABLE instead. > In ascii2fts (line 64) Warning: FINTS will be removed in a future release. Use TIMETABLE instead. > In fints/display (line 66)
whrl =


Fill any missing values denoted with NaNs in whrl with values calculated using the linear method:

```
f_whrl = fillts(whrl)
```

Warning: FINTS will be removed in a future release. Use TIMETABLE instead. > In fints/fillts (line 213)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead. > In fints/display (line 66)
f_whrl =

| desc: Filled Whi <br> freq: Unknown (0 |  | (WHR) |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 'dates: (1108)' | 'Open : | (1108)' | 'High: | (1108) ' | 'Low: | (1108) |
| '03-Jan-1995' | [ | 50.25] | [ | $50.88]$ | [ | 50.13] |
| '04-Jan-1995' | [ | $50.00]$ | [ | 50.13] | [ | $49.63]$ |
| '05-Jan-1995' | [ | $50.00]$ | [ | $50.00]$ | [ | $49.25]$ |
| '06-Jan-1995' | [ | $50.00]$ | [ | $50.25]$ | [ | $49.88]$ |
| '09-Jan-1995' | [ | $49.88]$ | [ | 50.381 | [ | 49.63] |


| '10-Jan-1995' | [ | $51.25]$ | [ | $51.25]$ | [ | $50.25]$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| '11-Jan-1995' | [ | $50.38]$ | [ | 50.63] | [ | $49.75]$ |
| '12-Jan-1995' | [ | $49.88]$ | [ | $50.13]$ | [ | $49.25]$ |
| '13-Jan-1995' | [ | $50.00]$ | [ | $50.25]$ | [ | $49.75]$ |
| '16-Jan-1995' | [ | $50.25]$ | [ | 50.38] | [ | $50.00]$ |
| '17-Jan-1995' | [ | $51.00]$ | [ | $51.00]$ |  | $50.00]$ |
| '18-Jan-1995' |  | $50.50]$ |  | $51.00]$ |  | $50.13]$ |
| '19-Jan-1995' |  | $50.25]$ |  | $50.50]$ |  | $50.00]$ |
| '20-Jan-1995' |  | $50.50]$ |  | $51.25]$ |  | $50.50]$ |

Transform the nonnormally distributed filled data series f_whrl into a normally distributed one using Box-Cox transformation:
bc_whrl = boxcox(f_whrl)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead. > In fints/boxcox (line 36)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead. > In fints/display (line 66)
bc_whrl =
desc: Box-Cox: Filled Whirlpool (WHR)
freq: Unknown (0)

| 'dates: (1108)' | 'Open: | (1108) ${ }^{\prime}$ | 'High: | (1108) ${ }^{\prime}$ | 'Low: | (1108) ${ }^{\prime}$ | 'Close |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| '03-Jan-1995' | [ | $0.56]$ | [ | $0.51]$ | [ | $0.60]$ |  |
| '04-Jan-1995' | [ | $0.56]$ | [ | $0.51]$ | [ | $0.60]$ | [ |
| '05-Jan-1995' | [ | $0.56]$ | [ | $0.51]$ | [ | $0.60]$ | [ |
| '06-Jan-1995' | [ | $0.56]$ | [ | $0.51]$ | [ | $0.60]$ | [ |
| '09-Jan-1995 ' | [ | $0.56]$ | [ | $0.51]$ | , | $0.60]$ | [ |
| '10-Jan-1995' | [ | $0.56]$ | [ | $0.51]$ | [ | $0.60]$ | [ |
| '11-Jan-1995' | [ | $0.56]$ | [ | $0.51]$ | [ | $0.60]$ | [ |
| '12-Jan-1995' | [ | $0.56]$ | [ | $0.51]$ | [ | $0.60]$ | [ |
| '13-Jan-1995' | [ | $0.56]$ | [ | $0.51]$ | [ | $0.60]$ | [ |
| '16-Jan-1995' | [ | $0.56]$ | [ | $0.51]$ | [ | $0.60]$ | [ |
| '17-Jan-1995' | [ | $0.56]$ | [ | $0.51]$ | [ | $0.60]$ | [ |
| '18-Jan-1995' | [ | $0.56]$ | [ | $0.51]$ | [ | $0.60]$ | [ |
| '19-Jan-1995' | [ | $0.56]$ | [ | $0.51]$ | [ | $0.60]$ | [ |

Compare the result of the Close data series with a normal (Gaussian) probability distribution function and the nonnormally distributed f_whrl:
subplot(2, 1, 1);
hist(f_whrl.Close);

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```
grid; title('Nonnormally Distributed Data');
subplot(2, 1, 2);
hist(bc_whrl.Close);
grid; tītle('Box-Cox Transformed Data');
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/subsref (line 106)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/hist (line 37)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/subsref (line 106)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/hist (line 37)
```



## Box-Cox Transformation

The bar chart on the top represents the probability distribution function of the filled data series, f_whrl, which is the original data series whrl with the missing values interpolated using the linear method. The distribution is skewed toward the left (not normally distributed). The bar chart on the bottom is less skewed to the left. If you plot a

Gaussian probability distribution function (PDF) with similar mean and standard deviation, the distribution of the transformed data is very close to normal (Gaussian).

When you examine the contents of the resulting object bc_whrl, you find an identical object to the original object whrl but the contents are the transformed data series. If you have the Statistics and Machine Learning Toolbox software, you can generate a Gaussian PDF with mean and standard deviation equal to those of the transformed data series and plot it as an overlay to the second bar chart. In the next figure, you can see that it is an approximately normal distribution.


## Overlay of Gaussian PDF

The next example uses the smoothts function to smooth a time series.
To begin, transform ibm9599. dat, a supplied data file, into a financial time series object:

```
ibm = ascii2fts('ibm9599.dat', 1, 3, 2)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In ascii2fts (line 64)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/display (line 66)
ibm =
    desc: International Business Machines Corporation (IBM)
    freq: Unknown (0)
    'dates: (1108)' 'OPEN: (1108)' 'HIGH: (1108)' 'LOW: (1108)'
```



Fill the missing data for holidays with data interpolated using the fillts function and the Spline fill method:

```
f ibm = fillts(ibm, 'Spline')
```

Warning: FINTS will be removed in a future release. Use TIMETABLE instead. > In fints/fillts (line 213)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead. > In fints/display (line 66)
f_ibm =

| 'dates: (1108)' | ' OPEN: | (1108) ' | 'HIGH: | (1108) ' | 'LOW: | (1108) | CLOSE |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| '03-Jan-1995' | [ | 36.75] | [ | 36.91] | [ | $36.66]$ |  |
| '04-Jan-1995' | [ | $37.00]$ | [ | $37.28]$ | [ | $36.63]$ |  |
| '05-Jan-1995' | [ | $37.13]$ | [ | $37.63]$ | [ | $36.88]$ | [ |
| '06-Jan-1995' | [ | $37.25]$ | [ | 37.88] | [ | 37.031 | [ |
| '09-Jan-1995' | [ | 37.50] | [ | $37.91]$ | [ | 37.41] | [ |
| '10-Jan-1995' | [ | 38.00] | [ | 38.63] | [ | 37.78 ] | [ |
| '11-Jan-1995' | [ | 38.53] | [ | 38.53] | [ | $37.63]$ | [ |
| '12-Jan-1995' | [ | $37.91]$ | [ | $38.16]$ | [ | $37.75]$ | [ |
| '13-Jan-1995' | [ | 38.25] | [ | $38.41]$ | [ | $37.88]$ | [ |
| '16-Jan-1995' | [ | 38.16] | [ | 39.00] | [ | 38.16] | [ |
| '17-Jan-1995' | [ | $38.63]$ | [ | $39.00]$ | [ | $38.53]$ | [ |
| '18-Jan-1995' | [ | 38.88 ] | [ | $38.91]$ | [ | 38.28] | [ |
| ' 19-Jan-1995' | [ | 38.53] | [ | 38.78] | [ | $38.16]$ | [ |
| '20-Jan-1995' | [ | 38.13] | [ | 38.25] | [ | $37.50]$ | [ |

Smooth the filled data series using the default Box (rectangular window) method:

```
sm_ibm = smoothts(f_ibm)
```

Warning: FINTS will be removed in a future release. Use TIMETABLE instead. > In fints/smoothts (line 44) Warning: FINTS will be removed in a future release. Use TIMETABLE instead. > In fints/display (line 66)
sm_ibm =
desc: Box-smoothed of Filled International Business Machines Corporation (IBM)
freq: Unknown (0)

| 'dates: (1108)' | ' OPEN: | (1108)' | 'HIGH: | (1108) ${ }^{\prime}$ | 'LOW: | (1108) ${ }^{\prime}$ | CLOSE |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| '03-Jan-1995' | [ | $22.18]$ | [ | $22.36]$ | [ | $22.03]$ | [ |
| ' 04-Jan-1995' | [ | 29.63] | [ | 29.94] | [ | $29.44]$ | [ |
| ' 05-Jan-1995' | [ | $37.13]$ | [ | $37.52]$ | [ | $36.92]$ | [ |
| ' 06-Jan-1995' | [ | $37.38]$ | [ | $37.86]$ | [ | $37.14]$ | [ |
| ' 09-Jan-1995' |  | 37.68 ] | [ | $38.11]$ | [ | $37.34]$ | [ |
| '10-Jan-1995' | [ | $37.84]$ | [ | $38.22]$ | [ | $37.52]$ | [ |
| '11-Jan-1995' | [ | $38.04]$ | [ | $38.33]$ | [ | $37.69]$ | [ |
| '12-Jan-1995' | [ | $38.17]$ | [ | $38.54]$ | [ | $37.84]$ | [ |
| '13-Jan-1995' | [ | $38.29]$ | [ | 38.62 ] | [ | $37.99]$ | [ |
| '16-Jan-1995' | [ | $38.36]$ | [ | $38.69]$ | [ | $38.12]$ | [ |
| '17-Jan-1995' |  | $38.49]$ |  | $38.82]$ | [ | $38.20]$ |  |

Now, plot the original and smoothed closing price series for IBM $^{\circledR}$ stock:

```
plot(f ibm.CLOSE('11/01/97::02/28/98'), 'r')
datetic}k('x', 'mmmyy'
hold on
plot(sm ibm.CLOSE('11/01/97::02/28/98'), 'b')
hold off
datetick('x', 'mmmyy')
legend('Filled', 'Smoothed')
title('Filled IBM Close Price vs. Smoothed Series')
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/subsref (line 106)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/plot (line 63)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/subsref (line 106)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/plot (line 63)
```

13-19


## Smoothed Data Series

These examples give you an idea of what you can do with a financial time series object. This toolbox provides some MATLAB functions that have been overloaded to work directly with these objects. The overloaded functions are those most commonly needed to work with time series data.

## See Also

ascii2fts | boxcox| convertto| datestr|diff|fillts|filter|fints| fts2mat|ftsbound|lagts|leadts|peravg|resamplets|smoothts| toannual | todaily|tomonthly|toquarterly|tosemi |toweekly|tsmovavg

## Related Examples

- "Creating Financial Time Series Objects" on page 11-2
- "Visualizing Financial Time Series Objects" on page 11-31
- "Using the Financial Time Series App" on page 14-11


## Indexing a Financial Time Series Object

You can also index into the object as with any other MATLAB variable or structure. A financial time series object lets you use a date character vector on page 13-22, a cell array of date character vectors, a date character vector range on page 13-24, or normal integer on page 13-25 indexing. You cannot, however, index into the object using serial dates. If you have serial dates, you must first use the MATLAB datestr command to convert them into date character vectors.

When indexing by date character vector, note that

- Each date character vector must contain the day, month, and year. Valid formats are
- 'ddmmmyy hh:mm' or 'ddmmmyyyy hh:mm'
- 'mm/dd/yy hh:mm' or 'mm/dd/yyyy hh:mm'
- 'dd-mmm-yy hh:mm' or 'dd-mmm-yyyy hh:mm'
- 'mmm.dd,yy hh:mm' or 'mmm.dd, yyyy hh:mm'
- All data falls at the end of the indicated time period, that is, weekly data falls on Fridays, monthly data falls on the end of each month, and so on, whenever the data has gone through a frequency conversion.


## Indexing with Date Character Vectors

With date character vector indexing, you get the values in a financial time series object for a specific date using a date character vector as the index into the object. Similarly, if you want values for multiple dates in the object, you can put those date character vectors into a cell array of character vectors and use the cell array as the index to the object. Here are some examples.

This example extracts all values for May 11, 1999 from myfts:

```
format short
myftsl('05/11/99')
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/subsref (line 106)
ans =
    desc: (none)
    freq: Unknown (0)
```

```
'dates: (1)' 'series1: (1)' 'series2: (1)'
'11-May-1999' [ 2.8108] [ 0.9323]
```

The next example extracts only series2 values for May 11, 1999 from myfts:

```
myfts1.series2('05/11/99')
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/subsref (line 106)
ans =
    desc: (none)
    freq: Unknown (0)
    'dates: (1)' 'series2: (1)'
    '11-May-1999' [ 0.9323]
```

The third example extracts all values for three different dates:

```
myfts1({'05/11/99', '05/21/99', '05/31/99'})
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/subsref (line 106)
ans =
    desc: (none)
    freq: Unknown (0)
    'dates: (3)' 'series1: (3)' 'series2: (3)'
    '11-May-1999' [ 2.8108] [ 0.9323]
    21-May-1999' [ 0.9050] [ 1.2445]
    '31-May-1999' [ 1.4266] [ 0.6470]
```

The next example extracts only series2 values for the same three dates:

```
myfts1.series2({'05/11/99', '05/21/99', '05/31/99'})
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/subsref (line 106)
ans =
    desc: (none)
    freq: Unknown (0)
    'dates: (3)' 'series2: (3)'
```

| '11-May-1999' | [ | $0.9323]$ |
| :--- | :--- | :--- |
| '21-May-1999' | [ | $1.2445]$ |
| '31-May-1999' | [ | $0.6470]$ |

## Indexing with Date Character Vector Range

A financial time series is unique because it allows you to index into the object using a date character vector range. A date character vector range consists of two date character vector separated by two colons (: :). In MATLAB this separator is called the double-colon operator. An example of a MATLAB date character vector range is '05/11/99: :05/31/99'. The operator gives you all data points available between those dates, including the start and end dates.

Here are some date character vector range examples:

```
myfts1 ('05/11/99::05/15/99')
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/subsref (line 106)
ans =
    desc: (none)
    freq: Unknown (0)
    'dates: (5)' 'series1: (5)' 'series2: (5)'
    '11-May-1999' [ 2.8108] [ 0.9323]
    '12-May-1999' [ 0.2454] [ 0.5608]
    '13-May-1999' [ 0.3568] [ 1.5989]
    '14-May-1999' [ 0.5255] [ 3.6682]
    '15-May-1999' [ 1.1862] [ 5.1284]
myfts1.series2('05/11/99::05/15/99')
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/subsref (line 106)
ans =
    desc: (none)
    freq: Unknown (0)
    'dates: (5)' 'series2: (5)'
    '11-May-1999' [ 0.9323]
    '12-May-1999' [ 0.5608]
    '13-May-1999' [ 1.5989]
```

| '14-May-1999' | $[$ | $3.6682]$ |
| :--- | :--- | :--- |
| '15-May-1999' |  |  |

As with any other MATLAB variable or structure, you can assign the output to another object variable:

```
nfts = myfts1.series2('05/11/99::05/20/99');
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/subsref (line 106)
```

nfts is the same as ans in the second example.
If one of the dates does not exist in the object, an error message indicates that one or both date indexes are out of the range of the available dates in the object. You can either display the contents of the object or use the command ftsbound to determine the first and last dates in the object.

## Indexing with Integers

Integer indexing is the normal form of indexing in MATLAB. Indexing starts at 1 (not 0); index $=1$ corresponds to the first element, index $=2$ to the second element, index $=3$ to the third element, and so on. Here are some examples with and without data series reference.

Get the first item in series2:

```
myfts1.series2(1)
```

```
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/subsref (line 106)
ans =
    desc: (none)
    freq: Unknown (0)
    'dates: (1)' 'series2: (1)'
    '11-May-1999' [ 0.9323]
```

Get the first, third, and fifth items in series2:
myfts1.series2([1, 3, 5])

```
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/subsref (line 106)
ans =
    desc: (none)
    freq: Unknown (0)
    'dates: (3)' 'series2: (3)'
    '11-May-1999' [ 0.9323]
    '13-May-1999' [ 1.5989]
    '15-May-1999' [ 5.1284]
```


## Get items 16 through 20 in series2:

myfts1.series2(16:20)

```
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/subsref (line 106)
ans =
    desc: (none)
    freq: Unknown (0)
    'dates: (5)' 'series2: (5)'
    '26-May-1999' [ 0.2105]
    '27-May-1999' [ 1.8916]
    '28-May-1999' [ 0.6673]
    '29-May-1999' [ 0.6681]
    '30-May-1999' [ 1.0877]
```

Get items 16 through 20 in the financial time series object myfts:
myfts1(16:20)

```
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/subsref (line 106)
ans =
    desc: (none)
    freq: Unknown (0)
    'dates: (5)' 'series1: (5)' 'series2: (5)'
    '26-May-1999' [ 0.7571] [ 0.2105]
    '27-May-1999' [ 1.2425] [ 1.8916]
    '28-May-1999' [ 1.8790] [ 0.6673]
```

| '29-May-1999' | $[$ | $0.5778]$ |  |  |
| :---: | :---: | :---: | :---: | :---: |
| '30-May-1999' | $[$ | $1.2581]$ | $[$ | $0.6681]$ |

Get the last item in myfts1:

```
myfts1(end)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/end (line 57)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/subsref (line 106)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/display (line 66)
ans =
    desc: (none)
    freq: Unknown (0)
\begin{tabular}{|c|c|c|}
\hline dates: (1)' & 'seriesl: (1)' & 'series2: (1)' \\
\hline ' 19-Aug-1999' & [ 1.4692] & \(3.4238]\) \\
\hline
\end{tabular}
```

This example uses the MATLAB special variable end, which points to the last element of the object when used as an index. The example returns an object whose contents are the values in the object myfts on the last date entry.

## Indexing When Time-of-Day Data Is Present

Both integer and date character vector indexing are permitted when time-of-day information is present in the financial time series object. You can index into the object with both date and time specifications, but not with time of day alone. To show how indexing works with time-of-day data present, create a financial time series object called timeday containing a time specification:

```
dates = ['01-Jan-2001';'01-Jan-2001'; '02-Jan-2001'; ...
    02-Jan-2001'; '03-Jan-2001';'03-Jan-2001'];
times = ['11:00';'12:00';'11:00';'12:00';'11:00';'12:00'];
dates_times = cellstr([dates, repmat(' ',size(dates,1),1),...
    times]);
timeday = fints(dates_times,(1:6)',{'Datal'},1,'My first FINTS')
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints (line 165)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/display (line 66)
timeday =
    desc: My first FINTS
    freq: Daily (1)
```

| 'dates: (6)' | 'times: | (6) ${ }^{\prime}$ | 'Datal: | (6) ${ }^{\prime}$ |
| :---: | :---: | :---: | :---: | :---: |
| '01-Jan-2001' | '11:00' |  | [ | $1]$ |
| ' " ' | '12:00' |  | [ | 2] |
| '02-Jan-2001' | '11:00' |  | [ | $3]$ |
| " ' | '12:00' |  | [ | 4] |
| '03-Jan-2001' | '11:00' |  | [ | $5]$ |
| ' " ' | '12:00' |  | [ | $6]$ |

Use integer indexing to extract the second and third data items from timeday:

```
timeday(2:3)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints (line 165)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/display (line 66)
ans =
    desc: My first FINTS
    freq: Daily (1)
    'dates: (2)' 'times: (2)' 'Datal: (2)'
    01-Jan-2001' '12:00' [ 2]
    '02-Jan-2001' '11:00' [ [ 3]
```

For date character vector indexing, enclose the date and time character vectors in one pair of quotation marks. If there is one date with multiple times, indexing with only the date returns the data for all the times for that specific date. For example, the command timeday ('01-Jan-2001') returns the data for all times on January 1, 2001:

```
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints (line 165)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/display (line 66)
ans =
    desc: My first FINTS
    freq: Daily (1)
    'dates: (2)' 'times: (2)' 'Data1: (2)
    '01-Jan-2001' '11:00' [ 1]
```

You can also indicate a specific date and time:
timeday('01-Jan-2001 12:00')

```
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints (line 165)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/display (line 66)
ans =
    desc: My first FINTS
    freq: Daily (1)
    'dates: (1)' 'times: (1)' 'Datal: (1)'
    '01-Jan-2001' '12:00' [ 2]
```

Use the double-colon operator : : to specify a range of dates and times:

```
timeday('01-Jan-2001 12:00::03-Jan-2001 11:00')
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints (line 165)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/display (line 66)
ans =
    desc: My first FINTS
    freq: Daily (1)
\begin{tabular}{|c|c|c|c|}
\hline 'dates: (4)' & 'times: & \((4)^{\prime}\) & 'Datal: (4)' \\
\hline '01-Jan-2001' & ' 12:00' & & \(2]\) \\
\hline '02-Jan-2001' & '11:00' & & \(3]\) \\
\hline ' " ' & ' 12:00' & & \(4]\) \\
\hline '03-Jan-2001' & '11:00' & & \(5]\) \\
\hline
\end{tabular}
```

Treat timeday as a MATLAB structure if you want to obtain the contents of a specific field. For example, to find the times of day included in this object, enter

```
datestr(timeday.times)
```

Warning: FINTS will be removed in a future release. Use TIMETABLE instead. > In fints/subsref (line 106)
ans =
11:00 AM
12:00 PM

```
11:00 AM
12:00 PM
11:00 AM
12:00 PM
```


## See Also

ascii2fts | boxcox| convertto|datestr|diff|fillts|filter|fints| fts2mat | ftsbound | lagts | leadts | peravg| resamplets | smoothts| toannual | todaily|tomonthly| toquarterly|tosemi|toweekly|tsmovavg

## Related Examples

- "Creating Financial Time Series Objects" on page 11-2
- "Visualizing Financial Time Series Objects" on page 11-31
- "Using the Financial Time Series App" on page 14-11


## Financial Time Series App

- "What Is the Financial Time Series App?" on page 14-2
- "Getting Started with the Financial Time Series App" on page 14-4
- "Loading Data with the Financial Time Series App" on page 14-7
- "Using the Financial Time Series App" on page 14-11
- "Using the Financial Time Series App with GUIs" on page 14-19


## What Is the Financial Time Series App?

Note The Financial Time Series app will be removed in a future release and will no longer accept a fints object.

Use fts2timetable to convert a fints object to a timetable object. For more information, see "Convert Financial Time Series Objects fints to Timetables" on page 122.

The Financial Time Series app enables you to create and manage financial time series (fints) objects. The Financial Time Series app interoperates with the Financial Time Series Graphical User Interface (ftsgui) and Interactive Chart (chartfts). In addition, you can use Datafeed Toolbox ${ }^{\mathrm{TM}}$ software to connect to external data sources.

A financial time series object minimally consists of:

- Desc, which is the description field.
- Freq, which is a frequency indicator field.
- Dates, which is a date vector field. If the date vector incorporates time-of-day information, the object contains an additional field named times.
- In addition, you can have at least one data series vector. You can specify names for any data series vectors. If you do not specify names, the object uses the default names series1, series2, series3, and so on.

In general, the workflow for using the Financial Time Series app is:
1 Acquire data.
2 Create a variable.
3 Convert the variable to fints.
4 Convert fints to a MATLAB double object.
To obtain the data for the Financial Time Series app, you need to use a MATLAB double object or a financial time series (fints) object. You can use previously stored internal data on your computer or you can connect to external data sources using Datafeed Toolbox software.

Note You must obtain a license for these products from MathWorks before you can use Datafeed Toolbox.

After creating a financial time series object, you can use the Financial Time Series app to change the characteristics of the time series object, including merging with other financial time series objects, removing rows or columns, and changing the frequency. You can also use the Financial Time Series app to generate various forms of plotted output and you can reconvert a fints object to a MATLAB double-precision matrix.

## See Also

ascii2fts | boxcox| convertto|datestr|diff|fillts|filter|fints| fts2mat | ftsbound | lagts | leadts | peravg| resamplets| smoothts| toannual | todaily|tomonthly| toquarterly|tosemi|toweekly|tsmovavg

## Related Examples

- "Getting Started with the Financial Time Series App" on page 14-4
- "Loading Data with the Financial Time Series App" on page 14-7
- "Using the Financial Time Series App" on page 14-11
- "Using the Financial Time Series App with GUIs" on page 14-19
- "Creating Financial Time Series Objects" on page 11-2
- "Visualizing Financial Time Series Objects" on page 11-31
- "Working with Financial Time Series Objects" on page 13-2


## Getting Started with the Financial Time Series App

To start the Financial Time Series app:
1 On the MATLAB desktop toolstrip, click the Apps tab and in the apps gallery, under Computational Finance, click Financial Time Series.

The Financial Time Series app opens. For an overview of the Financial Time Series app, see "What Is the Financial Time Series App?" on page 14-2.
2 To load data with the Financial Time Series app, see "Loading Data with the Financial Time Series App" on page 14-7.

If you plan to load data from Datafeed Toolbox software, ensure that you have a license. For more information on this toolbox, see the Datafeed Toolbox documentation.

3 For more information on the tasks supported by the Financial Time Series app, see "Using the Financial Time Series App" on page 14-11 and "Using the Financial Time Series App with GUIs" on page 14-19.


## See Also

ascii2fts | boxcox| convertto|datestr|diff|fillts|filter|fints| fts2mat | ftsbound | lagts | leadts | peravg| resamplets| smoothts| toannual | todaily | tomonthly | toquarterly|tosemi| toweekly|tsmovavg

## Related Examples

- "What Is the Financial Time Series App?" on page 14-2
- "Loading Data with the Financial Time Series App" on page 14-7
- "Using the Financial Time Series App" on page 14-11
- "Using the Financial Time Series App with GUIs" on page 14-19
- "Creating Financial Time Series Objects" on page 11-2
- "Visualizing Financial Time Series Objects" on page 11-31
- "Working with Financial Time Series Objects" on page 13-2


## Loading Data with the Financial Time Series App

```
In this section...
"Overview" on page 14-7
"Obtaining Internal Data" on page 14-7
"Viewing the MATLAB Workspace" on page 14-8
```


## Overview

The Data source pane in the Financial Time Series app lets you do the following:

- Load data you previously obtained and stored in a file.
- View data contained within the MATLAB workspace.


## Obtaining Internal Data

You can use the Financial Time Series app to load data from files previously stored on your computer. The types of data files you can load are as follows:

- MATLAB .mat files, with or without fints objects
- ASCII text files (.dat or .txt suffixes)
- Excel .xls files

To obtain internal data:
1 From the Financial Time Series app, select File > Load $>$ File to open the Load a MAT, ASCII, .XLS File dialog box.

2 Select the data you want to load into the Financial Time Series app.

- If you load a MATLAB MAT-file, the variables in the file are placed into the MATLAB workspace. The MATLAB Workspace Variables list box shows the variables that have been added to the workspace. For example, if you load the file disney.mat, which is distributed with the toolbox, the MATLAB Workspace Variables list box displays the variables in that MAT-file.

Note The Financial Time Series app automatically generates a line plot for each workspace variable unless you disable this feature by resetting the default action under File > Preferences > Generate line plot on load.

- If you load a . dat or an ASCII .txt file, the ASCII File Parameters dialog box opens. Use this dialog box to transform a text data file into a MATLAB financial time series fints object. The format for the ascii data must be:
- Dates must be in a valid date character vector format:
- 'ddmmmyy' or 'ddmmmyyyy'
- 'mm/dd/yy' or 'mm/dd/yyyy'
- 'dd-mmm-yy' or 'dd-mmm-yyyy'
- 'mmm.dd, yy' or 'mmm.dd, yyyy'
- Time information must be in 'hh:mm' format.
- Each column must be separated either by spaces or a tab.

For more information on converting ascii data to a fints object, see ascii2fts.

- If you load an Excel .xls file, the Excel File Parameters dialog box opens. Use this dialog box to transform Excel worksheet data into a MATLAB financial time series (fints) object.
3 From the Financial Time Series app, select File > Save to save the data you loaded from an internal file.


## Viewing the MATLAB Workspace

The MATLAB Workspace Variables list box displays all existing MATLAB workspace variables. Double-click any variable to display the data in the Data Table. You can only display financial time series (fints) objects, MATLAB doubles, and cell arrays of double data in the Data Table.


In addition, you can click Refresh variable list to refresh the MATLAB Workspace Variables list box. You need to refresh this list periodically because it is refreshed automatically only for operations performed with the Financial Time Series app, not for operations performed within MATLAB itself.

Click Remove variable(s) to remove variable from the MATLAB Workspace Variables list and from the MATLAB workspace.

## See Also

ascii2fts | boxcox | convertto|datestr|diff|fillts|filter|fints| fts2mat|ftsbound|lagts|leadts|peravg|resamplets|smoothts|toannual | todaily|tomonthly|toquarterly|tosemi|toweekly|tsmovavg

## Related Examples

- "What Is the Financial Time Series App?" on page 14-2
- "Getting Started with the Financial Time Series App" on page 14-4
- "Using the Financial Time Series App" on page 14-11
- "Using the Financial Time Series App with GUIs" on page 14-19
- "Creating Financial Time Series Objects" on page 11-2
- "Visualizing Financial Time Series Objects" on page 11-31
- "Working with Financial Time Series Objects" on page 13-2


# Using the Financial Time Series App 

```
In this section...
"Creating a Financial Time Series Object" on page 14-11
"Merge Financial Time Series Objects" on page 14-12
"Converting a Financial Time Series Object to a MATLAB Double-Precision Matrix" on
page 14-12
"Plotting the Output in Several Formats" on page 14-13
"Viewing Data for a Financial Time Series Object in the Data Table" on page 14-14
"Modifying Data for a Financial Time Series Object in the Data Table" on page 14-15
"Viewing and Modifying the Properties for a FINTS Object" on page 14-17
```


## Creating a Financial Time Series Object

Using the Create tab in the FINTS Objects and Outputs pane for the Financial Time Series app, you can create a financial time series (fints) object from one or more selected variables.


#### Abstract

Note When you first start the Financial Time Series app, the Create tab appears on top, unless you reset the default using File > Preferences > Show Create tab when ftstool starts.


To create a financial time series (fints) object from one or more selected variables:
1 Load data into the Financial Time Series app from either an external data source using Datafeed Toolbox software or an internal data source using File > Load > File.
2 Select one or more variables from the MATLAB Workspace Variables list.
3 Click the Create tab and then click Active variable.
When combining multiple variables, you can type a new variable name for the combined variables in the MATLAB workspace variable box. The new variable name is added to the MATLAB Workspace Variables list. (If you do not choose a name for the MATLAB workspace variable, the Financial Time Series app uses the default name myFts.)

4 Click Create FINTS object to display the result in the Data Table.

## Merge Financial Time Series Objects

Using the Create tab in the FINTS Objects and Outputs pane for the Financial Time Series app, you can create a new financial time series object by merging (joining) multiple existing financial time series objects.

Note When you first start the Financial Time Series app, the Create tab appears on top, unless you reset the default using File > Preferences.

To create a financial time series (fints) object by merging multiple existing financial time series objects:

1 Load data into the Financial Time Series app from either an external data source using Datafeed Toolbox software or an internal data source using File > Load > File.
2 To merge multiple existing financial time series objects, click the Create tab, click Components, and then select a value for the Time vector source and one or more items from the Data sources list.

Note You can merge at once multiple financial time series objects. For more information on merging fints objects, see merge.
3 Click Create FINTS object to display the result in the Data Table.

## Converting a Financial Time Series Object to a MATLAB Double-Precision Matrix

Using the Convert tab in the FINTS Objects and Outputs pane for the Financial Time Series app, you can convert a financial time series (fints) object to a MATLAB doubleprecision matrix.

To create a financial time series object from one or more selected variables:
1 Load data into the Financial Time Series app from either an external data source using Datafeed Toolbox software or an internal data source using File > Load > File.
2 Select a variable from the MATLAB Workspace Variables list box.

3 Click the Convert tab and then determine whether to include or exclude dates in the conversion by clicking Include dates or Exclude dates.
4 Type a variable name in the Output variable name box. (If you do not choose a variable name, the Financial Time Series app uses the default name myDbl.)
5 Click Convert FINTS to double matrix. (This operation is equivalent to performing fts2mat on a financial time series object.)

## Plotting the Output in Several Formats

Using the Plot tab in the FINTS Objects and Outputs pane for the Financial Time Series app, you can create several forms of plotted output by using a selection list. You can create four types of bar charts, candle plots, high-low plots, line plots, and interactive charts (the latter is created by using the interoperation of the Financial Time Series app with the function chartfts).

The set of plots supported by the Financial Time Series app are identical to the set provided by the Graphs menu of the Financial Time Series GUI. (See "Graphs Menu" on page 15-14.) You can find more detailed information for the supported plots by consulting the reference page for each individual type of plot.

To create a plotted output:
1 Load data into the Financial Time Series app from either an external data source using Datafeed Toolbox software or an internal data source using File > Load > File.
2 Select a variable from the MATLAB Workspace Variables list box or select data from the Data Table.
3 Click the Plot tab and indicate whether you are plotting based on a workspace variable or data from the Data Table.
4 From the Type drop-down list, select the type of plot.
5 Click Plot. The plot is displayed.

Note If the selected workspace variable that you are plotting is not a fints object, a fints object is created when you click Plot. The new fints object uses the name designated by the MATLAB workspace variable box on the Create tab.

## Viewing Data for a Financial Time Series Object in the Data Table

Once a financial time series (fints) object is created, the Financial Time Series app Data Table displays user-designated data, including financial time series objects, MATLAB double-precision variables, and cell arrays of doubles.

When displaying double variables (or a cell array of doubles) in the Data Table, the column headings for a double variable or cell array of doubles displayed in the Data Table are labeled A, B, C, and so on.

## Overwriting Data in the Data Table Display

If you use the command line to overwrite data previously retrieved using Datafeed Toolbox software, two events could occur:

- If the new data contains the same number of columns as before, the headers remain unchanged when you attempt to create a financial time series (fints) object using the modified data.
- If the data contains a different number of columns, a warning dialog box appears.

For example, assume that you use Datafeed Toolbox software to obtain Close, Date, High, Low, Open, and Volume data for the equity GOOG. You store the data in the MATLAB workspace with the variable name curl. From the command line, if you redefine the variable cur1, eliminating the second column (Close)
cur1(:,2) = [ ]
and then return to the Financial Time Series app and attempt to create a financial time series object, a warning dialog box appears.


## Modifying Data for a Financial Time Series Object in the Data Table

The Financial Time Series app lets you update your data displayed in the Data Table by adding or removing rows or columns.

Note Modifying data in the Data Table will not update the MATLAB workspace variable. To update the workspace variable after modifying the Data Table, click Update workspace variable.

## Adding and Removing Rows

To add a row of data displayed in the Data Table:
1 Select a row from the Data Table display where you want to add a row. Click Additional options to open the Data Table Options dialog box.
2 Click Add row. The default is to add up the row. To add a row down, select Insertion option and then click Add down. In addition, you can select the Insertion option of

Date to designate a specific date. (If a date is not specified, the added row contains a date that is chronologically in order with respect to the initial row.)

When you add rows, the Data Table display is immediately updated.
To remove a row of data from the Data Table:
1 Select one or more rows in the Data Table display that you want to remove. Click Additional options to open the Data Table Options dialog box.
2 Click Remove row(s). The default is to remove the selected rows. In addition, to remove selected rows, select Removal options and then select other options for row removal from the Remove rows list box. You can specify a Start and End date or you can click the Non-uniform range setting option to designate a range.

When you remove rows, the Data Table display is updated immediately.

## Adding and Removing Columns

To add a column of data displayed in the Data Table:
1 Select a column from the Data Table display where you want to add a column. Click Additional options to open the Data Table Options dialog box.
2 Click Add column. The default is to add the column to the left of the selected column.

Note For time series objects, you cannot add a column to the left of the Date/Times column; there is no restriction for double data.

To add a column to the right, select Insertion option and then click Add right. In addition, you can use the Insertion option of New Column Name to designate a specific column name. (If a New Column Name is not specified, an added column contains a column name of series1, series2, and so on.)

When you add columns, the Data Table display is updated immediately.
To remove a column of data displayed in the Data Table:
1 Select one or more columns in the Data Table display that you want to remove. Click Additional options to open the Data Table Options dialog box.

2 Click Remove column(s). The default is to remove the selected rows. In addition, to remove selected columns, select Removal options and then select columns for removal from the Remove columns list box.

When you remove columns, the Data Table display is updated immediately.

## Viewing and Modifying the Properties for a FINTS Object

The FINTS Object Properties pane in the Financial Time Series app lets you modify financial time series (fints) object properties. This area becomes active whenever the Data Table displays a financial time series object.

To modify the properties for a fints object:
1 After you create a fints object, double-click the object name in the MATLAB Workspace Variables list box to open the Data Table and display the fints object properties.
2 Click to modify the Description, Frequency, or Series Names fields.
The Frequency drop-down list supports the following conversion functions:

| Function | New Frequency |
| :--- | :--- |
| toannual | Annual |
| todaily | Daily |
| tomonthy | Monthly |
| toquarterly | Quarterly |
| tosemi | Semiannually |
| toweekly | Weekly |

3 Click Update properties to save the changes. This action also updates the associated workspace variable.

```
See Also
ascii2fts | boxcox| convertto|datestr|diff|fillts|filter|fints| fts2mat | ftsbound | lagts | leadts | peravg| resamplets| smoothts| toannual | todaily|tomonthly|toquarterly|tosemi|toweekly|tsmovavg
```


## Related Examples

- "What Is the Financial Time Series App?" on page 14-2
- "Getting Started with the Financial Time Series App" on page 14-4
- "Using the Financial Time Series App with GUIs" on page 14-19
- "Creating Financial Time Series Objects" on page 11-2
- "Visualizing Financial Time Series Objects" on page 11-31
- "Working with Financial Time Series Objects" on page 13-2


## Using the Financial Time Series App with GUIs

The Financial Time Series app works with Datafeed Toolbox software to load data. In addition, the Financial Time Series app interoperates with chartfts to display an interactive plot and ftsgui to perform further time series data analysis.

The workflow for using the Financial Time Series app with chartfts is:
1 After loading data from either Datafeed Toolbox software or an internal file, select a variable from the MATLAB Workspace Variables list box.
2 Click the Plot tab, click Type, and then select Interactive Chart.
3 Click Plot. The interactive plot is displayed in chartfts. You can then use chartfts menu items for further display options.

For more information on chartfts, select Help > Graphics Help.
The workflow for using the Financial Time Series app with the Financial Time Series GUI (ftsgui) is:

1 After loading data from either Datafeed Toolbox software or an internal file, select a variable from the MATLAB Workspace Variables list box.
2 Select Tools > FTSGUI to open the Financial Time Series GUI window.
3 Select a variable from the MATLAB Workspace Variables list box. Click the Plot tab and then select one of the following from the Type drop-down list: Line Plot, High-Low Plot, or Candlestick Plot.

4 Click Plot. The plot is displayed in a MATLAB graphic window. In addition, the Financial Time Series GUI window displays an entry for the plotted fints object. You can then use the menu items in the Financial Time Series GUI window to perform further analysis.

For more information on ftsgui, select Help > Help on Financial Time Series GUI.

Note If the selected workspace variable that you are plotting is not a fints object, a fints object is created when you click Plot. The new fints object uses the name designated by the MATLAB workspace variable box on the Create tab.

## See Also

ascii2fts | boxcox| convertto|datestr|diff|fillts|filter|fints| fts2mat|ftsbound|lagts|leadts|peravg|resamplets|smoothts|toannual | todaily|tomonthly|toquarterly|tosemi|toweekly|tsmovavg

## Related Examples

- "What Is the Financial Time Series App?" on page 14-2
- "Getting Started with the Financial Time Series App" on page 14-4
- "Using the Financial Time Series App" on page 14-11
- "Creating Financial Time Series Objects" on page 11-2
- "Visualizing Financial Time Series Objects" on page 11-31
- "Working with Financial Time Series Objects" on page 13-2


## Financial Time Series User Interface

- "Financial Time Series User Interface" on page 15-2
- "Using the Financial Time Series GUI" on page 15-7


## Financial Time Series User Interface

> Note The Financial Time Series User Interface will be removed in a future release and will no longer accept a fints object.

> Use fts2timetable to convert a fints object to a timetable object. For more information, see "Convert Financial Time Series Objects fints to Timetables" on page 122.

Use the Financial Time Series User Interface to analyze your time series data and display the results graphically without resorting to the command line. The Financial Time Series User Interface lets you visualize the data and the results at the same time.
"Using the Financial Time Series GUI" on page 15-7 discusses how to use this user interface.

## Main Window

Start the Financial Time Series User Interface with the command

## ftsgui

The Financial Time Series GUI window opens.


The title bar acts as an active time series object indicator (indicates the currently active financial time series object). For example, if you load the file disney.mat and want to use the time series data in the file dis, the title bar on the main GUI would read as shown.


The menu bar consists of six menu items: File on page 15-3, Data on page 15-4, Analysis on page 15-4, Graphs on page 15-5, Window on page 15-6, and Help on page 15-6. Under the menu bar is a status box that displays the steps you are doing.

## File Menu



The File menu contains the commands for input and output. You can read and save (Load, Save, and Save As) MATLAB MAT-files, ASCII (text) data files. To load MATLAB MAT-files, the MAT-file must contain a fints object. You can also import (Import) Excel XLS files.

The File menu also contains the printing suite (Page Setup, Print Preview, and Print). Lastly, from this menu you can close the GUI itself (Close FTS GUI) and quit MATLAB (Exit MATLAB).

## Data Menu



The Data menu provides a collection of data manipulation functions and data conversion functions.

| Resample data |  |
| :--- | :--- |
| Convert Data Frequency To | To Daily |
|  | To Weekly |
|  | To Monthly |
|  | To Quarterly |
|  | To Semi-annual |
|  | To Annual |

To use any of the functions here, make sure that the correct financial time series object is displayed in the title bar of the main GUI window.

## Analysis Menu



## The Analysis menu provides

- A set of exponentiation and logarithmic functions.
- Statistical tools (Basic Statistics), which calculate and display the minimum, maximum, average (mean), standard deviation, and variance of the current (active) time series object; these basic statistics numbers are displayed in a dialog box.
- Data difference (Difference) and periodic average (Periodic Average) calculations. Data difference generates a vector of data that is the difference between the first data point and the second, the second, and the third, and so on. The periodic average function calculates the average per defined length period, for example, averages of every five days.
- Technical analysis functions. See "Chart Technical Indicators" for a list of the technical analysis functions.

As with the Data menu, to use any of the Analysis menu functions, make sure that the correct financial time series object is displayed in the title bar of the main GUI window.

## Graphs Menu



The Graphs menu contains functions that graphically display the current (active) financial time series object. You can also start up the interactive charting function (chartfts) from this menu.

## Window Menu



The Window menu lists open windows under the current MATLAB session.

## Help Menu

| 4 Financial Time Series GUI, Active: Walt Disney Company (DIS) |  | - 回区 |
| :---: | :---: | :---: |
| File Data Analysis Graphs Window | Help |  |
| Started: Main FTS GUI Window <br> Loaded file: J:\disney.mat <br> Displaying: Walt Disney Company (DIS) <br> Displaying: TOQUARTERLY: Walt Disney Company (DIS <br> Displaying: Walt Disney Company (DIS) - No VOLUME | Help on Financial Time Series GUI <br> Help on Financial Time Series Toolbox | $\wedge$ |
|  | Help on MATLAB |  |
|  | MATLAB Demos | $\square$ |
| About Financial Time Series GUI |  |  |
|  | About MATLAB |  |

The Help menu provides a standard set of Help menu links.

## See Also

ftsgui|ftstool

## Related Examples

- "Using the Financial Time Series GUI" on page 15-7
- "Working with Financial Time Series Objects" on page 13-2


## Using the Financial Time Series GUI

```
In this section...
"Getting Started" on page 15-7
"Data Menu" on page 15-9
"Analysis Menu" on page 15-13
"Graphs Menu" on page 15-14
"Saving Time Series Data" on page 15-18
```


## Getting Started

To use the Financial Time Series GUI, start the financial time series user interface with the command ftsgui. Then load (or import) the time series data.

For example, if your data is in a MATLAB MAT-file, select Load from the File menu.



For illustration purposes, choose the file ftsdata.mat from the dialog box presented.
If you do not see the MAT-file, look in the folder matlabroot $\backslash$ toolbox $\backslash$ finance \findemos, where matlabroot is the MATLAB root folder (the folder where MATLAB is installed).

Note Data loaded through the Financial Time Series GUI is not available in the MATLAB workspace. You can access this data only through the GUI itself, not with any MATLAB command-line functions.

Each financial time series object inside the MAT-file is presented as a line plot in a separate window. The status window is updated accordingly.


Whirlpool (WHR) is the last plot displayed, as indicated on the title bar of the main window.

## Data Menu

The Data menu provides functions that manipulate time series data.


Here are some example tasks that illustrate the use of the functions on this menu.

## Fill Missing Data

First, look at filling missing data. The Fill Missing Data item uses the toolbox function fillts. With the data loaded from the file ftsdata, you have three time series: IBM

Corp. (IBM), Walt Disney Co. (DIS), and Whirlpool (WHR). Click the window that shows the time series data for Walt Disney Co. (DIS).


To view any missing data in this time series data set, zoom into the plot using the Zoom tool (the magnifying glass icon with the plus sign) from the toolbar and select a region.


The gaps represent the missing data in the series. To fill these gaps, select Data > Fill Missing Data. This selection automatically fills the gaps and generates a new plot that displays the filled time series data.


You cannot see the filled gaps when you display the entire data set. However, when you zoom into the plot, you see that the gaps have been eliminated. The title bar has changed; the title has been prefixed with the word Filled to reflect the filled time series data.

## Frequency Conversion

The Data menu also provides access to frequency conversion functions.

| Resample data |  |
| :--- | :--- |
| Convert Data Frequency To | To Daily |
|  | To Weekly |
|  | To Monthly |
|  | To Quarterly |
|  | To Semi-annual |
|  | To Annual |

This example changes the DIS time series data frequency from daily to monthly. Close the Filled Walt Disney Company (DIS) window, and click the Walt Disney Company (DIS) window to make it active (current) again. Then, from the Data menu, select Convert Data Frequency To and To Monthly.

A new figure window displays the result of this conversion.


The title reflects that the data displayed had its frequency changed to monthly.

## Analysis Menu

The Analysis menu provides functions that analyze time series data, including the technical analysis functions. (See "Chart Technical Indicators" for a complete list of the technical analysis functions and several usage examples.)

For example, you can use the Analysis menu to calculate the natural logarithm (log) of the data contained within the data set ftsdata.mat. This data file provides time series data for IBM (IBM), Walt Disney (DIS), and Whirlpool (WHR). Click the window displaying the data for IBM Corporation (IBM) to make it active (current). Then select the Analysis menu, followed by $\log (. .$.$) . The result appears in its own window.$


Close the above window and click again on the IBM data window to make it active (current).

Note Before proceeding with any time series analysis, make certain that the title bar confirms that the active data series is the correct one.

From the Analysis menu on the main window on page 15-2, select Technical Analysis and MACD. The result, again, is displayed in its own window.


Other analysis functions work similarly.

## Graphs Menu

The Graphs menu displays time series data using the provided graphics functions. Included in the Graphs menu are several types of bar charts (bar, barh and bar3, bar3h), line plot (plot), candle plot (candle), and High-Low plot (highlow). The Graphs menu also provides access to the interactive charting function, chartfts.

## Candle Plot

For example, you can display the candle plot of a set of time series data and start up the interactive chart on the same data set.

Load the ftsdata.mat data set, and click the window that displays the Whirlpool (WHR) time series data to make it active (current). From the main window on page 15-2, select the Graphs menu and then Candle Plot.


The result is shown below.


This does not look much like a candle plot because there are too many data points in the data set. All the candles are too compressed for effective viewing. However, when you zoom into a region of this plot, the candles become apparent.


## Interactive Chart

To create an interactive chart (chartfts) on the Whirlpool data, click the window that displays the Whirlpool (WHR) data to make it active (current). Then, go to the Graphs menu and select Interactive Chart.


The chart that results is shown below.


You can use this interactive chart as if you had invoked it with the chartfts command from the MATLAB command line. For a tutorial on the use of chartfts, see "Visualizing Financial Time Series Objects" on page 11-31.

## Saving Time Series Data

The Save and Save As items on the main window File menu let you save the time series data that results from your analyses and computations. These items save all time series data that has been loaded or processed during the current session, even if the window displaying the results of a computation has previously been dismissed.

Note The Save and Save As items on the File menu of the individual plot windows will not save the time series data, but will save the actual plot.

You can save your time series data in two ways:

- Into the latest MAT-file loaded (Save on page 15-19)
- Into a MAT-file chosen (or named) from the window (Save As on page 15-20)

To illustrate this, start by loading the data file testftsdata.mat (located in matlabroot/toolbox/finance/findemos). Then, convert the Disney (DIS) data from daily (the original frequency) to monthly data. Next, run the MACD analysis on the Whirlpool (WHR) data. You now have a set of five open figure windows.


## Saving into the Original File (Save)

To save the data back into the original file (testftsdata.mat), select Save from the File menu.

A confirmation window appears. It confirms that the data has been saved in the latest MAT-file loaded (testftsdata.mat in this example).


## Saving into a New File (Save As)

To save the data in a different file, choose Save As from the File menu.
The dialog box that appears lets you choose an existing MAT-file from a list or type in the name of a new MAT-file you want to create.


After you click the Save button, another confirmation window appears.


This confirmation window indicates that the data has been saved in a new file named myftstestdata.mat.

## See Also

ftsgui|ftstool

## Related Examples

- "Working with Financial Time Series Objects" on page 13-2


## More About

- "Financial Time Series User Interface" on page 15-2


## Trading Date Utilities

- "Trading Calendars User Interface" on page 16-2
- "UICalendar User Interface" on page 16-4


## Trading Calendars User Interface

Use the createholidays function to open the Trading Calendars user interface.


The createholidays function supports https://www.FinancialCalendar.com trading calendars. This function can be used from the command line or from the Trading Calendars user interface. To use createholidays or the Trading Calendars user interface, you must obtain data, codes, and info files from https:// www. FinancialCalendar. com trading calendars. For more information on using the command line to programmatically generate the market-specific holidays.m files without displaying the interface, see createholidays.

To use the Trading Calendars user interface:
1 From the command line, type the following command to open the Trading Calendars user interface.
createholidays

2 Click Choose data file to select the data file.
3 Click Choose codes file to select the codes file.
4 Click Choose info file to select the info file.
5 Click Choose directory for writing holiday files to select the output folder.
6 Select Include weekends to include weekends in the holiday list and click Prompt for target directory to be prompted for the file location for each holidays.m file that is created.

7 Click Create holiday files to convert FinancialCalendar. com financial center holiday data into market-specific holidays.m files.

The market-specific holidays.m files can be used in place of the standard holidays.m that ships with Financial Toolbox software.

## See Also

createholidays|holidays|nyseclosures

## Related Examples

- "Handle and Convert Dates" on page 2-2
- "UICalendar User Interface" on page 16-4


## UICalendar User Interface

## In this section...

"Using UICalendar in Standalone Mode" on page 16-4
"Using UICalendar with an Application" on page 16-4

## Using UICalendar in Standalone Mode

You can use the UICalendar user interface in standalone mode to look up any date. To use the standalone mode:

1 Type the following command to open the UICalendar GUI:
uicalendar
The UICalendar interface is displayed:


2 Click the date and year controls to locate any date.

## Using UICalendar with an Application

You can use the UICalendar user interface with an application to look up any date. To use the UICalendar graphical interface with an application, use the following command:

```
uicalendar('PARAM1', VALUE1, 'PARAM2', VALUE2', ...)
```

For more information, see uicalendar.

## Example of Using UICalendar with an Application

The UICalendar example creates a function that displays a user interface that lets you select a date from the UICalendar user interface and fill in a text field with that date.

1 Create a figure.
function uicalendarGUIExample
f = figure('Name', 'uicalendarGUIExample');
2 Add a text control field.

```
dateTextHandle = uicontrol(f, 'Style', 'Text', ...
'String', 'Date:', ...
'Horizontalalignment', 'left', ...
'Position', [100 200 50 20]);
```

3 Add an uicontrol editable text field to display the selected date.

```
dateEditBoxHandle = uicontrol(f, 'Style', 'Edit', ...
```

'Position', [140 200100 20], ...
'BackgroundColor', 'w');

4 Create a push button that starts up the UICalendar.

```
calendarButtonHandle = uicontrol(f, 'Style', 'PushButton', ...
'String', 'Select a single date', ...
'Position', [275 200 200 20], ...
'callback', @pushbutton_cb);
```

5 To start up UICalendar, create a nested function (callback function) for the push button.
function pushbutton_cb(hcbo, eventStruct)
\% Create a UICALENDĀR with the following properties:
\% 1) Highlight weekend dates.
\% 2) Only allow a single date to be selected at a time.
\% 3) Send the selected date to the edit box uicontrol. uicalendar('Weekend', [1 00000 1], ...
'SelectionType', 1, ...
'DestinationUI', dateEditBoxHandle);
end
end
6 Run the function uicalendarGUIExample to display the application interface:


7 Click Select a single date to display the UICalendar user interface:


8 Select a date and click $\mathbf{O K}$ to display the date in the text field:


## See Also

createholidays|holidays | nyseclosures

## Related Examples

- "Trading Calendars User Interface" on page 16-2
- "Handle and Convert Dates" on page 2-2


## Technical Analysis

- "Technical Indicators" on page 17-2
- "Technical Analysis Examples" on page 17-4


## Technical Indicators

Technical analysis (or charting) is used by some investment managers to help manage portfolios. Technical analysis relies heavily on the availability of historical data. Investment managers calculate different indicators from available data and plot them as charts. Observations of price, direction, and volume on the charts assist managers in making decisions on their investment portfolios.

The technical analysis functions in Financial Toolbox are tools to help analyze your investments. The functions in themselves will not make any suggestions or perform any qualitative analysis of your investment.

Technical Analysis: Oscillators

| Function | Type |
| :--- | :--- |
| adosc | Accumulation/distribution oscillator |
| chaikosc | Chaikin oscillator |
| macd | Moving Average Convergence/Divergence |
| stochosc | Stochastic oscillator |
| tsaccel | Acceleration |
| tsmom | Momentum |

Technical Analysis: Stochastics

| Function | Type |
| :--- | :--- |
| chaikvolat | Chaikin volatility |
| fpctkd | Fast stochastics |
| spctkd | Slow stochastics |
| willpctr | Williams \%R |
| Technical Analysis: Indexes |  |
| Function | Type |
| negvolidx | Negative volume index |
| posvolidx |  |
| rsindex | Positive volume index <br> Relative strength index |

## Technical Analysis: Indicators

| Function | Type |
| :--- | :--- |
| adline | Accumulation/distribution line |
| bollinger | Bollinger band |
| hhigh | Highest high |
| llow | Lowest low |
| medprice | Median price |
| onbalvol | On balance volume |
| prcroc | Price rate of change |
| pvtrend | Price-volume trend |
| typprice | Typical price |
| volroc | Volume rate of change |
| wclose | Weighted close |
| willad | Williams accumulation/distribution |

## See Also

adline | adosc | bollinger | chaikosc| chaikvolat|fpctkd|hhigh|llow|macd | medprice|negvolidx| onbalvol| posvolidx|prcroc|pvtrend|rsindex| spctkd|stochosc|tsaccel|tsmom|typprice|volroc|wclose|willad| willpctr

## Technical Analysis Examples

In this section...<br>"Overview" on page 17-4<br>"Moving Average Convergence/Divergence (MACD)" on page 17-4<br>"Williams \%R" on page 17-6<br>"Relative Strength Index (RSI)" on page 17-8<br>"On-Balance Volume (OBV)" on page 17-10

## Overview

To illustrate some of the technical analysis functions, this section uses the IBM stock price data contained in the supplied file ibm9599. dat. First create a financial time series object from the data using ascii2fts:

```
ibm = ascii2fts('ibm9599.dat', 1, 3, 2)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In ascii2fts (line 64)
```

The time series data contains the open, close, high, and low prices, and the volume traded on each day. The time series dates start on January 3, 1995, and end on April 1, 1999, with some values missing for weekday holidays; weekend dates are not included.

## Moving Average Convergence/Divergence (MACD)

Moving Average Convergence/Divergence (MACD) is an oscillator function used by technical analysts to spot overbought and oversold conditions. Use the IBM® stock price data contained in the supplied file ibm9599. dat. First, create a financial time series object from the data using ascii2fts. Look at the portion of the time series covering the 3-month period between October 1, 1995 and December 31, 1995. At the same time fill any missing values due to holidays within the time period specified:

```
ibm = ascii2fts('ibm9599.dat', 1, 3, 2);
Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in
part_ibm = fillts(ibm('10/01/95::12/31/95'));
Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in
```

Calculate the MACD, which when plotted produces two lines; the first line is the MACD line itself and the second is the nine-period moving average line:
macd_ibm $=$ macd(part_ibm);
Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in
When you call macd without giving it a second input argument to specify a particular data series name, it searches for a closing price series named Close (in all combinations of letter cases).

Plot the MACD lines and the High-Low plot of the IBM stock prices in two separate plots in one window.

```
subplot(2, 1, 1);
plot(macd_ibm);
Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in
title('MACD of IBM Close Stock Prices, 10/01/95-12/31/95');
datetick('x', 'mm/dd/yy');
subplot(2, 1, 2);
highlow(part_ibm);
Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in
title('IBM Stock Prices, 10/01/95-12/31/95');
datetick('x', 'mm/dd/yy')
```



IBM Stock Prices, 10/01/95-12/31/95


## Williams \%R

Williams \%R is an indicator that measures overbought and oversold levels. The function willpctr is from the stochastics category. All the technical analysis functions can accept a different name for a required data series. If, for example, a function needs the high, low, and closing price series but your time series object does not have the data series names exactly as High, Low, and Close, you can specify the correct names as follows:

```
wpr = willpctr(tsobj,
14,'HighName','Hi','LowName','Lo','CloseName','Closing').
```

The function willpctr now assumes that your high price series is named Hi , low price series is named Lo, and closing price series is named Closing. Use the IBM® stock price data contained in the supplied file ibm9599. dat. First, create a financial time series object from the data using ascii2fts. Look at the portion of the time series covering the 3-month period between October 1, 1995 and December 31, 1995. At the same time fill any missing values due to holidays within the time period specified:

```
ibm = ascii2fts('ibm9599.dat', 1, 3, 2);
Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in
part_ibm = fillts(ibm('10/01/95::12/31/95'));
Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in
```

Since the time series object part_ibm has its data series names identical to the required names, name adjustments are not needed. The input argument to the function is only the name of the time series object itself.

Calculate and plot the Williams \%R indicator for IBM stock along with the price range using these commands:

```
wpctr_ibm = willpctr(part_ibm);
Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in
subplot(2, 1, 1);
plot(wpctr_ibm);
Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in
title('Williams %R of IBM stock, 10/01/95-12/31/95');
datetick('x', 'mm/dd/yy');
hold on;
plot(wpctr_ibm.dates, -80*ones(1, length(wpctr_ibm)),...
'color', [0.5 0 0], 'linewidth', 2)
plot(wpctr_ibm.dates, -20*ones(1, length(wpctr_ibm)),...
```



```
subplot(2, 1, 2);
highlow(part_ibm);
Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in
title('IBM Stock Prices, 10/01/95-12/31/95');
datetick('x', 'mm/dd/yy');
```



IBM Stock Prices, 10/01/95-12/31/95


The top plot has the Williams \%R line plus two lines at $-20 \%$ and $-80 \%$. The bottom plot is the High-Low plot of the IBM stock price for the corresponding time period.

## Relative Strength Index (RSI)

The Relative Strength Index (RSI) is a momentum indicator that measures an equity's price relative to itself and its past performance. The function name is rsindex. The rsindex function needs a series that contains the closing price of a stock. The default period length for the RSI calculation is 14 periods. This length can be changed by providing a second input argument to the function. First, create a financial time series object from the data using ascii2fts. Look at the portion of the time series covering the

3-month period between October 1, 1995 and December 31, 1995. At the same time fill any missing values due to holidays within the time period specified:
ibm = ascii2fts('ibm9599.dat', 1, 3, 2);
Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in part_ibm = fillts(ibm('10/01/95::12/31/95'));

Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in
Calculate and plot the RSI for IBM® stock along with the price range using these commands:

```
rsi_ibm = rsindex(part_ibm);
```

Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in

```
subplot(2, 1, 1);
```

plot(rsi_ibm);
Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in

```
title('RSI of IBM stock, 10/01/95-12/31/95');
```

datetick('x', 'mm/dd/yy');
hold on;
wpctr_ibm = willpctr(part_ibm);
Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in
plot(rsi_ibm.dates, 30*ones(1, length(wpctr_ibm)),...
'color', [0.5 0 0], 'linewidth', 2)
plot(rsi ibm.dates, 70*ones(1, length(wpctr_ibm)),...
'color', [0 0.5 0], 'linewidth', 2)
subplot(2, 1, 2);
highlow(part_ibm);

Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in

```
title('IBM Stock Prices, 10/01/95-12/31/95');
```

datetick('x', 'mm/dd/yy');


## On-Balance Volume (OBV)

On-Balance Volume (OBV) relates volume to price change. The function onbalvol requires you to have the closing price (Close) series and the volume traded (Volume) series. First, create a financial time series object from the data using ascii2fts. Look at the portion of the time series covering the 3-month period between October 1, 1995 and December 31, 1995. At the same time fill any missing values due to holidays within the time period specified:

```
ibm = ascii2fts('ibm9599.dat', 1, 3, 2);
```

Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in
part_ibm = fillts(ibm('10/01/95::12/31/95'));
Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in
Calculate and plot the OBV for $\operatorname{IBM}{ }^{\circledR}$ stock along with the price range using these commands:
obv_ibm = onbalvol(part_ibm);
Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in

```
subplot(2, 1, 1);
```

plot(obv_ibm);

Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in
title('On-Balance Volume of IBM Stock, 10/01/95-12/31/95');
datetick('x', 'mm/dd/yy');
subplot(2, 1, 2);
highlow(part_ibm);
Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in
title('IBM Stock Prices, 10/01/95-12/31/95');
datetick('x', 'mm/dd/yy');


## See Also

adline | adosc | bollinger | chaikosc| chaikvolat |fpctkd|hhigh|llow|macd | medprice | negvolidx | onbalvol | posvolidx| prcroc | pvtrend | rsindex| spctkd|stochosc|tsaccel|tsmom|typprice|volroc|wclose|willad| willpctr

## Stochastic Differential Equations

- "SDEs" on page $18-2$
- "SDE Class Hierarchy" on page 18-5
- "SDE Models" on page 18-9
- "Base SDE Models" on page 18-17
- "Drift and Diffusion Models" on page 18-20
- "Linear Drift Models" on page 18-24
- "Parametric Models" on page 18-26
- "Simulating Equity Prices" on page 18-35
- "Simulating Interest Rates" on page 18-60
- "Stratified Sampling" on page 18-71
- "Performance Considerations" on page 18-77
- "Pricing American Basket Options by Monte Carlo Simulation" on page 18-85
- "Improving Performance of Monte Carlo Simulation with Parallel Computing" on page 18-108


## SDEs

## In this section...

"SDE Modeling" on page 18-2
"Trials vs. Paths" on page 18-3
"NTRIALS, NPERIODS, and NSTEPS" on page 18-4

## SDE Modeling

Financial Toolbox enables you to model dependent financial and economic variables, such as interest rates and equity prices, by performing Monte Carlo simulation of stochastic differential equations (SDEs). The flexible architecture of the SDE engine provides efficient simulation methods that allow you to create new simulation and derivative pricing methods.

The following table lists tasks you can perform using the SDE functionality.

| To perform this task ... | Use these types of models. |
| :---: | :---: |
| "Simulating Equity Prices" on page 18-35 | - Geometric Brownian Motion (GBM) on page 18-28 <br> - Constant Elasticity of Variance (CEV) on page 18-27 <br> - Stochastic Differential Equation (SDE) on page 18-17 <br> - Stochastic Differential Equations from Drift and Diffusion Objects (SDEDDO) on page 18-20 <br> - Stochastic Differential Equations from Linear Drift (SDELD) on page 18-24 <br> - Heston Stochastic Volatility (Heston) on page 18-33 |


| To perform this task ... | Use these types of models ... |
| :---: | :---: |
| "Simulating Interest Rates" on page 18-60 | - Hull-White-Vasicek (HWV) on page 1831 <br> - Cox-Ingersoll-Ross (CIR) on page 18-30 <br> - Stochastic Differential Equation (SDE) on page 18-17 <br> - Stochastic Differential Equations from Drift and Diffusion Objects (SDEDDO) on page 18-20 <br> - Stochastic Differential Equations from Mean-Reverting Drift (SDEMRD) Models on page 18-29 |
| "Pricing Equity Options" on page 18-56 | Geometric Brownian Motion (GBM) on page 18-28 |
| "Stratified Sampling" on page 18-71 | All supported models on page 18-15 |
| "Performance Considerations" on page 1877 | All supported models on page 18-15 |

## Trials vs. Paths

Monte Carlo simulation literature often uses different terminology for the evolution of the simulated variables of interest, such as trials and paths. The following sections use the terms trial and path interchangeably.

However, there are situations where you should distinguish between these terms. Specifically, the term trial often implies the result of an independent random experiment (for example, the evolution of the price of a single stock or portfolio of stocks). Such an experiment computes the average or expected value of a variable of interest (for example, the price of a derivative security) and its associated confidence interval.

By contrast, the term path implies the result of a random experiment that is different or unique from other results, but that may or may not be independent.

The distinction between these terms is unimportant. It may, however, be useful when applied to variance reduction techniques that attempt to increase the efficiency of Monte Carlo simulation by inducing dependence across sample paths. A classic example involves
pairwise dependence induced by antithetic sampling, and applies to more sophisticated variance reduction techniques, such as stratified sampling.

## NTRIALS, NPERIODS, and NSTEPS

SDE functions in the Financial Toolbox software use the parameters NTRIALS, NPERIODS, and NSTEPS as follows:

- The input argument NTRIALS specifies the number of simulated trials or sample paths to generate. This argument always determines the size of the third dimension (the number of pages) of the output three-dimensional time series array Paths. Indeed, in a traditional Monte Carlo simulation of one or more variables, each sample path is independent and represents an independent trial.
- The parameters NPERIODS and NSTEPS represent the number of simulation periods and time steps, respectively. Both periods and time steps are related to time increments that determine the exact sequence of observed sample times. The distinction between these terms applies only to issues of accuracy and memory management. For more information, see "Optimizing Accuracy: About Solution Precision and Error" on page 18-79 and "Managing Memory" on page 18-77.


## See Also

| bm | cev | cir|diffusion |drift | gbm | heston | hwv | interpolate| sde | sdeddo|sdeld|sdemrd|simByEuler| simBySolution|simulate|ts2func

## Related Examples

- "Base SDE Models" on page 18-17
- "Drift and Diffusion Models" on page 18-20
- "Linear Drift Models" on page 18-24
- "Parametric Models" on page 18-26


## More About

- "SDE Class Hierarchy" on page 18-5
- "SDE Models" on page 18-9


## SDE Class Hierarchy

The Financial Toolbox SDE class structure represents a generalization and specialization hierarchy. The top-level class provides the most general model interface and offers the default Monte Carlo simulation and interpolation methods. In turn, derived classes offer restricted interfaces that simplify model creation and manipulation while providing detail regarding model structure.

The following table lists the SDE classes. The introductory examples in "Available Models" on page 18-15 show how to use these classes to create objects associated with univariate models. Although the Financial Toolbox SDE engine supports multivariate models, univariate models facilitate object creation and display, and allow you to easily associate inputs with object parameters.

## SDE Classes

| Class Name | For More Information, See ... |
| :--- | :--- |
| SDE | sde and "Base SDE Models" on page 18-17 |
| Drift, Diffusion | drift, diffusion, and "Overview" on page 18- <br> 20 |
| SDEDDO | sdeddo and "Drift and Diffusion Models" on <br> page 18-20 |
| SDELD | sdeld and "Linear Drift Models" on page 18-24 |
| CEV | cev and "Creating Constant Elasticity of <br> Variance (CEV) Models" on page 18-27 |
| BM | bm and "Creating Brownian Motion (BM) <br> Models" on page 18-26 |
| SDEMRD | sdemrd and "Creating Stochastic Differential <br> Equations from Mean-Reverting Drift (SDEMRD) <br> Models" on page 18-29 |
| GBM | gbm and "Creating Geometric Brownian Motion <br> (GBM) Models" on page 18-28 |
| HWV | hwv and "Creating Hull-White/Vasicek (HWV) <br> Gaussian Diffusion Models" on page 18-31 |
| CIR | cir and "Creating Cox-Ingersoll-Ross (CIR) <br> Square Root Diffusion Models" on page 18-30 |
| Heston | heston and "Creating Heston Stochastic <br> Volatility Models" on page 18-33 |

The following figure illustrates the inheritance relationships among SDE classes.


## See Also

bm | cev |cir|diffusion|drift | gbm| heston | hwv | interpolate| sde| sdeddo | sdeld|sdemrd|simByEuler| simBySolution|simBySolution|simulate| ts2func

## Related Examples

- "Base SDE Models" on page 18-17
- "Drift and Diffusion Models" on page 18-20
- "Linear Drift Models" on page 18-24
- "Parametric Models" on page 18-26


## More About

- "SDEs" on page 18-2
- "SDE Models" on page 18-9


## SDE Models

```
In this section...
"Introduction" on page 18-9
"Creating SDE Objects" on page 18-9
"Drift and Diffusion" on page 18-14
"Available Models" on page 18-15
"SDE Simulation and Interpolation Methods" on page 18-16
"" on page 18-16
```


## Introduction

Most models and utilities available with Monte Carlo Simulation of SDEs are represented as MATLAB objects. Therefore, this documentation often uses the terms model and object interchangeably.

However, although all models are represented as objects, not all objects represent models. In particular, drift and diffusion objects are used in model specification, but neither of these types of objects in and of themselves makes up a complete model. Usually, you do not need to create drift and diffusion objects directly, so you do not need to differentiate between objects and models. It is important, however, to understand the distinction between these terms.

In many of the following examples, most model parameters are evaluated or invoked like any MATLAB function. Although it is helpful to examine and access model parameters as you would data structures, think of these parameters as functions that perform actions.

## Creating SDE Objects

- "Creating Objects" on page 18-10
- "Displaying Objects" on page 18-10
- "Assigning and Referencing Object Parameters" on page 18-10
- "Creating and Evaluating Models" on page 18-10
- "Specifying SDE Simulation Parameters" on page 18-11


## Creating Objects

For examples and more information on creating SDE objects, see:

- "Available Models" on page 18-15
- "Simulating Equity Prices" on page 18-35
- "Simulating Interest Rates" on page 18-60


## Displaying Objects

- Objects display like traditional MATLAB data structures.
- Displayed object parameters appear as nouns that begin with capital letters. In contrast, parameters such as simulate and interpolate appear as verbs that begin with lowercase letters, which indicate tasks to perform.


## Assigning and Referencing Object Parameters

- Objects support referencing similar to data structures. For example, statements like the following are valid:

A = obj. $A$

- Objects support complete parameter assignment similar to data structures. For example, statements like the following are valid:
obj. $A=3$
- Objects do not support partial parameter assignment as data structures do. Therefore, statements like the following are invalid:

```
obj.A(i,j) = 0.3
```


## Creating and Evaluating Models

- You can create objects of any model class only if enough information is available to determine unambiguously the dimensionality of the model. Because each object offers unique input interfaces, some models require additional information to resolve model dimensionality.
- You need only enter required input parameters in placeholder format, where a given input argument is associated with a specific position in an argument list. You can enter optional inputs in any order as parameter name-value pairs, where the name of a given parameter appears in single quotation marks and precedes its corresponding value.
- Association of dynamic (time-variable) behavior with function evaluation, where time and state $\left(t, X_{t}\right)$ are passed to a common, published interface, is pervasive throughout the SDE class system. You can use this function evaluation approach to model or construct powerful analytics. For a simple example, see "Example: Univariate GBM Models" on page 18-29.


## Specifying SDE Simulation Parameters

The SDE engine allows the simulation of generalized multivariate stochastic processes, and provides a flexible and powerful simulation architecture. The framework also provides you with utilities and model classes that offer various parametric specifications and interfaces. The architecture is fully multidimensional in both the state vector and the Brownian motion, and offers both linear and mean-reverting drift-rate specifications.

You can specify most parameters as MATLAB arrays or as functions accessible by a common interface, that supports general dynamic/nonlinear relationships common in SDE simulation. Specifically, you can simulate correlated paths of any number of state variables driven by a vector-valued Brownian motion of arbitrary dimensionality. This simulation approximates the underlying multivariate continuous-time process using a vector-valued stochastic difference equation.

Consider the following general stochastic differential equation:

$$
\begin{equation*}
d X_{t}=F\left(t, X_{t}\right) d t+G\left(t, X_{t}\right) d W_{t} \tag{18-1}
\end{equation*}
$$

where:

- $X$ is an NVARS-by- 1 state vector of process variables (for example, short rates or equity prices) to simulate.
- $W$ is an NBROWNS-by-1 Brownian motion vector.
- $F$ is an NVARS-by-1 vector-valued drift-rate function.
- $G$ is an NVARS-by-NBROWNS matrix-valued diffusion-rate function.

The drift and diffusion rates, $F$ and $G$, respectively, are general functions of a real-valued scalar sample time $t$ and state vector $X_{t}$. Also, static (non-time-variable) coefficients are simply a special case of the more general dynamic (time-variable) situation, just as a function can be a trivial constant; for example, $f\left(t, X_{t}\right)=4$. The SDE in "Equation 18-1" is useful in implementing derived classes that impose additional structure on the drift and diffusion-rate functions.

## Specifying User-Defined Functions as Model Parameters

Several examples in this documentation emphasize the evaluation of object parameters as functions accessible by a common interface. In fact, you can evaluate object parameters by passing to them time and state, regardless of whether the underlying user-specified parameter is a function. However, it is helpful to compare the behavior of object parameters that are specified as functions to that of user-specified noise and end-ofperiod processing functions.

Model parameters that are specified as functions are evaluated in the same way as userspecified random number (noise) generation functions. (For more information, see "Evaluating Different Types of Functions" on page 18-13.) Model parameters that are specified as functions are inputs to remove object constructors. User-specified noise and processing functions are optional inputs to simulation methods.

Because class constructors offer unique interfaces, and simulation methods of any given model have different implementation details, models often call parameter functions for validation purposes a different number of times, or in a different order, during object creation, simulation, and interpolation.

Therefore, although parameter functions, user-specified noise generation functions, and end-of-period processing functions all share the interface and are validated at the same initial time and state (obj.StartTime and obj.StartState), parameter functions are not guaranteed to be invoked only once before simulation as noise generation and end-ofperiod processing functions are. In fact, parameter functions might not even be invoked the same number of times during a given Monte Carlo simulation process.

In most applications in which you specify parameters as functions, they are simple, deterministic functions of time and/or state. There is no need to count periods, count trials, or otherwise accumulate information or synchronize time.

However, if parameter functions require more sophisticated bookkeeping, the correct way to determine when a simulation has begun (or equivalently, to determine when model validation is complete) is to determine when the input time and/or state differs from the initial time and state (obj.StartTime and obj.StartState, respectively). Because the input time is a known scalar, detecting a change from the initial time is likely the best choice in most situations. This is a general mechanism that you can apply to any type of user-defined function.

## Evaluating Different Types of Functions

It is useful to compare the evaluation rules of user-specified noise generation functions to those of end-of-period processing functions. These functions have the following in common:

- They both share the same general interface, returning a column vector of appropriate length when evaluated at the current time and state:

$$
\begin{aligned}
& X_{t}=f\left(t, X_{t}\right) \\
& z_{t}=Z\left(t, X_{t}\right)
\end{aligned}
$$

- Before simulation, the simulation method itself calls each function once to validate the size of the output at the initial time and state, obj. StartTime, and obj. StartState, respectively.
- During simulation, the simulation method calls each function the same number of times: NPERIODS * NSTEPS.

However, there is an important distinction regarding the timing between these two types of functions. It is most clearly drawn directly from the generic SDE model:

$$
d X_{t}=F\left(t, X_{t}\right) d t+G\left(t, X_{t}\right) d W_{t}
$$

This equation is expressed in continuous time, but the simulation methods approximate the model in discrete time as:

$$
X_{t+\Delta t}=X_{t}+F\left(t, X_{t}\right) \Delta t+G\left(t, X_{t}\right) \sqrt{\Delta t} Z\left(t, X_{t}\right)
$$

where $\Delta t>0$ is a small (and not necessarily equal) period or time increment into the future. This equation is often referred to as a Euler approximation. All functions on the rightmost side are evaluated at the current time and state $\left(t, X_{t}\right)$.

In other words, over the next small time increment, the simulation evolves the state vector based only on information available at the current time and state. In this sense, you can think of the noise function as a beginning-of-period function, or as a function evaluated from the left. This is also true for any user-supplied drift or diffusion function.

In contrast, user-specified end-of-period processing functions are applied only at the end of each simulation period or time increment. For more information about processing functions, see "Pricing Equity Options" on page 18-56.

Therefore, all simulation methods evaluate noise generation functions as:

$$
z_{t}=Z\left(t, X_{t}\right)
$$

for $t=t_{0}, t_{0}+\Delta t, t_{0}+2 \Delta t, \ldots, T-\Delta t$.
Yet simulation methods evaluate end-of-period processing functions as:

$$
\begin{gathered}
X_{t}=f\left(t, X_{t}\right) \\
\text { for } t=t_{0}+\Delta t, t_{0}+2 \Delta t, \ldots, T
\end{gathered}
$$

where $t_{0}$ and $T$ are the initial time (taken from the object) and the terminal time (derived from inputs to the simulation method), respectively. These evaluations occur on all sample paths. Therefore, during simulation, noise functions are never evaluated at the final (terminal) time, and end-of-period processing functions are never evaluated at the initial (starting) time.

## Drift and Diffusion

For example, an SDE with a linear drift rate has the form:

$$
\begin{equation*}
F\left(t, X_{t}\right)=A(t)+B(t) X_{t} \tag{18-2}
\end{equation*}
$$

where $A$ is an NVARS-by-1 vector-valued function and $B$ is an $N V A R S$-by-NVARS matrixvalued function.

As an alternative, consider a drift-rate specification expressed in mean-reverting form:

$$
\begin{equation*}
F\left(t, X_{t}\right)=S(t)\left[L(t)-X_{t}\right] \tag{18-3}
\end{equation*}
$$

where $S$ is an NVARS-by-NVARS matrix-valued function of mean reversion speeds (that is, rates of mean reversion), and $L$ is an NVARS-by-1 vector-valued function of mean reversion levels (that is, long run average level).

Similarly, consider the following diffusion-rate specification:

$$
\begin{equation*}
G\left(t, X_{t}\right)=D\left(t, X_{t}^{\alpha(t)}\right) V(t) \tag{18-4}
\end{equation*}
$$

where $D$ is an NVARS-by-NVARS diagonal matrix-valued function. Each diagonal element of $D$ is the corresponding element of the state vector raised to the corresponding element of an exponent Alpha, which is also an NVARS-by-1 vector-valued function. $V$ is an NVARS-by-NBROWNS matrix-valued function of instantaneous volatility rates. Each row of $V$ corresponds to a particular state variable, and each column corresponds to a particular Brownian source of uncertainty. $V$ associates the exposure of state variables with sources of risk.

The parametric specifications for the drift and diffusion-rate functions associate parametric restrictions with familiar models derived from the general SDE class, and provide coverage for many models.

The class system and hierarchy of the SDE engine use industry-standard terminology to provide simplified interfaces for many models by placing user-transparent restrictions on drift and diffusion specifications. This design allows you to mix and match existing models, and customize drift-rate or diffusion-rate functions.

## Available Models

For example, the following models are special cases of the general SDE model.

## SDE Models

| Model Name | Specification |
| :--- | :--- |
| Brownian Motion (BM) | $d X_{t}=A(t) d t+V(t) d W_{t}$ |
| Geometric Brownian Motion (GBM) | $d X_{t}=B(t) X_{t} d t+V(t) X_{t} d W_{t}$ |
| Constant Elasticity of Variance (CEV) | $d X_{t}=B(t) X_{t} d t+V(t) X_{t}^{\alpha(t)} d W_{t}$ |
| Cox-Ingersoll-Ross (CIR) | $d X_{t}=S(t)\left(L(t)-X_{t}\right) d t+V(t) X_{t}^{\frac{1}{2}} d W_{t}$ |
| Hull-White/Vasicek (HWV) | $d X_{t}=S(t)\left(L(t)-X_{t}\right) d t+V(t) d W_{t}$ |
| Heston | $d X_{1 t}=B(t) X_{1 t} d t+\sqrt{X_{2 t}} X_{1 t} d W_{1 t}$ |
|  | $d X_{2 t}=S(t)\left[L(t)-X_{2 t}\right] d t+V(t) \sqrt{X_{2 t}} d W_{2 t}$ |

## SDE Simulation and Interpolation Methods

The sde class provides default simulation and interpolation methods for all derived classes:

- simulate: High-level wrapper around the user-specified simulation method stored in the Simulation property
- simByEuler: Default Euler approximation simulation method
- interpolate: Stochastic interpolation method (that is, Brownian bridge)

The gbm and hwv classes feature an additional function, simBySolution for a gbm object and simBySolution for an hwv object, that simulates approximate solutions of diagonaldrift processes.

In addition, using simByTransition with a cir object, you can approximates a continuous-time Cox-Ingersoll-Ross (CIR) model by an approximation of the transition density function.

## See Also

bm | cev |cir|diffusion|drift | gbm| heston | hwv | interpolate| sde|sdeddo | sdeld | sdemrd | simByEuler | simBySolution | simBySolution | simulate | ts2func

## Related Examples

- "Base SDE Models" on page 18-17
- "Drift and Diffusion Models" on page 18-20
- "Linear Drift Models" on page 18-24
- "Parametric Models" on page 18-26


## More About

- "SDEs" on page 18-2
- "SDE Class Hierarchy" on page 18-5


## Base SDE Models

In this section...
"Overview" on page 18-17
"Example: Base SDE Models" on page 18-17

## Overview

The base sde object

$$
d X_{t}=F\left(t, X_{t}\right) d t+G\left(t, X_{t}\right) d W_{t}
$$

represents the most general model.

Tip The sde class is not an abstract class. You can instantiate sde objects directly to extend the set of core models.

Creating an sde object using sde requires the following inputs:

- A drift-rate function F. This function returns an NVARS-by-1 drift-rate vector when run with the following inputs:
- A real-valued scalar observation time $t$.
- An NVARS-by-1 state vector $X_{t}$.
- A diffusion-rate function G. This function returns an NVARS-by-NBROWNS diffusion-rate matrix when run with the inputs $t$ and $X_{t}$.

Evaluating object parameters by passing $\left(t, X_{t}\right)$ to a common, published interface allows most parameters to be referenced by a common input argument list that reinforces common method programming. You can use this simple function evaluation approach to model or construct powerful analytics, as in the following example.

## Example: Base SDE Models

Create an sde object using sde to represent a univariate geometric Brownian Motion model of the form:

$$
d X_{t}=0.1 X_{t} d t+0.3 X_{t} d W_{t}
$$

1 Create drift and diffusion functions that are accessible by the common $\left(t, X_{t}\right)$ interface:

```
F = @(t,X) 0.1 * X;
G = @(t,X) 0.3 * X;
```

2 Pass the functions to sde to create an sde object:

```
obj = sde(F, G) % dX = F(t,X)dt + G(t,X)dW
obj =
    Class SDE: Stochastic Differential Equation
    Dimensions: State = 1, Brownian = 1
        StartTime: 0
        StartState: 1
        Correlation: 1
            Drift: drift rate function F(t,X(t))
        Diffusion: diffusion rate function G(t,X(t))
        Simulation: simulation method/function simByEuler
```

The sde object displays like a MATLAB structure, with the following information:

- The object's class
- A brief description of the object
- A summary of the dimensionality of the model

The object's displayed parameters are as follows:

- StartTime: The initial observation time (real-valued scalar)
- StartState: The initial state vector (NVARS-by-1 column vector)
- Correlation: The correlation structure between Brownian process
- Drift: The drift-rate function $F\left(t, X_{t}\right)$
- Diffusion: The diffusion-rate function $G\left(t, X_{t}\right)$
- Simulation: The simulation method or function.

Of these displayed parameters, only Drift and Diffusion are required inputs.
The only exception to the $\left(t, X_{t}\right)$ evaluation interface is Correlation. Specifically, when you enter Correlation as a function, the SDE engine assumes that it is a deterministic
function of time, $C(t)$. This restriction on Correlation as a deterministic function of time allows Cholesky factors to be computed and stored before the formal simulation. This inconsistency dramatically improves run-time performance for dynamic correlation structures. If Correlation is stochastic, you can also include it within the simulation architecture as part of a more general random number generation function.

## See Also

bm | cev |cir|diffusion|drift | gbm| heston | hwv | interpolate| sde|sdeddo | sdeld|sdemrd|simByEuler| simBySolution|simBySolution|simulate| ts2func

## Related Examples

- "Drift and Diffusion Models" on page 18-20
- "Linear Drift Models" on page 18-24
- "Parametric Models" on page 18-26


## More About

- "SDEs" on page 18-2
- "SDE Models" on page 18-9
- "SDE Class Hierarchy" on page 18-5


## Drift and Diffusion Models

In this section...<br>"Overview" on page 18-20<br>"Example: Drift and Diffusion Rates" on page 18-21<br>"Example: SDEDDO Models" on page 18-22

## Overview

Because base-level sde objects accept drift and diffusion objects in lieu of functions accessible by $\left(t, X_{t}\right)$, you can create sde objects with combinations of customized drift or diffusion functions and objects. The drift and diffusion rate objects encapsulate the details of input parameters to optimize run-time efficiency for any given combination of input parameters.

Although drift and diffusion objects differ in the details of their representation, they are identical in their basic implementation and interface. They look, feel like, and are evaluated as functions:

- The drift object allows you to create drift-rate objects of the form:

$$
F\left(t, X_{t}\right)=A(t)+B(t) X_{t}
$$

where:

- A is an NVARS-by-1 vector-valued function accessible using the $\left(t, X_{t}\right)$ interface.
- B is an NVARS-by-NVARS matrix-valued function accessible using the $\left(t, X_{t}\right)$ interface.
- Similarly, the diffusion object allows you to create diffusion-rate objects:

$$
G\left(t, X_{t}\right)=D\left(t, X_{t}^{\alpha(t)}\right) V(t)
$$

where:

- D is an NVARS-by-NVARS diagonal matrix-valued function.
- Each diagonal element of $D$ is the corresponding element of the state vector raised to the corresponding element of an exponent Alpha, which is an NVARS-by-1 vector-valued function.
- $V$ is an NVARS-by-NBROWNS matrix-valued volatility rate function Sigma.
- Alpha and Sigma are also accessible using the $\left(t, X_{t}\right)$ interface.

Note You can express drift and diffusion objects in the most general form to emphasize the functional $\left(t, X_{t}\right)$ interface. However, you can specify the components A and B as functions that adhere to the common $\left(t, X_{t}\right)$ interface, or as MATLAB arrays of appropriate dimension.

## Example: Drift and Diffusion Rates

In this example, you create drift and diffusion rate objects to create the same model as in "Example: Base SDE Models" on page 18-17.

Create a drift-rate function F and a diffusion-rate function G :

```
F = drift(0, 0.1) % Drift rate function F(t,X)
F =
    Class DRIFT: Drift Rate Specification
        Rate: drift rate function F(t,X(t))
            A: 0
            B: 0.1
G = diffusion(1, 0.3) % Diffusion rate function G(t,X)
G =
    Class DIFFUSION: Diffusion Rate Specification
        Rate: diffusion rate function G(t,X(t))
        Alpha: 1
        Sigma: 0.3
```

Each object displays like a MATLAB structure and contains supplemental information, namely, the object's class and a brief description. However, in contrast to the SDE representation, a summary of the dimensionality of the model does not appear, because drift and diffusion objects create model components rather than models. Neither F nor $G$ contains enough information to characterize the dimensionality of a problem.

The drift object's displayed parameters are:

- Rate: The drift-rate function, $F\left(t, X_{t}\right)$
- A: The intercept term, $A\left(t, X_{t}\right)$, of $F\left(t, X_{t}\right)$
- B: The first order term, $B\left(t, X_{t}\right)$, of $F\left(t, X_{t}\right)$
$A$ and $B$ enable you to query the original inputs. The function stored in Rate fully encapsulates the combined effect of $A$ and $B$.

The diffusion object's displayed parameters are:

- Rate: The diffusion-rate function, $G\left(t, X_{t}\right)$.
- Alpha: The state vector exponent, which determines the format of $D\left(t, X_{t}\right)$ of $G\left(t, X_{t}\right)$.
- Sigma: The volatility rate, $V\left(t, X_{t}\right)$, of $G\left(t, X_{t}\right)$.

Again, Alpha and Sigma enable you to query the original inputs. (The combined effect of the individual Alpha and Sigma parameters is fully encapsulated by the function stored in Rate.) The Rate functions are the calculation engines for the drift and diffusion objects, and are the only parameters required for simulation.

## Example: SDEDDO Models

The sdeddo object derives from the basesde object. To use this object, you must pass drift and diffusion-rate objects to sdeddo.

1 Create drift and diffusion rate objects:

```
F = drift(0, 0.1); % Drift rate function F(t,X)
G = diffusion(1, 0.3); % Diffusion rate function G(t,X)
```

2 Pass these objects to the sdeddo object:

```
obj = sdeddo(F, G) % dX = F(t,X)dt + G(t,X)dW
obj =
    Class SDEDDO: SDE from Drift and Diffusion Objects
        Dimensions: State = 1, Brownian = 1
            StartTime: 0
        StartState: 1
        Correlation: 1
            Drift: drift rate function F(t,X(t))
            Diffusion: diffusion rate function G(t,X(t))
```

```
Simulation: simulation method/function simByEuler
    A: 0
    B: 0.1
    Alpha: 1
    Sigma: 0.3
```

In this example, the object displays the additional parameters associated with input drift and diffusion objects.

## See Also

bm | cev |cir|diffusion|drift | gbm| heston | hwv | interpolate| sde|sdeddo | sdeld|sdemrd|simByEuler| simBySolution|simBySolution|simulate| ts2func

## Related Examples

- "Base SDE Models" on page 18-17
- "Linear Drift Models" on page 18-24
- "Parametric Models" on page 18-26


## More About

- "SDEs" on page 18-2
- "SDE Models" on page 18-9
- "SDE Class Hierarchy" on page 18-5


## Linear Drift Models

```
In this section...
"Overview" on page 18-24
"Example: SDELD Models" on page 18-24
```


## Overview

The sdeld class derives from the sdeddo class. The sdeld objects allow you to simulate correlated paths of NVARS state variables expressed in linear drift-rate form:

$$
d X_{t}=\left(A(t)+B(t) X_{t}\right) d t+D\left(t, X_{t}^{\alpha(t)}\right) V(t) d W_{t}
$$

sdeld objects provide a parametric alternative to the mean-reverting drift form, as discussed in "Example: SDEMRD Models" on page 18-30. They also provide an alternative interface to the sdeddo parent class, because you can create an object without first having to create its drift and diffusion-rate components.

## Example: SDELD Models

Create the same model as in "Example: Base SDE Models" on page 18-17 using sdeld:

```
obj = sdeld(0, 0.1, 1, 0.3) % (A, B, Alpha, Sigma)
obj =
    Class SDELD: SDE with Linear Drift
        Dimensions: State = 1, Brownian = 1
        StartTime: 0
        StartState: 1
    Correlation: 1
            Drift: drift rate function F(t,X(t))
            Diffusion: diffusion rate function G(t,X(t))
        Simulation: simulation method/function simByEuler
            A: 0
            B: 0.1
            Alpha: 1
            Sigma: 0.3
```


## See Also

bm | cev |cir|diffusion|drift|gbm| heston | hwv | interpolate| sde| sdeddo | sdeld | sdemrd | simByEuler | simBySolution | simBySolution|simulate| ts2func

## Related Examples

- "Base SDE Models" on page 18-17
- "Drift and Diffusion Models" on page 18-20
- "Parametric Models" on page 18-26


## More About

- "SDEs" on page 18-2
- "SDE Models" on page 18-9
- "SDE Class Hierarchy" on page 18-5


## Parametric Models

```
In this section...
"Creating Brownian Motion (BM) Models" on page 18-26
"Example: BM Models" on page 18-26
"Creating Constant Elasticity of Variance (CEV) Models" on page 18-27
"Creating Geometric Brownian Motion (GBM) Models" on page 18-28
"Creating Stochastic Differential Equations from Mean-Reverting Drift (SDEMRD)
Models" on page 18-29
"Creating Cox-Ingersoll-Ross (CIR) Square Root Diffusion Models" on page 18-30
"Creating Hull-White/Vasicek (HWV) Gaussian Diffusion Models" on page 18-31
"Creating Heston Stochastic Volatility Models" on page 18-33
```


## Creating Brownian Motion (BM) Models

The Brownian Motion (BM) model (bm) derives directly from the linear drift (sdeld) model:

$$
d X_{t}=\mu(t) d t+V(t) d W_{t}
$$

## Example: BM Models

Create a univariate Brownian motion (bm) object to represent the model using bm:

```
        dX = 0.3dWt.
obj = bm(0, 0.3) % (A = Mu, Sigma)
obj =
    Class BM: Brownian Motion
    Dimensions: State = 1, Brownian = 1
        StartTime: 0
        StartState: 0
        Correlation: 1
```

```
        Drift: drift rate function F(t,X(t))
    Diffusion: diffusion rate function G(t,X(t))
Simulation: simulation method/function simByEuler
    Mu: 0
    Sigma: 0.3
```

bm objects display the parameter A as the more familiar Mu.
The bm object also provides an overloaded Euler simulation method that improves runtime performance in certain common situations. This specialized method is invoked automatically only if all the following conditions are met:

- The expected drift, or trend, rate Mu is a column vector.
- The volatility rate, Sigma, is a matrix.
- No end-of-period adjustments and/or processes are made.
- If specified, the random noise process Z is a three-dimensional array.
- If $Z$ is unspecified, the assumed Gaussian correlation structure is a double matrix.


## Creating Constant Elasticity of Variance (CEV) Models

The Constant Elasticity of Variance (CEV) model (cev) also derives directly from the linear drift (sdeld) model:

$$
d X_{t}=\mu(t) X_{t} d t+D\left(t, X_{t}^{\alpha(t)}\right) V(t) d W_{t}
$$

The cev object constrains $A$ to an NVARS-by-1 vector of zeros. $D$ is a diagonal matrix whose elements are the corresponding element of the state vector $X$, raised to an exponent $\alpha(t)$.

## Example: Univariate CEV Models

Create a univariate cev object to represent the model using cev:

```
    \(d X_{t}=0.25 X_{t}+0.3 X_{t}^{\frac{1}{2}} d W_{t}\).
obj \(=\operatorname{cev}(0.25,0.5,0.3) \%(B=\) Return, Alpha, Sigma)
obj =
    Class CEV: Constant Elasticity of Variance
```

```
    Dimensions: State = 1, Brownian = 1
        StartTime: 0
    StartState: 1
Correlation: 1
            Drift: drift rate function F(t,X(t))
        Diffusion: diffusion rate function G(t,X(t))
    Simulation: simulation method/function simByEuler
        Return: 0.25
        Alpha: 0.5
        Sigma: 0.3
```

cev and gbm objects display the parameter B as the more familiar Return.

## Creating Geometric Brownian Motion (GBM) Models

The Geometric Brownian Motion (GBM) model (gbm) derives directly from the CEV (cev) model:

$$
d X_{t}=\mu(t) X_{t} d t+D\left(t, X_{t}\right) V(t) d W_{t}
$$

Compared to the cev object, a gbm object constrains all elements of the alpha exponent vector to one such that $D$ is now a diagonal matrix with the state vector $X$ along the main diagonal.

The gbm object also provides two simulation methods that can be used by separable models:

- An overloaded Euler simulation method that improves run-time performance in certain common situations. This specialized method is invoked automatically only if all the following conditions are true:
- The expected rate of return (Return) is a diagonal matrix.
- The volatility rate (Sigma) is a matrix.
- No end-of-period adjustments/processes are made.
- If specified, the random noise process $Z$ is a three-dimensional array.
- If $Z$ is unspecified, the assumed Gaussian correlation structure is a double matrix.
- An approximate analytic solution (simBySolution) obtained by applying a Euler approach to the transformed (using Ito's formula) logarithmic process. In general, this
is not the exact solution to this GBM model, as the probability distributions of the simulated and true state vectors are identical only for piecewise constant parameters. If the model parameters are piecewise constant over each observation period, the state vector $X_{t}$ is lognormally distributed and the simulated process is exact for the observation times at which $X_{t}$ is sampled.


## Example: Univariate GBM Models

Create a univariate gbm object to represent the model using gbm:

```
    dX t}=0.25\mp@subsup{X}{t}{}dt+0.3\mp@subsup{X}{t}{}d\mp@subsup{W}{t}{
obj = gbm(0.25, 0.3) % (B = Return, Sigma)
obj =
    Class GBM: Generalized Geometric Brownian Motion
    Dimensions: State = 1, Brownian = 1
    StartTime: 0
    StartState: 1
    Correlation: 1
        Drift: drift rate function F(t,X(t))
        Diffusion: diffusion rate function G(t,X(t))
    Simulation: simulation method/function simByEuler
        Return: 0.25
            Sigma: 0.3
```


## Creating Stochastic Differential Equations from MeanReverting Drift (SDEMRD) Models

The sdemrd object derives directly from the sdeddo object. It provides an interface in which the drift-rate function is expressed in mean-reverting drift form:

$$
d X_{t}=S(t)\left[L(t)-X_{t}\right] d t+D\left(t, X_{t}^{\alpha(t)}\right) V(t) d W_{t}
$$

sdemrd objects provide a parametric alternative to the linear drift form by reparameterizing the general linear drift such that:

$$
A(t)=S(t) L(t), B(t)=-S(t)
$$

## Example: SDEMRD Models

Create an sdemrd object using sdemrd with a square root exponent to represent the model:

```
        dX t}=0.2(0.1-\mp@subsup{X}{t}{})dt+0.05\mp@subsup{X}{t}{\frac{1}{2}}d\mp@subsup{W}{t}{}
obj = sdemrd(0.2, 0.1, 0.5, 0.05)
obj =
    Class SDEMRD: SDE with Mean-Reverting Drift
    Dimensions: State = 1, Brownian = 1
        StartTime: 0
        StartState: 1
    Correlation: 1
        Drift: drift rate function F(t,X(t))
        Diffusion: diffusion rate function G(t,X(t))
    Simulation: simulation method/function simByEuler
        Alpha: 0.5
        Sigma: 0.05
        Level: 0.1
        Speed: 0.2
    % (Speed, Level, Alpha, Sigma)
```

sdemrd objects display the familiar Speed and Level parameters instead of A and B.

## Creating Cox-Ingersoll-Ross (CIR) Square Root Diffusion Models

The Cox-Ingersoll-Ross (CIR) short rate object, cir, derives directly from the SDE with mean-reverting drift (sdemrd) class:

$$
d X_{t}=S(t)\left[L(t)-X_{t}\right] d t+D\left(t, X_{t}^{\frac{1}{2}}\right) V(t) d W_{t}
$$

where $D$ is a diagonal matrix whose elements are the square root of the corresponding element of the state vector.

## Example: CIR Models

Create a cir object using cir to represent the same model as in "Example: SDEMRD Models" on page 18-30:

```
obj = cir(0.2, 0.1, 0.05) % (Speed, Level, Sigma)
obj =
    Class CIR: Cox-Ingersoll-Ross
        Dimensions: State = 1, Brownian = 1
        StartTime: 0
        StartState: 1
    Correlation: 1
        Drift: drift rate function F(t,X(t))
        Diffusion: diffusion rate function G(t,X(t))
        Simulation: simulation method/function simByEuler
        Sigma: 0.05
        Level: 0.1
        Speed: 0.2
```

Although the last two objects are of different classes, they represent the same mathematical model. They differ in that you create the cir object by specifying only three input arguments. This distinction is reinforced by the fact that the Alpha parameter does not display - it is defined to be $1 / 2$.

## Creating Hull-White/Vasicek (HWV) Gaussian Diffusion Models

The Hull-White/Vasicek (HWV) short rate object, hwv, derives directly from SDE with mean-reverting drift (sdemrd) class:

$$
d X_{t}=S(t)\left[L(t)-X_{t}\right] d t+V(t) d W_{t}
$$

## Example: HWV Models

Using the same parameters as in the previous example, create an hwv object using hwv to represent the model:

$$
\begin{aligned}
& \quad d X_{t}=0.2\left(0.1-X_{t}\right) d t+0.05 d W_{t} . \\
& \text { obj }=\operatorname{hwv}(0.2,0.1,0.05) \quad \%(\text { Speed, Level, Sigma) }
\end{aligned}
$$

```
obj =
    Class HWV: Hull-White/Vasicek
        Dimensions: State = 1, Brownian = 1
        StartTime: 0
        StartState: 1
    Correlation: 1
        Drift: drift rate function F(t,X(t))
        Diffusion: diffusion rate function G(t,X(t))
        Simulation: simulation method/function simByEuler
        Sigma: 0.05
        Level: 0.1
        Speed: 0.2
```

cir and hwv share the same interface and display methods. The only distinction is that cir and hwv model objects constrain Alpha exponents to $1 / 2$ and 0 , respectively. Furthermore, the hwv object also provides an additional method that simulates approximate analytic solutions (simBySolution) of separable models. This method simulates the state vector $X_{t}$ using an approximation of the closed-form solution of diagonal drift HWV models. Each element of the state vector $X_{t}$ is expressed as the sum of NBROWNS correlated Gaussian random draws added to a deterministic time-variable drift.

When evaluating expressions, all model parameters are assumed piecewise constant over each simulation period. In general, this is not the exact solution to this hwv model, because the probability distributions of the simulated and true state vectors are identical only for piecewise constant parameters. If $S\left(t, X_{t}\right), L\left(t, X_{t}\right)$, and $V\left(t, X_{t}\right)$ are piecewise constant over each observation period, the state vector $X_{t}$ is normally distributed, and the simulated process is exact for the observation times at which $X_{t}$ is sampled.

## Hull-White vs. Vasicek Models

Many references differentiate between Vasicek models and Hull-White models. Where such distinctions are made, Vasicek parameters are constrained to be constants, while Hull-White parameters vary deterministically with time. Think of Vasicek models in this context as constant-coefficient Hull-White models and equivalently, Hull-White models as time-varying Vasicek models. However, from an architectural perspective, the distinction between static and dynamic parameters is trivial. Since both models share the same general parametric specification as previously described, a single hwv object encompasses the models.

## Creating Heston Stochastic Volatility Models

The Heston (heston) object derives directly from SDE from the Drift and Diffusion (sdeddo) class. Each Heston model is a bivariate composite model, consisting of two coupled univariate models:

$$
\begin{align*}
& d X_{1 t}=B(t) X_{1 t} d t+\sqrt{X_{2 t}} X_{1 t} d W_{1 t}  \tag{18-5}\\
& d X_{2 t}=S(t)\left[L(t)-X_{2 t}\right] d t+V(t) \sqrt{X_{2 t}} d W_{2 t} \tag{18-6}
\end{align*}
$$

"Equation 18-5" is typically associated with a price process. "Equation 18-6" represents the evolution of the price process' variance. Models of type heston are typically used to price equity options.

## Example: Heston Models

Create a heston object using heston to represent the model:

```
    dX 1t =0.1 X 1t dt + \sqrt{}{\mp@subsup{X}{2t}{}}\mp@subsup{X}{1t}{}d\mp@subsup{W}{1t}{}
    dX 2t = 0.2[0.1- X 2t ]dt+0.05\sqrt{}{\mp@subsup{X}{2t}{}}d\mp@subsup{W}{2t}{}
obj = heston (0.1, 0.2, 0.1, 0.05)
obj =
    Class HESTON: Heston Bivariate Stochastic Volatility
    Dimensions: State = 2, Brownian = 2
        StartTime: 0
        StartState: 1 (2x1 double array)
        Correlation: 2x2 diagonal double array
            Drift: drift rate function F(t,X(t))
        Diffusion: diffusion rate function G(t,X(t))
        Simulation: simulation method/function simByEuler
    Return: 0.1
        Speed: 0.2
        Level: 0.1
        Volatility: 0.05
```


## See Also

bm | cev |cir|diffusion|drift | gbm| heston | hwv | interpolate| sde|sdeddo | sdeld | sdemrd | simByEuler|simBySolution|simBySolution|simulate| ts2func

## Related Examples

- "Base SDE Models" on page 18-17
- "Drift and Diffusion Models" on page 18-20
- "Linear Drift Models" on page 18-24


## More About

- "SDEs" on page 18-2
- "SDE Models" on page 18-9
- "SDE Class Hierarchy" on page 18-5


## Simulating Equity Prices

```
In this section...
"Simulating Multidimensional Market Models" on page 18-35
"Inducing Dependence and Correlation" on page 18-49
"Dynamic Behavior of Market Parameters" on page 18-51
"Pricing Equity Options" on page 18-56
```


## Simulating Multidimensional Market Models

This example compares alternative implementations of a separable multivariate geometric Brownian motion process that is often referred to as a multidimensional market model. It simulates sample paths of an equity index portfolio using sde, sdeddo, sdeld, cev, and gbm objects.

The market model to simulate is:

$$
\begin{equation*}
d X_{t}=\mu X_{t} d t+D\left(X_{t}\right) \sigma d W_{t} \tag{18-7}
\end{equation*}
$$

where:

- $\quad \mu$ is a diagonal matrix of expected index returns.
- $D$ is a diagonal matrix with $X_{t}$ along the diagonal.
- $\sigma$ is a diagonal matrix of standard deviations of index returns.


## Representing Market Models Using SDE Objects

Create an sde object using sde to represent the equity market model.
1 Load the Data_GlobalIdx2 data set:

```
load Data_GlobalIdx2
prices = [Dataset.TSX Dataset.CAC Dataset.DAX ...
Dataset.NIK Dataset.FTSE Dataset.SP];
```

2 Convert daily prices to returns:
returns = tick2ret(prices);

3 Compute data statistics to input to simulation methods:

```
nVariables = size(returns, 2);
expReturn = mean(returns);
sigma = std(returns);
correlation = corrcoef(returns);
t = 0;
X = 100;
X = X(ones(nVariables,1));
```

4 Create simple anonymous drift and diffusion functions accessible by $\left(t, X_{t}\right)$ :

```
F = @(t,X) diag(expReturn) * X;
G = @(t,X) diag(X) * diag(sigma);
```

5 Use these functions with sde to create an sde object to represent the market model in "Equation 18-7":

```
SDE = sde(F, G, 'Correlation', correlation, 'StartState', X)
SDE =
    Class SDE: Stochastic Differential Equation
        Dimensions: State = 6, Brownian = 6
            StartTime: 0
        StartState: 100 (6x1 double array)
        Correlation: 6x6 double array
            Drift: drift rate function F(t,X(t))
            Diffusion: diffusion rate function G(t,X(t))
        Simulation: simulation method/function simByEuler
```

The sde object requires additional information to determine the dimensionality of the model, because the functions passed to thesde object are known only by their ( $t, X_{t}$ ) interface. In other words, thesde object requires only two inputs: a drift-rate function and a diffusion-rate function, both accessible by passing the sample time and the corresponding state vector $\left(t, X_{t}\right)$.

In this case, this information is insufficient to determine unambiguously the dimensionality of the state vector and Brownian motion. You resolve the dimensionality by specifying an initial state vector, StartState. The SDE engine has assigned the default simulation method, simByEuler, to the Simulation parameter.

## Representing Market Models Using SDEDDO Objects

Create an sdeddo object using sdeddo to represent the market model in "Equation 18-7":

1 Load the Data_GlobalIdx2 data set:
load Data GlobalIdx2
prices = [Dataset.TSX Dataset.CAC Dataset.DAX ... Dataset.NIK Dataset.FTSE Dataset.SP];
2 Convert daily prices to returns:

```
returns = tick2ret(prices);
```

3 Compute data statistics to input to simulation methods:
nVariables = size(returns, 2);
expReturn = mean(returns);
sigma = std(returns);
correlation = corrcoef(returns);
4 Create drift and diffusion objects using drift and diffusion:

```
F = drift(zeros(nVariables,1), diag(expReturn))
F =
    Class DRIFT: Drift Rate Specification
        Rate: drift rate function F(t,X(t))
            A: 6xl double array
            B: 6x6 diagonal double array
```

G = diffusion(ones(nVariables,1), diag(sigma))
G =
Class DIFFUSION: Diffusion Rate Specification
Rate: diffusion rate function $\mathrm{G}(\mathrm{t}, \mathrm{X}(\mathrm{t}))$
Alpha: $6 x 1$ double array
Sigma: 6x6 diagonal double array

5 Pass the drift and diffusion objects to sdeddo:
SDEDDO = sdeddo(F, G, 'Correlation', correlation, ...
'StartState', 100)
SDEDDO =
Class SDEDDO: SDE from Drift and Diffusion Objects

```
    Dimensions: State = 6, Brownian = 6
        StartTime: 0
    StartState: 100 (6x1 double array)
Correlation: 6x6 double array
            Drift: drift rate function F(t,X(t))
        Diffusion: diffusion rate function G(t,X(t))
Simulation: simulation method/function simByEuler
            A: 6x1 double array
            B: 6x6 diagonal double array
            Alpha: 6x1 double array
            Sigma: 6x6 diagonal double array
```

The sdeddo object requires two input objects that provide more information than the two functions from step 4 of "Representing Market Models Using SDE Objects" on page 18-35. Thus, the dimensionality is more easily resolved. In fact, the initial price of each index is as a scalar (StartState = 100). This is in contrast to the sde object, which required an explicit state vector to uniquely determine the dimensionality of the problem.

Once again, the class of each object is clearly identified, and parameters display like fields of a structure. In particular, the Rate parameter of drift and diffusion objects is identified as a callable function of time and state, $F\left(t, X_{t}\right)$ and $G\left(t, X_{t}\right)$, respectively. The additional parameters, A, B, Alpha, and Sigma, are arrays of appropriate dimension, indicating static (non-time-varying) parameters. In other words, $A\left(t, X_{t}\right), B\left(t, X_{t}\right)$, $\operatorname{Alpha}\left(t, X_{t}\right)$, and $\operatorname{Sigma}\left(t, X_{t}\right)$ are constant functions of time and state.

## Representing Market Models Using SDELD, CEV, and GBM Objects

Create sdeld, cev, and gbm objects to represent the market model in "Equation 18-7".
1 Load the Data_GlobalIdx2 data set:
load Data GlobalIdx2
prices $=$ - [Dataset.TSX Dataset.CAC Dataset.DAX ...
Dataset.NIK Dataset.FTSE Dataset.SP];
2 Convert daily prices to returns:
returns = tick2ret(prices);
3 Compute data statistics to input to simulation methods:
nVariables = size(returns, 2);
expReturn = mean(returns);

```
sigma = std(returns);
correlation = corrcoef(returns);
t = 0;
X = 100;
X = X(ones(nVariables,1));
```

4 Create an sdeld object using sdeld:
SDELD $=$ sdeld(zeros(nVariables,1), diag(expReturn), ...
ones(nVariables,1), diag(sigma), 'Correlation', ...
correlation, 'StartState', X)
SDELD =
Class SDELD: SDE with Linear Drift
Dimensions: State $=6$, Brownian $=6$
StartTime: 0
StartState: 100 (6x1 double array)
Correlation: 6x6 double array
Drift: drift rate function $F(t, X(t))$
Diffusion: diffusion rate function $G(t, X(t))$
Simulation: simulation method/function simByEuler
A: $6 x 1$ double array
B: $6 \times 6$ diagonal double array
Alpha: 6x1 double array
Sigma: 6x6 diagonal double array

5 Create a cev object using cev:

```
CEV = cev(diag(expReturn), ones(nVariables,1), ...
    diag(sigma), 'Correlation', correlation, ...
    'StartState', X)
CEV =
    Class CEV: Constant Elasticity of Variance
    Dimensions: State = 6, Brownian = 6
    StartTime: 0
    StartState: 100 (6x1 double array)
    Correlation: 6x6 double array
                            Drift: drift rate function F(t,X(t))
            Diffusion: diffusion rate function G(t,X(t))
        Simulation: simulation method/function simByEuler
            Return: 6x6 diagonal double array
```

```
Alpha: 6x1 double array
Sigma: 6x6 diagonal double array
```

6 Create a gbm object using gbm:

```
GBM = gbm(diag(expReturn), diag(sigma), 'Correlation', ...
    correlation, 'StartState', X)
GBM =
    Class GBM: Generalized Geometric Brownian Motion
        Dimensions: State = 6, Brownian = 6
        StartTime: 0
        StartState: 100 (6x1 double array)
    Correlation: 6x6 double array
            Drift: drift rate function F(t,X(t))
        Diffusion: diffusion rate function G(t,X(t))
        Simulation: simulation method/function simByEuler
        Return: 6x6 diagonal double array
            Sigma: 6x6 diagonal double array
```

Note the succession of interface restrictions:

- sdeld objects require you to specify A, B, Alpha, and Sigma.
- cev objects require you to specify Return, Alpha, and Sigma.
- gbm objects require you to specify only Return and Sigma.

However, all three objects represent the same multidimensional market model.
Also, cev and gbm objects display the underlying parameter B derived from the sdeld object as Return. This is an intuitive name commonly associated with equity models.

## Simulating Equity Markets Using the Default Simulate Method

1 Load the Data_GlobalIdx2 data set and use sde to specify the SDE model as in "Representing Market Models Using SDE Objects" on page 18-35.

```
load Data_GlobalIdx2
prices =- [Dataset.TSX Dataset.CAC Dataset.DAX ...
    Dataset.NIK Dataset.FTSE Dataset.SP];
returns = tick2ret(prices);
```

```
nVariables = size(returns,2);
expReturn = mean(returns);
sigma = std(returns);
correlation = corrcoef(returns);
t = 0;
X = 100;
X = X(ones(nVariables,1));
F = @(t,X) diag(expReturn)* X;
G = @(t,X) diag(X) * diag(sigma);
SDE = sde(F, G, 'Correlation', ...
    correlation, 'StartState', X);
```

2 Simulate a single path of correlated equity index prices over one calendar year (defined as approximately 250 trading days) using the defaultsimulate method:

```
nPeriods = 249; % # of simulated observations
dt = 1; % time increment = 1 day
rng(142857,'twister')
[S,T] = simulate(SDE, nPeriods, 'DeltaTime', dt);
whos S
    Name Size Bytes Class Attributes
    S 250x6 12000 double
```

The output array S is a 250 -by- $6=($ NPERIODS +1 -by-nVariables-by-1) array with the same initial value, 100, for all indices. Each row of $S$ is an observation of the state vector $X_{t}$ at time $t$.
3 Plot the simulated paths.

```
plot(T, S), xlabel('Trading Day'), ylabel('Price')
title('Single Path of Multi-Dimensional Market Model')
legend({'Canada' 'France' 'Germany' 'Japan' 'UK' 'US'}, ...
    'Location', 'Best')
```



## Simulating Equity Markets Using the SimByEuler Method

Because simByEuler is a valid simulation method, you can call it directly, overriding the Simulation parameter's current method or function (which in this case is simByEuler). The following statements produce the same price paths as generated in "Simulating Equity Markets Using the Default Simulate Method" on page 18-40:

1 Load the Data_GlobalIdx2 data set and use sde to specify the SDE model as in "Representing Market Models Using SDE Objects" on page 18-35.

```
load Data_GlobalIdx2
prices =- [Dataset.TSX Dataset.CAC Dataset.DAX ...
        Dataset.NIK Dataset.FTSE Dataset.SP];
```

```
returns = tick2ret(prices);
nVariables = size(returns,2);
expReturn = mean(returns);
sigma = std(returns);
correlation = corrcoef(returns);
t = 0;
X = 100;
X = X(ones(nVariables,1));
F = @(t,X) diag(expReturn)* X;
G = @(t,X) diag(X) * diag(sigma);
SDE = sde(F, G, 'Correlation', ...
    correlation, 'StartState', X);
```

2 Simulate a single path using simByEuler.

```
nPeriods = 249; % # of simulated observations
dt = 1; % time increment = 1 day
rng(142857,'twister')
[S,T] = simByEuler(SDE, nPeriods, 'DeltaTime', dt);
```

3 Simulate 10 trials with the same initial conditions, and examine S :

```
rng(142857,'twister')
[S,T] = simulate(SDE, nPeriods, 'DeltaTime', dt, 'nTrials', 10);
whos S
Name Size Bytes Class Attributes
    S 250x6x10 120000 double
```

Now the output array S is an NPERIODS + 1-by-nVariables-by-nTrials time series array.
4 Plot the first paths.

```
plot(T, S(:,:,1)), xlabel('Trading Day'), ylabel('Price')
title('First Path of Multi-Dimensional Market Model')
legend({'Canada' 'France' 'Germany' 'Japan' 'UK' 'US'},...
    'Location', 'Best')
```



The first realization of $S$ is identical to the paths in the plot.

## Simulating Equity Markets Using GBM Simulation Methods

Finally, consider simulation using GBM simulation methods. Separable GBM models have two specific simulation methods:

- An overloaded Euler simulation method, simByEuler, designed for optimal performance
- A function, simBySolution, provides an approximate solution of the underlying stochastic differential equation, designed for accuracy

1 Load the Data_GlobalIdx2 data set and use sde to specify the SDE model as in "Representing Market Models Using SDE Objects" on page 18-35, and the GBM
model as in "Representing Market Models Using SDELD, CEV, and GBM Objects" on page 18-38.

```
load Data GlobalIdx2
prices =- [Dataset.TSX Dataset.CAC Dataset.DAX ...
    Dataset.NIK Dataset.FTSE Dataset.SP];
returns = tick2ret(prices);
nVariables = size(returns,2);
expReturn = mean(returns);
sigma = std(returns);
correlation = corrcoef(returns);
t = 0;
X = 100;
X = X(ones(nVariables,1));
F = @(t,X) diag(expReturn)* X;
G = @(t,X) diag(X) * diag(sigma);
SDE = sde(F, G, 'Correlation', ...
    correlation, 'StartState', X);
GBM = gbm(diag(expReturn),diag(sigma), 'Correlation', ...
    correlation, 'StartState', X);
```

2 To illustrate the performance benefit of the overloaded Euler approximation method, increase the number of trials to 10000.

```
nPeriods = 249; % # of simulated observations
dt = 1; % time increment = 1 day
rng(142857,'twister')
[X,T] = simulate(GBM, nPeriods, 'DeltaTime', dt, ...
    'nTrials', 10000);
whos X
```

| Name | Size | Bytes | Class |
| :--- | :---: | ---: | :--- | Attributes

The output $X$ is a much larger time series array.
3 Using this sample size, examine the terminal distribution of Canada's TSX Composite to verify qualitatively the lognormal character of the data.
histogram(squeeze(X(end,1,:)), 30), xlabel('Price'), ylabel('Frequency') title('Histogram of Prices after One Year: Canada (TSX Composite)')


4 Simulate 10 trials of the solution and plot the first trial:

```
rng(142857,'twister')
[S,T] = simulate(SDE, nPeriods, 'DeltaTime', dt, 'nTrials', 10);
rng(142857,'twister')
[X,T] = simBySolution(GBM, nPeriods,...
            'DeltaTime', dt, 'nTrials', 10);
subplot(2,1,1)
plot(T, S(:,:,1)), xlabel('Trading Day'),ylabel('Price')
title('1st Path of Multi-Dim Market Model:Euler Approximation')
subplot(2,1,2)
```



In this example, all parameters are constants, and simBySolution does indeed sample the exact solution. The details of a single index for any given trial show that the price paths of the Euler approximation and the exact solution are close, but not identical.

5 The following plot illustrates the difference between the two methods:

```
subplot(1,1,1)
plot(T, S(:,1,1) - X(:,1,1), 'blue'), grid('on')
xlabel('Trading Day'), ylabel('Price Difference')
title('Euler Approx Minus Exact Solution:Canada(TSX Composite)')
```



The simByEuler Euler approximation literally evaluates the stochastic differential equation directly from the equation of motion, for some suitable value of the dt time increment. This simple approximation suffers from discretization error. This error can be attributed to the discrepancy between the choice of the dt time increment and what in theory is a continuous-time parameter.

The discrete-time approximation improves as DeltaTime approaches zero. The Euler method is often the least accurate and most general method available. All models shipped in the simulation suite have this method.

In contrast, thesimBySolution method provides a more accurate description of the underlying model. This method simulates the price paths by an approximation of the closed-form solution of separable models. Specifically, it applies a Euler approach to a
transformed process, which in general is not the exact solution to this GBM model. This is because the probability distributions of the simulated and true state vectors are identical only for piecewise constant parameters.

When all model parameters are piecewise constant over each observation period, the simulated process is exact for the observation times at which the state vector is sampled. Since all parameters are constants in this example,simBySolution does indeed sample the exact solution.

For an example of how to use simBySolution to optimize the accuracy of solutions, see "Optimizing Accuracy: About Solution Precision and Error" on page 18-79.

## Inducing Dependence and Correlation

This example illustrates two techniques that induce dependence between individual elements of a state vector. It also illustrates the interaction between Sigma and Correlation.

The first technique generates correlated Gaussian variates to form a Brownian motion process with dependent components. These components are then weighted by a diagonal volatility or exposure matrix Sigma.

The second technique generates independent Gaussian variates to form a standard Brownian motion process, which is then weighted by the lower Cholesky factor of the desired covariance matrix. Although these techniques can be used on many models, the relationship between them is most easily illustrated by working with a separable GBM model (see Simulating Equity Prices Using GBM Simulation Methods on page 18-44). The market model to simulate is:

$$
d X_{t}=\mu X_{t} d t+\sigma X_{t} d W_{t}
$$

where $\mu$ is a diagonal matrix.
1 Load the data set:

```
load Data GlobalIdx2
prices = [Dataset.TSX Dataset.CAC Dataset.DAX ...
    Dataset.NIK Dataset.FTSE Dataset.SP];
```

2 Convert the daily prices to returns:

```
returns = tick2ret(prices);
```

3 Specify Sigma and Correlation using the first technique:
a Using the first technique, specify Sigma as a diagonal matrix of asset return standard deviations:

```
expReturn = diag(mean(returns)); % expected return vector
sigma = diag(std(returns)); % volatility of returns
```

b Specify Correlation as the sample correlation matrix of those returns. In this case, the components of the Brownian motion are dependent:

```
correlation = corrcoef(returns);
GBM1 = gbm(expReturn,sigma,'Correlation',...
    correlation);
```

4 Specify Sigma and Correlation using the second technique:
a Using the second technique, specify Sigma as the lower Cholesky factor of the asset return covariance matrix:

```
covariance = cov(returns);
```

sigma = cholcov(covariance)';
b Set Correlation to an identity matrix:
GBM2 $=$ gbm(expReturn,sigma);
Here, sigma captures both the correlation and magnitude of the asset return uncertainty. In contrast to the first technique, the components of the Brownian motion are independent. Also, this technique accepts the default assignment of an identity matrix to Correlation, and is more straightforward.
5 Simulate a single trial of 1000 observations (roughly four years of daily data) using both techniques. By default, all state variables start at 1:

```
rng(22814,'twister')
[X1,T] = simByEuler(GBM1,1000); % correlated Brownian motion
rng(22814,'twister')
[X2,T] = simByEuler(GBM2,1000); % standard Brownian motion
```

When based on the same initial random number state, each technique generates identical asset price paths:

```
subplot(2,1,1)
```

plot(T, X1)
title('Sample Paths from Correlated Brownian Motion')
ylabel('Asset Price')

```
subplot(2,1,2)
plot(T, X2)
title('Sample Paths from Standard Brownian Motion')
xlabel('Trading Day')
ylabel('Asset Price')
```




## Dynamic Behavior of Market Parameters

As discussed in "Creating SDE Objects" on page 18-9, object parameters may be evaluated as if they are MATLAB functions accessible by a common interface. This accessibility provides the impression of dynamic behavior regardless of whether the underlying parameters are truly time-varying. Furthermore, because parameters are
accessible by a common interface, seemingly simple linear constructs may in fact represent complex, nonlinear designs.

For example, consider a univariate geometric Brownian motion (GBM) model of the form:

$$
d X_{t}=\mu(t) X_{t} d t+\sigma(t) X_{t} d W_{t}
$$

In this model, the return, $\mu(t)$, and volatility, $\sigma(t)$, are dynamic parameters of time alone. However, when creating a gbm object to represent the underlying model, such dynamic behavior must be accessible by the common $\left(t, X_{t}\right)$ interface. This reflects the fact that GBM models (and others) are restricted parameterizations that derive from the general SDE class.

As a convenience, you can specify parameters of restricted models, such as GBM models, as traditional MATLAB arrays of appropriate dimension. In this case, such arrays represent a static special case of the more general dynamic situation accessible by the $(t$, $X_{t}$ ) interface.

Moreover, when you enter parameters as functions, object constructors can verify that they return arrays of correct size by evaluating them at the initial time and state. Otherwise, object constructors have no knowledge of any particular functional form.

The following example illustrates a technique that includes dynamic behavior by mapping a traditional MATLAB time series array to a callable function with a $\left(t, X_{t}\right)$ interface. It also compares the function with an otherwise identical model with constant parameters.

Because time series arrays represent dynamic behavior that must be captured by functions accessible by the ( $t, X_{t}$ ) interface, you need utilities to convert traditional time series arrays into callable functions of time and state. The following example shows how to do this using the conversion function ts2func (time series to function).

1 Load the data. Load a daily historical data set containing three-month Euribor rates and closing index levels of France's CAC 40 spanning the time interval February 7, 2001 to April 24, 2006:

```
load Data_GlobalIdx2
```

2 Simulate risk-neutral sample paths. Simulate risk-neutral sample paths of the CAC 40 index using a geometric Brownian motion (GBM) model:

$$
d X_{t}=r(t) X_{t} d t+\sigma X_{t} d W_{t}
$$

where $r(t)$ represents evolution of the risk-free rate of return.
Furthermore, assume that you need to annualize the relevant information derived from the daily data (annualizing the data is optional, but is useful for comparison to other examples), and that each calendar year comprises 250 trading days:

```
dt = 1/250;
returns = tick2ret(Dataset.CAC);
sigma = std(returns)*sqrt(250);
yields = Dataset.EB3M;
yields = 360*log(1 + yields);
```

3 Compare the sample paths from two risk-neutral historical simulation approaches. Compare the resulting sample paths obtained from two risk-neutral historical simulation approaches, where the daily Euribor yields serve as a proxy for the risk-free rate of return.
a The first approach specifies the risk-neutral return as the sample average of Euribor yields, and therefore assumes a constant (non-dynamic) risk-free return:

```
nPeriods = length(yields); % Simulated observations
rng(5713,'twister')
obj = gbm(mean(yields),diag(sigma),'StartState',100)
obj =
    Class GBM: Generalized Geometric Brownian Motion
        Dimensions: State = 1, Brownian = 1
        StartTime: 0
        StartState: 100
        Correlation: 1
            Drift: drift rate function F(t,X(t))
        Diffusion: diffusion rate function G(t,X(t))
        Simulation: simulation method/function simByEuler
            Return: 0.0278117
            Sigma: 0.231906
[X1,T] = simulate(obj,nPeriods,'DeltaTime',dt);
```

b In contrast, the second approach specifies the risk-neutral return as the historical time series of Euribor yields. It therefore assumes a dynamic, yet deterministic, rate of return; this example does not illustrate stochastic interest rates. To illustrate this dynamic effect, use the ts2func utility:

```
r = ts2func(yields,'Times',(0:nPeriods - 1)');
```

ts2func packages a specified time series array inside a callable function of time and state, and synchronizes it with an optional time vector. For instance:
$r(0,100)$
ans $=0.0470$
evaluates the function at ( $t=0, X_{t}=100$ ) and returns the first observed Euribor yield. However, you can also evaluate the resulting function at any intermediate time $t$ and state $X_{t}$ :
$r(7.5,200)$
ans $=0.0472$
Furthermore, the following command produces the same result when called with time alone:
$r(7.5)$
ans $=0.0472$
The equivalence of these last two commands highlights some important features.
When you specify parameters as functions, they must evaluate properly when passed a scalar, real-valued sample time ( $t$ ), and an NVARS-by-1 state vector ( $X_{t}$ ). They must also generate an array of appropriate dimensions, which in the first case is a scalar constant, and in the second case is a scalar, piecewise constant function of time alone.

You are not required to use either time $(t)$ or state $\left(X_{t}\right)$. In the current example, the function evaluates properly when passed time followed by state, thereby satisfying the minimal requirements. The fact that it also evaluates correctly when passed only time simply indicates that the function does not require the state vector $X_{t}$. The important point to make is that it works when you pass it $(t$, $X_{t}$ ).

Furthermore, the ts2func function performs a zero-order-hold (ZOH) piecewise constant interpolation. The notion of piecewise constant parameters is pervasive throughout the SDE architecture, and is discussed in more detail in "Optimizing Accuracy: About Solution Precision and Error" on page 18-79.

4 Perform a second simulation using the same initial random number state.
Complete the comparison by performing the second simulation using the same initial random number state:

```
rng(5713,'twister')
obj = gbm(r, diag(sigma),'StartState',100)
obj =
    Class GBM: Generalized Geometric Brownian Motion
        Dimensions: State = 1, Brownian = 1
        StartTime: 0
        StartState: 100
        Correlation: 1
            Drift: drift rate function F(t,X(t))
        Diffusion: diffusion rate function G(t,X(t))
        Simulation: simulation method/function simByEuler
        Return: function ts2func/vector2Function
        Sigma: 0.231906
X2 = simulate(obj,nPeriods,'DeltaTime',dt);
```

5 Compare the two simulation trials. Plot the series of risk-free reference rates to compare the two simulation trials:

```
subplot(2,1,1)
plot(dates,100*yields)
datetick('x')
xlabel('Date')
ylabel('Annualized Yield (%)')
title('Risk Free Rate(3-Mo Euribor Continuously-Compounded)')
subplot(2,1,2)
plot(T,X1,'red',T,X2,'blue')
xlabel('Time (Years)')
ylabel('Index Level')
title('Constant vs. Dynamic Rate of Return: CAC 40')
legend({'Constant Interest Rates' 'Dynamic Interest Rates'},...
    'Location', 'Best')
```



The paths are close but not exact. The blue line in the last plot uses all the historical Euribor data, and illustrates a single trial of a historical simulation.

## Pricing Equity Options

As discussed in "Ensuring Positive Interest Rates" on page 18-67, all simulation and interpolation methods allow you to specify one or more functions of the form:

$$
X_{t}=f\left(t, X_{t}\right)
$$

to evaluate at the end of every sample time.

The related example illustrates a simple, common end-of-period processing function to ensure nonnegative interest rates. This example illustrates a processing function that allows you to avoid simulation outputs altogether.

Consider pricing European stock options by Monte Carlo simulation within a Black-Scholes-Merton framework. Assume that the stock has the following characteristics:

- The stock currently trades at 100.
- The stock pays no dividends.
- The stock's volatility is $50 \%$ per annum.
- The option strike price is 95 .
- The option expires in three months.
- The risk-free rate is constant at $10 \%$ per annum.

To solve this problem, model the evolution of the underlying stock by a univariate geometric Brownian motion (GBM) model with constant parameters:

$$
d X_{t}=0.1 X_{t} d t+0.5 X_{t} d W_{t}
$$

Furthermore, assume that the stock price is simulated daily, and that each calendar month comprises 21 trading days:

```
strike = 95;
rate = 0.1;
sigma = 0.5;
dt = 1/252;
nPeriods = 63;
T = nPeriods*dt;
obj = gbm(rate,sigma,'StartState',100);
```

The goal is to simulate independent paths of daily stock prices, and calculate the price of European options as the risk-neutral sample average of the discounted terminal option payoff at expiration 63 days from now. This example calculates option prices by two approaches:

- A Monte Carlo simulation that explicitly requests the simulated stock paths as an output. The output paths are then used to price the options.
- An end-of-period processing function, accessible by time and state, that records the terminal stock price of each sample path. This processing function is implemented as a
nested function with access to shared information. For more information, see Example_BlackScholes.m.

1 Before simulation, invoke the example file to access the end-of-period processing function:

```
nTrials = 10000; % Number of independent trials (i.e., paths)
f = Example_BlackScholes(nPeriods,nTrials)
f = struct with fields:
    BlackScholes: @Example_BlackScholes/saveTerminalStockPrice
        CallPrice: @Example_BlackScholes/getCallPrice
            PutPrice: @Example_BlackScholes/getPutPrice
```

2 Simulate 10000 independent trials (sample paths). Request the simulated stock price paths as an output, and specify an end-of-period processing function:

```
rng(88161,'twister')
X = simBySolution(obj,nPeriods,'DeltaTime',dt,...
    'nTrials',nTrials,'Processes',f.BlackScholes);
```

3 Calculate the option prices directly from the simulated stock price paths. Because these are European options, ignore all intermediate stock prices:

```
call = mean(exp(-rate*T)*max(squeeze(X(end,:,:)) - strike, 0))
call = 13.9342
put = mean(exp(-rate*T)*max(strike - squeeze(X(end,:,:)), 0))
put = 6.4166
```

4 Price the options indirectly by invoking the nested functions:

```
f.CallPrice(strike,rate)
ans = 13.9342
f.PutPrice(strike,rate)
ans = 6.4166
```

For reference, the theoretical call and put prices computed from the Black-Scholes option formulas are 13.6953 and 6.3497 , respectively.
5 Although steps 3 and 4 produce the same option prices, the latter approach works directly with the terminal stock prices of each sample path. Therefore, it is much
more memory efficient. In this example, there is no compelling reason to request an output.

## See Also

bm | cev | cir|diffusion|drift|gbm| heston| hwv | interpolate| sde| sdeddo | sdeld | sdemrd | simByEuler| simBySolution|simBySolution| simulate| ts2func

## Related Examples

- "Simulating Interest Rates" on page 18-60
- "Stratified Sampling" on page 18-71
- "Pricing American Basket Options by Monte Carlo Simulation"
- "Improving Performance of Monte Carlo Simulation with Parallel Computing" on page 18-108
- "Base SDE Models" on page 18-17
- "Drift and Diffusion Models" on page 18-20
- "Linear Drift Models" on page 18-24
- "Parametric Models" on page 18-26


## More About

- "SDEs" on page 18-2
- "SDE Models" on page 18-9
- "SDE Class Hierarchy" on page 18-5
- "Performance Considerations" on page 18-77


## Simulating Interest Rates

```
In this section...
"Simulating Interest Rates" on page 18-60
"Ensuring Positive Interest Rates" on page 18-67
```


## Simulating Interest Rates

All simulation methods require that you specify a time grid by specifying the number of periods (NPERIODS). You can also optionally specify a scalar or vector of strictly positive time increments (DeltaTime) and intermediate time steps (NSTEPS). These parameters, along with an initial sample time associated with the object (StartTime), uniquely determine the sequence of times at which the state vector is sampled. Thus, simulation methods allow you to traverse the time grid from beginning to end (that is, from left to right).

In contrast, interpolation methods allow you to traverse the time grid in any order, allowing both forward and backward movements in time. They allow you to specify a vector of interpolation times whose elements do not have to be unique.

Many references define the Brownian Bridge as a conditional simulation combined with a scheme for traversing the time grid, effectively merging two distinct algorithms. In contrast, the interpolation method offered here provides additional flexibility by intentionally separating the algorithms. In this method for moving about a time grid, you perform an initial Monte Carlo simulation to sample the state at the terminal time, and then successively sample intermediate states by stochastic interpolation. The first few samples determine the overall behavior of the paths, while later samples progressively refine the structure. Such algorithms are often called variance reduction techniques. This algorithm is simple when the number of interpolation times is a power of 2 . In this case, each interpolation falls midway between two known states, refining the interpolation using a method like bisection. This example highlights the flexibility of refined interpolation by implementing this power-of-two algorithm.

1 Load the data. Load a historical data set of three-month Euribor rates, observed daily, and corresponding trading dates spanning the time interval from February 7, 2001 through April 24, 2006:
load Data GlobalIdx2
plot(dates, 100 * Dataset.EB3M)

```
datetick('x'), xlabel('Date'), ylabel('Daily Yield (%)')
title('3-Month Euribor as a Daily Effective Yield')
```



2 Fit a model to the data. Now fit a simple univariate Vasicek model to the daily equivalent yields of the three-month Euribor data:

$$
d X_{t}=S\left(L-X_{t}\right) d t+\sigma d W_{t}
$$

Given initial conditions, the distribution of the short rate at some time $T$ in the future is Gaussian with mean:

$$
E\left(X_{T}\right)=X_{0} e^{-S T}+L\left(1-e^{-S T}\right)
$$

and variance:

$$
\operatorname{Var}\left(X_{T}\right)=\sigma^{2}\left(1-e^{-S T}\right) / 2 S
$$

To calibrate this simple short rate model, rewrite it in more familiar regression format:

$$
y_{t}=\alpha+\beta x_{t}+\varepsilon_{t}
$$

where:

$$
y_{t}=d X_{t}, \alpha=S L d t, \beta=-S d t
$$

perform an ordinary linear regression where the model volatility is proportional to the standard error of the residuals:

```
\sigma=\sqrt{}{\operatorname{Var}(\mp@subsup{\varepsilon}{t}{})/dt}
yields = Dataset.EB3M;
regressors = [ones(length(yields) - 1, 1) yields(1:end-1)];
[coefficients, intervals, residuals] = ...
    regress(diff(yields), regressors);
dt = 1; % time increment = 1 day
speed = -coefficients(2)/dt;
level = -coefficients(1)/coefficients(2);
sigma = std(residuals)/sqrt(dt);
```

3 Create an object and set its initial StartState. Create an hwv object with StartState set to the most recently observed short rate:

```
obj = hwv(speed, level, sigma, 'StartState', yields(end))
obj =
    Class HWV: Hull-White/Vasicek
        Dimensions: State = 1, Brownian = 1
        StartTime: 0
        StartState: 7.70408e-05
        Correlation: 1
            Drift: drift rate function F(t,X(t))
        Diffusion: diffusion rate function G(t,X(t))
```

```
Simulation: simulation method/function simByEuler
    Sigma: 4.77637e-07
    Level: 6.00424e-05
    Speed: 0.00228854
```

4 Simulate the fitted model. Assume, for example, that you simulate the fitted model over $64\left(2^{6}\right)$ trading days, using a refined Brownian bridge with the power-of-two algorithm instead of the usual beginning-to-end Monte Carlo simulation approach. Furthermore, assume that the initial time and state coincide with those of the last available observation of the historical data, and that the terminal state is the expected value of the Vasicek model 64 days into the future. In this case, you can assess the behavior of various paths that all share the same initial and terminal states, perhaps to support pricing path-dependent interest rate options over a threemonth interval.

Create a vector of interpolation times to traverse the time grid by moving both forward and backward in time. Specifically, the first interpolation time is set to the most recent short rate observation time, the second interpolation time is set to the terminal time, and subsequent interpolation times successively sample intermediate states:

```
T = 64;
times = (1:T)';
t = NaN(length(times) + 1, 1);
t(1) = obj.StartTime;
t(2) = T;
delta = T;
jMax = 1;
iCount = 3;
for k = 1:log2(T)
    i = delta / 2;
    for j = 1:jMax
        t(iCount) = times(i);
        i = i + delta;
        iCount = iCount + 1;
    end
    jMax = 2 * jMax;
    delta = delta / 2;
end
```

5 Plot the interpolation times. Examine the sequence of interpolation times generated by this algorithm:

```
stem(1:length(t), t, 'filled')
xlabel('Index'), ylabel('Interpolation Time (Days)')
title ('Sampling Scheme for the Power-of-Two Algorithm')
```



The first few samples are widely separated in time and determine the course structure of the paths. Later samples are closely spaced and progressively refine the detailed structure.
6 Initialize the time series grid. Now that you have generated the sequence of interpolation times, initialize a course time series grid to begin the interpolation. The sampling process begins at the last observed time and state taken from the historical short rate series, and ends 64 days into the future at the expected value of the Vasicek model derived from the calibrated parameters:

```
average = obj.StartState * exp(-speed * T) + level * ...
(1 - exp(-speed * T));
X = [obj.StartState ; average];
```

7 Generate five sample paths. Generate five sample paths, setting the Refine input flag to TRUE to insert each new interpolated state into the time series grid as it becomes available. Perform interpolation on a trial-by-trial basis. Because the input time series $X$ has five trials (where each page of the three-dimensional time series represents an independent trial), the interpolated output series $Y$ also has five pages:

```
nTrials = 5;
rng(63349,'twister')
Y = obj.interpolate(t, X(:,:,ones(nTrials,1)), ...
'Times',[obj.StartTime T], 'Refine', true);
```

8 Plot the resulting sample paths. Because the interpolation times do not monotonically increase, sort the times and reorder the corresponding short rates:

```
[t,i] = sort(t);
Y = squeeze(Y);
Y = Y(i,:);
plot(t, 100 * Y), hold('on')
plot(t([1 end]), 100 * Y([1 end],1),'. black','MarkerSize',20)
xlabel('Interpolation Time (Days into the Future)')
ylabel('Yield (%)'), hold('off')
title ('Euribor Yields from Brownian Bridge Interpolation')
```



The short rates in this plot represent alternative sample paths that share the same initial and terminal values. They illustrate a special, though simplistic, case of a broader sampling technique known as stratified sampling. For a more sophisticated example of stratified sampling, see "Stratified Sampling" on page 18-71.

Although this simple example simulated a univariate Vasicek interest rate model, it applies to problems of any dimensionality.

Tip Brownian-bridge methods also apply more general variance-reduction techniques. For more information, see "Stratified Sampling" on page 18-71.

## Ensuring Positive Interest Rates

All simulation and interpolation methods allow you to specify a sequence of functions, or background processes, to evaluate at the end of every sample time period. This period includes any intermediate time steps determined by the optional NSTEPS input, as discussed in "Optimizing Accuracy: About Solution Precision and Error" on page 18-79. These functions are specified as callable functions of time and state, and must return an updated state vector $X_{t}$ :

$$
X_{t}=f\left(t, X_{t}\right)
$$

You must specify multiple processing functions as a cell array of functions. These functions are invoked in the order in which they appear in the cell array.

Processing functions are not required to use time $(t)$ or state $\left(X_{t}\right)$. They are also not required to update or change the input state vector. In fact, simulation and interpolation methods have no knowledge of any implementation details, and in this respect, they only adhere to a published interface.

Such processing functions provide a powerful modeling tool that can solve various problems. Such functions allow you to, for example, specify boundary conditions, accumulate statistics, plot graphs, and price path-dependent options.

Except for Brownian motion (BM) models, the individual components of the simulated state vector typically represent variables whose real-world counterparts are inherently positive quantities, such as asset prices or interest rates. However, by default, most of the simulation and interpolation methods provided here model the transition between successive sample times as a scaled (possibly multivariate) Gaussian draw. So, when approximating a continuous-time process in discrete time, the state vector may not remain positive. The only exception is simBySolution for gbm objects and simBySolution for hwv objects, a logarithmic transform of separable geometric Brownian motion models. Moreover, by default, none of the simulation and interpolation methods make adjustments to the state vector. Therefore, you are responsible for ensuring that all components of the state vector remain positive as appropriate.

Fortunately, specifying nonnegative states ensures a simple end-of-period processing adjustment. Although this adjustment is widely applicable, it is revealing when applied to a univariate cir square-root diffusion model:

$$
d X_{t}=0.25\left(0.1-X_{t}\right) d t+0.2 X_{t}^{\frac{1}{2}} d W_{t}=S\left(L-X_{t}\right) d t+\sigma X_{t}^{\frac{1}{2}} d W_{t}
$$

Perhaps the primary appeal of univariate cir models where:

$$
2 S L \geq \sigma^{2}
$$

is that the short rate remains positive. However, the positivity of short rates only holds for the underlying continuous-time model.

1 Simulate daily short rates of the cir model. To illustrate the latter statement, simulate daily short rates of the cir model, using cir, over one calendar year (approximately 250 trading days):

```
rng(14151617,'twister')
obj = cir(0.25,@(t,X)0.1,0.2,'StartState',0.02);
[X,T] = simByEuler(obj,250,'DeltaTime',1/250,'nTrials',5);
sprintf('%0.4f\t%0.4f+i%0.4f\n',[T(195:205)';...
    real(X(195:205,1,4))'; imag(X(195:205,1,4))'])
ans =
    '0.7760 0.0003+i0.0000
    0.7800 0.0004+i0.0000
    0.7840 0.0002+i0.0000
    0.7880 -0.0000+i0.0000
    0.7920 0.0001+i0.0000
    0.7960 0.0002+i0.0000
    0.8000 0.0002+i0.0000
    0.8040 0.0008+i0.0001
    0.8080 0.0004+i0.0001
    0.8120 0.0008+i0.0001
    0.8160 0.0008+i0.0001
```

Interest rates can become negative if the resulting paths are simulated in discrete time. Moreover, since cir models incorporate a square root diffusion term, the short rates might even become complex.
2 Repeat the simulation with a processing function. Repeat the simulation, this time specifying a processing function that takes the absolute magnitude of the short rate at the end of each period. You can access the processing function by time and state $\left(t, X_{t}\right)$, but it only uses the state vector of short rates $X_{t}$ :
rng(14151617,'twister')
[Y,T] = simByEuler(obj,250,'DeltaTime',1/250,...
'nTrials', 5,'Processes',@(t,X)abs(X));

3 Compare the adjusted and non-adjusted paths. Graphically compare the magnitude of the unadjusted path (with negative and complex numbers!) to the corresponding path kept positive by using an end-of-period processing function over the time span of interest:

```
clf
plot(T,100*abs(X(:,1,4)),'red',T,100*Y(:,1,4),'blue')
axis([0.75 1 0 0.4])
xlabel('Time (Years)'), ylabel('Short Rate (%)')
title('Univariate CIR Short Rates')
legend({'Negative/Complex Rates' 'Positive Rates'}, ...
    'Location', 'Best')
```



Tip You can use this method to obtain more accurate SDE solutions. For more information, see "Performance Considerations" on page 18-77.

## See Also

bm | cev |cir|diffusion|drift|gbm| heston | hwv | interpolate| sde| sdeddo | sdeld|sdemrd|simByEuler| simBySolution|simBySolution|simulate| ts2func

## Related Examples

- "Simulating Equity Prices" on page 18-35
- "Stratified Sampling" on page 18-71
- "Pricing American Basket Options by Monte Carlo Simulation"
- "Improving Performance of Monte Carlo Simulation with Parallel Computing" on page 18-108
- "Base SDE Models" on page 18-17
- "Drift and Diffusion Models" on page 18-20
- "Linear Drift Models" on page 18-24
- "Parametric Models" on page 18-26


## More About

- "SDEs" on page 18-2
- "SDE Models" on page 18-9
- "SDE Class Hierarchy" on page 18-5
- "Performance Considerations" on page 18-77


## Stratified Sampling

Simulation methods allow you to specify a noise process directly, as a callable function of time and state:

$$
z_{t}=Z\left(t, X_{t}\right)
$$

Stratified sampling is a variance reduction technique that constrains a proportion of sample paths to specific subsets (or strata) of the sample space.

This example specifies a noise function to stratify the terminal value of a univariate equity price series. Starting from known initial conditions, the function first stratifies the terminal value of a standard Brownian motion, and then samples the process from beginning to end by drawing conditional Gaussian samples using a Brownian bridge.

The stratification process assumes that each path is associated with a single stratified terminal value such that the number of paths is equal to the number of strata. This technique is called proportional sampling. This example is similar to, yet more sophisticated than, the one discussed in "Simulating Interest Rates" on page 18-60. Since stratified sampling requires knowledge of the future, it also requires more sophisticated time synchronization; specifically, the function in this example requires knowledge of the entire sequence of sample times. For more information, see the example stratifiedExample.m.

The function implements proportional sampling by partitioning the unit interval into bins of equal probability by first drawing a random number uniformly distributed in each bin. The inverse cumulative distribution function of a standard $N(0,1)$ Gaussian distribution then transforms these stratified uniform draws. Finally, the resulting stratified Gaussian draws are scaled by the square root of the terminal time to stratify the terminal value of the Brownian motion.

The noise function does not return the actual Brownian paths, but rather the Gaussian draws $Z\left(t, X_{t}\right)$ that drive it.

This example first stratifies the terminal value of a univariate, zero-drift, unit-variancerate Brownian motion (bm) model:

$$
d X_{t}=d W_{t}
$$

1 Assume that 10 paths of the process are simulated daily over a three-month period. Also assume that each calendar month and year consist of 21 and 252 trading days, respectively:

```
rng(10203,'twister')
dt = 1 / 252; % 1 day = 1/252 years
nPeriods = 63; % 3 months = 63 trading days
T = nPeriods * dt; % 3 months = 0.25 years
nPaths = 10; % # of simulated paths
obj = bm(0, 1, 'StartState', 0);
sampleTimes = cumsum([obj.StartTime; ...
        dt(ones(nPeriods,1))]);
z = Example_StratifiedRNG(nPaths, sampleTimes);
```

2 Simulate the standard Brownian paths by explicitly passing the stratified sampling function to the simulation method:

```
X = obj.simulate(nPeriods, 'DeltaTime', dt, ...
    'nTrials', nPaths, 'Z', z);
```

3 For convenience, reorder the output sample paths by reordering the threedimensional output to a two-dimensional equivalent array:

X = squeeze( X );
4 Verify the stratification:
a Recreate the uniform draws with proportional sampling:

```
rng(10203,'twister')
U = ((1:nPaths)' - 1 + rand(nPaths,1))/nPaths;
```

b Transform them to obtain the terminal values of standard Brownian motion:

```
WT = norminv(U) * sqrt(T); % Stratified Brownian motion.
```

c Plot the terminal values and output paths on the same figure:

```
plot(sampleTimes, X), hold('on')
xlabel('Time (Years)'), ylabel('Brownian State')
title('Terminal Stratification: Standard Brownian Motion')
plot(T, WT, '. black', T, WT, 'o black')
hold('off')
```



The last value of each sample path (the last row of the output array X ) coincides with the corresponding element of the stratified terminal value of the Brownian motion. This occurs because the simulated model and the noise generation function both represent the same standard Brownian motion.

However, you can use the same stratified sampling function to stratify the terminal price of constant-parameter geometric Brownian motion models. In fact, you can use the stratified sampling function to stratify the terminal value of any constant-parameter model driven by Brownian motion if the model's terminal value is a monotonic transformation of the terminal value of the Brownian motion.

To illustrate this, load the data set and simulate risk-neutral sample paths of the FTSE 100 index using a geometric Brownian motion (GBM) model with constant parameters:

$$
d X_{t}=r X_{t} d t+\sigma X_{t} d W_{t}
$$

where the average Euribor yield represents the risk-free rate of return.
1 Assume that the relevant information derived from the daily data is annualized, and that each calendar year comprises 252 trading days:

```
load Data_GlobalIdx2
returns = tick2ret(Dataset.FTSE);
sigma = std(returns) * sqrt(252);
rate = Dataset.EB3M;
rate = mean(360 * log(1 + rate));
```

2 Create the GBM model using gbm, assuming the FTSE 100 starts at 100:

```
obj = gbm(rate, sigma, 'StartState', 100);
```

3 Determine the sample time and simulate the price paths.
In what follows, NSTEPS specifies the number of intermediate time steps within each time increment DeltaTime. Each increment DeltaTime is partitioned into NSTEPS subintervals of length DeltaTime/nSteps each, refining the simulation by evaluating the simulated state vector at NSTEPS-1 intermediate points. This refinement improves accuracy by allowing the simulation to more closely approximate the underlying continuous-time process without storing the intermediate information:

```
nSteps = 1;
sampleTimes = cumsum([obj.StartTime ; ...
dt(ones(nPeriods * nSteps,1))/nSteps]);
z = Example_StratifiedRNG(nPaths, sampleTimes);
rng(10203,'twister')
[Y, Times] = obj.simBySolution(nPeriods, 'nTrials', nPaths,...
'DeltaTime', dt, 'nSteps', nSteps, 'Z', z);
Y = squeeze(Y); % Reorder to a 2-D array
plot(Times, Y)
xlabel('Time (Years)'), ylabel('Index Level')
title('FTSE 100 Terminal Stratification:Geometric Brownian Motion')
```



Although the terminal value of the Brownian motion shown in the latter plot is normally distributed, and the terminal price in the previous plot is lognormally distributed, the corresponding paths of each graph are similar.

Tip For another example of variance reduction techniques, see "Simulating Interest Rates" on page 18-60.

## See Also

bm | cev |cir|diffusion|drift|gbm| heston | hwv | interpolate| sde| sdeddo |sdeld|sdemrd|simByEuler|simBySolution|simBySolution|simulate| ts2func

## Related Examples

- "Simulating Equity Prices" on page 18-35
- "Simulating Interest Rates" on page 18-60
- "Pricing American Basket Options by Monte Carlo Simulation"
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- "Performance Considerations" on page 18-77


## Performance Considerations

In this section...<br>"Managing Memory" on page 18-77<br>"Enhancing Performance" on page 18-78<br>"Optimizing Accuracy: About Solution Precision and Error" on page 18-79

## Managing Memory

There are two general approaches for managing memory when solving most problems supported by the SDE engine:

- "Managing Memory with Outputs" on page 18-77
- "Managing Memory Using End-of-Period Processing Functions" on page 18-78


## Managing Memory with Outputs

Perform a traditional simulation to simulate the underlying variables of interest, specifically requesting and then manipulating the output arrays.

This approach is straightforward and the best choice for small or medium-sized problems. Since its outputs are arrays, it is convenient to manipulate simulated results in the MATLAB matrix-based language. However, as the scale of the problem increases, the benefit of this approach decreases, because the output arrays must store large quantities of possibly extraneous information.

For example, consider pricing a European option in which the terminal price of the underlying asset is the only value of interest. To ease the memory burden of the traditional approach, reduce the number of simulated periods specified by the required input NPERIODS and specify the optional input NSTEPS. This enables you to manage memory without sacrificing accuracy (see "Optimizing Accuracy: About Solution Precision and Error" on page 18-79).

In addition, simulation methods can determine the number of output arguments and allocate memory accordingly. Specifically, all simulation methods support the same output argument list:
[Paths,Times,Z]
where Paths and $Z$ can be large, three-dimensional time series arrays. However, the underlying noise array is typically unnecessary, and is only stored if requested as an output. In other words, Z is stored only at your request; do not request it if you do not need it.

If you need the output noise array Z, but do not need the Paths time series array, then you can avoid storing Paths two ways:

- It is best practice to use the $\sim$ output argument placeholder. For example, use the following output argument list to store Z and Times, but not Paths:
[~,Times, Z]
- Use the optional input flag StorePaths, which all simulation methods support. By default, Paths is stored (StorePaths = true). However, setting StorePaths to false returns Paths as an empty matrix.


## Managing Memory Using End-of-Period Processing Functions

Specify one or more end-of-period processing functions to manage and store only the information of interest, avoiding simulation outputs altogether.

This approach requires you to specify one or more end-of-period processing functions, and is often the preferred approach for large-scale problems. This approach allows you to avoid simulation outputs altogether. Since no outputs are requested, the threedimensional time series arrays Paths and $Z$ are not stored.

This approach often requires more effort, but is far more elegant and allows you to customize tasks and dramatically reduce memory usage. See "Pricing Equity Options" on page 18-56.

## Enhancing Performance

The following approaches improve performance when solving SDE problems:

- Specifying model parameters as traditional MATLAB arrays and functions, in various combinations. This provides a flexible interface that can support virtually any general nonlinear relationship. However, while functions offer a convenient and elegant solution for many problems, simulations typically run faster when you specify parameters as double-precision vectors or matrices. Thus, it is a good practice to specify model parameters as arrays when possible.
- Use models that have overloaded Euler simulation methods, when possible. Using Brownian motion (BM) and geometric Brownian motion (GBM) models that provide overloaded Euler simulation methods take advantage of separable, constantparameter models. These specialized methods are exceptionally fast, but are only available to models with constant parameters that are simulated without specifying end-of-period processing and noise generation functions.
- Replace the simulation of a constant-parameter, univariate model derived from the SDEDDO class with that of a diagonal multivariate model. Treat the multivariate model as a portfolio of univariate models. This increases the dimensionality of the model and enhances performance by decreasing the effective number of simulation trials.

Note This technique is applicable only to constant-parameter univariate models without specifying end-of-period processing and noise generation functions.

- Take advantage of the fact that simulation methods are designed to detect the presence of NaN (not a number) conditions returned from end-of-period processing functions. A NaN represents the result of an undefined numerical calculation, and any subsequent calculation based on a NaN produces another NaN. This helps improve performance in certain situations. For example, consider simulating paths of the underlier of a knock-out barrier option (that is, an option that becomes worthless when the price of the underlying asset crosses some prescribed barrier). Your end-of-period function could detect a barrier crossing and return a NaN to signal early termination of the current trial.


## Optimizing Accuracy: About Solution Precision and Error

The simulation architecture does not, in general, simulate exact solutions to any SDE. Instead, the simulation architecture provides a discrete-time approximation of the underlying continuous-time process, a simulation technique often known as a Euler approximation.

In the most general case, a given simulation derives directly from an SDE. Therefore, the simulated discrete-time process approaches the underlying continuous-time process only in the limit as the time increment $d t$ approaches zero. In other words, the simulation architecture places more importance on ensuring that the probability distributions of the discrete-time and continuous-time processes are close, than on the pathwise proximity of the processes.

Before illustrating techniques to improve the approximation of solutions, it is helpful to understand the source of error. Throughout this architecture, all simulation methods assume that model parameters are piecewise constant over any time interval of length $d t$. In fact, the methods even evaluate dynamic parameters at the beginning of each time interval and hold them fixed for the duration of the interval. This sampling approach introduces discretization error.

However, there are certain models for which the piecewise constant approach provides exact solutions:

- "Creating Brownian Motion (BM) Models" on page 18-26 with constant parameters, simulated by Euler approximation (simByEuler).
- "Creating Geometric Brownian Motion (GBM) Models" on page 18-28 with constant parameters, simulated by closed-form solution (simBySolution).
- "Creating Hull-White/Vasicek (HWV) Gaussian Diffusion Models" on page 18-31 with constant parameters, simulated by closed-form solution (simBySolution)

More generally, you can simulate the exact solutions for these models even if the parameters vary with time, if they vary in a piecewise constant way such that parameter changes coincide with the specified sampling times. However, such exact coincidence is unlikely; therefore, the previously discussed constant parameter condition is commonly used in practice.

One obvious way to improve accuracy involves sampling the discrete-time process more frequently. This decreases the time increment ( $d t$ ), causing the sampled process to more closely approximate the underlying continuous-time process. Although decreasing the time increment is universally applicable, however, there is a tradeoff among accuracy, run-time performance, and memory usage.

To manage this tradeoff, specify an optional input argument, NSTEPS, for all simulation methods. NSTEPS indicates the number of intermediate time steps within each time increment $d t$, at which the process is sampled but not reported.

It is important and convenient at this point to emphasize the relationship of the inputs NSTEPS, NPERIODS, and DeltaTime to the output vector Times, which represents the actual observation times at which the simulated paths are reported.

- NPERIODS, a required input, indicates the number of simulation periods of length DeltaTime, and determines the number of rows in the simulated three-dimensional Paths time series array (if an output is requested).
- DeltaTime is optional, and indicates the corresponding NPERIODS-length vector of positive time increments between successive samples. It represents the familiar $d t$ found in stochastic differential equations. If DeltaTime is unspecified, the default value of 1 is used.
- NSTEPS is also optional, and is only loosely related to NPERIODS and DeltaTime. NSTEPS specifies the number of intermediate time steps within each time increment DeltaTime.

Specifically, each time increment DeltaTime is partitioned into NSTEPS subintervals of length DeltaTime/NSTEPS each, and refines the simulation by evaluating the simulated state vector at (NSTEPS - 1) intermediate times. Although the output state vector (if requested) is not reported at these intermediate times, this refinement improves accuracy by causing the simulation to more closely approximate the underlying continuous-time process. If NSTEPS is unspecified, the default is 1 (to indicate no intermediate evaluation).

- The output Times is an NPERIODS + 1-length column vector of observation times associated with the simulated paths. Each element of Times is associated with a corresponding row of Paths.

The following example illustrates this intermediate sampling by comparing the difference between a closed-form solution and a sequence of Euler approximations derived from various values of NSTEPS.

## Example: Improving Solution Accuracy

Consider a univariate geometric Brownian motion (GBM) model using gbm with constant parameters:

$$
d X_{t}=0.1 X_{t} d t+0.4 X_{t} d W_{t} .
$$

Assume that the expected rate of return and volatility parameters are annualized, and that a calendar year comprises 250 trading days.

1 Simulate approximately four years of univariate prices for both the exact solution and the Euler approximation for various values of NSTEPS:

```
nPeriods = 1000;
dt = 1/250;
obj = gbm(0.1,0.4,'StartState',100);
rng(575,'twister')
[X1,T1] = simBySolution(obj,nPeriods,'DeltaTime',dt);
```

```
rng(575,'twister')
[Y1,T1] = simByEuler(obj,nPeriods,'DeltaTime',dt);
rng(575,'twister')
[X2,T2] = simBySolution(obj,nPeriods,'DeltaTime',...
    dt,'nSteps',2);
rng(575,'twister')
[Y2,T2] = simByEuler(obj,nPeriods,'DeltaTime',...
    dt,'nSteps',2);
rng(575,'twister')
[X3,T3] = simBySolution(obj,nPeriods, 'DeltaTime',...
    dt,'nSteps',10);
rng(575,'twister')
[Y3,T3] = simByEuler(obj,nPeriods,'DeltaTime',...
    dt,'nSteps',10);
rng(575,'twister')
[X4,T4] = simBySolution(obj,nPeriods,'DeltaTime',...
    dt,'nSteps',100);
rng(575,'twister')
[Y4,T4] = simByEuler(obj,nPeriods,'DeltaTime',...
    dt,'nSteps',100);
```

2 Compare the error (the difference between the exact solution and the Euler approximation) graphically:

```
clf;
plot(T1,X1 - Y1,'red')
hold on;
plot(T2,X2 - Y2,'blue')
plot(T3,X3 - Y3,'green')
plot(T4,X4 - Y4,'black')
hold off
xlabel('Time (Years)')
ylabel('Price Difference')
title('Exact Solution Minus Euler Approximation')
legend({'# of Steps = 1' '# of Steps = 2' ...
    '# of Steps = 10' '# of Steps = 100'},...
    'Location', 'Best')
hold off
```


whos T X Y
As expected, the simulation error decreases as the number of intermediate time steps increases. Because the intermediate states are not reported, all simulated time series have the same number of observations regardless of the actual value of NSTEPS.

Furthermore, since the previously simulated exact solutions are correct for any number of intermediate time steps, additional computations are not needed for this example. In fact, this assessment is correct. The exact solutions are sampled at intermediate times to ensure that the simulation uses the same sequence of Gaussian random variates in the same order. Without this assurance, there is no way to compare simulated prices on a
pathwise basis. However, there might be valid reasons for sampling exact solutions at closely spaced intervals, such as pricing path-dependent options.

## See Also

bm | cev | cir|diffusion|drift|gbm| heston| hwv|interpolate| sde| sdeddo | sdeld | sdemrd | simByEuler| simBySolution|simBySolution|simulate| ts2func

## Related Examples

- "Simulating Equity Prices" on page 18-35
- "Simulating Interest Rates" on page 18-60
- "Pricing American Basket Options by Monte Carlo Simulation"
- "Improving Performance of Monte Carlo Simulation with Parallel Computing" on page 18-108
- "Base SDE Models" on page 18-17
- "Drift and Diffusion Models" on page 18-20
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## More About

- "SDEs" on page 18-2
- "SDE Models" on page 18-9
- "SDE Class Hierarchy" on page 18-5


## Pricing American Basket Options by Monte Carlo Simulation

This example shows how to model the fat-tailed behavior of asset returns and assess the impact of alternative joint distributions on basket option prices. Using various implementations of a separable multivariate Geometric Brownian Motion (GBM) process, often referred to as a multi-dimensional market model, the example simulates risk-neutral sample paths of an equity index portfolio and prices basket put options using the technique of Longstaff \& Schwartz.

In addition, this example also illustrates salient features of the Stochastic Differential Equation (SDE) architecture, including

- Customized random number generation functions that compare Brownian motion and Brownian copulas
- End-of-period processing functions that form an equity index basket and price American options on the underlying basket based on the least squares method of Longstaff \& Schwartz
- Piecewise probability distributions and Extreme Value Theory (EVT)

This example also highlights important issues of volatility and interest rate scaling. It illustrates how equivalent results can be achieved by working with daily or annualized data. For more information about EVT and copulas, see "Using Extreme Value Theory and Copulas to Evaluate Market Risk" (Econometrics Toolbox).

## Overview of the Modeling Framework

The ultimate objective of this example is to compare basket option prices derived from different noise processes. The first noise process is a traditional Brownian motion model whose index portfolio price process is driven by correlated Gaussian random draws. As an alternative, the Brownian motion benchmark is compared to noise processes driven by Gaussian and Student's $t$ copulas, referred to collectively as a Brownian copula.

A copula is a multivariate cumulative distribution function (CDF) with uniformlydistributed margins. Although the theoretical foundations were established decades ago, copulas have experienced a tremendous surge in popularity over the last few years, primarily as a technique for modeling non-Gaussian portfolio risks.

Although numerous families exist, all copulas represent a statistical device for modeling the dependence structure between two or more random variables. In addition, important
statistics, such as rank correlation and tail dependence are properties of a given copula and are unchanged by monotonic transforms of their margins.

These copula draws produce dependent random variables, which are subsequently transformed to individual variables (margins). This transformation is achieved with a semi-parametric probability distribution with generalized Pareto tails.

The risk-neutral market model to simulate is

$$
d X_{t}=r X_{t} d t+\sigma X_{t} d W_{t}
$$

where the risk-free rate, $r$, is assumed constant over the life of the option. Because this is a separable multivariate model, the risk-free return is a diagonal matrix in which the same riskless return is applied to all indices. Dividend yields are ignored to simplify the model and its associated data collection.

In contrast, the specification of the exposure matrix, sigma, depends on how the driving source of uncertainty is modeled. You can model it directly as a Brownian motion (correlated Gaussian random numbers implicitly mapped to Gaussian margins) or model it as a Brownian copula (correlated Gaussian or $t$ random numbers explicitly mapped to semi-parametric margins).

Because the CDF and inverse CDF (quantile function) of univariate distributions are both monotonic transforms, a copula provides a convenient way to simulate dependent random variables whose margins are dissimilar and arbitrarily distributed. Moreover, because a copula defines a given dependence structure regardless of its margins, copula parameter calibration is typically easier than estimation of the joint distribution function.

Once you have simulated sample paths, options are priced by the least squares regression method of Longstaff \& Schwartz (see Valuing American Options by Simulation: A Simple Least-Squares Approach, The Review of Financial Studies, Spring 2001). This approach uses least squares to estimate the expected payoff of an option if it is not immediately exercised. It does so by regressing the discounted option cash flows received in the future on the current price of the underlier associated with all in-the-money sample paths. The continuation function is estimated by a simple third-order polynomial, in which all cash flows and prices in the regression are normalized by the option strike price, improving numerical stability.

## Import the Supporting Historical Dataset

Load a daily historical dataset of 3-month Euribor, the trading dates spanning the interval 07-Feb-2001 to 24-Apr-2006, and the closing index levels of the following representative large-cap equity indices:

- TSX Composite (Canada)
- CAC 40 (France)
- DAX (Germany)
- Nikkei 225 (Japan)
- FTSE 100 (UK)
- S\&P 500 (US)

```
clear
load Data GlobalIdx2
```

The following plots illustrate this data. Specifically, the plots show the relative price movements of each index and the Euribor risk-free rate proxy. The initial level of each index has been normalized to unity to facilitate the comparison of relative performance over the historical record.

```
nIndices = size(Data,2)-1; % # of indices
prices = Data(:,1:end-1);
yields = Data(:,end); % daily effective yields
yields = 360 * log(1 + yields); % continuously-compounded, annualized yield
plot(dates, ret2tick(tick2ret(prices,[],'continuous'),[],[],[],'continuous'))
datetick('x')
xlabel('Date')
ylabel('Index Value')
title ('Normalized Daily Index Closings')
legend(series{1:end-1}, 'Location', 'NorthWest')
Warning: This syntax will be removed in a future release. See the documentation
for recommended usage.
Warning: This syntax will be removed in a future release. See the documentation
for recommended usage.
```




## Extreme Value Theory \& Piecewise Probability Distributions

To prepare for copula modeling, characterize individually the distribution of returns of each index. Although the distribution of each return series may be characterized parametrically, it is useful to fit a semi-parametric model using a piecewise distribution with generalized Pareto tails. This uses Extreme Value Theory to better characterize the behavior in each tail.

The Statistics and Machine Learning Toolbox ${ }^{\mathrm{TM}}$ software currently supports two univariate probability distributions related to EVT, a statistical tool for modeling the fattailed behavior of financial data such as asset returns and insurance losses:

- Generalized Extreme Value (GEV) distribution, which uses a modeling technique known as the block maxima or minima method. This approach, divides a historical dataset into a set of sub-intervals, or blocks, and the largest or smallest observation in each block is recorded and fitted to a GEV distribution.
- Generalized Pareto (GP) distribution, uses a modeling technique known as the distribution of exceedances or peaks over threshold method. This approach sorts a historical dataset and fits the amount by which those observations that exceed a specified threshold to a GP distribution.

The following analysis highlights the Pareto distribution, which is more widely used in risk management applications.

Suppose we want to create a complete statistical description of the probability distribution of daily asset returns of any one of the equity indices. Assume that this description is provided by a piecewise semi-parametric distribution, where the asymptotic behavior in each tail is characterized by a generalized Pareto distribution.

Ultimately, a copula will be used to generate random numbers to drive the simulations. The CDF and inverse CDF transforms will capture the volatility of simulated returns as part of the diffusion term of the SDE. The mean return of each index is governed by the riskless rate and incorporated in the drift term of the SDE. The following code segment centers the returns (that is, extracts the mean) of each index.

Because the following analysis uses extreme value theory to characterize the distribution of each equity index return series, it is helpful to examine details for a particular country:

```
returns = tick2ret(prices,[],'continuous'); % convert prices to returns
returns = bsxfun(@minus, returns, mean(returns)); % center the returns
index = 3; % Germany stored in column 3
plot(dates(2:end), returns(:,index))
datetick('x')
xlabel('Date')
ylabel('Return')
title(['Daily Logarithmic Centered Returns: ' series{index}])
Warning: This syntax will be removed in a future release. See the documentation
for recommended usage.
```



Note that this code segment can be changed to examine details for any country.
Using these centered returns, estimate the empirical, or non-parametric, CDF of each index with a Gaussian kernel. This smoothes the CDF estimates, eliminating the staircase pattern of unsmoothed sample CDFs. Although non-parametric kernel CDF estimates are well-suited for the interior of the distribution, where most of the data is found, they tend to perform poorly when applied to the upper and lower tails. To better estimate the tails of the distribution, apply EVT to the returns that fall in each tail.

Specifically, find the upper and lower thresholds such that $10 \%$ of the returns are reserved for each tail. Then fit the amount by which the extreme returns in each tail fall beyond the associated threshold to a Pareto distribution by maximum likelihood.

The following code segment creates one object of type paretotails for each index return series. These Pareto tail objects encapsulate the estimates of the parametric Pareto lower tail, the non-parametric kernel-smoothed interior, and the parametric Pareto upper tail to construct a composite semi-parametric CDF for each index.

```
tailFraction = 0.1; % decimal fraction allocated to each tail
tails = cell(nIndices,1); % cell array of Pareto tail objects
for i = 1:nIndices
    tails{i} = paretotails(returns(:,i), tailFraction, 1 - tailFraction, 'kernel');
end
```

The resulting piecewise distribution object allows interpolation within the interior of the CDF and extrapolation (function evaluation) in each tail. Extrapolation allows estimation of quantiles outside the historical record, which is invaluable for risk management applications.

Pareto tail objects also provide methods to evaluate the CDF and inverse CDF (quantile function), and to query the cumulative probabilities and quantiles of the boundaries between each segment of the piecewise distribution.

Now that three distinct regions of the piecewise distribution have been estimated, graphically concatenate and display the result.

The following code calls the CDF and inverse CDF methods of the Pareto tails object of interest with data other than that upon which the fit is based. The referenced methods have access to the fitted state. They are now invoked to select and analyze specific regions of the probability curve, acting as a powerful data filtering mechanism.

For reference, the plot also includes a zero-mean Gaussian CDF of the same standard deviation. To a degree, the variation in options prices reflect the extent to which the distribution of each asset differs from this normal curve.

```
minProbability = cdf(tails{index}, (min(returns(:,index))));
maxProbability = cdf(tails{index}, (max(returns(:,index))));
pLowerTail = linspace(minProbability , tailFraction , 200); % lower tail
pUpperTail = linspace(1 - tailFraction, maxProbability , 200); % upper tail
pInterior = linspace(tailFraction , 1 - tailFraction, 200); % interior
plot(icdf(tails{index}, pLowerTail), pLowerTail, 'red' , 'LineWidth', 2)
hold on
grid on
```

```
plot(icdf(tails{index}, pInterior) , pInterior, 'black', 'LineWidth', 2)
plot(icdf(tails{index}, pUpperTail), pUpperTail, 'blue' , 'LineWidth', 2)
limits = axis;
x = linspace(limits(1), limits(2));
plot(x, normcdf(x, 0, std(returns(:,index))), 'green', 'LineWidth', 2)
fig = gcf;
fig.Color = [1 1 1];
hold off
xlabel('Centered Return')
ylabel('Probability')
title (['Semi-Parametric/Piecewise CDF: ' series{index}])
legend({'Pareto Lower Tail' 'Kernel Smoothed Interior' ...
    'Pareto Upper Tail' 'Gaussian with Same \sigma'}, 'Location', 'NorthWest')
```



The lower and upper tail regions, displayed in red and blue, respectively, are suitable for extrapolation, while the kernel-smoothed interior, in black, is suitable for interpolation.

## Copula Calibration

The Statistics and Machine Learning Toolbox software includes functionality that calibrates and simulates Gaussian and $t$ copulas.

Using the daily index returns, estimate the parameters of the Gaussian and $t$ copulas using the function copulafit. Since a $t$ copula becomes a Gaussian copula as the scalar degrees of freedom parameter (DoF) becomes infinitely large, the two copulas are really of the same family, and therefore share a linear correlation matrix as a fundamental parameter.

Although calibration of the linear correlation matrix of a Gaussian copula is straightforward, the calibration of a $t$ copula is not. For this reason, the Statistics and Machine Learning Toolbox software offers two techniques to calibrate a $t$ copula:

- The first technique performs maximum likelihood estimation (MLE) in a two-step process. The inner step maximizes the log-likelihood with respect to the linear correlation matrix, given a fixed value for the degrees of freedom. This conditional maximization is placed within a 1-D maximization with respect to the degrees of freedom, thus maximizing the log-likelihood over all parameters. The function being maximized in this outer step is known as the profile log-likelihood for the degrees of freedom.
- The second technique is derived by differentiating the log-likelihood function with respect to the linear correlation matrix, assuming the degrees of freedom is a fixed constant. The resulting expression is a non-linear equation that can be solved iteratively for the correlation matrix. This technique approximates the profile loglikelihood for the degrees of freedom parameter for large sample sizes. This technique is usually significantly faster than the true maximum likelihood technique outlined above; however, you should not use it with small or moderate sample sizes as the estimates and confidence limits may not be accurate.

When the uniform variates are transformed by the empirical CDF of each margin, the calibration method is often known as canonical maximum likelihood (CML). The following code segment first transforms the daily centered returns to uniform variates by the piecewise, semi-parametric CDFs derived above. It then fits the Gaussian and $t$ copulas to the transformed data:

```
U = zeros(size(returns));
for i = 1:nIndices
    U(:,i) = cdf(tails{i}, returns(:,i)); % transform each margin to uniform
end
options = statset('Display', 'off', 'TolX', le-4);
[rhoT, DoF] = copulafit('t', U, 'Method', 'ApproximateML', 'Options', options);
rhoG = copulafit('Gaussian', U);
```

The estimated correlation matrices are quite similar but not identical.

```
corrcoef(returns) % linear correlation matrix of daily returns
```

```
\begin{tabular}{llllll}
1.0000 & 0.4813 & 0.5058 & 0.1854 & 0.4573 & 0.6526 \\
0.4813 & 1.0000 & 0.8485 & 0.2261 & 0.8575 & 0.5102 \\
0.5058 & 0.8485 & 1.0000 & 0.2001 & 0.7650 & 0.6136 \\
0.1854 & 0.2261 & 0.2001 & 1.0000 & 0.2295 & 0.1439 \\
0.4573 & 0.8575 & 0.7650 & 0.2295 & 1.0000 & 0.4617 \\
0.6526 & 0.5102 & 0.6136 & 0.1439 & 0.4617 & 1.0000
\end{tabular}
rhoG \% linear correlation matrix of the optimized Gaussian copula
rhoG =
\begin{tabular}{llllll}
1.0000 & 0.4745 & 0.5018 & 0.1857 & 0.4721 & 0.6622 \\
0.4745 & 1.0000 & 0.8606 & 0.2393 & 0.8459 & 0.4912 \\
0.5018 & 0.8606 & 1.0000 & 0.2126 & 0.7608 & 0.5811 \\
0.1857 & 0.2393 & 0.2126 & 1.0000 & 0.2396 & 0.1494 \\
0.4721 & 0.8459 & 0.7608 & 0.2396 & 1.0000 & 0.4518 \\
0.6622 & 0.4912 & 0.5811 & 0.1494 & 0.4518 & 1.0000
\end{tabular}
rhot \% linear correlation matrix of the optimized \(t\) copula
\begin{tabular}{cccccc} 
rhoT \(=\) & & & & \\
1.0000 & 0.4671 & 0.4858 & 0.1907 & 0.4734 & 0.6521 \\
0.4671 & 1.0000 & 0.8871 & 0.2567 & 0.8500 & 0.5122 \\
0.4858 & 0.8871 & 1.0000 & 0.2326 & 0.7723 & 0.5877 \\
0.1907 & 0.2567 & 0.2326 & 1.0000 & 0.2503 & 0.1539 \\
0.4734 & 0.8500 & 0.7723 & 0.2503 & 1.0000 & 0.4769 \\
0.6521 & 0.5122 & 0.5877 & 0.1539 & 0.4769 & 1.0000
\end{tabular}
Note the relatively low degrees of freedom parameter obtained from the \(t\) copula calibration, indicating a significant departure from a Gaussian situation.
DoF
\% scalar degrees of freedom parameter of the optimized \(t\) copula
DoF =
4.8613
```


## Copula Simulation

Now that the copula parameters have been estimated, simulate jointly-dependent uniform variates using the function copularnd.

Then, by extrapolating the Pareto tails and interpolating the smoothed interior, transform the uniform variates derived from copularnd to daily centered returns via the inverse CDF of each index. These simulated centered returns are consistent with those obtained from the historical dataset. The returns are assumed to be independent in time, but at any point in time possess the dependence and rank correlation induced by the given copula.

The following code segment illustrates the dependence structure by simulating centered returns using the $t$ copula. It then plots a 2-D scatter plot with marginal histograms for the French CAC 40 and German DAX using the Statistics and Machine Learning Toolbox scatterhist function. The French and German indices were chosen simply because they have the highest correlation of the available data.

```
nPoints = 10000; % # of simulated observations
s = RandStream.getGlobalStream();
reset(s)
R = zeros(nPoints, nIndices); % pre-allocate simulated returns array
U = copularnd('t', rhoT, DoF, nPoints); % simulate U(0,1) from t copula
for j = 1:nIndices
    R(:,j) = icdf(tails{j}, U(:,j));
end
h = scatterhist(R(:,2), R(:,3),'Color','r','Marker','.','MarkerSize',1);
fig = gcf;
fig.Color = [llll
y1 = ylim(h(1));
y3 = ylim(h(3));
xlim(h(1), [-.1 .1])
ylim(h(1), [-.1 .1])
xlim(h(2), [-.1 .1])
ylim(h(3), [(y3(1) + (-0.1 - y1(1))) (y3(2) + (0.1 - y1(2)))])
xlabel('France')
ylabel('Germany')
title(['t Copula (\nu = ' num2str(DoF,2) ')'])
```



Now simulate and plot centered returns using the Gaussian copula.
reset(s)
$\mathrm{R}=$ zeros(nPoints, nIndices); $\quad$ \% pre-allocate simulated returns array
U = copularnd('Gaussian', rhoG, nPoints); \% simulate U(0,1) from Gaussian copula
for $\mathrm{j}=1:$ nIndices
$R(:, j)=i c d f(t a i l s\{j\}, ~ U(:, j)) ;$
end
h = scatterhist(R(:,2), R(:,3),'Color','r','Marker','.','MarkerSize',1); fig = gcf;
fig.Color = [1 1 1];
$y 1=y l i m(h(1)) ;$

```
y3 = ylim(h(3));
xlim(h(1), [-.1 .1])
ylim(h(1), [-.1 .1])
xlim(h(2), [-.1 .1])
ylim(h(3), [(y3(1) + (-0.1 - y1(1))) (y3(2) + (0.1 - y1(2)))])
xlabel('France')
ylabel('Germany')
title('Gaussian Copula')
```

Gaussian Copula



Examine these two figures. There is a strong similarity between the miniature histograms on the corresponding axes of each figure. This similarity is not coincidental.

Both copulas simulate uniform random variables, which are then transformed to daily centered returns by the inverse CDF of the piecewise distribution of each index.

Therefore, the simulated returns of any given index are identically distributed regardless of the copula.

However, the scatter graph of each figure indicates the dependence structure associated with the given copula, and in contrast to the univariate margins shown in the histograms, the scatter graphs are distinct.

Once again, the copula defines a dependence structure regardless of its margins, and therefore offers many features not limited to calibration alone.

For reference, simulate and plot centered returns using the Gaussian distribution, which underlies the traditional Brownian motion model.

```
reset(s)
R = mvnrnd(zeros(1,nIndices), cov(returns), nPoints);
h = scatterhist(R(:,2), R(:,3),'Color','r','Marker','.','MarkerSize',1);
fig = gcf;
fig.Color = [llll}11]
y1 = ylim(h(1));
y3 = ylim(h(3));
xlim(h(1), [-.1 .1])
ylim(h(1), [-.1 .1])
xlim(h(2), [-.1 .1])
ylim(h(3), [(y3(1) + (-0.1 - y1(1))) (y3(2) + (0.1 - y1(2)))])
xlabel('France')
ylabel('Germany')
title('Gaussian Distribution')
```



## American Option Pricing Using the Longstaff \& Schwartz Approach

Now that the copulas have been calibrated, compare the prices of at-the-money American basket options derived from various approaches. To simply the analysis, assume that:

- All indices begin at 100 .
- The portfolio holds a single unit, or share, of each index such that the value of the portfolio at any time is the sum of the values of the individual indices.
- The option expires in 3 months.
- The information derived from the daily data is annualized.
- Each calendar year is composed of 252 trading days.
- Index levels are simulated daily.
- The option may be exercised at the end of every trading day and approximates the American option as a Bermudan option.

Now compute the parameters common to all simulation methods:

```
dt = 1 / 252; % time increment = 1 day = 1/252 years
yields = Data(:,end); % daily effective yields
yields = 360 * log(1 + yields); % continuously-compounded, annualized yields
r = mean(yields); % historical 3M Euribor average
X = repmat(100, nIndices, 1); % initial state vector
strike = sum(X); % initialize an at-the-money basket
nTrials = 100; % # of independent trials
nPeriods = 63; % # of simulation periods: 63/252 = 0.25 years = 3 months
```

Now create two separable multi-dimensional market models in which the riskless return and volatility exposure matrices are both diagonal.

While both are diagonal GBM models with identical risk-neutral returns, the first is driven by a correlated Brownian motion and explicitly specifies the sample linear correlation matrix of centered returns. This correlated Brownian motion process is then weighted by a diagonal matrix of annualized index volatilities or standard deviations.

As an alternative, the same model could be driven by an uncorrelated Brownian motion (standard Brownian motion) by specifying correlation as an identity matrix, or by simply accepting the default value. In this case, the exposure matrix sigma is specified as the lower Cholesky factor of the index return covariance matrix. Because the copulabased approaches simulate dependent random numbers, the diagonal exposure form is chosen for consistency. For further details, see Alternatives for Inducing Dependence \& Correlation.

```
sigma = std(returns) * sqrt(252); % annualized volatility
correlation = corrcoef(returns); % correlated Gaussian disturbances
GBM1 = gbm(diag(r(ones(1,nIndices))), diag(sigma), 'StartState', X, ...
    'Correlation' , correlation);
```

Now create the second model driven by the Brownian copula with an identity matrix sigma.

GBM2 = gbm(diag(r(ones(1,nIndices))), eye(nIndices), 'StartState', X);
The newly created model may seem unusual, but it highlights the flexibility of the SDE architecture.

When working with copulas, it is often convenient to allow the random number generator function $\mathrm{Z}(\mathrm{t}, \mathrm{X})$ to induce dependence (of which the traditional notion of linear correlation is a special case) with the copula, and to induce magnitude or scale of variation (similar to volatility or standard deviation) with the semi-parametric CDF and inverse CDF transforms. Since the CDF and inverse CDF transforms of each index inherit the characteristics of historical returns, this also explains why the returns are now centered.

In the following sections, statements like:

```
z = Example_CopulaRNG(returns * sqrt(252), nPeriods, 'Gaussian');
or
z = Example_CopulaRNG(returns * sqrt(252), nPeriods, 't');
```

fit the Gaussian and $t$ copula dependence structures, respectively, and the semiparametric margins to the centered returns scaled by the square root of the number of trading days per year (252). This scaling does not annualize the daily centered returns. Instead, it scales them such that the volatility remains consistent with the diagonal annualized exposure matrix sigma of the traditional Brownian motion model (GBM1) created previously.

In this example, you also specify an end-of-period processing function that accepts time followed by state ( $\mathrm{t}, \mathrm{X}$ ), and records the sample times and value of the portfolio as the single-unit weighted average of all indices. This function also shares this information with other functions designed to price American options with a constant riskless rate using the least squares regression approach of Longstaff \& Schwartz.

```
f = Example_LongstaffSchwartz(nPeriods, nTrials)
f =
    struct with fields:
    LongstaffSchwartz: @Example_LongstaffSchwartz/saveBasketPrices
        CallPrice: @Example_LongstaffSchwartz/getCallPrice
            PutPrice: @Example_LongstaffSchwartz/getPutPrice
                Prices: @Example_LongstaffSchwartz/getBasketPrices
```

Now simulate independent trials of equity index prices over 3 calendar months using the default simByEuler method. No outputs are requested from the simulation methods; in
fact, the simulated prices of the individual indices which comprise the basket are unnecessary. Call option prices are reported for convenience:

```
reset(s)
simByEuler(GBM1, nPeriods, 'nTrials' , nTrials, 'DeltaTime', dt, ...
    'Processes', f.LongstaffSchwartz);
BrownianMotionCallPrice = f.CallPrice(strike, r);
BrownianMotionPutPrice = f.PutPrice (strike, r);
reset(s)
z = Example_CopulaRNG(returns * sqrt(252), nPeriods, 'Gaussian');
f = Example_LongstaffSchwartz(nPeriods, nTrials);
simByEuler(GBM2, nPeriods, 'nTrials' , nTrials, 'DeltaTime', dt, ...
    'Processes', f.LongstaffSchwartz, 'Z', z);
GaussianCopulaCallPrice = f.CallPrice(strike, r);
GaussianCopulaPutPrice = f.PutPrice (strike, r);
```

Now repeat the copula simulation with the $t$ copula dependence structure. You use the same model object for both copulas; only the random number generator and option pricing functions need to be re-initialized.

```
reset(s)
z = Example_CopulaRNG(returns * sqrt(252), nPeriods, 't');
f = Example_LongstaffSchwartz(nPeriods, nTrials);
simByEuler(GBM2, nPeriods, 'nTrials' , nTrials, 'DeltaTime', dt, ...
    'Processes', f.LongstaffSchwartz, 'Z', z);
tCopulaCallPrice = f.CallPrice(strike, r);
tCopulaPutPrice = f.PutPrice (strike, r);
```

Finally, compare the American put and call option prices obtained from all models.

```
disp(' ')
fprintf(' # of Monte Carlo Trials: %8d\n' , nTrials)
fprintf(' # of Time Periods/Trial: %8d\n\n' , nPeriods)
fprintf(' Brownian Motion American Call Basket Price: %8.4f\n' , BrownianMotionCallPr
fprintf(' Brownian Motion American Put Basket Price: %8.4f\n\n', BrownianMotionPutPri
fprintf(' Gaussian Copula American Call Basket Price: %8.4f\n' , GaussianCopulaCallPr
```

```
fprintf(' Gaussian Copula American Put Basket Price: %8.4f\n\n', GaussianCopulaPutPri
fprintf(' t Copula American Call Basket Price: %8.4f\n' , tCopulaCallPrice)
fprintf(' t Copula American Put Basket Price: %8.4f\n' , tCopulaPutPrice)
# of Monte Carlo Trials: 100
# of Time Periods/Trial: 63
Brownian Motion American Call Basket Price: 25.9456
Brownian Motion American Put Basket Price: 16.4132
Gaussian Copula American Call Basket Price: 24.5711
Gaussian Copula American Put Basket Price: 17.4229
    t Copula American Call Basket Price: 22.6220
    t Copula American Put Basket Price: 20.9983
```

This analysis represents only a small-scale simulation. If the simulation is repeated with 100,000 trials, the following results are obtained:

```
# of Monte Carlo Trials: 100000
# of Time Periods/Trial: 63
```

Brownian Motion American Call Basket Price: 20.2214
Brownian Motion American Put Basket Price: 16.5355
Gaussian Copula American Call Basket Price: 20.6097
Gaussian Copula American Put Basket Price: 16.5539
t Copula American Call Basket Price: 21.1273
t Copula American Put Basket Price: 16.6873

Interestingly, the results agree closely. Put option prices obtained from copulas exceed those of Brownian motion by less than $1 \%$.

## A Note on Volatility and Interest Rate Scaling

The same option prices could also be obtained by working with unannualized (in this case, daily) centered returns and riskless rates, where the time increment $d t=1$ day rather than $1 / 252$ years. In other words, portfolio prices would still be simulated every trading day; the data is simply scaled differently.

Although not executed, and by first resetting the random stream to its initial internal state, the following code segments work with daily centered returns and riskless rates and produce the same option prices.

## Gaussian Distribution/Brownian Motion \& Daily Data:

```
reset(s)
f = Example_LongstaffSchwartz(nPeriods, nTrials);
GBM1 = gbm(diag}(r(ones(1,nIndices))/252), diag(std(returns)), 'StartState', X, ...
    'Correlation', correlation);
simByEuler(GBM1, nPeriods, 'nTrials' , nTrials, 'DeltaTime', 1, ...
    'Processes', f.LongstaffSchwartz);
BrownianMotionCallPrice = f.CallPrice(strike, r/252)
BrownianMotionPutPrice = f.PutPrice (strike, r/252)
```


## Gaussian Copula \& Daily Data:

```
reset(s)
```

z = Example_CopulaRNG(returns, nPeriods, 'Gaussian');
$\mathrm{f}=$ Example_LongstaffSchwartz(nPeriods, nTrials);
GBM2 = gbm(diag(r(ones(1,nIndices))/252), eye(nIndices), 'StartState', X);
simByEuler(GBM2, nPeriods, 'nTrials' , nTrials, 'DeltaTime', 1, ...
'Processes', f.LongstaffSchwartz , 'Z', z);
GaussianCopulaCallPrice = f.CallPrice(strike, r/252)
GaussianCopulaPutPrice = f.PutPrice (strike, r/252)

## t Copula \& Daily Data:

```
reset(s)
z = Example_CopulaRNG(returns, nPeriods, 't');
f = Example_LongstaffSchwartz(nPeriods, nTrials);
simByEuler(GBM2, nPeriods, 'nTrials' , nTrials, 'DeltaTime', 1, ...
    'Processes', f.LongstaffSchwartz , 'Z', z);
tCopulaCallPrice = f.CallPrice(strike, r/252)
tCopulaPutPrice = f.PutPrice (strike, r/252)
```


## See Also

bm | cev |cir|diffusion |drift | gbm | heston | hwv | interpolate| sde| sdeddo | sdeld | sdemrd | simByEuler| simBySolution|simBySolution|simulate| ts2func

## Related Examples

- "Simulating Equity Prices" on page 18-35
- "Simulating Interest Rates" on page 18-60
- "Pricing American Basket Options by Monte Carlo Simulation"
- "Improving Performance of Monte Carlo Simulation with Parallel Computing" on page 18-108
- "Base SDE Models" on page 18-17
- "Drift and Diffusion Models" on page 18-20
- "Linear Drift Models" on page 18-24
- "Parametric Models" on page 18-26


## More About

- "SDEs" on page 18-2
- "SDE Models" on page 18-9
- "SDE Class Hierarchy" on page 18-5


## Improving Performance of Monte Carlo Simulation with Parallel Computing

This example shows how to improve the performance of a Monte Carlo simulation using Parallel Computing Toolbox.

Consider a geometric Brownian motion (GBM) process in which you want to incorporate alternative asset price dynamics. Specifically, suppose that you want to include a timevarying short rate and a volatility surface. The process to simulate is written as

$$
d S(t)=r(t) S(t) d t+V(t, S(t)) S(t) d W(t)
$$

for stock price $S(t)$, rate of return $r(t)$, volatility $V(t, S(t))$, and Brownian motion $W(t)$. In this example, the rate of return is a deterministic function of time and the volatility is a function of both time and current stock price. Both the return and volatility are defined on a discrete grid such that intermediate values is obtained by linear interpolation. For example, such a simulation could be used to support the pricing of thinly traded options.

To include a time series of riskless short rates, suppose that you derive the following deterministic short rate process as a function of time.

```
times = [0 0.25 0.5 1 2 3 4 5 6 7 8 9 10]; % in years
rates =[0.1 0.2 0.3 0.4 0.5 0.8 1.25 1.75 2.0 2.2 2.25 2.50 2.75]/100;
```

Suppose that you then derive the following volatility surface whose columns correspond to simple relative moneyness, or the ratio of the spot price to strike price, and whose rows correspond to time to maturity, or tenor.

```
surface = [l28.1 25.3 20.6 16.3 11.2 6.2 4.9 4.9 4.9 4.9.9
    22.7 19.8 15.4 12.6 9.6 6.7 5.2 5.2 5.2 5.2 5.2 
    21.7 17.6 13.7 11.5 9.4 7.3 5.7 5.4 5.4 5.4 5.4 5.4 5.4
    19.8 16.4 12.9 11.1 9.3 7.6 6.2 5.6 5.6 5.6 5.6 5.6
    18.6 15.6 12.5 10.8
    17.4 13.8 11.7 10.8 9.9 9.1 8.5 7.9 7.4 7.3 7.3 7.3
    17.1 13.7 12.0 11.2 10.6 10.0 9.5 9.1 8.8 8.8 8.6 8.4 8.0
    17.5 13.9 12.5 11.9 11.4 10.9 10.5 10.2 9.9 9.6 9.4 9.0
    18.3 14.9 13.7 13.2 12.8 12.4 12.0 11.7 11.4 11.2 11.0 10.8
    19.2 19.6 14.2 13.9 13.4 13.0 13.2 12.5 12.1 11.9 11.8 11.4]/100;
tenor = [0 0.25 0.50 0.75 1 2 3 5 7 10]; % in years
moneyness = [0.25 0.5 0.75 0.8 0.9 1 1.10 1.25 1.50 2 3 5];
```

Set the simulation parameters. The following assumes that the price of the underlying asset is initially equal to the strike price and that the price of the underlying asset is simulated monthly for 10 years, or 120 months. As a simple illustration, 100 sample paths are simulated.
price = 100;
strike = 100;
dt = 1/12;
nPeriods = 120;
nTrials = 100;
For reproducibility, set the random number generator to its default, and draw the Gaussian random variates that drive the simulation. Generating the random variates is not necessary to incur the performance improvement of parallel computation, but doing so allows the resulting simulated paths to match those of the conventional (that is, nonparallelized) simulation. Moreover, generating independent Gaussian random variates as inputs also guarantees that all simulated paths are independent.

```
rng default
Z = randn(nPeriods,1,nTrials);
```

Create the return and volatility functions and the GBM model using gbm. Notice that the rate of return is a deterministic function of time, and therefore accepts simulation time as its only input argument. In contrast, the volatility must account for the moneyness and is a function of both time and stock price. Moreover, since the volatility surface is defined as a function of time to maturity rather than simulation time, the volatility function subtracts the current simulation time from the last time at which the price process is simulated (10 years). This ensures that as the simulation time approaches its terminal value, the time to maturity of the volatility surface approaches zero. Although far more elaborate return and volatility functions could be used if desired, the following assumes simple linear interpolation.

```
mu = @(t) interp1(times,rates,t);
sigma = @(t,S) interp2(moneyness,tenor,surface,S/strike,tenor(end)-t);
mdl = gbm(mu,sigma,'StartState',price);
```

Simulate the paths of the underlying geometric Brownian motion without parallelization.

```
tStart = tic;
paths = simBySolution(mdl,nPeriods,'nTrials',nTrials,'DeltaTime',dt,'Z',Z);
timel = toc(tStart);
```

Simulate the paths in parallel using a parfor loop. For users licensed to access the Parallel Computing Toolbox, the following code segment automatically creates a parallel
pool using the default local profile. If desired, this behavior can be changed by first calling the parpool function. If a parallel pool is not already created, the following simulation will likely be slower than the previous simulation without parallelization. In this case, rerun the following simulation to assess the true benefits of parallelization.

```
tStart = tic;
parPaths = zeros(nPeriods+1,1,nTrials);
parfor i = 1:nTrials
    parPaths(:,:,i) = simBySolution(mdl,nPeriods,'DeltaTime',dt,'Z',Z(:,:,i));
end
time2 = toc(tStart);
```

If you examine any given path obtained without parallelization to the corresponding path with parallelization, you see that they are identical. Moreover, although relative performance varies, the results obtained with parallelization will generally incur a significant improvement. To assess the performance improvement, examine the runtime of each approach in seconds and speedup gained from simulating the paths in parallel.

```
time1 % elapsed time of conventional simulation, in seconds
time2 % elapsed time of parallel simulation, in seconds
speedup = timel/time2 % speedup factor
```

time1 =
6.1329
time2 =
2.5918
speedup =
2.3663

## See Also

bm | cev |cir|diffusion|drift|gbm| heston| hwv | interpolate| parpool| sde | sdeddo| sdeld \| sdemrd | simByEuler | simBySolution | simBySolution | simulate|ts2func

## Related Examples

- "Simulating Equity Prices" on page 18-35
- "Simulating Interest Rates" on page 18-60
- "Pricing American Basket Options by Monte Carlo Simulation"
- "Base SDE Models" on page 18-17
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## More About

- "SDEs" on page 18-2
- "SDE Models" on page 18-9
- "SDE Class Hierarchy" on page 18-5
- "Performance Considerations" on page 18-77

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Functions - Alphabetical List

## bm

Brownian motion models

## Description

Creates and displays Brownian motion (sometimes called arithmetic Brownian motion or generalized Wiener process) bm objects that derive from the sdeld (SDE with drift rate expressed in linear form) class.

Use bm objects to simulate sample paths of NVARS state variables driven by NBROWNS sources of risk over NPERIODS consecutive observation periods, approximating continuous-time Brownian motion stochastic processes. This enables you to transform a vector of NBROWNS uncorrelated, zero-drift, unit-variance rate Brownian components into a vector of NVARS Brownian components with arbitrary drift, variance rate, and correlation structure.

Use bm to simulate any vector-valued BM process of the form:

$$
d X_{t}=\mu(t) d t+V(t) d W_{t}
$$

where:

- $X_{t}$ is an NVARS-by-1 state vector of process variables.
- $\mu$ is an NVARS-by- 1 drift-rate vector.
- $V$ is an NVARS-by-NBROWNS instantaneous volatility rate matrix.
- $d W_{t}$ is an NBROWNS-by-1 vector of (possibly) correlated zero-drift/unit-variance rate Brownian components.


## Creation

## Syntax

$B M=b m(M u, S i g m a)$
$B M=b m($ $\qquad$ , Name, Value)

## Description

$B M=b m(M u, S i g m a)$ creates a default BM object.
Specify required input parameters as one of the following types:

- A MATLAB array. Specifying an array indicates a static (non-time-varying) parametric specification. This array fully captures all implementation details, which are clearly associated with a parametric form.
- A MATLAB function. Specifying a function provides indirect support for virtually any static, dynamic, linear, or nonlinear model. This parameter is supported via an interface, because all implementation details are hidden and fully encapsulated by the function.

Note You can specify combinations of array and function input parameters as needed.
Moreover, a parameter is identified as a deterministic function of time if the function accepts a scalar time $t$ as its only input argument. Otherwise, a parameter is assumed to be a function of time $t$ and state $X(t)$ and is invoked with both input arguments.

BM = bm( __ , Name, Value) creates a bm object with additional options specified by one or more Name, Value pair arguments.

Name is a property name and Value is its corresponding value. Name must appear inside single quotes (' '). You can specify several name-value pair arguments in any order as Name1, Value1, ..., NameN , ValueN

The BM object has the following "Properties" on page 19-5:

- StartTime - Initial observation time
- StartState - Initial state at time StartTime
- Correlation - Access function for the Correlation input argument, callable as a function of time
- Drift - Composite drift-rate function, callable as a function of time and state
- Diffusion - Composite diffusion-rate function, callable as a function of time and state
- Simulation - A simulation function or method


## Input Arguments

## Mu - Mu represents the parameter $\mu$

array or deterministic function of time | deterministic function of time and state
Mu represents the parameter $\mu$, specified as an array or deterministic function of time.
If you specify Mu as an array, it must be an NVARS-by-1 column vector representing the drift rate (the expected instantaneous rate of drift, or time trend).

As a deterministic function of time, when $M u$ is called with a real-valued scalar time $t$ as its only input, Mu must produce an NVARS-by-NVARS matrix. If you specify Mu as a function of time and state, it calculates the expected instantaneous rate of drift. This function must generate an NVARS-by-1 column vector when invoked with two inputs:

- A real-valued scalar observation time $t$.
- An NVARS-by-1 state vector $X_{t}$.

Data Types: double | function_handle

## Sigma - Sigma represents the parameter V <br> array or deterministic function of time | deterministic function of time and state

Sigma represents the parameter $V$, specified as an array or deterministic function of time.

If you specify Sigma as an array, it must be an NVARS-by-NBROWNS matrix of instantaneous volatility rates or as a deterministic function of time. In this case, each row of Sigma corresponds to a particular state variable. Each column corresponds to a particular Brownian source of uncertainty, and associates the magnitude of the exposure of state variables with sources of uncertainty.

As a deterministic function of time, when Sigma is called with a real-valued scalar time $t$ as its only input, Sigma must produce an NVARS-by-NBROWNS matrix. If you specify Sigma as a function of time and state, it must return an NVARS-by-NBROWNS matrix of volatility rates when invoked with two inputs:

- A real-valued scalar observation time $t$.
- An NVARS-by-1 state vector $X_{t}$.

Although the gbm constructor enforces no restrictions on the sign of Sigma volatilities, they are specified as positive values.
Data Types: double | function_handle

## Properties

## StartTime - Starting time of first observation, applied to all state variables <br> 0 (default) | scalar

Starting time of first observation, applied to all state variables, specified as a scalar

## Data Types: double

## StartState - Initial values of state variables

1 (default) | scalar, column vector, or matrix
Initial values of state variables, specified as a scalar, column vector, or matrix.
If StartState is a scalar, bm applies the same initial value to all state variables on all trials.

If StartState is a column vector, bm applies a unique initial value to each state variable on all trials.

If StartState is a matrix, bm applies a unique initial value to each state variable on each trial.

Data Types: double

## Correlation - Correlation between Gaussian random variates drawn to generate the Brownian motion vector (Wiener processes) <br> NBROWNS-by-NBROWNS identity matrix representing independent Gaussian processes (default) | positive semidefinite matrix | deterministic function

Correlation between Gaussian random variates drawn to generate the Brownian motion vector (Wiener processes), specified as an NBROWNS-by-NBROWNS positive semidefinite matrix, or as a deterministic function $C(t)$ that accepts the current time $t$ and returns an NBROWNS-by-NBROWNS positive semidefinite correlation matrix.

A Correlation matrix represents a static condition.

As a deterministic function of time, Correlation allows you to specify a dynamic correlation structure.

Data Types: double

## Simulation - User-defined simulation function or SDE simulation method simulation by Euler approximation (simByEuler) (default) | function \| SDE simulation method

User-defined simulation function or SDE simulation method, specified as a function or SDE simulation method.

Data Types: function_handle
Drift - Drift rate component of continuous-time stochastic differential equations (SDEs)
value stored from drift-rate function (default) | drift object or function accessible by ( $t, X_{t}$ )
This property is read-only.
Drift rate component of continuous-time stochastic differential equations (SDEs), specified as a drift object or function accessible by ( $t, X_{t}$.

The drift rate specification supports the simulation of sample paths of NVARS state variables driven by NBROWNS Brownian motion sources of risk over NPERIODS consecutive observation periods, approximating continuous-time stochastic processes.

The drift class allows you to create drift-rate objects using drift of the form:

$$
F\left(t, X_{t}\right)=A(t)+B(t) X_{t}
$$

where:

- A is an NVARS-by-1 vector-valued function accessible using the $\left(t, X_{t}\right)$ interface.
- B is an NVARS-by-NVARS matrix-valued function accessible using the $\left(t, X_{t}\right)$ interface.

The displayed parameters for a drift object are:

- Rate: The drift-rate function, $F\left(t, X_{t}\right)$
- A: The intercept term, $A\left(t, X_{t}\right)$, of $F\left(t, X_{t}\right)$
- B: The first order term, $B\left(t, X_{t}\right)$, of $F\left(t, X_{t}\right)$
$A$ and $B$ enable you to query the original inputs. The function stored in Rate fully encapsulates the combined effect of $A$ and $B$.

When specified as MATLAB double arrays, the inputs $A$ and $B$ are clearly associated with a linear drift rate parametric form. However, specifying either A or B as a function allows you to customize virtually any drift rate specification.

Note You can express drift and diffusion classes in the most general form to emphasize the functional $\left(t, X_{t}\right)$ interface. However, you can specify the components A and $B$ as functions that adhere to the common $\left(t, X_{t}\right)$ interface, or as MATLAB arrays of appropriate dimension.

Example: F = drift(0, 0.1) \% Drift rate function $F(t, X)$
Data Types: object

## Diffusion - Diffusion rate component of continuous-time stochastic differential equations (SDEs)

value stored from diffusion-rate function (default) | diffusion object or functions accessible by $\left(t, X_{t}\right)$

This property is read-only.
Diffusion rate component of continuous-time stochastic differential equations (SDEs), specified as a drift object or function accessible by $\left(t, X_{t}\right.$.

The diffusion rate specification supports the simulation of sample paths of NVARS state variables driven by NBROWNS Brownian motion sources of risk over NPERIODS consecutive observation periods, approximating continuous-time stochastic processes.

The diffusion class allows you to create diffusion-rate objects using diffusion:

$$
G\left(t, X_{t}\right)=D\left(t, X_{t}^{\alpha(t)}\right) V(t)
$$

where:

- D is an NVARS-by-NVARS diagonal matrix-valued function.
- Each diagonal element of $D$ is the corresponding element of the state vector raised to the corresponding element of an exponent Alpha, which is an NVARS-by-1 vectorvalued function.
- $V$ is an NVARS-by-NBROWNS matrix-valued volatility rate function Sigma.
- Alpha and Sigma are also accessible using the $\left(t, X_{t}\right)$ interface.

The displayed parameters for a diffusion object are:

- Rate: The diffusion-rate function, $G\left(t, X_{t}\right)$.
- Alpha: The state vector exponent, which determines the format of $D\left(t, X_{t}\right)$ of $G\left(t, X_{t}\right)$.
- Sigma: The volatility rate, $V\left(t, X_{t}\right)$, of $G\left(t, X_{t}\right)$.

Alpha and Sigma enable you to query the original inputs. (The combined effect of the individual Alpha and Sigma parameters is fully encapsulated by the function stored in Rate.) The Rate functions are the calculation engines for the drift and diffusion objects, and are the only parameters required for simulation.

Note You can express drift and diffusion classes in the most general form to emphasize the functional $\left(t, X_{t}\right)$ interface. However, you can specify the components A and $B$ as functions that adhere to the common $\left(t, X_{t}\right)$ interface, or as MATLAB arrays of appropriate dimension.

Example: $G=$ diffusion(1, 0.3) \% Diffusion rate function $G(t, X)$
Data Types: object

## Object Functions

interpolate Brownian interpolation of stochastic differential equations simulate Simulate multivariate stochastic differential equations (SDEs)
simByEuler Euler simulation of stochastic differential equations (SDEs)

## Examples

## Create a bm Object

Create a univariate Brownian motion (bm) object to represent the model: $d X_{t}=0.3 d W_{t}$.

```
obj = bm(0, 0.3) % (A = Mu, Sigma)
```

obj $=$

```
Class BM: Brownian Motion
    Dimensions: State = 1, Brownian = 1
    StartTime: 0
    StartState: 0
    Correlation: 1
        Drift: drift rate function F(t,X(t))
        Diffusion: diffusion rate function G(t,X(t))
        Simulation: simulation method/function simByEuler
        Mu: 0
        Sigma: 0.3
```

bm objects display the parameter A as the more familiar Mu.
The bm class also provides an overloaded Euler simulation method that improves run-time performance in certain common situations. This specialized method is invoked automatically only if all the following conditions are met:

- The expected drift, or trend, rate Mu is a column vector.
- The volatility rate, Sigma, is a matrix.
- No end-of-period adjustments and/or processes are made.
- If specified, the random noise process $Z$ is a three-dimensional array.
- If $Z$ is unspecified, the assumed Gaussian correlation structure is a double matrix.


## Definitions

## Instance Hierarchy

There are inheritance relationships among the SDE classes.
The following figure illustrates the inheritance relationships.


For more information, see "SDE Class Hierarchy" on page 18-5.

## Algorithms

When you specify the required input parameters as arrays, they are associated with a specific parametric form. By contrast, when you specify either required input parameter as a function, you can customize virtually any specification.

Accessing the output parameters with no inputs simply returns the original input specification. Thus, when you invoke these parameters with no inputs, they behave like simple properties and allow you to test the data type (double vs. function, or equivalently, static vs. dynamic) of the original input specification. This is useful for validating and designing methods.

When you invoke these parameters with inputs, they behave like functions, giving the impression of dynamic behavior. The parameters accept the observation time $t$ and a state vector $X_{t}$, and return an array of appropriate dimension. Even if you originally specified
an input as an array, bm treats it as a static function of time and state, by that means guaranteeing that all parameters are accessible by the same interface.

## References

[1] Ait-Sahalia, Y. "Testing Continuous-Time Models of the Spot Interest Rate." The Review of Financial Studies, Spring 1996, Vol. 9, No. 2, pp. 385-426.
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[3] Glasserman, P. Monte Carlo Methods in Financial Engineering. New York, SpringerVerlag, 2004.
[4] Hull, J. C. Options, Futures, and Other Derivatives, 5th ed. Englewood Cliffs, NJ: Prentice Hall, 2002.
[5] Johnson, N. L., S. Kotz, and N. Balakrishnan. Continuous Univariate Distributions. Vol. 2, 2nd ed. New York, John Wiley \& Sons, 1995.
[6] Shreve, S. E. Stochastic Calculus for Finance II: Continuous-Time Models. New York: Springer-Verlag, 2004.

## See Also

diffusion|drift|interpolate| sdeld|simByEuler|simulate

## Topics

"Simulating Equity Prices" on page 18-35
"Simulating Interest Rates" on page 18-60
"Stratified Sampling" on page 18-71
"Pricing American Basket Options by Monte Carlo Simulation" on page 18-85
"Base SDE Models" on page 18-17
"Drift and Diffusion Models" on page 18-20
"Linear Drift Models" on page 18-24
"Parametric Models" on page 18-26
"SDEs" on page 18-2
"SDE Models" on page 18-9
"SDE Class Hierarchy" on page 18-5
"Performance Considerations" on page 18-77

Introduced in R2008a

## cev

Constant Elasticity of Variance (CEV) model

## Description

Creates and displays cev objects, which derive from the sdeld (SDE with drift rate expressed in linear form) class.

Use cev objects to simulate sample paths of NVARS state variables driven by NBROWNS Brownian motion sources of risk over NPERIODS consecutive observation periods, approximating continuous-time stochastic processes.

This model allows you to simulate any vector-valued CEV of the form:

$$
d X_{t}=\mu(t) X_{t} d t+D\left(t, X_{t}^{\alpha(t)}\right) V(t) d W_{t}
$$

where:

- $X_{t}$ is an NVARS-by-1 state vector of process variables.
- $\mu$ is an NVARS-by-NVARS (generalized) expected instantaneous rate of return matrix.
- $D$ is an NVARS-by-NVARS diagonal matrix, where each element along the main diagonal is the corresponding element of the state vector raised to the corresponding power of $\alpha$.
- $V$ is an NVARS-by-NBROWNS instantaneous volatility rate matrix.
- $d W_{t}$ is an NBROWNS-by-1 Brownian motion vector.


## Creation

## Syntax

```
CEV = cev(Return,Alpha,Sigma)
CEV = cev( ___,Name,Value)
```


## Description

CEV $=\operatorname{cev}($ Return,Alpha,Sigma) creates a default CEV object.
Specify required input parameters as one of the following types:

- A MATLAB array. Specifying an array indicates a static (non-time-varying) parametric specification. This array fully captures all implementation details, which are clearly associated with a parametric form.
- A MATLAB function. Specifying a function provides indirect support for virtually any static, dynamic, linear, or nonlinear model. This parameter is supported via an interface, because all implementation details are hidden and fully encapsulated by the function.

Note You can specify combinations of array and function input parameters as needed.
Moreover, a parameter is identified as a deterministic function of time if the function accepts a scalar time $t$ as its only input argument. Otherwise, a parameter is assumed to be a function of time $t$ and state $X(t)$ and is invoked with both input arguments.

CEV $=\operatorname{cev}(\ldots \quad$, Name, Value) creates a CEV object with additional options specified by one or more Name, Value pair arguments.

Name is a property name and Value is its corresponding value. Name must appear inside single quotes (' ' ). You can specify several name-value pair arguments in any order as Name1, Value1, ..., NameN , ValueN

The CEV object has the following "Properties" on page 19-16:

- StartTime - Initial observation time
- StartState - Initial state at time StartTime
- Correlation - Access function for the Correlation input argument, callable as a function of time
- Drift - Composite drift-rate function, callable as a function of time and state
- Diffusion - Composite diffusion-rate function, callable as a function of time and state
- Simulation - A simulation function or method
- Return - Access function for the input argument Return, callable as a function of time and state
- Alpha - Access function for the input argument Alpha, callable as a function of time and state
- Sigma - Access function for the input argument Sigma, callable as a function of time and state


## Input Arguments

## Return - Return represents the parameter $\boldsymbol{\mu}$

 array or deterministic function of time or deterministic function of time and stateReturn represents the parameter $\mu$, specified as an array or deterministic function of time.

If you specify Return as an array, it must be an NVARS-by-NVARS matrix representing the expected (mean) instantaneous rate of return.

As a deterministic function of time, when Return is called with a real-valued scalar time $t$ as its only input, Return must produce an NVARS-by-NVARS matrix. If you specify Return as a function of time and state, it must return an NVARS-by-NVARS matrix when invoked with two inputs:

- A real-valued scalar observation time $t$.
- An NVARS-by-1 state vector $X_{t}$.

Data Types: double | function_handle

## Alpha - Return represents the parameter D <br> array or deterministic function of time or deterministic function of time and state

Alpha represents the parameter $D$, specified as an array or deterministic function of time.

If you specify Alpha as an array, it represents an NVARS-by-1 column vector of exponents.
As a deterministic function of time, when Alpha is called with a real-valued scalar time $t$ as its only input, Alpha must produce an NVARS-by-1 matrix.

If you specify it as a function of time and state, Alpha must return an NVARS-by-1 column vector of exponents when invoked with two inputs:

- A real-valued scalar observation time $t$.
- An NVARS-by-1 state vector $X_{t}$.


## Data Types: double | function_handle

## Sigma - Sigma represents the parameter V array or deterministic function of time or deterministic function of time and state

Sigma represents the parameter $V$, specified as an array or a deterministic function of time.

If you specify Sigma as an array, it must be an NVARS-by-NBROWNS matrix of instantaneous volatility rates. In this case, each row of Sigma corresponds to a particular state variable. Each column corresponds to a particular Brownian source of uncertainty, and associates the magnitude of the exposure of state variables with sources of uncertainty.

As a deterministic function of time, when Sigma is called with a real-valued scalar time $t$ as its only input, Sigma must produce an NVARS-by-NBROWNS matrix. If you specify Sigma as a function of time and state, it must return an NVARS-by-NBROWNS matrix of volatility rates when invoked with two inputs:

- A real-valued scalar observation time $t$.
- An NVARS-by-1 state vector $X_{t}$.

Data Types: double | function_handle

Note Although cev does not enforce restrictions on the signs of these input arguments, each argument is specified as a positive value.

## Properties

StartTime - Starting time of first observation, applied to all state variables 0 (default) | scalar

Starting time of first observation, applied to all state variables, specified as a scalar
Data Types: double

## StartState - Initial values of state variables

1 (default) | scalar, column vector, or matrix
Initial values of state variables, specified as a scalar, column vector, or matrix.
If StartState is a scalar, cev applies the same initial value to all state variables on all trials.

If StartState is a column vector, cev applies a unique initial value to each state variable on all trials.

If StartState is a matrix, cev applies a unique initial value to each state variable on each trial.

Data Types: double

## Correlation - Correlation between Gaussian random variates drawn to generate the Brownian motion vector (Wiener processes)

NBROWNS-by-NBROWNS identity matrix representing independent Gaussian processes (default) | positive semidefinite matrix | deterministic function

Correlation between Gaussian random variates drawn to generate the Brownian motion vector (Wiener processes), specified as an NBROWNS-by-NBROWNS positive semidefinite matrix, or as a deterministic function $C(t)$ that accepts the current time $t$ and returns an NBROWNS-by-NBROWNS positive semidefinite correlation matrix.

A Correlation matrix represents a static condition.
As a deterministic function of time, Correlation allows you to specify a dynamic correlation structure.

Data Types: double

## Simulation - User-defined simulation function or SDE simulation method simulation by Euler approximation (simByEuler) (default) | function | SDE simulation method

User-defined simulation function or SDE simulation method, specified as a function or SDE simulation method.

Data Types: function_handle

## Drift - Drift rate component of continuous-time stochastic differential equations (SDEs)

value stored from drift-rate function (default) | drift object or function accessible by $\left(t, X_{t}\right)$
This property is read-only.
Drift rate component of continuous-time stochastic differential equations (SDEs), specified as a drift object or function accessible by ( $t, X_{t}$.

The drift rate specification supports the simulation of sample paths of NVARS state variables driven by NBROWNS Brownian motion sources of risk over NPERIODS consecutive observation periods, approximating continuous-time stochastic processes.

The drift class allows you to create drift-rate objects using drift of the form:

$$
F\left(t, X_{t}\right)=A(t)+B(t) X_{t}
$$

where:

- A is an NVARS-by-1 vector-valued function accessible using the $\left(t, X_{t}\right)$ interface.
- B is an NVARS-by-NVARS matrix-valued function accessible using the $\left(t, X_{t}\right)$ interface.

The displayed parameters for a drift object are:

- Rate: The drift-rate function, $F\left(t, X_{t}\right)$
- A: The intercept term, $A\left(t, X_{t}\right)$, of $F\left(t, X_{t}\right)$
- B: The first order term, $B\left(t, X_{t}\right)$, of $F\left(t, X_{t}\right)$
$A$ and $B$ enable you to query the original inputs. The function stored in Rate fully encapsulates the combined effect of $A$ and $B$.

When specified as MATLAB double arrays, the inputs $A$ and $B$ are clearly associated with a linear drift rate parametric form. However, specifying either A or B as a function allows you to customize virtually any drift rate specification.

Note You can express drift and diffusion classes in the most general form to emphasize the functional $\left(t, X_{t}\right)$ interface. However, you can specify the components A and $B$ as functions that adhere to the common $\left(t, X_{t}\right)$ interface, or as MATLAB arrays of appropriate dimension.

## Example: $F=\operatorname{drift}(0,0.1) \%$ Drift rate function $F(t, X)$ <br> Data Types: struct | double

## Diffusion - Diffusion rate component of continuous-time stochastic differential equations (SDEs)

value stored from diffusion-rate function (default) | diffusion object or functions accessible by $\left(t, X_{t}\right)$

This property is read-only.
Diffusion rate component of continuous-time stochastic differential equations (SDEs), specified as an object or function accessible by $\left(t, X_{t}\right.$.

The diffusion rate specification supports the simulation of sample paths of NVARS state variables driven by NBROWNS Brownian motion sources of risk over NPERIODS consecutive observation periods, approximating continuous-time stochastic processes.

The diffusion class allows you to create diffusion-rate objects using diffusion:

$$
G\left(t, X_{t}\right)=D\left(t, X_{t}^{\alpha(t)}\right) V(t)
$$

where:

- D is an NVARS-by-NVARS diagonal matrix-valued function.
- Each diagonal element of $D$ is the corresponding element of the state vector raised to the corresponding element of an exponent Alpha, which is an NVARS-by-1 vectorvalued function.
- $V$ is an NVARS-by-NBROWNS matrix-valued volatility rate function Sigma.
- Alpha and Sigma are also accessible using the $\left(t, X_{t}\right)$ interface.

The displayed parameters for a diffusion object are:

- Rate: The diffusion-rate function, $G\left(t, X_{t}\right)$.
- Alpha: The state vector exponent, which determines the format of $D\left(t, X_{t}\right)$ of $G\left(t, X_{t}\right)$.
- Sigma: The volatility rate, $V\left(t, X_{t}\right)$, of $G\left(t, X_{t}\right)$.

Alpha and Sigma enable you to query the original inputs. (The combined effect of the individual Alpha and Sigma parameters is fully encapsulated by the function stored in Rate.) The Rate functions are the calculation engines for the drift and diffusion objects, and are the only parameters required for simulation.

Note You can express drift and diffusion classes in the most general form to emphasize the functional $\left(t, X_{t}\right)$ interface. However, you can specify the components A and B as functions that adhere to the common ( $t, X_{t}$ ) interface, or as MATLAB arrays of appropriate dimension.

Example: $G=$ diffusion(1, 0.3) \% Diffusion rate function $G(t, X)$
Data Types: struct \| double

## Object Functions

interpolate Brownian interpolation of stochastic differential equations simulate Simulate multivariate stochastic differential equations (SDEs) simByEuler Euler simulation of stochastic differential equations (SDEs)

## Examples

## Create a cev Object

Create a univariate cev object to represent the model: $d X_{t}=0.25 X_{t}+0.3 X_{t}^{\frac{1}{2}} d W_{t}$.

```
obj = cev(0.25, 0.5, 0.3) % (B = Return, Alpha, Sigma)
```

obj =
Class CEV: Constant Elasticity of Variance
Dimensions: State $=1$, Brownian $=1$
StartTime: 0
StartState: 1
Correlation: 1
Drift: drift rate function $\mathrm{F}(\mathrm{t}, \mathrm{X}(\mathrm{t}))$
Diffusion: diffusion rate function $\mathrm{G}(\mathrm{t}, \mathrm{X}(\mathrm{t})$ )
Simulation: simulation method/function simByEuler
Return: 0.25
Alpha: 0.5
Sigma: 0.3
cev objects display the parameter $B$ as the more familiar Return

## Definitions

## Instance Hierarchy

There are inheritance relationships among the SDE classes.
The following figure illustrates the inheritance relationships.


For more information, see "SDE Class Hierarchy" on page 18-5.

## Algorithms

When you specify the required input parameters as arrays, they are associated with a specific parametric form. By contrast, when you specify either required input parameter as a function, you can customize virtually any specification.

Accessing the output parameters with no inputs simply returns the original input specification. Thus, when you invoke these parameters with no inputs, they behave like simple properties and allow you to test the data type (double vs. function, or equivalently, static vs. dynamic) of the original input specification. This is useful for validating and designing methods.

When you invoke these parameters with inputs, they behave like functions, giving the impression of dynamic behavior. The parameters accept the observation time $t$ and a state vector $X_{t}$, and return an array of appropriate dimension. Even if you originally specified an input as an array, cev treats it as a static function of time and state, by that means guaranteeing that all parameters are accessible by the same interface.

## References

[1] Ait-Sahalia, Y. "Testing Continuous-Time Models of the Spot Interest Rate." The Review of Financial Studies, Spring 1996, Vol. 9, No. 2, pp. 385-426.
[2] Ait-Sahalia, Y. "Transition Densities for Interest Rate and Other Nonlinear Diffusions." The Journal of Finance, Vol. 54, No. 4, August 1999.
[3] Glasserman, P. Monte Carlo Methods in Financial Engineering. New York, SpringerVerlag, 2004.
[4] Hull, J. C. Options, Futures, and Other Derivatives, 5th ed. Englewood Cliffs, NJ: Prentice Hall, 2002.
[5] Johnson, N. L., S. Kotz, and N. Balakrishnan. Continuous Univariate Distributions. Vol. 2, 2nd ed. New York, John Wiley \& Sons, 1995.
[6] Shreve, S. E. Stochastic Calculus for Finance II: Continuous-Time Models. New York: Springer-Verlag, 2004.

## See Also

diffusion|drift|interpolate| sdeld|simByEuler|simulate

## Topics

"Creating Constant Elasticity of Variance (CEV) Models" on page 18-27
Implementing Multidimensional Equity Market Models, Implementation 3: Using SDELD,
CEV, and GBM Objects on page 18-38
"Simulating Equity Prices" on page 18-35
"Simulating Interest Rates" on page 18-60
"Stratified Sampling" on page 18-71
"Pricing American Basket Options by Monte Carlo Simulation" on page 18-85
"Base SDE Models" on page 18-17
"Drift and Diffusion Models" on page 18-20
"Linear Drift Models" on page 18-24
"Parametric Models" on page 18-26
"SDEs" on page 18-2
"SDE Models" on page 18-9
"SDE Class Hierarchy" on page 18-5
"Performance Considerations" on page 18-77

## Introduced in R2008a

## cir

Cox-Ingersoll-Ross mean-reverting square root diffusion model

## Description

Creates and displays cir objects, which derive from the sdemrd (SDE with drift rate expressed in mean-reverting form) class.

Use cir objects to simulate sample paths of NVARS state variables expressed in meanreverting drift-rate form. These state variables are driven by NBROWNS Brownian motion sources of risk over NPERIODS consecutive observation periods, approximating continuous-time CIR stochastic processes with square root diffusions.

You can simulate any vector-valued CIR process of the form:

$$
d X_{t}=S(t)\left[L(t)-X_{t}\right] d t+D\left(t, X_{t}^{\frac{1}{2}}\right) V(t) d W_{t}
$$

where:

- $X_{t}$ is an NVARS-by-1 state vector of process variables.
- $S$ is an NVARS-by-NVARS matrix of mean reversion speeds (the rate of mean reversion).
- $L$ is an NVARS-by-1 vector of mean reversion levels (long-run mean or level).
- $D$ is an NVARS-by-NVARS diagonal matrix, where each element along the main diagonal is the square root of the corresponding element of the state vector.
- $V$ is an NVARS-by-NBROWNS instantaneous volatility rate matrix.
- $d W_{t}$ is an NBROWNS-by-1 Brownian motion vector.


## Creation

## Syntax

CIR = cir(Speed,Level,Sigma)

```
CIR = cir(___ ,Name,Value)
```


## Description

CIR $=$ cir(Speed,Level,Sigma) creates a default CIR object.
Specify required input parameters as one of the following types:

- A MATLAB array. Specifying an array indicates a static (non-time-varying) parametric specification. This array fully captures all implementation details, which are clearly associated with a parametric form.
- A MATLAB function. Specifying a function provides indirect support for virtually any static, dynamic, linear, or nonlinear model. This parameter is supported via an interface, because all implementation details are hidden and fully encapsulated by the function.

Note You can specify combinations of array and function input parameters as needed.
Moreover, a parameter is identified as a deterministic function of time if the function accepts a scalar time $t$ as its only input argument. Otherwise, a parameter is assumed to be a function of time $t$ and state $X(t)$ and is invoked with both input arguments.

CIR $=\operatorname{cir}(\ldots \quad$, Name, Value) creates a CIR object with additional options specified by one or more Name, Value pair arguments.

Name is a property name and Value is its corresponding value. Name must appear inside single quotes (' ' ). You can specify several name-value pair arguments in any order as Name1, Value1, ..., NameN, ValueN

The CIR object has the following "Properties" on page 19-27:

- StartTime - Initial observation time
- StartState - Initial state at time StartTime
- Correlation - Access function for the Correlation input argument, callable as a function of time
- Drift - Composite drift-rate function, callable as a function of time and state
- Diffusion - Composite diffusion-rate function, callable as a function of time and state
- Simulation - A simulation function or method
- Speed - Access function for the input argument Speed, callable as a function of time and state
- Level - Access function for the input argument Level, callable as a function of time and state
- Sigma - Access function for the input argument Sigma, callable as a function of time and state


## Input Arguments

## Speed - Speed represents the parameter S

array or deterministic function of time or deterministic function of time and state
Speed represents the parameter $S$, specified as an array or deterministic function of time.
If you specify Speed as an array, it must be an NVARS-by-NVARS matrix of mean-reversion speeds (the rate at which the state vector reverts to its long-run average Level).

As a deterministic function of time, when Speed is called with a real-valued scalar time $t$ as its only input, Speed must produce an NVARS-by-NVARS matrix. If you specify Speed as a function of time and state, it calculates the speed of mean reversion. This function must generate an NVARS-by-NVARS matrix of reversion rates when called with two inputs:

- A real-valued scalar observation time $t$.
- An NVARS-by-1 state vector $X_{t}$.

Data Types: double | function_handle

## Level - Level represents the parameter $\mathbf{L}$

array or deterministic function of time or deterministic function of time and state
Level represents the parameter $L$, specified as an array or deterministic function of time.
If you specify Level as an array, it must be an NVARS-by-1 column vector of reversion levels.

As a deterministic function of time, when Level is called with a real-valued scalar time $t$ as its only input, Level must produce an NVARS-by-1 column vector. If you specify Level as a function of time and state, it must generate an NVARS-by-1 column vector of reversion levels when called with two inputs:

- A real-valued scalar observation time $t$.
- An NVARS-by-1 state vector $X_{t}$.


## Data Types: double | function_handle

## Sigma - Sigma represents the parameter V array or deterministic function of time or deterministic function of time and state

Sigma represents the parameter $V$, specified as an array or a deterministic function of time.

If you specify Sigma as an array, it must be an NVARS-by-NBROWNS matrix of instantaneous volatility rates or as a deterministic function of time. In this case, each row of Sigma corresponds to a particular state variable. Each column corresponds to a particular Brownian source of uncertainty, and associates the magnitude of the exposure of state variables with sources of uncertainty.

As a deterministic function of time, when Sigma is called with a real-valued scalar time $t$ as its only input, Sigma must produce an NVARS-by-NBROWNS matrix. If you specify Sigma as a function of time and state, it must return an NVARS-by-NBROWNS matrix of volatility rates when invoked with two inputs:

- A real-valued scalar observation time $t$.
- An NVARS-by-1 state vector $X_{t}$.

Data Types: double | function_handle

Note Although cir does not enforce restrictions on the signs of these input arguments, each argument is specified as a positive value.

## Properties

## StartTime - Starting time of first observation, applied to all state variables

 0 (default) | scalarStarting time of first observation, applied to all state variables, specified as a scalar
Data Types: double

## StartState - Initial values of state variables <br> 1 (default) | scalar, column vector, or matrix

Initial values of state variables, specified as a scalar, column vector, or matrix.
If StartState is a scalar, cir applies the same initial value to all state variables on all trials.

If StartState is a column vector, cir applies a unique initial value to each state variable on all trials.

If StartState is a matrix, cir applies a unique initial value to each state variable on each trial.

Data Types: double

## Correlation - Correlation between Gaussian random variates drawn to generate the Brownian motion vector (Wiener processes)

NBROWNS-by-NBROWNS identity matrix representing independent Gaussian processes (default) | positive semidefinite matrix | deterministic function

Correlation between Gaussian random variates drawn to generate the Brownian motion vector (Wiener processes), specified as an NBROWNS-by-NBROWNS positive semidefinite matrix, or as a deterministic function $C(t)$ that accepts the current time $t$ and returns an NBROWNS-by-NBROWNS positive semidefinite correlation matrix.

A Correlation matrix represents a static condition.
As a deterministic function of time, Correlation allows you to specify a dynamic correlation structure.
Data Types: double
Simulation - User-defined simulation function or SDE simulation method simulation by Euler approximation (simByEuler) (default) | function | SDE simulation method

User-defined simulation function or SDE simulation method, specified as a function or SDE simulation method.

Data Types: function_handle

## Drift - Drift rate component of continuous-time stochastic differential equations (SDEs)

value stored from drift-rate function (default) | drift object or function accessible by ( $t, X_{t}$ )
This property is read-only.
Drift rate component of continuous-time stochastic differential equations (SDEs), specified as a drift object or function accessible by $\left(t, X_{t}\right.$.

The drift rate specification supports the simulation of sample paths of NVARS state variables driven by NBROWNS Brownian motion sources of risk over NPERIODS consecutive observation periods, approximating continuous-time stochastic processes.

The drift class allows you to create drift-rate objects using drift of the form:

$$
F\left(t, X_{t}\right)=A(t)+B(t) X_{t}
$$

where:

- A is an NVARS-by-1 vector-valued function accessible using the $\left(t, X_{t}\right)$ interface.
- B is an NVARS-by-NVARS matrix-valued function accessible using the $\left(t, X_{t}\right)$ interface.

The displayed parameters for a drift object are:

- Rate: The drift-rate function, $F\left(t, X_{t}\right)$
- A: The intercept term, $A\left(t, X_{t}\right)$, of $F\left(t, X_{t}\right)$
- B: The first order term, $B\left(t, X_{t}\right)$, of $F\left(t, X_{t}\right)$
$A$ and $B$ enable you to query the original inputs. The function stored in Rate fully encapsulates the combined effect of $A$ and $B$.

When specified as MATLAB double arrays, the inputs $A$ and $B$ are clearly associated with a linear drift rate parametric form. However, specifying either A or B as a function allows you to customize virtually any drift rate specification.

Note You can express drift and diffusion classes in the most general form to emphasize the functional $\left(t, X_{t}\right)$ interface. However, you can specify the components A and B as functions that adhere to the common ( $t, X_{t}$ ) interface, or as MATLAB arrays of appropriate dimension.

## Example: $F=\operatorname{drift}(0,0.1) \%$ Drift rate function $F(t, X)$ <br> Data Types: struct \| double

## Diffusion - Diffusion rate component of continuous-time stochastic differential equations (SDEs)

value stored from diffusion-rate function (default) | diffusion object or functions accessible by $\left(t, X_{t}\right)$

This property is read-only.
Diffusion rate component of continuous-time stochastic differential equations (SDEs), specified as a drift object or function accessible by ( $t, X_{t}$.

The diffusion rate specification supports the simulation of sample paths of NVARS state variables driven by NBROWNS Brownian motion sources of risk over NPERIODS consecutive observation periods, approximating continuous-time stochastic processes.

The diffusion class allows you to create diffusion-rate objects using diffusion:

$$
G\left(t, X_{t}\right)=D\left(t, X_{t}^{\alpha(t)}\right) V(t)
$$

where:

- D is an NVARS-by-NVARS diagonal matrix-valued function.
- Each diagonal element of $D$ is the corresponding element of the state vector raised to the corresponding element of an exponent Alpha, which is an NVARS-by-1 vectorvalued function.
- $V$ is an NVARS-by-NBROWNS matrix-valued volatility rate function Sigma.
- Alpha and Sigma are also accessible using the $\left(t, X_{t}\right)$ interface.

The displayed parameters for a diffusion object are:

- Rate: The diffusion-rate function, $G\left(t, X_{t}\right)$.
- Alpha: The state vector exponent, which determines the format of $D\left(t, X_{t}\right)$ of $G\left(t, X_{t}\right)$.
- Sigma: The volatility rate, $V\left(t, X_{t}\right)$, of $G\left(t, X_{t}\right)$.

Alpha and Sigma enable you to query the original inputs. (The combined effect of the individual Alpha and Sigma parameters is fully encapsulated by the function stored in Rate.) The Rate functions are the calculation engines for the drift and diffusion objects, and are the only parameters required for simulation.

Note You can express drift and diffusion classes in the most general form to emphasize the functional $\left(t, X_{t}\right)$ interface. However, you can specify the components A and B as functions that adhere to the common ( $t, X_{t}$ ) interface, or as MATLAB arrays of appropriate dimension.

Example: $G=$ diffusion(1, 0.3 ) \% Diffusion rate function $G(t, X)$
Data Types: struct | double

## Object Functions

interpolate simulate simByEuler simByTransition

Brownian interpolation of stochastic differential equations
Simulate multivariate stochastic differential equations (SDEs)
Euler simulation of stochastic differential equations (SDEs)
Simulate Cox-Ingersoll-Ross sample paths with transition density

## Examples

## Create a cir Object

The Cox-Ingersoll-Ross (CIR) short rate class derives directly from SDE with mean-
reverting drift (SDEMRD): $d X_{t}=S(t)\left[L(t)-X_{t}\right] d t+D\left(t, X_{t}^{\frac{1}{2}}\right) V(t) d W$
where $D$ is a diagonal matrix whose elements are the square root of the corresponding element of the state vector.

Create a cir object to represent the model: $d X_{t}=0.2\left(0.1-X_{t}\right) d t+0.05 X_{t}^{\frac{1}{2}} d W$.

```
obj = cir(0.2, 0.1, 0.05) % (Speed, Level, Sigma)
```

obj =
Class CIR: Cox-Ingersoll-Ross
Dimensions: State $=1$, Brownian $=1$

```
        StartTime: 0
    StartState: 1
Correlation: 1
    Drift: drift rate function F(t,X(t))
    Diffusion: diffusion rate function G(t,X(t))
Simulation: simulation method/function simByEuler
    Sigma: 0.05
    Level: 0.1
    Speed: 0.2
```

Although the last two objects are of different classes, they represent the same mathematical model. They differ in that you create the cir object by specifying only three input arguments. This distinction is reinforced by the fact that the Alpha parameter does not display - it is defined to be $1 / 2$.

## Definitions

## Instance Hierarchy

There are inheritance relationships among the SDE classes.
The following figure illustrates the inheritance relationships.


For more information, see "SDE Class Hierarchy" on page 18-5.

## Algorithms

When you specify the required input parameters as arrays, they are associated with a specific parametric form. By contrast, when you specify either required input parameter as a function, you can customize virtually any specification.

Accessing the output parameters with no inputs simply returns the original input specification. Thus, when you invoke these parameters with no inputs, they behave like simple properties and allow you to test the data type (double vs. function, or equivalently, static vs. dynamic) of the original input specification. This is useful for validating and designing methods.

When you invoke these parameters with inputs, they behave like functions, giving the impression of dynamic behavior. The parameters accept the observation time $t$ and a state vector $X_{t}$, and return an array of appropriate dimension. Even if you originally specified
an input as an array, cir treats it as a static function of time and state, by that means guaranteeing that all parameters are accessible by the same interface.

## References

[1] Ait-Sahalia, Y. "Testing Continuous-Time Models of the Spot Interest Rate." The Review of Financial Studies, Spring 1996, Vol. 9, No. 2, pp. 385-426.
[2] Ait-Sahalia, Y. "Transition Densities for Interest Rate and Other Nonlinear Diffusions." The Journal of Finance, Vol. 54, No. 4, August 1999.
[3] Glasserman, P. Monte Carlo Methods in Financial Engineering. New York, SpringerVerlag, 2004.
[4] Hull, J. C. Options, Futures, and Other Derivatives, 5th ed. Englewood Cliffs, NJ: Prentice Hall, 2002.
[5] Johnson, N. L., S. Kotz, and N. Balakrishnan. Continuous Univariate Distributions. Vol. 2, 2nd ed. New York, John Wiley \& Sons, 1995.
[6] Shreve, S. E. Stochastic Calculus for Finance II: Continuous-Time Models. New York: Springer-Verlag, 2004.

## See Also

diffusion|drift|interpolate|sdeddo|simByEuler|simByTransition| simulate

## Topics

"Creating Cox-Ingersoll-Ross (CIR) Square Root Diffusion Models" on page 18-30
"Simulating Equity Prices" on page 18-35
"Simulating Interest Rates" on page 18-60
"Stratified Sampling" on page 18-71
"Pricing American Basket Options by Monte Carlo Simulation" on page 18-85
"Base SDE Models" on page 18-17
"Drift and Diffusion Models" on page 18-20
"Linear Drift Models" on page 18-24
"Parametric Models" on page 18-26
"SDEs" on page 18-2
"SDE Models" on page 18-9
"SDE Class Hierarchy" on page 18-5
"Performance Considerations" on page 18-77

## Introduced in R2008a

## diffusion

Diffusion-rate model component

## Description

The diffusion object specifies the diffusion-rate component of continuous-time stochastic differential equations (SDEs).

The diffusion-rate specification supports the simulation of sample paths of NVARS state variables driven by NBROWNS Brownian motion sources of risk over NPERIODS consecutive observation periods, approximating continuous-time stochastic processes.

The diffusion-rate specification can be any NVARS-by-NBROWNS matrix-valued function $G$ of the general form:

$$
\begin{equation*}
G\left(t, X_{t}\right)=D\left(t, X_{t}^{\alpha(t)}\right) V(t) \tag{19-1}
\end{equation*}
$$

where:

- D is an NVARS-by-NVARS diagonal matrix-valued function.
- Each diagonal element of $D$ is the corresponding element of the state vector raised to the corresponding element of an exponent Alpha, which is an NVARS-by-1 vectorvalued function.
- $V$ is an NVARS-by-NBROWNS matrix-valued volatility rate function Sigma.
- Alpha and Sigma are also accessible using the $\left(t, X_{t}\right)$ interface.

And a diffusion-rate specification is associated with a vector-valued SDE of the form:

$$
d X_{t}=F\left(t, X_{t}\right) d t+G\left(t, X_{t}\right) d W_{t}
$$

where:

- $X_{t}$ is an NVARS-by-1 state vector of process variables.
- $d W_{t}$ is an NBROWNS-by-1 Brownian motion vector.
- $D$ is an NVARS-by-NVARS diagonal matrix, in which each element along the main diagonal is the corresponding element of the state vector raised to the corresponding power of $\alpha$.
- $\quad V$ is an NVARS-by-NBROWNS matrix-valued volatility rate function Sigma.

The diffusion-rate specification is flexible, and provides direct parametric support for static volatilities and state vector exponents. It is also extensible, and provides indirect support for dynamic/nonlinear models via an interface. This enables you to specify virtually any diffusion-rate specification.

## Creation

## Syntax

DiffusionRate = diffusion(Alpha,Sigma)

## Description

DiffusionRate = diffusion(Alpha,Sigma) creates default DiffusionRate model component.

Specify required input parameters $A$ and $B$ as one of the following types:

- A MATLAB array. Specifying an array indicates a static (non-time-varying) parametric specification. This array fully captures all implementation details, which are clearly associated with a parametric form.
- A MATLAB function. Specifying a function provides indirect support for virtually any static, dynamic, linear, or nonlinear model. This parameter is supported via an interface, because all implementation details are hidden and fully encapsulated by the function.

Note You can specify combinations of array and function input parameters as needed.
Moreover, a parameter is identified as a deterministic function of time if the function accepts a scalar time $t$ as its only input argument. Otherwise, a parameter is assumed to be a function of time $t$ and state $X(t)$ and is invoked with both input arguments.

The diffusion object that you create encapsulates the composite drift-rate specification and returns the following displayed parameters:

- Rate - The diffusion-rate function, G. Rate is the diffusion-rate calculation engine. It accepts the current time $t$ and an NVARS-by- 1 state vector $X_{t}$ as inputs, and returns an NVARS-by-1 diffusion-rate vector.
- Alpha - Access function for the input argument Alpha.
- Sigma - Access function for the input argument Sigma.


## Input Arguments

## Alpha - Return represents the parameter D <br> array or deterministic function of time

Alpha represents the parameter $D$, specified as an array or deterministic function of time.

If you specify Alpha as an array, it represents an NVARS-by-1 column vector of exponents.
As a deterministic function of time, when Alpha is called with a real-valued scalar time t as its only input, Alpha must produce an NVARS-by-1 matrix.

If you specify it as a function of time and state, Alpha must return an NVARS-by-1 column vector of exponents when invoked with two inputs:

- A real-valued scalar observation time $t$.
- An NVARS-by-1 state vector $X_{t}$.

Data Types: double | function_handle

## Sigma - Sigma represents the parameter V array or deterministic function of time

Sigma represents the parameter $V$, specified as an array or a deterministic function of time.

If you specify Sigma as an array, it must be an NVARS-by-NBROWNS two-dimensional matrix of instantaneous volatility rates. In this case, each row of Sigma corresponds to a particular state variable. Each column corresponds to a particular Brownian source of uncertainty, and associates the magnitude of the exposure of state variables with sources of uncertainty.

As a deterministic function of time, when Sigma is called with a real-valued scalar time $t$ as its only input, Sigma must produce an NVARS-by-NBROWNS matrix. If you specify Sigma as a function of time and state, it must return an NVARS-by-NBROWNS matrix of volatility rates when invoked with two inputs:

- A real-valued scalar observation time $t$.
- An NVARS-by-1 state vector $X_{t}$.

Data Types: double | function_handle

Note Although diffusion enforces no restrictions on the signs of these volatility parameters, each parameter is specified as a positive value.

## Properties

## Rate - Composite diffusion-rate function

value stored from diffusion-rate function (default) | function accessible by $\left(t, X_{t}\right)$
This property is read-only.
Composite diffusion-rate function, specified as: $G\left(t, X_{t}\right)$ ). The function stored in Rate fully encapsulates the combined effect of Alpha and Sigma where:

- Alpha is the state vector exponent, which determines the format of $D\left(t, X_{t}\right)$ of $G\left(t, X_{t}\right)$.
- Sigma is the volatility rate, $V\left(t, X_{t}\right)$, of $G\left(t, X_{t}\right)$.

Data Types: struct | double

## Examples

## Create a diffusion Object

Create a diffusion-rate function G :
G = diffusion(1, 0.3) \% Diffusion rate function G(t,X)
G =
Class DIFFUSION: Diffusion Rate Specification

Rate: diffusion rate function $\mathrm{G}(\mathrm{t}, \mathrm{X}(\mathrm{t}))$
Alpha: 1
Sigma: 0.3
The diffusion object displays like a MATLAB® structure and contains supplemental information, namely, the object's class and a brief description. However, in contrast to the SDE representation, a summary of the dimensionality of the model does not appear, because the diffusion class creates a model component rather than a model. G does not contain enough information to characterize the dimensionality of a problem.

## Definitions

## Instance Hierarchy

There are inheritance relationships among the SDE classes.
The following figure illustrates the inheritance relationships.


For more information, see "SDE Class Hierarchy" on page 18-5.

## Algorithms

When you specify the input arguments Alpha and Sigma as MATLAB arrays, they are associated with a specific parametric form. By contrast, when you specify either Alpha or Sigma as a function, you can customize virtually any diffusion-rate specification.

Accessing the output diffusion-rate parameters Alpha and Sigma with no inputs simply returns the original input specification. Thus, when you invoke diffusion-rate parameters with no inputs, they behave like simple properties and allow you to test the data type (double vs. function, or equivalently, static vs. dynamic) of the original input specification. This is useful for validating and designing methods.

When you invoke diffusion-rate parameters with inputs, they behave like functions, giving the impression of dynamic behavior. The parameters Alpha and Sigma accept the observation time $t$ and a state vector $X_{t}$, and return an array of appropriate dimension.

Specifically, parameters Alpha and Sigma evaluate the corresponding diffusion-rate component. Even if you originally specified an input as an array, diffusion treats it as a static function of time and state, by that means guaranteeing that all parameters are accessible by the same interface.

## References

[1] Ait-Sahalia, Y. "Testing Continuous-Time Models of the Spot Interest Rate." The Review of Financial Studies, Spring 1996, Vol. 9, No. 2, pp. 385-426.
[2] Ait-Sahalia, Y. "Transition Densities for Interest Rate and Other Nonlinear Diffusions." The Journal of Finance, Vol. 54, No. 4, August 1999.
[3] Glasserman, P. Monte Carlo Methods in Financial Engineering. New York, SpringerVerlag, 2004.
[4] Hull, J. C. Options, Futures, and Other Derivatives, 5th ed. Englewood Cliffs, NJ: Prentice Hall, 2002.
[5] Johnson, N. L., S. Kotz, and N. Balakrishnan. Continuous Univariate Distributions. Vol. 2, 2nd ed. New York, John Wiley \& Sons, 1995.
[6] Shreve, S. E. Stochastic Calculus for Finance II: Continuous-Time Models. New York: Springer-Verlag, 2004.

## See Also

drift|sdeddo

## Topics

"Simulating Equity Prices" on page 18-35
"Simulating Interest Rates" on page 18-60
"Stratified Sampling" on page 18-71
"Pricing American Basket Options by Monte Carlo Simulation" on page 18-85
"Base SDE Models" on page 18-17
"Drift and Diffusion Models" on page 18-20
"Linear Drift Models" on page 18-24
"Parametric Models" on page 18-26
"SDEs" on page 18-2
"SDE Models" on page 18-9
"SDE Class Hierarchy" on page 18-5
"Performance Considerations" on page 18-77

## Introduced in R2008a

## drift

Drift-rate model component

## Description

The drift object specifies the drift-rate component of continuous-time stochastic differential equations (SDEs).

The drift-rate specification supports the simulation of sample paths of NVARS state variables driven by NBROWNS Brownian motion sources of risk over NPERIODS consecutive observation periods, approximating continuous-time stochastic processes.

The drift-rate specification can be any NVARS-by-1 vector-valued function $F$ of the general form:

$$
F\left(t, X_{t}\right)=A(t)+B(t) X_{t}
$$

where:

- A is an NVARS-by-1 vector-valued function accessible using the $\left(t, X_{t}\right)$ interface.
- B is an NVARS-by-NVARS matrix-valued function accessible using the $\left(t, X_{t}\right)$ interface.

And a drift-rate specification is associated with a vector-valued SDE of the form

$$
d X_{t}=F\left(t, X_{t}\right) d t+G\left(t, X_{t}\right) d W_{t}
$$

where:

- $X_{t}$ is an NVARS-by-1 state vector of process variables.
- $d W_{t}$ is an NBROWNS-by-1 Brownian motion vector.
- $A$ and $B$ are model parameters.

The drift-rate specification is flexible, and provides direct parametric support for static/ linear drift models. It is also extensible, and provides indirect support for dynamic/ nonlinear models via an interface. This enables you to specify virtually any drift-rate specification.

## Creation

## Syntax

DriftRate = drift(A,B)

## Description

DriftRate $=\operatorname{drift}(A, B)$ creates a default DriftRate model component.
Specify required input parameters $A$ and $B$ as one of the following types:

- A MATLAB array. Specifying an array indicates a static (non-time-varying) parametric specification. This array fully captures all implementation details, which are clearly associated with a parametric form.
- A MATLAB function. Specifying a function provides indirect support for virtually any static, dynamic, linear, or nonlinear model. This parameter is supported via an interface, because all implementation details are hidden and fully encapsulated by the function.

Note You can specify combinations of array and function input parameters as needed.
Moreover, a parameter is identified as a deterministic function of time if the function accepts a scalar time $t$ as its only input argument. Otherwise, a parameter is assumed to be a function of time $t$ and state $X(t)$ and is invoked with both input arguments.

The drift object that you create encapsulates the composite drift-rate specification and returns the following displayed parameters:

- Rate - The drift-rate function, $F$. Rate is the drift-rate calculation engine. It accepts the current time $t$ and an NVARS-by-1 state vector $X_{t}$ as inputs, and returns an NVARS-by-1 drift-rate vector.
- A - Access function for the input argument A.
- $B-$ Access function for the input argument $B$.


## Input Arguments

## A - A represents the parameter A

array or deterministic function of time
A represents the parameter $A$, specified as an array or deterministic function of time.
If you specify A as an array, it must be an NVARS-by-1 column vector of intercepts.
As a deterministic function of time, when $A$ is called with a real-valued scalar time $t$ as its only input, A must produce an NVARS-by-1 column vector. If you specify A as a function of time and state, it must generate an NVARS-by-1 column vector of intercepts when invoked with two inputs:

- A real-valued scalar observation time $t$.
- An NVARS-by-1 state vector $X_{t}$.

Data Types: double | function_handle

## B - B represents the parameter B <br> array or deterministic function of time

$B$ represents the parameter $B$, specified as an array or deterministic function of time.
If you specify B as an array, it must be an NVARS-by-NVARS two-dimensional matrix of state vector coefficients.

As a deterministic function of time, when $B$ is called with a real-valued scalar time $t$ as its only input, B must produce an NVARS-by-NVARS matrix. If you specify B as a function of time and state, it must generate an NVARS-by-NVARS matrix of state vector coefficients when invoked with two inputs:

- A real-valued scalar observation time $t$.
- An NVARS-by-1 state vector $X_{t}$.

Data Types: double | function_handle

## Properties

## Rate - Composite drift-rate function

value stored from drift-rate function (default) | function accessible by $F\left(t, X_{t}\right)$

## This property is read-only.

Composite drift-rate function, specified as $F\left(t, X_{t}\right)$. The function stored in Rate fully encapsulates the combined effect of $A$ and $B$, where $A$ and $B$ are:

- A: The intercept term, $A\left(t, X_{t}\right)$, of $F\left(t, X_{t}\right)$
- B: The first order term, $B\left(t, X_{t}\right)$, of $F\left(t, X_{t}\right)$

Data Types: struct | double

## Examples

## Create a drift Object

Create a drift-rate function F :

```
F = drift(0, 0.1) % Drift rate function F(t,X)
```

$\mathrm{F}=$
Class DRIFT: Drift Rate Specification
Rate: drift rate function $F(t, X(t))$
A: 0
B: 0.1
The drift object displays like a MATLAB® structure and contains supplemental information, namely, the object's class and a brief description. However, in contrast to the SDE representation, a summary of the dimensionality of the model does not appear, because the drift class creates a model component rather than a model. F does not contain enough information to characterize the dimensionality of a problem.

## Definitions

## Instance Hierarchy

There are inheritance relationships among the SDE classes.
The following figure illustrates the inheritance relationships.


For more information, see "SDE Class Hierarchy" on page 18-5.

## Algorithms

When you specify the input arguments $A$ and $B$ as MATLAB arrays, they are associated with a linear drift parametric form. By contrast, when you specify either A or B as a function, you can customize virtually any drift-rate specification.

Accessing the output drift-rate parameters A and B with no inputs simply returns the original input specification. Thus, when you invoke drift-rate parameters with no inputs, they behave like simple properties and allow you to test the data type (double vs. function, or equivalently, static vs. dynamic) of the original input specification. This is useful for validating and designing methods.

When you invoke drift-rate parameters with inputs, they behave like functions, giving the impression of dynamic behavior. The parameters A and B accept the observation time $t$ and a state vector $X_{t}$, and return an array of appropriate dimension. Specifically,
parameters $A$ and $B$ evaluate the corresponding drift-rate component. Even if you originally specified an input as an array, drift treats it as a static function of time and state, by that means guaranteeing that all parameters are accessible by the same interface.

## References

[1] Ait-Sahalia, Y. "Testing Continuous-Time Models of the Spot Interest Rate." The Review of Financial Studies, Spring 1996, Vol. 9, No. 2, pp. 385-426.
[2] Ait-Sahalia, Y. "Transition Densities for Interest Rate and Other Nonlinear Diffusions." The Journal of Finance, Vol. 54, No. 4, August 1999.
[3] Glasserman, P. Monte Carlo Methods in Financial Engineering. New York, SpringerVerlag, 2004.
[4] Hull, J. C. Options, Futures, and Other Derivatives, 5th ed. Englewood Cliffs, NJ: Prentice Hall, 2002.
[5] Johnson, N. L., S. Kotz, and N. Balakrishnan. Continuous Univariate Distributions. Vol. 2, 2nd ed. New York, John Wiley \& Sons, 1995.
[6] Shreve, S. E. Stochastic Calculus for Finance II: Continuous-Time Models. New York: Springer-Verlag, 2004.

## See Also

diffusion|sdeddo

## Topics

"Simulating Equity Prices" on page 18-35
"Simulating Interest Rates" on page 18-60
"Stratified Sampling" on page 18-71
"Pricing American Basket Options by Monte Carlo Simulation" on page 18-85
"Base SDE Models" on page 18-17
"Drift and Diffusion Models" on page 18-20
"Linear Drift Models" on page 18-24
"Parametric Models" on page 18-26
"SDEs" on page 18-2
"SDE Models" on page 18-9
"SDE Class Hierarchy" on page 18-5
"Performance Considerations" on page 18-77
Introduced in R2008a

## Financial Time Series

Create and manage Financial Time Series

Note The Financial Time Series app will be removed in a future release and will no longer accept a fints object.

Use fts2timetable to convert a fints object to a timetable object. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Description

The Financial Time Series app enables you to create and manage Financial Time Series (fints) objects.

## Open the Financial Time Series App

- MATLAB Toolstrip: On the Apps tab, under Financial Time Series, click the app icon.
- MATLAB command prompt: Enter ftstool.


## Examples

- "Loading Data with the Financial Time Series App" on page 14-7
- "Using the Financial Time Series App" on page 14-11
- "Using the Financial Time Series App with GUIs" on page 14-19
- "Getting Started with the Financial Time Series App" on page 14-4


## Programmatic Use

ftstool

## See Also

## Functions

ftsgui

## Topics

"Loading Data with the Financial Time Series App" on page 14-7
"Using the Financial Time Series App" on page 14-11
"Using the Financial Time Series App with GUIs" on page 14-19
"Getting Started with the Financial Time Series App" on page 14-4
"What Is the Financial Time Series App?" on page 14-2

## Introduced in R2006b

## gbm

Geometric Brownian motion model

## Description

Creates and displays geometric Brownian motion (GBM) models, which derive from the cev (constant elasticity of variance) class.

Geometric Brownian motion (GBM) models allow you to simulate sample paths of NVARS state variables driven by NBROWNS Brownian motion sources of risk over NPERIODS consecutive observation periods, approximating continuous-time GBM stochastic processes. Specifically, this model allows the simulation of vector-valued GBM processes of the form

$$
d X_{t}=\mu(t) X_{t} d t+D\left(t, X_{t}\right) V(t) d W_{t}
$$

where:

- $X_{t}$ is an NVARS-by-1 state vector of process variables.
- $\mu$ is an NVARS-by-NVARS generalized expected instantaneous rate of return matrix.
- $D$ is an NVARS-by-NVARS diagonal matrix, where each element along the main diagonal is the corresponding element of the state vector $X_{t}$.
- $V$ is an NVARS-by-NBROWNS instantaneous volatility rate matrix.
- $d W_{t}$ is an NBROWNS-by-1 Brownian motion vector.


## Creation

## Syntax

```
GBM = gbm(Return,Sigma)
GBM = gbm( ___ ,Name,Value)
```


## Description

GBM $=$ gbm(Return, Sigma) creates a default GBM object.
Specify required input parameters as one of the following types:

- A MATLAB array. Specifying an array indicates a static (non-time-varying) parametric specification. This array fully captures all implementation details, which are clearly associated with a parametric form.
- A MATLAB function. Specifying a function provides indirect support for virtually any static, dynamic, linear, or nonlinear model. This parameter is supported via an interface, because all implementation details are hidden and fully encapsulated by the function.

Note You can specify combinations of array and function input parameters as needed.
Moreover, a parameter is identified as a deterministic function of time if the function accepts a scalar time $t$ as its only input argument. Otherwise, a parameter is assumed to be a function of time $t$ and state $X(t)$ and is invoked with both input arguments.

GBM $=$ gbm ( $\qquad$ , Name, Value) creates a GBM object with additional options specified by one or more Name, Value pair arguments.

Name is a property name and Value is its corresponding value. Name must appear inside single quotes (' ' ). You can specify several name-value pair arguments in any order as Name1, Value1, ..., NameN , ValueN

The GBM object has the following "Properties" on page 19-56:

- StartTime - Initial observation time
- StartState - Initial state at StartTime
- Correlation - Access function for the Correlation input, callable as a function of time
- Drift - Composite drift-rate function, callable as a function of time and state
- Diffusion - Composite diffusion-rate function, callable as a function of time and state
- Simulation - A simulation function or method
- Return - Access function for the input argument Return, callable as a function of time and state
- Sigma - Access function for the input argument Sigma, callable as a function of time and state


## Input Arguments

Return - Return represents the parameter $\boldsymbol{\mu}$ array or deterministic function of time or deterministic function of time and state

Return represents the parameter $\mu$, specified as an array or deterministic function of time.

If you specify Return as an array, it must be an NVARS-by-NVARS matrix representing the expected (mean) instantaneous rate of return.

As a deterministic function of time, when Return is called with a real-valued scalar time $t$ as its only input, Return must produce an NVARS-by-NVARS matrix. If you specify Return as a function of time and state, it must return an NVARS-by-NVARS matrix when invoked with two inputs:

- A real-valued scalar observation time $t$.
- An NVARS-by-1 state vector $X_{t}$.

Data Types: double | function_handle

## Sigma - Sigma represents the parameter V array or deterministic function of time or deterministic function of time and state

Sigma represents the parameter $V$, specified as an array or a deterministic function of time.

If you specify Sigma as an array, it must be an NVARS-by-NBROWNS matrix of instantaneous volatility rates or as a deterministic function of time. In this case, each row of Sigma corresponds to a particular state variable. Each column corresponds to a particular Brownian source of uncertainty, and associates the magnitude of the exposure of state variables with sources of uncertainty.

As a deterministic function of time, when Sigma is called with a real-valued scalar time $t$ as its only input, Sigma must produce an NVARS-by-NBROWNS matrix. If you specify Sigma
as a function of time and state, it must return an NVARS-by-NBROWNS matrix of volatility rates when invoked with two inputs:

- A real-valued scalar observation time $t$.
- An NVARS-by-1 state vector $X_{t}$.

Although gbm enforces no restrictions on the sign of Sigma volatilities, they are specified as positive values.

```
Data Types: double| function_handle
```


## Properties

## StartTime - Starting time of first observation, applied to all state variables 0 (default) | scalar

Starting time of first observation, applied to all state variables, specified as a scalar Data Types: double

StartState - Initial values of state variables<br>1 (default) | scalar, column vector, or matrix

Initial values of state variables, specified as a scalar, column vector, or matrix.
If StartState is a scalar, gbm applies the same initial value to all state variables on all trials.

If StartState is a column vector, gbm applies a unique initial value to each state variable on all trials.

If StartState is a matrix, gbm applies a unique initial value to each state variable on each trial.

## Data Types: double

## Correlation - Correlation between Gaussian random variates drawn to generate the Brownian motion vector (Wiener processes) <br> NBROWNS-by-NBROWNS identity matrix representing independent Gaussian processes (default) | positive semidefinite matrix | deterministic function

Correlation between Gaussian random variates drawn to generate the Brownian motion vector (Wiener processes), specified as an NBROWNS-by-NBROWNS positive semidefinite
matrix, or as a deterministic function $C(t)$ that accepts the current time $t$ and returns an NBROWNS-by-NBROWNS positive semidefinite correlation matrix.

A Correlation matrix represents a static condition.
As a deterministic function of time, Correlation allows you to specify a dynamic correlation structure.

Data Types: double

## Simulation - User-defined simulation function or SDE simulation method simulation by Euler approximation (simByEuler) (default)|function | SDE simulation method

User-defined simulation function or SDE simulation method, specified as a function or SDE simulation method.

Data Types: function_handle

## Drift - Drift rate component of continuous-time stochastic differential equations (SDEs)

value stored from drift-rate function (default) | drift object or function accessible by $\left(t, X_{t}\right)$
This property is read-only.
Drift rate component of continuous-time stochastic differential equations (SDEs), specified as a drift object or function accessible by ( $t, X_{t}$.

The drift rate specification supports the simulation of sample paths of NVARS state variables driven by NBROWNS Brownian motion sources of risk over NPERIODS consecutive observation periods, approximating continuous-time stochastic processes.

The drift class allows you to create drift-rate objects (using drift) of the form:

$$
F\left(t, X_{t}\right)=A(t)+B(t) X_{t}
$$

where:

- A is an NVARS-by-1 vector-valued function accessible using the $\left(t, X_{t}\right)$ interface.
- B is an NVARS-by-NVARS matrix-valued function accessible using the $\left(t, X_{t}\right)$ interface.

The displayed parameters for a drift object are:

- Rate: The drift-rate function, $F\left(t, X_{t}\right)$
- A: The intercept term, $A\left(t, X_{t}\right)$, of $F\left(t, X_{t}\right)$
- B: The first order term, $B\left(t, X_{t}\right)$, of $F\left(t, X_{t}\right)$
$A$ and $B$ enable you to query the original inputs. The function stored in Rate fully encapsulates the combined effect of $A$ and $B$.

When specified as MATLAB double arrays, the inputs A and B are clearly associated with a linear drift rate parametric form. However, specifying either A or B as a function allows you to customize virtually any drift rate specification.

Note You can express drift and diffusion classes in the most general form to emphasize the functional $\left(t, X_{t}\right)$ interface. However, you can specify the components A and B as functions that adhere to the common $\left(t, X_{t}\right)$ interface, or as MATLAB arrays of appropriate dimension.

Example: $F=\operatorname{drift}(0,0.1)$ \% Drift rate function $F(t, X)$
Data Types: struct \| double
Diffusion - Diffusion rate component of continuous-time stochastic differential
equations (SDEs)
value stored from diffusion-rate function (default) | diffusion object or functions accessible by $\left(t, X_{t}\right)$

Diffusion rate component of continuous-time stochastic differential equations (SDEs), specified as a drift object or function accessible by ( $t, X_{t}$.

The diffusion rate specification supports the simulation of sample paths of NVARS state variables driven by NBROWNS Brownian motion sources of risk over NPERIODS consecutive observation periods, approximating continuous-time stochastic processes.

The diffusion class allows you to create diffusion-rate objects (using diffusion):

$$
G\left(t, X_{t}\right)=D\left(t, X_{t}^{\alpha(t)}\right) V(t)
$$

where:

- D is an NVARS-by-NVARS diagonal matrix-valued function.
- Each diagonal element of $D$ is the corresponding element of the state vector raised to the corresponding element of an exponent Alpha, which is an NVARS-by-1 vectorvalued function.
- $V$ is an NVARS-by-NBROWNS matrix-valued volatility rate function Sigma.
- Alpha and Sigma are also accessible using the $\left(t, X_{t}\right)$ interface.

The diffusion object's displayed parameters are:

- Rate: The diffusion-rate function, $G\left(t, X_{t}\right)$.
- Alpha: The state vector exponent, which determines the format of $D\left(t, X_{t}\right)$ of $G\left(t, X_{t}\right)$.
- Sigma: The volatility rate, $V\left(t, X_{t}\right)$, of $G\left(t, X_{t}\right)$.

Alpha and Sigma enable you to query the original inputs. (The combined effect of the individual Alpha and Sigma parameters is fully encapsulated by the function stored in Rate.) The Rate functions are the calculation engines for the drift and diffusion objects, and are the only parameters required for simulation.

Note You can express drift and diffusion classes in the most general form to emphasize the functional $\left(t, X_{t}\right)$ interface. However, you can specify the components A and B as functions that adhere to the common $\left(t, X_{t}\right)$ interface, or as MATLAB arrays of appropriate dimension.

Example: $G=$ diffusion(1, 0.3) \% Diffusion rate function $G(t, X)$ Data Types: struct | double

## Object Functions

interpolate Brownian interpolation of stochastic differential equations simulate $\quad$ Simulate multivariate stochastic differential equations (SDEs)
simByEuler Euler simulation of stochastic differential equations (SDEs)
simBySolution Simulate approximate solution of diagonal-drift GBM processes

## Examples

## Create a gbm Object

Create a univariate gbm object to represent the model: $d X_{t}=0.25 X_{t} d t+0.3 X_{t} d W_{t}$. obj $=\operatorname{gbm}(0.25,0.3) \%(B=$ Return, Sigma)
obj =

```
Class GBM: Generalized Geometric Brownian Motion
```

Dimensions: State $=1$, Brownian $=1$
StartTime: 0
StartState: 1
Correlation: 1
Drift: drift rate function $\mathrm{F}(\mathrm{t}, \mathrm{X}(\mathrm{t}))$
Diffusion: diffusion rate function $\mathrm{G}(\mathrm{t}, \mathrm{X}(\mathrm{t}))$
Simulation: simulation method/function simByEuler
Return: 0.25
Sigma: 0.3
gbm objects display the parameter $B$ as the more familiar Return

## Definitions

## Instance Hierarchy

There are inheritance relationships among the SDE classes.
The following figure illustrates the inheritance relationships.


For more information, see "SDE Class Hierarchy" on page 18-5.

## Algorithms

When you specify the required input parameters as arrays, they are associated with a specific parametric form. By contrast, when you specify either required input parameter as a function, you can customize virtually any specification.

Accessing the output parameters with no inputs simply returns the original input specification. Thus, when you invoke these parameters with no inputs, they behave like simple properties and allow you to test the data type (double vs. function, or equivalently, static vs. dynamic) of the original input specification. This is useful for validating and designing methods.

When you invoke these parameters with inputs, they behave like functions, giving the impression of dynamic behavior. The parameters accept the observation time $t$ and a state vector $X_{t}$, and return an array of appropriate dimension. Even if you originally specified
an input as an array, gbm treats it as a static function of time and state, by that means guaranteeing that all parameters are accessible by the same interface.

## References

[1] Ait-Sahalia, Y. "Testing Continuous-Time Models of the Spot Interest Rate." The Review of Financial Studies, Spring 1996, Vol. 9, No. 2, pp. 385-426.
[2] Ait-Sahalia, Y. "Transition Densities for Interest Rate and Other Nonlinear Diffusions." The Journal of Finance, Vol. 54, No. 4, August 1999.
[3] Glasserman, P. Monte Carlo Methods in Financial Engineering. New York, SpringerVerlag, 2004.
[4] Hull, J. C. Options, Futures, and Other Derivatives, 5th ed. Englewood Cliffs, NJ: Prentice Hall, 2002.
[5] Johnson, N. L., S. Kotz, and N. Balakrishnan. Continuous Univariate Distributions. Vol. 2, 2nd ed. New York, John Wiley \& Sons, 1995.
[6] Shreve, S. E. Stochastic Calculus for Finance II: Continuous-Time Models. New York: Springer-Verlag, 2004.

## See Also

bm | cev | diffusion | drift | interpolate | simByEuler| simulate

## Topics

"Creating Geometric Brownian Motion (GBM) Models" on page 18-28
"Representing Market Models Using SDELD, CEV, and GBM Objects" on page 18-38
"Simulating Equity Prices" on page 18-35
"Simulating Interest Rates" on page 18-60
"Stratified Sampling" on page 18-71
"Pricing American Basket Options by Monte Carlo Simulation" on page 18-85
"Base SDE Models" on page 18-17
"Drift and Diffusion Models" on page 18-20
"Linear Drift Models" on page 18-24
"Parametric Models" on page 18-26
"SDEs" on page 18-2
"SDE Models" on page 18-9
"SDE Class Hierarchy" on page 18-5
"Performance Considerations" on page 18-77

## Introduced in R2008a

## simBySolution

Simulate approximate solution of diagonal-drift GBM processes

## Syntax

```
[Paths,Times,Z] = simBySolution(MDL,NPeriods)
[Paths,Times,Z] = simBySolution( ___ ,Name,Value)
```


## Description

[Paths,Times,Z] = simBySolution(MDL,NPeriods) simulates approximate solution of diagonal-drift for geometric Brownian motion (GBM) processes.
[Paths,Times,Z] = simBySolution( $\qquad$ ,Name, Value) adds optional name-value pair arguments.

## Examples

## Simulating Equity Markets Using GBM Simulation Functions

Use GBM simulation functions. Separable GBM models have two specific simulation functions:

- An overloaded Euler simulation function (simulate), designed for optimal performance.
- A simBySolution function that provides an approximate solution of the underlying stochastic differential equation, designed for accuracy.

Load the Data_GlobalIdx2 data set and specify the SDE model as in "Representing Market Models Using SDE Objects" on page 18-35, and the GBM model as in "Representing Market Models Using SDELD, CEV, and GBM Objects" on page 18-38.
load Data GlobalIdx2
prices =-[Dataset.TSX Dataset.CAC Dataset.DAX ...

```
    Dataset.NIK Dataset.FTSE Dataset.SP];
returns = tick2ret(prices);
nVariables = size(returns,2);
expReturn = mean(returns);
sigma = std(returns);
correlation = corrcoef(returns);
t = 0;
X = 100;
X = X(ones(nVariables,1));
F = @(t,X) diag(expReturn)* X;
G = @(t,X) diag(X) * diag(sigma);
SDE = sde(F, G, 'Correlation', ...
    correlation, 'StartState', X);
GBM = gbm(diag(expReturn),diag(sigma), 'Correlation', ...
    correlation, 'StartState', X);
```

To illustrate the performance benefit of the overloaded Euler approximation function (simulate), increase the number of trials to 10000.

```
nPeriods = 249; % # of simulated observations
dt = 1; % time increment = 1 day
rng(142857,'twister')
[X,T] = simulate(GBM, nPeriods, 'DeltaTime', dt, ...
    'nTrials', 10000);
whos X
    Name Size Bytes Class Attributes
    X 250x6x10000 120000000 double
```

Using this sample size, examine the terminal distribution of Canada's TSX Composite to verify qualitatively the lognormal character of the data.

```
histogram(squeeze(X(end,1,:)), 30), xlabel('Price'), ylabel('Frequency')
title('Histogram of Prices after One Year: Canada (TSX Composite)')
```



Simulate 10 trials of the solution and plot the first trial:

```
rng('default')
[S,T] = simulate(SDE, nPeriods, 'DeltaTime', dt, 'nTrials', 10);
rng('default')
[X,T] = simBySolution(GBM, nPeriods,...
    'DeltaTime', dt, 'nTrials', 10);
subplot(2,1,1)
plot(T, S(:,:,1)), xlabel('Trading Day'),ylabel('Price')
title('1st Path of Multi-Dim Market Model:Euler Approximation')
subplot(2,1,2)
plot(T, X(:,:,1)), xlabel('Trading Day'),ylabel('Price')
title('1st Path of Multi-Dim Market Model:Analytic Solution')
```



In this example, all parameters are constants, and simBySolution does indeed sample the exact solution. The details of a single index for any given trial show that the price paths of the Euler approximation and the exact solution are close, but not identical.

The following plot illustrates the difference between the two functions:

```
subplot(1,1,1)
plot(T, S(:,1,1) - X(:,1,1), 'blue'), grid('on')
xlabel('Trading Day'), ylabel('Price Difference')
title('Euler Approx Minus Exact Solution:Canada(TSX Composite)')
```



The simByEuler Euler approximation literally evaluates the stochastic differential equation directly from the equation of motion, for some suitable value of the dt time increment. This simple approximation suffers from discretization error. This error can be attributed to the discrepancy between the choice of the $d t$ time increment and what in theory is a continuous-time parameter.

The discrete-time approximation improves as DeltaTime approaches zero. The Euler function is often the least accurate and most general method available. All models shipped in the simulation suite have the simByEuler function.

In contrast, the simBySolution function provides a more accurate description of the underlying model. This function simulates the price paths by an approximation of the closed-form solution of separable models. Specifically, it applies a Euler approach to a
transformed process, which in general is not the exact solution to this GBM model. This is because the probability distributions of the simulated and true state vectors are identical only for piecewise constant parameters.

When all model parameters are piecewise constant over each observation period, the simulated process is exact for the observation times at which the state vector is sampled. Since all parameters are constants in this example, simBySolution does indeed sample the exact solution.

For an example of how to use simBySolution to optimize the accuracy of solutions, see "Optimizing Accuracy: About Solution Precision and Error" on page 18-79.

## Input Arguments

## MDL - Geometric Brownian motion (GBM) model

gbm object
Geometric Brownian motion (GBM) model, specified as a gbm object that is created using gbm.

Data Types: object

## NPeriods - Number of simulation periods

positive integer
Number of simulation periods, specified as a positive scalar integer. The value of NPeriods determines the number of rows of the simulated output series.

## Data Types: double

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1,Value1, ... , NameN, ValueN.

Example: [Paths,Times,Z] = simBySolution(GBM,NPeriods,'DeltaTime',dt,'NTrials',10)

## NTrials - Simulated trials (sample paths) of NPERIODS observations each 1 (single path of correlated state variables) (default) | positive integer

Simulated trials (sample paths) of NPERIODS observations each, specified as the commaseparated pair consisting of 'NTrials' and a positive scalar integer.
Data Types: double
DeltaTimes - Positive time increments between observations
1 (default) | scalar | column vector
Positive time increments between observations, specified as the comma-separated pair consisting of 'DeltaTimes ' and a scalar or a NPERIODS-by-1 column vector.

DeltaTime represents the familiar $d t$ found in stochastic differential equations, and determines the times at which the simulated paths of the output state variables are reported.

Data Types: double

## NSteps - Number of intermediate time steps within each time increment dt (specified as DeltaTime)

1 (indicating no intermediate evaluation) (default) | positive integer
Number of intermediate time steps within each time increment $d t$ (specified as DeltaTime), specified as the comma-separated pair consisting of 'NSteps ' and a positive scalar integer.

The simBySolution function partitions each time increment $d t$ into NSteps subintervals of length $d t /$ NSteps, and refines the simulation by evaluating the simulated state vector at NSteps - 1 intermediate points. Although simBySolution does not report the output state vector at these intermediate points, the refinement improves accuracy by allowing the simulation to more closely approximate the underlying continuous-time process.
Data Types: double
Antithetic - Flag to indicate whether simBySolution uses antithetic sampling to generate the Gaussian random variates
False (no antithetic sampling) (default) | logical with values True or False
Flag to indicate whether simBySolution uses antithetic sampling to generate the Gaussian random variates that drive the Brownian motion vector (Wiener processes),
specified as the comma-separated pair consisting of 'Antithetic' and a scalar logical flag with a value of True or False.

When you specify True, simBySolution performs sampling such that all primary and antithetic paths are simulated and stored in successive matching pairs:

- Odd trials $(1,3,5, \ldots)$ correspond to the primary Gaussian paths.
- Even trials $(2,4,6, \ldots)$ are the matching antithetic paths of each pair derived by negating the Gaussian draws of the corresponding primary (odd) trial.

Note If you specify an input noise process (see Z), simBySolution ignores the value of Antithetic.

Data Types: logical

## Z - Direct specification of the dependent random noise process used to generate the Brownian motion vector

generates correlated Gaussian variates based on the Correlation member of the SDE object (default) | function | three-dimensional array of dependent random variates

Direct specification of the dependent random noise process used to generate the Brownian motion vector (Wiener process) that drives the simulation, specified as the comma-separated pair consisting of ' $Z$ ' and a function or as an (NPERIODS * NSTEPS)-by-NBROWNS-by-NTRIALS three-dimensional array of dependent random variates.

The input argument $Z$ allows you to directly specify the noise generation process. This process takes precedence over the Correlation parameter of the input gbm object and the value of the Antithetic input flag.

Note If you specify $Z$ as a function, it must return an NBROWNS-by-1 column vector, and you must call it with two inputs:

- A real-valued scalar observation time $t$.
- An NVARS-by-1 state vector $X_{t}$.

[^2]
## StorePaths - Flag that indicates how the output array Paths is stored and returned

True (default) | logical with values True or False
Flag that indicates how the output array Paths is stored and returned, specified as the comma-separated pair consisting of 'StorePaths' and a scalar logical flag with a value of True or False.

If StorePaths is True (the default value) or is unspecified, simBySolution returns Paths as a three-dimensional time series array.

If StorePaths is False (logical 0), simBySolution returns the Paths output array as an empty matrix.
Data Types: logical

## Processes - Sequence of end-of-period processes or state vector adjustments of the form

simBySolution makes no adjustments and performs no processing (default) | function | cell array of functions

Sequence of end-of-period processes or state vector adjustments of the form, specified as the comma-separated pair consisting of 'Processes ' and a function or cell array of functions of the form

$$
X_{t}=P\left(t, X_{t}\right)
$$

The simBySolution function runs processing functions at each interpolation time. They must accept the current interpolation time $t$, and the current state vector $X_{t}$, and return a state vector that may be an adjustment to the input state.
simBySolution applies processing functions at the end of each observation period. These functions must accept the current observation time $t$ and the current state vector $X_{t}$, and return a state vector that may be an adjustment to the input state.

The end-of-period Processes argument allows you to terminate a given trial early. At the end of each time step, simBySolution tests the state vector $X_{t}$ for an all-NaN condition. Thus, to signal an early termination of a given trial, all elements of the state vector $X_{t}$ must be NaN. This test enables a user-defined Processes function to signal early termination of a trial, and offers significant performance benefits in some situations (for example, pricing down-and-out barrier options).

If you specify more than one processing function, simBySolution invokes the functions in the order in which they appear in the cell array. You can use this argument to specify boundary conditions, prevent negative prices, accumulate statistics, plot graphs, and more.

Data Types: cell | function

## Output Arguments

## Paths - Simulated paths of correlated state variables <br> array

Simulated paths of correlated state variables, returned as a (NPERIODS + 1)-by-NVARS-by-NTRIALS three-dimensional time series array.

For a given trial, each row of Paths is the transpose of the state vector $X_{t}$ at time $t$. When the input flag StorePaths = False, simBySolution returns Paths as an empty matrix.

## Times - Observation times associated with the simulated paths column vector

Observation times associated with the simulated paths, returned as a (NPERIODS + 1)-by-1 column vector. Each element of Times is associated with the corresponding row of Paths.

## Z - Dependent random variates used to generate the Brownian motion vector array

Dependent random variates used to generate the Brownian motion vector (Wiener processes) that drive the simulation, returned as a (NPERIODS * NSTEPS) -by-NBROWNS-by-NTRIALS three-dimensional time series array.

## Definitions

## Antithetic Sampling

Simulation methods allow you to specify a popular variance reduction technique called antithetic sampling.

This technique attempts to replace one sequence of random observations with another of the same expected value, but smaller variance. In a typical Monte Carlo simulation, each sample path is independent and represents an independent trial. However, antithetic sampling generates sample paths in pairs. The first path of the pair is referred to as the primary path, and the second as the antithetic path. Any given pair is independent of any other pair, but the two paths within each pair are highly correlated. Antithetic sampling literature often recommends averaging the discounted payoffs of each pair, effectively halving the number of Monte Carlo trials.

This technique attempts to reduce variance by inducing negative dependence between paired input samples, ideally resulting in negative dependence between paired output samples. The greater the extent of negative dependence, the more effective antithetic sampling is.

## Algorithms

The simBySolution function simulates NTRIALS sample paths of NVARS correlated state variables, driven by NBROWNS Brownian motion sources of risk over NPERIODS consecutive observation periods, approximating continuous-time GBM short-rate models by an approximation of the closed-form solution.

Consider a separable, vector-valued GBM model of the form:

$$
d X_{t}=\mu(t) X_{t} d t+D\left(t, X_{t}\right) V(t) d W_{t}
$$

where:

- $X_{t}$ is an NVARS-by- 1 state vector of process variables.
- $\mu$ is an NVARS-by-NVARS generalized expected instantaneous rate of return matrix.
- $V$ is an NVARS-by-NBROWNS instantaneous volatility rate matrix.
- $d W_{t}$ is an NBROWNS-by-1 Brownian motion vector.

The simBySolution function simulates the state vector $X_{t}$ using an approximation of the closed-form solution of diagonal-drift models.

When evaluating the expressions, simBySolution assumes that all model parameters are piecewise-constant over each simulation period.

In general, this is not the exact solution to the models, because the probability distributions of the simulated and true state vectors are identical only for piecewiseconstant parameters.

When parameters are piecewise-constant over each observation period, the simulated process is exact for the observation times at which $X_{t}$ is sampled.

Gaussian diffusion models, such as hwv, allow negative states. By default, simBySolution does nothing to prevent negative states, nor does it guarantee that the model be strictly mean-reverting. Thus, the model may exhibit erratic or explosive growth.

## References

[1] Ait-Sahalia, Y. "Testing Continuous-Time Models of the Spot Interest Rate." The Review of Financial Studies, Spring 1996, Vol. 9, No. 2, pp. 385-426.
[2] Ait-Sahalia, Y. "Transition Densities for Interest Rate and Other Nonlinear Diffusions." The Journal of Finance, Vol. 54, No. 4, August 1999.
[3] Glasserman, P. Monte Carlo Methods in Financial Engineering. New York, SpringerVerlag, 2004.
[4] Hull, J. C. Options, Futures, and Other Derivatives, 5th ed. Englewood Cliffs, NJ: Prentice Hall, 2002.
[5] Johnson, N. L., S. Kotz, and N. Balakrishnan. Continuous Univariate Distributions. Vol. 2, 2nd ed. New York, John Wiley \& Sons, 1995.
[6] Shreve, S. E. Stochastic Calculus for Finance II: Continuous-Time Models. New York: Springer-Verlag, 2004.

## See Also

gbm | simByEuler| simBySolution | simulate

## Topics

"Simulating Equity Prices" on page 18-35
"Simulating Interest Rates" on page 18-60
"Stratified Sampling" on page 18-71
"Pricing American Basket Options by Monte Carlo Simulation" on page 18-85
"Base SDE Models" on page 18-17
"Drift and Diffusion Models" on page 18-20
"Linear Drift Models" on page 18-24
"Parametric Models" on page 18-26
"SDEs" on page 18-2
"SDE Models" on page 18-9
"SDE Class Hierarchy" on page 18-5
"Performance Considerations" on page 18-77

## Introduced in R2008a

## simBySolution

Simulate approximate solution of diagonal-drift HWV processes

## Syntax

[Paths,Times,Z] = simBySolution(MDL,NPeriods)
[Paths,Times,Z] = simBySolution( $\qquad$ ,Name, Value)

## Description

[Paths,Times,Z] = simBySolution(MDL,NPeriods) simulates approximate solution of diagonal-drift for Hull-White/Vasicek Gaussian Diffusion (HWV) processes.
[Paths,Times,Z] = simBySolution( $\qquad$ ,Name, Value) adds optional name-value pair arguments.

## Examples

## Use simBySolution with an hwv Object

Create an hwv object to represent the model:

```
    dX 
hwv = hwv(0.2, 0.1, 0.05) %(Speed, Level, Sigma)
hwv =
    Class HWV: Hull-White/Vasicek
        Dimensions: State = 1, Brownian = 1
        StartTime: 0
        StartState: 1
```

```
Correlation: 1
    Drift: drift rate function F(t,X(t))
    Diffusion: diffusion rate function G(t,X(t))
    Simulation: simulation method/function simByEuler
    Sigma: 0.05
    Level: 0.1
    Speed: 0.2
```

The simBySolution function simulates the state vector $X_{t}$ using an approximation of the closed-form solution of diagonal drift HWV models. Each element of the state vector $X_{t}$ is expressed as the sum of NBROWNS correlated Gaussian random draws added to a deterministic time-variable drift.
nPeriods $=100$
[Paths,Times,Z] = simBySolution(hwv, nPeriods,'nTrials', 10);

## Input Arguments

## MDL - Hull-White/Vasicek (HWV) model

hwv object
Hull-White/Vasicek (HWV) mode, specified as a hwv object that is created using hwv.
Data Types: object

## NPeriods - Number of simulation periods

positive integer
Number of simulation periods, specified as a positive scalar integer. The value of NPeriods determines the number of rows of the simulated output series.

Data Types: double

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Example: [Paths,Times,Z] = simBySolution(HWV,NPeriods,'DeltaTime',dt,'NTrials',10)

## NTrials - Simulated trials (sample paths) of NPERIODS observations each 1 (single path of correlated state variables) (default) | positive integer

Simulated trials (sample paths) of NPERIODS observations each, specified as the commaseparated pair consisting of 'NTrials' and a positive scalar integer.

Data Types: double
DeltaTimes - Positive time increments between observations
1 (default) | scalar | column vector
Positive time increments between observations, specified as the comma-separated pair consisting of 'DeltaTimes ' and a scalar or a NPERIODS-by-1 column vector.

DeltaTime represents the familiar $d t$ found in stochastic differential equations, and determines the times at which the simulated paths of the output state variables are reported.

## Data Types: double

## NSteps - Number of intermediate time steps within each time increment dt (specified as DeltaTime)

1 (indicating no intermediate evaluation) (default) | positive integer
Number of intermediate time steps within each time increment $d t$ (specified as DeltaTime), specified as the comma-separated pair consisting of 'NSteps ' and a positive scalar integer.

The simBySolution function partitions each time increment $d t$ into NSteps subintervals of length $d t /$ NSteps, and refines the simulation by evaluating the simulated state vector at NSteps - 1 intermediate points. Although simBySolution does not report the output state vector at these intermediate points, the refinement improves accuracy by allowing the simulation to more closely approximate the underlying continuous-time process.

Data Types: double
Antithetic - Flag to indicate whether simBySolution uses antithetic sampling to generate the Gaussian random variates
False (no antithetic sampling) (default) | logical with values True or False
Flag to indicate whether simBySolution uses antithetic sampling to generate the Gaussian random variates that drive the Brownian motion vector (Wiener processes),
specified as the comma-separated pair consisting of 'Antithetic' and a scalar logical flag with a value of True or False.

When you specify True, simBySolution performs sampling such that all primary and antithetic paths are simulated and stored in successive matching pairs:

- Odd trials $(1,3,5, \ldots)$ correspond to the primary Gaussian paths.
- Even trials ( $2,4,6, \ldots$ ) are the matching antithetic paths of each pair derived by negating the Gaussian draws of the corresponding primary (odd) trial.

Note If you specify an input noise process (see Z), simBySolution ignores the value of Antithetic.

## Data Types: logical

## Z - Direct specification of the dependent random noise process used to generate the Brownian motion vector

generates correlated Gaussian variates based on the Correlation member of the SDE object (default) | function | three-dimensional array of dependent random variates

Direct specification of the dependent random noise process used to generate the Brownian motion vector (Wiener process) that drives the simulation, specified as the comma-separated pair consisting of ' $Z$ ' and a function or as an (NPERIODS $*$ NSTEPS)-by-NBROWNS-by-NTRIALS three-dimensional array of dependent random variates.

The input argument $Z$ allows you to directly specify the noise generation process. This process takes precedence over the Correlation parameter of the input gbm object and the value of the Antithetic input flag.

Note If you specify Z as a function, it must return an NBROWNS-by-1 column vector, and you must call it with two inputs:

- A real-valued scalar observation time $t$.
- An NVARS-by-1 state vector $X_{t}$.

[^3]
## StorePaths - Flag that indicates how the output array Paths is stored and returned

True (default) | logical with values True or False
Flag that indicates how the output array Paths is stored and returned, specified as the comma-separated pair consisting of 'StorePaths' and a scalar logical flag with a value of True or False.

If StorePaths is True (the default value) or is unspecified, simBySolution returns Paths as a three-dimensional time series array.

If StorePaths is False (logical 0), simBySolution returns the Paths output array as an empty matrix.

Data Types: logical

## Processes - Sequence of end-of-period processes or state vector adjustments of the form

simBySolution makes no adjustments and performs no processing (default) | function | cell array of functions

Sequence of end-of-period processes or state vector adjustments of the form, specified as the comma-separated pair consisting of 'Processes ' and a function or cell array of functions of the form

$$
X_{t}=P\left(t, X_{t}\right)
$$

The simBySolution function runs processing functions at each interpolation time. They must accept the current interpolation time $t$, and the current state vector $X_{t}$, and return a state vector that may be an adjustment to the input state.
simBySolution applies processing functions at the end of each observation period. These functions must accept the current observation time $t$ and the current state vector $X_{t}$, and return a state vector that may be an adjustment to the input state.

The end-of-period Processes argument allows you to terminate a given trial early. At the end of each time step, simBySolution tests the state vector $X_{t}$ for an all-NaN condition. Thus, to signal an early termination of a given trial, all elements of the state vector $X_{t}$ must be NaN. This test enables a user-defined Processes function to signal early termination of a trial, and offers significant performance benefits in some situations (for example, pricing down-and-out barrier options).

If you specify more than one processing function, simBySolution invokes the functions in the order in which they appear in the cell array. You can use this argument to specify boundary conditions, prevent negative prices, accumulate statistics, plot graphs, and more.

Data Types: cell|function

## Output Arguments

## Paths - Simulated paths of correlated state variables

array
Simulated paths of correlated state variables, returned as a (NPERIODS + 1) -by-NVARS-by-NTRIALS three-dimensional time series array.

For a given trial, each row of Paths is the transpose of the state vector $X_{t}$ at time $t$. When the input flag StorePaths = False, simBySolution returns Paths as an empty matrix.

## Times - Observation times associated with the simulated paths column vector

Observation times associated with the simulated paths, returned as a (NPERIODS +1 )-by-1 column vector. Each element of Times is associated with the corresponding row of Paths.

## Z - Dependent random variates used to generate the Brownian motion vector array

Dependent random variates used to generate the Brownian motion vector (Wiener processes) that drive the simulation, returned as a (NPERIODS * NSTEPS) -by-NBROWNS-by-NTRIALS three-dimensional time series array.

## Definitions

## Antithetic Sampling

Simulation methods allow you to specify a popular variance reduction technique called antithetic sampling.

This technique attempts to replace one sequence of random observations with another of the same expected value, but smaller variance. In a typical Monte Carlo simulation, each sample path is independent and represents an independent trial. However, antithetic sampling generates sample paths in pairs. The first path of the pair is referred to as the primary path, and the second as the antithetic path. Any given pair is independent of any other pair, but the two paths within each pair are highly correlated. Antithetic sampling literature often recommends averaging the discounted payoffs of each pair, effectively halving the number of Monte Carlo trials.

This technique attempts to reduce variance by inducing negative dependence between paired input samples, ideally resulting in negative dependence between paired output samples. The greater the extent of negative dependence, the more effective antithetic sampling is.

## Algorithms

The simBySolution method simulates NTRIALS sample paths of NVARS correlated state variables, driven by NBROWNS Brownian motion sources of risk over NPERIODS consecutive observation periods, approximating continuous-time Hull-White/Vasicek (HWV) by an approximation of the closed-form solution.

Consider a separable, vector-valued HWV model of the form:

$$
d X_{t}=S(t)\left[L(t)-X_{t}\right] d t+V(t) d W_{t}
$$

where:

- $X$ is an $N V A R S$-by- 1 state vector of process variables.
- $S$ is an $N V A R S$-by-NVARS matrix of mean reversion speeds (the rate of mean reversion).
- $L$ is an NVARS-by-1 vector of mean reversion levels (long-run mean or level).
- $V$ is an NVARS-by-NBROWNS instantaneous volatility rate matrix.
- $W$ is an NBROWNS-by-1 Brownian motion vector.

The simBySolution method simulates the state vector $X_{t}$ using an approximation of the closed-form solution of diagonal-drift models.

When evaluating the expressions, simBySolution assumes that all model parameters are piecewise-constant over each simulation period.

In general, this is not the exact solution to the models, because the probability distributions of the simulated and true state vectors are identical only for piecewiseconstant parameters.

When parameters are piecewise-constant over each observation period, the simulated process is exact for the observation times at which $X_{t}$ is sampled.

Gaussian diffusion models, such as hwv, allow negative states. By default, simBySolution does nothing to prevent negative states, nor does it guarantee that the model be strictly mean-reverting. Thus, the model may exhibit erratic or explosive growth.

## References

[1] Ait-Sahalia, Y. "Testing Continuous-Time Models of the Spot Interest Rate." The Review of Financial Studies, Spring 1996, Vol. 9, No. 2, pp. 385-426.
[2] Ait-Sahalia, Y. "Transition Densities for Interest Rate and Other Nonlinear Diffusions." The Journal of Finance, Vol. 54, No. 4, August 1999.
[3] Glasserman, P. Monte Carlo Methods in Financial Engineering. New York, SpringerVerlag, 2004.
[4] Hull, J. C. Options, Futures, and Other Derivatives, 5th ed. Englewood Cliffs, NJ: Prentice Hall, 2002.
[5] Johnson, N. L., S. Kotz, and N. Balakrishnan. Continuous Univariate Distributions. Vol. 2, 2nd ed. New York, John Wiley \& Sons, 1995.
[6] Shreve, S. E. Stochastic Calculus for Finance II: Continuous-Time Models. New York: Springer-Verlag, 2004.

## See Also

hwv | simByEuler| simBySolution | simulate

## Topics

"Creating Hull-White/Vasicek (HWV) Gaussian Diffusion Models" on page 18-31
"Simulating Equity Prices" on page 18-35
"Simulating Interest Rates" on page 18-60
"Stratified Sampling" on page 18-71
"Pricing American Basket Options by Monte Carlo Simulation" on page 18-85
"Base SDE Models" on page 18-17
"Drift and Diffusion Models" on page 18-20
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"Parametric Models" on page 18-26
"SDEs" on page 18-2
"SDE Models" on page 18-9
"SDE Class Hierarchy" on page 18-5
"Performance Considerations" on page 18-77

## Introduced in R2008a

## heston

Heston model

## Description

Creates and displays heston objects, which derive from thesdeddo (SDE from drift and diffusion objects).

Use heston objects to simulate sample paths of two state variables. Each state variable is driven by a single Brownian motion source of risk over NPERIODS consecutive observation periods, approximating continuous-time stochastic volatility processes.

Heston models are bivariate composite models. Each Heston model consists of two coupled univariate models:

- A geometric Brownian motion (gbm) model with a stochastic volatility function.

$$
d X_{1 t}=B(t) X_{1 t} d t+\sqrt{X_{2 t}} X_{1 t} d W_{1 t}
$$

This model usually corresponds to a price process whose volatility (variance rate) is governed by the second univariate model.

- A Cox-Ingersoll-Ross (cir) square root diffusion model.

$$
d X_{2 t}=S(t)\left[L(t)-X_{2 t}\right] d t+V(t) \sqrt{X_{2 t}} d W_{2 t}
$$

This model describes the evolution of the variance rate of the coupled GBM price process.

## Creation

## Syntax

heston $=$ heston(Return,Level,Speed,Volatility)

```
heston = heston(
```

$\qquad$

``` , Name, Value)
```


## Description

heston $=$ heston(Return,Level, Speed, Volatility) creates a default heston object.

Specify required input parameters as one of the following types:

- A MATLAB array. Specifying an array indicates a static (non-time-varying) parametric specification. This array fully captures all implementation details, which are clearly associated with a parametric form.
- A MATLAB function. Specifying a function provides indirect support for virtually any static, dynamic, linear, or nonlinear model. This parameter is supported via an interface, because all implementation details are hidden and fully encapsulated by the function.

Note You can specify combinations of array and function input parameters as needed.
Moreover, a parameter is identified as a deterministic function of time if the function accepts a scalar time $t$ as its only input argument. Otherwise, a parameter is assumed to be a function of time $t$ and state $X(t)$ and is invoked with both input arguments.
heston = heston( ___ , Name, Value) constructs a heston object with additional options specified by one or more Name, Value pair arguments.

Name is a property name and Value is its corresponding value. Name must appear inside single quotes (' ' ). You can specify several name-value pair arguments in any order as Name1, Value1, ..., NameN , ValueN

The heston object has the following "Properties" on page 19-90:

- StartTime - Initial observation time
- StartState - Initial state at StartTime
- Correlation - Access function for the Correlation input, callable as a function of time
- Drift - Composite drift-rate function, callable as a function of time and state
- Diffusion - Composite diffusion-rate function, callable as a function of time and state
- Simulation - A simulation function or method
- Return - Access function for the input argument Return, callable as a function of time and state
- Speed - Access function for the input argument Speed, callable as a function of time and state
- Level - Access function for the input argument Level, callable as a function of time and state
- Volatility - Access function for the input argument Volatility, callable as a function of time and state


## Input Arguments

## Return - Return represents the parameter $\boldsymbol{\mu}$

 array or deterministic function of time or deterministic function of time and stateReturn represents the parameter $\mu$, specified as an array or deterministic function of time.

If you specify Return as an array, it must be an NVARS-by-NVARS matrix representing the expected (mean) instantaneous rate of return.

As a deterministic function of time, when Return is called with a real-valued scalar time t as its only input, Return must produce an NVARS-by-NVARS matrix. If you specify Return as a function of time and state, it must return an NVARS-by-NVARS matrix when invoked with two inputs:

- A real-valued scalar observation time $t$.
- An NVARS-by-1 state vector $X_{t}$.

Data Types: double | function_handle

## Level - Level represents the parameter L

array or deterministic function of time or deterministic function of time and state
Level represents the parameter $L$, specified as an array or deterministic function of time.
If you specify Level as an array, it must be an NVARS-by-1 column vector of reversion levels.

As a deterministic function of time, when Level is called with a real-valued scalar time $t$ as its only input, Level must produce an NVARS-by-1 column vector. If you specify Level as a function of time and state, it must generate an NVARS-by-1 column vector of reversion levels when called with two inputs:

- A real-valued scalar observation time $t$.
- An NVARS-by-1 state vector $X_{t}$.


## Data Types: double | function_handle

## Speed - Return represents the parameter S

array or deterministic function of time or deterministic function of time and state
Speed represents the parameter $S$, specified as an array or deterministic function of time.
If you specify Speed as an array, it must be an NVARS-by-NVARS matrix of mean-reversion speeds (the rate at which the state vector reverts to its long-run average Level).

As a deterministic function of time, when Speed is called with a real-valued scalar time $t$ as its only input, Speed must produce an NVARS-by-NVARS matrix. If you specify Speed as a function of time and state, it calculates the speed of mean reversion. This function must generate an NVARS-by-NVARS matrix of reversion rates when called with two inputs:

- A real-valued scalar observation time $t$.
- An NVARS-by-1 state vector $X_{t}$.

Data Types: double | function_handle

## Volatility - Volatility represents the instantaneous volatility of the CIR stochastic variance model

scalar or deterministic function of time or deterministic function of time and state
Volatility (often called the volatility of volatility or volatility of variance) represents the instantaneous volatility of the CIR stochastic variance model, specified as a scalar, or deterministic function of time.

If you specifyVolatility as a scalar, it represents the instantaneous volatility of the CIR stochastic variance model.

As a deterministic function of time, when Volatility is called with a real-valued scalar time $t$ as its only input, Volatility must produce a scalar. If you specify it as a function time and state, Volatility generates a scalar when invoked with two inputs:

- A real-valued scalar observation time $t$.
- A 2-by-1 state vector $X_{t}$.

Data Types: double | function_handle

Note Although heston does not enforce restrictions on the signs of any of these input arguments, each argument is specified as a positive value.

## Properties

## StartTime - Starting time of first observation, applied to all state variables 0 (default) | scalar

Starting time of first observation, applied to all state variables, specified as a scalar Data Types: double

## StartState - Initial values of state variables

1 (default) | scalar, column vector, or matrix
Initial values of state variables, specified as a scalar, column vector, or matrix.
If StartState is a scalar, heston applies the same initial value to all state variables on all trials.

If StartState is a column vector, heston applies a unique initial value to each state variable on all trials.

If StartState is a matrix, heston applies a unique initial value to each state variable on each trial.

## Data Types: double

## Correlation - Correlation between Gaussian random variates drawn to generate the Brownian motion vector (Wiener processes)

NBROWNS-by-NBROWNS identity matrix representing independent Gaussian processes (default) | positive semidefinite matrix | deterministic function

Correlation between Gaussian random variates drawn to generate the Brownian motion vector (Wiener processes), specified as an NBROWNS-by-NBROWNS positive semidefinite
matrix, or as a deterministic function $C(t)$ that accepts the current time $t$ and returns an NBROWNS-by-NBROWNS positive semidefinite correlation matrix.

A Correlation matrix represents a static condition.
As a deterministic function of time, Correlation allows you to specify a dynamic correlation structure.

Data Types: double

## Simulation - User-defined simulation function or SDE simulation method simulation by Euler approximation (simByEuler) (default) | function \| SDE simulation method

User-defined simulation function or SDE simulation method, specified as a function or SDE simulation method.

Data Types: function_handle

## Drift - Drift rate component of continuous-time stochastic differential equations (SDEs)

value stored from drift-rate function (default) | drift object or function accessible by ( $t, X_{t}$ )
This property is read-only.
Drift rate component of continuous-time stochastic differential equations (SDEs), specified as a drift object or function accessible by ( $t, X_{t}$.

The drift rate specification supports the simulation of sample paths of NVARS state variables driven by NBROWNS Brownian motion sources of risk over NPERIODS consecutive observation periods, approximating continuous-time stochastic processes.

The drift class allows you to create drift-rate objects using drift of the form:

$$
F\left(t, X_{t}\right)=A(t)+B(t) X_{t}
$$

where:

- A is an NVARS-by-1 vector-valued function accessible using the $\left(t, X_{t}\right)$ interface.
- B is an NVARS-by-NVARS matrix-valued function accessible using the $\left(t, X_{t}\right)$ interface.

The displayed parameters for a drift object are:

- Rate: The drift-rate function, $F\left(t, X_{t}\right)$
- A: The intercept term, $A\left(t, X_{t}\right)$, of $F\left(t, X_{t}\right)$
- B: The first order term, $B\left(t, X_{t}\right)$, of $F\left(t, X_{t}\right)$
$A$ and $B$ enable you to query the original inputs. The function stored in Rate fully encapsulates the combined effect of $A$ and $B$.

When specified as MATLAB double arrays, the inputs A and B are clearly associated with a linear drift rate parametric form. However, specifying either A or B as a function allows you to customize virtually any drift rate specification.

Note You can express drift and diffusion classes in the most general form to emphasize the functional $\left(t, X_{t}\right)$ interface. However, you can specify the components A and $B$ as functions that adhere to the common $\left(t, X_{t}\right)$ interface, or as MATLAB arrays of appropriate dimension.

Example: $F=\operatorname{drift}(0,0.1) \%$ Drift rate function $F(t, X)$
Data Types: struct \| double
Diffusion - Diffusion rate component of continuous-time stochastic differential
equations (SDEs)
value stored from diffusion-rate function (default) | diffusion object or functions accessible by $\left(t, X_{t}\right)$

This property is read-only.
Diffusion rate component of continuous-time stochastic differential equations (SDEs), specified as a drift object or function accessible by ( $t, X_{t}$.

The diffusion rate specification supports the simulation of sample paths of NVARS state variables driven by NBROWNS Brownian motion sources of risk over NPERIODS consecutive observation periods, approximating continuous-time stochastic processes.

The diffusion class allows you to create diffusion-rate objects using diffusion:

$$
G\left(t, X_{t}\right)=D\left(t, X_{t}^{\alpha(t)}\right) V(t)
$$

where:

- D is an NVARS-by-NVARS diagonal matrix-valued function.
- Each diagonal element of $D$ is the corresponding element of the state vector raised to the corresponding element of an exponent Alpha, which is an NVARS-by-1 vectorvalued function.
- $V$ is an NVARS-by-NBROWNS matrix-valued volatility rate function Sigma.
- Alpha and Sigma are also accessible using the $\left(t, X_{t}\right)$ interface.

The displayed parameters for a diffusion object are:

- Rate: The diffusion-rate function, $G\left(t, X_{t}\right)$.
- Alpha: The state vector exponent, which determines the format of $D\left(t, X_{t}\right)$ of $G\left(t, X_{t}\right)$.
- Sigma: The volatility rate, $V\left(t, X_{t}\right)$, of $G\left(t, X_{t}\right)$.

Alpha and Sigma enable you to query the original inputs. (The combined effect of the individual Alpha and Sigma parameters is fully encapsulated by the function stored in Rate.) The Rate functions are the calculation engines for the drift and diffusion objects, and are the only parameters required for simulation.

Note You can express drift and diffusion classes in the most general form to emphasize the functional $\left(t, X_{t}\right)$ interface. However, you can specify the components A and B as functions that adhere to the common $\left(t, X_{t}\right)$ interface, or as MATLAB arrays of appropriate dimension.

Example: G = diffusion(1, 0.3) \% Diffusion rate function G(t,X)
Data Types: struct|double

## Object Functions

interpolate Brownian interpolation of stochastic differential equations simulate Simulate multivariate stochastic differential equations (SDEs) simByEuler Euler simulation of stochastic differential equations (SDEs)

## Examples

## Create a heston Object

The Heston (heston) class derives directly from SDE from Drift and Diffusion (sdeddo). Each Heston model is a bivariate composite model, consisting of two coupled univariate models:

$$
\begin{aligned}
& d X_{1 t}=B(t) X_{1 t} d t+\sqrt{X_{2 t}} X_{1 t} d W_{1 t} \\
& d X_{2 t}=S(t)\left[L(t)-X_{2 t}\right] d t+V(t) \sqrt{X_{2 t}} d W_{2 t}
\end{aligned}
$$

Create a heston object to represent the model:

```
        dX XIt}=0.1\mp@subsup{X}{1t}{}dt+\sqrt{}{\mp@subsup{X}{2t}{}}\mp@subsup{X}{1t}{}d\mp@subsup{W}{1t}{
        dX 2t =0.2[0.1-\mp@subsup{X}{2t}{}]dt+0.05\sqrt{}{\mp@subsup{X}{2t}{}}d\mp@subsup{W}{2t}{}
obj = heston (0.1, 0.2, 0.1, 0.05) % (Return, Speed, Level, Volatility)
obj =
    Class HESTON: Heston Bivariate Stochastic Volatility
    Dimensions: State = 2, Brownian = 2
        StartTime: 0
    StartState: 1 (2x1 double array)
Correlation: 2x2 diagonal double array
        Drift: drift rate function F(t,X(t))
            Diffusion: diffusion rate function G(t,X(t))
    Simulation: simulation method/function simByEuler
        Return: 0.1
            Speed: 0.2
            Level: 0.1
    Volatility: 0.05
```


## Definitions

## Instance Hierarchy

There are inheritance relationships among the SDE classes.
The following figure illustrates the inheritance relationships.


For more information, see "SDE Class Hierarchy" on page 18-5.

## Algorithms

When you specify the required input parameters as arrays, they are associated with a specific parametric form. By contrast, when you specify either required input parameter as a function, you can customize virtually any specification.

Accessing the output parameters with no inputs simply returns the original input specification. Thus, when you invoke these parameters with no inputs, they behave like simple properties and allow you to test the data type (double vs. function, or equivalently, static vs. dynamic) of the original input specification. This is useful for validating and designing methods.

When you invoke these parameters with inputs, they behave like functions, giving the impression of dynamic behavior. The parameters accept the observation time $t$ and a state vector $X_{t}$, and return an array of appropriate dimension. Even if you originally specified an input as an array, heston treats it as a static function of time and state, by that means guaranteeing that all parameters are accessible by the same interface.

## References

[1] Ait-Sahalia, Y. "Testing Continuous-Time Models of the Spot Interest Rate." The Review of Financial Studies, Spring 1996, Vol. 9, No. 2, pp. 385-426.
[2] Ait-Sahalia, Y. "Transition Densities for Interest Rate and Other Nonlinear Diffusions." The Journal of Finance, Vol. 54, No. 4, August 1999.
[3] Glasserman, P. Monte Carlo Methods in Financial Engineering. New York, SpringerVerlag, 2004.
[4] Hull, J. C. Options, Futures, and Other Derivatives, 5th ed. Englewood Cliffs, NJ: Prentice Hall, 2002.
[5] Johnson, N. L., S. Kotz, and N. Balakrishnan. Continuous Univariate Distributions. Vol. 2, 2nd ed. New York, John Wiley \& Sons, 1995.
[6] Shreve, S. E. Stochastic Calculus for Finance II: Continuous-Time Models. New York: Springer-Verlag, 2004.

## See Also

diffusion|drift|interpolate|sdeddo|simByEuler|simulate

## Topics

"Creating Heston Stochastic Volatility Models" on page 18-33
"Simulating Equity Prices" on page 18-35
"Simulating Interest Rates" on page 18-60
"Stratified Sampling" on page 18-71
"Pricing American Basket Options by Monte Carlo Simulation" on page 18-85
"Base SDE Models" on page 18-17
"Drift and Diffusion Models" on page 18-20
"Linear Drift Models" on page 18-24
"Parametric Models" on page 18-26
"SDEs" on page 18-2
"SDE Models" on page 18-9
"SDE Class Hierarchy" on page 18-5
"Performance Considerations" on page 18-77

## Introduced in R2008a

## hwv

Hull-White/Vasicek Gaussian Diffusion model

## Description

Create and displays hwv objects, which derive from the sdemrd (SDE with drift rate expressed in mean-reverting form) class.

Use hwv objects to simulate sample paths of NVARS state variables expressed in meanreverting drift-rate form. These state variables are driven by NBROWNS Brownian motion sources of risk over NPERIODS consecutive observation periods, approximating continuous-time Hull-White/Vasicek stochastic processes with Gaussian diffusions.

This model allows you to simulate vector-valued Hull-White/Vasicek processes of the form:

$$
\begin{equation*}
d X_{t}=S(t)\left[L(t)-X_{t}\right] d t+V(t) d W_{t} \tag{19-2}
\end{equation*}
$$

where:

- $X_{t}$ is an NVARS-by-1 state vector of process variables.
- $S$ is an NVARS-by-NVARS of mean reversion speeds (the rate of mean reversion).
- $L$ is an NVARS-by-1 vector of mean reversion levels (long-run mean or level).
- $V$ is an NVARS-by-NBROWNS instantaneous volatility rate matrix.
- $d W_{t}$ is an NBROWNS-by-1 Brownian motion vector.


## Creation

## Syntax

```
HWV = hwv(Speed,Level,Sigma)
HWV = hwv(___,Name,Value)
```


## Description

HWV $=$ hwv(Speed,Level,Sigma) creates a default HWV object.
Specify required input parameters as one of the following types:

- A MATLAB array. Specifying an array indicates a static (non-time-varying) parametric specification. This array fully captures all implementation details, which are clearly associated with a parametric form.
- A MATLAB function. Specifying a function provides indirect support for virtually any static, dynamic, linear, or nonlinear model. This parameter is supported via an interface, because all implementation details are hidden and fully encapsulated by the function.

Note You can specify combinations of array and function input parameters as needed.
Moreover, a parameter is identified as a deterministic function of time if the function accepts a scalar time $t$ as its only input argument. Otherwise, a parameter is assumed to be a function of time $t$ and state $X(t)$ and is invoked with both input arguments.

HWV = hwv(__ , Name, Value) creates a HWV object with additional options specified by one or more Name, Value pair arguments.

Name is a property name and Value is its corresponding value. Name must appear inside single quotes (' ' ). You can specify several name-value pair arguments in any order as Name1, Value1, ..., NameN , ValueN

The HWV object has the following "Properties" on page 19-101:

- StartTime - Initial observation time
- StartState - Initial state at StartTime
- Correlation - Access function for the Correlation input, callable as a function of time
- Drift - Composite drift-rate function, callable as a function of time and state
- Diffusion - Composite diffusion-rate function, callable as a function of time and state
- Simulation - A simulation function or method
- Speed - Access function for the input argument Speed, callable as a function of time and state
- Level - Access function for the input argument Level, callable as a function of time and state
- Sigma - Access function for the input argument Sigma, callable as a function of time and state


## Input Arguments

## Speed - Speed represents the parameter S <br> array or deterministic function of time or deterministic function of time and state

Speed represents the parameter $S$, specified as an array or deterministic function of time.
If you specify Speed as an array, it must be an NVARS-by-NVARS matrix of mean-reversion speeds (the rate at which the state vector reverts to its long-run average Level).

As a deterministic function of time, when Speed is called with a real-valued scalar time $t$ as its only input, Speed must produce an NVARS-by-NVARS matrix. If you specify Speed as a function of time and state, it calculates the speed of mean reversion. This function must generate an NVARS-by-NVARS matrix of reversion rates when called with two inputs:

- A real-valued scalar observation time $t$.
- An NVARS-by-1 state vector $X_{t}$.

Data Types: double | function_handle

## Level - Level represents the parameter L

array or deterministic function of time or deterministic function of time and state
Level represents the parameter $L$, specified as an array or deterministic function of time.
If you specify Level as an array, it must be an NVARS-by-1 column vector of reversion levels.

As a deterministic function of time, when Level is called with a real-valued scalar time $t$ as its only input, Level must produce an NVARS-by-1 column vector. If you specify Level as a function of time and state, it must generate an NVARS-by-1 column vector of reversion levels when called with two inputs:

- A real-valued scalar observation time $t$.
- An NVARS-by-1 state vector $X_{t}$.

Data Types: double | function_handle

## Sigma - Sigma represents the parameter V array or deterministic function of time or deterministic function of time and state

Sigma represents the parameter $V$, specified as an array or a deterministic function of time.

If you specify Sigma as an array, it must be an NVARS-by-NBROWNS matrix of instantaneous volatility rates or as a deterministic function of time. In this case, each row of Sigma corresponds to a particular state variable. Each column corresponds to a particular Brownian source of uncertainty, and associates the magnitude of the exposure of state variables with sources of uncertainty.

As a deterministic function of time, when Sigma is called with a real-valued scalar time $t$ as its only input, Sigma must produce an NVARS-by-NBROWNS matrix. If you specify Sigma as a function of time and state, it must return an NVARS-by-NBROWNS matrix of volatility rates when invoked with two inputs:

- A real-valued scalar observation time $t$.
- An NVARS-by-1 state vector $X_{t}$.

Although hwv does not enforce restrictions on the signs of any of these input arguments, each argument is specified as a positive value.
Data Types: double | function_handle

Note Although hwv does not enforce restrictions on the signs of any of these input arguments, each argument is specified as a positive value.

## Properties

## StartTime - Starting time of first observation, applied to all state variables 0 (default) | scalar

Starting time of first observation, applied to all state variables, specified as a scalar

## Data Types: double

## StartState - Initial values of state variables <br> 1 (default) | scalar, column vector, or matrix

Initial values of state variables, specified as a scalar, column vector, or matrix.
If StartState is a scalar, hwv applies the same initial value to all state variables on all trials.

If StartState is a column vector, hwv applies a unique initial value to each state variable on all trials.

If StartState is a matrix, hwv applies a unique initial value to each state variable on each trial.

Data Types: double

## Correlation - Correlation between Gaussian random variates drawn to generate the Brownian motion vector (Wiener processes)

NBROWNS-by-NBROWNS identity matrix representing independent Gaussian processes (default) | positive semidefinite matrix | deterministic function

Correlation between Gaussian random variates drawn to generate the Brownian motion vector (Wiener processes), specified as an NBROWNS-by-NBROWNS positive semidefinite matrix, or as a deterministic function $C(t)$ that accepts the current time $t$ and returns an NBROWNS-by-NBROWNS positive semidefinite correlation matrix.

A Correlation matrix represents a static condition.
As a deterministic function of time, Correlation allows you to specify a dynamic correlation structure.
Data Types: double
Simulation - User-defined simulation function or SDE simulation method simulation by Euler approximation (simByEuler) (default) \| function \| SDE simulation method

User-defined simulation function or SDE simulation method, specified as a function or SDE simulation method.

Data Types: function_handle

## Drift - Drift rate component of continuous-time stochastic differential equations (SDEs)

value stored from drift-rate function (default) | drift object or function accessible by ( $t, X_{t}$ )
This property is read-only.
Drift rate component of continuous-time stochastic differential equations (SDEs), specified as a drift object or function accessible by $\left(t, X_{t}\right.$.

The drift rate specification supports the simulation of sample paths of NVARS state variables driven by NBROWNS Brownian motion sources of risk over NPERIODS consecutive observation periods, approximating continuous-time stochastic processes.

The drift class allows you to create drift-rate objects using drift of the form:

$$
F\left(t, X_{t}\right)=A(t)+B(t) X_{t}
$$

where:

- A is an NVARS-by-1 vector-valued function accessible using the $\left(t, X_{t}\right)$ interface.
- B is an NVARS-by-NVARS matrix-valued function accessible using the $\left(t, X_{t}\right)$ interface.

The displayed parameters for a drift object are:

- Rate: The drift-rate function, $F\left(t, X_{t}\right)$
- A: The intercept term, $A\left(t, X_{t}\right)$, of $F\left(t, X_{t}\right)$
- B: The first order term, $B\left(t, X_{t}\right)$, of $F\left(t, X_{t}\right)$
$A$ and $B$ enable you to query the original inputs. The function stored in Rate fully encapsulates the combined effect of $A$ and $B$.

When specified as MATLAB double arrays, the inputs $A$ and $B$ are clearly associated with a linear drift rate parametric form. However, specifying either A or B as a function allows you to customize virtually any drift rate specification.

Note You can express drift and diffusion classes in the most general form to emphasize the functional $\left(t, X_{t}\right)$ interface. However, you can specify the components A and B as functions that adhere to the common ( $t, X_{t}$ ) interface, or as MATLAB arrays of appropriate dimension.

## Example: $F=\operatorname{drift}(0,0.1) \%$ Drift rate function $F(t, X)$ <br> Data Types: struct | double

## Diffusion - Diffusion rate component of continuous-time stochastic differential equations (SDEs)

value stored from diffusion-rate function (default) | diffusion object or functions accessible by $\left(t, X_{t}\right)$

This property is read-only.
Diffusion rate component of continuous-time stochastic differential equations (SDEs), specified as a drift object or function accessible by ( $t, X_{t}$.

The diffusion rate specification supports the simulation of sample paths of NVARS state variables driven by NBROWNS Brownian motion sources of risk over NPERIODS consecutive observation periods, approximating continuous-time stochastic processes.

The diffusion class allows you to create diffusion-rate objects using diffusion:

$$
G\left(t, X_{t}\right)=D\left(t, X_{t}^{\alpha(t)}\right) V(t)
$$

where:

- D is an NVARS-by-NVARS diagonal matrix-valued function.
- Each diagonal element of $D$ is the corresponding element of the state vector raised to the corresponding element of an exponent Alpha, which is an NVARS-by-1 vectorvalued function.
- $V$ is an NVARS-by-NBROWNS matrix-valued volatility rate function Sigma.
- Alpha and Sigma are also accessible using the $\left(t, X_{t}\right)$ interface.

The displayed parameters for a diffusion object are:

- Rate: The diffusion-rate function, $G\left(t, X_{t}\right)$.
- Alpha: The state vector exponent, which determines the format of $D\left(t, X_{t}\right)$ of $G\left(t, X_{t}\right)$.
- Sigma: The volatility rate, $V\left(t, X_{t}\right)$, of $G\left(t, X_{t}\right)$.

Alpha and Sigma enable you to query the original inputs. (The combined effect of the individual Alpha and Sigma parameters is fully encapsulated by the function stored in Rate.) The Rate functions are the calculation engines for the drift and diffusion objects, and are the only parameters required for simulation.

Note You can express drift and diffusion classes in the most general form to emphasize the functional $\left(t, X_{t}\right)$ interface. However, you can specify the components A and $B$ as functions that adhere to the common $\left(t, X_{t}\right)$ interface, or as MATLAB arrays of appropriate dimension.

Example: $G=$ diffusion(1, 0.3 ) \% Diffusion rate function $G(t, X)$
Data Types: struct|double

## Object Functions

interpolate simulate simByEuler simBySolution Simulate approximate solution of diagonal-drift HWV processes

## Examples

## Create a hwv Object

The Hull-White/Vasicek (HWV) short rate class derives directly from SDE with meanreverting drift (that is, sdemrd): $d X_{t}=S(t)\left[L(t)-X_{t L}\right] d t+V(t) d W_{t}$.

Create an hwv object to represent the model: $d X_{t}=0.2\left(0.1-X_{t}\right) d t+0.05 d W_{t}$.

```
obj = hwv(0.2, 0.1, 0.05) %(Speed, Level, Sigma)
obj =
    Class HWV: Hull-White/Vasicek
    Dimensions: State = 1, Brownian = 1
        StartTime: 0
        StartState: 1
    Correlation: 1
            Drift: drift rate function F(t,X(t))
            Diffusion: diffusion rate function G(t,X(t))
            Simulation: simulation method/function simByEuler
```

```
Sigma: 0.05
Level: 0.1
Speed: 0.2
```


## Definitions

## Instance Hierarchy

There are inheritance relationships among the SDE classes.
The following figure illustrates the inheritance relationships.


For more information, see "SDE Class Hierarchy" on page 18-5.

## Algorithms

When you specify the required input parameters as arrays, they are associated with a specific parametric form. By contrast, when you specify either required input parameter as a function, you can customize virtually any specification.

Accessing the output parameters with no inputs simply returns the original input specification. Thus, when you invoke these parameters with no inputs, they behave like simple properties and allow you to test the data type (double vs. function, or equivalently, static vs. dynamic) of the original input specification. This is useful for validating and designing methods.

When you invoke these parameters with inputs, they behave like functions, giving the impression of dynamic behavior. The parameters accept the observation time $t$ and a state vector $X_{t}$, and return an array of appropriate dimension. Even if you originally specified an input as an array, hwv treats it as a static function of time and state, by that means guaranteeing that all parameters are accessible by the same interface.

## References

[1] Ait-Sahalia, Y. "Testing Continuous-Time Models of the Spot Interest Rate." The Review of Financial Studies, Spring 1996, Vol. 9, No. 2, pp. 385-426.
[2] Ait-Sahalia, Y. "Transition Densities for Interest Rate and Other Nonlinear Diffusions." The Journal of Finance, Vol. 54, No. 4, August 1999.
[3] Glasserman, P. Monte Carlo Methods in Financial Engineering. New York, SpringerVerlag, 2004.
[4] Hull, J. C. Options, Futures, and Other Derivatives, 5th ed. Englewood Cliffs, NJ: Prentice Hall, 2002.
[5] Johnson, N. L., S. Kotz, and N. Balakrishnan. Continuous Univariate Distributions. Vol. 2, 2nd ed. New York, John Wiley \& Sons, 1995.
[6] Shreve, S. E. Stochastic Calculus for Finance II: Continuous-Time Models. New York: Springer-Verlag, 2004.

## See Also

diffusion|drift|interpolate|sdeddo|simByEuler|simulate
Topics"Creating Hull-White/Vasicek (HWV) Gaussian Diffusion Models" on page 18-31"Simulating Equity Prices" on page 18-35
"Simulating Interest Rates" on page 18-60
"Stratified Sampling" on page 18-71
"Pricing American Basket Options by Monte Carlo Simulation" on page 18-85
"Base SDE Models" on page 18-17
"Drift and Diffusion Models" on page 18-20
"Linear Drift Models" on page 18-24
"Parametric Models" on page 18-26
"SDEs" on page 18-2
"SDE Models" on page 18-9
"SDE Class Hierarchy" on page 18-5
"Performance Considerations" on page 18-77
Introduced in R2008a

## interpolate

Brownian interpolation of stochastic differential equations

## Syntax

[ $\mathrm{XT}, \mathrm{T}]=$ interpolate(MDL,Times,Paths)
[XT,T] = interpolate( $\qquad$ ,Name, Value)

## Description

[XT,T] = interpolate(MDL,Times,Paths) performs a Brownian interpolation into a user-specified time series array, based on a piecewise-constant Euler sampling approach.
[XT,T] = interpolate( $\qquad$ ,Name, Value) adds optional name-value pair arguments.

## Examples

## Stochastic Interpolation Without Refinement

Many applications require knowledge of the state vector at intermediate sample times that are initially unavailable. One way to approximate these intermediate states is to perform a deterministic interpolation. However, deterministic interpolation techniques fail to capture the correct probability distribution at these intermediate times. Brownian (or stochastic) interpolation captures the correct joint distribution by sampling from a conditional Gaussian distribution. This sampling technique is sometimes referred to as a Brownian Bridge.

The default stochastic interpolation technique is designed to interpolate into an existing time series and ignore new interpolated states as additional information becomes available. This technique is the usual notion of interpolation, which is called Interpolation without refinement.

Alternatively, the interpolation technique may insert new interpolated states into the existing time series upon which subsequent interpolation is based, by that means refining
information available at subsequent interpolation times. This technique is called interpolation with refinement.

Interpolation without refinement is a more traditional technique, and is most useful when the input series is closely spaced in time. In this situation, interpolation without refinement is a good technique for inferring data in the presence of missing information, but is inappropriate for extrapolation. Interpolation with refinement is more suitable when the input series is widely spaced in time, and is useful for extrapolation.

The stochastic interpolation method is available to any model. It is best illustrated, however, by way of a constant-parameter Brownian motion process. Consider a correlated, bivariate Brownian motion (BM) model of the form:

$$
\begin{aligned}
& d X_{1 t}=0.3 d t+0.2 d W_{1 t}-0.1 d W_{2 t} \\
& d X_{2 t}=0.4 d t+0.1 d W_{1 t}-0.2 d W_{2 t} \\
& E\left[d W_{1 t} d W_{2 t}\right]=\rho d t=0.5 d t
\end{aligned}
$$

1 Create a bm object to represent the bivariate model:

```
mu = [0.3; 0.4];
sigma = [0.2 -0.1; 0.1 -0.2];
rho = [1 0.5; 0.5 1];
obj = bm(mu,sigma,'Correlation',rho);
```

2 Assuming that the drift (Mu) and diffusion (Sigma) parameters are annualized, simulate a single Monte Carlo trial of daily observations for one calendar year (250 trading days):

```
rng default % make output reproducible
dt = 1/250; % 1 trading day = 1/250 years
[X,T] = simulate(obj,250,'DeltaTime',dt);
```

3 It is helpful to examine a small interval in detail.
a Interpolate into the simulated time series with a Brownian bridge:

```
t = ((T(1) + dt/2):(dt/2):(T(end) - dt/2));
x = interpolate(obj,t,X,'Times',T);
```

b Plot both the simulated and interpolated values:

```
plot(T,X(:,1),'.-r',T,X(:,2),'.-b')
grid on;
hold on;
```

```
        plot(t,x(:,1),'or',t,x(:,2),'ob')
        hold off;
        xlabel('Time (Years)')
        ylabel('State')
        title('Bi-Variate Brownian Motion: \rho = 0.5')
        axis([0.4999 0.6001 0.25 0.4])
```

Bi -Variate Brownian Motion: $\rho=\mathbf{0 . 5}$


In this plot:

- The solid red and blue dots indicate the simulated states of the bivariate model.
- The straight lines that connect the solid dots indicate intermediate states that would be obtained from a deterministic linear interpolation.
- Open circles indicate interpolated states.
- Open circles associated with every other interpolated state encircle solid dots associated with the corresponding simulated state. However, interpolated states at the midpoint of each time increment typically deviate from the straight line connecting each solid dot.


## Simulation of Conditional Gaussian Distributions

You can gain additional insight into the behavior of stochastic interpolation by regarding a Brownian bridge as a Monte Carlo simulation of a conditional Gaussian distribution.

This example examines the behavior of a Brownian bridge over a single time increment.
1 Divide a single time increment of length dt into 10 subintervals:

```
mu = [0.3; 0.4];
sigma = [0.2 -0.1; 0.1 -0.2];
rho = [1 0.5; 0.5 1];
obj = bm(mu,sigma,'Correlation',rho);
rng default; % make output reproducible
dt = 1/250; % 1 trading day = 1/250 years
[X,T] = simulate(obj,250,'DeltaTime',dt);
n = 125; % index of simulated state near middle
times = (T(n):(dt/10):T(n + 1));
nTrials = 25000; % # of Trials at each time
```

2 In each subinterval, take 25000 independent draws from a Gaussian distribution, conditioned on the simulated states to the left, and right:

```
average = zeros(length(times),1);
variance = zeros(length(times),1);
for i = 1:length(times)
    t = times(i);
    x = interpolate(obj,t(ones(nTrials,1)),...
        X,'Times',T);
    average(i) = mean(x(:,1));
    variance(i) = var(x(:,1));
end
```

3 Plot the sample mean and variance of each state variable:

Note The following graph plots the sample statistics of the first state variable only, but similar results hold for any state variable.

```
subplot(2,1,1);
hold on;
grid on;
plot([T(n) T(n + 1)],[X(n,1) X(n + 1,1)],'.-b')
plot(times, average, 'or')
hold off;
title('Brownian Bridge without Refinement: Sample Mean')
ylabel('Mean')
limits = axis;
axis([T(n) T(n + 1) limits(3:4)]);
subplot(2,1,2)
hold on;
grid on;
plot(T(n),0,'.-b',T(n + 1),0,'.-b')
plot(times, variance, '.-r')
hold('off');
title('Brownian Bridge without Refinement: Sample Variance')
xlabel('Time (Years)')
ylabel('Variance')
limits = axis;
axis([T(n) T(n + 1) limits(3:4)]);
```



The Brownian interpolation within the chosen interval, $d t$, illustrates the following:

- The conditional mean of each state variable lies on a straight-line segment between the original simulated states at each endpoint.
- The conditional variance of each state variable is a quadratic function. This function attains its maximum midway between the interval endpoints, and is zero at each endpoint.
- The maximum variance, although dependent upon the actual model diffusion-rate function $G(t, X)$, is the variance of the sum of NBROWNS correlated Gaussian variates scaled by the factor $d t / 4$.

The previous plot highlights interpolation without refinement, in that none of the interpolated states take into account new information as it becomes available. If you
had performed interpolation with refinement, new interpolated states would have been inserted into the time series and made available to subsequent interpolations on a trial-by-trial basis. In this case, all random draws for any given interpolation time would be identical. Also, the plot of the sample mean would exhibit greater variability, but would still cluster around the straight-line segment between the original simulated states at each endpoint. The plot of the sample variance, however, would be zero for all interpolation times, exhibiting no variability.

## Input Arguments

## MDL - Stochastic differential equation model

 objectStochastic differential equation model, specified as an sde, bm, gbm, cev, cir, hwv, heston, sdeddo, sdeld, or sdemrd object.

All MDL parameters are assumed piecewise constant, evaluated from the most recent observation time in Times that precedes a specified interpolation time in T. This is consistent with the Euler approach of Monte Carlo simulation.

Data Types: object

## Times - Interpolation times <br> vector

Interpolation times, specified as a NTIMES element vector. The length of this vector determines the number of rows in the interpolated output time series XT.
Data Types: double

## Paths - Sample paths of correlated state variables

time series array
Sample paths of correlated state variables, specified as a NPERIODS-by-NVARS-byNTRIALS time series array.

For a given trial, each row of this array is the transpose of the state vector $X_{t}$ at time $t$. Paths is the initial time series array into which the interpolate function performs the Brownian interpolation.

Data Types: double

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.
Example: $[\mathrm{XT}, \mathrm{T}]=$ interpolate(MDL,T,Paths,'Times', t$)$

## Times - Observation times associated with the time series input Paths zero-based, unit-increment column vector of length NPERIODS (default) | column vector

Observation times associated with the time series input Paths, specified as the commaseparated pair consisting of 'Times' and a column vector.
Data Types: double

## Refine - Flag that indicates whether interpolate uses the interpolation times you request

False (interpolate bases the interpolation only on the state information specified in Paths) (default) | logical with values True or False

Flag that indicates whether interpolate uses the interpolation times you request (see T ) to refine the interpolation as new information becomes available, specified as the comma-separated pair consisting of 'Refine' and a logical with a value of True or False.
Data Types: logical

## Processes - Sequence of background processes or state vector adjustments

 interpolate makes no adjustments and performs no processing (default) | function | cell array of functionsSequence of background processes or state vector adjustments, specified as the commaseparated pair consisting of 'Processes' and a function or cell array of functions of the form

$$
X_{t}=P\left(t, X_{t}\right)
$$

The interpolate function runs processing functions at each interpolation time. They must accept the current interpolation time $t$, and the current state vector $X_{t}$, and return a state vector that may be an adjustment to the input state.

If you specify more than one processing function, interpolate invokes the functions in the order in which they appear in the cell array. You can use this argument to specify boundary conditions, prevent negative prices, accumulate statistics, plot graphs, and so on.

Data Types: cell|function

## Output Arguments

## XT - Interpolated state variables

array
Interpolated state variables, returned as a NTIMES-by-NVARS-by-NTRIALS time series array.

For a given trial, each row of this array is the transpose of the interpolated state vector $X_{t}$ at time $t$. XT is the interpolated time series formed by interpolating into the input Paths time series array.

## T - Interpolation times associated with the output time series XT <br> column vector

Interpolation times associated with the output time series XT, returned as a NTIMES-by-1 column vector.

If the input interpolation time vector Times contains no missing observations (NaNs), the output of T is the same time vector as Times, but with the NaNs removed. This reduces the length of T and the number of rows of XT.

## Algorithms

This function performs a Brownian interpolation into a user-specified time series array, based on a piecewise-constant Euler sampling approach.

Consider a vector-valued SDE of the form:

$$
d X_{t}=F\left(t, X_{t}\right) d t+G\left(t, X_{t}\right) d W_{t}
$$

where:

- $X$ is an $N V A R S$-by- 1 state vector.
- $F$ is an NVARS-by-1 drift-rate vector-valued function.
- $G$ is an $N V A R S$-by-NBROWNS diffusion-rate matrix-valued function.
- $W$ is an $N B R O W N S$-by- 1 Brownian motion vector.

Given a user-specified time series array associated with this equation, this function performs a Brownian (stochastic) interpolation by sampling from a conditional Gaussian distribution. This sampling technique is sometimes called a Brownian bridge.

Note Unlike simulation methods, the interpolation function does not support userspecified noise processes.

- The interpolate function assumes that all model parameters are piecewiseconstant, and evaluates them from the most recent observation time in Times that precedes a specified interpolation time in T. This is consistent with the Euler approach of Monte Carlo simulation.
- When an interpolation time falls outside the interval specified by Times, a Euler simulation extrapolates the time series by using the nearest available observation.
- The user-defined time series Paths and corresponding observation Times must be fully observed (no missing observations denoted by NaNs ).
- The interpolate function assumes that the user-specified time series array Paths is associated with the sde object. For example, the Times and Paths input pair are the result of an initial course-grained simulation. However, the interpolation ignores the initial conditions of the sde object (StartTime and StartState), allowing the userspecified Times and Paths input series to take precedence.


## References

[1] Ait-Sahalia, Y. "Testing Continuous-Time Models of the Spot Interest Rate." The Review of Financial Studies, Spring 1996, Vol. 9, No. 2, pp. 385-426.
[2] Ait-Sahalia, Y. "Transition Densities for Interest Rate and Other Nonlinear Diffusions." The Journal of Finance, Vol. 54, No. 4, August 1999.
[3] Glasserman, P. Monte Carlo Methods in Financial Engineering. New York, SpringerVerlag, 2004.
[4] Hull, J. C. Options, Futures, and Other Derivatives, 5th ed. Englewood Cliffs, NJ: Prentice Hall, 2002.
[5] Johnson, N. L., S. Kotz, and N. Balakrishnan. Continuous Univariate Distributions. Vol. 2, 2nd ed. New York, John Wiley \& Sons, 1995.
[6] Shreve, S. E. Stochastic Calculus for Finance II: Continuous-Time Models. New York: Springer-Verlag, 2004.

## See Also

sde | simulate

## Topics

"Simulating Equity Prices" on page 18-35
"Simulating Interest Rates" on page 18-60
"Stratified Sampling" on page 18-71
"Pricing American Basket Options by Monte Carlo Simulation" on page 18-85
"Base SDE Models" on page 18-17
"Drift and Diffusion Models" on page 18-20
"Linear Drift Models" on page 18-24
"Parametric Models" on page 18-26
"SDEs" on page 18-2
"SDE Models" on page 18-9
"SDE Class Hierarchy" on page 18-5
"Performance Considerations" on page 18-77

## Introduced in R2008a

## sde

Stochastic Differential Equation (SDE) model

## Description

Creates and displays general stochastic differential equation (SDE) models from userdefined drift and diffusion rate functions.

Use sde objects to simulate sample paths of NVARS state variables driven by NBROWNS Brownian motion sources of risk over NPERIODS consecutive observation periods, approximating continuous-time stochastic processes.

An sde object enables you to simulate any vector-valued SDE of the form:

$$
d X_{t}=F\left(t, X_{t}\right) d t+G\left(t, X_{t}\right) d W_{t}
$$

where:

- $X_{t}$ is an NVARS-by- 1 state vector of process variables.
- $d W_{t}$ is an NBROWNS-by-1 Brownian motion vector.
- $F$ is an NVARS-by-1 vector-valued drift-rate function.
- $G$ is an NVARS-by-NBROWNS matrix-valued diffusion-rate function.


## Creation

## Syntax

SDE = sde(DriftRate, DiffusionRate)
SDE = sde( $\qquad$ , Name, Value)

## Description

SDE = sde(DriftRate,DiffusionRate) creates a default SDE object.

SDE = sde (__ , Name, Value) creates a SDE object with additional options specified by one or more Name, Value pair arguments.

Name is a property name and Value is its corresponding value. Name must appear inside single quotes (' ' ). You can specify several name-value pair arguments in any order as Name1, Value1, ..., NameN, ValueN.

The SDE object has the following "Properties" on page 19-122:

- StartTime - Initial observation time
- StartState - Initial state at time StartTime
- Correlation - Access function for the Correlation input argument, callable as a function of time
- Drift - Composite drift-rate function, callable as a function of time and state
- Diffusion - Composite diffusion-rate function, callable as a function of time and state
- Simulation - A simulation function or method


## Input Arguments

## DriftRate - DriftRate is a user-defined drift-rate function and represents the parameter $F$ <br> vector or object of class drift

DriftRate is a user-defined drift-rate function and represents the parameter $F$, specified as a vector or object of class drift.

DriftRate is a function that returns an NVARS-by-1 drift-rate vector when called with two inputs:

- A real-valued scalar observation time $t$.
- An NVARS-by-1 state vector $X_{t}$.

Alternatively, DriftRate can also be an object of class drift that encapsulates the driftrate specification. In this case, however, sde uses only the Rate parameter of the object. For more information on the drift object, see drift.

Data Types: double|object

## DiffusionRate - DiffusionRate is a user-defined drift-rate function and represents the parameter $\boldsymbol{G}$

matrix or object of class diffusion
DiffusionRate is a user-defined drift-rate function and represents the parameter $G$, specified as a matrix or object of class diffusion.

DiffusionRate is a function that returns an NVARS-by-NBROWNS diffusion-rate matrix when called with two inputs:

- A real-valued scalar observation time $t$.
- An NVARS-by- 1 state vector $X_{t}$.

Alternatively, DiffusionRate can also be an object of class diffusion that encapsulates the diffusion-rate specification. In this case, however, sde uses only the Rate parameter of the object. For more information on the diffusion object, see diffusion.

Data Types: double|object

## Properties

## StartTime - Starting time of first observation, applied to all state variables

 0 (default) | scalarStarting time of first observation, applied to all state variables, specified as a scalar
Data Types: double
StartState - Initial values of state variables
1 (default) | scalar, column vector, or matrix
Initial values of state variables, specified as a scalar, column vector, or matrix.
If StartState is a scalar, sde applies the same initial value to all state variables on all trials.

If StartState is a column vector, sde applies a unique initial value to each state variable on all trials.

If StartState is a matrix, sde applies a unique initial value to each state variable on each trial.

## Data Types: double

## Correlation - Correlation between Gaussian random variates drawn to generate the Brownian motion vector (Wiener processes) NBROWNS-by-NBROWNS identity matrix representing independent Gaussian processes (default) | positive semidefinite matrix | deterministic function

Correlation between Gaussian random variates drawn to generate the Brownian motion vector (Wiener processes), specified as an NBROWNS-by-NBROWNS positive semidefinite matrix, or as a deterministic function $C(t)$ that accepts the current time $t$ and returns an NBROWNS-by-NBROWNS positive semidefinite correlation matrix.

A Correlation matrix represents a static condition.
As a deterministic function of time, Correlation allows you to specify a dynamic correlation structure.

## Data Types: double

## Simulation - User-defined simulation function or SDE simulation method simulation by Euler approximation (simByEuler) (default) | function | SDE simulation method

User-defined simulation function or SDE simulation method, specified as a function or SDE simulation method.

Data Types: function_handle

## Drift - Drift rate component of continuous-time stochastic differential equations (SDEs)

value stored from drift-rate function (default) | drift object or function accessible by $\left(t, X_{t}\right)$
This property is read-only.
Drift rate component of continuous-time stochastic differential equations (SDEs), specified as a drift object or function accessible by ( $t, X_{t}$.

The drift rate specification supports the simulation of sample paths of NVARS state variables driven by NBROWNS Brownian motion sources of risk over NPERIODS consecutive observation periods, approximating continuous-time stochastic processes.

The drift class allows you to create drift-rate objects (using drift) of the form:

$$
F\left(t, X_{t}\right)=A(t)+B(t) X_{t}
$$

where:

- A is an NVARS-by-1 vector-valued function accessible using the $\left(t, X_{t}\right)$ interface.
- B is an NVARS-by-NVARS matrix-valued function accessible using the $\left(t, X_{t}\right)$ interface.

The displayed parameters for a drift object are:

- Rate: The drift-rate function, $F\left(t, X_{t}\right)$
- A: The intercept term, $A\left(t, X_{t}\right)$, of $F\left(t, X_{t}\right)$
- B: The first order term, $B\left(t, X_{t}\right)$, of $F\left(t, X_{t}\right)$
$A$ and $B$ enable you to query the original inputs. The function stored in Rate fully encapsulates the combined effect of $A$ and $B$.

When specified as MATLAB double arrays, the inputs $A$ and $B$ are clearly associated with a linear drift rate parametric form. However, specifying either A or B as a function allows you to customize virtually any drift rate specification.

Note You can express drift and diffusion classes in the most general form to emphasize the functional $\left(t, X_{t}\right)$ interface. However, you can specify the components A and $B$ as functions that adhere to the common $\left(t, X_{t}\right)$ interface, or as MATLAB arrays of appropriate dimension.

```
Example: F = drift(0, 0.1) % Drift rate function F(t,X)
```

Data Types: object

## Diffusion - Diffusion rate component of continuous-time stochastic differential equations (SDEs)

value stored from diffusion-rate function (default) | diffusion object or functions accessible by $\left(t, X_{t}\right)$

This property is read-only.
Diffusion rate component of continuous-time stochastic differential equations (SDEs), specified as a drift object or function accessible by ( $t, X_{t}$.

The diffusion rate specification supports the simulation of sample paths of NVARS state variables driven by NBROWNS Brownian motion sources of risk over NPERIODS consecutive observation periods, approximating continuous-time stochastic processes.

The diffusion class allows you to create diffusion-rate objects (using diffusion):

$$
G\left(t, X_{t}\right)=D\left(t, X_{t}^{\alpha(t)}\right) V(t)
$$

where:

- D is an NVARS-by-NVARS diagonal matrix-valued function.
- Each diagonal element of $D$ is the corresponding element of the state vector raised to the corresponding element of an exponent Alpha, which is an NVARS-by-1 vectorvalued function.
- $V$ is an NVARS-by-NBROWNS matrix-valued volatility rate function Sigma.
- Alpha and Sigma are also accessible using the $\left(t, X_{t}\right)$ interface.

The displayed parameters for a diffusion object are:

- Rate: The diffusion-rate function, $G\left(t, X_{t}\right)$.
- Alpha: The state vector exponent, which determines the format of $D\left(t, X_{t}\right)$ of $G\left(t, X_{t}\right)$.
- Sigma: The volatility rate, $V\left(t, X_{t}\right)$, of $G\left(t, X_{t}\right)$.

Alpha and Sigma enable you to query the original inputs. (The combined effect of the individual Alpha and Sigma parameters is fully encapsulated by the function stored in Rate.) The Rate functions are the calculation engines for the drift and diffusion objects, and are the only parameters required for simulation.

Note You can express drift and diffusion classes in the most general form to emphasize the functional $\left(t, X_{t}\right)$ interface. However, you can specify the components A and B as functions that adhere to the common $\left(t, X_{t}\right)$ interface, or as MATLAB arrays of appropriate dimension.

Example: G = diffusion(1, 0.3) \% Diffusion rate function G(t,X)
Data Types: object

## Object Functions

interpolate Brownian interpolation of stochastic differential equations simulate Simulate multivariate stochastic differential equations (SDEs) simByEuler Euler simulation of stochastic differential equations (SDEs)

## Examples

## Create an SDE Object

Construct an SDE object obj to represent a univariate geometric Brownian Motion model of the form: $d X_{t}=0.1 X_{t} d t+0.3 X_{t} d W_{t}$.

Create drift and diffusion functions that are accessible by the common $\left(t, X_{t}\right)$ interface:

```
F = @(t,X) 0.1 * X;
G = @(t,X) 0.3 * X;
```

Pass the functions to sde to create an object (obj) of class sde:

```
obj = sde(F, G) % dX = F(t,X)dt + G(t,X)dW
obj =
    Class SDE: Stochastic Differential Equation
        Dimensions: State = 1, Brownian = 1
        StartTime: 0
        StartState: 1
        Correlation: 1
            Drift: drift rate function F(t,X(t))
        Diffusion: diffusion rate function G(t,X(t))
        Simulation: simulation method/function simByEuler
```

obj displays like a MATLAB® structure, with the following information:

- The object's class
- A brief description of the object
- A summary of the dimensionality of the model

The object's displayed parameters are as follows:

- StartTime: The initial observation time (real-valued scalar)
- StartState: The initial state vector (NVARS-by-1 column vector)
- Correlation: The correlation structure between Brownian process
- Drift: The drift-rate function $F\left(t, X_{t}\right)$
- Diffusion: The diffusion-rate function $G\left(t, X_{t}\right)$
- Simulation: The simulation method or function.

Of these displayed parameters, only Drift and Diffusion are required inputs.
The only exception to the $\left({ }^{t}, X_{t}\right)$ evaluation interface is Correlation. Specifically, when you enter Correlation as a function, the SDE engine assumes that it is a deterministic function of time, $C(t)$. This restriction on Correlation as a deterministic function of time allows Cholesky factors to be computed and stored before the formal simulation. This inconsistency dramatically improves run-time performance for dynamic correlation structures. If Correlation is stochastic, you can also include it within the simulation architecture as part of a more general random number generation function.

## Definitions

## Instance Hierarchy

There are inheritance relationships among the SDE classes.
The following figure illustrates the inheritance relationships.


For more information, see "SDE Class Hierarchy" on page 18-5.

## Algorithms

When you specify the required input parameters as arrays, they are associated with a specific parametric form. By contrast, when you specify either required input parameter as a function, you can customize virtually any specification.

Accessing the output parameters with no inputs simply returns the original input specification. Thus, when you invoke these parameters with no inputs, they behave like simple properties and allow you to test the data type (double vs. function, or equivalently, static vs. dynamic) of the original input specification. This is useful for validating and designing methods.

When you invoke these parameters with inputs, they behave like functions, giving the impression of dynamic behavior. The parameters accept the observation time $t$ and a state vector $X_{t}$, and return an array of appropriate dimension. Even if you originally specified
an input as an array, sde treats it as a static function of time and state, by that means guaranteeing that all parameters are accessible by the same interface.

## References

[1] Ait-Sahalia, Y. "Testing Continuous-Time Models of the Spot Interest Rate." The Review of Financial Studies, Spring 1996, Vol. 9, No. 2, pp. 385-426.
[2] Ait-Sahalia, Y. "Transition Densities for Interest Rate and Other Nonlinear Diffusions." The Journal of Finance, Vol. 54, No. 4, August 1999.
[3] Glasserman, P. Monte Carlo Methods in Financial Engineering. New York, SpringerVerlag, 2004.
[4] Hull, J. C. Options, Futures, and Other Derivatives, 5th ed. Englewood Cliffs, NJ: Prentice Hall, 2002.
[5] Johnson, N. L., S. Kotz, and N. Balakrishnan. Continuous Univariate Distributions. Vol. 2, 2nd ed. New York, John Wiley \& Sons, 1995.
[6] Shreve, S. E. Stochastic Calculus for Finance II: Continuous-Time Models. New York: Springer-Verlag, 2004.

## See Also

diffusion|drift

## Topics

"Base SDE Models" on page 18-17
Representing Market Models Using SDE Objects on page 18-35
"Simulating Equity Prices" on page 18-35
"Simulating Interest Rates" on page 18-60
"Stratified Sampling" on page 18-71
"Pricing American Basket Options by Monte Carlo Simulation" on page 18-85
"Base SDE Models" on page 18-17
"Drift and Diffusion Models" on page 18-20
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"SDEs" on page 18-2
"SDE Models" on page 18-9
"SDE Class Hierarchy" on page 18-5
"Performance Considerations" on page 18-77
Introduced in R2008a

19-130

## sdeddo

Stochastic Differential Equation (SDE) model from Drift and Diffusion components

## Description

Creates and displays sdeddo objects, instantiated with objects of classdrift and diffusion. These restricted sdeddo objects contain the input drift and diffusion objects; therefore, you can directly access their displayed parameters.

This abstraction also generalizes the notion of drift and diffusion-rate objects as functions that sdeddo evaluates for specific values of time $t$ and state $X_{t}$. Like sde objects, sdeddo objects allow you to simulate sample paths of NVARS state variables driven by NBROWNS Brownian motion sources of risk over NPERIODS consecutive observation periods, approximating continuous-time stochastic processes.

This method enables you to simulate any vector-valued SDEDDO of the form:

$$
\begin{equation*}
d X_{t}=F\left(t, X_{t}\right) d t+G\left(t, X_{t}\right) d W_{t} \tag{19-3}
\end{equation*}
$$

where:

- $X_{t}$ is an NVARS-by-1 state vector of process variables.
- $d W_{t}$ is an NBROWNS-by-1 Brownian motion vector.
- $F$ is an NVARS-by-1 vector-valued drift-rate function.
- $G$ is an NVARS-by-NBROWNS matrix-valued diffusion-rate function.


## Creation

## Syntax

```
SDEDDO = sdeddo(DriftRate,DiffusionRate)
SDEDDO = sdeddo( ___ ,Name,Value)
```


## Description

SDEDDO = sdeddo(DriftRate,DiffusionRate) creates a default SDEDDO object.
SDEDDO = sdeddo (__ , Name, Value) creates a SDEDDO object with additional options specified by one or more Name, Value pair arguments.

Name is a property name and Value is its corresponding value. Name must appear inside single quotes (' ' ). You can specify several name-value pair arguments in any order as Name1, Value1, ..., NameN , ValueN.

The SDEDDO object has the following displayed "Properties" on page 19-133:

- StartTime - Initial observation time
- StartState - Initial state at time StartTime
- Correlation - Access function for the Correlation input argument, callable as a function of time
- Drift - Composite drift-rate function, callable as a function of time and state
- Diffusion - Composite diffusion-rate function, callable as a function of time and state
- A - Access function for the drift-rate property $A$, callable as a function of time and state
- B - Access function for the drift-rate property B, callable as a function of time and state
- Alpha - Access function for the diffusion-rate property Alpha, callable as a function of time and state
- Sigma - Access function for the diffusion-rate property Sigma, callable as a function of time and state
- Simulation - A simulation function or method


## Input Arguments

## DriftRate - DriftRate is a user-defined drift-rate function and represents the parameter $F$ <br> vector or object of class Drift

DriftRate is a user-defined drift-rate function and represents the parameter $F$, specified as a vector or object of class drift.

DriftRate is a function that returns an NVARS-by-1 drift-rate vector when called with two inputs:

- A real-valued scalar observation time $t$.
- An NVARS-by-1 state vector $X_{t}$.

Alternatively, DriftRate can also be an object of class drift that encapsulates the driftrate specification. In this case, however, sde uses only the Rate parameter of the object. For more information on the drift object, see drift.

## Data Types: double

## DiffusionRate - DiffusionRate is a user-defined drift-rate function and represents the parameter $\boldsymbol{G}$

matrix or object of class Diffusion
DiffusionRate is a user-defined drift-rate function and represents the parameter $G$, specified as a matrix or object of class diffusion.

DiffusionRate is a function that returns an NVARS-by-NBROWNS diffusion-rate matrix when called with two inputs:

- A real-valued scalar observation time $t$.
- An NVARS-by-1 state vector $X_{t}$.

Alternatively, DiffusionRate can also be an object of class diffusion that encapsulates the diffusion-rate specification. In this case, however, sde uses only the Rate parameter of the object. For more information on the diffusion object, see diffusion.

Data Types: double

## Properties

StartTime - Starting time of first observation, applied to all state variables 0 (default) | scalar

Starting time of first observation, applied to all state variables, specified as a scalar
Data Types: double

## StartState - Initial values of state variables <br> 1 (default) | scalar, column vector, or matrix

Initial values of state variables, specified as a scalar, column vector, or matrix.
If StartState is a scalar, sdeddo applies the same initial value to all state variables on all trials.

If StartState is a column vector, sdeddo applies a unique initial value to each state variable on all trials.

If StartState is a matrix, sdeddo applies a unique initial value to each state variable on each trial.

Data Types: double

## Correlation - Correlation between Gaussian random variates drawn to generate the Brownian motion vector (Wiener processes)

NBROWNS-by-NBROWNS identity matrix representing independent Gaussian processes (default) | positive semidefinite matrix | deterministic function

Correlation between Gaussian random variates drawn to generate the Brownian motion vector (Wiener processes), specified as an NBROWNS-by-NBROWNS positive semidefinite matrix, or as a deterministic function $C(t)$ that accepts the current time $t$ and returns an NBROWNS-by-NBROWNS positive semidefinite correlation matrix.

A Correlation matrix represents a static condition.
As a deterministic function of time, Correlation allows you to specify a dynamic correlation structure.
Data Types: double

## Simulation - User-defined simulation function or SDE simulation method simulation by Euler approximation (simByEuler) (default) | function | SDE simulation method

User-defined simulation function or SDE simulation method, specified as a function or SDE simulation method.

Data Types: function_handle

## Drift - Drift rate component of continuous-time stochastic differential equations (SDEs)

value stored from drift-rate function (default) | drift object or function accessible by ( $t, X_{t}$ )
This property is read-only.
Drift rate component of continuous-time stochastic differential equations (SDEs), specified as a drift object or function accessible by ( $t, X_{t}$.

The drift rate specification supports the simulation of sample paths of NVARS state variables driven by NBROWNS Brownian motion sources of risk over NPERIODS consecutive observation periods, approximating continuous-time stochastic processes.

The drift class allows you to create drift-rate objects using drift of the form:

$$
F\left(t, X_{t}\right)=A(t)+B(t) X_{t}
$$

where:

- A is an NVARS-by-1 vector-valued function accessible using the $\left(t, X_{t}\right)$ interface.
- B is an NVARS-by-NVARS matrix-valued function accessible using the $\left(t, X_{t}\right)$ interface.

The displayed parameters for a drift object are:

- Rate: The drift-rate function, $F\left(t, X_{t}\right)$
- A: The intercept term, $A\left(t, X_{t}\right)$, of $F\left(t, X_{t}\right)$
- B: The first order term, $B\left(t, X_{t}\right)$, of $F\left(t, X_{t}\right)$
$A$ and $B$ enable you to query the original inputs. The function stored in Rate fully encapsulates the combined effect of $A$ and $B$.

When specified as MATLAB double arrays, the inputs $A$ and $B$ are clearly associated with a linear drift rate parametric form. However, specifying either A or B as a function allows you to customize virtually any drift rate specification.

Note You can express drift and diffusion classes in the most general form to emphasize the functional $\left(t, X_{t}\right)$ interface. However, you can specify the components A and B as functions that adhere to the common ( $t, X_{t}$ ) interface, or as MATLAB arrays of appropriate dimension.

## Example: $F=\operatorname{drift}(0,0.1) \%$ Drift rate function $F(t, X)$ <br> Data Types: struct \| double

## Diffusion - Diffusion rate component of continuous-time stochastic differential equations (SDEs)

value stored from diffusion-rate function (default) | diffusion object or functions accessible by $\left(t, X_{t}\right)$

This property is read-only.
Diffusion rate component of continuous-time stochastic differential equations (SDEs), specified as a drift object or function accessible by ( $t, X_{t}$.

The diffusion rate specification supports the simulation of sample paths of NVARS state variables driven by NBROWNS Brownian motion sources of risk over NPERIODS consecutive observation periods, approximating continuous-time stochastic processes.

The diffusion class allows you to create diffusion-rate objects using diffusion:

$$
G\left(t, X_{t}\right)=D\left(t, X_{t}^{\alpha(t)}\right) V(t)
$$

where:

- D is an NVARS-by-NVARS diagonal matrix-valued function.
- Each diagonal element of $D$ is the corresponding element of the state vector raised to the corresponding element of an exponent Alpha, which is an NVARS-by-1 vectorvalued function.
- $V$ is an NVARS-by-NBROWNS matrix-valued volatility rate function Sigma.
- Alpha and Sigma are also accessible using the $\left(t, X_{t}\right)$ interface.

The displayed parameters for a diffusion object are:

- Rate: The diffusion-rate function, $G\left(t, X_{t}\right)$.
- Alpha: The state vector exponent, which determines the format of $D\left(t, X_{t}\right)$ of $G\left(t, X_{t}\right)$.
- Sigma: The volatility rate, $V\left(t, X_{t}\right)$, of $G\left(t, X_{t}\right)$.

Alpha and Sigma enable you to query the original inputs. (The combined effect of the individual Alpha and Sigma parameters is fully encapsulated by the function stored in Rate.) The Rate functions are the calculation engines for the drift and diffusion objects, and are the only parameters required for simulation.

Note You can express drift and diffusion classes in the most general form to emphasize the functional $\left(t, X_{t}\right)$ interface. However, you can specify the components A and B as functions that adhere to the common $\left(t, X_{t}\right)$ interface, or as MATLAB arrays of appropriate dimension.

Example: $G=$ diffusion(1, 0.3 ) \% Diffusion rate function $G(t, X)$
Data Types: struct | double

## Object Functions

interpolate Brownian interpolation of stochastic differential equations simulate Simulate multivariate stochastic differential equations (SDEs) simByEuler Euler simulation of stochastic differential equations (SDEs)

## Examples

## Create a sdeddo Object

The sdeddo class derives from the base sde class. To use this class, you must pass drift and diffusion-rate objects to the sdeddo function.

Create drift and diffusion rate objects:

```
F = drift(0, 0.1); % Drift rate function F(t,X)
G = diffusion(1, 0.3); % Diffusion rate function G(t,X)
```

Pass the functions to the sdeddo function to create an object obj of class sdeddo:

```
obj = sdeddo(F, G) % dX = F(t,X)dt + G(t,X)dW
obj =
    Class SDEDDO: SDE from Drift and Diffusion Objects
    Dimensions: State = 1, Brownian = 1
        StartTime: 0
        StartState: 1
        Correlation: 1
            Drift: drift rate function F(t,X(t))
```

```
Diffusion: diffusion rate function G(t,X(t))
Simulation: simulation method/function simByEuler
    A: 0
    B: 0.1
    Alpha: 1
    Sigma: 0.3
```

In this example, the object displays the additional parameters associated with input drift and diffusion objects.

## Definitions

## Instance Hierarchy

There are inheritance relationships among the SDE classes.
The following figure illustrates the inheritance relationships.


For more information, see "SDE Class Hierarchy" on page 18-5.

## Algorithms

When you specify the required input parameters as arrays, they are associated with a specific parametric form. By contrast, when you specify either required input parameter as a function, you can customize virtually any specification.

Accessing the output parameters with no inputs simply returns the original input specification. Thus, when you invoke these parameters with no inputs, they behave like simple properties and allow you to test the data type (double vs. function, or equivalently, static vs. dynamic) of the original input specification. This is useful for validating and designing methods.

When you invoke these parameters with inputs, they behave like functions, giving the impression of dynamic behavior. The parameters accept the observation time $t$ and a state vector $X_{t}$, and return an array of appropriate dimension. Even if you originally specified an input as an array, sdeddo treats it as a static function of time and state, by that means guaranteeing that all parameters are accessible by the same interface.

## References

[1] Ait-Sahalia, Y. "Testing Continuous-Time Models of the Spot Interest Rate." The Review of Financial Studies, Spring 1996, Vol. 9, No. 2, pp. 385-426.
[2] Ait-Sahalia, Y. "Transition Densities for Interest Rate and Other Nonlinear Diffusions." The Journal of Finance, Vol. 54, No. 4, August 1999.
[3] Glasserman, P. Monte Carlo Methods in Financial Engineering. New York, SpringerVerlag, 2004.
[4] Hull, J. C. Options, Futures, and Other Derivatives, 5th ed. Englewood Cliffs, NJ: Prentice Hall, 2002.
[5] Johnson, N. L., S. Kotz, and N. Balakrishnan. Continuous Univariate Distributions. Vol. 2, 2nd ed. New York, John Wiley \& Sons, 1995.
[6] Shreve, S. E. Stochastic Calculus for Finance II: Continuous-Time Models. New York: Springer-Verlag, 2004.

## See Also

diffusion|drift|interpolate| sdeld|simByEuler|simulate

## Topics

"Drift and Diffusion Models" on page 18-20
"Representing Market Models Using SDEDDO Objects" on page 18-37
"Representing Market Models Using SDE Objects" on page 18-35
"Simulating Equity Prices" on page 18-35
"Simulating Interest Rates" on page 18-60
"Stratified Sampling" on page 18-71
"Pricing American Basket Options by Monte Carlo Simulation" on page 18-85
"Base SDE Models" on page 18-17
"Drift and Diffusion Models" on page 18-20
"Linear Drift Models" on page 18-24
"Parametric Models" on page 18-26
"SDEs" on page 18-2
"SDE Models" on page 18-9
"SDE Class Hierarchy" on page 18-5
"Performance Considerations" on page 18-77

## Introduced in R2008a

## sdeld

SDE with Linear Drift model

## Description

Creates and displays SDE objects whose drift rate is expressed in linear drift-rate form and that derive from the sdeddo (SDE from drift and diffusion objects class).

Use sdeld objects to simulate sample paths of NVARS state variables expressed in linear drift-rate form. They provide a parametric alternative to the mean-reverting drift form (see sdemrd).

These state variables are driven by NBROWNS Brownian motion sources of risk over NPERIODS consecutive observation periods, approximating continuous-time stochastic processes with linear drift-rate functions.

The sdeld object allows you to simulate any vector-valued SDELD of the form:

$$
d X_{t}=\left(A(t)+B(t) X_{t}\right) d t+D\left(t, X_{t}^{\alpha(t)}\right) V(t) d W_{t}
$$

where:

- $X_{t}$ is an NVARS-by-1 state vector of process variables.
- $A$ is an NVARS-by-1 vector.
- $B$ is an NVARS-by-NVARS matrix.
- $D$ is an NVARS-by-NVARS diagonal matrix, where each element along the main diagonal is the corresponding element of the state vector raised to the corresponding power of $\alpha$.
- $V$ is an NVARS-by-NBROWNS instantaneous volatility rate matrix.
- $d W_{t}$ is an NBROWNS-by-1 Brownian motion vector.


## Creation

## Syntax

SDELD $=$ sdeld (A, B, Alpha, Sigma)
SDELD = sdeld( $\qquad$ , Name, Value)

## Description

SDELD $=\operatorname{sdeld}(A, B, A l p h a, S i g m a)$ creates a default SDELD object.
SDELD = sdeld( _ , , Name, Value) creates a SDELD object with additional options specified by one or more Name, Value pair arguments.

Name is a property name and Value is its corresponding value. Name must appear inside single quotes (' ' ). You can specify several name-value pair arguments in any order as Name1, Value1, ..., NameN, ValueN.

The SDELD object has the following displayed "Properties" on page 19-145:

- StartTime - Initial observation time
- StartState - Initial state at time StartTime
- Correlation - Access function for the Correlation input argument, callable as a function of time
- Drift - Composite drift-rate function, callable as a function of time and state
- Diffusion - Composite diffusion-rate function, callable as a function of time and state
- A - Access function for the input argument A, callable as a function of time and state
- B - Access function for the input argument B, callable as a function of time and state
- Alpha - Access function for the input argument Alpha, callable as a function of time and state
- Sigma - Access function for the input argument Sigma, callable as a function of time and state
- Simulation - A simulation function or method


## Input Arguments

## A - A represents the parameter A

array or deterministic function of time or deterministic function of time and state
A represents the parameter $A$, specified as an array or deterministic function of time.
If you specify A as an array, it must be an NVARS-by-1 column vector of intercepts.
As a deterministic function of time, when $A$ is called with a real-valued scalar time $t$ as its only input, A must produce an NVARS-by- 1 column vector. If you specify A as a function of time and state, it must generate an NVARS-by-1 column vector of intercepts when invoked with two inputs:

- A real-valued scalar observation time $t$.
- An NVARS-by-1 state vector $X_{t}$.

Data Types: double | function_handle

## B - B represents the parameter B

array or deterministic function of time or deterministic function of time and state
$B$ represents the parameter $B$, specified as an array or deterministic function of time.
If you specify A as an array, it must be an NVARS-by-NVARS matrix of state vector coefficients.

As a deterministic function of time, when $B$ is called with a real-valued scalar time $t$ as its only input, B must produce an NVARS-by-NVARS matrix. If you specify B as a function of time and state, it must generate an NVARS-by-NVARS matrix of state vector coefficients when invoked with two inputs:

- A real-valued scalar observation time $t$.
- An NVARS-by- 1 state vector $X_{t}$.

Data Types: double | function_handle

## Alpha - Alpha represents the parameter D <br> array or deterministic function of time or deterministic function of time and state

Alpha represents the parameter $D$, specified as an array or deterministic function of time.

If you specify Alpha as an array, it represents an NVARS-by-1 column vector of exponents.
As a deterministic function of time, when Alpha is called with a real-valued scalar time $t$ as its only input, Alpha must produce an NVARS-by-1 matrix.

If you specify it as a function of time and state, Alpha must return an NVARS-by-1 column vector of exponents when invoked with two inputs:

- A real-valued scalar observation time $t$.
- An NVARS-by-1 state vector $X_{t}$.


## Data Types: double | function_handle

## Sigma - Sigma represents the parameter V array or deterministic function of time or deterministic function of time and state

Sigma represents the parameter $V$, specified as an array or a deterministic function of time.

If you specify Sigma as an array, it must be an NVARS-by-NBROWNS matrix of instantaneous volatility rates or as a deterministic function of time. In this case, each row of Sigma corresponds to a particular state variable. Each column corresponds to a particular Brownian source of uncertainty, and associates the magnitude of the exposure of state variables with sources of uncertainty.

As a deterministic function of time, when Sigma is called with a real-valued scalar time $t$ as its only input, Sigma must produce an NVARS-by-NBROWNS matrix. If you specify Sigma as a function of time and state, it must return an NVARS-by-NBROWNS matrix of volatility rates when invoked with two inputs:

- A real-valued scalar observation time $t$.
- An NVARS-by-1 state vector $X_{t}$.

Although thegbm constructor enforces no restrictions on the sign of Sigma volatilities, they are specified as positive values.

Data Types: double | function_handle

Note Although sdeld does not enforce restrictions on the signs of Alpha or Sigma, each parameter is specified as a positive value.

## Properties

## StartTime - Starting time of first observation, applied to all state variables 0 (default) | scalar

Starting time of first observation, applied to all state variables, specified as a scalar Data Types: double

## StartState - Initial values of state variables

1 (default) | scalar, column vector, or matrix
Initial values of state variables, specified as a scalar, column vector, or matrix.
If StartState is a scalar, sdeld applies the same initial value to all state variables on all trials.

If StartState is a column vector, sdeld applies a unique initial value to each state variable on all trials.

If StartState is a matrix, sdeld applies a unique initial value to each state variable on each trial.

## Data Types: double

## Correlation - Correlation between Gaussian random variates drawn to generate the Brownian motion vector (Wiener processes)

NBROWNS-by-NBROWNS identity matrix representing independent Gaussian processes (default) | positive semidefinite matrix | deterministic function

Correlation between Gaussian random variates drawn to generate the Brownian motion vector (Wiener processes), specified as an NBROWNS-by-NBROWNS positive semidefinite matrix, or as a deterministic function $C(t)$ that accepts the current time $t$ and returns an NBROWNS-by-NBROWNS positive semidefinite correlation matrix.

A Correlation matrix represents a static condition.
As a deterministic function of time, Correlation allows you to specify a dynamic correlation structure.

Data Types: double

## Simulation - User-defined simulation function or SDE simulation method simulation by Euler approximation (simByEuler) (default) | function \| SDE simulation method

User-defined simulation function or SDE simulation method, specified as a function or SDE simulation method.

## Data Types: function_handle

## Drift - Drift rate component of continuous-time stochastic differential equations (SDEs)

value stored from drift-rate function (default) | drift object or function accessible by $\left(t, X_{t}\right)$
This property is read-only.
Drift rate component of continuous-time stochastic differential equations (SDEs), specified as a drift object or function accessible by ( $t, X_{t}$.

The drift rate specification supports the simulation of sample paths of NVARS state variables driven by NBROWNS Brownian motion sources of risk over NPERIODS consecutive observation periods, approximating continuous-time stochastic processes.

The drift class allows you to create drift-rate objects using drift of the form:

$$
F\left(t, X_{t}\right)=A(t)+B(t) X_{t}
$$

where:

- A is an NVARS-by-1 vector-valued function accessible using the $\left(t, X_{t}\right)$ interface.
- B is an NVARS-by-NVARS matrix-valued function accessible using the $\left(t, X_{t}\right)$ interface.

The displayed parameters for a drift object are:

- Rate: The drift-rate function, $F\left(t, X_{t}\right)$
- A: The intercept term, $A\left(t, X_{t}\right)$, of $F\left(t, X_{t}\right)$
- B: The first order term, $B\left(t, X_{t}\right)$, of $F\left(t, X_{t}\right)$
$A$ and $B$ enable you to query the original inputs. The function stored in Rate fully encapsulates the combined effect of $A$ and $B$.

When specified as MATLAB double arrays, the inputs $A$ and $B$ are clearly associated with a linear drift rate parametric form. However, specifying either A or B as a function allows you to customize virtually any drift rate specification.

Note You can express drift and diffusion classes in the most general form to emphasize the functional $\left(t, X_{t}\right)$ interface. However, you can specify the components A and $B$ as functions that adhere to the common $\left(t, X_{t}\right)$ interface, or as MATLAB arrays of appropriate dimension.

## Example: $F=\operatorname{drift}(0,0.1) \%$ Drift rate function $F(t, X)$ <br> Data Types: struct | double

## Diffusion - Diffusion rate component of continuous-time stochastic differential equations (SDEs)

value stored from diffusion-rate function (default) | diffusion object or functions accessible by $\left(t, X_{t}\right)$

This property is read-only.
Diffusion rate component of continuous-time stochastic differential equations (SDEs), specified as a drift object or function accessible by $\left(t, X_{t}\right.$.

The diffusion rate specification supports the simulation of sample paths of NVARS state variables driven by NBROWNS Brownian motion sources of risk over NPERIODS consecutive observation periods, approximating continuous-time stochastic processes.

The diffusion class allows you to create diffusion-rate objects using diffusion:

$$
G\left(t, X_{t}\right)=D\left(t, X_{t}^{\alpha(t)}\right) V(t)
$$

where:

- D is an NVARS-by-NVARS diagonal matrix-valued function.
- Each diagonal element of $D$ is the corresponding element of the state vector raised to the corresponding element of an exponent Alpha, which is an NVARS-by-1 vectorvalued function.
- V is an NVARS-by-NBROWNS matrix-valued volatility rate function Sigma.
- Alpha and Sigma are also accessible using the $\left(t, X_{t}\right)$ interface.

The displayed parameters for a diffusion object are:

- Rate: The diffusion-rate function, $G\left(t, X_{t}\right)$.
- Alpha: The state vector exponent, which determines the format of $D\left(t, X_{t}\right)$ of $G\left(t, X_{t}\right)$.
- Sigma: The volatility rate, $V\left(t, X_{t}\right)$, of $G\left(t, X_{t}\right)$.

Alpha and Sigma enable you to query the original inputs. (The combined effect of the individual Alpha and Sigma parameters is fully encapsulated by the function stored in Rate.) The Rate functions are the calculation engines for the drift and diffusion objects, and are the only parameters required for simulation.

Note You can express drift and diffusion classes in the most general form to emphasize the functional $\left(t, X_{t}\right)$ interface. However, you can specify the components A and $B$ as functions that adhere to the common $\left(t, X_{t}\right)$ interface, or as MATLAB arrays of appropriate dimension.

Example: $G=$ diffusion(1, 0.3) \% Diffusion rate function $G(t, X)$
Data Types: struct \| double

## Object Functions

interpolate Brownian interpolation of stochastic differential equations
simulate Simulate multivariate stochastic differential equations (SDEs)
simByEuler Euler simulation of stochastic differential equations (SDEs)

## Examples

## Create a sdeld Object

The sdeld class derives from the sdeddo class. These objects allow you to simulate correlated paths of NVARS state variables expressed in linear drift-rate form:

```
dX}\mp@subsup{X}{t}{}=(A(t)+B(t)\mp@subsup{X}{t}{})dt+D(t,\mp@subsup{X}{t}{\alpha(t)})V(t)d\mp@subsup{W}{t}{}
obj = sdeld(0, 0.1, 1, 0.3) % (A, B, Alpha, Sigma)
```

```
obj =
Class SDELD: SDE with Linear Drift
    Dimensions: State = 1, Brownian = 1
        StartTime: 0
    StartState: 1
    Correlation: 1
        Drift: drift rate function F(t,X(t))
        Diffusion: diffusion rate function G(t,X(t))
        Simulation: simulation method/function simByEuler
            A: 0
            B: 0.1
            Alpha: 1
            Sigma: 0.3
```

sdeld objects provide a parametric alternative to the mean-reverting drift form and also provide an alternative interface to the sdeddo parent class, because you can create an object without first having to create its drift and diffusion-rate components.

## Definitions

## Instance Hierarchy

There are inheritance relationships among the SDE classes.
The following figure illustrates the inheritance relationships.


For more information, see "SDE Class Hierarchy" on page 18-5.

## Algorithms

When you specify the required input parameters as arrays, they are associated with a specific parametric form. By contrast, when you specify either required input parameter as a function, you can customize virtually any specification.

Accessing the output parameters with no inputs simply returns the original input specification. Thus, when you invoke these parameters with no inputs, they behave like simple properties and allow you to test the data type (double vs. function, or equivalently, static vs. dynamic) of the original input specification. This is useful for validating and designing methods.

When you invoke these parameters with inputs, they behave like functions, giving the impression of dynamic behavior. The parameters accept the observation time $t$ and a state vector $X_{t}$, and return an array of appropriate dimension. Even if you originally specified
an input as an array, sdeld treats it as a static function of time and state, by that means guaranteeing that all parameters are accessible by the same interface.

## References

[1] Ait-Sahalia, Y. "Testing Continuous-Time Models of the Spot Interest Rate." The Review of Financial Studies, Spring 1996, Vol. 9, No. 2, pp. 385-426.
[2] Ait-Sahalia, Y. "Transition Densities for Interest Rate and Other Nonlinear Diffusions." The Journal of Finance, Vol. 54, No. 4, August 1999.
[3] Glasserman, P. Monte Carlo Methods in Financial Engineering. New York, SpringerVerlag, 2004.
[4] Hull, J. C. Options, Futures, and Other Derivatives, 5th ed. Englewood Cliffs, NJ: Prentice Hall, 2002.
[5] Johnson, N. L., S. Kotz, and N. Balakrishnan. Continuous Univariate Distributions. Vol. 2, 2nd ed. New York, John Wiley \& Sons, 1995.
[6] Shreve, S. E. Stochastic Calculus for Finance II: Continuous-Time Models. New York: Springer-Verlag, 2004.

## See Also

diffusion|drift|sdeddo|simByEuler

## Topics

"Linear Drift Models" on page 18-24
Implementing Multidimensional Equity Market Models, Implementation 3: Using SDELD,
CEV, and GBM Objects on page 18-38
"Simulating Equity Prices" on page 18-35
"Simulating Interest Rates" on page 18-60
"Stratified Sampling" on page 18-71
"Pricing American Basket Options by Monte Carlo Simulation" on page 18-85
"Base SDE Models" on page 18-17
"Drift and Diffusion Models" on page 18-20
"Linear Drift Models" on page 18-24
"Parametric Models" on page 18-26
"SDEs" on page 18-2
"SDE Models" on page 18-9
"SDE Class Hierarchy" on page 18-5
"Performance Considerations" on page 18-77

## Introduced in R2008a

## sdemrd

SDE with Mean-Reverting Drift model

## Description

Creates and displays SDE objects whose drift rate is expressed in mean-reverting driftrate form and which derive from the sdeddo class (SDE from drift and diffusion objects).

Use sdemrd objects to simulate of sample paths of NVARS state variables expressed in mean-reverting drift-rate form, and provide a parametric alternative to the linear drift form (see sdeld). These state variables are driven by NBROWNS Brownian motion sources of risk over NPERIODS consecutive observation periods, approximating continuous-time stochastic processes with mean-reverting drift-rate functions.

The sdemrd object allows you to simulate any vector-valued SDEMRD of the form:

$$
d X_{t}=S(t)\left[L(t)-X_{t}\right] d t+D\left(t, X_{t}^{\alpha(t)}\right) V(t) d W_{t}
$$

where:

- $X_{t}$ is an NVARS-by-1 state vector of process variables.
- $S$ is an NVARS-by-NVARS matrix of mean reversion speeds.
- $L$ is an NVARS-by-1 vector of mean reversion levels.
- $D$ is an NVARS-by-NVARS diagonal matrix, where each element along the main diagonal is the corresponding element of the state vector raised to the corresponding power of $\alpha$.
- $V$ is an NVARS-by-NBROWNS instantaneous volatility rate matrix.
- $d W_{t}$ is an NBROWNS-by-1 Brownian motion vector.


## Creation

## Syntax

```
SDEMRD = sdemrd(Speed,Level,Alpha,Sigma)
SDEMRD = sdemrd( , Name, Value)
```


## Description

SDEMRD = sdemrd(Speed,Level,Alpha,Sigma) creates a default SDEMRD object.
SDEMRD = sdemrd (___ , Name, Value) creates a SDEMRD object with additional options specified by one or more Name, Value pair arguments.

Name is a property name and Value is its corresponding value. Name must appear inside single quotes (' ' ). You can specify several name-value pair arguments in any order as Name1, Value1, ..., NameN, ValueN.

The SDELD object has the following displayed "Properties" on page 19-157:

- StartTime - Initial observation time
- StartState - Initial state at time StartTime
- Correlation - Access function for the Correlation input argument, callable as a function of time
- Drift - Composite drift-rate function, callable as a function of time and state
- Diffusion - Composite diffusion-rate function, callable as a function of time and state
- Speed - Access function for the input argument Speed, callable as a function of time and state
- Level - Access function for the input argument Level, callable as a function of time and state
- Alpha - Access function for the input argument Alpha, callable as a function of time and state
- Sigma - Access function for the input argument Sigma, callable as a function of time and state
- Simulation - A simulation function or method


## Input Arguments

## Speed - Speed represents the parameter S array or deterministic function of time or deterministic function of time and state

Speed represents the parameter $S$, specified as an array or deterministic function of time.
If you specify Speed as an array, it must be an NVARS-by-NVARS matrix of mean-reversion speeds (the rate at which the state vector reverts to its long-run average Level).

As a deterministic function of time, when Speed is called with a real-valued scalar time t as its only input, Speed must produce an NVARS-by-NVARS matrix. If you specify Speed as a function of time and state, it calculates the speed of mean reversion. This function must generate an NVARS-by-NVARS matrix of reversion rates when called with two inputs:

- A real-valued scalar observation time $t$.
- An NVARS-by-1 state vector $X_{t}$.

Data Types: double | function_handle

## Level - Level represents the parameter L

array or deterministic function of time or deterministic function of time and state
Level represents the parameter $L$, specified as an array or deterministic function of time.
If you specify Level as an array, it must be an NVARS-by-1 column vector of reversion levels.

As a deterministic function of time, when Level is called with a real-valued scalar time $t$ as its only input, Level must produce an NVARS-by-1 column vector. If you specify Level as a function of time and state, it must generate an NVARS-by-1 column vector of reversion levels when called with two inputs:

- A real-valued scalar observation time $t$.
- An NVARS-by-1 state vector $X_{t}$.

Data Types: double | function_handle

## Alpha - Alpha represents the parameter D <br> array or deterministic function of time or deterministic function of time and state

Alpha represents the parameter $D$, specified as an array or deterministic function of time.

If you specify Alpha as an array, it represents an NVARS-by-1 column vector of exponents.
As a deterministic function of time, when Alpha is called with a real-valued scalar time $t$ as its only input, Alpha must produce an NVARS-by-1 matrix.

If you specify it as a function of time and state, Alpha must return an NVARS-by-1 column vector of exponents when invoked with two inputs:

- A real-valued scalar observation time $t$.
- An NVARS-by-1 state vector $X_{t}$.

Data Types: double | function_handle

## Sigma - Sigma represents the parameter V

array or deterministic function of time or deterministic function of time and state
Sigma represents the parameter $V$, specified as an array or a deterministic function of time.

If you specify Sigma as an array, it must be an NVARS-by-NBROWNS matrix of instantaneous volatility rates or as a deterministic function of time. In this case, each row of Sigma corresponds to a particular state variable. Each column corresponds to a particular Brownian source of uncertainty, and associates the magnitude of the exposure of state variables with sources of uncertainty.

As a deterministic function of time, when Sigma is called with a real-valued scalar time $t$ as its only input, Sigma must produce an NVARS-by-NBROWNS matrix. If you specify Sigma as a function of time and state, it must return an NVARS-by-NBROWNS matrix of volatility rates when invoked with two inputs:

- A real-valued scalar observation time $t$.
- An NVARS-by-1 state vector $X_{t}$.

Data Types: double | function_handle

Note Although sdemrd does not enforce restrictions on the signs of Alpha or Sigma, each parameter is specified as a positive value.

## Properties

## StartTime - Starting time of first observation, applied to all state variables 0 (default) | scalar

Starting time of first observation, applied to all state variables, specified as a scalar Data Types: double

## StartState - Initial values of state variables

1 (default) | scalar, column vector, or matrix
Initial values of state variables, specified as a scalar, column vector, or matrix.
If StartState is a scalar, sdemrd applies the same initial value to all state variables on all trials.

If StartState is a column vector, sdemrd applies a unique initial value to each state variable on all trials.

If StartState is a matrix, sdemrd applies a unique initial value to each state variable on each trial.

## Data Types: double

## Correlation - Correlation between Gaussian random variates drawn to generate the Brownian motion vector (Wiener processes)

NBROWNS-by-NBROWNS identity matrix representing independent Gaussian processes (default) | positive semidefinite matrix | deterministic function

Correlation between Gaussian random variates drawn to generate the Brownian motion vector (Wiener processes), specified as an NBROWNS-by-NBROWNS positive semidefinite matrix, or as a deterministic function $C(t)$ that accepts the current time $t$ and returns an NBROWNS-by-NBROWNS positive semidefinite correlation matrix.

A Correlation matrix represents a static condition.
As a deterministic function of time, Correlation allows you to specify a dynamic correlation structure.

Data Types: double

## Simulation - User-defined simulation function or SDE simulation method simulation by Euler approximation (simByEuler) (default) | function \| SDE simulation method

User-defined simulation function or SDE simulation method, specified as a function or SDE simulation method.

## Data Types: function_handle

## Drift - Drift rate component of continuous-time stochastic differential equations (SDEs)

value stored from drift-rate function (default) | drift object or function accessible by $\left(t, X_{t}\right)$
This property is read-only.
Drift rate component of continuous-time stochastic differential equations (SDEs), specified as a drift object or function accessible by ( $t, X_{t}$.

The drift rate specification supports the simulation of sample paths of NVARS state variables driven by NBROWNS Brownian motion sources of risk over NPERIODS consecutive observation periods, approximating continuous-time stochastic processes.

The drift class allows you to create drift-rate objects using drift of the form:

$$
F\left(t, X_{t}\right)=A(t)+B(t) X_{t}
$$

where:

- A is an NVARS-by-1 vector-valued function accessible using the $\left(t, X_{t}\right)$ interface.
- B is an NVARS-by-NVARS matrix-valued function accessible using the $\left(t, X_{t}\right)$ interface.

The displayed parameters for a drift object are:

- Rate: The drift-rate function, $F\left(t, X_{t}\right)$
- A: The intercept term, $A\left(t, X_{t}\right)$, of $F\left(t, X_{t}\right)$
- B: The first order term, $B\left(t, X_{t}\right)$, of $F\left(t, X_{t}\right)$
$A$ and $B$ enable you to query the original inputs. The function stored in Rate fully encapsulates the combined effect of $A$ and $B$.

When specified as MATLAB double arrays, the inputs $A$ and $B$ are clearly associated with a linear drift rate parametric form. However, specifying either A or B as a function allows you to customize virtually any drift rate specification.

Note You can express drift and diffusion classes in the most general form to emphasize the functional $\left(t, X_{t}\right)$ interface. However, you can specify the components A and $B$ as functions that adhere to the common $\left(t, X_{t}\right)$ interface, or as MATLAB arrays of appropriate dimension.

```
Example: F = drift(0, 0.1) % Drift rate function F(t,X)
Data Types: struct | double
```


## Diffusion - Diffusion rate component of continuous-time stochastic differential equations (SDEs)

value stored from diffusion-rate function (default) | diffusion object or functions accessible by $\left(t, X_{t}\right)$

This property is read-only.
Diffusion rate component of continuous-time stochastic differential equations (SDEs), specified as a drift object or function accessible by ( $t, X_{t}$.

The diffusion rate specification supports the simulation of sample paths of NVARS state variables driven by NBROWNS Brownian motion sources of risk over NPERIODS consecutive observation periods, approximating continuous-time stochastic processes.

The diffusion class allows you to create diffusion-rate objects using diffusion:

$$
G\left(t, X_{t}\right)=D\left(t, X_{t}^{\alpha(t)}\right) V(t)
$$

where:

- D is an NVARS-by-NVARS diagonal matrix-valued function.
- Each diagonal element of $D$ is the corresponding element of the state vector raised to the corresponding element of an exponent Alpha, which is an NVARS-by-1 vectorvalued function.
- $V$ is an NVARS-by-NBROWNS matrix-valued volatility rate function Sigma.
- Alpha and Sigma are also accessible using the $\left(t, X_{t}\right)$ interface.

The displayed parameters for a diffusion object are:

- Rate: The diffusion-rate function, $G\left(t, X_{t}\right)$.
- Alpha: The state vector exponent, which determines the format of $D\left(t, X_{t}\right)$ of $G\left(t, X_{t}\right)$.
- Sigma: The volatility rate, $V\left(t, X_{t}\right)$, of $G\left(t, X_{t}\right)$.

Alpha and Sigma enable you to query the original inputs. (The combined effect of the individual Alpha and Sigma parameters is fully encapsulated by the function stored in Rate.) The Rate functions are the calculation engines for the drift and diffusion objects, and are the only parameters required for simulation.

Note You can express drift and diffusion classes in the most general form to emphasize the functional $\left(t, X_{t}\right)$ interface. However, you can specify the components A and $B$ as functions that adhere to the common $\left(t, X_{t}\right)$ interface, or as MATLAB arrays of appropriate dimension.

Example: G = diffusion(1, 0.3) \% Diffusion rate function G(t,X)
Data Types: struct | double

## Object Functions

interpolate Brownian interpolation of stochastic differential equations simulate Simulate multivariate stochastic differential equations (SDEs) simByEuler Euler simulation of stochastic differential equations (SDEs)

## Examples

## Create a sdemrd Object

The sdemrd class derives directly from the sdeddo class. It provides an interface in which the drift-rate function is expressed in mean-reverting drift form:

$$
d X_{t}=S(t)\left[L(t)-X_{t}\right] d t+D\left(t, X_{t}^{\alpha(t)}\right) V(t) d W_{t} .
$$

sdemrd objects provide a parametric alternative to the linear drift form by reparameterizing the general linear drift such that: $A(t)=S(t) L(t), B(t)=-S(t)$.

Create an sdemrd object obj with a square root exponent to represent the model:

```
dX }=0.2(0.1-\mp@subsup{X}{t}{})dt+0.05\mp@subsup{X}{t}{\frac{1}{2}}d\mp@subsup{W}{t}{}
obj = sdemrd(0.2, 0.1, 0.5, 0.05) %(Speed, Level, Alpha, Sigma)
obj =
    Class SDEMRD: SDE with Mean-Reverting Drift
        Dimensions: State = 1, Brownian = 1
        StartTime: 0
        StartState: 1
        Correlation: 1
            Drift: drift rate function F(t,X(t))
            Diffusion: diffusion rate function G(t,X(t))
        Simulation: simulation method/function simByEuler
            Alpha: 0.5
            Sigma: 0.05
            Level: 0.1
            Speed: 0.2
```

sdemrd objects display the familiar Speed and Level parameters instead of A and B.

## Definitions

## Instance Hierarchy

There are inheritance relationships among the SDE classes.
The following figure illustrates the inheritance relationships.


For more information, see "SDE Class Hierarchy" on page 18-5.

## Algorithms

When you specify the required input parameters as arrays, they are associated with a specific parametric form. By contrast, when you specify either required input parameter as a function, you can customize virtually any specification.

Accessing the output parameters with no inputs simply returns the original input specification. Thus, when you invoke these parameters with no inputs, they behave like simple properties and allow you to test the data type (double vs. function, or equivalently, static vs. dynamic) of the original input specification. This is useful for validating and designing methods.

When you invoke these parameters with inputs, they behave like functions, giving the impression of dynamic behavior. The parameters accept the observation time $t$ and a state vector $X_{t}$, and return an array of appropriate dimension. Even if you originally specified
an input as an array, sdemrd treats it as a static function of time and state, by that means guaranteeing that all parameters are accessible by the same interface.

## References

[1] Ait-Sahalia, Y. "Testing Continuous-Time Models of the Spot Interest Rate." The Review of Financial Studies, Spring 1996, Vol. 9, No. 2, pp. 385-426.
[2] Ait-Sahalia, Y. "Transition Densities for Interest Rate and Other Nonlinear Diffusions." The Journal of Finance, Vol. 54, No. 4, August 1999.
[3] Glasserman, P. Monte Carlo Methods in Financial Engineering. New York, SpringerVerlag, 2004.
[4] Hull, J. C. Options, Futures, and Other Derivatives, 5th ed. Englewood Cliffs, NJ: Prentice Hall, 2002.
[5] Johnson, N. L., S. Kotz, and N. Balakrishnan. Continuous Univariate Distributions. Vol. 2, 2nd ed. New York, John Wiley \& Sons, 1995.
[6] Shreve, S. E. Stochastic Calculus for Finance II: Continuous-Time Models. New York: Springer-Verlag, 2004.

## See Also

diffusion|drift|sdeddo|simByEuler

## Topics

"Creating Stochastic Differential Equations from Mean-Reverting Drift (SDEMRD) Models" on page 18-29
"Simulating Equity Prices" on page 18-35
"Simulating Interest Rates" on page 18-60
"Stratified Sampling" on page 18-71
"Pricing American Basket Options by Monte Carlo Simulation" on page 18-85
"Base SDE Models" on page 18-17
"Drift and Diffusion Models" on page 18-20
"Linear Drift Models" on page 18-24
"Parametric Models" on page 18-26
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"SDE Models" on page 18-9
"SDE Class Hierarchy" on page 18-5
"Performance Considerations" on page 18-77
Introduced in R2008a

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## simByEuler

Euler simulation of stochastic differential equations (SDEs)

## Syntax

```
[Paths,Times,Z] = simByEuler(MDL,NPeriods)
[Paths,Times,Z] = simByEuler(
```

$\qquad$

``` ,Name, Value)
```


## Description

[Paths,Times,Z] = simByEuler(MDL,NPeriods) simulates NTRIALS sample paths of NVARS correlated state variables driven by NBROWNS Brownian motion sources of risk over NPERIODS consecutive observation periods. simByEuler uses the Euler approach to approximate continuous-time stochastic processes.
[Paths,Times,Z] = simByEuler( $\qquad$ ,Name, Value) specifies options using one or more name-value pair arguments in addition to the input arguments in the previous syntax.

## Examples

## Simulate Equity Markets Using simByEuler

## Load the Data and Specify the SDE Model

```
load Data GlobalIdx2
prices = [Dataset.TSX Dataset.CAC Dataset.DAX ...
    Dataset.NIK Dataset.FTSE Dataset.SP];
returns = tick2ret(prices);
nVariables = size(returns,2);
expReturn = mean(returns);
sigma = std(returns);
```

```
correlation = corrcoef(returns);
t = 0;
X = 100;
X = X(ones(nVariables,1));
F = @(t,X) diag(expReturn)* X;
G = @(t,X) diag(X) * diag(sigma);
SDE = sde(F, G, 'Correlation', ...
    correlation, 'StartState', X);
```


## Simulate a Single Path Over a Year

```
nPeriods = 249; % # of simulated observations
dt = 1; % time increment = 1 day
rng(142857,'twister')
[S,T] = simByEuler(SDE, nPeriods, 'DeltaTime', dt);
```


## Simulate 10 trials and examine the SDE model

```
rng(142857,'twister')
[S,T] = simulate(SDE, nPeriods, 'DeltaTime', dt, 'nTrials', 10);
whos S
    Name Size Bytes Class Attributes
    S 250x6x10 120000 double
```


## Plot the paths

```
plot(T, S(:,:,1)), xlabel('Trading Day'), ylabel('Price')
title('First Path of Multi-Dimensional Market Model')
legend({'Canada' 'France' 'Germany' 'Japan' 'UK' 'US'},...
    'Location', 'Best')
```



## Euler Simulation for a CIR Object

The Cox-Ingersoll-Ross (CIR) short rate class derives directly from SDE with meanreverting drift (SDEMRD): $d X_{t}=S(t)\left[L(t)-X_{t}\right] d t+D\left(t, X_{t}^{\frac{1}{2}}\right) V(t) d W$
where $D$ is a diagonal matrix whose elements are the square root of the corresponding element of the state vector.

```
Create a cir object to represent the model: }d\mp@subsup{X}{t}{}=0.2(0.1-\mp@subsup{X}{t}{})dt+0.05\mp@subsup{X}{t}{\frac{1}{2}}d
CIR = cir(0.2, 0.1, 0.05) % (Speed, Level, Sigma)
CIR =
    Class CIR: Cox-Ingersoll-Ross
        Dimensions: State = 1, Brownian = 1
        StartTime: 0
        StartState: 1
    Correlation: 1
            Drift: drift rate function F(t,X(t))
            Diffusion: diffusion rate function G(t,X(t))
            Simulation: simulation method/function simByEuler
            Sigma: 0.05
            Level: 0.1
            Speed: 0.2
```

Simulate a single path over a year using simByEuler.

```
nPeriods = 249; % # of simulated observations
dt = 1; % time increment = 1 day
rng(142857,'twister')
[Paths,Times] = simByEuler(CIR,nPeriods,'Method','higham-mao','DeltaTime', dt)
```

Paths = 250×1
1.0000
0.8613
0.7245
0.6349
0.4741
0.3853
0.3374
0.2549
0.1859
0.1814
Times $=250 \times 1$
0

## Input Arguments

## MDL - Stochastic differential equation model object

Stochastic differential equation model, specified as an sde, bm, gbm, cev, cir, hwv, heston, sdeddo, sdeld, or sdemrd object.

Data Types: object

## NPeriods - Number of simulation periods

positive scalar integer
Number of simulation periods, specified as a positive scalar integer. The value of NPeriods determines the number of rows of the simulated output series.

## Data Types: double

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Example: [Paths,Times,Z] = simByEuler(SDE,NPeriods,'DeltaTime',dt)

## Method - Method to handle negative values

'basic' (default)|character vector with values'basic', 'absorption',

```
string with values "basic", "absorption", "reflection", "partial-truncation",
"full-truncation", or "higham-mao"
```

Method to handle negative values, specified as the comma-separated pair consisting of 'Method ' and a character vector or string with a supported value.

Note The Method argument is only supported when using a CIR object. For more information on creating a CIR object, see cir.

## Data Types: char|string

NTrials - Simulated trials (sample paths) of NPERIODS observations each 1 (single path of correlated state variables) (default) | positive scalar integer

Simulated trials (sample paths) of NPERIODS observations each, specified as the commaseparated pair consisting of 'NTrials' and a positive scalar integer.

Data Types: double

## DeltaTimes - Positive time increments between observations 1 (default) | scalar | column vector

Positive time increments between observations, specified as the comma-separated pair consisting of 'DeltaTimes ' and a scalar or a NPERIODS-by-1 column vector.

DeltaTime represents the familiar $d t$ found in stochastic differential equations, and determines the times at which the simulated paths of the output state variables are reported.

Data Types: double

## NSteps - Number of intermediate time steps within each time increment $\boldsymbol{d t}$ (specified as DeltaTime)

1 (indicating no intermediate evaluation) (default) | positive scalar integer
Number of intermediate time steps within each time increment $d t$ (specified as DeltaTime), specified as the comma-separated pair consisting of 'NSteps ' and a positive scalar integer.

The simByEuler function partitions each time increment $d t$ into NSteps subintervals of length $d t / N S t e p s$, and refines the simulation by evaluating the simulated state vector at

NSteps - 1 intermediate points. Although simByEuler does not report the output state vector at these intermediate points, the refinement improves accuracy by allowing the simulation to more closely approximate the underlying continuous-time process.
Data Types: double

## Antithetic - Flag to indicate whether simByEuler uses antithetic sampling to generate the Gaussian random variates

False (no antithetic sampling) (default) | logical with values True or False
Flag indicates whether simByEuler uses antithetic sampling to generate the Gaussian random variates that drive the Brownian motion vector (Wiener processes). This argument is specified as the comma-separated pair consisting of 'Antithetic' and a scalar logical flag with a value of True or False.

When you specify True, simByEuler performs sampling such that all primary and antithetic paths are simulated and stored in successive matching pairs:

- Odd trials $(1,3,5, \ldots)$ correspond to the primary Gaussian paths.
- Even trials $(2,4,6, \ldots)$ are the matching antithetic paths of each pair derived by negating the Gaussian draws of the corresponding primary (odd) trial.

Note If you specify an input noise process (see Z), simByEuler ignores the value of Antithetic.

## Data Types: logical

## Z - Direct specification of the dependent random noise process used to generate the Brownian motion vector

generates correlated Gaussian variates based on the Correlation member of the SDE object (default) | function | three-dimensional array of dependent random variates

Direct specification of the dependent random noise process used to generate the Brownian motion vector (Wiener process) that drives the simulation. This argument is specified as the comma-separated pair consisting of ' $Z$ ' and a function or as an (NPERIODS $]$ NSTEPS)-by-NBROWNS-by-NTRIALS three-dimensional array of dependent random variates.

Note If you specify Z as a function, it must return an NBROWNS-by-1 column vector, and you must call it with two inputs:

- A real-valued scalar observation time $t$.
- An NVARS-by-1 state vector $X_{t}$.


## Data Types: double | function

## StorePaths - Flag that indicates how the output array Paths is stored and returned

True (default) | logical with values True or False
Flag that indicates how the output array Paths is stored and returned, specified as the comma-separated pair consisting of 'StorePaths' and a scalar logical flag with a value of True or False.

If StorePaths is True (the default value) or is unspecified, simByEuler returns Paths as a three-dimensional time series array.

If StorePaths is False (logical 0), simByEuler returns the Paths output array as an empty matrix.

## Data Types: logical

## Processes - Sequence of end-of-period processes or state vector adjustments

simByEuler makes no adjustments and performs no processing (default) | function | cell array of functions

Sequence of end-of-period processes or state vector adjustments of the form, specified as the comma-separated pair consisting of 'Processes ' and a function or cell array of functions of the form

$$
X_{t}=P\left(t, X_{t}\right)
$$

The simByEuler function runs processing functions at each interpolation time. They must accept the current interpolation time $t$, and the current state vector $X_{t}$, and return a state vector that may be an adjustment to the input state.

If you specify more than one processing function, simByEuler invokes the functions in the order in which they appear in the cell array. You can use this argument to specify boundary conditions, prevent negative prices, accumulate statistics, plot graphs, and more.

Data Types: cell|function

## Output Arguments

## Paths - Simulated paths of correlated state variables array

Simulated paths of correlated state variables, returned as a (NPERIODS + 1)-by-NVARS-by-NTRIALS three-dimensional time series array.

For a given trial, each row of Paths is the transpose of the state vector $X_{t}$ at time $t$. When the input flag StorePaths $=$ False, simByEuler returns Paths as an empty matrix.

## Times - Observation times associated with the simulated paths column vector

Observation times associated with the simulated paths, returned as a (NPERIODS + 1)-by-1 column vector. Each element of Times is associated with the corresponding row of Paths.

## Z - Dependent random variates used to generate the Brownian motion vector array

Dependent random variates used to generate the Brownian motion vector (Wiener processes) that drive the simulation, returned as a (NPERIODS $\square$ NSTEPS) -by-NBROWNS-by-NTRIALS three-dimensional time series array.

## Definitions

## Antithetic Sampling

Simulation methods allow you to specify a popular variance reduction technique called antithetic sampling.

This technique attempts to replace one sequence of random observations with another of the same expected value, but smaller variance. In a typical Monte Carlo simulation, each sample path is independent and represents an independent trial. However, antithetic sampling generates sample paths in pairs. The first path of the pair is referred to as the primary path, and the second as the antithetic path. Any given pair is independent of any other pair, but the two paths within each pair are highly correlated. Antithetic sampling literature often recommends averaging the discounted payoffs of each pair, effectively halving the number of Monte Carlo trials.

This technique attempts to reduce variance by inducing negative dependence between paired input samples, ideally resulting in negative dependence between paired output samples. The greater the extent of negative dependence, the more effective antithetic sampling is.

## Algorithms

This function simulates any vector-valued SDE of the form

$$
d X_{t}=F\left(t, X_{t}\right) d t+G\left(t, X_{t}\right) d W_{t}
$$

where:

- $X$ is an NVARS-by-1 state vector of process variables (for example, short rates or equity prices) to simulate.
- $W$ is an NBROWNS-by-1 Brownian motion vector.
- $F$ is an NVARS-by-1 vector-valued drift-rate function.
- $G$ is an $N V A R S$-by-NBROWNS matrix-valued diffusion-rate function.
simByEuler simulates NTRIALS sample paths of NVARS correlated state variables driven by NBROWNS Brownian motion sources of risk over NPERIODS consecutive observation periods, using the Euler approach to approximate continuous-time stochastic processes.
- This simulation engine provides a discrete-time approximation of the underlying generalized continuous-time process. The simulation is derived directly from the stochastic differential equation of motion. Thus, the discrete-time process approaches the true continuous-time process only as DeltaTime approaches zero.
- The input argument $Z$ allows you to directly specify the noise-generation process. This process takes precedence over the Correlation parameter of the sde object and the value of the Antithetic input flag. If you do not specify a value for $Z$, simByEuler generates correlated Gaussian variates, with or without antithetic sampling as requested.
- The end-of-period Processes argument allows you to terminate a given trial early. At the end of each time step, simByEuler tests the state vector $X_{t}$ for an all-NaN condition. Thus, to signal an early termination of a given trial, all elements of the state vector $X_{t}$ must be NaN. This test enables a user-defined Processes function to signal early termination of a trial, and offers significant performance benefits in some situations (for example, pricing down-and-out barrier options).


## References

[1] Deelstra, G. and F. Delbaen. "Convergence of discretized stochastic (interest rate) processes with stochastic drift term." Applied Stochastic Models and Data Analysis., 1998, Vol. 14, Number 1, pp. 77-84.
[2] Higham, D. and X. Mao. "Convergence of Monte Carlo simulations involving the meanreverting square root process." Journal of Computational Finance., 2005, Vol. 8, Number 3, pp. 35-61.
[3] Lord, R., R. Koekkoek, and D. Van Dijk. "A comparison of biased simulation schemes for stochastic volatility models." Quantitative Finance., 2010, Vol. 10, Number 2, pp. 177-194.

## See Also

| bm | cev | cir| gbm | heston | hwv | sde | sdeddo | sdeld| sdemrd | simBySolution |simByTransition|simulate

## Topics

Implementing Multidimensional Equity Market Models, Implementation 5: Using the simByEuler Method on page 18-42
"Simulating Equity Prices" on page 18-35
"Simulating Interest Rates" on page 18-60
"Stratified Sampling" on page 18-71
"Pricing American Basket Options by Monte Carlo Simulation" on page 18-85
"Base SDE Models" on page 18-17
"Drift and Diffusion Models" on page 18-20
"Linear Drift Models" on page 18-24
"Parametric Models" on page 18-26
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"SDE Class Hierarchy" on page 18-5
"Performance Considerations" on page 18-77

## Introduced in R2008a

## simByTransition

Simulate Cox-Ingersoll-Ross sample paths with transition density

## Syntax

[Paths,Times] = simByTransition(MDL,NPeriods)
[Paths,Times] = simByTransition( $\qquad$

## Description

[Paths,Times] = simByTransition(MDL,NPeriods) simulates NTRIALS sample paths of NVARS independent state variables driven by the Cox-Ingersoll-Ross (CIR) process sources of risk over NPERIODS consecutive observation periods. simByTransition approximates a continuous-time CIR model using an approximation of the transition density function.
[Paths,Times] = simByTransition( $\qquad$ ,Name, Value) specifies options using one or more name-value pair arguments in addition to the input arguments in the previous syntax.

## Examples

## Simulate Future Term Structures Using a CIR Model

Using the short rate, simulate the rate dynamics and term structures in the future using a CIR model. The CIR model is expressed as

$$
d r(t)=\alpha(b-r(t)) d t+\sigma \sqrt{r(t)} d W(t)
$$

The exponential affine form of the bond price is

$$
B(t, T)=e^{-A(t, T) r(t)+C(t, T)}
$$

where

$$
\begin{aligned}
& A(t, T)=\frac{2\left(e^{\gamma(T-t)}-1\right)}{(\gamma+\alpha)\left(e^{\gamma(T-t)}-1\right)+2 \gamma} \\
& B(t, T)=\frac{2 \alpha b}{\sigma^{2}} \log \left(\frac{2 \gamma e^{(\alpha+\gamma)(T-t) / 2}}{(\gamma+\alpha)\left(e^{\gamma(T-t)}-1\right)+2 \gamma}\right)
\end{aligned}
$$

and

$$
\gamma=\sqrt{\alpha^{2}+2 \sigma^{2}}
$$

Define the parameters for the cir object.

```
alpha = .1;
```

b = . 05;
sigma = .05;
r0 = . 04 ;

Define the function for bond prices.

```
gamma = sqrt(alpha^2 + 2*sigma^2);
A_func = @(t, T) ...
    2*(exp(gamma*(T-t))-1)/((alpha+gamma)*(exp(gamma*(T-t))-1)+2*gamma);
C func = @(t, T) ...
    (2*alpha*b/sigma^2)*log(2*gamma*exp((alpha+gamma)*(T-t)/2)/((alpha+gamma)*(exp(gamr
P_func = @(t,T,r_t) exp(-A_func(t,T)*r_t+C_func(t,T));
```

Create a cir object.

```
obj = cir(alpha,b,sigma,'StartState',r0)
```

obj =
Class CIR: Cox-Ingersoll-Ross
Dimensions: State $=1$, Brownian $=1$
StartTime: 0
StartState: 0.04
Correlation: 1
Drift: drift rate function $\mathrm{F}(\mathrm{t}, \mathrm{X}(\mathrm{t}))$
Diffusion: diffusion rate function $G(t, X(t))$

```
Simulation: simulation method/function simByEuler
    Sigma: 0.05
    Level: 0.05
    Speed: 0.1
```

Define the simulation parameters.

```
nTrials = 100;
nPeriods = 5; % Simulate future short over the next five years
nSteps = 12; % Set intermediate steps to improve the accuracy
```

Simulate the short rates. The returning path is a (NPERIODS + 1)-by-NVARS-by-NTRIALS three-dimensional time series array. For this example, the size of the output is 6 -by-1-by-100.

```
rng('default'); % Reproduce the same result
rPaths = simByTransition(obj,nPeriods,'nTrials',nTrials,'nSteps',nSteps);
size(rPaths)
ans = 1\times3
```

    61100
    rPathsExp $=$ mean(rPaths,3);

Determine the term structure over the next 30 years.

```
maturity = 30;
T = 1:maturity;
futuresTimes = 1:nPeriods+1;
```

```
% Preallocate simTermStruc
```

% Preallocate simTermStruc
simTermStructure = zeros(nPeriods+1,30);
simTermStructure = zeros(nPeriods+1,30);
for i = futuresTimes
for i = futuresTimes
for t = T
for t = T
bondPrice = P func(i,i+t,rPathsExp(i));
bondPrice = P func(i,i+t,rPathsExp(i));
simTermStructure(i,t) = - log(bondPrice)/t;
simTermStructure(i,t) = - log(bondPrice)/t;
end
end
end
end
plot(simTermStructure')
plot(simTermStructure')
legend('Current','1-year','2-year','3-year','4-year','5-year')
legend('Current','1-year','2-year','3-year','4-year','5-year')
title('Projected Term Strucutre for Next 5 Years')
title('Projected Term Strucutre for Next 5 Years')
ylabel('Long Rate Maturity R(t,T)')
ylabel('Long Rate Maturity R(t,T)')
xlabel('Time')

```
xlabel('Time')
```



## Input Arguments

## MDL - Stochastic differential equation model

object
Stochastic differential equation model, specified as a cir object. For more information on creating a CIR object, see cir.
Data Types: object

## NPeriods - Number of simulation periods

positive scalar integer

Number of simulation periods, specified as a positive scalar integer. The value of NPeriods determines the number of rows of the simulated output series.

## Data Types: double

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Example: [Paths,Times] = simByTransition(CIR,NPeriods,'DeltaTime',dt)

## NTrials - Simulated trials (sample paths) <br> 1 (single path of correlated state variables) (default) | positive integer

Simulated trials (sample paths) of NPERIODS observations each, specified as the commaseparated pair consisting of 'NTrials' and a positive scalar integer.
Data Types: double
DeltaTimes - Positive time increments between observations

## 1 (default) | scalar | column vector

Positive time increments between observations, specified as the comma-separated pair consisting of 'DeltaTimes ' and a scalar or a NPERIODS-by-1 column vector.

DeltaTime represents the familiar $d t$ found in stochastic differential equations, and determines the times at which the simulated paths of the output state variables are reported.

Data Types: double

## NSteps - Number of intermediate time steps

1 (indicating no intermediate evaluation) (default) | positive integer
Number of intermediate time steps within each time increment $d t$ (defined as DeltaTimes), specified as the comma-separated pair consisting of 'NSteps ' and a positive scalar integer.

The simByTransition function partitions each time increment $d t$ into NSteps subintervals of length $d t /$ NSteps, and refines the simulation by evaluating the simulated
state vector at NSteps - 1 intermediate points. Although simByTransition does not report the output state vector at these intermediate points, the refinement improves accuracy by enabling the simulation to more closely approximate the underlying continuous-time process.

Data Types: double

## StorePaths - Flag for storage and return method <br> True (default) | logical with values True or False

Flag for storage and return method that indicates how the output array Paths is stored and returned, specified as the comma-separated pair consisting of 'StorePaths' and a scalar logical flag with a value of True or False.

- If StorePaths is True (the default value) or is unspecified, then simByTransition returns Paths as a three-dimensional time series array.
- If StorePaths is False (logical 0), then simByTransition returns the Paths output array as an empty matrix.

Data Types: logical

## Processes - Sequence of end-of-period processes or state vector adjustments

 simByEuler makes no adjustments and performs no processing (default) | function | cell array of functionsSequence of end-of-period processes or state vector adjustments, specified as the commaseparated pair consisting of 'Processes' and a function or cell array of functions of the form

$$
X_{t}=P\left(t, X_{t}\right)
$$

simByTransition applies processing functions at the end of each observation period. The processing functions accept the current observation time $t$ and the current state vector $X_{t}$, and return a state vector that may adjust the input state.

If you specify more than one processing function, simByTransition invokes the functions in the order in which they appear in the cell array.

Data Types: cell|function

## Output Arguments

## Paths - Simulated paths of correlated state variables

array
Simulated paths of correlated state variables, returned as an (NPERIODS + 1)-by-NVARS-by-NTRIALS three-dimensional time series array.

For a given trial, each row of Paths is the transpose of the state vector $X_{t}$ at time $t$. When the input flag StorePaths $=$ False, simByTransition returns Paths as an empty matrix.

## Times - Observation times associated with the simulated paths column vector

Observation times associated with the simulated paths, returned as an (NPERIODS + 1)-by-1 column vector. Each element of Times is associated with the corresponding row of Paths.

## Definitions

## Transition Density Simulation

The SDE has no solution such that $r(t)=f(r(0), \cdots)$.
In other words, the equation is not explicitly solvable. However, the transition density for the process is known.

The exact simulation for the distribution of $r\left(t_{-} 1\right), \cdots, r\left(t_{-} n\right)$ is that of the process at times $t \_1, \cdots, t \_n$ for the same value of $r(0)$. The transition density for this process is known and is expressed as

$$
r(t)=\frac{\sigma^{2}\left(1-e^{-\alpha(t-u)}\right.}{4 \alpha} x_{d}^{2}\left(\frac{4 \alpha e^{-\alpha(t-u)}}{\sigma^{2}\left(1-e^{-\alpha(t-u)}\right)} r(u)\right), t>u
$$

where

$$
d \equiv \frac{4 b \alpha}{\sigma^{2}}
$$

## Algorithms

Use the simByTransition function to simulate any vector-valued CIR process of the form

$$
d X_{t}=S(t)\left[L(t)-X_{t}\right] d t+D\left(t, X_{t}^{\frac{1}{2}}\right) V(t) d W_{t}
$$

where

- $X_{t}$ is an NVARS-by-1 state vector of process variables.
- $S$ is an NVARS-by-NVARS matrix of mean reversion speeds (the rate of mean reversion).
- $L$ is an NVARS-by-1 vector of mean reversion levels (long-run mean or level).
- $D$ is an NVARS-by-NVARS diagonal matrix, where each element along the main diagonal is the square root of the corresponding element of the state vector.
- $V$ is an NVARS-by-NBROWNS instantaneous volatility rate matrix.
- $d W_{t}$ is an NBROWNS-by-1 Brownian motion vector.


## References

[1] Glasserman, P. Monte Carlo Methods in Financial Engineering. New York: SpringerVerlag, 2004.

## See Also

cir|simByEuler

## Topics

"SDEs" on page 18-2
"SDE Models" on page 18-9
"SDE Class Hierarchy" on page 18-5

Introduced in R2018b

## simulate

Simulate multivariate stochastic differential equations (SDEs)

## Syntax

```
[Paths,Times,Z] = simulate(MDL)
[Paths,Times,Z] = simulate(___,Optional)
```


## Description

[Paths,Times,Z] = simulate(MDL) simulates NTRIALS sample paths of NVARS correlated state variables, driven by NBROWNS Brownian motion sources of risk over NPERIODS consecutive observation periods, approximating continuous-time stochastic processes.
simulate accepts any variable-length list of input arguments that the simulation method or function referenced by the SDE. Simulation parameter requires or accepts. It passes this input list directly to the appropriate SDE simulation method or user-defined simulation function.
[Paths,Times,Z] = simulate( __ , Optional) adds optional input arguments.

## Examples

## Antithetic Sampling to a Path-Dependent Barrier Option

Consider a European up-and-in call option on a single underlying stock. The evolution of this stock's price is governed by a Geometric Brownian Motion (GBM) model with constant parameters:

$$
d X_{t}=0.05 X_{t} d t+0.3 X_{t} d W_{t}
$$

Assume the following characteristics:

- The stock currently trades at 105.
- The stock pays no dividends.
- The stock volatility is $30 \%$ per annum.
- The option strike price is 100.
- The option expires in three months.
- The option barrier is 120 .
- The risk-free rate is constant at $5 \%$ per annum.

The goal is to simulate various paths of daily stock prices, and calculate the price of the barrier option as the risk-neutral sample average of the discounted terminal option payoff. Since this is a barrier option, you must also determine if and when the barrier is crossed.

This example performs antithetic sampling by explicitly setting the Antithetic flag to true, and then specifies an end-of-period processing function to record the maximum and terminal stock prices on a path-by-path basis.

Create a GBM model using gbm.

```
barrier = 120; % barrier
strike = 100; % exercise price
rate = 0.05; % annualized risk-free rate
sigma = 0.3; % annualized volatility
nPeriods = 63; % 63 trading days
dt = 1 / 252; % time increment = 252 days
T = nPeriods * dt; % expiration time = 0.25 years
obj = gbm(rate, sigma, 'StartState', 105);
```

Perform a small-scale simulation that explicitly returns two simulated paths.

```
rng('default') % make output reproducible
[X, T] = obj.simBySolution(nPeriods, 'DeltaTime', dt, ...
    'nTrials', 2, 'Antithetic', true);
```

Perform antithetic sampling such that all primary and antithetic paths are simulated and stored in successive matching pairs. Odd paths ( $1,3,5, \ldots$ ) correspond to the primary Gaussian paths. Even paths ( $2,4,6, \ldots$ ) are the matching antithetic paths of each pair, derived by negating the Gaussian draws of the corresponding primary (odd) path. Verify this by examining the matching paths of the primary/antithetic pair.

```
plot(T, X(:,:,1), 'blue', T, X(:,:,2), 'red')
xlabel('Time (Years)'), ylabel('Stock Price'), ...
```



To price the European barrier option, specify an end-of-period processing function to record the maximum and terminal stock prices. This processing function is accessible by time and state, and is implemented as a nested function with access to shared information that allows the option price and corresponding standard error to be calculated. For more information on using an end-of-period processing function, see "Pricing Equity Options" on page 18-56.

Simulate 200 paths using the processing function method.

```
rng('default') % make output reproducible
barrier = 120; % barrier
strike = 100; % exercise price
rate = 0.05; % annualized risk-free rate
sigma = 0.3; % annualized volatility
nPeriods = 63; % 63 trading days
dt = 1 / 252; % time increment = 252 days
T = nPeriods * dt; % expiration time = 0.25 years
obj = gbm(rate, sigma, 'StartState', 105);
nPaths = 200; % # of paths = 100 sets of pairs
f = Example BarrierOption(nPeriods, nPaths);
simulate(obj, nPeriods, 'DeltaTime' , dt, ...
    'nTrials', nPaths, 'Antithetic', true, ...
    'Processes', f.SaveMaxLast);
```

Approximate the option price with a 95\% confidence interval.

```
optionPrice = f.OptionPrice(strike, rate, barrier);
standardError = f.StandardError(strike, rate, barrier,...
                                    true);
lowerBound = optionPrice - 1.96 * standardError;
upperBound = optionPrice + 1.96 * standardError;
```

displaySummary(optionPrice, standardError, lowerBound, upperBound);
Up-and-In Barrier Option Price: 6.6572
Standard Error of Price: 0.7292
Confidence Interval Lower Bound: 5.2280
Confidence Interval Upper Bound: 8.0864

## Utility Function

function displaySummary(optionPrice, standardError, lowerBound, upperBound)
fprintf(' Up-and-In Barrier Option Price: \%8.4f\n', ... optionPrice);
fprintf(' Standard Error of Price: \%8.4f\n', ... standardError);
fprintf(' Confidence Interval Lower Bound: \%8.4f\n', ... lowerBound);
fprintf(' Confidence Interval Upper Bound: \%8.4f\n', ...

## Input Arguments

## MDL - Stochastic differential equation model object

Stochastic differential equation model, specified as an sde, bm, gbm, cev, cir, hwv, heston, sdeddo, sdeld, or sdemrd object.
Data Types: object
Optional - (Optional) Any variable-length list of input arguments that the simulation method or function referenced by the SDE. Simulation parameter requires or accepts
input arguments
Any variable-length list of input arguments that the simulation method or function referenced by the SDE. Simulation parameter requires or accepts, specified as a variable-length list of input arguments. This input list is passed directly to the appropriate SDE simulation method or user-defined simulation function.

Data Types: double

## Output Arguments

Paths - Three-dimensional time series array, consisting of simulated paths of correlated state variables
array
Three-dimensional time series array, consisting of simulated paths of correlated state variables, returned as an (NPERIODS + 1)-by-NVARS-by-NTRIALS array.

For a given trial, each row of Paths is the transpose of the state vector $X_{t}$ at time $t$.

## Times - Observation times associated with the simulated paths column vector

Observation times associated with the simulated paths, returned as a (NPERIODS + 1)-by-1 column vector.

## Z - Three-dimensional time series array of dependent random variates used to generate the Brownian motion vector (Wiener processes) array | matrix | table | timetable

Three-dimensional time series array of dependent random variates used to generate the Brownian motion vector (Wiener processes) that drove the simulated results found in Paths, returned as a NTIMES-by-NBROWNS-by-NTRIALS array.

NTIMES is the number of time steps at which the simulate function samples the state vector. NTIMES includes intermediate times designed to improve accuracy, which simulate does not necessarily report in the Paths output time series.

## Definitions

## Antithetic Sampling

Simulation methods allow you to specify a popular variance reduction technique called antithetic sampling.

This technique attempts to replace one sequence of random observations with another of the same expected value, but smaller variance. In a typical Monte Carlo simulation, each sample path is independent and represents an independent trial. However, antithetic sampling generates sample paths in pairs. The first path of the pair is referred to as the primary path, and the second as the antithetic path. Any given pair is independent of any other pair, but the two paths within each pair are highly correlated. Antithetic sampling literature often recommends averaging the discounted payoffs of each pair, effectively halving the number of Monte Carlo trials.

This technique attempts to reduce variance by inducing negative dependence between paired input samples, ideally resulting in negative dependence between paired output samples. The greater the extent of negative dependence, the more effective antithetic sampling is.

## Algorithms

This function simulates any vector-valued SDE of the form:

$$
\begin{equation*}
d X_{t}=F\left(t, X_{t}\right) d t+G\left(t, X_{t}\right) d W_{t} \tag{19-4}
\end{equation*}
$$

where:

- $X$ is an $N V A R S$-by- 1 state vector of process variables (for example, short rates or equity prices) to simulate.
- $W$ is an NBROWNS-by-1 Brownian motion vector.
- $F$ is an NVARS-by-1 vector-valued drift-rate function.
- $G$ is an NVARS-by-NBROWNS matrix-valued diffusion-rate function.


## References

[1] Ait-Sahalia, Y. "Testing Continuous-Time Models of the Spot Interest Rate." The Review of Financial Studies, Spring 1996, Vol. 9, No. 2, pp. 385-426.
[2] Ait-Sahalia, Y. "Transition Densities for Interest Rate and Other Nonlinear Diffusions." The Journal of Finance, Vol. 54, No. 4, August 1999.
[3] Glasserman, P. Monte Carlo Methods in Financial Engineering. New York, SpringerVerlag, 2004.
[4] Hull, J. C. Options, Futures, and Other Derivatives, 5th ed. Englewood Cliffs, NJ: Prentice Hall, 2002.
[5] Johnson, N. L., S. Kotz, and N. Balakrishnan. Continuous Univariate Distributions. Vol. 2, 2nd ed. New York, John Wiley \& Sons, 1995.
[6] Shreve, S. E. Stochastic Calculus for Finance II: Continuous-Time Models. New York: Springer-Verlag, 2004.

## See Also

bm | cev |cir|gbm| heston | hwv | sde | sdeddo | sdeld | sdemrd| simByEuler | simBySolution|simBySolution
Topics
"Simulating Equity Prices" on page 18-35"Simulating Interest Rates" on page 18-60"Stratified Sampling" on page 18-71
"Pricing American Basket Options by Monte Carlo Simulation" on page 18-85
"Base SDE Models" on page 18-17
"Drift and Diffusion Models" on page 18-20
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"Performance Considerations" on page 18-77
Introduced in R2008a

## ts2func

Convert time series arrays to functions of time and state

## Syntax

```
F = ts2func(Array)
F = ts2func(
,Name,Value)
```


## Description

$F=t s 2 f u n c(A r r a y)$ encapsulates a time series array associated with a vector of realvalued observation times within a MATLAB function suitable for Monte Carlo simulation of an NVARS-by-1 state vector $X_{t}$.
$n$ periods.
$F=$ ts2func $($ $\qquad$ , Name, Value) adds optional name-value pair arguments.

## Examples

Illustrate the Dynamic Behavior of Market Parameters
Load the data.
load Data_GlobalIdx2

## Simulate risk-neutral sample paths.

dt $\quad=1 / 250$;
returns = tick2ret(Dataset.CAC);
sigma $=$ std(returns)*sqrt(250);
yields = Dataset.EB3M;
yields $=360 * \log (1+$ yields);

```
Simulate paths using a constant, risk-free return
nPeriods = length(yields); % Simulated observations
rng(5713,'twister')
obj = gbm(mean(yields),diag(sigma),'StartState',100)
obj =
    Class GBM: Generalized Geometric Brownian Motion
        Dimensions: State = 1, Brownian = 1
        StartTime: 0
    StartState: 100
    Correlation: 1
    Drift: drift rate function F(t,X(t))
        Diffusion: diffusion rate function G(t,X(t))
        Simulation: simulation method/function simByEuler
        Return: 0.0278117
    Sigma: 0.231906
[X1,T] = simulate(obj,nPeriods,'DeltaTime',dt);
```

Simulate paths using a dynamic, deterministic rate of return (get r) $r=$ ts2func(yields,'Times',(0:nPeriods - 1)');

Simulate paths using a dynamic, deterministic rate of return (r output 1) r(0,100)
ans $=0.0470$
Simulate paths using a dynamic, deterministic rate of return (r output 2)
$r(7.5,200)$
ans $=0.0472$
Simulate paths using a dynamic, deterministic rate of return (r output 3)
$r(7.5)$
ans $=0.0472$

## Simulate paths using a dynamic, deterministic rate of return

```
rng(5713,'twister')
obj = gbm(r, diag(sigma),'StartState',100)
obj =
    Class GBM: Generalized Geometric Brownian Motion
    Dimensions: State = 1, Brownian = 1
        StartTime: 0
        StartState: 100
        Correlation: 1
            Drift: drift rate function F(t,X(t))
            Diffusion: diffusion rate function G(t,X(t))
            Simulation: simulation method/function simByEuler
            Return: function ts2func/vector2Function
            Sigma: 0.231906
X2 = simulate(obj,nPeriods,'DeltaTime',dt);
```


## Compare the two simulation trials.

```
subplot(2,1,1)
plot(dates,100*yields)
datetick('x')
xlabel('Date')
ylabel('Annualized Yield (%)')
title('Risk Free Rate(3-Mo Euribor Continuously-Compounded)')
subplot(2,1,2)
plot(T,X1,'red',T,X2,'blue')
xlabel('Time (Years)')
ylabel('Index Level')
title('Constant vs. Dynamic Rate of Return: CAC 40')
legend({'Constant Interest Rates' 'Dynamic Interest Rates'},...
    'Location', 'Best')
```



## Input Arguments

## Array - Time series array to encapsulate within a callable function of time and state <br> vector | 2 -dimensional matrix | three-dimensional array

Time series array to encapsulate within a callable function of time and state, specified as a vector, two-dimensional matrix, or three-dimensional array
Data Types: double

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, ... , NameN, ValueN.

```
Example: F = ts2func(yields,'Times',(0:nPeriods - 1)')
```


## Times - Monotonically increasing observation times associated with the time series input array Array

zero-based, unit-increment vector of the same length as that of the dimension of Array (default) | vector

Monotonically increasing observation times associated with the time series input array (Array), specified as the comma-separated pair consisting of 'Times ' and a vector.

Data Types: double

## TimeDimension - Specifies which dimension of the input time series array Array is associated with time

1 (indicates that time is associated with the rows of Array) (default) | scalar integer
Specifies which dimension of the input time series array (Array) is associated with time, specified as the comma-separated pair consisting of 'TimeDimension' and a scalar integer.

Data Types: double
StateDimension - Specifies which dimension of the input time series array
Array is associated with the NVARS state variables
first dimension of Array not already associated with time is associated with first available dimension of Array not already assigned to TimeDimension (default) | positive scalar integer

Specifies which dimension of the input time series array (Array) is associated with the NVARS state variables, specified as the comma-separated pair consisting of 'StateDimension' and a positive scalar integer.

Data Types: double

## Deterministic - Flag to indicate whether the output function is a deterministic function of time alone

false (meaning $F$ is a callable function of time and state, $F(t, X)$ ) (default) | scalar logical

Flag to indicate whether the output function is a deterministic function of time alone, specified as the comma-separated pair consisting of 'Determininistic' and a scalar integer flag.

If Deterministic is true, the output function F is a deterministic function of time, $F(t)$, and the only input it accepts is a scalar, real-valued time $t$. If Deterministic is false, the output function F accepts two inputs, a scalar, real-valued time $t$ followed by an NVARS-by-1 state vector $X(t)$.

Data Types: logical

## Output Arguments

## F-Callable function $F(t)$ of a real-valued scalar observation time $t$ vector | matrix | table | timetable

Callable function $F(t)$ of a real-valued scalar observation time $t$, returned as a function.
If the optional input argument Deterministic is true, F is a deterministic function of time, $F(t)$, and the only input it accepts is a scalar, real-valued time $t$. Otherwise, if Deterministic is false (the default), F accepts a scalar, real-valued time $t$ followed by an NVARS-by-1 state vector $X(t)$.

Note You can invoke F with a second input (such as an NVARS-by-1 state vector $X$ ), which is a placeholder that ts2func ignores. For example, while $F(t)$ and $F(t, X)$ produce identical results, the latter directly supports SDE simulation methods.

## Algorithms

- When you specify Array as a scalar or a vector (row or column), ts2func assumes that it represents a univariate time series.
- F returns an array with one less dimension than the input time series array Array with which $F$ is associated. Thus, when Array is a vector, a 2 -dimensional matrix, or a three-dimensional array, F returns a scalar, vector, or 2-dimensional matrix, respectively.
- When the scalar time $t$ at which ts2func evaluates the function F does not coincide with an observation time in Times, F performs a zero-order-hold interpolation. The
only exception is if $t$ precedes the first element of Times, in which case $F(t)=$ $F($ Times(1)).
- To support Monte Carlo simulation methods, the output function F returns an NVARS-by-1 column vector or a two-dimensional matrix with NVARS rows.
- The output function $F$ is always a deterministic function of time, $F(t)$, and may always be called with a single input regardless of the Deterministic flag. The distinction is that when Deterministic is false, the function $F$ may also be called with a second input, an NVARS-by-1 state vector $X(t)$, which is a placeholder and ignored. While $F(t)$ and $F(t, X)$ produce identical results, the former specifically indicates that the function is a deterministic function of time, and may offer significant performance benefits in some situations.


## References

[1] Ait-Sahalia, Y. "Testing Continuous-Time Models of the Spot Interest Rate." The Review of Financial Studies, Spring 1996, Vol. 9, No. 2, pp. 385-426.
[2] Ait-Sahalia, Y. "Transition Densities for Interest Rate and Other Nonlinear Diffusions." The Journal of Finance, Vol. 54, No. 4, August 1999.
[3] Glasserman, P. Monte Carlo Methods in Financial Engineering. New York, SpringerVerlag, 2004.
[4] Hull, J. C. Options, Futures, and Other Derivatives, 5th ed. Englewood Cliffs, NJ: Prentice Hall, 2002.
[5] Johnson, N. L., S. Kotz, and N. Balakrishnan. Continuous Univariate Distributions. Vol. 2, 2nd ed. New York, John Wiley \& Sons, 1995.
[6] Shreve, S. E. Stochastic Calculus for Finance II: Continuous-Time Models. New York: Springer-Verlag, 2004.

## See Also

simByEuler|simulate

## Topics

"Simulating Equity Prices" on page 18-35
"Simulating Interest Rates" on page 18-60
"Stratified Sampling" on page 18-71
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## Introduced in R2008a

## abs2active

Convert constraints from absolute to active format

## Syntax

ActiveConSet $=$ abs2active(AbsConSet,Index)

## Description

ActiveConSet $=$ abs2active(AbsConSet, Index) transforms a constraint matrix to an equivalent matrix expressed in active weight format (relative to the index).

## Input Arguments

## AbsConSet

Portfolio linear inequality constraint matrix expressed in absolute weight format. AbsConSet is formatted as [A b] such that $A * W<=b$, where $A$ is a number of constraints (NCONSTRAINTS) by number of assets (NASSETS) weight coefficient matrix, and $b$ and $w$ are column vectors of length NASSETS. The value $w$ represents a vector of absolute asset weights whose elements sum to the total portfolio value. See the output ConSet from portcons for additional details about constraint matrices.

## Index

NASSETS-by-1 vector of index portfolio weights. The sum of the index weights must equal the total portfolio value (for example, a standard portfolio optimization imposes a sum-toone budget constraint).

## Output Arguments

## ActiveConSet

The transformed portfolio linear inequality constraint matrix expressed in active weight format, also of the form [A b] such that $A * w<=b$. The value $w$ represents a vector of active asset weights (relative to the index portfolio) whose elements sum to zero.

## Examples

Set up constraints for a portfolio optimization for portfolio w0 with constraints in the form $A * W<=b$, where $w$ is absolute portfolio weights. (Absolute weights do not depend on the tracking portfolio.) Use abs2active to convert constraints in terms of absolute weights into constraints in terms of active portfolio weights, defined relative to the tracking portfolio $w 0$. Assume three assets with the following mean and covariance of asset returns:

```
m = [ 0.14; 0.10; 0.05 ];
C = [ 0.29^2 0.4*0.29*0.17 0.1*0.29*0.08; 0.4*0.29*0.17 0.17^2 0.3*0.17*0.08;...
0.1*0.29*0.08 0.3*0.17*0.08 0.08^2 ];
```

Absolute portfolio constraints are the typical ones (weights sum to 1 and fall from 0 through 1), create the $A$ and $b$ matrices using portcons:

```
AbsCons = portcons('PortValue',1,3,'AssetLims', [0; 0; 0], [1; 1; 1;]);
```

Use the Portfolio object to determine the efficient frontier:

```
p = Portfolio('AssetMean', m, 'AssetCovar', C);
p = p.setInequality(AbsCons(:,1:end-1), AbsCons(:,end));
p.plotFrontier;
```



The tracking portfolio w0 is:
w0 = [ 0.1; 0.55; 0.35 ];
Use abs2active to compute the constraints for active portfolio weights:
ActCons = abs2active(AbsCons, we)
This returns:
ActCons =

| 1.0000 | 1.0000 | 1.0000 | 0 |
| ---: | ---: | ---: | ---: |
| -1.0000 | -1.0000 | -1.0000 | 0 |
| 1.0000 | 0 | 0 | 0.9000 |
| 0 | 1.0000 | 0 | 0.4500 |
| 0 | 0 | 1.0000 | 0.6500 |
| -1.0000 | 0 | 0 | 0.1000 |
| 0 | -1.0000 | 0 | 0.5500 |
| 0 | 0 | -1.0000 | 0.3500 |

Use the Portfolio object p and its efficient frontier to demonstrate expected returns and risk relative to the tracking portfolio w0:
$\mathrm{p}=\mathrm{p} . \operatorname{set}$ Inequality(ActCons(:,1:end-1), ActCons(:,end));
p.plotFrontier;


Note, when using abs2active to compute "active constraints" for use with a Portfolio object, don't use the Portfolio object's default constraints because the relative weights can be positive or negative (the setDefaultConstraints function specifies weights to be nonnegative).

## Algorithms

abs2active transforms a constraint matrix to an equivalent matrix expressed in active weight format (relative to the index). The transformation equation is

$$
A w_{\text {absolute }}=A\left(w_{\text {active }}+w_{\text {index }}\right) \leq b_{\text {absolute }}
$$

Therefore

$$
A w_{\text {active }} \leq b_{\text {absolute }}-A w_{\text {index }}=b_{\text {active }}
$$

The initial constraint matrix consists of NCONSTRAINTS portfolio linear inequality constraints expressed in absolute weight format. The index portfolio vector contains NASSETS assets.

## See Also

Portfolio| active2abs|pcalims | pcglims | pcpval|portcons| setInequality

## Topics

"Data Transformation and Frequency Conversion" on page 13-13

## Introduced before R2006a

## accrfrac

Fraction of coupon period before settlement

## Syntax

Fraction = accrfrac(Settle,Maturity)
Fraction = accrfrac( _ , Period,Basis,EndMonthRule,IssueDate, FirstCouponDate,LastCouponDate)

## Description

Fraction $=$ accrfrac(Settle, Maturity) returns the fraction of the coupon period before settlement.

Use accrfrac for computing accrued interest. accrfrac calculates accrued interest for bonds with regular or odd first or last coupon periods.

Required input arguments must be number of bonds, NUMBONDS-by-1 or 1-by-NUMBONDS, conforming vectors or scalars.

Fraction = accrfrac (__ ,Period,Basis,EndMonthRule,IssueDate, FirstCouponDate, LastCouponDate) returns the fraction of the coupon period before settlement with optional inputs.

Optional input arguments must be either NUMBONDS-by-1 or 1-by-NUMBONDS conforming vectors, scalars, or empty matrices.

## Examples

## Find Accrued Interest for a Bond

This example shows how to find the accrued interest for given bond data.
Settle = '14-Mar-1997';
Maturity = ['30-Nov-2000'

```
    '31-Dec-2000'
    '31-Jan-2001'];
Period = 2;
Basis = 0;
EndMonthRule = 1;
Fraction = accrfrac(Settle, Maturity, Period, Basis,...
EndMonthRule)
Fraction = 3×1
    0.5714
    0.4033
    0.2320
```


## Find Accrued Interest for a Bond Using a datetime Array

This example shows how to find the accrued interest for a given bond's data using a datetime array.

Fraction = accrfrac(datetime('14-Mar-1997', 'Locale', 'en_US'), ['30-Nov-2000'; '31-De
Fraction $=3 \times 1$
0.5714
0.4033
0.2320

## Input Arguments

## Settle - Settlement date

serial date numbers | date character vector | datetime object
Settlement date, specified as a vector of serial date number, date character vector, or datetime array. Settle must be earlier than Maturity.

Data Types: double|char|datetime

## Maturity - Maturity date <br> serial date number | date character vector | datetime array

Maturity date, specified as a vector of serial date numbers, date character vectors, or datetime arrays.

## Data Types: double | char | datetime

## Period - Coupons per year of the bond

2 (semiannual) (default) | vector of positive integers from the set [1, 2, 3, 4, 6, 12]
Coupons per year of the bond, specified as a vector of positive integers from the set [1, 2, 3, 4, 6, 12].

Data Types: single | double

## Basis - Day-count basis of the instrument

0 (actual/actual) (default) | numeric with value 0 through $13 \mid$ vector of numerics with values 0 through 13

Day-count basis of the instrument, specified as an integer with a value of 0 through 13 or a N -by-1 vector of integers with values of 0 through 13.

- $0=$ actual/actual (default)
- $1=30 / 360$ (SIA)
- 2 = actual/360
- 3 = actual/365
- $4=30 / 360$ (BMA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- $9=$ actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- $12=$ actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-0

Data Types: single | double

## EndMonthRule - End-of-month rule flag for month having 30 or fewer days 1 (in effect) (default) | nonnegative integer [0,1]

End-of-month rule flag for month having 30 or fewer days, specified as a nonnegative integer [0, 1] using a N -by-1 vector of values. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days.

- $0=$ Ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month.
- $1=$ Set rule on, meaning that a bond's coupon payment date is always the last actual day of the month.


## Data Types: logical

## IssueDate - Bond issue date <br> serial date number | date character vector | datetime array

Bond issue date, specified as a serial date number, date character vector, or datetime array.

Data Types: double|char|datetime

## FirstCouponDate - Date when bond makes first coupon payment serial date number | date character vector | datetime array

Date when a bond makes its first coupon payment, specified as a serial date number, date character vector, or datetime array.

FirstCouponDate is used when a bond has an irregular first coupon period. When FirstCouponDate and LastCouponDate are both specified, FirstCouponDate takes precedence in determining the coupon payment structure. If you do not specify a FirstCouponDate, the cash flow payment dates are determined from other inputs.
Data Types: double | char | datetime

## LastCouponDate - Last coupon date of bond before maturity date

serial date number | date character vector | datetime array
Last coupon date of a bond before maturity date, specified as a serial date number, date character vector, or datetime array.

LastCouponDate is used when a bond has an irregular last coupon period. In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated at the LastCouponDate, regardless of where it falls, and is followed only by the bond's maturity cash flow date. If you do not specify a LastCouponDate, the cash flow payment dates are determined from other inputs.

Data Types: double | char | datetime

## Output Arguments

## Fraction - Fraction of coupon period before settlement <br> vector

Fraction of the coupon period before settlement, returned as an NUMBONDS-by-1 vector.

## See Also

cfamounts | cfdates | cpncount | cpndaten | cpndatenq| cpndatep|cpndatepq| cpndaysn| cpndaysp|cpnpersz|datetime

## Topics

"Pricing and Computing Yields for Fixed-Income Securities" on page 2-22
Introduced before R2006a

## acrubond

Accrued interest of security with periodic interest payments

## Syntax

AccruInterest = acrubond(IssueDate,Settle,FirstCouponDate,Face, CouponRate)
AccruInterest = acrubond( $\qquad$ ,Period,Basis)

## Description

AccruInterest = acrubond(IssueDate,Settle,FirstCouponDate,Face, CouponRate) returns the accrued interest for a security with periodic interest payments. acrubond computes the accrued interest for securities with standard, short, and long first coupon periods.

Note cfamounts or accrfrac is recommended when calculating accrued interest beyond the first period.

AccruInterest = acrubond( $\qquad$ , Period, Basis) adds optional arguments for Period and Basis.

## Examples

## Find Accrued Interest of a Bond with Periodic Interest Payments

This example shows how to find the accrued interest for a bond with semiannual interest payments.

```
AccruInterest = acrubond('31-jan-1983', '1-mar-1993', ...
    '31-jul-1983', 100, 0.1, 2, 0)
```

AccruInterest $=0.8011$

## Find Accrued Interest of a Bond with Periodic Interest Payments Using datetime Inputs

This example shows how to use datetime inputs to find the accrued interest for a bond with semiannual interest payments.

AccruInterest = acrubond(datetime('31-jan-1983','Locale','en_US'),datetime('1-mar-1993 100, 0.1, 2, 0)

AccruInterest $=0.8011$

## Input Arguments

## IssueDate - Issue date of security <br> serial date number | date character vector | datetime

Issue date of the security, specified as a scalar or a NINST-by-1 vector of serial date numbers, date character vectors, or datetime arrays.

Data Types: double | char | datetime

## Settle - Settlement date of security <br> serial date number | date character vector | datetime

Settlement date of the security, specified as a scalar or a NINST-by-1 vector of serial date numbers, date character vectors, or datetime arrays. The Settle date must be before the Maturity date.

Data Types: double | char | datetime

## FirstCouponDate - First coupon date of security <br> serial date number | date character vector | datetime

First coupon date of the security, specified as a scalar or a NINST-by-1 vector of serial date numbers, date character vectors, or datetime arrays.

Data Types: double| char|datetime

## Face - Redemption value of security

numeric

Redemption value (par value) of the security, specified as a scalar or a NINST-by-1 vector.

## Data Types: double

## CouponRate - Coupon rate of security <br> decimal fraction

Coupon rate of the security, specified as a scalar or a NINST-by-1 vector of decimal fraction values.

## Data Types: double

## Period - Number of coupon payments per year

2 (default) | numeric with values $0,1,2,3,4,6$ or 12
(Optional) Number of coupon payments per year for the security, specified as scalar or a NINST-by-1 vector using the values: $0,1,2,3,4,6$, or 12.

## Data Types: double

## Basis - Day-count basis

0 (actual/actual) (default) | integers of the set [0...13] | vector of integers of the set [0...13]
(Optional) Day-count basis for the security, specified as a scalar or a NINST-by-1 vector. Values are:

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- $2=$ actual/360
- 3 = actual/365
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- $9=$ actual/360 (ICMA)
- $10=$ actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-0
Data Types: double

## Output Arguments

## AccruInterest - Accrued interest

numeric
Accrued interest for the security, returned as a scalar or a NINST-by-1 vector.

See Also<br>accrfrac|acrudisc| bndprice| bndyield|cfamounts | datenum | datetime<br>\section*{Topics}<br>"Coupon Date Calculations" on page 2-31<br>"Fixed-Income Terminology" on page 2-22<br>Introduced before R2006a

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## acrudisc

Accrued interest of discount security paying at maturity

## Syntax

AccruInterest $=$ acrudisc(Settle, Maturity, Face, Discount)
AccruInterest $=$ acrudisc( $\qquad$ , Period,Basis)

## Description

AccruInterest $=$ acrudisc(Settle, Maturity, Face, Discount) returns the accrued interest of a discount security paid at maturity.

AccruInterest = acrudisc( $\qquad$ , Period,Basis) adds optional arguments for Period and Basis.

## Examples

## Find Accrued Interest of a Discount Security Paid at Maturity

This example shows how to find the accrued interest of a discount security paid at maturity.

```
AccruInterest = acrudisc('05/01/1992', '07/15/1992', ...
    100, 0.1, 2, 0)
```

AccruInterest $=2.0604$

## Find Accrued Interest of a Discount Security Paid at Maturity Using datetime Inputs

This example shows how to use datetime inputs to find the accrued interest of a discount security paid at maturity.

```
AccruInterest = acrudisc(datetime('1-May-1992','Locale','en_US'),datetime('15-Jul-1992
```

100, 0.1, 2, 0)

AccruInterest $=2.0604$

## Input Arguments

## Settle - Settlement date of security

serial date number | date character vector | datetime
Settlement date of the security, specified as a scalar or a NINST-by-1 vector of serial date numbers, date character vectors, or datetime arrays. The Settle date must be before the Maturity date.

Data Types: double | char|datetime

## Maturity - Maturity date of security <br> serial date number | date character vector | datetime

Maturity date of the security, specified as a scalar or a NINST-by-1 vector of serial date numbers, date character vectors, or datetime arrays.

Data Types: double | char | datetime

## Face - Redemption value of security

numeric
Redemption value (par value) of the security, specified as a scalar or a NINST-by-1 vector.
Data Types: double

## Discount - Discount rate of security

decimal fraction
Discount rate of the security, specified as a scalar or a NINST-by-1 vector of decimal fraction values.

## Data Types: double

## Period - Number of coupon payments per year

2 (default) | numeric with values $0,1,2,3,4,6$ or 12
(Optional) Number of coupon payments per year for security, specified as scalar or a NINST-by-1 vector using the values: $0,1,2,3,4,6$, or 12.

## Data Types: double

## Basis - Day-count basis

0 (actual/actual) (default) | integers of the set [0...13] | vector of integers of the set [0...13]
(Optional) Day-count basis for security, specified as a scalar or a NINST-by-1 vector. Values are:

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- 2 = actual/360
- 3 = actual/365
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- $7=$ actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- $9=$ actual/360 (ICMA)
- $10=$ actual/365 (ICMA)
- $11=30 / 360 \mathrm{E}$ (ICMA)
- $12=$ actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-0
Data Types: double

## Output Arguments

## AccruInterest - Accrued interest

numeric
Accrued interest, returned as a scalar or a NINST-by-1 vector.

## References

[1] Mayle, J. Standard Securities Calculation Methods. Volumes I-II, 3rd edition. Formula D.

See Also<br>acrubond | datetime $\mid$ prdisc $\mid$ prmat $|y l d d i s c| y l d m a t$<br>\section*{Topics}<br>"Coupon Date Calculations" on page 2-31<br>"Fixed-Income Terminology" on page 2-22<br>Introduced before R2006a

## active2abs

Convert constraints from active to absolute format

## Syntax

AbsConSet = active2abs(ActiveConSet,Index)

## Arguments

| ActiveConSet | Portfolio linear inequality constraint matrix expressed in <br> active weight format. ActiveConSet is formatted as [A b] <br> such that $A *$ w < b where A is a number of constraints <br> (NCONSTRAINTS) by number of assets (NASSETS) weight <br> coefficient matrix, and b and w are column vectors of length <br> NASSETS. The value w represents a vector of active asset <br> weights (relative to the index portfolio) whose elements sum <br> to 0. <br> See the output ConSet from portcons for additional details <br> about constraint matrices. |
| :--- | :--- |
| Index | NASSETS-by-1 vector of index portfolio weights. The sum of <br> the index weights must equal the total portfolio value (for <br> example, a standard portfolio optimization imposes a sum-to- <br> one budget constraint). |

## Description

AbsConSet $=$ active2abs(ActiveConSet,Index) transforms a constraint matrix to an equivalent matrix expressed in absolute weight format. The transformation equation is

$$
A w_{\text {active }}=A\left(w_{\text {absolute }}-w_{\text {index }}\right) \leq b_{\text {active }}
$$

Therefore

$$
A w_{\text {absolute }} \leq b_{\text {active }}+A w_{\text {index }}=b_{\text {absolute }}
$$

The initial constraint matrix consists of NCONSTRAINTS portfolio linear inequality constraints expressed in active weight format (relative to the index portfolio). The index portfolio vector contains NASSETS assets.

AbsConSet is the transformed portfolio linear inequality constraint matrix expressed in absolute weight format, also of the form [A b] such that $A * w<=b$. The value $w$ represents a vector of active asset weights (relative to the index portfolio) whose elements sum to the total portfolio value.

## See Also

abs2active|pcalims | pcgcomp| pcglims | pcpval|portcons

## Topics

"Data Transformation and Frequency Conversion" on page 13-13

## Introduced before R2006a

## addEquality

Add linear equality constraints for portfolio weights to existing constraints

## Syntax

```
obj = addEquality(obj,AEquality,bEquality)
```


## Description

obj = addEquality(obj,AEquality,bEquality) adds linear equality constraints for portfolio weights to existing constraints for Portfolio, PortfolioCVaR, or PortfolioMAD objects. For details on the respective workflows when using these different objects, see "Portfolio Object Workflow" on page 4-21, "PortfolioCVaR Object Workflow" on page 5-20, and "PortfolioMAD Object Workflow" on page 6-19.

Given a linear equality constraint matrix AEquality and vector bEquality, every weight in a portfolio Port must satisfy the following:

```
AEquality * Port = bEquality
```

This function "stacks" additional linear equality constraints onto any existing linear equality constraints that exist in the input portfolio object. If no constraints exist, this method is the same as setEquality.

## Examples

## Add a Linear Equality Constraint for a Portfolio Object

Use the addEquality method to create linear equality constraints. Add another linear equality constraint to ensure that the last three assets constitute $50 \%$ of a portfolio.

```
p = Portfolio;
A = [ 1 1 1 0 0 ]; % First equality constraint
b = 0.5;
```

```
p = setEquality(p, A, b);
A = [ 0 0 1 1 1 1 ]; % Second equality constraint
b = 0.5;
p = addEquality(p, A, b);
disp(p.NumAssets);
    5
disp(p.AEquality);
    lllll
disp(p.bEquality);
    0.5000
    0.5000
```


## Add a Linear Equality Constraint for a PortfolioCVaR Object

Use the addEquality method to create linear equality constraints. Add another linear equality constraint to ensure that the last three assets constitute $50 \%$ of a portfolio.

```
p = PortfolioCVaR;
A = [ 1 1 1 0 0 ]; % First equality constraint
b = 0.5;
p = setEquality(p, A, b);
A = [ 0 0 1 1 1 ]; % Second equality constraint
b = 0.5;
p = addEquality(p, A, b);
disp(p.NumAssets);
    5
disp(p.AEquality);
    1
disp(p.bEquality);
```

0.5000
0.5000

## Add a Linear Equality Constraint for a PortfolioMAD Object

Use the addEquality method to create linear equality constraints. Add another linear equality constraint to ensure that the last three assets constitute 50\% of a portfolio.

```
p = PortfolioMAD;
A = [ 1 1 1 0 0 ]; % First equality constraint
b = 0.5;
p = setEquality(p, A, b);
A = [ 0 0 1 1 l ]; % Second equality constraint
b = 0.5;
p = addEquality(p, A, b);
disp(p.NumAssets);
    5
disp(p.AEquality);
\begin{tabular}{lllll}
1 & 1 & 1 & 0 & 0 \\
0 & 0 & 1 & 1 & 1
\end{tabular}
disp(p.bEquality);
0.5000
0.5000
```


## Input Arguments

## obj - Object for portfolio

object
Object for portfolio, specified using Portfolio, PortfolioCVaR, or PortfolioMAD object. For more information on creating a portfolio object, see

- Portfolio
- PortfolioCVaR
- PortfolioMAD

Data Types: object

## AEquality - Linear equality constraints formed from matrix matrix

Linear equality constraints, specified as a matrix.

Note An error results if AEquality is empty and bEquality is nonempty.

## Data Types: double

## bEquality - Linear equality constraints formed from vector

 vectorLinear equality constraints, specified as a vector.

Note An error results if bEquality is empty and AEquality is nonempty.

Data Types: double

## Output Arguments

obj - Updated portfolio object
object for portfolio
Updated portfolio object, returned as a Portfolio, PortfolioCVaR, or PortfolioMAD object. For more information on creating a portfolio object, see

- Portfolio
- PortfolioCVaR
- PortfolioMAD


## Tips

- You can also use dot notation to add the linear equality constraints for portfolio weights.

```
obj = obj.addEquality(AEquality, bEquality)
```

- You can also remove linear equality constraints from a portfolio object using dot notation.

```
obj = obj.setEquality([ ], [ ])
```


## See Also

setEquality

## Topics

"Working with Linear Equality Constraints Using Portfolio Object" on page 4-88
"Working with Linear Equality Constraints Using PortfolioCVaR Object" on page 5-82
"Setting Linear Inequality Constraints Using the setInequality and addInequality Functions" on page 6-82
"Portfolio Optimization Examples" on page 4-167
"Portfolio Set for Optimization Using Portfolio Object" on page 4-10
"Portfolio Set for Optimization Using PortfolioCVaR Object" on page 5-10
"Portfolio Set for Optimization Using PortfolioMAD Object" on page 6-10

## Introduced in R2011a

## addGroupRatio

Add group ratio constraints for portfolio weights to existing group ratio constraints

## Syntax

obj = addGroupRatio(obj,GroupA,GroupB,LowerRatio)
obj = addGroupRatio(obj,GroupA,GroupB,LowerRatio,UpperRatio)

## Description

obj = addGroupRatio(obj,GroupA,GroupB,LowerRatio) adds group ratio constraints for portfolio weights to existing group ratio constraints for Portfolio, PortfolioCVaR, or PortfolioMAD objects. For details on the respective workflows when using these different objects, see "Portfolio Object Workflow" on page 4-21, "PortfolioCVaR Object Workflow" on page 5-20, and "PortfolioMAD Object Workflow" on page 6-19.

Given base and comparison group matrices GroupA and GroupB and, either LowerRatio, or UpperRatio bounds, group ratio constraints require any portfolio in Port to satisfy the following:
(GroupB * Port) .* LowerRatio <= GroupA * Port <= (GroupB * Port) .* UpperRatio

Note This collection of constraints usually requires that portfolio weights be nonnegative and that the products GroupA * Port and GroupB * Port are always nonnegative. Although negative portfolio weights and non-Boolean group ratio matrices are supported, use with caution.
obj = addGroupRatio(obj,GroupA,GroupB,LowerRatio,UpperRatio) adds group ratio constraints for portfolio weights to existing group ratio constraints with an additional option for UpperRatio.

Given base and comparison group matrices GroupA and GroupB and, either LowerRatio, or UpperRatio bounds, group ratio constraints require any portfolio in Port to satisfy the following:

```
(GroupB * Port) .* LowerRatio <= GroupA * Port <= (GroupB * Port) .* UpperRatio
```

Note This collection of constraints usually requires that portfolio weights be nonnegative and that the products GroupA * Port and GroupB * Port are always nonnegative. Although negative portfolio weights and non-Boolean group ratio matrices are supported, use with caution.

## Examples

## Add Group Ratio Constraints to a Portfolio Object

Set a group ratio constraint to ensure that the weight in financial assets does not exceed $50 \%$ of the weight in nonfinancial assets. Then add another group ratio constraint to ensure that the weight in financial assets constitute at least $20 \%$ of the weight in nonfinancial assets of the portfolio.

```
p = Portfolio;
GA = [ true true true false false false ]; % financial companies
GB = [ false false false true true true ]; % nonfinancial companies
p = setGroupRatio(p, GA, GB, [], 0.5);
GA = [ true false true false true false ]; % odd-numbered companies
GB = [ false false false true true true ]; % nonfinancial companies
p = addGroupRatio(p, GA, GB, 0.2);
disp(p.NumAssets);
    6
disp(p.GroupA);
    1
disp(p.GroupB);
    0
    0
disp(p.LowerRatio);
```

- Inf
0.2000
disp(p.UpperRatio);
0.5000

Inf

## Add Group Ratio Constraints to a PortfolioCVaR Object

Set a group ratio constraint to ensure that the weight in financial assets does not exceed $50 \%$ of the weight in nonfinancial assets. Then add another group ratio constraint to ensure that the weight in financial assets constitute at least $20 \%$ of the weight in nonfinancial assets of the portfolio.

```
p = PortfolioCVaR;
GA = [ true true true false false false ]; % financial companies
GB = [ false false false true true true ]; % nonfinancial companies
p = setGroupRatio(p, GA, GB, [], 0.5);
GA = [ true false true false true false ]; % odd-numbered companies
GB = [ false false false true true true ]; % nonfinancial companies
p = addGroupRatio(p, GA, GB, 0.2);
disp(p.NumAssets);
    6
disp(p.GroupA);
    1
    1 0
disp(p.GroupB);
\begin{tabular}{llllll}
0 & 0 & 0 & 1 & 1 & 1 \\
0 & 0 & 0 & 1 & 1 & 1
\end{tabular}
disp(p.LowerRatio);
    -Inf
    0.2000
disp(p.UpperRatio);
```


### 0.5000

Inf

## Add Group Ratio Constraints to a PortfolioMAD Object

Set a group ratio constraint to ensure that the weight in financial assets does not exceed $50 \%$ of the weight in nonfinancial assets. Then add another group ratio constraint to ensure that the weight in financial assets constitute at least $20 \%$ of the weight in nonfinancial assets of the portfolio.

```
p = PortfolioMAD;
GA = [ true true true false false false ]; % financial companies
GB = [ false false false true true true ]; % nonfinancial companies
p = setGroupRatio(p, GA, GB, [], 0.5);
GA = [ true false true false true false ]; % odd-numbered companies
GB = [ false false false true true true ]; % nonfinancial companies
p = addGroupRatio(p, GA, GB, 0.2);
disp(p.NumAssets);
    6
disp(p.GroupA);
\begin{tabular}{llllll}
1 & 1 & 1 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 & 1 & 0
\end{tabular}
disp(p.GroupB);
\begin{tabular}{llllll}
0 & 0 & 0 & 1 & 1 & 1 \\
0 & 0 & 0 & 1 & 1 & 1
\end{tabular}
disp(p.LowerRatio);
        -Inf
    0.2000
disp(p.UpperRatio);
    0.5000
        Inf
```


## Input Arguments

## obj - Object for portfolio

object
Object for portfolio, specified using Portfolio, PortfolioCVaR, or PortfolioMAD object. For more information on creating a portfolio object, see

- Portfolio
- PortfolioCVaR
- PortfolioMAD

Data Types: object

## GroupA - Base groups for comparison

matrix of logical or numerical arrays
Base groups for comparison, specified as a matrix of logical or numerical arrays.

Note The group matrices GroupA and GroupB are usually indicators of membership in groups, which means that their elements are usually either 0 or 1 . Because of this interpretation, the GroupA and GroupB matrices can be logical or numerical arrays.

## Data Types: double

## GroupB - Comparison group

matrix of logical or numerical arrays
Comparison group, specified as a matrix of logical or numerical arrays.

Note The group matrices GroupA and GroupB are usually indicators of membership in groups, which means that their elements are usually either 0 or 1 . Because of this interpretation, the GroupA and GroupB matrices can be logical or numerical arrays.

## Data Types: double

## LowerRatio - Lower-bound for ratio of GroupB groups to GroupA groups vector

Lower-bound for ratio of GroupB groups to GroupA groups, specified as a vector.

Note If input is scalar, LowerRatio undergoes scalar expansion to be conformable with the group matrices.

## Data Types: double

## UpperRatio - Upper-bound for ratio of GroupB groups to GroupA groups vector

Upper-bound for ratio of GroupB groups to GroupA groups, specified as a vector.

Note If input is scalar, UpperRatio undergoes scalar expansion to be conformable with the group matrices.

## Data Types: double

## Output Arguments

## obj - Updated portfolio object

object for portfolio
Updated portfolio object, returned as a Portfolio, PortfolioCVaR, or PortfolioMAD object. For more information on creating a portfolio object, see

- Portfolio
- PortfolioCVaR
- PortfolioMAD


## Tips

- You can also use dot notation to add group ratio constraints for the portfolio weights to existing group ratio constraints.
obj = obj.addGroupRatio(GroupA, GroupB, LowerRatio, UpperRatio)
- To remove group ratio constraints from any of the portfolio objects using dot notation, enter empty arrays for the corresponding arrays.


## See Also

setGroupRatio

## Topics

"Working with Group Ratio Constraints Using Portfolio Object" on page 4-84
"Working with Group Ratio Constraints Using PortfolioCVaR Object" on page 5-78
"Working with Group Ratio Constraints Using PortfolioMAD Object" on page 6-75
"Portfolio Optimization Examples" on page 4-167
"Portfolio Set for Optimization Using Portfolio Object" on page 4-10
"Portfolio Set for Optimization Using PortfolioCVaR Object" on page 5-10
"Portfolio Set for Optimization Using PortfolioMAD Object" on page 6-10

## Introduced in R2011a

## addGroups

Add group constraints for portfolio weights to existing group constraints

## Syntax

obj = addGroups(obj,GroupMatrix,LowerGroup)
obj = addGroups(obj,GroupMatrix,LowerGroup,UpperGroup)

## Description

obj = addGroups(obj,GroupMatrix, LowerGroup) adds group constraints for portfolio weights to existing group constraints for Portfolio, PortfolioCVaR, or PortfolioMAD objects. For details on the respective workflows when using these different objects, see "Portfolio Object Workflow" on page 4-21, "PortfolioCVaR Object Workflow" on page 5-20, and "PortfolioMAD Object Workflow" on page 6-19.

Given GroupMatrix and either LowerGroup or UpperGroup, a portfolio Port must satisfy the following:

LowerGroup <= GroupMatrix * Port <= UpperGroup
obj = addGroups(obj,GroupMatrix,LowerGroup,UpperGroup) adds group constraints for portfolio weights to existing group constraints with an additional option for UpperGroup.

Given GroupMatrix and either LowerGroup or UpperGroup, a portfolio Port must satisfy the following:

LowerGroup <= GroupMatrix * Port <= UpperGroup

## Examples

## Add Group Constraints to a Portfolio Object

Set a group constraint to ensure that the first three assets constitute at most $30 \%$ of a portfolio. Then add another group constraint to ensure that the odd-numbered assets constitute at least $20 \%$ of a portfolio.

```
p = Portfolio;
G = [ true true true false false ]; % group matrix for first group constraint
p = setGroups(p, G, [], 0.3);
G = [ true false true false true ]; % group matrix for second group constraint
p = addGroups(p, G, 0.2);
disp(p.NumAssets);
    5
disp(p.GroupMatrix);
\begin{tabular}{lllll}
1 & 1 & 1 & 0 & 0 \\
1 & 0 & 1 & 0 & 1
\end{tabular}
disp(p.LowerGroup);
    -Inf
    0.2000
disp(p.UpperGroup);
    0.3000
        Inf
```


## Add Group Constraints to a PortfolioCVaR Object

Set a group constraint to ensure that the first three assets constitute at most $30 \%$ of a portfolio. Then add another group constraint to ensure that the odd-numbered assets constitute at least $20 \%$ of a portfolio.

```
p = PortfolioCVaR;
G = [ true true true false false ]; % group matrix for first group constraint
p = setGroups(p, G, [], 0.3);
G = [ true false true false true ]; % group matrix for second group constraint
p = addGroups(p, G, 0.2);
disp(p.NumAssets);
```

5
disp(p.GroupMatrix);

| 1 | 1 | 1 | 0 | 0 |
| :--- | :--- | :--- | :--- | :--- |
| 1 | 0 | 1 | 0 | 1 |

disp(p.LowerGroup);
-Inf
0. 2000
disp(p.UpperGroup);
0.3000

Inf

## Add Group Constraints to a PortfolioMAD Object

Set a group constraint to ensure that the first three assets constitute at most $30 \%$ of a portfolio. Then add another group constraint to ensure that the odd-numbered assets constitute at least $20 \%$ of a portfolio.

```
p = PortfolioMAD;
```

G = [ true true true false false ]; \% group matrix for first group constraint
$\mathrm{p}=\operatorname{set} \operatorname{roups}(\mathrm{p}, \mathrm{G}, \mathrm{[ }], 0.3)$;
$\mathrm{G}=[$ true false true false true ]; \% group matrix for second group constraint
$\mathrm{p}=\operatorname{addGroups}(\mathrm{p}, \mathrm{G}, 0.2)$;
disp(p.NumAssets);

5
disp(p.GroupMatrix);

| 1 | 1 | 1 | 0 | 0 |
| :--- | :--- | :--- | :--- | :--- |

disp(p.LowerGroup);
-Inf
0.2000
disp(p.UpperGroup);

### 0.3000

## Input Arguments

## obj - Object for portfolio

object
Object for portfolio, specified using Portfolio, PortfolioCVaR, or PortfolioMAD object. For more information on creating a portfolio object, see

- Portfolio
- PortfolioCVaR
- PortfolioMAD

Data Types: object

## GroupMatrix - Group constraint matrix

matrix
Group constraint matrix, specified as a matrix.

Note The group matrix GroupMatrix often indicates membership in groups, which means that its elements are usually either 0 or 1 . Because of this interpretation,GroupMatrix can be a logical or numerical matrix.

## Data Types: double

## LowerGroup - Lower bound for group constraints

vector
Lower bound for group constraints, specified as a vector.

Note If input is scalar, LowerGroup undergoes scalar expansion to be conformable with GroupMatrix.

## Data Types: double

## UpperGroup - Upper bound for group constraints vector

Upper bound for group constraints, specified as a vector.

Note If input is scalar, UpperGroup undergoes scalar expansion to be conformable with GroupMatrix.

## Data Types: double

## Output Arguments

## obj - Updated portfolio object

object for portfolio
Updated portfolio object, returned as a Portfolio, PortfolioCVaR, or PortfolioMAD object. For more information on creating a portfolio object, see

- Portfolio
- PortfolioCVaR
- PortfolioMAD


## Tips

- You can also use dot notation to add group constraints for portfolio weights. obj = obj.addGroups(GroupMatrix, LowerGroup, UpperGroup)
- To remove group constraints from any of the portfolio objects using dot notation, enter empty arrays for the corresponding arrays.


## See Also

setGroups

## Topics

"Working with Group Constraints Using Portfolio Object" on page 4-80
"Working with Group Constraints Using PortfolioCVaR Object" on page 5-74
"Working with Group Constraints Using PortfolioMAD Object" on page 6-71
"Portfolio Optimization Examples" on page 4-167
"Portfolio Set for Optimization Using Portfolio Object" on page 4-10
"Portfolio Set for Optimization Using PortfolioCVaR Object" on page 5-10
"Portfolio Set for Optimization Using PortfolioMAD Object" on page 6-10

## Introduced in R2011a

## addInequality

Add linear inequality constraints for portfolio weights to existing constraints

## Syntax

```
obj = addInequality(obj,AInequality,bInequality)
```


## Description

obj = addInequality(obj,AInequality,bInequality) adds linear inequality constraints for portfolio weights to existing constraints for Portfolio, PortfolioCVaR, or PortfolioMAD objects. For details on the respective workflows when using these different objects, see "Portfolio Object Workflow" on page 4-21, "PortfolioCVaR Object Workflow" on page 5-20, and "PortfolioMAD Object Workflow" on page 6-19.

Given a linear inequality constraint matrix AInequality and vector bInequality, every weight in a portfolio Port must satisfy the following:

```
AInequality * Port = bInequality
```

This function "stacks" additional linear inequality constraints onto any existing linear inequality constraints that exist in the input portfolio object. If no constraints exist, this function is the same as setInequality.

## Examples

## Add Linear Inequality Constraint to a Portfolio Object

Set a linear inequality constraint to ensure that the first three assets constitute at most $50 \%$ of a portfolio. Then add another linear inequality constraint to ensure that the last three assets constitute at least $50 \%$ of a portfolio.

```
p = Portfolio;
A = [ 1 1 1 0 0 ]; % first inequality constraint
```

```
b = 0.5;
p = setInequality(p, A, b);
A = [ 0 0 -1 -1 -1 ]; % second inequality constraint
b = -0.5;
p = addInequality(p, A, b);
disp(p.NumAssets);
    5
disp(p.AInequality);
    1
disp(p.bInequality);
    0.5000
    -0.5000
```


## Add Linear Inequality Constraint to a PortfolioCVaR Object

Set a linear inequality constraint to ensure that the first three assets constitute at most $50 \%$ of a portfolio. Then add another linear inequality constraint to ensure that the last three assets constitute at least 50\% of a portfolio.

```
p = PortfolioCVaR;
A = [ 1 1 1 0 0 ]; % first inequality constraint
b = 0.5;
p = setInequality(p, A, b);
A = [ 0 0 -1 -1 -1 ]; % second inequality constraint
b = -0.5;
p = addInequality(p, A, b);
disp(p.NumAssets);
    5
disp(p.AInequality);
```

| 1 | 1 | 1 | 0 | 0 |
| ---: | ---: | ---: | ---: | ---: |
| 0 | 0 | -1 | -1 | -1 |

```
disp(p.bInequality);
    0.5000
    -0.5000
```


## Add Linear Inequality Constraint to a PortfolioMAD Object

Set a linear inequality constraint to ensure that the first three assets constitute at most $50 \%$ of a portfolio. Then add another linear inequality constraint to ensure that the last three assets constitute at least $50 \%$ of a portfolio.

```
p = PortfolioMAD;
A = [ 1 1 1 0 0 ]; % first inequality constraint
b = 0.5;
p = setInequality(p, A, b);
A = [ 0 0 -1 -1 -1 ]; % second inequality constraint
b = -0.5;
p = addInequality(p, A, b);
disp(p.NumAssets);
    5
disp(p.AInequality);
    1
    0 0
disp(p.bInequality);
    0.5000
    -0.5000
```


## Input Arguments

## obj - Object for portfolio

object

Object for portfolio, specified using Portfolio, PortfolioCVaR, or PortfolioMAD object. For more information on creating a portfolio object, see

- Portfolio
- PortfolioCVaR
- PortfolioMAD

Data Types: object
AInequality - Linear inequality constraints formed from matrix matrix

Linear inequality constraints, specified as a matrix.

Note An error results if AInequality is empty and bInequality is nonempty.

## Data Types: double

## bInequality - Linear inequality constraints formed from vector

 vectorLinear inequality constraints, specified as a vector.

Note An error results if bInequality is empty and AInequality is nonempty.

## Data Types: double

## Output Arguments

## obj - Updated portfolio object

object for portfolio
Updated portfolio object, returned as a Portfolio, PortfolioCVaR, or PortfolioMAD object. For more information on creating a portfolio object, see

- Portfolio
- PortfolioCVaR
- PortfolioMAD


## Tips

- You can also use dot notation to add the linear inequality constraints for portfolio weights.

```
obj = obj.addInequality(AInequality, bInequality)
```

- You can also remove linear inequality constraints from any of the portfolio objects using dot notation.

```
obj = obj.setInequality([ ], [ ])
```


## See Also

## setInequality

## Topics

"Working with Linear Inequality Constraints Using Portfolio Object" on page 4-91
"Working with Linear Inequality Constraints Using PortfolioCVaR Object" on page 5-85
"Working with Linear Inequality Constraints Using PortfolioMAD Object" on page 6-82
"Portfolio Optimization Examples" on page 4-167
"Portfolio Set for Optimization Using Portfolio Object" on page 4-10
"Portfolio Set for Optimization Using PortfolioCVaR Object" on page 5-10
"Portfolio Set for Optimization Using PortfolioMAD Object" on page 6-10

## Introduced in R2011a

## adline

Accumulation/Distribution line

Note adline has been partially removed and will no longer accept a fints object (tsobj) argument. Use a matrix, timetable, or table instead for financial time series.

Use fts2timetable to convert a fints object to a timetable object.

## Syntax

ADline = adline(Data)

## Description

ADline = adline(Data) calculates the Accumulation/Distribution Line from a set of high, low, closing prices, and volume traded of a security.

## Examples

## Calculate the Accumulation/Distribution Line for a Data Series for a Stock

Load the file SimulatedStock. mat, which provides a timetable (TMW) for financial data for TMW stock.

```
load SimulatedStock.mat
line = adline(TMW);
plot(line.Time,line.ADLine)
title('Accumulation/Distribution Line for TMW')
```



## Input Arguments

Data - Data with high, low, closing prices, and volume traded matrix | table | timetable

Data with high, low, closing prices and volume traded, specified as a matrix, table, or timetable. For matrix input, Data is M-by-4 with high, low, closing prices, and volume traded. Timetables and tables with M rows must contain a variable named 'High', 'Low', 'Close', and 'Volume' (case insensitive).

Data Types: double|table|timetable

## Output Arguments

## ADline - Accumulation/Distribution line matrix | table | timetable

Accumulation/Distribution line, returned with the same number of rows (M) and the same type (matrix, table, or timetable) as the input Data.

## References

[1] Achelis, S. B. Technical Analysis from A to Z. Second Edition. McGraw-Hill, 1995, pp. 52-53.

## See Also

adosc|table|timetable|willad|willpctr

## Topics

"Using Timetables in Finance"
"Convert Financial Time Series Objects fints to Timetables" on page 12-2

Introduced before R2006a

## adosc

Accumulation/Distribution oscillator

Note adosc has been partially removed and will no longer accept a fints object (tsobj) argument. Use a matrix, timetable, or table instead for financial time series.

Use fts2timetable to convert a fints object to a timetable object.

## Syntax

ado $=\operatorname{adosc}($ Data $)$

## Description

ado $=$ adosc(Data) calculates the Accumulation/Distribution (A/D) oscillator.

## Examples

## Calculate the Accumulation/Distribution Oscillator for a Stock

Load the file SimulatedStock.mat, which provides a timetable (TMW) for financial data for TMW stock.

```
load SimulatedStock.mat
ADOsc = adosc(TMW);
plot(ADOsc.dates, ADOsc.ADOscillator)
title('A/D Oscillator for TMW')
```



## Input Arguments

## Data - Data with high, low, open, close information

matrix | table | timetable
Data with high, low, open, close information, specified as a matrix, table, or timetable. For matrix input, Data is an M-by-4 matrix of high, low, opening, and closing prices.
Timetables and tables with M rows must contain variables named 'High', 'Low', 'Open ', and 'Close' (case insensitive).

Data Types: double| table| timetable

## Output Arguments

## ado - Accumulation/Distribution oscillator <br> matrix | table | timetable

Accumulation/Distribution oscillator, returned with the same number of rows (M) and type (matrix, table, or timetable) as the input Data.

## References

[1] Kaufman, P. J. The New Commodity Trading Systems and Methods. John Wiley and Sons, New York, 1987.

## See Also

adline|table|timetable|willad

## Topics

"Using Timetables in Finance"
"Convert Financial Time Series Objects fints to Timetables" on page 12-2
Introduced before R2006a

## amortize

Amortization schedule

## Syntax

[Principal,Interest,Balance,Payment] = amortize(Rate,NumPeriods,PresentValue,FutureVal

## Arguments

| Rate | Interest rate per period, as a decimal fraction. |
| :--- | :--- |
| NumPeriods | Number of payment periods. |
| PresentValue | Present value of the loan. |
| FutureValue | (Optional) Future value of the loan. Default $=0$. |
| Due | (Optional) When payments are due: $0=$ end of period <br> (default), or $1=$ beginning of period. |

## Description

[Principal,Interest,Balance, Payment] =
amortize(Rate, NumPeriods, PresentValue, FutureValue, Due) returns the principal and interest payments of a loan, the remaining balance of the original loan amount, and the periodic payment.

| Principal | Principal paid in each period. A 1-by-NumPeriods vector. |
| :--- | :--- |
| Interest | Interest paid in each period. A 1-by-NumPeriods vector. |
| Balance | Remaining balance of the loan in each payment period. A 1- <br> by-NumPeriods vector. |
| Payment | Payment per period. A scalar. |

## Examples

## Compute an Amortization Schedule for a Conventional 30-Year, Fixed-Rate Mortgage With Fixed Monthly Payments

Compute an amortization schedule for a conventional 30-year, fixed-rate mortgage with fixed monthly payments and assume a fixed rate of $12 \%$ APR and an initial loan amount of \$100,000.

```
Rate = 0.12/12; % 12 percent APR = 1 percent per month
NumPeriods = 30*12; % 30 years = 360 months
PresentValue = 100000;
[Principal, Interest, Balance, Payment] = amortize(Rate, ...
NumPeriods, PresentValue);
```

The output argument Payment contains the fixed monthly payment.
format bank
Payment
Payment =
1028.61

Summarize the amortization schedule graphically by plotting the current outstanding loan balance, the cumulative principal, and the interest payments over the life of the mortgage. In particular, note that total interest paid over the life of the mortgage exceeds $\$ 270,000$, far in excess of the original loan amount.

```
plot(Balance,'b'), hold('on')
plot(cumsum(Principal),'--k')
plot(cumsum(Interest),':r')
xlabel('Payment Month')
ylabel('Dollars')
grid('on')
title('Outstanding Balance, Cumulative Principal & Interest')
legend('Outstanding Balance', 'Cumulative Principal', ...
'Cumulative Interest')
```



The solid blue line represents the declining principal over the 30-year period. The dotted red line indicates the increasing cumulative interest payments. Finally, the dashed black line represents the cumulative principal payments, reaching \$100,000 after 30 years.

## See Also

annurate | annuterm | payadv | payodd | payper

## Topics

"Analyzing and Computing Cash Flows" on page 2-18

Introduced before R2006a

## annurate

Periodic interest rate of annuity

## Syntax

Rate = annurate(NumPeriods,Payment,PresentValue,FutureValue, Due)

## Arguments

| NumPeriods | Number of payment periods. |
| :--- | :--- |
| Payment | Payment per period. |
| PresentValue | Present value of the loan or annuity. |
| FutureValue | (Optional) Future value of the loan or annuity. Default $=0$. |
| Due | (Optional) When payments are due: $0=$ end of period <br> (default), or $1=$ beginning of period. |

## Description

Rate = annurate(NumPeriods,Payment,PresentValue,FutureValue,Due) returns the periodic interest rate paid on a loan or annuity.

## Examples

## Calculate the Periodic Interest Rate Paid on a Loan or Annuity

This example shows how to find the periodic interest rate of a four-year, \$5000 loan with a $\$ 130$ monthly payment made at the end of each month.

Rate = annurate(4*12, 130, 5000, 0, 0)
Rate = 0.0094

Rate multiplied by 12 gives an annual interest rate of $11.32 \%$ on the loan.

See Also<br>amortize|annuterm|bndyield|irr<br>Topics<br>"Analyzing and Computing Cash Flows" on page 2-18<br>Introduced before R2006a

## annuterm

Number of periods to obtain value

## Syntax

NumPeriods = annuterm(Rate, Payment, PresentValue, FutureValue, Due)

## Arguments

| Rate | Interest rate per period, as a decimal fraction. |
| :--- | :--- |
| Payment | Payment per period. |
| PresentValue | Present value. |
| FutureValue | (Optional) Future value. Default $=0$. |
| Due | (Optional) When payments are due: $0=$ end of period <br> (default), or $1=$ beginning of period. |

## Description

NumPeriods = annuterm(Rate, Payment, PresentValue,FutureValue, Due) calculates the number of periods needed to obtain a future value. To calculate the number of periods needed to pay off a loan, enter the payment or the present value as a negative value.

## Examples

## Calculate the Number of Periods Needed to Obtain a Future Value

This example shows a savings account with a starting balance of $\$ 1500$. $\$ 200$ is added at the end of each month and the account pays $9 \%$ interest, compounded monthly. How many years will it take to save $\$ 5,000$ ?

```
NumPeriods = annuterm(0.09/12, 200, 1500, 5000, 0)
NumPeriods = 15.6752
```


## See Also

amortize| annurate|fvfix|pvfix

## Topics

"Analyzing and Computing Cash Flows" on page 2-18

## Introduced before R2006a

## arith2geom

Arithmetic to geometric moments of asset returns

## Syntax

[mg, Cg] = arith2geom(ma,Ca)
[mg, Cg] = arith2geom(ma,Ca,t)

## Arguments

| ma | Arithmetic mean of asset-return data (n-vector). |
| :--- | :--- |
| Ca | Arithmetic covariance of asset-return data, an n-by-n symmetric, <br> positive-semidefinite matrix. |
| t | (Optional) Target period of geometric moments in terms of <br> periodicity of arithmetic moments with default value 1 (scalar). |

## Description

arith2geom transforms moments associated with a simple Brownian motion into equivalent continuously compounded moments associated with a geometric Brownian motion with a possible change in periodicity.
[mg,Cg] = arith2geom(ma, Ca,t) returns mg, continuously compounded or "geometric" mean of asset returns over the target period ( n -vector), and Cg , which is a continuously compounded or "geometric" covariance of asset returns over the target period ( $n$-by-n matrix).

Arithmetic returns over period $t_{A}$ are modeled as multivariate normal random variables with moments

$$
E[\mathrm{X}]=\mathrm{m}_{A}
$$

and

$$
\operatorname{cov}(\mathrm{X})=\mathrm{C}_{A}
$$

Geometric returns over period $t_{G}$ are modeled as multivariate lognormal random variables with moments

$$
\begin{aligned}
& E[\mathrm{Y}]=1+\mathrm{m}_{G} \\
& \operatorname{cov}(\mathrm{Y})=\mathrm{C}_{G}
\end{aligned}
$$

Given $t=t_{G} / t_{A}$, the transformation from geometric to arithmetic moments is

$$
\begin{aligned}
& 1+\mathrm{m}_{G_{i}}=\exp \left(t \mathrm{~m}_{A_{i}}+\frac{1}{2} t \mathrm{C}_{A_{i i}}\right) \\
& \mathrm{C}_{G_{i j}}=\left(1+\mathrm{m}_{G_{i}}\right)\left(1+\mathrm{m}_{G_{\mathrm{j}}}\right)\left(\exp \left(t \mathrm{C}_{A i j}\right)-1\right)
\end{aligned}
$$

For $i, j=1, \ldots, n$.

## Note If $t=1$, then $\mathbf{Y}=\exp (\mathbf{X})$.

This function has no restriction on the input mean ma but requires the input covariance Ca to be a symmetric positive-semidefinite matrix.

The functions arith2geom and geom2arith are complementary so that, given m, C, and $t$, the sequence
[mg, Cg] = arith2geom(m,C,t);
[ma,Ca] = geom2arith (mg, Cg,1/t);
yields $m a=m$ and $C a=C$.

## Examples

Example 1. Given arithmetic mean $m$ and covariance $C$ of monthly total returns, obtain annual geometric mean mg and covariance Cg . In this case, the output period (1 year) is 12 times the input period (1 month) so that $t=12$ with

```
[mg, Cg] = arith2geom(m, C, 12);
```

Example 2. Given annual arithmetic mean $m$ and covariance $C$ of asset returns, obtain monthly geometric mean mg and covariance Cg . In this case, the output period (1 month) is $1 / 12$ times the input period ( 1 year) so that $\mathrm{t}=1 / 12$ with
[mg, Cg] = arith2geom(m, C, 1/12);
Example 3. Given arithmetic means $m$ and standard deviations $s$ of daily total returns (derived from 260 business days per year), obtain annualized continuously compounded mean mg and standard deviations sg with
[mg, Cg] = arith2geom(m, diag(s .^2), 260); $\mathrm{sg}=\mathrm{sqrt}(\mathrm{diag}(\mathrm{Cg}))$;

Example 4. Given arithmetic mean $m$ and covariance $C$ of monthly total returns, obtain quarterly continuously compounded return moments. In this case, the output is 3 of the input periods so that $t=3$ with
[mg, Cg] = arith2geom(m, C, 3);
Example 5. Given arithmetic mean m and covariance C of 1254 observations of daily total returns over a 5 -year period, obtain annualized continuously compounded return moments. Since the periodicity of the arithmetic data is based on 1254 observations for a 5 -year period, a 1-year period for geometric returns implies a target period of $t=$ 1254/5 so that
[mg, Cg] = arith2geom(m, C, 1254/5);

## See Also

geom2arith

## Topics

"Data Transformation and Frequency Conversion" on page 13-13

Introduced before R2006a

## ascii2fts

Create financial time series object from ASCII file

Note ascii2fts will be removed in a future release. Use timetable instead. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

tsobj = ascii2fts(filename, descrow,colheadrow,skiprows)
tsobj = ascii2fts(filename,timedata,descrow,colheadrow,skiprows)

## Arguments

| filename | ASCII data file |
| :--- | :--- |
| descrow | (Optional) Row number in the data file that contains the description <br> to be used for the description field of the financial time series object <br> (Optional) Row number that has the column headers/names |
| colheadrow | (Optional) Scalar or vector of row numbers to be skipped in the data <br> file |
| skiprows | Set to ' $\mathrm{T}^{\prime}$ ' if time-of-day data is present in the ASCII data file or to <br> 'NT' if no time-of-day data is present. |
| timedata |  |

## Description

tsobj = ascii2fts(filename, descrow, colheadrow, skiprows) creates a financial time series object tsobj from the ASCII file named filename. This form of the function can only read a data file without time-of-day information and create a financial time series object without time information. If time information is present in the ASCII file, an error message appears.

The general format of the text data file is

- Can contain header text lines.
- Can contain column header information. The column header information must immediately precede the data series columns unless skiprows is specified.
- Leftmost column must be the date column.
- Dates must be in a valid character vector date format:
- 'ddmmmyy' or 'ddmmmyyyy'
- 'mm/dd/yy' or 'mm/dd/yyyy'
- 'dd-mmm-yy' or 'dd-mmm-yyyy'
- 'mmm.dd,yy' or 'mmm.dd,yyyy'
- Time information must be in 'hh:mm' format.
- Each column must be separated either by spaces or a tab.
tsobj = ascii2fts(filename, timedata, descrow, colheadrow, skiprows) creates a financial time series object containing time-of-day data. Set timedata to ' $T$ ' to create a financial time series object containing time-of-day data. The ascii time information must be in 'hh:mm' format for ascii2fts.


## Examples

Example 1. If your data file contains no description or column header rows,

| $1 / 3 / 95$ | 36.75 | 36.9063 | 36.6563 | 36.875 | 1167900 |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $1 / 4 / 95$ | 37 | 37.2813 | 36.625 | 37.1563 | 1994700 | $\ldots$ |

you can create a financial time series object from it with the simplest form of the ascii2fts function:

```
myinc = ascii2fts('my_inc.dat')
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In ascii2fts (line 64)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/display (line 66)
myinc =
desc: my inc.dat
freq: Unk̄nown (0)
```

```
'dates: (2)' 'series1: (2)' 'series2: (2)' 'series3: (2)'...
'03-Jan-1995' [ 36.7500] [ 36.9063] [ 36.6563]
'04-Jan-1995' [ 37] [ 37.2813] [ 36.6250]
```

Example 2: If your data file contains description and column header information with the data series immediately following the column header row,

International Business Machines Corporation (IBM) Daily prices (1/3/95 to 4/5/99)

| DATE | OPEN | HIGH | LOW | CLOSE | VOLUME |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $1 / 3 / 95$ | 36.75 | 36.9063 | 36.6563 | 36.875 | 1167900 |  |
| $1 / 4 / 95$ | 37 | 37.2813 | 36.625 | 37.1563 | 1994700 | $\ldots$ |

you must specify the row numbers containing the description and column headers:

```
ibm = ascii2fts('ibm9599.dat', 1, 3)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In ascii2fts (line 64)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/display (line 66)
ibm =
desc: International Business Machines Corporation (IBM)
freq: Unknown (0)
'dates: (2)' 'OPEN: (2)' 'HIGH: (2)' 'LOW: (2)' ...
'03-Jan-1995' [ 36.7500] [ 36.9063] [ 36.6563]
'04-Jan-1995' [ 37] [ 37.2813] [ 36.6250]
```

Example 3: If your data file contains rows between the column headers and the data series, for example,

```
Staples, Inc. (SPLS)
Daily prices
DATE OPEN HIGH LOW CLOSE VOLUME
Starting date: 04/08/1996
Ending date: 04/07/1999
4/8/96 19.50 19.75 19.25 19.375 548500
4/9/96 19.75 20.125 19.375 20 1135900 ...
```

you need to indicate to ascii2fts the rows in the file that must be skipped. Assume that you have called the data file containing the Staples data (staples.dat).

```
spls = ascii2fts('staples.dat', 1, 3, [4 5])
```

The command above indicates that the fourth and fifth rows in the file should be skipped in creating the financial time series object:

```
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In ascii2fts (line 64)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/display (line 66)
spls =
desc: Staples, Inc. (SPLS)
freq: Unknown (0)
\begin{tabular}{|c|c|c|c|}
\hline s: (2)' & 'OPEN: (2)' & 'HIGH: (2)' & 'LOW: (2) \\
\hline '08-Apr-1996' & \(19.5000]\) & 19.7500] & [19.2500] \\
\hline '09-Apr-1996' & 19.7500] & 20.1250] & [19.3750] \\
\hline
\end{tabular}
```

Example 4: Create a financial time series object containing time-of-day information.
First create a data file with time information:

```
dates = ['01-Jan-2001';'01-Jan-2001'; '02-Jan-2001'; ...
'02-Jan-2001'; '03-Jan-2001';'03-Jan-2001'];
times = ['11:00';'12:00';'11:00';'12:00';'11:00';'12:00'];
serial_dates_times = [datenum(dates), datenum(times)];
data = round(10*rand(6,2));
stat = fts2ascii('myfts file2.txt',serial dates times,data, ...
{'dates';'times';'Data1';'Data2'},'My FTS with Time')
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fts2ascii (line 60)
stat =
    1
```

Now read the data file back and create a financial time series object:

```
MyFts = ascii2fts('myfts file2.txt','t',1,2,1)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In ascii2fts (line 64)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/display (line 66)
MyFts =
```

```
desc: My FTS with Time
freq: Unknown (0)
```

| 'dates: (6)' | 'times: | (6) ' | 'Datal: (6)' | 'Data2: (6)' |
| :---: | :---: | :---: | :---: | :---: |
| '01-Jan-2001' | '11:00' |  | $9]$ | 4] |
| " ' | '12:00' |  | 7] | $9]$ |
| '02-Jan-2001' | '11:00' |  | $2]$ | 1] |
|  | '12:00' |  | $4]$ | 4] |
| '03-Jan-2001' | '11:00' |  | $9]$ | 8] |
|  | '12:00' |  | [ 9] | $0]$ |

## See Also

fints|fts2ascii

## Topics

"Creating a Financial Time Series Object" on page 14-11
"Working with Financial Time Series Objects" on page 13-2

## Introduced before R2006a

## bar, barh

Bar chart

Note bar, barh will be removed in a future release. Use bar or barh instead.

## Syntax

```
bar(tsobj)
bar(tsobj,width)
bar(..., 'style')
hbar = bar(...)
barh(...)
hbarh = barh(...)
```


## Arguments

| tsobj | Financial time series object. |
| :--- | :--- |
| width | Width of the bars and separation of bars within a group. (Default $=$ <br> 0.8.$)$ If width is 1, the bars within a group touch one another. <br> Values > 1 produce overlapping bars. |
| style | 'grouped ' (default) or 'stacked '. |

## Description

bar, barh draw vertical and horizontal bar charts.
bar(tsobj) draws the columns of data series of the object tsobj. The number of data series dictates the number of vertical bars per group. Each group is the data for one particular date.
bar(tsobj, width) specifies the width of the bars.
bar(..., 'style') changes the style of the bar chart.
hbar $=\operatorname{bar}(. .$.$) returns a vector of bar handles.$
Use the MATLAB command shading faceted to put edges on the bars. Use shading flat to turn edges off.

## Examples

## Create a Bar Chart for a Stock

This example shows how to create a bar chart for Disney stock showing high, low, opening, and closing prices.
load disney
Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in bar(q_dis)

Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in title('Bar Chart of Disney Prices')

Bar Chart of Disney Prices


## Create a Horizontal Bar Chart for a Stock

This example shows how to create a horizontal bar chart for Disney stock showing high, low, opening, and closing prices.

```
load disney
barh(q_dis)
```

Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in title('Horizontal Bar Chart of Disney Prices')


See Also<br>bar3, bar3h|candle|highlow

## Introduced before R2006a

## bar3, bar3h

3-D bar chart

Note bar3, bar3h will be removed in a future release. Use bar3 or bar3h instead.

## Syntax

```
bar3(tsobj)
```

bar3(tsobj,width)
bar3(.... 'style')
hbar3 = bar3(...)
bar3h(...)
hbar3h = bar3h(...)

## Arguments

| tsobj | Financial time series object. |
| :--- | :--- |
| width | Width of the bars and separation of bars within a group. (Default = <br> 0.8.$)$ If width is 1, the bars within a group touch one another. <br> Values > 1 produce overlapping bars. |
| style | 'detached ' (default), 'grouped ' , or 'stacked '. |

## Description

bar3, bar3h draw three-dimensional vertical and horizontal bar charts.
bar3(tsobj) draws the columns of data series of the object tsobj. The number of data series dictates the number of vertical bars per group. Each group is the data for one particular date.
bar3(tsobj, width) specifies the width of the bars.
bar3(..., 'style') changes the style of the bar chart.
hbar3 $=\operatorname{bar} 3(. .$.$) returns a vector of bar handles.$
Use the MATLAB command shading faceted to put edges on the bars. Use shading flat to turn edges off.

## Examples

## Create a Three-Dimensional Bar Chart

This example shows how to create a three-dimensional bar chart for Disney stock showing high, low, opening, and closing prices.

```
load disney
bar3(q_dis, 'stacked')
```

Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in title('Three-Dimensional Bar Chart of Disney Prices')


## Create a Three-Dimensional (Stacked) Bar Chart

This example shows how to create a three-dimensional, stacked bar chart for Disney stock showing high, low, opening, and closing prices.

```
load disney
bar3(q_dis, 'stacked')
```

Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in title('Three-Dimensional Bar Chart of Disney Prices (Stacked)')


## See Also

bar, barh|candle|highlow
Introduced before R2006a

## beytbill

Bond equivalent yield for Treasury bill

## Syntax

```
Yield = beytbill(Settle,Maturity,Discount)
```


## Description

Yield = beytbill(Settle,Maturity,Discount) returns the bond equivalent yield for a Treasury bill.

## Examples

## Find the Bond Equivalent Yield for a Treasury Bill

This example shows how to find the bond equivalent yield for a Treasury bill that has a settlement date of February 11, 2000, a maturity date of August 7, 2000, and a discount rate is 5.77.

Yield = beytbill('2/11/2000', '8/7/2000', 0.0577)
Yield $=0.0602$

## Find the Bond Equivalent Yield for a Treasury Bill Using datetime Inputs

This example shows how to use datetime inputs to find the bond equivalent yield for a Treasury bill that has a settlement date of February 11, 2000, a maturity date of August 7, 2000, and the discount rate is 5.77 .

```
Yield = beytbill(datetime('11-Feb-2000','Locale','en US'), datetime('7-Aug-2000','Loca 0.0577)
```


## Input Arguments

## Settle - Settlement date of Treasury bill

serial date number | date character vector | datetime
Settlement date of the Treasury bill, specified as a scalar or a NTBILLS-by-1 vector of serial date numbers, date character vectors, or datetime arrays. Settle must be earlier than Maturity.

## Data Types: double | char | datetime

## Maturity - Maturity date of Treasury bill

serial date number | date character vector | datetime
Maturity date of the Treasury bill, specified as a scalar or a NTBILLS-by-1 vector of serial date numbers, date character vectors, or datetime arrays.

Data Types: double | char|datetime

## Discount - Discount rate of Treasury bill <br> decimal

Discount rate of the Treasury bill, specified as a scalar of a NTBILLS-by-1 vector of decimal fraction values.

Data Types: double

## Output Arguments

## Yield - Treasury bill yield <br> decimal

Treasury bill yield, returned as a scalar or NTBILLS-by-1 vector.

Note The number of days to maturity is typically quoted as: md - sd-1. A NaN is returned for all cases in which negative prices are implied by the discount rate, Discount, and the number of days between Settle and Maturity.
See Alsodatenum|datetime|prtbill|yldtbill
Topics
"Computing Treasury Bill Price and Yield" on page 2-38"Treasury Bills Defined" on page 2-37
Introduced before R2006a

## binprice

Binomial put and call American option pricing using Cox-Ross-Rubinstein model

## Syntax

[AssetPrice,OptionValue] = binprice(Price,Strike,Rate,Time, Increment, Volatility, Flag)
[AssetPrice,OptionValue] = binprice( __ , DividendRate, Dividend,ExDiv)

## Description

[AssetPrice,OptionValue] = binprice(Price,Strike,Rate,Time, Increment, Volatility, Flag) prices an American option using the Cox-RossRubinstein binomial pricing model.
[AssetPrice,OptionValue] = binprice( __ , DividendRate,Dividend,ExDiv) adds optional arguments for DividendRate,Dividend, and ExDiv.

## Examples

## Price an American Option Using the Cox-Ross-Rubinstein Binomial Pricing Model

This example shows how to price an American put option with an exercise price of $\$ 50$ that matures in 5 months. The current asset price is $\$ 52$, the risk-free interest rate is $10 \%$, and the volatility is $40 \%$. There is one dividend payment of $\$ 2.06$ in $3-1 / 2$ months. When specifying the input argument ExDiv in terms of number of periods, divide the exdividend date, specified in years, by the time Increment.

ExDiv $=(3.5 / 12) /(1 / 12)=3.5$
[Price, Option] = binprice(52, 50, 0.1, 5/12, 1/12, 0.4, 0, 0, 2.06, 3.5)
Price $=6 \times 6$

| 52.0000 | 58.1367 | 65.0226 | 72.7494 | 79.3515 | 89.0642 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 46.5642 | 52.0336 | 58.1706 | 62.9882 | 70.6980 |
| 0 | 0 | 41.7231 | 46.5981 | 49.9992 | 56.1192 |
| 0 | 0 | 0 | 37.4120 | 39.6887 | 44.5467 |
| 0 | 0 | 0 | 0 | 31.5044 | 35.3606 |
| 0 | 0 | 0 | 0 | 0 | 28.0688 |
| Option $=6 \times 6$ |  |  |  |  |  |
| 4.4404 | 2.1627 | 0.6361 | 0 | 0 | 0 |
| 0 | 6.8611 | 3.7715 | 1. 3018 | 0 | 0 |
| 0 | 0 | 10.1591 | 6.3785 | 2.6645 | 0 |
| 0 | 0 | 0 | 14.2245 | 10.3113 | 5.4533 |
| 0 | 0 | 0 | 0 | 18.4956 | 14.6394 |
| 0 | 0 | 0 | 0 | 0 | 21.9312 |

The output returned is the asset price and American option value at each node of the binary tree.

## Input Arguments

## Price - Current price of underlying asset

numeric
Current price of underlying asset, specified as a scalar numeric value.
Data Types: double

## Strike - Exercise price of the option

numeric
Exercise price of the option, specified as a scalar numeric value.
Data Types: double

## Rate - Risk-free interest rate <br> decimal

Risk-free interest rate, specified as scalar decimal fraction.
Data Types: double

## Time - Option time until maturity numeric

Option time until maturity, specified as a scalar for the number of years.

## Data Types: double

## Increment - Time increment

numeric
Time increment, specified as a scalar numeric. Increment is adjusted so that the length of each interval is consistent with the maturity time of the option. (Increment is adjusted so that Time divided by Increment equals an integer number of increments.)
Data Types: double
Volatility - Asset volatility
numeric
Asset volatility, specified as a scalar numeric.
Data Types: double

## Flag - Flag indicating whether option is a call or put

integer with values 0 or 1
Flag indicating whether option is a call or put, specified as a scalar Flag $=1$ for a call option, or Flag $=0$ for a put option.
Data Types: logical
DividendRate - Dividend rate
0 (default) | decimal
(Optional) Dividend rate, specified as a scalar decimal. If you enter a value for DividendRate, set Dividend and ExDiv = 0 or do not enter them. If you enter values for Dividend and ExDiv, set DividendRate $=0$

Data Types: double
Dividend - Dividend payment
0 (default) | numeric
(Optional) Dividend payment at an ex-dividend date (ExDiv), specified as a 1-by-N row vector. For each dividend payment, there must be a corresponding ex-dividend date. If you enter values for Dividend and ExDiv, set DividendRate $=0$.

## Data Types: double

## ExDiv - Ex-dividend date

0 (default) | numeric
(Optional) Ex-dividend date, specified as a 1-by-N vector row vector for the number of periods.

Data Types: double

## Output Arguments

## AssetPrice - Asset price

vector
Asset price, returned as a vector that represents each node of the Cox-Ross-Rubinstein (CRR) binary tree.

OptionValue - Option value
vector
Option value, returned as a vector that represents each node of the Cox-Ross-Rubinstein (CRR) binary tree.

## References

[1] Cox, J., S. Ross, and M. Rubenstein. "Option Pricing: A Simplified Approach." Journal of Financial Economics. Vol. 7, Sept. 1979, pp. 229-263.
[2] Hull, John C. Options, Futures, and Other Derivative Securities. 2nd edition, Chapter 14.

## See Also

blkprice|blsprice

## Topics

"Pricing and Analyzing Equity Derivatives" on page 2-45
"Greek-Neutral Portfolios of European Stock Options" on page 10-18
"Plotting Sensitivities of an Option" on page 10-30
"Plotting Sensitivities of a Portfolio of Options" on page 10-33

## Introduced before R2006a

## blkimpv

Implied volatility for futures options from Black model

## Syntax

```
Volatility = blkipmv(Price,Strike,Rate,Time,Value)
Volatility = blkimpv(___,Name,Value)
```


## Description

Volatility = blkipmv(Price,Strike,Rate,Time,Value) computes the implied volatility of a futures price from the market value of European futures options using Black's model. If the Class name-value argument is empty or unspecified, the default is a call option

Note Any input argument can be a scalar, vector, or matrix. When a value is a scalar, that value is used to compute the implied volatility of all the options. If more than one input is a vector or matrix, the dimensions of all nonscalar inputs must be identical.

Ensure that Rate and Time are expressed in consistent units of time.

Volatility = blkimpv( $\qquad$ ,Name, Value) specifies options using one or more name-value pair arguments in addition to the input arguments in the previous syntax.

## Examples

## Find Implied Volatility for Futures Options from Black's Model

This example shows how to find the implied volatility for a European call futures option that expires in four months, trades at $\$ 1.1166$, and has an exercise price of $\$ 20$. Assume that the current underlying futures price is also $\$ 20$ and that the risk-free rate is $9 \%$ per annum. Furthermore, assume that you are interested in implied volatilities no greater
than 0.5 (50\% per annum). Under these conditions, the following commands all return an implied volatility of 0.25 , or $25 \%$ per annum.

```
Volatility = blkimpv(20, 20, 0.09, 4/12, 1.1166, 'Limit',0.5);
Volatility = blkimpv(20, 20, 0.09, 4/12, 1.1166, 'Limit',0.5,'Class',{'Call'});
Volatility = blkimpv(20, 20, 0.09, 4/12, 1.1166, 'Limit',0.5,'Class',true);
Volatility = blkimpv(20, 20, 0.09, 4/12, 1.1166, 'Limit',0.5,'Class',true,'Method','ja
Volatility = 0.2500
```


## Input Arguments

## Price - Current price of underlying asset scalar numeric

Current price of the underlying asset (that is, a futures contract), specified as a scalar numeric.
Data Types: double

## Strike - Exercise price of the futures option <br> scalar numeric

Exercise price of the futures option, specified as a scalar numeric.
Data Types: double

## Rate - Annualized continuously compounded risk-free rate of return over life of the option

## scalar positive decimal

Annualized continuously compounded risk-free rate of return over the life of the option, specified as a scalar positive decimal.

## Data Types: double

## Time - Time to expiration of the futures option <br> scalar numeric

Time to expiration of the futures option, specified as the number of years using a scalar numeric.

Data Types: double

## Value - Price of a European option from which implied volatility of underlying asset is derived

## scalar numeric

Price of a European futures option from which the implied volatility of the underlying asset is derived, specified as a scalar numeric.

Data Types: double

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Example: Volatility = blkimpv(Yield,CouponRate,Settle,Maturity,'Method','jackel2016')

## Limit - Upper bound of implied volatility search interval

10 (1000\% per annum) (default) | positive scalar numeric
Upper bound of the implied volatility search interval, specified as the comma-separated pair consisting of 'Limit' and a positive scalar numeric. If Limit is empty or unspecified, the default is 10 , or $1000 \%$ per annum.

Note If you are using Method with a value of 'jackel2016', the Limit argument is ignored.

## Data Types: double

## Tolerance - Implied volatility termination tolerance

1e-6 (default) | positive scalar numeric
Implied volatility termination tolerance, specified as the comma-separated pair consisting of 'Tolerance' and a positive scalar numeric. If empty or missing, the default is 1e-6.

Note If you are using Method with a value of 'jackel2016', the Tolerance argument is ignored.

## Data Types: double

## Class - Option class from which implied volatility is derived

true (call option) (default) | logical | cell array of character vectors | string array
Option class indicating option type (call or put) from which implied volatility is derived, specified as the comma-separated pair consisting of 'Class' and a logical indicator, cell array of character vectors, or string array.

To specify call options, set Class $=$ true or Class $=\{$ 'call' $\}$. To specify put options, set Class $=$ false or Class $=\{$ 'put' $\}$ or Class $=$ ["put"]. If Class is empty or unspecified, the default is a call option.
Data Types: logical|cell|string

## Method - Method for computing implied volatility

' jackel2016' (default)| character vector with values 'search' or 'jackel2016' | string with values "search" or "jackel2016"

Method for computing implied volatility, specified as the comma-separated pair consisting of 'Method ' and a character vector with a value of 'search' or 'jackel2016' or a string with a value of "search" or "jackel2016".
Data Types: char|string

## Output Arguments

## Volatility - Implied volatility of underlying asset derived from European futures option prices

decimal
Implied volatility of the underlying asset derived from European futures option prices, returned as a decimal. If no solution is found, blkimpv returns NaN .

## References

[1] Hull, John C. Options, Futures, and Other Derivatives. 5th edition, Prentice Hall, 2003, pp. 287-288.
[2] Jäckel, Peter. "Let's Be Rational." Wilmott Magazine., January, 2015 (https:// onlinelibrary.wiley.com/doi/pdf/10.1002/wilm.10395).
[3] Black, Fischer. "The Pricing of Commodity Contracts." Journal of Financial Economics. March 3, 1976, pp. 167-79.

See Also<br>blkprice|blsimpv|blsprice

## Topics

"Pricing and Analyzing Equity Derivatives" on page 2-45
"Greek-Neutral Portfolios of European Stock Options" on page 10-18
"Plotting Sensitivities of an Option" on page 10-30
"Plotting Sensitivities of a Portfolio of Options" on page 10-33

Introduced before R2006a

## blkprice

Black model for pricing futures options

## Syntax

[Call,Put] = blkprice(Price,Strike,Rate,Time,Volatility)

## Description

[Call,Put] = blkprice(Price,Strike,Rate,Time, Volatility) computes European put and call futures option prices using Black's model.

Note Any input argument can be a scalar, vector, or matrix. If a scalar, then that value is used to price all options. If more than one input is a vector or matrix, then the dimensions of those non-scalar inputs must be the same.

Ensure that Rate, Time, and Volatility are expressed in consistent units of time.

## Examples

## Compute European Put and Call Futures Option Prices Using Black's Model

This example shows how to price European futures options with exercise prices of \$20 that expire in four months. Assume that the current underlying futures price is also $\$ 20$ with a volatility of $25 \%$ per annum. The risk-free rate is $9 \%$ per annum.
[Call, Put] = blkprice(20, 20, 0.09, 4/12, 0.25)
$\mathrm{Call}=1.1166$
Put $=1.1166$

## Input Arguments

## Price - Current price of underlying asset

numeric
Current price of the underlying asset (that is, a futures contract), specified as a numeric value.

Data Types: double

## Strike - Exercise price of the futures option <br> numeric

Exercise price of the futures option, specified as a numeric value.
Data Types: double

## Rate - Annualized continuously compounded risk-free rate of return over life of the option <br> positive decimal

Annualized continuously compounded risk-free rate of return over the life of the option, specified as a positive decimal number.

## Data Types: double

## Time - Time to expiration of option numeric

Time to expiration of the option, specified as the number of years. Time must be greater than 0 .

Data Types: double

Volatility - Annualized asset price volatility<br>positive decimal

Annualized futures price volatility, specified as a positive decimal number.
Data Types: double

## Output Arguments

## Call - Price of a European call futures option matrix

Price of a European call futures option, returned as a matrix.

## Put - Price of a European put futures option matrix

Price of a European put futures option, returned as a matrix.

## References

[1] Hull, John C. Options, Futures, and Other Derivatives. 5th edition, Prentice Hall, , 2003, pp. 287-288.
[2] Black, Fischer. "The Pricing of Commodity Contracts." Journal of Financial Economics. March 3, 1976, pp. 167-79.

## See Also

binprice|blsprice

## Topics

"Pricing and Analyzing Equity Derivatives" on page 2-45
"Greek-Neutral Portfolios of European Stock Options" on page 10-18
"Plotting Sensitivities of an Option" on page 10-30
"Plotting Sensitivities of a Portfolio of Options" on page 10-33

Introduced before R2006a

## blsdelta

Black-Scholes sensitivity to underlying price change

## Syntax

[CallDelta,PutDelta] = blsdelta(Price,Strike,Rate,Time,Volatility) [CallDelta,PutDelta] = blsdelta( $\qquad$ ,Yield)

## Description

[CallDelta,PutDelta] = blsdelta(Price,Strike,Rate,Time,Volatility) returns delta, the sensitivity in option value to change in the underlying asset price. Delta is also known as the hedge ratio. blsdelta uses normcdf, the normal cumulative distribution function in the Statistics and Machine Learning Toolbox.

Note blsdelta can handle other types of underlies like Futures and Currencies. When pricing Futures (Black model), enter the input argument Yield as:

```
Yield = Rate
```

When pricing currencies (Garman-Kohlhagen model), enter the input argument Yield as:

```
Yield = ForeignRate
```

where ForeignRate is the continuously compounded, annualized risk-free interest rate in the foreign country.
[CallDelta,PutDelta] = blsdelta( $\qquad$ ,Yield) adds an optional argument for Yield.

## Examples

## Find the Sensitivity in Option Value to Change in the Underlying Asset Price

This example shows how to find the Black-Scholes delta sensitivity for an underlying asset price change.
[CallDelta, PutDelta] = blsdelta(50, 50, 0.1, 0.25, 0.3, 0)
CallDelta $=0.5955$
PutDelta $=-0.4045$

## Input Arguments

## Price - Current price of underlying asset

numeric
Current price of the underlying asset, specified as a numeric value.
Data Types: double

## Strike - Exercise price of option

numeric
Exercise price of the option, specified as a numeric value.
Data Types: double
Rate - Annualized, continuously compounded risk-free rate of return over life of option
positive decimal
Annualized, continuously compounded risk-free rate of return over the life of the option, specified as a positive decimal value.

## Data Types: double

## Time - Time (in years) to expiration of the option numeric

Time (in years) to expiration of the option, specified as a numeric value.
Data Types: double

## Volatility - Annualized asset price volatility

positive decimal
Annualized asset price volatility (annualized standard deviation of the continuously compounded asset return), specified as a positive decimal value.

## Data Types: double

## Yield - Annualized, continuously compounded yield of the underlying asset over life of option

## 0 (default) | decimal

(Optional) Annualized, continuously compounded yield of the underlying asset over the life of the option, specified as a decimal value. For example, for options written on stock indices, Yield could represent the dividend yield. For currency options, Yield could be the foreign risk-free interest rate.

Data Types: double

## Output Arguments

## CallDelta - Delta of call option

numeric
Delta of the call option, returned as a numeric value.

## PutDelta - Delta of put option

numeric

Delta of the put option, returned as a numeric.

## References

[1] Hull, John C. Options, Futures, and Other Derivatives. 5th edition, Prentice Hall, 2003.

## See Also

blsgamma|blslambda|blsprice|blsrho|blstheta|blsvega

## Topics

"Pricing and Analyzing Equity Derivatives" on page 2-45
"Greek-Neutral Portfolios of European Stock Options" on page 10-18
"Plotting Sensitivities of an Option" on page 10-30
"Plotting Sensitivities of a Portfolio of Options" on page 10-33

## Introduced before R2006a

## blsgamma

Black-Scholes sensitivity to underlying delta change

## Syntax

Gamma = blsgamma(Price,Strike, Rate, Time, Volatility)
Gamma = blsgamma( $\qquad$ ,Yield)

## Description

Gamma = blsgamma(Price,Strike,Rate,Time,Volatility) returns gamma, the sensitivity of delta to change in the underlying asset price. blsgamma uses normpdf, the probability density function in the Statistics and Machine Learning Toolbox.

Note blsgamma can handle other types of underlies like Futures and Currencies. When pricing Futures (Black model), enter the input argument Yield as:

Yield = Rate
When pricing currencies (Garman-Kohlhagen model), enter the input argument Yield as:

```
Yield = ForeignRate
```

where ForeignRate is the continuously compounded, annualized risk-free interest rate in the foreign country.

Gamma = blsgamma( __ , Yield) adds an optional argument for Yield.

## Examples

## Find Gamma for a Change in the Underlying Asset Price

This example shows how to find the gamma, the sensitivity of delta to a change in the underlying asset price.

Gamma = blsgamma(50, 50, 0.12, 0.25, 0.3, 0)
Gamma $=0.0512$

## Input Arguments

## Price - Current price of underlying asset

numeric
Current price of the underlying asset, specified as a numeric value.
Data Types: double

## Strike - Exercise price of option

numeric
Exercise price of the option, specified as a numeric value.
Data Types: double

## Rate - Annualized, continuously compounded risk-free rate of return over life of option <br> positive decimal

Annualized, continuously compounded risk-free rate of return over the life of the option, specified as a positive decimal value.
Data Types: double
Time - Time (in years) to expiration of the option
numeric
Time (in years) to expiration of the option, specified as a numeric value.
Data Types: double
Volatility - Annualized asset price volatility
positive decimal

Annualized asset price volatility (annualized standard deviation of the continuously compounded asset return), specified as a positive decimal value.

## Data Types: double

## Yield - Annualized, continuously compounded yield of the underlying asset over life of option

## 0 (default) | decimal

(Optional) Annualized, continuously compounded yield of the underlying asset over the life of the option, specified as a decimal value. For example, for options written on stock indices, Yield could represent the dividend yield. For currency options, Yield could be the foreign risk-free interest rate.

Data Types: double

## Output Arguments

## Gamma - Delta to change in underlying security price

numeric
Delta to change in underlying security price, returned as a numeric value.

## References

[1] Hull, John C. Options, Futures, and Other Derivatives. 5th edition, Prentice Hall, 2003.

See Also<br>blsdelta|blslambda|blsprice|blsrho|blstheta|blsvega<br>\section*{Topics}<br>"Pricing and Analyzing Equity Derivatives" on page 2-45<br>"Greek-Neutral Portfolios of European Stock Options" on page 10-18<br>"Plotting Sensitivities of an Option" on page 10-30<br>"Plotting Sensitivities of a Portfolio of Options" on page 10-33

Introduced before R2006a

## blsimpv

Black-Scholes implied volatility

## Syntax

```
Volatility = blsimpv(Price,Strike,Rate,Time,Value)
Volatility = blsimpv( __ ,Name,Value)
```


## Description

Volatility = blsimpv(Price,Strike,Rate,Time,Value) using a Black-Scholes model computes the implied volatility of an underlying asset from the market value of European options. If the Class name-value argument is empty or unspecified, the default is a call option

Note The input arguments Price, Strike, Rate, Time, Value, Yield, and Class can be scalars, vectors, or matrices. If scalars, then that value is used to compute the implied volatility from all options. If more than one of these inputs is a vector or matrix, then the dimensions of all non-scalar inputs must be the same.

Also, ensure that Rate, Time, and Yield are expressed in consistent units of time.

Volatility = blsimpv( $\qquad$ ,Name, Value) specifies options using one or more name-value pair arguments in addition to the input arguments in the previous syntax.

## Examples

## Compute the Implied Volatility of an Underlying Asset Using a Black-Scholes Model

This example shows how to compute the implied volatility for a European call option trading at $\$ 10$ with an exercise price of $\$ 95$ and three months until expiration. Assume
that the underlying stock pays no dividend and trades at $\$ 100$. The risk-free rate is $7.5 \%$ per annum. Furthermore, assume that you are interested in implied volatilities no greater than 0.5 ( $50 \%$ per annum). Under these conditions, the following statements all compute an implied volatility of 0.3130 , or $31.30 \%$ per annum.

```
Volatility = blsimpv(100, 95, 0.075, 0.25, 10, 'Limit', 0.5);
Volatility = blsimpv(100, 95, 0.075, 0.25, 10, 'Limit',0.5,'Yield',0,'Class', {'Call'}
Volatility = blsimpv(100, 95, 0.075, 0.25, 10, 'Limit',0.5,'Yield',0, 'Class', true);
Volatility = blsimpv(100, 95, 0.075, 0.25, 10, 'Limit',0.5,'Yield',0, 'Class', true,'Me
Volatility = 0.3130
```


## Input Arguments

## Price - Current price of underlying asset

scalar numeric
Current price of the underlying asset, specified as a scalar numeric.

## Data Types: double

## Strike - Exercise price of the option

scalar numeric
Exercise price of the option, specified as a scalar numeric.
Data Types: double

## Rate - Annualized continuously compounded risk-free rate of return over life of the option <br> scalar positive decimal

Annualized continuously compounded risk-free rate of return over the life of the option, specified as a scalar positive decimal.

Data Types: double

## Time - Time to expiration of option <br> scalar numeric

Time to expiration of the option, specified as the number of years using a scalar numeric.
Data Types: double

## Value - Price of a European option from which implied volatility of underlying asset is derived

scalar numeric
Price of a European option from which the implied volatility of the underlying asset is derived, specified as a scalar numeric.
Data Types: double

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Example: Volatility = blsimpv(Yield,CouponRate,Settle,Maturity,'Method','jackel2016')

## Limit - Upper bound of implied volatility search interval

10 (1000\% per annum) (default) | positive scalar numeric
Upper bound of the implied volatility search interval, specified as the comma-separated pair consisting of 'Limit' and a positive scalar numeric. If Limit is empty or unspecified, the default is 10 , or $1000 \%$ per annum.

Note If you are using Method with a value of 'jackel2016', the Limit argument is ignored.

## Data Types: double

## Yield - Annualized continuously compounded yield of underlying asset over life of the option

0 (default) | decimal
Annualized continuously compounded yield of the underlying asset over the life of the option,specified as the comma-separated pair consisting of 'Yield' and a decimal number. If Yield is empty or missing, the default value is 0.

For example, for options written on stock indices, Yield could represent the dividend yield. For currency options, Yield could be the foreign risk-free interest rate.

Note blsimpv can handle other types of underlies like Futures and Currencies. When pricing Futures (Black model), enter the input argument Yield as:

Yield = Rate
When pricing currencies (Garman-Kohlhagen model), enter the input argument Yield as:
Yield = ForeignRate
where ForeignRate is the continuously compounded, annualized risk-free interest rate in the foreign country.

## Data Types: double

## Tolerance - Implied volatility termination tolerance <br> 1e-6 (default) | positive scalar numeric

Implied volatility termination tolerance, specified as the comma-separated pair consisting of 'Tolerance' and a positive scalar numeric. If empty or missing, the default is 1e-6.

Note If you are using Method with a value of 'jackel2016', the Tolerance argument is ignored.

## Data Types: double

## Class - Option class from which implied volatility is derived

true (call option) (default) | logical | cell array of character vectors | string array
Option class indicating option type (call or put) from which implied volatility is derived, specified as the comma-separated pair consisting of 'Class' and a logical indicator, cell array of character vectors, or string array.

To specify call options, set Class $=$ true or Class $=\{$ 'call' $\}$. To specify put options, set Class $=$ false or Class $=\{$ 'put' $\}$ or Class $=$ ["put"]. If Class is empty or unspecified, the default is a call option.

Data Types: logical|cell|string

## Method - Method for computing implied volatility

[^4]Method for computing implied volatility, specified as the comma-separated pair consisting of 'Method ' and a character vector with a value of 'search' or 'jackel2016' or a string with a value of "search" or "jackel2016".

Data Types: char|string

## Output Arguments

## Volatility - Implied volatility of underlying asset derived from European option prices <br> decimal

Implied volatility of the underlying asset derived from European option prices, returned as a decimal. If no solution is found, blsimpv returns NaN.

## References

[1] Hull, John C. Options, Futures, and Other Derivatives. 5th edition, Prentice Hall, 2003.
[2] Jäckel, Peter. "Let's Be Rational." Wilmott Magazine., January, 2015 (https:// onlinelibrary.wiley.com/doi/pdf/10.1002/wilm.10395).
[3] Luenberger, David G. Investment Science. Oxford University Press, 1998.

## See Also

blsdelta|blsgamma|blslambda|blsprice|blsrho|blstheta|blsvega

## Topics

"Pricing and Analyzing Equity Derivatives" on page 2-45
"Greek-Neutral Portfolios of European Stock Options" on page 10-18
"Plotting Sensitivities of an Option" on page 10-30
"Plotting Sensitivities of a Portfolio of Options" on page 10-33
Introduced before R2006a

## blslambda

Black-Scholes elasticity

## Syntax

[CallEl,PutEl] = blslambda(Price,Strike,Rate,Time,Volatility)
[CallEl,PutEl] = blslambda( $\qquad$ , Yield)

## Description

[CallEl,PutEl] = blslambda(Price,Strike,Rate,Time,Volatility) returns the elasticity of an option. CallEl is the call option elasticity or leverage factor, and PutEl is the put option elasticity or leverage factor. Elasticity (the leverage of an option position) measures the percent change in an option price per 1 percent change in the underlying asset price. blslambda uses normcdf, the normal cumulative distribution function in the Statistics and Machine Learning Toolbox.

Note blslambda can handle other types of underlies like Futures and Currencies. When pricing Futures (Black model), enter the input argument Yield as:

Yield = Rate
When pricing currencies (Garman-Kohlhagen model), enter the input argument Yield as:
Yield = ForeignRate
where ForeignRate is the continuously compounded, annualized risk-free interest rate in the foreign country.
[Callel,PutEl] = blslambda( $\qquad$ , Yield) adds an optional argument for Yield.

## Examples

## Find the Black-Scholes Elasticity (Lambda) for an Option

This example shows how to find the Black-Scholes elasticity, or leverage, of an option position.
[Callel, PutEl] = blslambda(50, 50, 0.12, 0.25, 0.3)
Callel = 8.1274
PutEl $=-8.6466$

## Input Arguments

## Price - Current price of underlying asset

numeric
Current price of the underlying asset, specified as a numeric value.
Data Types: double

## Strike - Exercise price of option

numeric
Exercise price of the option, specified as a numeric value.
Data Types: double
Rate - Annualized, continuously compounded risk-free rate of return over life of option
positive decimal
Annualized, continuously compounded risk-free rate of return over the life of the option, specified as a positive decimal value.

## Data Types: double

## Time - Time (in years) to expiration of the option numeric

Time (in years) to expiration of the option, specified as a numeric value.
Data Types: double

## Volatility - Annualized asset price volatility

positive decimal
Annualized asset price volatility (annualized standard deviation of the continuously compounded asset return), specified as a positive decimal value.

Data Types: double

## Yield - Annualized, continuously compounded yield of the underlying asset over life of option

## 0 (default) | decimal

(Optional) Annualized, continuously compounded yield of the underlying asset over the life of the option, specified as a decimal value. For example, for options written on stock indices, Yield could represent the dividend yield. For currency options, Yield could be the foreign risk-free interest rate.

Data Types: double

## Output Arguments

## Callel - Call option elasticity

numeric
Call option elasticity or leverage factor, returned as a numeric value.

## PutEl - Put option elasticity

numeric
Put option elasticity or leverage factor, returned as a numeric value.

## References

[1] Daigler, R. Advanced Options Trading. McGraw-Hill, 1993.

See Also<br>blsdelta|blsgamma|blsprice|blsrho|blstheta|blsvega

## Topics

"Pricing and Analyzing Equity Derivatives" on page 2-45
"Greek-Neutral Portfolios of European Stock Options" on page 10-18
"Plotting Sensitivities of an Option" on page 10-30
"Plotting Sensitivities of a Portfolio of Options" on page 10-33

## Introduced before R2006a

## blsprice

Black-Scholes put and call option pricing

## Syntax

[Call,Put] = blsprice(Price,Strike,Rate,Time,Volatility) [Call,Put] = blsprice( $\qquad$ ,Yield)

## Description

[Call,Put] = blsprice(Price,Strike,Rate,Time,Volatility) computes European put and call option prices using a Black-Scholes model.

Note Any input argument can be a scalar, vector, or matrix. If a scalar, then that value is used to price all options. If more than one input is a vector or matrix, then the dimensions of those non-scalar inputs must be the same.

Ensure that Rate, Time, Volatility, and Yield are expressed in consistent units of time.
[Call,Put] = blsprice( $\qquad$ , Yield) adds an optional an argument for Yield.

## Examples

## Compute European Put and Call Option Prices Using a Black-Scholes Model

This example shows how to price European stock options that expire in three months with an exercise price of $\$ 95$. Assume that the underlying stock pays no dividend, trades at $\$ 100$, and has a volatility of $50 \%$ per annum. The risk-free rate is $10 \%$ per annum.
[Call, Put] = blsprice(100, 95, 0.1, 0.25, 0.5)
Call $=13.6953$

## Compute European Put and Call Option Prices on a Stock Index Using a BlackScholes Model

The S\&P 100 index is at 910 and has a volatility of $25 \%$ per annum. The risk-free rate of interest is $2 \%$ per annum and the index provides a dividend yield of $2.5 \%$ per annum. Calculate the value of a three-month European call and put with a strike price of 980.

```
[Call,Put] = blsprice(910,980,.02,.25,.25,.025)
Call = 19.6863
Put = 90.4683
```


## Price a European Call Option with the Garman-Kohlhagen Model

Price an FX option on buying GBP with USD.

```
S = 1.6; % spot exchange rate
X = 1.6; % strike
T = .3333;
r_d = .08; % USD interest rate
r_f = .11; % GBP interest rate
sigma = .2;
Price = blsprice(S,X,r_d,T,sigma,r_f)
Price = 0.0639
```


## Input Arguments

## Price - Current price of underlying asset <br> numeric

Current price of the underlying asset, specified as a numeric value.

## Data Types: double

## Strike - Exercise price of the option

numeric
Exercise price of the option, specified as a numeric value.
Data Types: double

## Rate - Annualized continuously compounded risk-free rate of return over life of the option <br> ```positive decimal```

Annualized continuously compounded risk-free rate of return over the life of the option, specified as a positive decimal number.

## Data Types: double

## Time - Time to expiration of option

numeric
Time to expiration of the option, specified as the number of years.

## Data Types: double

## Volatility - Annualized asset price volatility

 positive decimalAnnualized asset price volatility (that is, annualized standard deviation of the continuously compounded asset return), specified as a positive decimal number.
Data Types: double

## Yield - Annualized continuously compounded yield of underlying asset over life of the option <br> 0 (default) | decimal

(Optional) Annualized continuously compounded yield of the underlying asset over the life of the option, specified as a decimal number. If Yield is empty or missing, the default value is 0 .

For example, Yield could represent the dividend yield (annual dividend rate expressed as a percentage of the price of the security) or foreign risk-free interest rate for options written on stock indices and currencies.

Note blsprice can handle other types of underlies like Futures and Currencies. When pricing Futures (Black model), enter the input argument Yield as:

Yield = Rate
When pricing currencies (Garman-Kohlhagen model), enter the input argument Yield as:

## Yield = ForeignRate

where ForeignRate is the continuously compounded, annualized risk-free interest rate in the foreign country.

## Data Types: double

## Output Arguments

## Call - Price of a European call option

matrix
Price of a European call option, returned as a matrix.

## Put - Price of a European put option

matrix
Price of a European put option, returned as a matrix.

## References

[1] Hull, John C. Options, Futures, and Other Derivatives. 5th edition, Prentice Hall, 2003.
[2] Luenberger, David G. Investment Science. Oxford University Press, 1998.

## See Also

blkprice|blsdelta|blsgamma|blsimpv|blslambda|blsrho|blstheta| blsvega

## Topics

"Pricing and Analyzing Equity Derivatives" on page 2-45

[^5]
## blsrho

Black-Scholes sensitivity to interest-rate change

## Syntax

[CallRho, PutRho] = blsrho(Price,Strike,Rate,Time,Volatility)
[CallRho,PutRho] = blsrho( $\qquad$ , Yield)

## Description

[CallRho,PutRho] = blsrho(Price,Strike,Rate,Time,Volatility) returns the call option rho CallRho, and the put option rho PutRho. Rho is the rate of change in value of derivative securities with respect to interest rates. bls sho uses normcdf, the normal cumulative distribution function in the Statistics and Machine Learning Toolbox.

Note bls rho can also handle an underlying asset such as currencies. When pricing currencies (Garman-Kohlhagen model), enter the input argument Yield as:

Yield = ForeignRate
where ForeignRate is the continuously compounded, annualized risk-free interest rate in the foreign country.
[CallRho,PutRho] = blsrho(__ , Yield) adds an optional argument for Yield.

## Examples

Find the Black-Scholes Sensitivity (Rho) to Interest-Rate Change
This example shows how to find the Black-Scholes sensitivity, rho, to interest-rate change.
[CallRho, PutRho] = blsrho(50, 50, 0.12, 0.25, 0.3, 0)

CallRho $=6.6686$
PutRho $=-5.4619$

## Input Arguments

Price - Current price of underlying asset
numeric
Current price of the underlying asset, specified as a numeric value.
Data Types: double

## Strike - Exercise price of option <br> numeric

Exercise price of the option, specified as a numeric value.
Data Types: double

## Rate - Annualized, continuously compounded risk-free rate of return over life of option

positive decimal
Annualized, continuously compounded risk-free rate of return over the life of the option, specified as a positive decimal value.

## Data Types: double

## Time - Time (in years) to expiration of the option numeric

Time (in years) to expiration of the option, specified as a numeric value.
Data Types: double

## Volatility - Annualized asset price volatility

positive decimal
Annualized asset price volatility (annualized standard deviation of the continuously compounded asset return), specified as a positive decimal value.

Data Types: double

## Yield - Annualized, continuously compounded yield of the underlying asset over life of option

## 0 (default) | decimal

(Optional) Annualized, continuously compounded yield of the underlying asset over the life of the option, specified as a decimal value. For example, for options written on stock indices, Yield could represent the dividend yield. For currency options, Yield could be the foreign risk-free interest rate.

Data Types: double

## Output Arguments

## CallRho - Call option rho

numeric
Call option rho, returned as a numeric value.

## PutRho - Put option rho

numeric
Put option rho, returned as a numeric value.

## References

[1] Hull, John C. Options, Futures, and Other Derivatives. 5th edition, Prentice Hall, 2003.

## See Also

blsdelta|blsgamma|blslambda|blsprice|blstheta|blsvega

## Topics

"Pricing and Analyzing Equity Derivatives" on page 2-45
"Greek-Neutral Portfolios of European Stock Options" on page 10-18
"Plotting Sensitivities of an Option" on page 10-30
"Plotting Sensitivities of a Portfolio of Options" on page 10-33

Introduced before R2006a

19-312

## blstheta

Black-Scholes sensitivity to time-until-maturity change

## Syntax

[CallTheta,PutTheta] = blstheta(Price,Strike,Rate,Time,Volatility) [CallTheta,PutTheta] = blstheta( $\qquad$ ,Yield)

## Description

[CallTheta, PutTheta] = blstheta(Price,Strike,Rate, Time, Volatility) returns the call option theta CallTheta, and the put option theta PutTheta.

Theta is the sensitivity in option value with respect to time and is measured in years. CallTheta or PutTheta can be divided by 365 to get Theta per calendar day or by 252 to get Theta by trading day.
blstheta uses normcdf, the normal cumulative distribution function, and normpdf, the normal probability density function, in the Statistics and Machine Learning Toolbox.

Note blstheta can handle other types of underlies like Futures and Currencies. When pricing Futures (Black model), enter the input argument Yield as:

Yield = Rate
When pricing currencies (Garman-Kohlhagen model), enter the input argument Yield as:
Yield = ForeignRate
where ForeignRate is the continuously compounded, annualized risk-free interest rate in the foreign country.
[CallTheta,PutTheta] = blstheta( $\qquad$ ,Yield) adds an optional argument for Yield.

## Examples

## Compute the Black-Scholes Sensitivity to Time-Until-Maturity Change (Theta)

This example shows how to compute theta, the sensitivity in option value with respect to time.
[CallTheta, PutTheta] = blstheta(50, 50, 0.12, 0.25, 0.3, 0)
CallTheta $=-8.9630$
PutTheta $=-3.1404$

## Input Arguments

## Price - Current price of underlying asset

numeric
Current price of the underlying asset, specified as a numeric value.
Data Types: double

## Strike - Exercise price of option <br> numeric

Exercise price of the option, specified as a numeric value.

## Data Types: double

Rate - Annualized, continuously compounded risk-free rate of return over life of option

## positive decimal

Annualized, continuously compounded risk-free rate of return over the life of the option, specified as a positive decimal value.

## Data Types: double

## Time - Time (in years) to expiration of the option numeric

Time (in years) to expiration of the option, specified as a numeric value.

## Data Types: double

## Volatility - Annualized asset price volatility

positive decimal
Annualized asset price volatility (annualized standard deviation of the continuously compounded asset return), specified as a positive decimal value.

## Data Types: double

## Yield - Annualized, continuously compounded yield of the underlying asset over life of option

## 0 (default) | decimal

(Optional) Annualized, continuously compounded yield of the underlying asset over the life of the option, specified as a decimal value. For example, for options written on stock indices, Yield could represent the dividend yield. For currency options, Yield could be the foreign risk-free interest rate.
Data Types: double

## Output Arguments

## CallTheta - Call option theta

numeric
Call option theta, returned as a numeric value.

## PutTheta - Put option theta

numeric
Put option theta, returned as a numeric value.

## References

[1] Hull, John C. Options, Futures, and Other Derivatives. 5th edition, Prentice Hall, 2003.

See Also<br>blsdelta|blsgamma|blslambda|blsprice|blsrho|blsvega<br>\section*{Topics}<br>"Pricing and Analyzing Equity Derivatives" on page 2-45<br>"Greek-Neutral Portfolios of European Stock Options" on page 10-18<br>"Plotting Sensitivities of an Option" on page 10-30<br>"Plotting Sensitivities of a Portfolio of Options" on page 10-33<br>\section*{Introduced before R2006a}

## blsvega

Black-Scholes sensitivity to underlying price volatility

## Syntax

Vega = blsvega(Price,Strike,Rate,Time,Volatility)
Vega = blsvega( $\qquad$ ,Yield)

## Description

Vega = blsvega(Price, Strike,Rate, Time, Volatility) rate of change of the option value with respect to the volatility of the underlying asset. blsvega uses normpdf, the normal probability density function in the Statistics and Machine Learning Toolbox.

Note blsvega can handle other types of underlies like Futures and Currencies. When pricing Futures (Black model), enter the input argument Yield as:

Yield $=$ Rate
When pricing currencies (Garman-Kohlhagen model), enter the input argument Yield as:
Yield = ForeignRate
where ForeignRate is the continuously compounded, annualized risk-free interest rate in the foreign country.

Vega = blsvega( __ , Yield) adds an optional argument for Yield.

## Examples

## Compute Black-Scholes Sensitivity to Underlying Price Volatility (Vega)

This example shows how to compute vega, the rate of change of the option value with respect to the volatility of the underlying asset.

```
Vega = blsvega(50, 50, 0.12, 0.25, 0.3, 0)
Vega = 9.6035
```


## Input Arguments

## Price - Current price of underlying asset <br> numeric

Current price of the underlying asset, specified as a numeric value.

## Data Types: double

## Strike - Exercise price of option

numeric
Exercise price of the option, specified as a numeric value.
Data Types: double

## Rate - Annualized, continuously compounded risk-free rate of return over life of option <br> positive decimal

Annualized, continuously compounded risk-free rate of return over the life of the option, specified as a positive decimal value.
Data Types: double
Time - Time (in years) to expiration of the option
numeric
Time (in years) to expiration of the option, specified as a numeric value.
Data Types: double
Volatility - Annualized asset price volatility
positive decimal

Annualized asset price volatility (annualized standard deviation of the continuously compounded asset return), specified as a positive decimal value.

## Data Types: double

## Yield - Annualized, continuously compounded yield of the underlying asset over life of option

```
0 (default) | decimal
```

(Optional) Annualized, continuously compounded yield of the underlying asset over the life of the option, specified as a decimal value. For example, for options written on stock indices, Yield could represent the dividend yield. For currency options, Yield could be the foreign risk-free interest rate.
Data Types: double

## Output Arguments

## Vega - Rate of change of option value with respect to volatility of underlying asset

numeric
Rate of change of the option value with respect to the volatility of the underlying asset, returned as a numeric value.

## References

[1] Hull, John C. Options, Futures, and Other Derivatives. 5th edition, Prentice Hall, 2003.

See Also<br>blsdelta|blsgamma|blslambda|blsprice|blsrho|blstheta<br>\section*{Topics}<br>"Pricing and Analyzing Equity Derivatives" on page 2-45<br>"Greek-Neutral Portfolios of European Stock Options" on page 10-18<br>"Plotting Sensitivities of an Option" on page 10-30<br>"Plotting Sensitivities of a Portfolio of Options" on page 10-33

Introduced before R2006a

## bondDefaultBootstrap

Bootstrap default probability curve from bond prices

## Syntax

```
[ProbabilityData,HazardData] = bondDefaultBootstrap(ZeroData,
MarketData,Settle)
[ProbabilityData,HazardData] = bondDefaultBootstrap( ___ ,Name,Value)
```


## Description

[ProbabilityData,HazardData] = bondDefaultBootstrap(ZeroData, MarketData, Settle) bootstraps the default probability curve from bond prices.

Using bondDefaultBootstrap, you can:

- Extract discrete default probabilities for a certain period from market bond data.
- Interpolate these default probabilities to get the default probability curve for pricing and risk management purposes.
[ProbabilityData,HazardData] = bondDefaultBootstrap( $\qquad$ ,Name, Value) adds optional name-value pair arguments.


## Examples

## Determine the Default Probability and Hazard Rate Values for Treasury Bonds

Use the following bond data.

```
Settle = datenum('08-Jul-2016');
MarketDate = datenum({'06/15/2018', '01/08/2019', '02/01/2021', '03/18/2021', '08/04/2
CouponRate = [2.240 2.943 5.750 3.336 4.134]'/100;
MarketPrice = [101.300 103.020 115.423 104.683 108.642]';
MarketData = [MarketDate,MarketPrice,CouponRate];
```

Calculate the ProbabilityData and HazardData.

```
TreasuryParYield = [0.26 0.28 0.36 0.48 0.61 0.71 0.95 1.19 1.37 1.69 2.11]'/100;
TreasuryDates = datemnth(Settle, [[1 3 6], 12 * [1 2 3 5 7 10 20 30]]');
[ZeroRates, CurveDates] = pyld2zero(TreasuryParYield, TreasuryDates, Settle);
ZeroData = [CurveDates, ZeroRates];
format longg
[ProbabilityData,HazardData]=bondDefaultBootstrap(ZeroData,MarketData,Settle)
ProbabilityData = 5×2
```

$737226 \quad 0.0299675399937613$
$737433 \quad 0.0418832295824674$
$738188 \quad 0.0905183328842618$
$738233 \quad 0.101248065083714$
$739833 \quad 0.233002708031915$
HazardData $=5 \times 2$
$737226 \quad 0.0157077745460246$
$737433 \quad 0.0217939816590399$
$738188 \quad 0.0251849128247209$
$738233 \quad 0.096260871864083$
$739833 \quad 0.0361632398787917$
format

## Reprice a Bond Listed on the Default Probability Curve

Reprice one of the bonds from bonds list based on the default probability curve. The expected result of this repricing is a perfect match with the market quote.

Use the following Treasury data from US Department of the Treasury.

```
Settle = datetime('08-Jul-2016','Locale','en US');
TreasuryParYield = [0.26 0.28 0.36 0.48 0.61-0.71 0.95 1.19 1.37 1.69 2.11]'/100;
TreasuryDates = datemnth(Settle, [[1 3 6], 12 * [1 2 3 5 7 10 20 30]]');
```

Preview the bond date using semiannual coupon bonds with market quotes, coupon rates, and a settle date of July-08-2016.

```
MarketDate = datenum({'06/01/2017','06/01/2019','06/01/2020','06/01/2022'}','mm/dd/yyy
CouponRate = [7 8 9 10]'/100;
MarketPrice = [101.300 109.020 114.42 118.62]';
MarketData = [MarketDate, MarketPrice, CouponRate];
BondList = array2table(MarketData, 'VariableNames', {'Maturity', 'Price','Coupon'});
BondList.Maturity = datetime(BondList.Maturity,'Locale','en_US','ConvertFrom','datenum
BondList.Maturity.Format = 'MMM-dd-yyyy'
BondList=4\times3 table
\begin{tabular}{|c|c|c|}
\hline Maturity & Price & Coupon \\
\hline Jun-01-2017 & 101.3 & 0.07 \\
\hline Jun-01-2019 & 109.02 & 0.08 \\
\hline Jun-01-2020 & 114.42 & 0.09 \\
\hline Jun-01-2022 & 118.62 & 0.1 \\
\hline
\end{tabular}
```

Choose the second coupon bond as the one to be priced.

```
number = 2;
TestCase = BondList(number, :);
```

Preview the risk-free rate data provided here that is based on a continuous compound rate.

```
[ZeroRates, CurveDates] = pyld2zero(TreasuryParYield, TreasuryDates, Settle);
ZeroData = [datenum(CurveDates), ZeroRates];
RiskFreeRate = array2table(ZeroData, 'VariableNames', {'Date', 'Rate'});
RiskFreeRate.Date = datetime(RiskFreeRate.Date,'Locale','en_US','ConvertFrom','datenum
RiskFreeRate.Date.Format = 'MMM-dd-yyyy'
RiskFreeRate=11\times2 table
    Date Rate
Aug-08-2016 0.0026057
Oct-08-2016 0.0027914
Jan-08-2017 0.0035706
Jul-08-2017 0.0048014
Jul-08-2018 0.0061053
Jul-08-2019 0.0071115
Jul-08-2021 0.0095416
Jul-08-2023 0.012014
```

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```
Jul-08-2026 0.013883
Jul-08-2036 0.017359
Jul-08-2046 0.022704
```

Bootstrap the probability of default (PD) curve from the bonds.

```
format longg
[defaultProb1, hazard1] = bondDefaultBootstrap(ZeroData, MarketData, Settle)
defaultProb1 = 4×2
```

| 736847 | 0.0704863142317494 |
| ---: | ---: |
| 737577 | 0.162569420050034 |
| 737943 | 0.217308133826187 |
| 738673 | 0.38956773145021 |

```
hazard1 = 4×2
```

| 736847 | 0.0813390794774647 |
| ---: | ---: |
| 737577 | 0.0521615800986284 |
| 737943 | 0.0674145844133175 |
| 738673 | 0.12428587278862 |

## format

Reformat the default probability and hazard rate for a better representation.
DefProbHazard = [defaultProb1, hazard1(:,2)];
DefProbHazardTable = array2table(DefProbHazard, 'VariableNames', \{'Date', 'DefaultProb DefProbHazardTable.Date = datetime(DefProbHazardTable.Date,'Locale','en_US','ConvertFr DefProbHazardTable.Date.Format = 'MMM-dd-yyyy'

| DefProbHazardTable=Date <br> Dat <br>  <br> DefaultProbability |  |  |  |
| :---: | :---: | :---: | :---: |
| Haz-01-2017 |  |  |  |
| Jun-01-2019 | 0.070486 |  | 0.081339 |
| Jun-01-2020 | 0.16257 |  | 0.052162 |
| Jun-01-2022 | 0.21731 |  | 0.067415 |
|  | 0.38957 |  | 0.12429 |

Preview the selected bond to reprice based on the PD curve.

```
TestCase
TestCase=1\times3 table
    Maturity Price Coupon
    Jun-01-2019 109.02 0.08
```

To reprice the bond, first generate cash flows and payment dates.

```
[Payments, PaymentDates] = cfamounts(TestCase.Coupon, Settle, TestCase.Maturity);
AccInt=-Payments(1);
    % Truncate the payments as well as payment dates for calculation
    % PaymentDates(1) is the settle date, no need for following calculations
PaymentDates = PaymentDates(2:end)
PaymentDates = 1x6 datetime array
Columns 1 through 5
```

    01-Dec-2016 01-Jun-2017 01-Dec-2017 01-Jun-2018 01-Dec-2018
    Column 6

01-Jun-2019

```
Payments = Payments(2:end)
```

Payments = $1 \times 6$

| 4 | 4 | 4 | 4 | 4 | 104 |
| :--- | :--- | :--- | :--- | :--- | :--- |

Calculate the discount factors on the payment dates.
DF = zero2disc(interp1(RiskFreeRate.Date, RiskFreeRate.Rate, PaymentDates, 'linear',
DF $=1 \times 6$
0.9987
0.9959
0.9926
0.9887
0.9845
0.9799

Assume that the recovery amount is a fixed proportion of bond's face value. The bond's face value is 100, and the recovery ratio is set to $40 \%$ as assumed in bondDefaultBootstrap.

```
Num = length(Payments);
RecoveryAmount = repmat(100*0.4, 1, Num)
RecoveryAmount = 1×6
    40}4040 40 40 40 40 
```

Calculate the probability of default based on the default curve.
DefaultProb1 = bondDefaultBootstrap(ZeroData, MarketData, Settle, 'ZeroCompounding',
SurvivalProb = 1 - DefaultProb1(:,2)
SurvivalProb $=6 \times 1$
0.9680
0.9295
0.9055
0.8823
0.8595
0.8375

Calculate the model-based clean bond price.

```
DirtyPrice = DF * (SurvivalProb.*Payments') + (RecoveryAmount.*DF) * (-diff([1;Surviva
```

ModelPrice = DirtyPrice - AccInt
ModelPrice = 109.0200

Compare the repriced bond to the market quote.
ResultTable = TestCase;
ResultTable.ModelPrice = ModelPrice;
ResultTable.Difference = ModelPrice - TestCase.Price
ResultTable=1×5 table

|  |  |  | ModelPrice |  |
| :---: | :---: | :---: | :---: | :---: |
| Maturity | Price | Coupon | ModelPrice | Difference |
| Jun-01-2019 | 109.02 | 0. 08 | 109.02 | 0 |

## Input Arguments

ZeroData - Zero rate data<br>matrix | IRDataCurve object

Zero rate data, specified as an M-by-2 matrix of dates and zero rates or an IRDataCurve object of zero rates. For array input, the dates must be entered as serial date numbers, and discount rate must be in decimal form.

When ZeroData is an IRDataCurve object, ZeroCompounding and ZeroBasis are implicit in ZeroData and are redundant inside this function. In this case, specify these optional parameters when constructing the IRDataCurve object before using this bondDefaultBootstrap function.

For more information on an IRDataCurve object, see "Creating an IRDataCurve Object" (Financial Instruments Toolbox).

Data Types: double

## MarketData - Bond market data

matrix

Bond market data, specified as an N-by-3 matrix of maturity dates, market prices, and coupon rates for bonds. The dates must be entered as serial date numbers, market prices must be numeric values, and coupon rate must be in decimal form.

Note A warning is displayed when MarketData is not sorted in ascending order by time.

## Data Types: double

## Settle - Settlement date <br> serial date number | date character vector | datetime object | date string object

Settlement date, specified as a serial date number, a date character vector, a datetime object, or a date string object. Settle must be earlier than or equal to the maturity dates in MarketData.

Data Types: double|char|datetime | string

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Note Any optional input of size N -by-1 is also acceptable as an array of size 1-by-N, or as a single value applicable to all contracts.

## Example: [ProbabilityData,HazardData] =

bondDefaultBootstrap(ZeroData,MarketData, Settle, 'RecoveryRate' , Recov ery,'ZeroCompounding', - 1)

## RecoveryRate - Recovery rate

0.4 (default) | decimal

Recovery rate, specified as the comma-separated pair consisting of 'RecoveryRate' and a N -by- 1 vector of recovery rates, expressed as a decimal from 0 through 1.

## Data Types: double

## ProbabilityDates - Dates for output of default probability data

column of dates in MarketData (default) | serial date number | date character vector | datetime object | date string object

Dates for the output of default probability data, specified as the comma-separated pair consisting of 'ProbabilityDates ' and a P-by-1 vector, given as serial date numbers, datetime objects, date character vectors, or date string objects.
Data Types: double | char \| datetime | string

## ZeroCompounding - Compounding frequency of the zero curve

2 (semiannual) (default) | integer with value of $1,2,3,4,6,12$, or -1
Compounding frequency of the zero curve, specified as the comma-separated pair consisting of 'ZeroCompounding ' and a N-by-1 vector. Values are:

- 1 - Annual compounding
- 2-Semiannual compounding
- 3-Compounding three times per year
- 4 - Quarterly compounding
- 6 - Bimonthly compounding
- 12 - Monthly compounding
- -1 - Continuous compounding


## Data Types: double

## ZeroBasis - Basis of the zero curve

0 (actual/actual) (default) | integer with value of 0 to 13
Basis of the zero curve, specified as the comma-separated pair consisting of 'ZeroBasis' and the same values listed for Basis.

Data Types: double

## RecoveryMethod - Recovery method

'facevalue' (default)| character vector with value of 'presentvalue' or
'facevalue' | string object with value of 'presentvalue' or 'facevalue'
Recovery method, specified as the comma-separated pair consisting of
'RecoveryMethod' and a character vector or a string with a value of 'presentvalue' or 'facevalue'.

- 'presentvalue' assumes that upon default, a bond is valued at a given fraction to the hypothetical present value of its remaining cash flows, discounted at risk-free rate.
- 'facevalue ' assumes that a bond recovers a given fraction of its face value upon recovery.


## Data Types: char|string

## Face - Face or par value

100 (default) | numeric
Face or par value, specified as the comma-separated pair consisting of 'Face' and a NINST-by-1 vector of bonds.

Data Types: double

## Period - Payment frequency

2 (default) | numeric with values $0,1,2,3,4,6$ or 12

Payment frequency, specified as the comma-separated pair consisting of 'Period ' and a N -by-1 vector with values of $0,1,2,3,4,6$, or 12.

## Data Types: double

## Basis - Day-count basis of the instrument

0 (actual/actual) (default) | integers of the set [0...13] | vector of integers of the set [0...13]

Day-count basis of the instrument, specified as the comma-separated pair consisting of 'Basis' and a positive integer using a NINST-by-1 vector. Values are:

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- 2 = actual/360
- $3=$ actual $/ 365$
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- $9=$ actual/360 (ICMA)
- $10=$ actual/365 (ICMA)
- $11=30 / 360 E$ (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-0
Data Types: double

## EndMonthRule - End-of-month rule flag

1 (in effect) (default) | nonnegative integer 0 or 1
End-of-month rule flag, specified as the comma-separated pair consisting of 'EndMonthRule' and a nonnegative integer, 0 or 1, using a NINST-by-1 vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days.

- $0=$ Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

Data Types: double

## IssueDate - Bond issue date

if IssueDate not specified, cash flow payment dates determined from other inputs (default) | serial date number | date character vector | datetime object | date string object

Bond issue date, specified as the comma-separated pair consisting of 'IssueDate' and a N -by- 1 vector, given as serial date numbers, datetime objects, date character vectors, or date string objects.
Data Types: double | char|datetime | string

## FirstCouponDate - First actual coupon date

if you do not specify a FirstCouponDate, cash flow payment dates are determined from other inputs (default) | serial date number

First actual coupon date, specified as the comma-separated pair consisting of 'FirstCouponDate' and a serial date number. FirstCouponDate is used when a bond has an irregular first coupon period. When FirstCouponDate and LastCouponDate are both specified, FirstCouponDate takes precedence in determining the coupon payment structure.

Data Types: double

## LastCouponDate - Last actual coupon date

if you do not specify a LastCouponDate, cash flow payment dates are determined from other inputs (default) | scalar for serial date number

Last actual coupon date, specified as the comma-separated pair consisting of 'LastCouponDate' and a serial date number. LastCouponDate is used when a bond has an irregular last coupon period. In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated at the LastCouponDate, regardless of where it falls, and is followed only by the bond's maturity cash flow date.

Data Types: double

## StartDate - Forward starting date of payments <br> if you do not specify StartDate, effective start date is Settle date (default) | serial date number

Forward starting date of payments, specified as the comma-separated pair consisting of 'StartDate' and a serial date number. StartDate is when a bond actually starts (the date from which a bond cash flow is considered). To make an instrument forward-starting, specify this date as a future date.

## Data Types: double

## BusinessDayConvention - Business day conventions

'actual' (default)| character vector or string object with values'actual', 'follow', 'modifiedfollow', 'previous'or 'modifiedprevious'

Business day conventions, specified as the comma-separated pair consisting of 'BusinessDayConvention' and a character vector or a string object. The selection for business day convention determines how nonbusiness days are treated. Nonbusiness days are defined as weekends plus any other date that businesses are not open (for example, statutory holidays). Values are:

- 'actual' - Nonbusiness days are effectively ignored. Cash flows that fall on nonbusiness days are assumed to be distributed on the actual date.
- 'follow' - Cash flows that fall on a nonbusiness day are assumed to be distributed on the following business day.
- 'modifiedfollow' - Cash flows that fall on a non-business day are assumed to be distributed on the following business day. However if the following business day is in a different month, the previous business day is adopted instead.
- 'previous ' - Cash flows that fall on a nonbusiness day are assumed to be distributed on the previous business day.
- 'modifiedprevious' - Cash flows that fall on a nonbusiness day are assumed to be distributed on the previous business day. However if the previous business day is in a different month, the following business day is adopted instead.

Data Types: char | cell|string

## Output Arguments

## ProbabilityData - Default probability values

matrix

Default probability values, returned as a P-by-2 matrix with dates and corresponding cumulative default probability values. The dates match those in MarketData, unless the optional input parameter ProbabilityDates is provided.

## HazardData - Hazard rate values

## matrix

Hazard rate values, returned as an N -by-2 matrix with dates and corresponding hazard rate values for the survival probability model. The dates match those in MarketData.

Note A warning is displayed when nonmonotone default probabilities (that is, negative hazard rates) are found.

## References

[1] Jarrow, Robert A., and Stuart Turnbull. "Pricing Derivatives on Financial Securities Subject to Credit Risk." Journal of Finance. 50.1, 1995, pp. 53-85.
[2] Berd, A., Mashal, R. and Peili Wang. "Defining, Estimating and Using Credit Term Structures." Research report, Lehman Brothers, 2004.

## See Also

IRDataCurve|cdsbootstrap

## Topics

"Creating an IRDataCurve Object" (Financial Instruments Toolbox)

## Introduced in R2017a

## bndconvp

Bond convexity given price

Note In R2017b, the specification of optional input arguments has changed. While the previous ordered inputs syntax is still supported, it may no longer be supported in a future release. Use the optional name-value pair inputs: Period, Basis, EndMonthRule, IssueDate,FirstCouponDate, LastCouponDate, StartDate,Face, CompoundingFrequency, DiscountBasis, and LastCouponInterest.

## Syntax

[YearConvexity,PerConvexity] = bndconvp(Price,CouponRate,Settle, Maturity)
[YearConvexity,PerConvexity] = bndconvp( $\qquad$ ,Name, Value)

## Description

[YearConvexity,PerConvexity] = bndconvp(Price,CouponRate, Settle, Maturity) computes the convexity of NUMBONDS fixed income securities given a clean price for each bond.
bndconvp determines the convexity for a bond whether the first or last coupon periods in the coupon structure are short or long (that is, whether the coupon structure is synchronized to maturity). bndconvp also determines the convexity of a zero coupon bond.
[YearConvexity,PerConvexity] = bndconvp( $\qquad$ ,Name, Value) adds optional name-value pair arguments.

## Examples

## Find Bond Convexity Given Price

This example shows how to compute the convexity of three bonds given their prices.

```
Price = [106; 100; 98];
CouponRate = 0.055;
Settle = '02-Aug-1999';
Maturity = '15-Jun-2004';
Period = 2;
Basis = 0;
[YearConvexity, PerConvexity] = bndconvp(Price,...
CouponRate,Settle, Maturity, Period, Basis)
```

YearConvexity $=3 \times 1$
21.4447
21.0363
20.8951
PerConvexity $=3 \times 1$
85.7788
84.1454
83.5803

## Find Bond Convexity Given Price Using datetime Inputs

This example shows how to compute the convexity of three bonds given their prices using datetime inputs.

Price = [106; 100; 98];
CouponRate = 0.055;
Period = 2;
Basis = 0;
Settle = datetime('02-Aug-1999', 'Locale','en_US');
Maturity = datetime('15-Jun-2004','Locale','ēn_US');
[YearConvexity, PerConvexity] = bndconvp(Price,....
CouponRate, Settle, Maturity, Period, Basis)

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```
YearConvexity = 3×1
```

21.4447
21.0363
20.8951

```
PerConvexity = 3×1
```

85.7788
84.1454
83.5803

## Input Arguments

## Price - Clean price (excludes accrued interest)

numeric
Clean price (excludes accrued interest), specified as numeric value using a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector.

## Data Types: double

## CouponRate - Annual percentage rate used to determine coupons payable on a bond

decimal
Annual percentage rate used to determine the coupons payable on a bond, specified as decimal value using a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector.
Data Types: double

## Settle - Settlement date for certificate of deposit

serial date number | date character vector | datetime
Settlement date for the certificate of deposit, specified as a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector using serial date numbers, date character vectors, or datetime arrays. The Settle date must be before the Maturity date.

Data Types: double | char |datetime

## Maturity - Maturity date for certificate of deposit <br> serial date number | date character vector | datetime

Maturity date for the certificate of deposit, specified as a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector using serial date numbers, date character vectors, or datetime arrays.

Data Types: double|char|datetime

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Example: [YearConvexity,PerConvexity] = bndconvp(Price,CouponRate,Settle, Maturity,'Period',4,'Basis',7)

## Period - Number of coupon payments per year

2 (default) | numeric with values $0,1,2,3,4$, 6 or 12
Number of coupon payments per year, specified as the comma-separated pair consisting of 'Period ' and a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector using the values: $0,1,2,3,4,6$, or 12 .

## Data Types: double

## Basis - Day-count basis of instrument

0 (default) | numeric values: $0,1,2,3,4,6,7,8,9,10,11,12,13$
Day-count of the instrument, specified as the comma-separated pair consisting of 'Basis' and a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector using a supported value:

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- 2 = actual/360
- 3 = actual/365
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- $9=\mathrm{actual} / 360$ (ICMA)
- $10=$ actual/365 (ICMA)
- $11=30 / 360 \mathrm{E}$ (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-0

## Data Types: double

## EndMonthRule - End-of-month rule flag

1 (in effect) (default) | nonnegative integer 0 or 1
End-of-month rule flag, specified as the comma-separated pair consisting of
'EndMonthRule' and a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days.

- $0=$ Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- $1=$ Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.


## Data Types: logical

## IssueDate - Bond issue date

serial date number | date character vector | datetime
Bond Issue date, specified as the comma-separated pair consisting of 'IssueDate' and a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector using serial date numbers, date character vectors, or datetime arrays.

If you do not specify an IssueDate, the cash flow payment dates are determined from other inputs.
Data Types: double | char | datetime

## FirstCouponDate - Irregular or normal first coupon date serial date number | date character vector | datetime

Irregular or normal first coupon date, specified as the comma-separated pair consisting of 'FirstCouponDate' and a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector using serial date numbers, date character vectors, or datetime arrays.

If you do not specify a FirstCouponDate, the cash flow payment dates are determined from other inputs.

## Data Types: double | char|datetime

## LastCouponDate - Irregular or normal last coupon date serial date number | date character vector | datetime

Irregular or normal last coupon date, specified as the comma-separated pair consisting of 'LastCouponDate' and a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector using serial date numbers, date character vectors, or datetime arrays.

If you do not specify a LastCouponDate, the cash flow payment dates are determined from other inputs.

Data Types: double|char|datetime

## StartDate - Forward starting date of payments serial date number | date character vector | datetime

Forward starting date of payments, specified as the comma-separated pair consisting of 'StartDate' and a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector using serial date numbers, date character vectors, or datetime arrays. The StartDate is when a bond actually starts (the date from which a bond cash flow is considered). To make an instrument forward-starting, specify this date as a future date.

If you do not specify a StartDate, the effective start date is the Settle date.
Data Types: double | char|datetime

## Face - Face value of bond

100 (default) | numeric
Face value of the bond, specified as the comma-separated pair consisting of 'Face' and a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector.

Data Types: double

## CompoundingFrequency - Compounding frequency for yield calculation

SIA bases uses2, ICMA bases uses 1 (default) | integer with value of 1, 2, 3, 4, 6, or 12
Compounding frequency for yield calculation, specified as the comma-separated pair consisting of 'CompundingFrequency' and a scalar or a NUMBONDS-by-1 or 1-byNUMBONDS vector.

- 1 - Annual compounding
- 2-Semiannual compounding
- 3-Compounding three times per year
- 4-Quarterly compounding
- 6-Bimonthly compounding
- 12 - Monthly compounding

Note By default, SIA bases (0-7) and BUS/252 use a semiannual compounding convention and ICMA bases (8-12) use an annual compounding convention.

## Data Types: double

## DiscountBasis - Basis used to compute the discount factors for computing the yield

SIA uses 0 (default) | integers of the set [0...13] | vector of integers of the set [0...13]

Basis used to compute the discount factors for computing the yield, specified as the comma-separated pair consisting of 'DiscountBasis' and a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector. Values are:

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- $2=$ actual $/ 360$
- $3=$ actual $/ 365$
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- $12=$ actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-0

Note If a SIA day-count basis is defined in the Basis input argument and there is no value assigned for DiscountBasis, the default behavior is for SIA bases to use the actual/actual day count to compute discount factors.

If an ICMA day-count basis or BUS/252 is defined in the Basis input argument and there is no value assigned for DiscountBasis, the specified bases from the Basis input argument are used.

## Data Types: double

## LastCouponInterest - Compounding convention for computing yield of a bond in last coupon period

## compound (default) | values are simple or compound

Compounding convention for computing the yield of a bond in the last coupon period, specified as the comma-separated pair consisting of 'LastCouponInterest' and a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector. LastCouponInterest is based on only the last coupon and the face value to be repaid. Acceptable values are:

- simple
- compound

Data Types: char | cell

## Output Arguments

## YearConvexity - Yearly (annualized) convexity <br> numeric

Yearly (annualized) convexity, returned as a NUMBONDS-by-1 vector.

## PerConvexity - Periodic convexity reported on semiannual bond basis numeric

Periodic convexity reported on a semiannual bond basis (in accordance with SIA convention), returned as a NUMBONDS-by-1 vector.

## References

[1] Krgin, D. Handbook of Global Fixed Income Calculations. Wiley, 2002.
[2] Mayle, J. "Standard Securities Calculations Methods: Fixed Income Securities Formulas for Analytic Measures." SIA, Vol 2, Jan 1994.
[3] Stigum, M., Robinson, F. Money Market and Bond Calculation. McGraw-Hill, 1996.

See Also<br>bndconvy | bnddurp | bnddury | cfconv | cfdur | datetime

## Topics

"Bond Portfolio for Hedging Duration and Convexity" on page 10-6
"Yield Conventions" on page 2-31
Introduced before R2006a

## bndconvy

Bond convexity given yield


#### Abstract

Note In R2017b, the specification of optional input arguments has changed. While the previous ordered inputs syntax is still supported, it may no longer be supported in a future release. Use the optional name-value pair inputs: Period, Basis, EndMonthRule, IssueDate,FirstCouponDate, LastCouponDate, StartDate,Face, CompoundingFrequency, DiscountBasis, and LastCouponInterest.


## Syntax

```
[YearConvexity,PerConvexity] = bndconvy(Yield,CouponRate,Settle,
Maturity)
[YearConvexity,PerConvexity] = bndconvy(
```

$\qquad$

``` ,Name,Value)
```


## Description

[YearConvexity,PerConvexity] = bndconvy(Yield,CouponRate,Settle, Maturity) computes the convexity of NUMBONDS fixed income securities given a clean price for each bond.
bndconvy determines the convexity for a bond whether the first or last coupon periods in the coupon structure are short or long (that is, whether the coupon structure is synchronized to maturity). bndconvy also determines the convexity of a zero coupon bond.
[YearConvexity,PerConvexity] = bndconvy( $\qquad$ ,Name, Value) adds optional name-value pair arguments.

## Examples

## Find Bond Convexity Given Yield

This example shows how to compute the convexity of a bond at three different yield values.

```
Yield = [0.04; 0.055; 0.06];
CouponRate = 0.055;
Settle = '02-Aug-1999';
Maturity = '15-Jun-2004';
Period = 2;
Basis = 0;
```

[YearConvexity, PerConvexity]=bndconvy(Yield, CouponRate,... Settle, Maturity, Period, Basis)

YearConvexity = 3×1
21.4825
21.0358
20.8885

PerConvexity $=3 \times 1$
85.9298
84.1434
83.5541

## Find Bond Convexity Given Yield Using datetime Inputs

This example shows how to use datetime inputs to compute the convexity of a bond at three different yield values.

```
Yield = [0.04; 0.055; 0.06];
CouponRate = 0.055;
Settle = datetime('02-Aug-1999','Locale','en_US');
Maturity = datetime('15-Jun-2004','Locale','en_US');
Period = 2;
Basis = 0;
[YearConvexity, PerConvexity]=bndconvy(Yield, CouponRate,...
Settle, Maturity, Period, Basis)
```

```
YearConvexity = 3×1
```

21.4825
21.0358
20.8885

```
PerConvexity = 3x1
```

85.9298
84.1434
83.5541

## Input Arguments

## Yield - Yield to maturity on semiannual basis <br> numeric

Yield to maturity on a semiannual basis, specified as numeric value using a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector.

## Data Types: double

## CouponRate - Annual percentage rate used to determine coupons payable on a bond

decimal
Annual percentage rate used to determine the coupons payable on a bond, specified as decimal value using a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector.

Data Types: double

## Settle - Settlement date for certificate of deposit <br> serial date number | date character vector | datetime

Settlement date for the certificate of deposit, specified as a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector using serial date numbers, date character vectors, or datetime arrays. The Settle date must be before the Maturity date.

Data Types: double |char|datetime

## Maturity - Maturity date for certificate of deposit serial date number | date character vector | datetime

Maturity date for the certificate of deposit, specified as a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector using serial date numbers, date character vectors, or datetime arrays.

Data Types: double | char|datetime

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Example: [YearConvexity,PerConvexity] = bndconvy(Yield,CouponRate,Settle, Maturity,'Period',4,'Basis',7)

## Period - Number of coupon payments per year

2 (default) | numeric with values $0,1,2,3,4,6$ or 12
Number of coupon payments per year, specified as the comma-separated pair consisting of 'Period ' and a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector using the values: $0,1,2,3,4,6$, or 12 .

Data Types: double

## Basis - Day-count basis of instrument

0 (default) | numeric values: $0,1,2,3,4,6,7,8,9,10,11,12,13$
Day-count of the instrument, specified as the comma-separated pair consisting of 'Basis' and a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector using a supported value:

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- 2 = actual/360
- 3 = actual/365
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- $9=$ actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- $12=$ actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-0

## Data Types: double

## EndMonthRule - End-of-month rule flag

1 (in effect) (default) | nonnegative integer 0 or 1
End-of-month rule flag, specified as the comma-separated pair consisting of 'EndMonthRule' and a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days.

- $0=$ Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- $1=$ Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.


## Data Types: logical

## IssueDate - Bond issue date

serial date number | date character vector | datetime
Bond Issue date, specified as the comma-separated pair consisting of 'IssueDate' and a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector using serial date numbers, date character vectors, or datetime arrays.

If you do not specify an IssueDate, the cash flow payment dates are determined from other inputs.

Data Types: double|char|datetime

## FirstCouponDate - Irregular or normal first coupon date serial date number | date character vector | datetime

Irregular or normal first coupon date, specified as the comma-separated pair consisting of 'FirstCouponDate' and a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector using serial date numbers, date character vectors, or datetime arrays.

If you do not specify a FirstCouponDate, the cash flow payment dates are determined from other inputs.
Data Types: double | char | datetime

## LastCouponDate - Irregular or normal last coupon date serial date number | date character vector | datetime

Irregular or normal last coupon date, specified as the comma-separated pair consisting of 'LastCouponDate' and a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector using serial date numbers, date character vectors, or datetime arrays.

If you do not specify a LastCouponDate, the cash flow payment dates are determined from other inputs.

Data Types: double | char | datetime

## StartDate - Forward starting date of payments serial date number | date character vector | datetime

Forward starting date of payments, specified as the comma-separated pair consisting of 'StartDate' and a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector using serial date numbers, date character vectors, or datetime arrays. The StartDate is when a bond actually starts (the date from which a bond cash flow is considered). To make an instrument forward-starting, specify this date as a future date.

If you do not specify a StartDate, the effective start date is the Settle date.
Data Types: double|char|datetime

## Face - Face value of bond

100 (default) | numeric
Face value of the bond, specified as the comma-separated pair consisting of 'Face' and a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector.

Data Types: double

## CompoundingFrequency - Compounding frequency for yield calculation

## SIA bases uses2, ICMA bases uses 1 (default) | integer with value of 1, 2, 3, 4, 6, or 12

Compounding frequency for yield calculation, specified as the comma-separated pair consisting of 'CompoundingFrequency ' and a scalar or a NUMBONDS-by-1 or 1-byNUMBONDS vector.

- 1 - Annual compounding
- 2-Semiannual compounding
- 3-Compounding three times per year
- 4-Quarterly compounding
- 6 - Bimonthly compounding
- 12 - Monthly compounding

Note By default, SIA bases (0-7) and BUS/252 use a semiannual compounding convention and ICMA bases (8-12) use an annual compounding convention.

## Data Types: double

## DiscountBasis - Basis used to compute the discount factors for computing the yield

SIA uses 0 (default) | integers of the set [0...13] | vector of integers of the set [0...13]

Basis used to compute the discount factors for computing the yield, specified as the comma-separated pair consisting of 'DiscountBasis' and a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector. Values are:

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- $2=$ actual/360
- 3 = actual/365
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- $11=30 / 360 E$ (ICMA)
- $12=$ actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-0

Note If a SIA day-count basis is defined in the Basis input argument and there is no value assigned for DiscountBasis, the default behavior is for SIA bases to use the actual/actual day count to compute discount factors.

If an ICMA day-count basis or BUS/252 is defined in the Basis input argument and there is no value assigned for DiscountBasis, the specified bases from the Basis input argument are used.

## Data Types: double

## LastCouponInterest - Compounding convention for computing yield of a bond in last coupon period

compound (default) | values are simple or compound
Compounding convention for computing the yield of a bond in the last coupon period, specified as the comma-separated pair consisting of 'LastCouponInterest' and a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector. LastCouponInterest is based on only the last coupon and the face value to be repaid. Acceptable values are:

- simple
- compound

Data Types: char | cell

## Output Arguments

## YearConvexity - Yearly (annualized) convexity <br> numeric

Yearly (annualized) convexity, returned as a NUMBONDS-by-1 vector.

## PerConvexity - Periodic convexity reported on semiannual bond basis numeric

Periodic convexity reported on a semiannual bond basis (in accordance with SIA convention), returned as a NUMBONDS-by-1 vector.

## References

[1] Krgin, D. Handbook of Global Fixed Income Calculations. Wiley, 2002.
[2] Mayle, J. "Standard Securities Calculations Methods: Fixed Income Securities Formulas for Analytic Measures." SIA, Vol 2, Jan 1994.
[3] Stigum, M., Robinson, F. Money Market and Bond Calculation. McGraw-Hill, 1996.

## See Also <br> bndconvp | bnddurp | bnddury | cfconv | cfdur | datetime

## Topics

on page 10-6
"Yield Conventions" on page 2-31

## Introduced before R2006a

## bnddurp

Bond duration given price

Note In R2017b, the specification of optional input arguments has changed. While the previous ordered inputs syntax is still supported, it may no longer be supported in a future release. Use the optional name-value pair inputs: Period, Basis, EndMonthRule, IssueDate,FirstCouponDate, LastCouponDate, StartDate,Face, CompoundingFrequency, DiscountBasis, and LastCouponInterest.

## Syntax

[ModDuration, YearDuration, PerDuration] = bnddurp(Price,CouponRate, Settle,Maturity)
[ModDuration, YearDuration,PerDuration] = bnddurp( $\qquad$ ,Name, Value)

## Description

[ModDuration, YearDuration, PerDuration] = bnddurp(Price,CouponRate, Settle, Maturity) computes the Macaulay and modified duration of NUMBONDS fixedincome securities given a clean price for each bond.
bnddurp determines the Macaulay and modified duration for a bond whether the first or last coupon periods in the coupon structure are short or long (that is, whether the coupon structure is synchronized to maturity). bnddurp also determines the Macaulay and modified duration for a zero coupon bond.
[ModDuration,YearDuration,PerDuration] = bnddurp( $\qquad$ ,Name, Value) adds optional name-value pair arguments.

## Examples

## Find Bond Duration Given Price

This example shows how to compute the duration of three bonds given their prices.

```
Price = [106; 100; 98];
CouponRate = 0.055;
Settle = '02-Aug-1999';
Maturity = '15-Jun-2004';
Period = 2;
Basis = 0;
```

[ModDuration, YearDuration, PerDuration] = bnddurp(Price,...
CouponRate, Settle, Maturity, Period, Basis)
ModDuration $=3 \times 1$
4.2400
4.1925
4.1759
YearDuration $=3 \times 1$
4.3275
4.3077
4.3007
PerDuration = $3 \times 1$
8.6549
8.6154
8.6014

## Find Bond Duration Given Price Using datetime Inputs

This example shows how to use datetime inputs to compute the duration of three bonds given their prices.

```
Price = [106; 100; 98];
```

CouponRate = 0.055;

```
Settle = datetime('02-Aug-1999','Locale','en_US');
Maturity = datetime('15-Jun-2004','Locale','ēn_US');
Period = 2;
Basis = 0;
[ModDuration, YearDuration, PerDuration] = bnddurp(Price,...
CouponRate, Settle, Maturity, Period, Basis)
ModDuration = 3×1
    4.2400
    4.1925
    4.1759
YearDuration = 3×1
    4.3275
    4.3077
    4.3007
PerDuration = 3×1
    8.6549
    8.6154
    8.6014
```


## Input Arguments

## Price - Clean price (excludes accrued interest) <br> numeric

Clean price (excludes accrued interest), specified as numeric value using a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector.

## Data Types: double

## CouponRate - Annual percentage rate used to determine coupons payable on a bond

decimal

Annual percentage rate used to determine the coupons payable on a bond, specified as decimal value using a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector.

Data Types: double

## Settle - Settlement date for certificate of deposit <br> serial date number | date character vector | datetime

Settlement date for the certificate of deposit, specified as a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector using serial date numbers, date character vectors, or datetime arrays. The Settle date must be before the Maturity date.
Data Types: double | char \| datetime
Maturity - Maturity date for certificate of deposit serial date number | date character vector | datetime

Maturity date for the certificate of deposit, specified as a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector using serial date numbers, date character vectors, or datetime arrays.
Data Types: double|char|datetime

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, ... , NameN, ValueN.

Example: [ModDuration,YearDuration,PerDuration] = bnddurp(Price, CouponRate, Settle, Maturity, 'Period' , 4, 'Basis' , 7)

## Period - Number of coupon payments per year

2 (default) | numeric with values $0,1,2,3,4,6$ or 12
Number of coupon payments per year, specified as the comma-separated pair consisting of 'Period ' and a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector using the values: $0,1,2,3,4,6$, or 12 .

Data Types: double

## Basis - Day-count basis of instrument

0 (default) | numeric values: $0,1,2,3,4,6,7,8,9,10,11,12,13$

Day-count of the instrument, specified as the comma-separated pair consisting of 'Basis' and a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector using a supported value:

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- 2 = actual/360
- $3=$ actual $/ 365$
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- $9=$ actual/360 (ICMA)
- $10=$ actual/365 (ICMA)
- $11=30 / 360 E$ (ICMA)
- $12=$ actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-0
Data Types: double

## EndMonthRule - End-of-month rule flag

1 (in effect) (default) | nonnegative integer 0 or 1
End-of-month rule flag, specified as the comma-separated pair consisting of 'EndMonthRule ' and a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days.

- $0=$ Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- $1=$ Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

Data Types: logical

## IssueDate - Bond issue date <br> serial date number | date character vector | datetime

Bond Issue date, specified as the comma-separated pair consisting of 'IssueDate' and a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector using serial date numbers, date character vectors, or datetime arrays.

If you do not specify an IssueDate, the cash flow payment dates are determined from other inputs.

Data Types: double | char | datetime

## FirstCouponDate - Irregular or normal first coupon date serial date number | date character vector | datetime

Irregular or normal first coupon date, specified as the comma-separated pair consisting of 'FirstCouponDate' and a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector using serial date numbers, date character vectors, or datetime arrays.

If you do not specify a FirstCouponDate, the cash flow payment dates are determined from other inputs.

Data Types: double | char | datetime

## LastCouponDate - Irregular or normal last coupon date <br> serial date number | date character vector | datetime

Irregular or normal last coupon date, specified as the comma-separated pair consisting of 'LastCouponDate' and a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector using serial date numbers, date character vectors, or datetime arrays.

If you do not specify a LastCouponDate, the cash flow payment dates are determined from other inputs.

Data Types: double|char|datetime

## StartDate - Forward starting date of payments <br> serial date number | date character vector | datetime

Forward starting date of payments, specified as the comma-separated pair consisting of 'StartDate' and a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector using serial date numbers, date character vectors, or datetime arrays. The StartDate is when a bond actually starts (the date from which a bond cash flow is considered). To make an instrument forward-starting, specify this date as a future date.

If you do not specify a StartDate, the effective start date is the Settle date.

## Data Types: double | char|datetime

## Face - Face value of bond

100 (default) | numeric
Face value of the bond, specified as the comma-separated pair consisting of 'Face' and a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector.

## Data Types: double

## CompoundingFrequency - Compounding frequency for yield calculation

SIA bases uses2, ICMA bases uses 1 (default) | integer with value of 1, 2, 3, 4, 6, or 12
Compounding frequency for yield calculation, specified as the comma-separated pair consisting of 'CompoundingFrequency ' and a scalar or a NUMBONDS-by-1 or 1-byNUMBONDS vector.

- 1 - Annual compounding
- 2-Semiannual compounding
- 3-Compounding three times per year
- 4-Quarterly compounding
- 6 - Bimonthly compounding
- 12 - Monthly compounding

Note By default, SIA bases (0-7) and BUS/252 use a semiannual compounding convention and ICMA bases (8-12) use an annual compounding convention.

## Data Types: double

## DiscountBasis - Basis used to compute the discount factors for computing the yield

```
SIA uses 0 (default)| integers of the set [0...13] | vector of integers of the set
[0...13]
```

Basis used to compute the discount factors for computing the yield, specified as the comma-separated pair consisting of 'DiscountBasis' and a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector. Values are:

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- 2 = actual/360
- 3 = actual/365
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- $11=30 / 360 E$ (ICMA)
- $12=$ actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-0

Note If a SIA day-count basis is defined in the Basis input argument and there is no value assigned for DiscountBasis, the default behavior is for SIA bases to use the actual/actual day count to compute discount factors.

If an ICMA day-count basis or BUS/252 is defined in the Basis input argument and there is no value assigned for DiscountBasis, the specified bases from the Basis input argument are used.

## Data Types: double

## LastCouponInterest - Compounding convention for computing yield of a bond in last coupon period

## compound (default)|values are simple or compound

Compounding convention for computing the yield of a bond in the last coupon period, specified as the comma-separated pair consisting of 'LastCouponInterest' and a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector. LastCouponInterest is based on only the last coupon and the face value to be repaid. Acceptable values are:

- simple
- compound

Data Types: char | cell

## Output Arguments

## ModDuration - Modified duration in years

numeric
Modified duration in years reported on a semiannual bond basis (in accordance with SIA convention), returned as a NUMBONDS-by-1 vector.

## YearDuration - Macaulay duration in years <br> numeric

Macaulay duration in years, returned as a NUMBONDS-by-1 vector.

## PerDuration - Periodic Macaulay duration <br> numeric

Periodic Macaulay duration reported on a semiannual bond basis (in accordance with SIA convention), returned as a NUMBONDS-by-1 vector.

## References

[1] Krgin, D. Handbook of Global Fixed Income Calculations. Wiley, 2002.
[2] Mayle, J. "Standard Securities Calculations Methods: Fixed Income Securities Formulas for Analytic Measures." SIA, Vol 2, Jan 1994.
[3] Stigum, M., Robinson, F. Money Market and Bond Calculation. McGraw-Hill, 1996.

## See Also

bndconvp | bndconvy | bnddury | bndkrdur | datetime

## Topics

"Bond Portfolio for Hedging Duration and Convexity" on page 10-6
"Yield Conventions" on page 2-31
Introduced before R2006a

## bnddury

Bond duration given yield

Note In R2017b, the specification of optional input arguments has changed. While the previous ordered inputs syntax is still supported, it may no longer be supported in a future release. Use the optional name-value pair inputs: Period, Basis, EndMonthRule, IssueDate,FirstCouponDate, LastCouponDate, StartDate,Face, CompoundingFrequency, DiscountBasis, and LastCouponInterest.

## Syntax

[ModDuration, YearDuration, PerDuration] = bnddury(Yield,CouponRate, Settle,Maturity)
[ModDuration,YearDuration,PerDuration] = bnddury( $\qquad$ ,Name, Value)

## Description

[ModDuration, YearDuration, PerDuration] = bnddury(Yield,CouponRate, Settle, Maturity) computes the Macaulay and modified duration of NUMBONDS fixed income securities given yield to maturity for each bond.
bnddury determines the Macaulay and modified duration for a bond whether the first or last coupon periods in the coupon structure are short or long (that is, whether the coupon structure is synchronized to maturity). bnddury also determines the Macaulay and modified duration for a zero coupon bond.
[ModDuration,YearDuration,PerDuration] = bnddury( $\qquad$ ,Name, Value) adds optional name-value pair arguments.

## Examples

## Find Bond Duration Given Yield

This example shows how to compute the duration of a bond at three different yield values.

```
Yield = [0.04; 0.055; 0.06];
CouponRate = 0.055;
Settle = '02-Aug-1999';
Maturity = '15-Jun-2004';
Period = 2;
Basis = 0;
[ModDuration, YearDuration, PerDuration]=bnddury(Yield,...
CouponRate, Settle, Maturity, Period, Basis)
ModDuration = 3×1
    4.2444
    4.1924
    4.1751
YearDuration = 3×1
    4.3292
    4.3077
    4.3004
PerDuration = 3×1
    8.6585
    8.6154
    8.6007
```


## Find Bond Duration Given Yield Using datetime Inputs

This example shows how to use datetime inputs to compute the duration of a bond at three different yield values.

Yield = [0.04; 0.055; 0.06];
CouponRate = 0.055;

```
Settle = datetime('02-Aug-1999','Locale','en_US');
Maturity = datetime('15-Jun-2004','Locale','ēn_US');
Period = 2;
Basis = 0;
[ModDuration,YearDuration,PerDuration]=bnddury(Yield,...
CouponRate, Settle, Maturity, Period, Basis)
ModDuration = 3×1
    4.2444
    4.1924
    4.1751
YearDuration = 3×1
    4.3292
    4.3077
    4.3004
PerDuration = 3×1
    8.6585
    8.6154
    8.6007
```


## Input Arguments

## Yield - Yield to maturity on a semiannual basis decimal

Yield to maturity on a semiannual basis, specified as a decimal value using a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector.

## Data Types: double

## CouponRate - Annual percentage rate used to determine coupons payable on a bond

decimal

Annual percentage rate used to determine the coupons payable on a bond, specified as decimal value using a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector.
Data Types: double

## Settle - Settlement date for certificate of deposit <br> serial date number | date character vector | datetime

Settlement date for the certificate of deposit, specified as a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector using serial date numbers, date character vectors, or datetime arrays. The Settle date must be before the Maturity date.
Data Types: double | char \| datetime
Maturity - Maturity date for certificate of deposit serial date number | date character vector | datetime

Maturity date for the certificate of deposit, specified as a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector using serial date numbers, date character vectors, or datetime arrays.

Data Types: double | char | datetime

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, ... , NameN, ValueN.

Example: [ModDuration,YearDuration,PerDuration] = bnddury (Yield,CouponRate,Settle,Maturity,'Period',4,'Basis', 7)

## Period - Number of coupon payments per year <br> 2 (default) | numeric with values $0,1,2,3,4,6$ or 12

Number of coupon payments per year, specified as the comma-separated pair consisting of 'Period ' and a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector using the values: $0,1,2,3,4,6$, or 12 .

Data Types: double

## Basis - Day-count basis of instrument

0 (default) | numeric values: $0,1,2,3,4,6,7,8,9,10,11,12,13$

Day-count of the instrument, specified as the comma-separated pair consisting of 'Basis' and a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector using a supported value:

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- 2 = actual/360
- $3=$ actual $/ 365$
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- $9=$ actual/360 (ICMA)
- $10=$ actual/365 (ICMA)
- $11=30 / 360 E$ (ICMA)
- $12=$ actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-0
Data Types: double

## EndMonthRule - End-of-month rule flag

1 (in effect) (default) | nonnegative integer 0 or 1
End-of-month rule flag, specified as the comma-separated pair consisting of 'EndMonthRule ' and a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days.

- $0=$ Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- $1=$ Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

Data Types: logical

## IssueDate - Bond issue date <br> serial date number | date character vector | datetime

Bond Issue date, specified as the comma-separated pair consisting of 'IssueDate' and a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector using serial date numbers, date character vectors, or datetime arrays.

If you do not specify an IssueDate, the cash flow payment dates are determined from other inputs.

Data Types: double | char | datetime

## FirstCouponDate - Irregular or normal first coupon date serial date number | date character vector | datetime

Irregular or normal first coupon date, specified as the comma-separated pair consisting of 'FirstCouponDate' and a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector using serial date numbers, date character vectors, or datetime arrays.

If you do not specify a FirstCouponDate, the cash flow payment dates are determined from other inputs.

Data Types: double | char | datetime

## LastCouponDate - Irregular or normal last coupon date <br> serial date number | date character vector | datetime

Irregular or normal last coupon date, specified as the comma-separated pair consisting of 'LastCouponDate' and a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector using serial date numbers, date character vectors, or datetime arrays.

If you do not specify a LastCouponDate, the cash flow payment dates are determined from other inputs.

Data Types: double|char|datetime

## StartDate - Forward starting date of payments <br> serial date number | date character vector | datetime

Forward starting date of payments, specified as the comma-separated pair consisting of 'StartDate' and a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector using serial date numbers, date character vectors, or datetime arrays. The StartDate is when a bond actually starts (the date from which a bond cash flow is considered). To make an instrument forward-starting, specify this date as a future date.

If you do not specify a StartDate, the effective start date is the Settle date.

## Data Types: double | char | datetime

## Face - Face value of bond

100 (default) | numeric
Face value of the bond, specified as the comma-separated pair consisting of 'Face' and a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector.

## Data Types: double

## CompoundingFrequency - Compounding frequency for yield calculation

SIA bases uses2, ICMA bases uses 1 (default) | integer with value of 1, 2, 3, 4, 6, or 12
Compounding frequency for yield calculation, specified as the comma-separated pair consisting of 'CompoundingFrequency ' and a scalar or a NUMBONDS-by-1 or 1-byNUMBONDS vector.

- 1 - Annual compounding
- 2-Semiannual compounding
- 3-Compounding three times per year
- 4-Quarterly compounding
- 6 - Bimonthly compounding
- 12 - Monthly compounding

Note By default, SIA bases (0-7) and BUS/252 use a semiannual compounding convention and ICMA bases (8-12) use an annual compounding convention.

## Data Types: double

## DiscountBasis - Basis used to compute the discount factors for computing the yield

```
SIA uses 0 (default)| integers of the set [0...13] | vector of integers of the set
[0...13]
```

Basis used to compute the discount factors for computing the yield, specified as the comma-separated pair consisting of 'DiscountBasis' and a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector. Values are:

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- 2 = actual/360
- 3 = actual/365
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- $11=30 / 360 E$ (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-0

Note If a SIA day-count basis is defined in the Basis input argument and there is no value assigned for DiscountBasis, the default behavior is for SIA bases to use the actual/actual day count to compute discount factors.

If an ICMA day-count basis or BUS/252 is defined in the Basis input argument and there is no value assigned for DiscountBasis, the specified bases from the Basis input argument are used.

## Data Types: double

## LastCouponInterest - Compounding convention for computing yield of a bond in last coupon period

## compound (default)|values are simple or compound

Compounding convention for computing the yield of a bond in the last coupon period, specified as the comma-separated pair consisting of 'LastCouponInterest' and a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector. LastCouponInterest is based on only the last coupon and the face value to be repaid. Acceptable values are:

- simple
- compound

Data Types: char | cell

## Output Arguments

## ModDuration - Modified duration in years

numeric
Modified duration in years reported on a semiannual bond basis (in accordance with SIA convention), returned as a NUMBONDS-by-1 vector.

## YearDuration - Macaulay duration in years <br> numeric

Macaulay duration in years, returned as a NUMBONDS-by-1 vector.

## PerDuration - Periodic Macaulay duration <br> numeric

Periodic Macaulay duration reported on a semiannual bond basis (in accordance with SIA convention), returned as a NUMBONDS-by-1 vector.

## References

[1] Krgin, D. Handbook of Global Fixed Income Calculations. Wiley, 2002.
[2] Mayle, J. "Standard Securities Calculations Methods: Fixed Income Securities Formulas for Analytic Measures." SIA, Vol 2, Jan 1994.
[3] Stigum, M., Robinson, F. Money Market and Bond Calculation. McGraw-Hill, 1996.

## See Also

bndconvp | bndconvy | bnddurp | bndkrdur | datetime

## Topics

"Bond Portfolio for Hedging Duration and Convexity" on page 10-6
"Yield Conventions" on page 2-31

## Introduced before R2006a

## bndkrdur

Bond key rate duration given zero curve

## Syntax

```
KeyRateDuration = bndkrdur(ZeroData,CouponRate,Settle,Maturity)
KeyRateDuration = bndkrdur(
```

$\qquad$

``` , Name, Value)
```


## Description

KeyRateDuration = bndkrdur(ZeroData, CouponRate, Settle, Maturity) computes the key rate durations for one or more bonds given a zero curve and a set of key rates.

KeyRateDuration = bndkrdur( $\qquad$ ,Name, Value) adds optional name-value pair arguments.

## Examples

## Find the Bond Key Rate Duration Given the Zero Curve

This example shows how to compute the key rate duration of a bond for key rate times of $2,5,10$, and 30 years.

```
ZeroRates = [0.0476 .0466 . 0465 .0468 . 0473 . 0478 ...
.0493 .0539 .0572 .0553 .0530]';
ZeroDates = daysadd('31-Dec-1998',[30 360 360*2 360*3 360*5 ...
360*7 360*10 360*15 360*20 360*25 360*30],1);
ZeroData = [ZeroDates ZeroRates];
krdur = bndkrdur(ZeroData,.0525,'12/31/1998',...
'11/15/2028','KeyRates',[2 5 10 30])
```

0.2986
0.8791
4.1353
9.5814

## Find the Bond Key Rate Duration Given the Zero Curve Using datetime Inputs

This example shows how to use datetime inputs for Settle and Maturity and also use a table for ZeroData to compute the key rate duration of a bond for key rate times of 2 , 5,10 , and 30 years.

```
ZeroRates = [0.0476 .0466 .0465 .0468 .0473 . 0478 ...
.0493 .0539 .0572 .0553 .0530]';
ZeroDates = daysadd('31-Dec-1998',[30 360 360*2 360*3 360*5 ...
360*7 360*10 360*15 360*20 360*25 360*30],1);
ZeroData = table(datetime(ZeroDates,'ConvertFrom','datenum','Locale','en_US'), ZeroRat
krdur = bndkrdur(ZeroData,.0525,datetime('12/31/1998','Locale','en US'),...
datetime('11/15/2028','Locale','en_US'),'KeyRates',[2 5 10 30])
krdur = 1×4
```

0.2986
0.8791
4.1353
9.5814

## Input Arguments

## ZeroData - Zero curve <br> matrix | table

Zero Curve, specified as a numRates-by-2 matrix or a numRates-by-2 table.
If ZeroData is represented as a numRates-by-2 matrix, the first column is a MATLAB serial date number and the second column is the accompanying zero rates.

If ZeroData is a table, the first column can be serial date numbers, date character vectors, or datetime arrays. The second column must be numeric data corresponding to the zero rates.

## Data Types: double | table

## CouponRate - Annual percentage rate used to determine coupons payable on a bond <br> decimal

Annual percentage rate used to determine the coupons payable on a bond, specified as decimal value using a scalar or a NUMBONDS-by-1 vector.

## Data Types: double

## Settle - Settlement date for all bonds and zero curve <br> serial date number | date character vector | datetime

Settlement date for all bonds and zero curve, specified as a scalar using a serial date number, date character vector, or datetime array. Settle must be the same settlement date for all the bonds and the zero curve.
Data Types: double | char | datetime
Maturity - Maturity date for bonds
serial date number | date character vector | datetime
Maturity date for bonds, specified as a scalar or a NUMBONDS-by-1 vector using serial date numbers, date character vectors, or datetime arrays.
Data Types: double | char \| datetime

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Example: KeyRateDuration = bndkrdur(ZeroData, . 0525,'12/31/1998','11/15/2028','KeyRates',[2 510 30])

[^6]Interpolation method used to obtain points from the zero curve, specified as the commaseparated pair consisting of 'InterpMethod ' and a character vector using one of the following values:

- 'linear' (default)
- 'cubic'
- 'pchip'


## Data Types: char

## ShiftValue - Value that zero curve is shifted up and down to compute duration . 01 (100 basis points) (default) | numeric

Value that zero curve is shifted up and down to compute duration, specified as the comma-separated pair consisting of 'ShiftValue' and a scalar numeric value.

## Data Types: double

## KeyRates - Rates to perform the duration calculation set to each of the zero dates (default) | numeric

Rates to perform the duration calculation, specified as the comma-separated pair consisting of 'KeyRates' and a time to maturity using a scalar or a NUMBONDS-by-1 vector.

Data Types: double
CurveCompounding - Compounding frequency of curve
2 (default) | integer with value of 1, 2, 3, 4, 6, or 12
Compounding frequency of the curve, specified as the comma-separated pair consisting of 'CurveCompounding' and a scalar using one of the following values:

- 1 - Annual compounding
- 2-Semiannual compounding
- 3 - Compounding three times per year
- 4-Quarterly compounding
- 6 - Bimonthly compounding
- 12 - Monthly compounding


## Data Types: double

## CurveBasis - Basis of the curve

0 (default) | numeric values: $0,1,2,3,4,6,7,8,9,10,11,12,13$
Basis of the curve, specified as the comma-separated pair consisting of 'CurveBasis ' and a scalar using one of the following values:

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- 2 = actual/360
- $3=$ actual/365
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- $7=$ actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- $9=$ actual/360 (ICMA)
- $10=$ actual/365 (ICMA)
- $11=30 / 360 E$ (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-0
Data Types: double

## Period - Number of coupon payments per year

2 (default) | numeric with values $0,1,2,3,4,6$ or 12
Number of coupon payments per year, specified as the comma-separated pair consisting of 'Period ' and a scalar or a NUMBONDS-by-1 vector using the values: $0,1,2,3,4,6$, or 12.

Data Types: double

## Basis - Day-count basis of instrument

0 (default) | numeric values: $0,1,2,3,4,6,7,8,9,10,11,12,13$

Day-count of the instrument, specified as the comma-separated pair consisting of ' Basis' and a scalar or a NUMBONDS-by-1 vector using a supported value:

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- 2 = actual/360
- 3 = actual/365
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- $9=$ actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-0
Data Types: double

## EndMonthRule - End-of-month rule flag

## 1 (in effect) (default) | nonnegative integer 0 or 1

End-of-month rule flag, specified as the comma-separated pair consisting of 'EndMonthRule' and a scalar or a NUMBONDS-by-1 vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days.

- $0=$ Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.


## Data Types: logical

## IssueDate - Bond issue date

serial date number | date character vector | datetime

Bond Issue date, specified as the comma-separated pair consisting of 'IssueDate' and a scalar or a NUMBONDS-by- 1 vector using serial date numbers, date character vectors, or datetime arrays.

If you do not specify an IssueDate, the cash flow payment dates are determined from other inputs.

Data Types: double | char | datetime

## FirstCouponDate - Irregular or normal first coupon date

serial date number | date character vector | datetime
Irregular or normal first coupon date, specified as the comma-separated pair consisting of 'FirstCouponDate' and a scalar or a NUMBONDS-by-1 vector using serial date numbers, date character vectors, or datetime arrays.

If you do not specify a FirstCouponDate, the cash flow payment dates are determined from other inputs.

Data Types: double | char |datetime

## LastCouponDate - Irregular or normal last coupon date <br> serial date number | date character vector | datetime

Irregular or normal last coupon date, specified as the comma-separated pair consisting of ' LastCouponDate' and a scalar or a NUMBONDS-by-1 vector using serial date numbers, date character vectors, or datetime arrays.

If you do not specify a LastCouponDate, the cash flow payment dates are determined from other inputs.
Data Types: double | char \| datetime

## StartDate - Forward starting date of payments serial date number | date character vector | datetime

Forward starting date of payments, specified as the comma-separated pair consisting of 'StartDate' and a scalar or a NUMBONDS-by-1 vector using serial date numbers, date character vectors, or datetime arrays. The StartDate is when a bond actually starts (the date from which a bond cash flow is considered). To make an instrument forward-starting, specify this date as a future date.

If you do not specify a StartDate, the effective start date is the Settle date.

## Data Types: double | char |datetime

## Face - Face value of bond

100 (default) | numeric
Face value of the bond, specified as the comma-separated pair consisting of 'Face' and a scalar or a NUMBONDS-by-1 vector.
Data Types: double

## Output Arguments

## KeyRateDuration - Key rate durations

matrix
Key rate durations, returned as a numBonds-by-numRates matrix.

## Algorithms

bndkrdur computes the key rate durations for one or more bonds given a zero curve and a set of key rates. By default, the key rates are each of the zero curve rates. For each key rate, the duration is computed by shifting the zero curve up and down by a specified amount (ShiftValue) at that particular key rate, computing the present value of the bond in each case with the new zero curves, and then evaluating the following:

$$
k r d u r_{i}=\frac{\left(P V_{\text {down }}-P V_{u p}\right)}{(P V \times \text { ShiftValue } \times 2)}
$$

Note The shift to the curve is computed by shifting the particular key rate by the ShiftValue and then interpolating the values of the curve in the interval between the previous and next key rates. For the first key rate, any curve values before the date are equal to the ShiftValue; likewise, for the last key rate, any curve values after the date are equal to the ShiftValue.

## References

[1] Golub, B., Tilman, L. Risk Management: Approaches for Fixed Income Markets. Wiley, 2000.
[2] Tuckman, B. Fixed Income Securities: Tools for Today's Markets. Wiley, 2002.

See Also<br>bndconvp | bndconvy | bnddurp | bnddury | datetime

## Topics

"Bond Portfolio for Hedging Duration and Convexity" on page 10-6
"Yield Conventions" on page 2-31

Introduced before R2006a

## bndprice

Price fixed-income security from yield to maturity

Note In R2017b, the specification of optional input arguments has changed. While the previous ordered inputs syntax is still supported, it may no longer be supported in a future release. Use the optional name-value pair inputs: Period, Basis, EndMonthRule, IssueDate,FirstCouponDate, LastCouponDate, StartDate,Face, CompoundingFrequency, DiscountBasis, and LastCouponInterest.

## Syntax

[Price,AccruedInt] = bndprice(Yield,CouponRate,Settle,Maturity) [Price,AccruedInt] = bndprice( $\qquad$ , Name, Value)

## Description

[Price,AccruedInt] = bndprice(Yield,CouponRate,Settle,Maturity) given bonds with SIA date parameters and yields to maturity, returns the clean prices and accrued interest due.
[Price,AccruedInt] = bndprice( $\qquad$ ,Name, Value) adds optional name-value pair arguments.

## Examples

## Price a Treasury Bond from Yield to Maturity

This example shows how to price a treasury bond at three different yield values.

```
Yield = [0.04; 0.05; 0.06];
CouponRate = 0.05;
Settle = '20-Jan-1997';
Maturity = '15-Jun-2002';
```

```
Period = 2;
Basis = 0;
[Price, AccruedInt] = bndprice(Yield, CouponRate, Settle,...
Maturity, Period, Basis)
Price = 3\times1
    104.8106
    99.9951
    95.4384
AccruedInt = 3\times1
    0.4945
    0.4945
    0.4945
```


## Price a Treasury Bond from Yield to Maturity Using datetime Inputs

This example shows how to use datetime inputs to price a treasury bond at three different yield values.

```
Yield = [0.04; 0.05; 0.06];
CouponRate = 0.05;
Settle = datetime('20-Jan-1997','Locale','en_US');
Maturity = datetime('15-Jun-2002','Locale','èn_US');
Period = 2;
Basis = 0;
[Price, AccruedInt] = bndprice(Yield, CouponRate, Settle,...
Maturity, Period, Basis)
Price = 3×1
    104.8106
    99.9951
    95.4384
AccruedInt = 3\times1
```

0.4945
0.4945
0.4945

## Price a Treasury Bond with Different Yield Values

This example shows how to price a Treasury bond at two different yield values that include parameter/value pairs for CompoundingFrequency, DiscountBasis, and LastCouponPeriodInterest.
bndprice(.04,0.08,'5/25/2004','4/21/2005','Period',1,'Basis',8, ... 'LastCouponInterest','simple')

## Input Arguments

## Yield - Bond yield to maturity

numeric
Bond yield to maturity is specified as a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector. Yield is on a semiannual basis for Basis values 0 through 7 and 13 and an annual basis for Basis values 8 through 12.

Data Types: double

## CouponRate - Annual percentage rate used to determine coupons payable on a bond <br> decimal

Annual percentage rate used to determine the coupons payable on a bond, specified as decimal using a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector.

Data Types: double

## Settle - Settlement date of bond

serial date number | date character vector | datetime

Settlement date of the bond, specified as a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector using serial date numbers, date character vectors, or datetime arrays. The Settle date must be before the Maturity date.
Data Types: double|char|datetime

## Maturity - Maturity date of bond

serial date number | date character vector | datetime
Maturity date of the bond, specified as a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector using serial date numbers, date character vectors, or datetime arrays.

Data Types: double | char | datetime

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN , ValueN.

Example: [Price,AccruedInt] = bndprice(Yield,CouponRate,Settle,Maturity, 'Period',4,'Basis', 9)

## Period - Number of coupon payments per year

2 (default) | numeric with values $0,1,2,3,4,6$ or 12
Number of coupon payments per year, specified as the comma-separated pair consisting of 'Period ' and a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector using the values: $0,1,2,3,4,6$, or 12 .

Data Types: double

## Basis - Day-count basis of instrument

0 (default) | numeric values: $0,1,2,3,4,6,7,8,9,10,11,12,13$
Day-count of the instrument, specified as the comma-separated pair consisting of 'Basis' and a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector using a supported value:

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- 2 = actual/360
- 3 = actual/365
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-0
Data Types: double

## EndMonthRule - End-of-month rule flag

1 (in effect) (default) | nonnegative integer 0 or 1
End-of-month rule flag, specified as the comma-separated pair consisting of 'EndMonthRule' and a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days.

- $0=$ Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.


## Data Types: logical

## IssueDate - Bond issue date

serial date number | date character vector | datetime
Bond Issue date, specified as the comma-separated pair consisting of 'IssueDate' and a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector using serial date numbers, date character vectors, or datetime arrays.

If you do not specify an IssueDate, the cash flow payment dates are determined from other inputs.

## Data Types: double | char | datetime

## FirstCouponDate - Irregular or normal first coupon date serial date number | date character vector | datetime

Irregular or normal first coupon date, specified as the comma-separated pair consisting of 'FirstCouponDate' and a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector using serial date numbers, date character vectors, or datetime arrays.

If you do not specify a FirstCouponDate, the cash flow payment dates are determined from other inputs.

Data Types: double | char | datetime

## LastCouponDate - Irregular or normal last coupon date <br> serial date number | date character vector | datetime

Irregular or normal last coupon date, specified as the comma-separated pair consisting of 'LastCouponDate' and a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector using serial date numbers, date character vectors, or datetime arrays.

If you do not specify a LastCouponDate, the cash flow payment dates are determined from other inputs.

Data Types: double | char | datetime

## StartDate - Forward starting date of payments <br> serial date number | date character vector | datetime

Forward starting date of payments, specified as the comma-separated pair consisting of ' StartDate ' and a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector using serial date numbers, date character vectors, or datetime arrays.

If you do not specify a StartDate, the effective start date is the Settle date.
Data Types: double | char | datetime

## Face - Face value of bond

100 (default) | numeric
Face value of the bond, specified as the comma-separated pair consisting of 'Face' and a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector.

## Data Types: double

## CompoundingFrequency - Compounding frequency for yield calculation

SIA bases uses2, ICMA bases uses 1 (default) | integer with value of $1,2,3,4,6$, or 12
Compounding frequency for yield calculation, specified as the comma-separated pair consisting of 'CompoundingFrequency' and a scalar or a NUMBONDS-by-1 or 1-byNUMBONDS vector.

- 1 - Annual compounding
- 2-Semiannual compounding
- 3-Compounding three times per year
- 4-Quarterly compounding
- 6 - Bimonthly compounding
- 12 - Monthly compounding

Note By default, SIA bases (0-7) and BUS/252 use a semiannual compounding convention and ICMA bases (8-12) use an annual compounding convention.

## Data Types: double

## DiscountBasis - Basis used to compute the discount factors for computing the yield <br> SIA bases uses 0 (default) | integers of the set [0...13] | vector of integers of the set [0...13]

Basis used to compute the discount factors for computing the yield, specified as the comma-separated pair consisting of 'DiscountBasis' and a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector. Values are:

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- 2 = actual/360
- 3 = actual/365
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- $7=$ actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- $9=\mathrm{actual} / 360$ (ICMA)
- $10=$ actual/365 (ICMA)
- $11=30 / 360 E$ (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-0

Note If a SIA day-count basis is defined in the Basis input argument and there is no value assigned for DiscountBasis, the default behavior is for SIA bases to use the actual/actual day count to compute discount factors.

If an ICMA day-count basis or BUS/252 is defined in the Basis input argument and there is no value assigned for DiscountBasis, the specified bases from the Basis input argument are used.

## Data Types: double

## LastCouponInterest - Compounding convention for computing yield of a bond in last coupon period

## compound (default) | values are simple or compound

Compounding convention for computing the yield of a bond in the last coupon period, specified as the comma-separated pair consisting of 'LastCouponInterest' and a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector. LastCouponInterest is based on only the last coupon and the face value to be repaid. Acceptable values are:

- simple
- compound

Data Types: char |cell

## Output Arguments

## Price - Clean price of bond

numeric
Clean price of bond, returned as a NUMBONDS-by-1 vector. The dirty price of the bond is the clean price plus the accrued interest. It equals the present value of the bond cash flows of the yield to maturity with semiannual compounding.

## AccruedInt - Accrued interest payable at settlement numeric

accrued interest payable at settlement, returned as a NUMBONDS-by-1 vector.

## Definitions

## Price and Yield Conventions

The Price and Yield are related to different formulae for SIA and ICMA conventions.
For SIA conventions, Price and Yield are related by the formula:

```
Price + Accrued Interest = sum(Cash_Flow*(1+Yield/2)^(-Time))
```

where the sum is over the bond's cash flows and corresponding times in units of semiannual coupon periods.

For ICMA conventions, the Price and Yield are related by the formula:

```
Price + Accrued Interest = sum(Cash_Flow*(1+Yield)^(-Time))
```


## Algorithms

For SIA conventions, the following formula defines bond price and yield:

$$
P V=\sum_{i=1}^{n}\left(\frac{C F}{\left(1+\frac{z}{f}\right)^{T F}}\right)
$$

where:

| $P V=$ | Present value of a cash flow. |
| :--- | :--- |
| $C F=$ | Cash flow amount. |
| $z=$ | Risk-adjusted annualized rate or yield corresponding to a given cash <br> flow. The yield is quoted on a semiannual basis. |
| $f=$ | Frequency of quotes for the yield. Default is 2 for Basis values 0 to 7 <br> and 13 and 1 for Basis values 8 to 12. The default can be overridden <br> by specifying the CompoundingFrequency name-value pair. |
| $T F=$ | Time factor for a given cash flow. The time factor is computed using the <br> compounding frequency and the discount basis. If these values are not <br> specified, then the defaults are as follows: CompoundingFrequency <br> default is 2 for Basis values 0 to 7 and 13 and 1 for Basis values 8 to <br> 12. DiscountBasis is 0 for Basis values 0 to 7 and 13 and the input <br> Basis for Basis values 8 to 12. |

Note The Basis is always used to compute accrued interest.

For ICMA conventions, the frequency of annual coupon payments determines bond price and yield.

## References

[1] Krgin, D. Handbook of Global Fixed Income Calculations. Wiley, 2002.
[2] Mayle, J. "Standard Securities Calculations Methods: Fixed Income Securities Formulas for Analytic Measures." SIA, Vol 2, Jan 1994.
[3] Stigum, M., Robinson, F. Money Market and Bond Calculation. McGraw-Hill, 1996.

## See Also

bndyield | cfamounts | datetime

## Topics

"Bond Portfolio for Hedging Duration and Convexity" on page 10-6
"Pricing Functions" on page 2-32
"Sensitivity of Bond Prices to Interest Rates" on page 10-2
"Yield Conventions" on page 2-31
Introduced before R2006a

## bndspread

Static spread over spot curve

Note In R2017b, the specification of optional input arguments has changed. While the previous ordered inputs syntax is still supported, it may no longer be supported in a future release. Use the optional name-value pair inputs: Period, Basis, EndMonthRule, IssueDate,FirstCouponDate, LastCouponDate, StartDate,Face, CompoundingFrequency, DiscountBasis, and LastCouponInterest.

## Syntax

Spread $=$ bndspread(SpotInfo,Price, Coupon,Settle,Maturity)
Spread = bndspread( $\qquad$ ,Name, Value)

## Description

Spread = bndspread(SpotInfo,Price,Coupon,Settle,Maturity) computes the static spread (Z-spread) to benchmark in basis points.

Spread = bndspread( ___ ,Name, Value) adds optional name-value pair arguments.

## Examples

## Compute the Static Spread Over a Spot Curve

This example shows how to compute a Federal National Mortgage Association (FNMA) 4 3/8 spread over a Treasury spot curve and plot the results.

```
RefMaturity = [datenum('02/27/2003');
    datenum('05/29/2003');
    datenum('10/31/2004');
    datenum('11/15/2007');
    datenum('11/15/2012');
```

```
datenum('02/15/2031')];
RefCpn = [0;
    0;
    2.125;
    3;
    4;
    5.375] / 100;
RefPrices = [99.6964;
        99.3572;
        100.3662;
        99.4511;
        99.4299;
        106.5756];
RefBonds = [RefPrices, RefMaturity, RefCpn];
Settle = datenum('26-Nov-2002');
[ZeroRates, CurveDates] = zbtprice(RefBonds(:, 2:end), ...
RefPrices, Settle)
ZeroRates = 6×1
    0.0121
    0.0127
    0.0194
    0.0317
    0.0423
    0.0550
CurveDates = 6×1
```

    731639
    731730
    732251
    733361
    735188
    741854
    ```
\% FNMA 4 3/8 maturing 10/06 at 4.30 pm Tuesday
Price \(=105.484\);
Coupon \(=0.04375\);
```

```
Maturity = datenum('15-0ct-2006');
% All optional inputs are supposed to be accounted by default,
% except the accrued interest under 30/360 (SIA), so:
Period = 2;
Basis = 1;
SpotInfo = [CurveDates, ZeroRates];
% Compute static spread over treasury curve, taking into account
% the shape of curve as derived by bootstrapping method embedded
% within bndspread.
SpreadInBP = bndspread(SpotInfo, Price, Coupon, Settle, ...
Maturity, Period, Basis)
SpreadInBP = 18.5669
plot(CurveDates, ZeroRates*100, 'b', CurveDates, ...
ZeroRates*100+SpreadInBP/100, 'r--')
legend({'Treasury'; 'FNMA 4 3/8'})
xlabel('Curve Dates')
ylabel('Spot Rate [%]')
grid;
```



## Compute the Static Spread Over a Spot Curve Using datetime Inputs

This example shows how to compute a Federal National Mortgage Association (FNMA) 4 3/8 spread over a Treasury spot curve using datetime inputs for Settle and Maturity and a table for SpotInfo and plot the results.

```
RefMaturity = [datenum('02/27/2003');
    datenum('05/29/2003');
    datenum('10/31/2004');
    datenum('11/15/2007');
    datenum('11/15/2012');
```

```
datenum('02/15/2031')];
RefCpn = [0;
    0;
    2.125;
    3;
    4;
    5.375] / 100;
RefPrices = [99.6964;
        99.3572;
        100.3662;
        99.4511;
        99.4299;
        106.5756];
RefBonds = [RefPrices, RefMaturity, RefCpn];
Settle = datetime('26-Nov-2002','Locale','en_US');
[ZeroRates, CurveDates] = zbtprice(RefBonds(:, 2:end), ...
RefPrices, Settle)
ZeroRates = 6×1
    0.0121
    0.0127
    0.0194
    0.0317
    0.0423
    0.0550
CurveDates = 6x1 datetime array
    27-Feb-2003
    29-May-2003
    31-0ct-2004
    15-Nov-2007
    15-Nov-2012
    15-Feb-2031
```

```
% FNMA 4 3/8 maturing 10/06 at 4.30 pm Tuesday
Price = 105.484;
Coupon = 0.04375;
Maturity = datetime('15-0ct-2006','Locale','en_US');
```

```
% All optional inputs are accounted by default,
% except the accrued interest under 30/360 (SIA), so:
Period = 2;
Basis = 1;
```

SpotInfo = table(datetime(CurveDates,'ConvertFrom','datenum','Locale','en_US'), ZeroRa
\% Compute static spread over treasury curve, taking into account
\% the shape of curve as derived by bootstrapping method embedded
\% within bndspread.
SpreadInBP = bndspread(SpotInfo, Price, Coupon, Settle, ...
Maturity, Period, Basis)
SpreadInBP = 18.5669
plot(CurveDates, ZeroRates*100, 'b', CurveDates, ...
ZeroRates*100+SpreadInBP/100, 'r--')
legend(\{'Treasury'; 'FNMA 4 3/8'\})
xlabel('Curve Dates')
ylabel('Spot Rate [\%]')
grid;


## Input Arguments

## SpotInfo - Spot-rates information

matrix | table | term structure
Spot-rates information, specified as matrix of two columns, an annualized term structure created by intenvset, or a table.

- Matrix of two columns- The first column is the SpotDate, and the second column, ZeroRates, is the zero-rate corresponding to maturities on the SpotDate. It is recommended that the spot-rates are spaced as evenly apart as possible, perhaps one
that is built from 3-months deposit rates. For example, using the 3-month deposit rates:

```
SpotInfo = ...
[datenum('2-Jan-2004') , 0.03840;
datenum('2-Jan-2005') , 0.04512;
datenum('2-Jan-2006') , 0.05086];
```

- Annualized term structure - Refer to intenvset to create an annualized term structure. For example:

```
Settle = datenum('1-Jan-2004');
Rates = [0.03840; 0.04512; 0.05086];
EndDates = [datenum('2-Jan-2004'); datenum('2-Jan-2005');...
    datenum('2-Jan-2006')];
SpotInfo = intenvset('StartDates' , Settle ,...
                            'Rates' , Rates ,...
                            'EndDates' , EndDates,...
                            'Compounding', 2
                            'Basis' , 0);
```

- Table - If SpotInfo is a table, the first column can be either a serial date number, date character vector, or datetime array. The second column is numerical data representing zero rates. For example:

```
ZeroRates = ... [0.012067955808764;0.012730933424479;0.019360902068703;0.031704525214251;0.042306085224510;0.05498741534
CurveDates = [731639;731730;732251;733361;735188;741854];
Settle = datenum('26-Nov-2002');
Price = 105.484;
Coupon = 0.04375;
Maturity = datenum('15-0ct-2006');
Period = 2;
Basis = 1;
SpotInfo = table(datestr(CurveDates), ZeroRates);
```

Data Types: double | table | struct

## Price - Price for every $\$ 100$ notional amount of bonds whose spreads are computed

numeric
Price for every $\$ 100$ notional amount of bonds whose spreads are computed, specified as numeric value using a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector.

Data Types: double

## Coupon - Annual coupon rate of bonds whose spreads are computed decimal

Annual coupon rate of bonds whose spreads are computed, specified as decimal value using a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector.

## Data Types: double

## Settle - Settlement date of bond <br> serial date number | date character vector | datetime

Settlement date of the bond, specified as a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector using serial date numbers, date character vectors, or datetime arrays. The Settle date must be before the Maturity date.

## Data Types: double | char | datetime

## Maturity - Maturity date of bond

serial date number | date character vector | datetime
Maturity date of the bond, specified as a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector using serial date numbers, date character vectors, or datetime arrays.
Data Types: double | char | datetime

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, ... , NameN, ValueN.

Example: Spread =
bndspread(SpotInfo,Price,Coupon,Settle,Maturity,'Period',4,'Basis', 7)

## Period - Number of coupon payments per year

2 (default) | numeric with values $0,1,2,3,4,6$ or 12
Number of coupon payments per year, specified as the comma-separated pair consisting of 'Period ' and a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector using the values: $0,1,2,3,4,6$, or 12 .

Data Types: double

## Basis - Day-count basis of instrument

0 (default) | numeric values: $0,1,2,3,4,6,7,8,9,10,11,12,13$

Day-count of the instrument, specified as the comma-separated pair consisting of 'Basis' and a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector using a supported value:

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- 2 = actual/360
- 3 = actual/365
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- $9=$ actual/360 (ICMA)
- $10=$ actual/365 (ICMA)
- $11=30 / 360 \mathrm{E}$ (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-0
Data Types: double

## EndMonthRule - End-of-month rule flag

1 (in effect) (default) | nonnegative integer 0 or 1
End-of-month rule flag, specified as the comma-separated pair consisting of 'EndMonthRule' and a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days.

- $0=$ Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- $1=$ Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

Data Types: logical

## IssueDate - Bond issue date <br> serial date number | date character vector | datetime

Bond Issue date, specified as the comma-separated pair consisting of 'IssueDate' and a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector using serial date numbers, date character vectors, or datetime arrays.

If you do not specify an IssueDate, the cash flow payment dates are determined from other inputs.

Data Types: double | char | datetime

## FirstCouponDate - Irregular or normal first coupon date serial date number | date character vector | datetime

Irregular or normal first coupon date, specified as the comma-separated pair consisting of 'FirstCouponDate' and a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector using serial date numbers, date character vectors, or datetime arrays.

If you do not specify a FirstCouponDate, the cash flow payment dates are determined from other inputs.

Data Types: double | char |datetime

## LastCouponDate - Irregular or normal last coupon date <br> serial date number | date character vector | datetime

Irregular or normal last coupon date, specified as the comma-separated pair consisting of 'LastCouponDate' and a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector using serial date numbers, date character vectors, or datetime arrays.

If you do not specify a LastCouponDate, the cash flow payment dates are determined from other inputs.

Data Types: double | char | datetime

## StartDate - Forward starting date of payments <br> serial date number | date character vector | datetime

Forward starting date of payments, specified as the comma-separated pair consisting of 'StartDate' and a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector using serial date numbers, date character vectors, or datetime arrays. The StartDate is when a bond actually starts (the date from which a bond cash flow is considered). To make an instrument forward-starting, specify this date as a future date.

If you do not specify a StartDate, the effective start date is the Settle date.

## Data Types: double | char|datetime

## Face - Face value of bond

100 (default) | numeric
Face value of the bond, specified as the comma-separated pair consisting of 'Face' and a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector.

## Data Types: double

## CompoundingFrequency - Compounding frequency for yield calculation

SIA bases uses2, ICMA bases uses 1 (default) | integer with value of 1, 2, 3, 4, 6, or 12
Compounding frequency for yield calculation, specified as the comma-separated pair consisting of 'CompoundingFrequency ' and a scalar or a NUMBONDS-by-1 or 1-byNUMBONDS vector.

- 1 - Annual compounding
- 2 - Semiannual compounding
- 3-Compounding three times per year
- 4-Quarterly compounding
- 6 - Bimonthly compounding
- 12 - Monthly compounding

Note By default, SIA bases (0-7) and BUS/252 use a semiannual compounding convention and ICMA bases (8-12) use an annual compounding convention.

## Data Types: double

## DiscountBasis - Basis used to compute the discount factors for computing the yield

```
SIA uses 0 (default)| integers of the set [0...13] | vector of integers of the set
[0...13]
```

Basis used to compute the discount factors for computing the yield, specified as the comma-separated pair consisting of 'DiscountBasis' and a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector. Values are:

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- 2 = actual/360
- 3 = actual/365
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- $7=$ actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- $9=$ actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- $11=30 / 360 E$ (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-0

Note If a SIA day-count basis is defined in the Basis input argument and there is no value assigned for DiscountBasis, the default behavior is for SIA bases to use the actual/actual day count to compute discount factors.

If an ICMA day-count basis or BUS/252 is defined in the Basis input argument and there is no value assigned for DiscountBasis, the specified bases from the Basis input argument are used.

## Data Types: double

## LastCouponInterest - Compounding convention for computing yield of a bond in last coupon period

## compound (default) | values are simple or compound

Compounding convention for computing the yield of a bond in the last coupon period, specified as the comma-separated pair consisting of 'LastCouponInterest' and a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector. LastCouponInterest is based on only the last coupon and the face value to be repaid. Acceptable values are:

- simple
- compound


## Data Types: char|cell

## Output Arguments

## Spread - Static spread to benchmark in basis points numeric

Static spread to benchmark, returned in basis points as a scalar or a NUMBONDS-by-1 vector.

## References

[1] Krgin, D. Handbook of Global Fixed Income Calculations. Wiley, 2002.
[2] Mayle, J. "Standard Securities Calculations Methods: Fixed Income Securities Formulas for Analytic Measures." SIA, Vol 2, Jan 1994.
[3] Stigum, M., Robinson, F. Money Market and Bond Calculation. McGraw-Hill, 1996.

## See Also

bndprice| bndyield|datetime

## Topics

"Bond Portfolio for Hedging Duration and Convexity" on page 10-6
"Pricing Functions" on page 2-32
"Sensitivity of Bond Prices to Interest Rates" on page 10-2
"Yield Conventions" on page 2-31

## Introduced before R2006a

## bndtotalreturn

Total return of fixed-coupon bond

## Syntax

```
[BondEquiv,EffectiveRate] = bndtotalreturn(Price,CouponRate,Settle,
Maturity,ReinvestRate)
[BondEquiv,EffectiveRate] = bndtotalreturn(
```

$\qquad$

``` ,Name, Value)
```


## Description

[BondEquiv,EffectiveRate] = bndtotalreturn(Price,CouponRate,Settle, Maturity, ReinvestRate) calculates the total return for fixed-coupon bonds to maturity or to a specific investment horizon.
[BondEquiv,EffectiveRate] = bndtotalreturn( __ , Name,Value) adds optional name-value pair arguments.

## Examples

## Compute the Total Return of a Fixed-Coupon Bond

Use bndtotal return to compute the total return for a fixed-coupon bond, given an investment horizon date.

Define fixed-coupon bond.
Price = 101;
CouponRate $=0.05$;
Settle = '15-Nov-2011';
Maturity $=$ '15-Nov-2031';
ReinvestRate $=0.04$;
Calculate the total return to maturity.

```
[BondEquiv, EffectiveRate] = bndtotalreturn(Price, CouponRate, ...
Settle, Maturity, ReinvestRate)
```

```
BondEquiv = 0.0460
```

EffectiveRate = 0.0466

Specify an investment horizon.

```
HorizonDate = '15-Nov-2021';
[BondEquiv, EffectiveRate] = bndtotalreturn(Price, CouponRate, ...
Settle, Maturity, ReinvestRate, 'HorizonDate', HorizonDate)
```

BondEquiv $=0.0521$
EffectiveRate = 0.0528

Perform scenario analysis on the reinvestment rate.
ReinvestRate = [0.03; 0.035; 0.04; 0.045; 0.05];
[BondEquiv, EffectiveRate] = bndtotalreturn(Price, CouponRate, ... Settle, Maturity, ReinvestRate, 'HorizonDate', HorizonDate)

BondEquiv = 5×1
0.0557
0.0538
0.0521
0.0505
0.0490

EffectiveRate = 5×1
0.0565
0.0546
0.0528
0.0511
0.0496

## Compute the Total Return of a Fixed-Coupon Bond Using datetime Inputs

Use bndtotal return with datetime inputs to compute the total return for a fixedcoupon bond, given an investment horizon date.

```
Price = 101;
CouponRate = 0.05;
Settle = datetime('15-Nov-2011','Locale','en_US');
Maturity = datetime('15-Nov-2031','Locale','en_US');
HorizonDate = datetime('15-Nov-2021','Locale','en_US');
ReinvestRate = 0.04;
[BondEquiv, EffectiveRate] = bndtotalreturn(Price, CouponRate, ...
Settle, Maturity, ReinvestRate, 'HorizonDate', HorizonDate)
BondEquiv = 0.0521
EffectiveRate = 0.0528
```


## Input Arguments

Price - Clean price at settlement date
matrix
Clean price at the settlement date, specified as a scalar or a NINST-by-1 vector.
Data Types: double

## CouponRate - Coupon rate <br> decimal

Coupon rate, specified as a scalar or a NINST-by-1 vector of decimal values.
Data Types: double

## Settle - Settlement date of fixed-coupon bond serial date number | date character vector | datetime

Settlement date of the fixed-coupon bond, specified as scalar or a NINST-by-1 vector of serial date numbers, date character vectors, or datetime arrays. If supplied as a NINST-by-1 vector of dates, settlement dates can be different, as long as they are before the Maturity date and HorizonDate.

Data Types: double|char|datetime

## Maturity - Maturity date of fixed-coupon bond <br> serial date number | date character vector | datetime

Maturity date of the fixed-coupon bond, specified as scalar or a NINST-by-1 vector of serial date numbers, date character vectors, or datetime arrays.

Data Types: double | char | datetime

## ReinvestRate - Reinvestment rate

decimal
Reinvestment rate (the rate earned by reinvesting the coupons), specified as scalar or a NINST-by-2 vector of decimal values.

Data Types: double

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1,Value1, ..., NameN, ValueN.

Example: [BondEquiv,EffectiveRate] = bndtotalreturn(Price,CouponRate,Settle,Maturity,ReinvestRate,'Horizo nDate','15-Nov-2021')

## HorizonDate - Investment horizon date

Maturity date (default) | serial date number | date character vector | datetime
Investment horizon date, specified as the comma-separated pair consisting of 'HorizonDate' and a scalar or a NINST-by-1 vector using serial date numbers, date character vectors, or datetime arrays.

If HorizonDate is unspecified, the total return is calculated to Maturity.
Data Types: double| char|datetime

## HorizonPrice - Price at investment horizon date

calculated based on ReinvestRate (default) | numeric
Price at investment horizon date, specified as the comma-separated pair consisting of 'HorizonPrice ' and a scalar or a NINST-by-1 vector.

If HorizonPrice is unspecified, the price at the HorizonDate is calculated based on the ReinvestRate. If the HorizonDate equals the Maturity date, the HorizonPrice is ignored and the total return to maturity is calculated based on the Face value.

## Data Types: double

## Period - Number of coupon payments per year

2 (default) | numeric with values $0,1,2,3,4,6$ or 12
Number of coupon payments per year, specified as the comma-separated pair consisting of 'Period ' and a scalar or a NINST-by-1 vector using the values: $0,1,2,3,4,6$, or 12.

## Data Types: double

## Basis - Day-count basis

0 (actual/actual) (default) | integers of the set [0...13] | vector of integers of the set [0...13]

Day-count basis, specified as the comma-separated pair consisting of 'Basis' and a scalar or a NINST-by-1 vector. Values are:

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- $2=$ actual/360
- $3=$ actual $/ 365$
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- $7=$ actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- $9=$ actual/360 (ICMA)
- $10=$ actual/365 (ICMA)
- $11=30 / 360 \mathrm{E}$ (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-0
Data Types: double

## EndMonthRule - End-of-month rule flag <br> 1 (in effect) (default) | nonnegative integer 0 or 1

End-of-month rule flag, specified as the comma-separated pair consisting of 'EndMonthRule' and a scalar or a NINST-by-1 vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days.

- $0=$ Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.


## Data Types: logical

## IssueDate - Bond issue date <br> serial date number | date character vector | datetime

Bond Issue date, specified as the comma-separated pair consisting of 'IssueDate' and a scalar or a NINST-by-1 vector using serial date numbers, date character vectors, or datetime arrays.

If you do not specify an IssueDate, the cash flow payment dates are determined from other inputs.

Data Types: double | char | datetime

## FirstCouponDate - Irregular or normal first coupon date <br> serial date number | date character vector | datetime

Irregular or normal first coupon date, specified as the comma-separated pair consisting of 'FirstCouponDate' and a scalar or a NINST-by-1 vector using serial date numbers, date character vectors, or datetime arrays.

If you do not specify a FirstCouponDate, the cash flow payment dates are determined from other inputs.
Data Types: double | char \| datetime
LastCouponDate - Irregular or normal last coupon date
serial date number | date character vector | datetime

Irregular or normal last coupon date, specified as the comma-separated pair consisting of 'LastCouponDate' and a scalar or a NINST-by-1 vector using serial date numbers, date character vectors, or datetime arrays.

If you do not specify a LastCouponDate, the cash flow payment dates are determined from other inputs.

Data Types: double|char|datetime

## StartDate - Forward starting date of payments <br> serial date number | date character vector | datetime

Forward starting date of payments, specified as the comma-separated pair consisting of 'StartDate ' and a scalar or a NINST-by-1 vector using serial date numbers, date character vectors, or datetime arrays.

If you do not specify a StartDate, the effective start date is the Settle date.
Data Types: double|char|datetime

## Face - Face value of bond

100 (default) | numeric
Face value of the bond, specified as the comma-separated pair consisting of 'Face' and a scalar or a NINST-by-1 vector.

Data Types: double

## CompoundingFrequency - Compounding frequency for yield calculation

 integer with value of $1,2,3,4,6$, or 12Compounding frequency for yield calculation, specified as the comma-separated pair consisting of 'CompoundingFrequency' and a scalar or a NINST-by-1 vector.

- 1 - Annual compounding
- 2-Semiannual compounding
- 3-Compounding three times per year
- 4-Quarterly compounding
- 6 - Bimonthly compounding
- 12 - Monthly compounding

Note By default, SIA bases (0-7) and BUS/252 use a semiannual compounding convention and ICMA bases (8-12) use an annual compounding convention.

## Data Types: double

## DiscountBasis - Basis used to compute the discount factors for computing the yield <br> integers of the set [0...13] | vector of integers of the set [0...13]

Basis used to compute the discount factors for computing the yield, specified as the comma-separated pair consisting of 'DiscountBasis' and a scalar or a NINST-by-1 vector. Values are:

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- 2 = actual/360
- 3 = actual/365
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- $9=$ actual/360 (ICMA)
- $10=$ actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- $12=$ actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-0

Note The default behavior is for SIA bases (0-7) to use the actual/actual day count to compute discount factors, and for ICMA day counts ( $8-12$ ) and BUS/252 to use the specified DiscountBasis.

Data Types: double

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## Output Arguments

## BondEquiv - Total return in bond equivalent basis

numeric
Total return in bond equivalent basis, returned as a NUMBONDS-by-1 vector.

## EffectiveRate - Total return in effective rate basis numeric

Total return in effective rate basis, returned as a NUMBONDS-by-1 vector.

## References

[1] Fabozzi, Frank J., Mann, Steven V. Introduction to Fixed Income Analytics: Relative Value Analysis, Risk Measures and Valuation. John Wiley and Sons, New York, 2010.

## See Also <br> bndprice | bndyield | cfamounts | datetime

## Topics

"Bond Portfolio for Hedging Duration and Convexity" on page 10-6
"Pricing Functions" on page 2-32
"Sensitivity of Bond Prices to Interest Rates" on page 10-2
"Yield Conventions" on page 2-31

## Introduced in R2012b

## bndyield

Yield to maturity for fixed-income security

Note In R2017b, the specification of optional input arguments has changed. While the previous ordered inputs syntax is still supported, it may no longer be supported in a future release. Use the optional name-value pair inputs: Period, Basis, EndMonthRule, IssueDate,FirstCouponDate, LastCouponDate, StartDate,Face, CompoundingFrequency, DiscountBasis, and LastCouponInterest.

## Syntax

Yield = bndyield(Price,CouponRate,Settle, Maturity)
Yield = bndyield( $\qquad$ , Name, Value)

## Description

Yield = bndyield(Price,CouponRate,Settle, Maturity) given NUMBONDS bonds with SIA date parameters and clean prices (excludes accrued interest), returns the bond equivalent yields to maturity.

Yield = bndyield( $\qquad$ , Name, Value) adds optional name-value arguments.

## Examples

## Compute Yield to Maturity for a Treasury Bond

This example shows how to compute the yield of a Treasury bond at three different price values.

```
Price = [95; 100; 105];
```

CouponRate $=0.05$;
Settle = '20-Jan-1997';
Maturity = '15-Jun-2002';

Period = 2;
Basis = 0;
Yield = bndyield(Price, CouponRate, Settle,...
Maturity, Period, Basis)
Yield $=3 \times 1$
0.0610
0.0500
0.0396

## Compute Yield to Maturity for a Treasury Bond Using datetime Inputs

This example shows how to use datetime inputs to compute the yield of a Treasury bond at three different price values.

```
Price = [95; 100; 105];
CouponRate = 0.05;
Settle = datetime('20-Jan-1997','Locale','en_US');
Maturity = datetime('15-Jun-2002','Locale','èn_US');
Period = 2;
Basis = 0;
Yield = bndyield(Price, CouponRate, Settle,...
Maturity, Period, Basis)
Yield = 3\times1
```

    0.0610
    0.0500
    0.0396
    
## Compute the Yield of a Treasury Bond Using the Same Basis for Discounting and Generating the Cash Flows

Compute the yield of a Treasury bond.

```
Price = [95; 100; 105];
CouponRate = 0.0345;
Settle = '15-May-2016';
Maturity = '02-Feb-2026';
Period = 2;
Basis = 1;
format long
Yield = bndyield(Price,CouponRate,Settle,Maturity,Period,Basis)
Yield = 3×1
    0.040764403932618
    0.034482347625316
    0.028554719853118
```

Using the same data, compute the yield of a Treasury bond using the same basis for discounting and generating the cash flows.

DiscountBasis = 1;
Yield = bndyield(Price,CouponRate,Settle,Maturity,'Period',Period,'Basis',Basis, ... 'DiscountBasis',DiscountBasis)

Yield = 3×1
0.040780176658036
0.034495592361619
0.028565614029497

## Input Arguments

## Price - Clean price of the bond

numeric

Clean price of the bond (current price without accrued interest), specified as a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector.

Data Types: double

## CouponRate - Annual percentage rate used to determine coupons payable on a bond <br> decimal

Annual percentage rate used to determine the coupons payable on a bond, specified as decimal using a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector.

## Data Types: double

## Settle - Settlement date of bond <br> serial date number | date character vector | datetime

Settlement date of the bond, specified as a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector using serial date numbers, date character vectors, or datetime arrays. The Settle date must be before the Maturity date.
Data Types: double | char \| datetime

## Maturity - Maturity date of bond

serial date number | date character vector | datetime
Maturity date of the bond, specified as a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector using serial date numbers, date character vectors, or datetime arrays.
Data Types: double | char | datetime

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Example: Yield = bndyield(Price,CouponRate,Settle,Maturity,'Period', 4,'Basis',9)

## Period - Number of coupon payments per year

2 (default) | numeric with values $0,1,2,3,4,6$ or 12
Number of coupon payments per year, specified as the comma-separated pair consisting of 'Period ' and a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector using the values: $0,1,2,3,4,6$, or 12 .

Data Types: double

## Basis - Day-count basis of instrument

## 0 (default) | numeric values: $0,1,2,3,4,6,7,8,9,10,11,12,13$

Day-count of the instrument, specified as the comma-separated pair consisting of 'Basis' and a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector using a supported value:

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- 2 = actual/360
- 3 = actual/365
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- $10=$ actual/365 (ICMA)
- $11=30 / 360 E$ (ICMA)
- $12=$ actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-0

## Data Types: double

## EndMonthRule - End-of-month rule flag

## 1 (in effect) (default) | nonnegative integer 0 or 1

End-of-month rule flag, specified as the comma-separated pair consisting of 'EndMonthRule' and a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days.

- $0=$ Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.


## Data Types: logical

## IssueDate - Bond issue date <br> serial date number | date character vector | datetime

Bond Issue date, specified as the comma-separated pair consisting of 'IssueDate' and a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector using serial date numbers, date character vectors, or datetime arrays.

If you do not specify an IssueDate, the cash flow payment dates are determined from other inputs.
Data Types: double | char | datetime

## FirstCouponDate - Irregular or normal first coupon date serial date number | date character vector | datetime

Irregular or normal first coupon date, specified as the comma-separated pair consisting of 'FirstCouponDate' and a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector using serial date numbers, date character vectors, or datetime arrays.

If you do not specify a FirstCouponDate, the cash flow payment dates are determined from other inputs.

Data Types: double | char |datetime

## LastCouponDate - Irregular or normal last coupon date <br> serial date number | date character vector | datetime

Irregular or normal last coupon date, specified as the comma-separated pair consisting of 'LastCouponDate' and a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector using serial date numbers, date character vectors, or datetime arrays.

If you do not specify a LastCouponDate, the cash flow payment dates are determined from other inputs.

Data Types: double | char | datetime

## StartDate - Forward starting date of payments <br> serial date number | date character vector | datetime

Forward starting date of payments, specified as the comma-separated pair consisting of 'StartDate ' and a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector using serial date numbers, date character vectors, or datetime arrays.

If you do not specify a StartDate, the effective start date is the Settle date.

## Data Types: double | char|datetime

## Face - Face value of bond

100 (default) | numeric
Face value of the bond, specified as the comma-separated pair consisting of 'Face' and a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector.

## Data Types: double

## CompoundingFrequency - Compounding frequency for yield calculation

SIA uses 2, ICMA uses 1 (default) | integer with value of 1, 2, 3, 4, 6, or 12

Compounding frequency for yield calculation, specified as the comma-separated pair consisting of 'CompoundingFrequency' and a scalar or a NUMBONDS-by-1 or 1-byNUMBONDS vector. Values are:

- 1 - Annual compounding
- 2 - Semiannual compounding
- 3-Compounding three times per year
- 4-Quarterly compounding
- 6 - Bimonthly compounding
- 12 - Monthly compounding

Note By default, SIA bases (0-7) and BUS/252 use a semiannual compounding convention and ICMA bases (8-12) use an annual compounding convention.

## Data Types: double

## DiscountBasis - Basis used to compute the discount factors for computing the yield

```
SIA uses 0 (default)| integers of the set [0...13] | vector of integers of the set
[0...13]
```

Basis used to compute the discount factors for computing the yield, specified as the comma-separated pair consisting of 'DiscountBasis' and a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector. Values are:

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- 2 = actual/360
- 3 = actual/365
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- $7=$ actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- $9=$ actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- $11=30 / 360 E$ (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-0

Note If a SIA day-count basis is defined in the Basis input argument and there is no value assigned for DiscountBasis, the default behavior is for SIA bases to use the actual/actual day count to compute discount factors.

If an ICMA day-count basis or BUS/252 is defined in the Basis input argument and there is no value assigned for DiscountBasis, the specified bases from the Basis input argument are used.

## Data Types: double

LastCouponInterest - Compounding convention for computing yield of a bond
in last coupon period
compound (default) | values are simple or compound
Compounding convention for computing the yield of a bond in the last coupon period, specified as the comma-separated pair consisting of 'LastCouponInterest' and a scalar or a NUMBONDS-by-1 or 1-by-NUMBONDS vector. This is based on only the last coupon and the face value to be repaid. Acceptable values are:

- simple
- compound


## Data Types: char|cell

## Output Arguments

## Yield - Yield to maturity with semiannual compounding numeric

Yield to maturity with semiannual compounding, returned as a NUMBONDS-by-1 vector.

## Definitions

## Price and Yield Conventions

The Price and Yield are related to different formulae for SIA and ICMA conventions.
For SIA conventions, Price and Yield are related by the formula:

```
Price + Accrued Interest = sum(Cash_Flow*(1+Yield/2)^(-Time))
```

where the sum is over the bond's cash flows and corresponding times in units of semiannual coupon periods.

For ICMA conventions, the Price and Yield are related by the formula:

```
Price + Accrued Interest = sum(Cash_Flow*(1+Yield)^(-Time))
```


## Algorithms

For SIA conventions, the following formula defines bond price and yield:

$$
P V=\frac{C F}{\left(1+\frac{z}{f}\right)^{T F}},
$$

where:

| $P V=$ | Present value of a cash flow. |
| :--- | :--- |
| $C F=$ | The cash flow amount. |
| $z=$ | The risk-adjusted annualized rate or yield corresponding to a given <br> cash flow. The yield is quoted on a semiannual basis. |
| $f=$ | The frequency of quotes for the yield. |
| $T F=$ | Time factor for a given cash flow. Time is measured in semiannual <br> periods from the settlement date to the cash flow date. In computing <br> time factors, use SIA actual/actual day count conventions for all <br> time factor calculations. |

For ICMA conventions, the frequency of annual coupon payments determines bond price and yield.

## References

[1] Krgin, D. Handbook of Global Fixed Income Calculations. Wiley, 2002.
[2] Mayle, J. "Standard Securities Calculations Methods: Fixed Income Securities Formulas for Analytic Measures." SIA, Vol 2, Jan 1994.
[3] Stigum, M., Robinson, F. Money Market and Bond Calculation. McGraw-Hill, 1996.

## See Also

bndprice | cfamounts | datetime

## Topics

"Bond Portfolio for Hedging Duration and Convexity" on page 10-6
"Pricing Functions" on page 2-32
"Sensitivity of Bond Prices to Interest Rates" on page 10-2
"Yield Conventions" on page 2-31

Introduced before R2006a

## bolling

Bollinger band chart

Note bolling will be removed in a future release. Use bollinger instead.

## Syntax

```
bolling(Asset,Samples,Alpha,Width)
```

[Movavgv,UpperBand,LowerBand] = bolling(Asset, Samples,Alpha, Width)

## Arguments

| Asset | Vector of asset data. |
| :--- | :--- |
| Samples | Number of samples to use in computing the moving average. |
| Alpha | (Optional) Exponent used to compute the element weights of the <br> moving average. Default = 0 (simple moving average). |
| Width | (Optional) Number of standard deviations to include in the <br> envelope. A multiplicative factor specifying how tight the bands <br> should be around the simple moving average. Default $=2$. |

## Description

bolling(Asset,Samples,Alpha,Width) plots Bollinger bands for given Asset data. This form of the function does not return any data.
[Movavgv,UpperBand,LowerBand] = bolling(Asset,Samples,Alpha,Width) returns Movavgv with the moving average of the Asset data, UpperBand with the upper band data, and LowerBand with the lower band data. This form of the function does not plot any data.

Note The standard deviations are normalized by $\mathrm{N}-1$, where $\mathrm{N}=$ the sequence length.

## Examples

If Asset is a column vector of closing stock prices

```
bolling(Asset, 20, 1)
```

plots linear 20-day moving average Bollinger bands based on the stock prices.
[Movavgv, UpperBand, LowerBand] = bolling(Asset, 20, 1)
returns Movavgv, UpperBand, and LowerBand as vectors containing the moving average, upper band, and lower band data, without plotting the data.

See Also<br>candle|dateaxis | highlow|movavg|movavg<br>Introduced before R2006a

## bollinger

Time series Bollinger band

Note bollinger has been partially removed and will no longer accept a fints object (tsobj) argument. Use a matrix, timetable, or table instead for financial time series.

Use fts2timetable to convert a fints object to a timetable object.

## Syntax

[middle,upper,lower] = bollinger(Data)
[middle, upper, lower] = bollinger( __ , Name, Value)

## Description

[middle, upper,lower] = bollinger(Data) calculates the middle, upper, and lower bands that make up the Bollinger bands from a series of data.
[middle,upper,lower] = bollinger( $\qquad$ ,Name, Value) adds optional name-value pair arguments.

## Examples

## Calculate the Bollinger Bands for a Data Series for a Stock

Load the file SimulatedStock.mat, which provides a timetable (TMW) for financial data for TMW stock.

```
load SimulatedStock.mat
[middle,upper,lower]= bollinger(TMW);
CloseBolling = [middle.Close, upper.Close,...
lower.Close];
plot(middle.Time,CloseBolling)
title('Bollinger Bands for TMW Closing Prices')
```



## Input Arguments

## Data - Data for market prices

matrix | table | timetable
Data for market prices, specified as a matrix, table, or timetable. For matrix input, Data must be column-oriented.

Data Types: double | table | timetable

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1,Value1, ..., NameN, ValueN.

Example: [middle,upper,lower] = bollinger(TMW_CLOSE,'WindowSize', 10,'Type',1)

## WindowSize - Number of observations of input series to include in moving average in periods

10 (default) | positive integer
Number of observations of the input series to include in the moving average in periods, specified as the comma-separated pair consisting of 'WindowSize' and a scalar positive integer.

## Data Types: double

## Type - Type of moving average to compute <br> 0 (simple) (default) | integer with value 0 or 1

Type of moving average to compute, specified as the comma-separated pair consisting of 'Type ' and a scalar integer with a value of 0 (simple) or 1 (linear).

## Data Types: double

## NumStd - Number of standard deviations for the upper and lower bounds 2 (default) | positive integer

Number of standard deviations for the upper and lower bounds, specified as the commaseparated pair consisting of 'NumStd ' and a scalar positive integer.
Data Types: double

## Output Arguments

## middle - Moving average series representing the middle band matrix | table | timetable

Moving average series representing the middle band, returned with the same number of rows $(\mathrm{M})$ and the same type (matrix, table, or timetable) as the input Data.

## upper - Moving average series representing the upper band matrix | table | timetable

Moving average series representing the upper band, returned with the same number of rows $(\mathrm{M})$ and the same type (matrix, table, or timetable) as the input Data.

## lower - Moving average series representing the lower band matrix | table | timetable

Moving average series representing the lower band, returned with the same number of rows $(\mathrm{M})$ and the same type (matrix, table, or timetable) as the input Data.

## References

[1] Achelis, S. B. Technical Analysis from A to Z. Second Edition. McGraw-Hill, 1995, pp. 72-74.

## See Also

movavg| table| timetable

## Topics

"Using Timetables in Finance"
"Convert Financial Time Series Objects fints to Timetables" on page 12-2

Introduced before R2006a

## boxcox

Box-Cox transformation

Note boxcox has been partially removed and will no longer accept a fints object (tsobj).

Replace all instances of a fints object (tsobj) for input with an array by using fts2timetable to convert a fints object to a timetable object and then use timetable2table and table2array.

## Syntax

[transdat,lambda] = boxcox(data)
[transfts,lambda] = boxcox(tsobj)
transdat $=$ boxcox(lambda,data)
transfts $=$ boxcox(lambda,tsobj)

## Arguments

| data | Data vector. Must be positive and specified as a column data vector. |
| :--- | :--- |
| tsobj | Financial time series object. |

## Description

boxcox transforms nonnormally distributed data to a set of data that has approximately normal distribution. The Box-Cox transformation is a family of power transformations.

If $\lambda$ is not $=0$, then

$$
\operatorname{data}(\lambda)=\frac{d a t a^{\lambda}-1}{\lambda}
$$

If $\lambda$ is $=0$, then

$$
\operatorname{data}(\lambda)=\log (d a t a)
$$

The logarithm is the natural logarithm (log base e). The algorithm calls for finding the $\lambda$ value that maximizes the Log-Likelihood Function (LLF). The search is conducted using fminsearch.
[transdat,lambda] = boxcox(data) transforms the data vector data using the BoxCox transformation method into transdat. It also estimates the transformation parameter $\lambda$.
[transfts,lambda] = boxcox(tsojb) transforms the financial time series object tsobj using the Box-Cox transformation method into transfts. It also estimates the transformation parameter $\lambda$.

If the input data is a vector, lambda is a scalar. If the input is a financial time series object, lambda is a structure with fields similar to the components of the object; for example, if the object contains series names Open and Close, lambda has fields lambda. Open and lambda. Close.
transdat $=$ boxcox(lambda, data) and transfts $=$ boxcox(lambda, tsobj) transform the data using a certain specified $\lambda$ for the Box-Cox transformation. This syntax does not find the optimum $\lambda$ that maximizes the LLF.

## Examples

## Transform a Data Series Contained in a Financial Times Series Object

Use boxcox to transform the data series contained in a financial time series object into another set of data series with relatively normal distributions.

Create a financial time series object from the supplied whirlpool. dat data file.

```
whrl = ascii2fts('whirlpool.dat', 1, 2, []);
```

Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in
Fill any missing values denoted with NaN's in whrl with values calculated using the linear method.
f_whrl = fillts(whrl);
Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in
Transform the nonnormally distributed filled data series f_whrl into a normally distributed one using Box-Cox transformation.
bc_whrl = boxcox(f_whrl);
Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in
Compare the result of the Close data series with a normal (Gaussian) probability distribution function and the nonnormally distributed f_whrl.

```
subplot(2, 1, 1);
hist(f_whrl.Close);
```

Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in

```
grid; title('Nonnormally Distributed Data');
```

subplot(2, 1, 2);
hist(bc_whrl.Close);

Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in
grid; title('Box-Cox Transformed Data');


The bar chart on the top represents the probability distribution function of the filled data series, f_whrl, which is the original data series whrl with the missing values interpolated using the linear method. The distribution is skewed toward the left (not normally distributed). The bar chart on the bottom is less skewed to the left. If you plot a Gaussian probability distribution function (PDF) with similar mean and standard deviation, the distribution of the transformed data is very close to normal (Gaussian). When you examine the contents of the resulting object bc_whrl, you find an identical object to the original object whrl but the contents are the transformed data series.

## See Also

fminsearch

## Topics

"Data Transformation and Frequency Conversion" on page 13-13

Introduced before R2006a

## busdate

Next or previous business day

## Syntax

Busday = busdate(Date)
Busday = busdate( ,DirFlag,Holiday, Weekend)

## Description

Busday $=$ busdate (Date) returns the scalar, vector, or matrix of the next or previous business days, depending on the definition for Holiday.

Busday = busdate( $\qquad$ ,DirFlag, Holiday, Weekend) returns the scalar, vector, or matrix of the next or previous business days, depending on the optional input arguments, including Holiday.

If both Date and Holiday are either serial date numbers or date character vectors, Busday is returned as a serial date number.

However, if either Date or Holiday are datetime arrays, Busday is returned as a datetime array.

Use the function datestr to convert serial date numbers to formatted date character vectors.

## Examples

Determine Business Days
Determine the next business day when Date is a character vector.
Busday = busdate('3-Jul-2001', 1)

Busday = 731037

```
datestr(Busday)
ans =
'05-Jul-2001'
```

Indicate that Saturday is a business day by appropriately setting the Weekend argument. July 4, 2003 falls on a Friday. Use busdate to verify that Saturday, July 5, is actually a business day.

```
Weekend = [1 0 0 0 0 0 0];
Date = datestr(busdate('3-Jul-2003', 1, [], Weekend))
Date =
'05-Jul-2003'
```

If either Date or Holiday are datetime arrays, Busday is returned as a datetime array.

```
Busday = busdate(datetime('3-Jul-2001','Locale','en_US'), 1)
Busday = datetime
    05-Jul-2001
```


## Input Arguments

## Date - Reference business date <br> serial date number | date character vector | datetime object

Reference business date, specified as a scalar, vector, or matrix using serial date numbers, date character vectors, or datetime arrays.

Data Types: double | char | datetime

## DirFlag - Business day convention

follow (default) | date character vector with values of follow, modifiedfollow, previous, or modifiedprevious | cell array of date character vectors with values of follow, modifiedfollow, previous, or modifiedprevious

Business day convention, specified date character vector or cell array of date character vectors with values of follow, modifiedfollow, previous, or modifiedprevious.

Also, DirFlag can be a scalar, vector, or matrix of search directions, where Next is DIREC = 1 (default) or Previous is DIREC $=-1$.

Data Types: double | char | datetime

## Holiday - Holidays and nontrading-day dates

non-trading day vector is determined by the routine holidays (default) | serial date number | date character vector | datetime object

Holidays and nontrading-day dates, specified as vector.
All dates in Holiday must be the same format: either serial date numbers, or date character vectors, or datetime arrays. (Using serial date numbers improves performance.)
Data Types: double | char | datetime

## Weekend - Weekend days

[1000001] (Saturday and Sunday form the weekend) (default) | vector of length 7 , containing 0 and 1 , where 1 indicates weekend days

Weekend days, specified as a vector of length 7, containing 0 and 1 , where 1 indicates weekend days and the first element of this vector corresponds to Sunday.
Data Types: double

## Output Arguments

## Busday - Next or previous business day

scalar | vector | matrix
Next or previous business day, returned as a scalar, vector, or matrix depending on the definition for Holiday. If Date is a datetime array, Busday returns a datetime array. Otherwise, Busday returns a serial date numbers.

## See Also

datetime|holidays

## Topics

"Handle and Convert Dates" on page 2-2
"Trading Calendars User Interface" on page 16-2

## Introduced before R2006a

## busdays

Business days for given period

## Syntax

```
bdates = busdays(sdate,edate)
bdates = busdays(
```

$\qquad$

``` , bdmode, holvec)
```


## Description

bdates = busdays(sdate, edate) generates a vector of business days between the last business date of the period that contains the start date (sdate), and the last business date of period that contains the end date (edate).
bdates = busdays (__ , bdmode, holvec) generates a vector of business days between the last business date of the period that contains the start date (sdate), and the last business date of period that contains the end date (edate) using optional input arguments. If holvec is not supplied, the dates are generated based on United States holidays. If you do not supply bdmode, bdates generates a daily vector.

## Examples

Determine Business Days for a Given Period
Determine the business days for a weekly period.

```
    bdates = datestr(busdays('1/2/01','1/9/01','weekly'))
bdates = 2x11 char array
    '05-Jan-2001'
    '12-Jan-2001'
```

The end of the week is considered to be a Friday. Between $1 / 2 / 01$ (Monday) and 1/9/01 (Tuesday), there is only one end-of-week day, 1/5/01 (Friday). Because 1/9/01 is part of the following week, the following Friday (1/12/01) is also reported.

Determine the business days for a weekly period using a datetime input for sdate.

```
bdates = busdays(datetime('2-Jan-2001','Locale','en_US'),'9-Jan-2001','weekly')
bdates = 2x1 datetime array
    05-Jan-2001
    12-Jan-2001
```

Determine the business days for a monthly period.

```
vec = datestr(busdays('1/8/16','3/1/16','monthly'))
vec = 3x11 char array
    '29-Jan-2016'
    '29-Feb-2016'
    '31-Mar-2016'
```

The start date $(1 / 8 / 16)$ is in the month of January, 2016. The last business day for the month of January is $1 / 29 / 16$ (Friday). The end date $(3 / 1 / 16)$ is in the month of March, 2016. The last business day for the month of March is $3 / 31 / 16$ (Thursday). The month of February, 2016 lies between the start date and the end date. The last business day for the month of February is 2/29/16 (Monday).

## Input Arguments

## sdate - Start date

serial date number | date character vector | datetime object
Start date, specified as a serial date numbers, date character vectors, or datetime arrays.
Data Types: double | char | datetime

## edate - End date

serial date number | date character vector | datetime object
End date, specified as serial date numbers, date character vectors, or datetime arrays.

## Data Types: double | char | datetime

## bdmode - Frequency of business days

DAILY (1) (default) | nonnegative numeric with values 1 through 5 | date character vector with values DAILY, WEEKLY, MONTHLY, QUARTERLY, SEMIANNUAL or ANNUAL

Frequency of business days, specified as a nonnegative numeric with values 1 through 5 or date character vector with values of DAILY, WEEKLY, MONTHLY, QUARTERLY, SEMIANNUAL, or ANNUAL

Valid periodicities include:

- DAILY, Daily, daily, D, d, 1 (default)
- WEEKLY, Weekly, weekly, W, w, 2
- MONTHLY, Monthly, monthly, M, m, 3
- QUARTERLY, Quarterly, quarterly, Q, q, 4
- SEMIANNUAL, Semiannual, semiannual, S, s, 5
- ANNUAL, Annual, annual, A, a, 6

Character vectors must be enclosed in single quotation marks.
For example, if bdmode is set to monthly, busdays returns end-of-month business dates for all full or partial months between the start date and end date inclusive.
Data Types: double \| char

## holvec - Holiday dates

if holvec is [ ] holiday dates used are based on United States holidays (default) | serial date number | date character vector | datetime object

Holiday dates, specified as a vector in date character vector, serial date, or datetime array format. If you specify holvec, you must also supply the frequency bdmode. Using a holvec value of NaN uses a holiday list that has no dates.

Data Types: double|char|datetime

## Output Arguments

## bdates - Business days

column vector

Business days, returned as a column vector of business dates, in serial date format (default) or datetime format (if sdate, edate, or holvec are in datetime format). Business dates can exist before and/or after the specified sdate and edate.

## See Also

datetime| holidays

## Topics

"Handle and Convert Dates" on page 2-2
"Trading Calendars User Interface" on page 16-2

## Introduced before R2006a

## candle

Candlestick chart

Note candle is updated to accept data input as a matrix, timetable, or table.

## Syntax

```
candle(Data)
candle(Data,Color)
h = candle(ax
```

$\qquad$

## Description

candle(Data) plots a candlestick chart from a series of opening, high, low, and closing prices of a security. If the closing price is greater than the opening price, the body (the region between the open and close price) is unfilled; otherwise the body is filled.
candle(Data, Color) adds an optional argument for Color.
$\mathrm{h}=$ candle(ax__ $)$ adds an optional argument for ax .

## Examples

## Generate a Candlestick Chart for a Data Series for a Stock

Load the file SimulatedStock.mat, which provides a timetable (TMW) for financial data for TMW stock. This is a candlestick chart with blue candles, for the most recent 21 days in SimulatedStock.mat.
load SimulatedStock.mat;
candle(TMW(end-20:end,:),'b');
title('Candlestick chart for TMW')


## Input Arguments

## Data - Data for opening, high, low, and closing prices

## matrix | table | timetable

Data for opening, high, low, and closing prices, specified as a matrix, table, or timetable. For matrix input, Data is an M-by-4 matrix of opening, high, low, and closing prices stored in the corresponding columns. Timetables and tables with $M$ rows must contain variables named 'Open', 'High', 'Low', and 'Close' (case insensitive).

Data Types: double| table| timetable

## Color - (Optional) Three element color vector <br> background color of figure window (default) | color vector [R G B] | string

Three element color vector, specified as a $\left[\begin{array}{ll}R & G\end{array}\right]$ ] color vector or a string specifying the color name. The default color differs depending on the background color of the figure window. For more information, see colorspec.

## Data Types: double | string

ax - Valid axis object
current axes (ax = gca) (default) | axes object
(Optional) Valid axis object, specified as an axes object. The candle plot is created in the axes specified by ax instead of in the current axes ( $a x=g c a$ ). The option ax can precede any of the input argument combinations.

Data Types: object

## Output Arguments

## h - Graphic handle of the figure <br> handle object

Graphic handle of the figure, returned as a handle object.

## See Also

highlow| kagi | linebreak|movavg|pointfig|priceandvol|renko|table| timetable|volarea

## Topics

"Using Timetables in Finance"
"Convert Financial Time Series Objects fints to Timetables" on page 12-2

Introduced before R2006a

## candle (fts)

Time series candle plot

Note candle (fts) will be removed in a future release. Use candle instead.
Use fts2timetable to convert a fints object to a timetable object.

## Syntax

```
candle(tsobj)
candle(tsobj,color)
candle(tsobj,color,dateform)
candle(tsobj,color,dateform,'ParameterName',ParameterValue, ...)
hcdl = candle(tsobj,color,dateform,'ParameterName',ParameterValue, ...)
```


## Arguments

| tsobj | Financial time series object |
| :--- | :--- |
| color | (Optional) A three-element row vector representing RGB or a color <br> identifier. (See plot.) |
| dateform | (Optional) Date character vector format used as the x-axis tick <br> labels. (See datetick.) You can specify a dateform only when <br> tsobj does not contain time-of-day data. If tsobj contains time-of- <br> day data, dateform is restricted to 'dd-mmm-yyyy HH: MM' . |

## Description

candle(tsobj) generates a candle plot of the data in the financial time series object tsobj. tsobj must contain at least four data series representing the high, low, open, and
closing prices. These series must have the names High, Low, Open, and Close (caseinsensitive).
candle(tsobj, color) additionally specifies the color of the candle box.
candle(tsobj, color, dateform) additionally specifies the date character vector format used as the $x$-axis tick labels. See datestr for a list of date character vector formats.
candle(tsobj, color,dateform,'ParameterName',ParameterValue, ...) indicates the actual names of the required data series if the data series do not have the default names. 'ParameterName' can be

- HighName: high prices series name
- LowName: low prices series name
- OpenName: open prices series name
- CloseName: closing prices series name
hcdl = candle(tsobj, color, dateform, 'ParameterName',
ParameterValue, ...) returns the handle to the patch objects and the line object that make up the candle plot. hdcl is a three-element column vector representing the handles to the two patches and one line that forms the candle plot.


## Examples

## Create a Candle Plot for a Financial Time Series Object

This example shows how to create a candle plot for Disney stock for the dates March 31, 1998 through April 30, 1998.
load disney.mat
candle(dis('3/31/98: :4/30/98'))
Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in
Warning: Low prices must be less than or equal to the corresponding opening prices.
title('Disney 3/31/98 to 4/30/98')


## See Also <br> candle|chartfts|highlow|plot

Introduced before R2006a

## cdai

Accrued interest on certificate of deposit

## Syntax

AccrInt = cdai(CouponRate,Settle, Maturity,IssueDate)
AccrInt = cdai( $\qquad$ ,Basis)

## Description

AccrInt = cdai(CouponRate,Settle,Maturity,IssueDate) computes the accrued interest on a certificate of deposit.
cdai assumes that the certificates of deposit pay interest at maturity. Because of the simple interest treatment of these securities, this function is best used for short-term maturities (less than 1 year). The default simple interest calculation uses the Basis for the actual/360 convention (2).

AccrInt = cdai( $\qquad$ ,Basis) adds an optional argument for Basis.

## Examples

## Find the Accrued Interest on a Certificate of Deposit

This example shows how to compute the accrued interest due, given a certificate of deposit with the following characteristics.

```
CouponRate = 0.05;
Settle = '02-Jan-02';
Maturity = '31-Mar-02';
IssueDate = '1-Oct-01';
AccrInt = cdai(CouponRate, Settle, Maturity, IssueDate)
AccrInt = 1.2917
```


## Find the Accrued Interest on a Certificate of Deposit Using datetime Inputs

This example shows how to use datetime inputs to compute the accrued interest due, given a certificate of deposit with the following characteristics.

```
CouponRate = 0.05;
Settle = datetime('02-Jan-02','Locale','en US');
Maturity = datetime('31-Mar-02','Locale','è_US');
IssueDate = datetime('1-0ct-01','Locale','en US');
AccrInt = cdai(CouponRate, Settle, Maturity, IssueDate)
AccrInt = 1.2917
```


## Input Arguments

## CouponRate - Annual interest rate <br> decimal

Annual interest rate, specified as decimal using a scalar or a NCDS-by-1 or 1-by-NCDS vector.

## Data Types: double

## Settle - Settlement date for certificate of deposit <br> serial date number | date character vector | datetime

Settlement date for the certificate of deposit, specified as a scalar or a NCDS-by-1 or 1-byNCDS vector using serial date numbers, date character vectors, or datetime arrays. The Settle date must be before the Maturity date.

Data Types: double|char|datetime

## Maturity - Maturity date for certificate of deposit <br> serial date number | date character vector | datetime

Maturity date for the certificate of deposit, specified as a scalar or a NCDS-by-1 or 1-byNCDS vector using serial date numbers, date character vectors, or datetime arrays.

Data Types: double| char|datetime

## IssueDate - Issue date for certificate of deposit <br> serial date number | date character vector | datetime

Issue date for the certificate of deposit, specified as a scalar or a NCDS-by-1 or 1-by-NCDS vector using serial date numbers, date character vectors, or datetime arrays.

## Data Types: double | char | datetime

## Basis - Day-count basis for certificate of deposit

2 (actual/360) (default) | integers of the set [0...13] | vector of integers of the set [0...13]
(Optional) Day-count basis for the certificate of deposit, specified as a scalar or a NINST-by-1 vector. Values are:

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- 2 = actual/360
- $3=$ actual $/ 365$
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- $9=$ actual/360 (ICMA)
- $10=$ actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-0
Data Types: double

## Output Arguments

## AccrInt - Accrued interest per $\$ 100$ of face value <br> numeric

Accrued interest per $\$ 100$ of face value, returned as a NCDS-by-1 or 1-by-NCDS vector.

See Also<br>accrfrac| bndyield|cdprice|cdyield|datetime|stepcpnyield|tbillyield | zeroyield

## Topics

"Coupon Date Calculations" on page 2-31
"Yield Conventions" on page 2-31

Introduced before R2006a

## cdprice

Price of certificate of deposit

## Syntax

```
[Price,AccrInt] = cdprice(Yield,CouponRate,Settle,Maturity,
IssueDate)
[PriceAccrInt] = cdprice(
```

$\qquad$

``` ,Basis)
```


## Description

[Price,AccrInt] = cdprice(Yield,CouponRate,Settle, Maturity, IssueDate) computes the price of a certificate of deposit given its yield.
cdprice assumes that the certificates of deposit pay interest at maturity. Because of the simple interest treatment of these securities, this function is best used for short-term maturities (less than 1 year). The default simple interest calculation uses the Basis for the actual/360 convention (2).

```
[PriceAccrInt] = cdprice(
```

$\qquad$

``` ,Basis) adds an optional argument for Basis.
```


## Examples

## Compute the Price and Accrued Interest for a Certificate of Deposit

This example shows how to compute the price and the accrued interest due on the settlement date, given a certificate of deposit with the following characteristics.

```
Yield = 0.0525;
CouponRate = 0.05;
Settle = '02-Jan-02';
Maturity = '31-Mar-02';
IssueDate = '1-0ct-01';
```

```
[Price, AccruedInt] = cdprice(Yield, CouponRate, Settle, ...
Maturity, IssueDate)
Price = 99.9233
AccruedInt = 1.2917
```


## Compute the Price and Accrued Interest for a Certificate of Deposit Using datetime Inputs

This example shows how to use datetime inputs to compute the price and the accrued interest due on the settlement date, given a certificate of deposit with the following characteristics.

```
Yield = 0.0525;
CouponRate = 0.05;
Settle = datetime('02-Jan-02','Locale','en US');
Maturity = datetime('31-Mar-02','Locale','en_US');
IssueDate = datetime('1-0ct-01','Locale','en_US');
```

[Price, AccruedInt] = cdprice(Yield, CouponRate, Settle, ...
Maturity, IssueDate)
Price = 99.9233
AccruedInt = 1.2917

## Input Arguments

## Yield - Simple yield to maturity over basis denominator numeric

Simple yield to maturity over the basis denominator, specified as a numeric value using a scalar or a NCDS-by-1 or 1-by-NCDS vector.

Data Types: double

## CouponRate - Coupon annual interest rate <br> decimal

Coupon annual interest rate, specified as decimal using a scalar or a NCDS-by-1 or 1-byNCDS vector.

## Data Types: double

## Settle - Settlement date for certificate of deposit serial date number | date character vector | datetime

Settlement date for the certificate of deposit, specified as a scalar or a NCDS-by-1 or 1-byNCDS vector using serial date numbers, date character vectors, or datetime arrays. The Settle date must be before the Maturity date.

Data Types: double | char|datetime

## Maturity - Maturity date for certificate of deposit

serial date number | date character vector | datetime
Maturity date for the certificate of deposit, specified as a scalar or a NCDS-by-1 or 1-byNCDS vector using serial date numbers, date character vectors, or datetime arrays.
Data Types: double | char \| datetime

## IssueDate - Issue date for certificate of deposit <br> serial date number | date character vector | datetime

Issue date for the certificate of deposit, specified as a scalar or a NCDS-by-1 or 1-by-NCDS vector using serial date numbers, date character vectors, or datetime arrays.

## Data Types: double | char | datetime

## Basis - Day-count basis for certificate of deposit

2 (actual/360) (default) | integers of the set [0...13] | vector of integers of the set [0...13]
(Optional) Day-count basis for the certificate of deposit, specified as a scalar or a NINST-by-1 vector. Values are:

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- $2=$ actual/360
- 3 = actual/365
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-0
Data Types: double

## Output Arguments

## Price - Clean price of certificate of deposit per \$100

numeric
Clean price of the certificate of deposit per \$100, returned as a NCDS-by-1 or 1-by-NCDS vector.

## AccrInt - Accrued interest payable at settlement per unit of face value numeric

Accrued interest payable at settlement per unit of face value, returned as a NCDS-by-1 or 1-by-NCDS vector.

## See Also

bndprice|cdai|cdyield|datetime| stepcpnprice|tbillprice

## Topics

"Coupon Date Calculations" on page 2-31
"Yield Conventions" on page 2-31

Introduced before R2006a

## cdsbootstrap

Bootstrap default probability curve from credit default swap market quotes

## Syntax

```
[ProbData,HazData] = cdsbootstrap(ZeroData,MarketData,Settle)
[ProbData,HazData] = cdsbootstrap(
```

$\qquad$

``` ,Name, Value)
```


## Description

[ProbData,HazData] = cdsbootstrap(ZeroData,MarketData,Settle) bootstraps the default probability curve using credit default swap (CDS) market quotes. The market quotes can be expressed as a list of maturity dates and corresponding CDS market spreads, or as a list of maturities and corresponding upfronts and standard spreads for standard CDS contracts. The estimation uses the standard model of the survival probability.
[ProbData,HazData] = cdsbootstrap( $\qquad$ ,Name,Value) adds optional namevalue pair arguments.

## Examples

## Bootstrap Default Probability Curve from Credit Default Swap Market Quotes

This example shows how to use cdsbootstrap with market quotes for CDS contracts to generate ProbData and HazData values.

```
Settle = '17-Jul-2009'; % valuation date for the CDS
Spread_Time = [1 2 3 5 7]';
Spread = [140 175 210 265 310]';
Market_Dates = daysadd(datenum(Settle),360*Spread_Time,1);
MarketDData = [Market Dates Spread];
Zero Time = [.5 1 2 3 4 5]';
Zero_Rate = [1.35 1.43 1.9 2.47 2.936 3.311]'/100;
```

```
Zero_Dates = daysadd(datenum(Settle),360*Zero_Time,1);
ZeroD
format long
[ProbData,HazData] = cdsbootstrap(ZeroData,MarketData,Settle)
ProbData = 5×2
105}
    7.343360000000000 0.000000233427859
    7.347010000000000 0.000000575839968
    7.350670000000000 0.000001021397017
    7.357970000000000 0.000002064539982
    7.3652800000000000 0.000003234110940
HazData = 5×2
105 x
    7.343360000000000 0.000000232959886
    7.347010000000000 0.000000352000512
    7.350670000000000 0.000000476383354
    7.357970000000000 0.000000609055766
    7.365280000000000 0.000000785241515
```


## Input Arguments

## ZeroData - Zero rate data <br> vector | IRDataCurve object

Zero rate data, specified as a M-by-2 vector of dates and zero rates or an IRDataCurve object of zero rates.

When ZeroData is an IRDataCurve object, ZeroCompounding and ZeroBasis are implicit in ZeroData and are redundant inside this function. In this case, specify these optional parameters when constructing the IRDataCurve object before using the cdsbootstrap function.

For more information on an IRDataCurve object, see "Creating an IRDataCurve Object" (Financial Instruments Toolbox).

## Data Types: double

## MarketData - Bond market data matrix

Bond market data, specified as a N -by-2 matrix of dates and corresponding market spreads or N -by-3 matrix of dates, upfronts, and standard spreads of CDS contracts. The dates must be entered as serial date numbers, upfronts must be numeric values between 0 and 1, and spreads must be in basis points.

## Data Types: double

## Settle - Settlement date

serial date number | date character vector
Settlement date, specified as a serial date number or a date character vector. The Settle date must be earlier than or equal to the dates in MarketData

## Data Types: double | char

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1,Value1, ... , NameN, ValueN.

Note Any optional input of size N -by-1 is also acceptable as an array of size 1-by-N, or as a single value applicable to all contracts. Single values are internally expanded to an array of size N -by-1.

Example: [ProbData,HazData] =
cdsbootstrap(ZeroData,MarketData,Settle, 'RecoveryRate' ,Recovery, 'Zer oCompounding ', -1)

## RecoveryRate - Recovery rate

0.4 (default) | decimal

Recovery rate, specified as the comma-separated pair consisting of 'RecoveryDate ' and a N -by- 1 vector of recovery rates, specified as a decimal from 0 to 1.

## Data Types: double

## Period - Premium payment frequency

4 (default) | numeric with values $1,2,3,4,6$ or 12
Premium payment frequency, specified as the comma-separated pair consisting of 'Period ' and a N -by-1 vector with values of $1,2,3,4,6$, or 12.

## Data Types: double

## Basis - Day-count basis of contract

2 (actual/360) (default) | integers of the set [0...13] | vector of integers of the set [0...13]

Day-count basis of the contract, specified as the comma-separated pair consisting of 'Basis' and a positive integer using a NINST-by-1 vector.

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- 2 = actual/360
- 3 = actual/365
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- $9=$ actual/360 (ICMA)
- $10=$ actual/365 (ICMA)
- $11=30 / 360 \mathrm{E}$ (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-0
Data Types: double

## BusDayConvention - Business day conventions

'actual' (default)| character vector

Business day conventions, specified as the comma-separated pair consisting of 'BusDayConvention' and a character vector. The selection for business day convention determines how non-business days are treated. Non-business days are defined as weekends plus any other date that businesses are not open (for example, statutory holidays). Values are:

- 'actual' - Non-business days are effectively ignored. Cash flows that fall on nonbusiness days are assumed to be distributed on the actual date.
- 'follow' - Cash flows that fall on a non-business day are assumed to be distributed on the following business day.
- 'modifiedfollow' - Cash flows that fall on a non-business day are assumed to be distributed on the following business day. However if the following business day is in a different month, the previous business day is adopted instead.
- 'previous ' - Cash flows that fall on a non-business day are assumed to be distributed on the previous business day.
- 'modifiedprevious' - Cash flows that fall on a non-business day are assumed to be distributed on the previous business day. However if the previous business day is in a different month, the following business day is adopted instead.


## Data Types: char

## PayAccruedPremium - Flag for accrued premiums paid upon default

 true (default) | integer with value 1 or 0Flag for accrued premiums paid upon default, specified as the comma-separated pair consisting of 'PayAccruedPremium' and a N-by-1 vector of Boolean flags that is true (default) if accrued premiums are paid upon default, false otherwise.

Data Types: logical

## TimeStep - Number of days as time step for numerical integration 10 (days) (default) | nonnegative integer

Number of days to take as time step for the numerical integration, specified as the comma-separated pair consisting of 'TimeStep' and a nonnegative integer.

Data Types: double

## ZeroCompounding - Compounding frequency of the zero curve

2 (semiannual) (default) | integer with value of 1,2,3,4,6,12, or -1

Compounding frequency of the zero curve, specified as the comma-separated pair consisting of 'ZeroCompounding' and an integer with values:

- 1 - Annual compounding
- 2 - Semiannual compounding
- 3-Compounding three times per year
- 4 - Quarterly compounding
- 6 - Bimonthly compounding
- 12 - Monthly compounding
- -1 - Continuous compounding


## Data Types: double

## ZeroBasis - Basis of the zero curve

0 (actual/actual) (default) | integer with value of 0 to 13
Basis of the zero curve, specified as the comma-separated pair consisting of 'ZeroBasis' and an integer with values that are identical to Basis.

## Data Types: double

## ProbDates - Dates for probability data

column of dates in MarketData (default) | serial date number | date character vector
Dates for probability data, specified as the comma-separated pair consisting of 'ProbDates ' and a P-by-1 vector of dates, given as serial date numbers or date character vectors.

Data Types: double | char

## Output Arguments

## ProbData - Default probability values

matrix
Default probability values, returned as a P-by-2 matrix with dates and corresponding cumulative default probability values. The dates match those in MarketData, unless the optional input parameter ProbDates is provided.

## HazData - Hazard rate values

matrix
Hazard rate values, returned as a N-by-2 matrix with dates and corresponding hazard rate values for the survival probability model. The dates match those in MarketData.

Note A warning is displayed when non-monotone default probabilities (that is, negative hazard rates) are found.

## Algorithms

If the time to default is denoted by $\tau$, the default probability curve, or function, $P D(t)$, and its complement, the survival function $Q(t)$, are given by:

$$
P D(t)=P[\tau \leq t]=1-P[\tau>t]=1-Q(t)
$$

In the standard model, the survival probability is defined in terms of a piecewise constant hazard rate $h(t)$. For example, if $h(t)=$
$\lambda_{1}$, for $0 \leq t \leq t_{1}$
$\lambda_{2}$, for $t_{1}<t \leq t_{2}$
$\lambda_{3}$, for $t_{2}<t$
then the survival function is given by $Q(t)=$

```
\(e^{-\lambda_{1} t}\), for \(0 \leq t \leq t_{1}\)
\(e^{-\lambda_{1} t-\lambda_{2}\left(t-t_{1}\right)}\), for \(t_{1}<t \leq t_{2}\)
\(e^{-\lambda_{1} t_{1}-\lambda_{2}\left(t_{2}-t_{1}\right)-\lambda_{3}\left(t-t_{2}\right)}\), for \(t_{2}<t\)
```

Given $n$ market dates $t_{1}, \ldots, t_{n}$ and corresponding market CDS spreads $S_{1}, \ldots, S_{n}$, cdsbootstrap calibrates the parameters $\lambda_{1}, \ldots, \lambda_{n}$ and evaluates $P D(t)$ on the market dates, or an optional user-defined set of dates.

## References

[1] Beumee, J., D. Brigo, D. Schiemert, and G. Stoyle. "Charting a Course Through the CDS Big Bang." Fitch Solutions, Quantitative Research, Global Special Report. April 7, 2009.
[2] Hull, J., and A. White. "Valuing Credit Default Swaps I: No Counterparty Default Risk." Journal of Derivatives. Vol. 8, pp. 29-40.
[3] O'Kane, D. and S. Turnbull. "Valuation of Credit Default Swaps." Lehman Brothers, Fixed Income Quantitative Credit Research, April 2003.

## See Also

IRDataCurve|cdsprice|cdsrpv01|cdsspread

## Topics

"Bootstrapping a Default Probability Curve" on page 8-129
"Bootstrapping from Inverted Market Curves" on page 8-141
"Credit Default Swap (CDS)" on page 8-128

## Introduced in R2010b

## cdsprice

Determine price for credit default swap

## Syntax

[Price,AccPrem, PaymentDates,PaymentTimes,PaymentCF] = cdsprice( ZeroData, ProbData,Settle, Maturity, ContractSpread)
[Price,AccPrem, PaymentDates,PaymentTimes,PaymentCF] = cdsprice( $\qquad$ , Name, Value)

## Description

[Price,AccPrem,PaymentDates,PaymentTimes,PaymentCF] = cdsprice( ZeroData, ProbData, Settle,Maturity, ContractSpread) computes the price, or the mark-to-market value for CDS instruments.
[Price,AccPrem,PaymentDates,PaymentTimes,PaymentCF] = cdsprice( $\qquad$ , Name, Value) adds optional name-value pair arguments.

## Examples

## Determine the Price for a Credit Default Swap

This example shows how to use cdsprice to compute the clean price for a CDS contract using the following data.

```
Settle = '17-Jul-2009'; % valuation date for the CDS
Zero_Time = [.5 1 2 3 4 5]';
Zero_Rate = [1.35 1.43 1.9 2.47 2.936 3.311]'/100;
Zero_Dates = daysadd(Settle,360*Zero_Time,1);
ZeroData = [Zero_Dates Zero_Rate];
ProbData = [daysadd(datenum(Settle),360,1), 0.0247];
Maturity = '20-Sep-2010';
```

```
ContractSpread = 135;
[Price,AccPrem] = cdsprice(ZeroData,ProbData,Settle,Maturity,ContractSpread)
Price = 1.5461e+04
AccPrem = 10500
```


## Input Arguments

## ZeroData - Zero rate data <br> vector | IRDataCurve object

Zero rate data, specified as a M-by-2 vector of dates and zero rates or an IRDataCurve object of zero rates.

When ZeroData is an IRDataCurve object, ZeroCompounding and ZeroBasis are implicit in ZeroData and are redundant inside this function. In this case, specify these optional parameters when constructing the IRDataCurve object before using the cdsprice function.

For more information on an IRDataCurve object, see "Creating an IRDataCurve Object" (Financial Instruments Toolbox).

Data Types: double | struct

## ProbData - Default probability values <br> matrix

Default probability values, specified as a P-by-2 matrix with dates and corresponding cumulative default probability values.
Data Types: double \| char
Settle - Settlement date
serial date number | date character vector
Settlement date, specified as a scalar serial date number or date character vector. The Settle date must be earlier than or equal to the dates in Maturity.

Data Types: double | char

## Maturity - Maturity date

## serial date number | date character vector

Maturity date, specified as a N -by- 1 vector of serial date numbers or date character vectors.

Data Types: double | char

## ContractSpread - Contract spreads

numeric
Contract spreads, specified as a N-by-1 vector of spreads, expressed in basis points.
Data Types: double

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Note Any optional input of size N -by-1 is also acceptable as an array of size 1-by-N, or as a single value applicable to all contracts. Single values are internally expanded to an array of size N -by-1.

```
Example:[Price,AccPrem] =
cdsprice(ZeroData,ProbData,Settle,Maturity,ContractSpread,'Basis',
7,'BusDayConvention','previous')
```


## RecoveryRate - Recovery rate

## 0.4 (default) | decimal

Recovery rate, specified as the comma-separated pair consisting of 'RecoveryRate ' and a N -by- 1 vector of recovery rates, specified as a decimal from 0 to 1.

## Data Types: double

## Period - Premium payment frequency

4 (default) | numeric with values 1, 2, 3, 4, 6 or 12

Premium payment frequency, specified as the comma-separated pair consisting of 'Period ' and a N-by-1 vector with values of $1,2,3,4,6$, or 12.

## Data Types: double

## Basis - Day-count basis of contract

2 (actual/360) (default) | positive integers of the set [1. . 13] | vector of positive integers of the set [1...13]

Day-count basis of the contract, specified as the comma-separated pair consisting of 'Basis' and a positive integer using a NINST-by-1 vector.

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- $2=$ actual/360
- $3=$ actual/365
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- $9=$ actual/360 (ICMA)
- $10=$ actual/365 (ICMA)
- $11=30 / 360 \mathrm{E}$ (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-0

## Data Types: double

## BusDayConvention - Business day conventions

actual (default) | character vector
Business day conventions, specified as the comma-separated pair consisting of 'BusDayConvention' and a character vector. The selection for business day convention determines how non-business days are treated. Non-business days are defined as
weekends plus any other date that businesses are not open (for example, statutory holidays). Values are:

- actual - Non-business days are effectively ignored. Cash flows that fall on nonbusiness days are assumed to be distributed on the actual date.
- follow - Cash flows that fall on a non-business day are assumed to be distributed on the following business day.
- modifiedfollow - Cash flows that fall on a non-business day are assumed to be distributed on the following business day. However if the following business day is in a different month, the previous business day is adopted instead.
- previous - Cash flows that fall on a non-business day are assumed to be distributed on the previous business day.
- modifiedprevious - Cash flows that fall on a non-business day are assumed to be distributed on the previous business day. However if the previous business day is in a different month, the following business day is adopted instead.


## Data Types: char

## PayAccruedPremium - Flag for accrued premiums paid upon default

 true (default) | integer with value 1 or 0Flag for accrued premiums paid upon default, specified as the comma-separated pair consisting of 'PayAccruedPremium' and a N-by-1 vector of Boolean flags that is true (default) if accrued premiums are paid upon default, false otherwise.

Data Types: logical

## Notional - Contract notional values

10MM (default) | positive or negative integer
Contract notional values, specified as the comma-separated pair consisting of 'Notional' and a N-by-1vector of integers. Use positive integer values for long positions and negative integer values for short positions.

Data Types: double

## TimeStep - Number of days as time step for numerical integration

10 (days) (default) | nonnegative integer

Number of days to take as time step for the numerical integration, specified as the comma-separated pair consisting of 'TimeStep' and a nonnegative integer.

## Data Types: double

## ZeroCompounding - Compounding frequency of the zero curve

2 (semiannual) (default) | integer with value of $1,2,3,4,6,12$, or -1
Compounding frequency of the zero curve, specified as the comma-separated pair consisting of 'ZeroCmpounding' and an integer with values:

- 1 - Annual compounding
- 2-Semiannual compounding
- 3-Compounding three times per year
- 4 - Quarterly compounding
- 6 - Bimonthly compounding
- 12 - Monthly compounding
- -1 - Continuous compounding

Data Types: double

## ZeroBasis - Basis of the zero curve

0 (actual/actual) (default) | integer with value of 0 to 13
Basis of the zero curve, specified as the comma-separated pair consisting of 'ZeroBasis' and an integer with values that are identical to Basis.
Data Types: double

## Output Arguments

## Price - CDS clean prices

vector
CDS clean prices, returned as a N -by- 1 vector.

## AccPrem - Accrued premiums

vector
Accrued premiums, returned as a N -by- 1 vector.

## PaymentDates - Payment dates <br> matrix

Payment dates, returned as a N -by-numCF matrix.

## PaymentTimes - Payment times

## matrix

Payment times, returned as a N -by-numCF matrix of accrual fractions.

## PaymentCF - Payments

matrix
Payments, returned as a N-by-numCF matrix.

## Definitions

## CDS Price

The price or mark-to-market (MtM) value of an existing CDS contract.
The CDS price is computed using the following formula:

```
CDS price = Notional * (Current Spread - Contract Spread) * RPV01
```

Current Spread is the current breakeven spread for a similar contract, according to current market conditions. RPV01 is the 'risky present value of a basis point,' the present value of the premium payments, considering the default probability. This formula assumes a long position, and the right side is multiplied by -1 for short positions.

## Algorithms

The premium leg is computed as the product of a spread $S$ and the risky present value of a basis point (RPV01). The RPV01 is given by:

$$
R P V 01=\sum_{j=1}^{N} Z\left(t_{j}\right) \Delta\left(t_{j-1}, t_{j}, B\right) Q\left(t_{j}\right)
$$

when no accrued premiums are paid upon default, and it can be approximated by

$$
R P V 01 \approx \frac{1}{2} \sum_{j=1}^{N} Z\left(t_{j}\right) \Delta\left(t_{j-1}, t_{j}, B\right)\left(Q\left(t_{j-1}\right)+Q\left(t_{j}\right)\right)
$$

when accrued premiums are paid upon default. Here, $t_{0}=0$ is the valuation date, and $t_{1}, \ldots, t_{n}=T$ are the premium payment dates over the life of the contract, $T$ is the maturity of the contract, $Z(t)$ is the discount factor for a payment received at time $t$, and $\Delta\left(t_{j-1}, t_{j}, B\right)$ is a day count between dates $t_{j-1}$ and $t_{j}$ corresponding to a basis $B$.

The protection leg of a CDS contract is given by the following formula:

$$
\begin{aligned}
& \text { ProtectionLeg }=\int_{0}^{T} Z(\tau)(1-R) d P D(\tau) \\
& \approx(1-R) \sum_{i=1}^{M} Z\left(\tau_{i}\right)\left(P D\left(\tau_{i}\right)-P D\left(\tau_{i-1}\right)\right) \\
& =(1-R) \sum_{i=1}^{M} Z\left(\tau_{i}\right)\left(Q\left(\tau_{i-1}\right)-Q\left(\tau_{i}\right)\right)
\end{aligned}
$$

where the integral is approximated with a finite sum over the discretization $\tau_{0}=0, \tau_{1}, \ldots, \tau_{M}$ $=T$.

If the spread of an existing CDS contract is $S_{C}$, and the current breakeven spread for a comparable contract is $S_{0}$, the current price, or mark-to-market value of the contract is given by:

MtM $=$ Notional $\left(S_{0}-S_{C}\right)$ RPV01
This assumes a long position from the protection standpoint (protection was bought). For short positions, the sign is reversed.

## References

[1] Beumee, J., D. Brigo, D. Schiemert, and G. Stoyle. "Charting a Course Through the CDS Big Bang." Fitch Solutions, Quantitative Research, Global Special Report. April 7, 2009.
[2] Hull, J., and A. White. "Valuing Credit Default Swaps I: No Counterparty Default Risk." Journal of Derivatives. Vol. 8, pp. 29-40.
[3] O'Kane, D. and S. Turnbull. "Valuation of Credit Default Swaps." Lehman Brothers, Fixed Income Quantitative Credit Research, April 2003.

## See Also

IRDataCurve|cdsbootstrap|cdsoptprice|cdsspread

## Topics

"Finding Breakeven Spread for New CDS Contract" on page 8-132
"Valuing an Existing CDS Contract" on page 8-135
"Converting from Running to Upfront" on page 8-138
"Credit Default Swap (CDS)" on page 8-128

## External Websites

Pricing and Valuation of Credit Default Swaps (4 min 22 sec)

Introduced in R2010b

## cdsspread

Determine spread of credit default swap

## Syntax

[Spread,PaymentDates,PaymentTimes,] = cdsspread(ZeroData,ProbData, Settle,Maturity,)
[Spread,PaymentDates,PaymentTimes,] = cdsspread( $\qquad$ ,Name, Value)

## Description

[Spread,PaymentDates,PaymentTimes,] = cdsspread(ZeroData,ProbData, Settle, Maturity,) computes the spread of the CDS.
[Spread,PaymentDates,PaymentTimes,] = cdsspread( $\qquad$ ,Name,Value) adds optional name-value pair arguments.

## Examples

## Determine the Spread of a Credit Default Swap

This example shows how to use cdsspread to compute the spread (in basis points) for a CDS contract with the following data.

```
Settle = '17-Jul-2009'; % valuation date for the CDS
Zero_Time = [.5 1 2 3 4 5]';
Zero_Rate = [1.35 1.43 1.9 2.47 2.936 3.311]'/100;
Zero-Dates = daysadd(Settle,360*Zero_Time,1);
ZeroData = [Zero_Dates Zero_Rate];
ProbData = [daysadd(datenum(Settle),360,1), 0.0247];
Maturity = '20-Sep-2010';
Spread = cdsspread(ZeroData,ProbData,Settle,Maturity)
Spread = 148.2705
```


## Input Arguments

ZeroData - Zero rate data<br>vector | IRDataCurve object

Zero rate data, specified as a M-by-2 vector of dates and zero rates or an IRDataCurve object of zero rates.

When ZeroData is an IRDataCurve object, ZeroCompounding and ZeroBasis are implicit in ZeroData and are redundant inside this function. In this case, specify these optional parameters when constructing the IRDataCurve object before using the cdsspread function.

For more information on an IRDataCurve object, see "Creating an IRDataCurve Object" (Financial Instruments Toolbox).

Data Types: double | struct

## ProbData - Default probability values <br> matrix

Default probability values, specified as a P-by-2 matrix with dates and corresponding cumulative default probability values.

Data Types: double | char

## Settle - Settlement date <br> serial date number | date character vector

Settlement date, specified as a scalar serial date number or date character vector. The Settle date must be earlier than or equal to the dates in Maturity.

Data Types: double | char

## Maturity - Maturity date <br> serial date number | date character vector

Maturity date, specified as a N -by- 1 vector of serial date numbers or date character vectors.

Data Types: double | char

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Note Any optional input of size N -by-1 is also acceptable as an array of size 1-by-N, or as a single value applicable to all contracts. Single values are internally expanded to an array of size N -by-1.

Example: Spread = cdsspread(ZeroData, ProbData,Settle,Maturity,'Basis', 7,'BusDayConvention','previous')

## RecoveryRate - Recovery rate

0.4 (default) | decimal

Recovery rate, specified as the comma-separated pair consisting of 'RecoveryRate' and a N -by-1 vector of recovery rates, specified as a decimal from 0 to 1.

Data Types: double

## Period - Premium payment frequency

4 (default) | numeric with values 1, 2, 3, 4, 6 or 12
Premium payment frequency, specified as the comma-separated pair consisting of 'Period ' and a N-by-1 vector with values of $1,2,3,4,6$, or 12.
Data Types: double

## Basis - Day-count basis of contract

2 (actual/360) (default) | positive integers of the set [1...13] | vector of positive integers of the set [1...13]

Day-count basis of the contract, specified as the comma-separated pair consisting of 'Basis' and a positive integer using a NINST-by-1 vector.

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- $2=$ actual/360
- 3 = actual/365
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- $12=$ actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-0
Data Types: double

## BusDayConvention - Business day conventions

actual (default)| character vector
Business day conventions, specified as the comma-separated pair consisting of 'BusDayConvention' and a character vector. The selection for business day convention determines how non-business days are treated. Non-business days are defined as weekends plus any other date that businesses are not open (for example, statutory holidays). Values are:

- actual - Non-business days are effectively ignored. Cash flows that fall on nonbusiness days are assumed to be distributed on the actual date.
- follow - Cash flows that fall on a non-business day are assumed to be distributed on the following business day.
- modifiedfollow - Cash flows that fall on a non-business day are assumed to be distributed on the following business day. However if the following business day is in a different month, the previous business day is adopted instead.
- previous - Cash flows that fall on a non-business day are assumed to be distributed on the previous business day.
- modifiedprevious - Cash flows that fall on a non-business day are assumed to be distributed on the previous business day. However if the previous business day is in a different month, the following business day is adopted instead.


## Data Types: char

## PayAccruedPremium - Flag for accrued premiums paid upon default true (default) | integer with value 1 or 0

Flag for accrued premiums paid upon default, specified as the comma-separated pair consisting of 'PayAccruedPremium' and a N-by-1 vector of Boolean flags that is true (default) if accrued premiums are paid upon default, false otherwise.
Data Types: logical

## TimeStep - Number of days as time step for numerical integration 10 (days) (default) | nonnegative integer

Number of days to take as time step for the numerical integration, specified as the comma-separated pair consisting of 'TimeStep' and a nonnegative integer.
Data Types: double

## ZeroCompounding - Compounding frequency of the zero curve

2 (semiannual) (default) | integer with value of $1,2,3,4,6,12$, or -1
Compounding frequency of the zero curve, specified as the comma-separated pair consisting of 'ZeroCompounding' and an integer with values:

- 1 - Annual compounding
- 2-Semiannual compounding
- 3-Compounding three times per year
- 4-Quarterly compounding
- 6 - Bimonthly compounding
- 12 - Monthly compounding
- -1 - Continuous compounding

Data Types: double

## ZeroBasis - Basis of the zero curve

0 (actual/actual) (default) | integer with value of 0 to 13
Basis of the zero curve, specified as the comma-separated pair consisting of 'ZeroBasis' and a positive integer with values that are identical to Basis.

Data Types: double

## Output Arguments

## Spread - Spreads (in basis points)

vector
Spreads (in basis points), returned as a N -by-1 vector.

## PaymentDates - Payment dates

 matrixPayment dates, returned as a N -by-numCF matrix.

## PaymentTimes - Payment times <br> matrix

Payment times, returned as a N -by-numCF matrix of accrual fractions.

## Definitions

## CDS Spread

The market, or breakeven, spread value of a CDS.
The CDS spread can be computed by equating the value of the protection leg with the value of the premium leg:

Market Spread * RPV01 = Value of Protection Leg
The left side corresponds to the value of the premium leg, and this has been decomposed as the product of the market or breakeven spread times the RPV01 or 'risky present value of a basis point' of the contract. The latter is the present value of the premium payments, considering the default probability. The Market Spread can be computed as the ratio of the value of the protection leg, to the RPV01 of the contract. cdsspread returns the resulting spread in basis points.

## Algorithms

The premium leg is computed as the product of a spread $S$ and the risky present value of a basis point (RPV01). The RPV01 is given by:

$$
R P V 01=\sum_{j=1}^{N} Z\left(t_{j}\right) \Delta\left(t_{j-1}, t_{j}, B\right) Q\left(t_{j}\right)
$$

when no accrued premiums are paid upon default, and it can be approximated by

$$
R P V 01 \approx \frac{1}{2} \sum_{j=1}^{N} Z\left(t_{j}\right) \Delta\left(t_{j-1}, t_{j}, B\right)\left(Q\left(t_{j-1}\right)+Q\left(t_{j}\right)\right)
$$

when accrued premiums are paid upon default. Here, $t_{0}=0$ is the valuation date, and $t_{1}, \ldots, t_{n}=T$ are the premium payment dates over the life of the contract, $T$ is the maturity of the contract, $Z(t)$ is the discount factor for a payment received at time $t$, and $\Delta\left(t_{j-1}, t_{j}, B\right)$ is a day count between dates $t_{j-1}$ and $t_{j}$ corresponding to a basis $B$.

The protection leg of a CDS contract is given by the following formula:

$$
\begin{aligned}
& \text { ProtectionLeg }=\int_{0}^{T} Z(\tau)(1-R) d P D(\tau) \\
& \approx(1-R) \sum_{i=1}^{M} Z\left(\tau_{i}\right)\left(P D\left(\tau_{i}\right)-P D\left(\tau_{i-1}\right)\right) \\
& =(1-R) \sum_{i=1}^{M} Z\left(\tau_{i}\right)\left(Q\left(\tau_{i-1}\right)-Q\left(\tau_{i}\right)\right)
\end{aligned}
$$

where the integral is approximated with a finite sum over the discretization $\tau_{0}=0, \tau_{1}, \ldots, \tau_{M}$ $=T$.

A breakeven spread $S_{0}$ makes the value of the premium and protection legs equal. It follows that:

$$
S_{0}=\frac{\text { ProtectionLeg }}{R P V 01}
$$

## References

[1] Beumee, J., D. Brigo, D. Schiemert, and G. Stoyle. "Charting a Course Through the CDS Big Bang." Fitch Solutions, Quantitative Research, Global Special Report. April 7, 2009.
[2] Hull, J., and A. White. "Valuing Credit Default Swaps I: No Counterparty Default Risk." Journal of Derivatives. Vol. 8, pp. 29-40.
[3] O'Kane, D. and S. Turnbull. "Valuation of Credit Default Swaps." Lehman Brothers, Fixed Income Quantitative Credit Research, April 2003.

## See Also

IRDataCurve|cdsbootstrap|cdsprice

## Topics

"Finding Breakeven Spread for New CDS Contract" on page 8-132
"Valuing an Existing CDS Contract" on page 8-135
"Converting from Running to Upfront" on page 8-138
"First-to-Default Swaps" (Financial Instruments Toolbox)
"Pricing a CDS Index Option" (Financial Instruments Toolbox)
"Credit Default Swap (CDS)" on page 8-128

## External Websites

Pricing and Valuation of Credit Default Swaps (4 min 22 sec)

## Introduced in R2010b

## cdsrpv01

Compute risky present value of a basis point for credit default swap

## Syntax

```
RPV01 = cdsrpv01(ZeroData,ProbData,Settle,Maturity)
RPV01 = cdsrpv01(
```

$\qquad$

``` ,Name,Value)
[RPV01,PaymentDates,PaymentTimes] = cdsrpv01(ZeroData,ProbData,
Settle,Maturity)
[RPV01,PaymentDates,PaymentTimes] = cdsrpv01(
```

$\qquad$

``` , Name, Value)
```


## Description

RPV01 = cdsrpv01(ZeroData, ProbData, Settle, Maturity) computes the risky present value of a basis point (RPV01) for a credit default swap (CDS).

RPV01 = cdsrpv01( $\qquad$ , Name, Value) adds optional name-value arguments.
[RPV01,PaymentDates,PaymentTimes] = cdsrpv01(ZeroData, ProbData, Settle, Maturity) computes the risky present value of a basis point (RPV01), PaymentDates, and PaymentTimes for a credit default swap (CDS).
[RPV01,PaymentDates,PaymentTimes] = cdsrpv01( $\qquad$ ,Name,Value) computes the risky present value of a basis point (RPV01), PaymentDates, and PaymentTimes for a credit default swap (CDS) using optional name-value pair arguments.

## Examples

## Calculate the RPV01 Value for a CDS

Calculate the RPV01 value, given the following specification for a CDS.

```
Settle = '17-Jul-2009'; % valuation date for the CDS
Zero_Time = [.5 1 2 3 4 5]';
```

```
Zero Rate = [1.35 1.43 1.9 2.47 2.936 3.311]'/100;
Zero_Dates = daysadd(Settle,360*Zero_Time,1);
ZeroData = [Zero Dates Zero Rate];
ProbData = [daysadd(datenum(Settle),360,1), 0.0247];
Maturity = '20-Sep-2010';
RPV01 = cdsrpv01(ZeroData,ProbData,Settle,Maturity)
RPV01 = 1.1651
```


## Input Arguments

## ZeroData - Dates and zero rates

object from IRDataCurve or vector of dates and zero rates
Dates and zero rates, specified by an M-by-2 vector of dates and zero rates or the object IRDataCurve for zero rates. For more information on an IRDataCurve object, see "Creating an IRDataCurve Object" (Financial Instruments Toolbox).

Data Types: struct | double

## ProbData - Dates and default probabilities

vector of dates and default probabilities
Dates and default probabilities, specified by a P-by-2 array.
Data Types: double

## Settle - Settlement date <br> serial date number | character vector | cell array of character vectors

Settlement date, specified by a serial date number or date character vector. This must be earlier than or equal to the dates in Maturity.

Data Types: char|cell|double

## Maturity - CDS maturity date

serial date number | character vector | cell array of character vectors
CDS maturity date, specified by an N -by-1 vector of serial date numbers or date character vectors containing the maturity dates. The CDS premium payment dates occur at regular intervals, and the last payment occurs on these maturity dates.

Data Types: char |cell|double

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.
Example: RPV01 = cdsrpv01(ZeroData, ProbData, Settle,Maturity,'Period', 1,'StartDate','20-Sep-2010', 'Basis',1, 'BusDayConvention', actual,'CleanRPV01',true,' PayAccruedPremium' ,true ,'ZeroCompounding',1,'ZeroBasis',1)

## Period - Number of premium payments per year

4 (default) | positive integer from the set [1, 2, 3, 4, 6, 12] | vector of positive integers from the set [1, 2, 3, 4, 6, 12]

Number of premium payments per year, specified as the comma-separated pair consisting of 'Period' and an N -by-1 vector. Values are 1, 2, 3, 4, 6, and 12.

Data Types: double

## StartDate - Dates the CDS premium leg starts

Settle date (default) | serial date number | character vector | cell array of character vectors

Dates when the CDS premium leg actually starts, specified as the comma-separated pair consisting of 'StartDate' and an N -by-1 vector of serial date numbers or date character vectors. Must be on or between the Settle and Maturity dates. For a forward-starting CDS, specify this date as a future date after Settle.

Data Types: double | char | cell

## Basis - Day-count basis of contract

2 (actual/360) (default) | positive integers of the set [1...13] | vector of positive integers of the set [1...13]

Day-count basis of the contract, specified as the comma-separated pair consisting of 'Basis' and a positive integer using a NINST-by-1 vector.

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- 2 = actual/360
- 3 = actual/365
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- $9=$ actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-0 .

## Data Types: double

## BusDayConvention - Business day conventions

## actual (default) | character vector | cell array of character vectors

Business day conventions, specified as the comma-separated pair consisting of 'BusDayConvention ' and a character vector or N -by-1 cell array of character vectors of business day conventions. The selection for business day convention determines how nonbusiness days are treated. Non-business days are defined as weekends plus any other date that businesses are not open (e.g. statutory holidays). Values are:

- actual - Non-business days are effectively ignored. Cash flows that fall on nonbusiness days are assumed to be distributed on the actual date.
- follow - Cash flows that fall on a non-business day are assumed to be distributed on the following business day.
- modifiedfollow - Cash flows that fall on a non-business day are assumed to be distributed on the following business day. However if the following business day is in a different month, the previous business day is adopted instead.
- previous - Cash flows that fall on a non-business day are assumed to be distributed on the previous business day.
- modifiedprevious - Cash flows that fall on a non-business day are assumed to be distributed on the previous business day. However if the previous business day is in a different month, the following business day is adopted instead.


## Data Types: char | cell

## CleanRPV01 - Flag for premium accrual <br> true (default) | boolean flag with value true or false

Flag for premium accrual, specified as the comma-separated pair consisting of 'CleanRPV01' and a N-by-1 vector of Boolean flags, which is true if the premium accrued at StartDate is excluded in the RPV01, and false otherwise.

Data Types: logical

## PayAccruedPremium - Flag for accrued premium payment

true (default) | boolean flag with value true or false
Flag for accrued premium payment, specified as the comma-separated pair consisting of ' PayAccruedPremium ' and a N -by-1 vector of Boolean flags, true if accrued premiums are paid upon default, false otherwise.

## Data Types: logical

## ZeroCompounding - Compounding frequency of zero curve

2 semiannual compounding (default) $\mid$ integer with acceptable value $[1,2,3,4,6,12$, 1]

Compounding frequency of the zero curve, specified as the comma-separated pair consisting of 'ZeroCompounding' and an integer with values:

- 1 - Annual compounding
- 2-Semiannual compounding
- 3-Compounding three times per year
- 4-Quarterly compounding
- 6 - Bimonthly compounding
- 12 - Monthly compounding
- -1 - Continuous compounding

Note When ZeroData is an IRDataCurve object, the arguments ZeroCompounding and ZeroBasis are implicit in ZeroData and are redundant inside this function. In that
case, specify these optional arguments when constructing the IRDataCurve object before calling this function.

## Data Types: double

## ZeroBasis - Basis of zero curve

0 (actual/actual) (default) | positive integers of the set [1...13] | vector of positive integers of the set [1...13]

Basis of the zero curve, specified as the comma-separated pair consisting of 'ZeroBasis ' and a positive integer using a NINST-by-1 vector.

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- 2 = actual/360
- 3 = actual/365
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- $9=$ actual/360 (ICMA)
- $10=$ actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-0
Data Types: double

## Output Arguments

## RPV01 - RPV01 value

scalar | vector

RPV01 value, returned as an N -by-1 vector.

## PaymentDates - Payment dates

scalar | vector
Payment dates, returned as an N -by-numCF matrix of dates.

## PaymentTimes - Payment times

scalar | vector
Payment times, returned as an N-by-numCF matrix of accrual fractions.

## Definitions

## RPV01

RPV01, associated with a CDS, is the value of a stream of 1-basis-point premiums according to the payment structure of the CDS contract, and considering the default probability over time.

For more information, see [3] and [4] for details.

## References

[1] Beumee, J., D. Brigo, D. Schiemert, and G. Stoyle. "Charting a Course Through the CDS Big Bang." Fitch Solutions, Quantitative Research. Global Special Report. April 7, 2009.
[2] Hull, J., and A. White. "Valuing Credit Default Swaps I: No Counterparty Default Risk." Journal of Derivatives. Vol. 8, pp. 29-40.
[3] O'Kane, D. and S. Turnbull. "Valuation of Credit Default Swaps." Lehman Brothers, Fixed Income Quantitative Credit Research. April, 2003.
[4] O'Kane, D. Modelling Single-name and Multi-name Credit Derivatives. Wiley Finance, 2008.
See Also
IRDataCurve|cdsbootstrap|cdsoptprice|cdsprice|cdsspread
Topics"Pricing a CDS Index Option" (Financial Instruments Toolbox)"Credit Default Swap Option" (Financial Instruments Toolbox)
External Websites
www.cdsmodel.com/cdsmodel/
Pricing and Valuation of Credit Default Swaps (4 min 22 sec)
Introduced in R2013b

## creditexposures

Compute credit exposures from contract values

## Syntax

[exposures, exposurecpty] = creditexposures(values,counterparties)
[exposures, exposurecpty] = creditexposures (__, Name,Value)
[exposures,exposurecpty,collateral] = creditexposures(__,
Name, Value)

## Description

[exposures,exposurecpty] = creditexposures(values,counterparties) computes the counterparty credit exposures from an array of mark-to-market OTC contract values. These exposures are used when calculating the CVA (credit value adjustment) for a portfolio.
[exposures,exposurecpty] = creditexposures(__, Name,Value) adds optional name-value arguments.
[exposures,exposurecpty,collateral] = creditexposures( $\qquad$ ,
Name, Value) computes the counterparty credit exposures from an array of mark-tomarket OTC contract values using optional name-value pair arguments for CollateralTable and Dates, the collateral output is returned for the simulated collateral amounts available to counterparties at each simulation date and over each scenario.

## Examples

## View Contract Values and Exposures Over Time for a Particular Counterparty

After computing the mark-to-market contract values for a portfolio of swaps over many scenarios, compute the credit exposure for a particular counterparty. View the contract
values and credit exposure over time. First, load data (ccr.mat) containing the mark-tomarket contract values for a portfolio of swaps over many scenarios.

```
load ccr.mat
% Look at one counterparty.
cpID = 4;
cpValues = squeeze(sum(values(:,swaps.Counterparty == cpID,:),2));
subplot(2,1,1)
plot(simulationDates,cpValues);
title(sprintf('Mark-to-Market Contract Values for Counterparty: %d',cpID));
datetick('x','mmmyy')
ylabel('Portfolio Value ($)')
% Compute the exposure by counterparty.
[exposures, expcpty] = creditexposures(values,swaps.Counterparty,...
'NettingID',swaps.NettingID);
% View the credit exposure over time for the counterparty.
subplot(2,1,2)
cpIdx = find(expcpty == cpID);
plot(simulationDates,squeeze(exposures(:,cpIdx,:)));
title(sprintf('Exposure for counterparty: %d',cpIdx));
datetick('x','mmmyy')
ylabel('Exposure ($)')
xlabel('Simulation Dates')
```



## Compute the Credit Exposure and Determine the Incremental Exposure for a New Trade

Load the data (ccr.mat) containing the mark-to-market contract values for a portfolio of swaps over many scenarios.
load ccr.mat
Look at one counterparty.
cpID = 4;
cpIdx = swaps.Counterparty == cpID;

```
        cpValues = values(:,cpIdx,:);
        plot(simulationDates,squeeze(sum(cpValues,2)));
        grid on;
        title(sprintf('Potential Mark-to-Market Portfolio Values for Counterparty: %d',cpID));
        datetick('x','mmmyy')
        ylabel('Portfolio Value ($)')
```



Compute the exposures.

```
netting = swaps.NettingID(cpIdx);
exposures = creditexposures(cpValues,cpID,'NettingID',netting);
```

View the credit exposure over time for the counterparty.
figure;
plot(simulationDates,squeeze(exposures));
grid on
title(sprintf('Exposure for counterparty: \%d',cpID));
datetick('x','mmmy')
ylabel('Exposure (\$)')
xlabel('Simulation Dates')


Compute the credit exposure profiles.

```
profilesBefore = exposureprofiles(simulationDates,exposures)
profilesBefore = struct with fields:
    Dates: [37x1 double]
        EE: [37x1 double]
```

```
        PFE: [37x1 double]
        MPFE: 2.1580e+05
EffEE: [37xl double]
    EPE: 2.8602e+04
EffEPE: 4.9579e+04
```

Consider a new trade with a counterparty. For this example, take another trade from the original swap portfolio and "copy" it for a new counterparty. This example is only for illustrative purposes.

```
newTradeIdx = 3;
newTradeValues = values(:,newTradeIdx,:);
% Append a new trade to your existing portfolio.
cpValues = [cpValues newTradeValues];
netting = [netting; cpID];
exposures = creditexposures(cpValues,cpID,'NettingID',netting);
```

Compute the new credit exposure profiles.

```
profilesAfter = exposureprofiles(simulationDates,exposures)
profilesAfter = struct with fields:
    Dates: [37x1 double]
            EE: [37x1 double]
            PFE: [37x1 double]
            MPFE: 2.4689e+05
    EffEE: [37x1 double]
        EPE: 3.1609e+04
    EffEPE: 5.6178e+04
```

Visualize the expected exposures and the new trade's incremental exposure. The incremental exposure is used to compute the incremental credit value adjustment (CVA) charge.

```
figure;
subplot(2,1,1)
plot(simulationDates,profilesBefore.EE,...
    simulationDates,profilesAfter.EE);
grid on;
legend({'EE before','EE with trade'})
datetick('x','mmmyy','keeplimits')
title('Expected Exposure before and after new trade');
```

```
ylabel('Exposure ($)')
subplot(2,1,2)
incrementalEE = profilesAfter.EE - profilesBefore.EE;
plot(simulationDates,incrementalEE);
grid on;
legend('incremental EE')
datetick('x','mmmyy','keeplimits')
ylabel('Exposure ($)')
xlabel('Simulation Dates')
```




## Compute Exposures for Counterparties Under Collateral Agreement

Load the data (ccr.mat) containing the mark-to-market contract values for a portfolio of swaps over many scenarios.

```
load ccr.mat
```

Only look at a single counterparty for this example.

```
cpID = 4;
cpIdx = swaps.Counterparty == cpID;
cpValues = values(:,cpIdx,:);
```

Compute the uncollateralized exposures.

```
exposures = creditexposures(cpValues,swaps.Counterparty(cpIdx),...
'NettingID',swaps.NettingID(cpIdx));
```

View the credit exposure over time for the counterparty.

```
plot(simulationDates,squeeze(exposures));
expYLim = get(gca,'YLim');
title(sprintf('Exposures for Counterparty: %d',cpID));
datetick('x','mmmyy')
ylabel('Exposure ($)')
xlabel('Simulation Dates')
```



Now add a collateral agreement for the counterparty. The 'CollateralTable' parameter is a MATLAB® table. You can create tables from spreadsheets or other data sources, in addition to building them inline as seen here. For more information, see table.

```
collateralVariables = {'Counterparty';'PeriodOfRisk';'Threshold';'MinimumTransfer'};
periodOfRisk = 14;
threshold = 100000;
minTransfer = 10000;
collateralTable = table(cpID,periodOfRisk,threshold,minTransfer,...
'VariableNames',collateralVariables)
collateralTable=1\times4 table
    Counterparty PeriodOfRisk Threshold MinimumTransfer
```

Compute the collateralized exposures.
[collatExp, collatcpty, collateral] = creditexposures(cpValues,... swaps.Counterparty(cpIdx),'NettingID', swaps.NettingID(cpIdx),... 'CollateralTable', collateralTable, 'Dates',simulationDates);

Plot the collateral levels and collateralized exposures.

```
figure;
subplot(2,1,1)
plot(simulationDates,squeeze(collateral));
set(gca,'YLim',expYLim);
title(sprintf('Collateral for counterparty: %d',cpID));
datetick('x','mmmyy')
ylabel('Collateral ($)')
xlabel('Simulation Dates')
subplot(2,1,2)
plot(simulationDates,squeeze(collatExp));
set(gca,'YLim',expYLim);
title(sprintf('Collateralized Exposure for Counterparty: %d',CPID));
datetick('x','mmmyy')
ylabel('Exposure ($)')
xlabel('Simulation Dates');
```



## Input Arguments

values - 3-D array of simulated mark-to-market values of portfolio of contracts array

3-D array of simulated mark-to-market values of a portfolio of contracts simulated over a series of simulation dates and across many scenarios, specified as a NumDates-by-NumContracts-by-NumScenarios "cube" of contract values. Each row represents a different simulation date, each column a different contract, and each "page" is a different scenario from a Monte-Carlo simulation.

## Data Types: double

## counterparties - Counterparties corresponding to each contract vector | cell array

Counterparties corresponding to each contract in values, specified as a NumContractselement vector of counterparties. Counterparties can be a vector of numeric IDs or a cell array of counterparty names. By default, each counterparty is assumed to have one netting set that covers all of its contracts. If counterparties are covered by multiple netting sets, then use the NettingID parameter. A value of NaN (or ' ' in a cell array) indicates that a contract is not included in any netting set unless otherwise specified by NettingID. counterparties is case insensitive and leading or trailing white spaces are removed.

Data Types: double | cell

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Example: [exposures,exposurecpty] = creditexposures(values,counterparties,'NettingID','10','ExposureType ','Additive')

## NettingID - Netting set IDs indicate which netting set each contract belongs vector | cell array

Netting set IDs to indicate to which netting set each contract in values belongs, specified by a NumContracts-element vector of netting set IDs. NettingID can be a vector of numeric IDs or else a cell array of character vector identifiers. The creditexposures function uses counterparties and NettingID to define each unique netting set (all contracts in a netting set must be with the same counterparty). By default, each counterparty has a single netting set which covers all of their contracts. A value of NaN (or ' ' in a cell array) indicates that a contract is not included in any netting set. NettingID is case insensitive and leading or trailing white spaces are removed.

Data Types: double | cell

## ExposureType - Calculation method for exposures

'Counterparty ' (default) | character vector with value of 'Counterparty' or 'Additive'

Calculation method for exposures, specified with values:

- 'Counterparty' - Compute exposures per counterparty.
- 'Additive' - Compute additive exposures at the contract level. Exposures are computed per contract and sum to the total counterparty exposure.


## Data Types: char

## CollateralTable - Table containing information on collateral agreements of counterparties

## MATLAB table

Table containing information on collateral agreements of counterparties, specified as a MATLAB table. The table consists of one entry (row) per collateralized counterparty and must have the following variables (columns):

- 'Counterparty ' - Counterparty name or ID. The Counterparty name or ID should match the parameter 'Counterparty' for the ExposureType argument.
- 'PeriodOfRisk' - Margin period of risk in days. The number of days from a margin call until the posted collateral is available from the counterparty.
- 'Threshold ' - Collateral threshold. When counterparty exposures exceed this amount, the counterparty must post collateral.
- 'MinimumTransfer' - Minimum transfer amount. The minimum amount over/under the threshold required to trigger transfer of collateral.

Note When computing collateralized exposures, both the CollateralTable parameter and the Dates parameter must be specified.

## Data Types: table

## Dates - Simulation dates corresponding to each row of the values array

vector of date numbers | cell array of character vectors
Simulation dates corresponding to each row of the values array, specified as a NUMDATES-by-1 vector of simulation dates. Dates is either a vector of MATLAB date
numbers or else a cell array of character vectors in a known date format. See datenum for known date formats.

Note When computing collateralized exposures, both the CollateralTable parameter and the Dates parameter must be specified.

## Data Types: double |cell

## Output Arguments

## exposures - 3-D array of credit exposures

array
3-D array of credit exposures representing the potential losses from each counterparty or contract at each date and over all scenarios. The size of exposures depends on the ExposureType input argument:

- When ExposureType is 'Counterparty', exposures returns a NumDates-by-NumCounterparties-by-NumScenarios "cube" of credit exposures representing potential losses that could be incurred over all dates, counterparties, and scenarios, if a counterparty defaulted (ignoring any post-default recovery).
- When ExposureType is 'Additive', exposures returns a NumDates-by-NumContracts-by-NumScenarios "cube," where each element is the additive exposure of each contract (over all dates and scenarios). Additive exposures sum to the counterparty-level exposure.


## exposurecpty - Counterparties that correspond to columns of exposures array

 vectorCounterparties that correspond to columns of the exposures array, returned as NumCounterparties or NumContracts elements depending on the ExposureType.

## collateral - Simulated collateral amounts available to counterparties at each simulation date and over each scenario <br> 3D array

Simulated collateral amounts available to counterparties at each simulation date and over each scenario, returned as a NumDates-by-NumCounterparties-by-NumScenarios 3D
array. Collateral amounts are calculated using a Brownian bridge to estimate contract values between simulation dates. For more information, see "Brownian Bridge" on page 19-506. If the CollateralTable was not specified, this output is empty.

## Definitions

## Brownian Bridge

A Brownian bridge is used to simulate portfolio values at intermediate dates to compute collateral available at the subsequent simulation dates.

For example, to estimate collateral available at a particular simulation date, $t_{i}$, you need to know the state of the portfolio at time $t_{i}-d t$, where $d t$ is the margin period of risk. Portfolio values are simulated at these intermediate dates by drawing from a distribution defined by the Brownian bridge between $t_{i}$ and the previous simulation date, $t_{i-1}$.

If the contract values at time $t_{i-1}$ and $t_{i}$ are known and you want to estimate the contract value at time $t_{c}$ (where $t_{c}$ is $t_{i}-d t$ ), then a sample from a normal distribution is used with variance:

$$
\frac{\left(t_{i}-t_{c}\right)\left(t_{c}-t_{i-1}\right)}{\left(t_{i}-t_{i-1}\right)}
$$

and with mean that is simply the linear interpolation of the contract values between the two simulation dates at time $t_{c}$. For more details, see References.

## References

[1] Lomibao, D., and S. Zhu. "A Conditional Valuation Approach for Path-Dependent Instruments." August 2005.
[2] Pykhtin M. "Modeling credit exposure for collateralized counterparties." December 2009.
[3] Pykhtin M., and S. Zhu. "A Guide to Modeling Counterparty Credit Risk." GARP, July/ August 2007, issue 37.
[4] Pykhtin, Michael., and Dan Rosen. "Pricing Counterparty Risk at the Trade Level and CVA Allocations." FEDS Working Paper No. 10., February 1, 2010.
See Alsodatenum | exposureprofiles|table
Topics"Counterparty Credit Risk and CVA" (Financial Instruments Toolbox)"Wrong Way Risk with Copulas" (Financial Instruments Toolbox)
Introduced in R2014a

## exposureprofiles

Compute exposure profiles from credit exposures

## Syntax

profilestructs = exposureprofiles(dates,exposures) profilestructs = exposureprofiles( $\qquad$ , Name, Value)

## Description

profilestructs = exposureprofiles(dates, exposures) computes common counterparty credit exposures profiles from an array of exposures.
profilestructs $=$ exposureprofiles (__ , Name, Value) adds optional namevalue arguments.

## Examples

## View Exposure Profiles of a Particular Counterparty

After computing the mark-to-market contract values for a portfolio of swaps over many scenarios, view the exposure profiles of a particular counterparty.

First, load data (ccr.mat) containing the mark-to-market contract values for a portfolio of swaps over many scenarios.

```
load ccr.mat
```

Compute the exposure by counterparty.

```
[exposures, expcpty] = creditexposures(values,swaps.Counterparty,...
'NettingID',swaps.NettingID);
```

Compute the credit exposure profiles for all counterparties.

```
cpProfiles = exposureprofiles(simulationDates,exposures)
cpProfiles = 5x1 struct array with fields:
    Dates
    EE
    PFE
    MPFE
    EffEE
    EPE
    EffEPE
```

Visualize the exposure profiles for a particular counterparty.

```
cpIdx = find(expcpty == 4);
numDates = numel(simulationDates);
plot(simulationDates,cpProfiles(cpIdx).PFE,...
    simulationDates,cpProfiles(cpIdx).MPFE * ones(numDates,1),...
    simulationDates,cpProfiles(cpIdx).EE,...
    simulationDates,cpProfiles(cpIdx).EPE * ones(numDates,1),...
    simulationDates,cpProfiles(cpIdx).EffEE,...
    simulationDates,cpProfiles(cpIdx).EffEPE * ones(numDates,1));
legend({'PFE (95%)','Max PFE','Exp Exposure (EE)',...
    'Time-Avg EE (EPE)','Max past EE (EffEE)',...
    'Time-Avg EffEE (EffEPE)'})
datetick('x','mmmyy','keeplimits')
title(sprintf('Counterparty %d Exposure Profiles',cpIdx));
ylabel('Exposure ($)')
xlabel('Simulation Dates')
```



## Input Arguments

## dates - Simulation dates

vector of date numbers | cell array of character vectors
Simulation dates, specified as vector of date numbers or a cell array of character vectors in a known date format. For more information for known date formats, see the function datenum.

Data Types: double | char | cell

## exposures - 3-D array of potential losses due to counterparty default array

3-D array of potential losses due to counterparty default on a set of instruments simulated over a series of simulation dates and across many scenarios, specified as a NumDates-by-NumCounterParties-by-NumScenarios "cube" of credit exposures. Each row represents a different simulation date, each column a different counterparty, and each "page" is a different scenario from a Monte-Carlo simulation.

## Data Types: double

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1,Value1, . . . ,NameN, ValueN.

Example: profilestructs = exposureprofiles(dates,exposures,'ProfileSpec','PFE','PFEProbability Level',.9)

## ProfileSpec - Exposure profiles

All (generate all profiles) (default) | character vector with possible values EE, PFE, MPE, Effee, EPE, EffePE, All | cell array of character vectors with possible values EE, PFE, MPE, EffEE, EPE, EffePE

Exposure profiles, specified as a character vector or cell array of character vectors with the following possible values:

- EE - Expected Exposure. The mean of the distribution of exposures at each date. A [NumDates-by-1] vector.
- PFE - Potential Future Exposure. A high percentile (default 95\%) of the distribution of possible exposures at each date. This is sometimes referred to as "Peak Exposure." A [NumDates-by-1] vector.
- MPFE - Maximum Potential Future Exposure. The maximum potential future exposure (PFE) over all dates
- EffEE - Effective Expected Exposure. The maximum expected exposure (at a specific date) that occurs at that date or any prior date. This is the expected exposure, but constrained to be nondecreasing over time. A [NumDates-by-1] vector.
- EPE - Expected Positive Exposure. The weighted average over time of expected exposures. A scalar.
- EffEPE - Effective Expected Positive Exposure. The weighted average over time of the effective expected exposure (EffEE). A scalar.
- All - Generate all the previous profiles.

> Note Exposure profiles are computed on a per-counterparty basis.

## Data Types: char | cell

## PFEProbabilityLevel - Level for potential future exposure (PFE) and maximum potential future exposure (MPFE)

.95 (the 95th percentile) (default) | scalar with value [0..1]
Level for potential future exposure (PFE) and maximum potential future exposure (MPFE), specified as a scalar with value [0..1].
Data Types: double

## Output Arguments

## profilestructs - Structure of credit exposure profiles

array of structs holding credit exposure profiles for each counterparty
Structure of credit exposure profiles, returned as an array of structs holding credit exposure profiles for each counterparty, returned as a struct, with the fields of the struct as the (abbreviated) names of every exposure profile. Profiles listed in the ProfileSpec (and their related profiles) are populated, while those not requested contain empty ([]). profilestructs contains the dates information as a vector of MATLAB date numbers requested in the ProfileSpec argument.

## References

[1] Basel II: International Convergence of Capital Measurement and Capital Standards: A Revised Framework - Comprehensive Version. at https://www.bis.org/publ/ bcbs128.htm, 2006.
See Alsocreditexposures|datenum
Topics"Counterparty Credit Risk and CVA" (Financial Instruments Toolbox)"Wrong Way Risk with Copulas" (Financial Instruments Toolbox)
Introduced in R2014a

## cdyield

Yield on certificate of deposit (CD)

## Syntax

```
Yield = cdyield(Price,CouponRate,Settle,Maturity,IssueDate)
Yield = cdyield(
```

$\qquad$

``` ,Basis)
```


## Description

Yield = cdyield(Price,CouponRate,Settle, Maturity,IssueDate) computes the yield to maturity of a certificate of deposit given its clean price.
cdyield assumes that the certificates of deposit pay interest at maturity. Because of the simple interest treatment of these securities, this function is best used for short-term maturities (less than 1 year). The default simple interest calculation uses the Basis for the actual/360 convention (2).

Yield $=$ cdyield( __ , Basis) adds an optional argument for Basis.

## Examples

## Compute the Yield to Maturity of a Certificate of Deposit

This example shows how to compute the yield on the certificate of deposit (CD), given a CD with the following characteristics.

```
Price = 101.125;
CouponRate = 0.05;
Settle = '02-Jan-02';
Maturity = '31-Mar-02';
IssueDate = '1-0ct-01';
Yield = cdyield(Price, CouponRate, Settle, Maturity, IssueDate)
```


## Compute the Yield to Maturity of a Certificate of Deposit Using datetime Inputs

This example shows how to use datetime inputs to compute the yield on the certificate of deposit (CD), given a CD with the following characteristics.

```
Price = 101.125;
CouponRate = 0.05;
Settle = datetime('02-Jan-02','Locale','en US');
Maturity = datetime('31-Mar-02','Locale','en US');
IssueDate = datetime('1-0ct-01','Locale','en_US');
Yield = cdyield(Price, CouponRate, Settle, Maturity, IssueDate)
Yield = 0.0039
```


## Input Arguments

## Price - Clean price of certificate of deposit per \$100 face

numeric
Clean price of the certificate of deposit per $\$ 100$ face, specified as a numeric value using a scalar or a NCDS-by-1 or 1-by-NCDS vector.

## Data Types: double

## CouponRate - Coupon annual interest rate decimal

Coupon annual interest rate, specified as decimal using a scalar or a NCDS-by-1 or 1-byNCDS vector.

## Data Types: double

## Settle - Settlement date of certificate of deposit <br> serial date number | date character vector | datetime

Settlement date of the certificate of deposit, specified as a scalar or a NCDS-by-1 or 1-byNCDS vector using serial date numbers, date character vectors, or datetime arrays. The Settle date must be before the Maturity date.
Data Types: double|char|datetime

## Maturity - Maturity date of certificate of deposit

serial date number | date character vector | datetime
Maturity date of the certificate of deposit, specified as a scalar or a NCDS-by-1 or 1-byNCDS vector using serial date numbers, date character vectors, or datetime arrays.

Data Types: double | char | datetime

## IssueDate - Issue date for certificate of deposit <br> serial date number | date character vector | datetime

Issue date for the certificate of deposit, specified as a scalar or a NCDS-by-1 or 1-by-NCDS vector using serial date numbers, date character vectors, or datetime arrays.

## Data Types: double|char|datetime

## Basis - Day-count basis for certificate of deposit

```
2 (actual/360) (default)| integers of the set [0. . .13] | vector of integers of the set [0...13]
```

(Optional) Day-count basis for the certificate of deposit, specified as a scalar or a NINST-by-1 vector. Values are:

- $0=$ actual/actual
- 1 = 30/360 (SIA)
- $2=$ actual/360
- $3=$ actual/365
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- $10=$ actual/365 (ICMA)
- $11=30 / 360 \mathrm{E}$ (ICMA)
- $12=$ actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-0
Data Types: double

## Output Arguments

## Yield - Simple yield to maturity of certificate of deposit <br> numeric

Simple yield to maturity of the certificate of deposit, returned as a NCDS-by-1 or 1-byNCDS vector.

## See Also

bndprice|cdai|cdprice|datetime| stepcpnprice|tbillprice

## Topics

"Coupon Date Calculations" on page 2-31
"Yield Conventions" on page 2-31

Introduced before R2006a

## cfamounts

Cash flow and time mapping for bond portfolio


#### Abstract

Note In R2017b, the specification of optional input arguments has changed. While the previous ordered inputs syntax is still supported, it may no longer be supported in a future release. Use the optional name-value pair inputs: Period, Basis, EndMonthRule, IssueDate,FirstCouponDate, LastCouponDate, StartDate,Face, AdjustCashFlowsBasis, BusinessDayConvention, CompoundingFrequency, DiscountBasis, Holidays, and PrincipalType.


## Syntax

[CFlowAmounts,CFlowDates,TFactors,CFlowFlags,CFPrincipal] = cfamounts(CouponRate, Settle, Maturity)
[CFlowAmounts,CFlowDates,TFactors,CFlowFlags,CFPrincipal] = cfamounts( $\qquad$ , Name, Value)

## Description

[CFlowAmounts,CFlowDates,TFactors,CFlowFlags,CFPrincipal] = cfamounts(CouponRate, Settle,Maturity) returns matrices of cash flow amounts, cash flow dates, time factors, and cash flow flags for a portfolio of NUMBONDS fixed-income securities.

The elements contained in the cfamounts outputs for the cash flow matrix, time factor matrix, and cash flow flag matrix correspond to the cash flow dates for each security. The first element of each row in the cash flow matrix is the accrued interest payable on each bond. This accrued interest is zero in the case of all zero coupon bonds. cfamounts determines all cash flows and time mappings for a bond whether or not the coupon structure contains odd first or last periods. All output matrices are padded with NaNs as necessary to ensure that all rows have the same number of elements.
[CFlowAmounts,CFlowDates,TFactors,CFlowFlags,CFPrincipal] = cfamounts( $\qquad$ , Name, Value) adds optional name-value arguments.

## Examples

## Compute the Cash Flow Structure and Time Factors for a Bond Portfolio

This example shows how to compute the cash flow structure and time factors for a bond portfolio that contains a corporate bond paying interest quarterly and a Treasury bond paying interest semiannually.

```
Settle = '01-Nov-1993';
Maturity = ['15-Dec-1994';'15-Jun-1995'];
CouponRate= [0.06; 0.05];
Period = [4; 2];
Basis = [1; 0];
[CFlowAmounts, CFlowDates, TFactors, CFlowFlags] = ...
cfamounts(CouponRate,Settle, Maturity, Period, Basis)
CFlowAmounts = 2×6
\begin{tabular}{rrrrrr}
-0.7667 & 1.5000 & 1.5000 & 1.5000 & 1.5000 & 101.5000 \\
-1.8989 & 2.5000 & 2.5000 & 2.5000 & 102.5000 & NaN
\end{tabular}
CFlowDates = 2×6
\begin{tabular}{rrrrrr}
728234 & 728278 & 728368 & 728460 & 728552 & 728643 \\
728234 & 728278 & 728460 & 728643 & 728825 & NaN
\end{tabular}
TFactors = 2×6
\begin{tabular}{rrrrrr}
0 & 0.2404 & 0.7403 & 1.2404 & 1.7403 & 2.2404 \\
0 & 0.2404 & 1.2404 & 2.2404 & 3.2404 & NaN
\end{tabular}
CFlowFlags = 2×6
\begin{tabular}{rrrrrr}
0 & 3 & 3 & 3 & 3 & 4 \\
0 & 3 & 3 & 3 & 4 & NaN
\end{tabular}
```

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## Compute the Cash Flow Structure and Time Factors for a Bond Portfolio and Return a datetime array for CFlowDates

This example shows how to compute the cash flow structure and time factors for a bond portfolio that contains a corporate bond paying interest quarterly and a Treasury bond paying interest semiannually and CFlowDates is returned as a datetime array.

```
Settle = datetime('01-Nov-1993','Locale','en US');
Maturity = ['15-Dec-1994';'15-Jun-1995'];
CouponRate= [0.06; 0.05];
Period = [4; 2];
Basis = [1; 0];
[CFlowAmounts, CFlowDates, TFactors, CFlowFlags] = cfamounts(CouponRate,...
Settle, Maturity, Period, Basis)
CFlowAmounts = 2×6
\begin{tabular}{rrrrrr}
-0.7667 & 1.5000 & 1.5000 & 1.5000 & 1.5000 & 101.5000 \\
-1.8989 & 2.5000 & 2.5000 & 2.5000 & 102.5000 & NaN
\end{tabular}
CFlowDates = 2x6 datetime array
Columns 1 through 5
01-Nov-1993 15-Dec-1993 15-Mar-1994 15-Jun-1994 15-Sep-1994
01-Nov-1993 15-Dec-1993 15-Jun-1994 15-Dec-1994 15-Jun-1995
Column 6
15-Dec-1994
NaT
TFactors = 2×6
\begin{tabular}{lllllr}
0 & 0.2404 & 0.7403 & 1.2404 & 1.7403 & 2.2404 \\
0 & 0.2404 & 1.2404 & 2.2404 & 3.2404 & NaN
\end{tabular}
CFlowFlags = \(2 \times 6\)
\begin{tabular}{rrrrrr}
0 & 3 & 3 & 3 & 3 & 4 \\
0 & 3 & 3 & 3 & 4 & NaN
\end{tabular}
```


## Compute the Cash Flow Structure and Time Factors for a Bond Portfolio Using Optional Name-Value Pairs

This example shows how to compute the cash flow structure and time factors for a bond portfolio that contains a corporate bond paying interest quarterly and a Treasury bond paying interest semiannually. This example uses the following Name-Value pairs for Period, Basis, BusinessDayConvention, and AdjustCashFlowsBasis.

```
Settle = '01-Jun-2010';
Maturity = ['15-Dec-2011';'15-Jun-2012'];
CouponRate= [0.06; 0.05];
Period = [4; 2];
Basis = [1; 0];
```

[CFlowAmounts, CFlowDates, TFactors, CFlowFlags] = ...
cfamounts(CouponRate, Settle, Maturity, 'Period',Period, ...
'Basis', Basis, 'AdjustCashFlowsBasis', true,...
'BusinessDayConvention','modifiedfollow')
CFlowAmounts = $2 \times 8$

| -1.2667 | 1.5000 | 1.5000 | 1.5000 | 1.5000 | 1.5000 | 1.5000 | 101.5000 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| -2.3077 | 2.4932 | 2.5068 | 2.4932 | 2.5000 | 102.5000 | NaN | NaN |

CFlowDates $=2 \times 8$

| 734290 | 734304 | 734396 | 734487 | 734577 | 734669 | 734761 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 734290 | 734304 | 734487 | 734669 | 734852 | 735035 | NaN |

TFactors $=2 \times 8$

| 0 | 0.0778 | 0.5778 | 1.0778 | 1.5778 | 2.0778 | 2.5778 | 3.0778 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 0 | 0.0769 | 1.0769 | 2.0769 | 3.0769 | 4.0769 | NaN | NaN |

CFlowFlags $=2 \times 8$

| 0 | 3 | 3 | 3 | 3 | 3 | 3 | 4 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 0 | 3 | 3 | 3 | 3 | 4 | NaN | NaN |

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## Use cfamounts With a CouponRate Schedule

This example shows how to use cfamounts with a CouponRate schedule. For CouponRate and Face that change over the life of the bond, schedules for CouponRate and Face can be specified with an NINST-by-1 cell array, where each element is a NumDates-by-2 matrix where the first column is dates and the second column is associated rates.

```
CouponSchedule = {[datenum('15-Mar-2012') .04;datenum('15- Mar - 2013') .05;...
```

datenum('15- Mar -2015') .06]\}
CouponSchedule = 1x1 cell array
$\{3 \times 2$ double $\}$
cfamounts(CouponSchedule,'01-Mar-2011','15-Mar-2015' )
ans $=1 \times 10$

| -1.8453 | 2.0000 | 2.0000 | 2.0000 | 2.5000 | 2.5000 | 3.0000 | 3.0000 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

## Use cfamounts With a Face Schedule

This example shows how to use cfamounts with a Face schedule. For CouponRate and Face that change over the life of the bond, schedules for CouponRate and Face can be specified with an NINST-by-1 cell array, where each element is a NumDates-by-2 matrix where the first column is dates and the second column is associated rates.

```
FaceSchedule = {[datenum('15-Mar-2012') 100;datenum('15- Mar -2013') 90;...
datenum('15- Mar -2015') 80]}
FaceSchedule = lx1 cell array
    {3\times2 double}
cfamounts(.05,'01-Mar-2011','15-Mar-2015', 'Face', FaceSchedule)
ans = 1\times10
```

$-2.3066$
2.5000
2.5000
12.5000
2.2500
12.2500
2.0000
2.0000
2.

## Use cfamounts to Generate the Cash Flows for a Sinking Bond

This example shows how to use cfamounts to generate the cash flows for a sinking bond.
[CFlowAmounts,CFDates,TFactors,CFFlags,CFPrincipal] = cfamounts(.05,'04-Nov-2010',... \{'15-Jul-2014';'15-Jul-2015'\},'Face',\{[datenum('15-Jul-2013') 100; datenum('15-Jul-2014 90;datenum('15-Jul-2015') 80]\})

CFlowAmounts = $2 \times 11$

| -1.5217 | 2.5000 | 2.5000 | 2.5000 | 2.5000 | 2.5000 | 12.5000 | 2.2500 | 92.2 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| -1.5217 | 2.5000 | 2.5000 | 2.5000 | 2.5000 | 2.5000 | 12.5000 | 2.2500 | 12.2 |

CFDates $=2 \times 11$

| 734446 | 734518 | 734699 | 734883 | 735065 | 735249 | 735430 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 734446 | 734518 | 734699 | 734883 | 735065 | 735249 | 735430 |

TFactors $=2 \times 11$

| 0 | 0.3913 | 1.3913 | 2.3913 | 3.3913 | 4.3913 | 5.3913 | 6.3913 | 7.3 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | 0.3913 | 1.3913 | 2.3913 | 3.3913 | 4.3913 | 5.3913 | 6.3913 | 7.3 |

CFFlags = $2 \times 11$
$\begin{array}{lllllllllll}0 & 3 & 3 & 3 & 3 & 3 & 13 & 3 & 4 & \mathrm{NaN} & \mathrm{NaN}\end{array}$
$\begin{array}{lllllllllll}0 & 3 & 3 & 3 & 3 & 3 & 13 & 3 & 13 & 3 & 4\end{array}$

CFPrincipal = 2×11

| 0 | 0 | 0 | 0 | 0 | 0 | 10 | 0 | 90 | NaN | NaN |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 0 | 0 | 0 | 0 | 0 | 0 | 10 | 0 | 10 | 0 | 80 |

## Input Arguments

## CouponRate - Annual percentage rate used to determine coupons payable on a bond

decimal
Annual percentage rate used to determine the coupons payable on a bond, specified as decimal using a scalar or a NBONDS-by-1 vector.

CouponRate is 0 for zero coupon bonds.


#### Abstract

Note CouponRate and Face can change over the life of the bond. Schedules for CouponRate and Face can be specified with an NBONDS-by-1 cell array, where each element is a NumDates-by-2 matrix or cell array, where the first column is dates (serial date numbers or character vectors) and the second column is associated rates. The date indicates the last day that the coupon rate or face value is valid. This means that the corresponding CouponRate and Face value applies "on or before" the specified ending date.


## Data Types: double | cell|char

## Settle - Settlement date of bond <br> serial date number | date character vector | datetime

Settlement date of the bond, specified as a scalar or a NBONDS-by-1 vector using serial date numbers, date character vectors, or datetime arrays. The Settle date must be before the Maturity date.

Data Types: double | char|datetime

```
Maturity - Maturity date of bond
serial date number | date character vector | datetime
```

Maturity date of the bond, specified as a scalar or a NBONDS-by-1 vector using serial date numbers, date character vectors, or datetime arrays.

Data Types: double|char|datetime

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Example: [CFlowAmounts, CFlowDates, TFactors, CFlowFlags] = ... cfamounts(CouponRate,Settle, Maturity,'Period',4,'Basis', 3,'AdjustCashFlowsBasis',true,'BusinessDayConvention','modifiedfollo W')

## Period - Number of coupon payments per year for bond

2 (default) | numeric with values $0,1,2,3,4,6$ or 12

Number of coupon payments per year for the bond, specified as the comma-separated pair consisting of 'Period ' and a scalar or a NBONDS-by-1 vector using the values: 0, 1, $2,3,4,6$, or 12 .

Data Types: double

## Basis - Day-count basis of bond

```
0 (default) | numeric values: 0,1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 13
```

Day-count basis of the bond, specified as the comma-separated pair consisting of 'Basis' and a scalar or a NBONDS-by-1 vector using a supported value:

- $0=$ actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- $11=30 / 360 \mathrm{E}$ (ICMA)
- $12=$ actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-0
Data Types: double

## EndMonthRule - End-of-month rule flag

1 (in effect) (default) | nonnegative integer 0 or 1
End-of-month rule flag, specified as the comma-separated pair consisting of 'EndMonthRule' and a scalar or a NBONDS-by-1 vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days.

- $0=$ Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- $1=$ Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.


## Data Types: logical

## IssueDate - Bond issue date <br> serial date number | date character vector | datetime

Bond issue date (the date the bond begins to accrue interest), specified as the commaseparated pair consisting of 'IssueDate' and a scalar or a NBONDS-by-1 vector using serial date numbers, date character vectors, or datetime arrays. The IssueDate cannot be after the Settle date.

If you do not specify an IssueDate, the cash flow payment dates are determined from other inputs.

Data Types: double | char | datetime

## FirstCouponDate - Irregular or normal first coupon date serial date number | date character vector | datetime

Irregular or normal first coupon date, specified as the comma-separated pair consisting of 'FirstCouponDate' and a scalar or a NBONDS-by-1 vector using serial date numbers, date character vectors, or datetime arrays.

If you do not specify a FirstCouponDate, the cash flow payment dates are determined from other inputs.

Note When FirstCouponDate and LastCouponDate
are both specified, the FirstCouponDate takes precedence in determining the coupon payment structure. If FirstCouponDate is not specified, then LastCouponDate determines the coupon structure of the bond.

## Data Types: double | char |datetime

## LastCouponDate - Irregular or normal last coupon date <br> serial date number | date character vector | datetime

Irregular or normal last coupon date, specified as the comma-separated pair consisting of ' LastCouponDate' and a scalar or a NBONDS-by-1 vector using serial date numbers, date character vectors, or datetime arrays.

Note When FirstCouponDate and LastCouponDate are both specified, the FirstCouponDate takes precedence in determining the coupon payment structure. If FirstCouponDate is not specified, then LastCouponDate determines the coupon structure of the bond.

## Data Types: double | char | datetime

## StartDate - Forward starting date of coupon payments

serial date number | date character vector | datetime
Forward starting date of coupon payments after the Settle date, specified as the comma-separated pair consisting of 'StartDate' and a scalar or a NBONDS-by-1 vector using serial date numbers, date character vectors, or datetime arrays.

Note To make an instrument forward starting, specify StartDate as a future date.

If you do not specify a StartDate, the effective start date is the Settle date.
Data Types: double|char|datetime

## Face - Face value of bond

100 (default) | numeric
Face value of the bond, specified as the comma-separated pair consisting of 'Face' and a scalar or a NBONDS-by-1 vector.

Note CouponRate and Face can change over the life of the bond. Schedules for CouponRate and Face can be specified with an NBONDS-by-1 cell array where each element is a NumDates-by-2 matrix or cell array, where the first column is dates (serial date numbers or character vectors) and the second column is associated rates. The date indicates the last day that the coupon rate or face value is valid. This means that the corresponding CouponRate and Face value applies "on or before" the specified ending date.

When the corresponding Face value is used to compute the coupon cashflow on the specified ending date. Three things happen on the specified ending date:

1 The last coupon corresponding to the current Face value is paid.
2 The principal differential (between the current and the next Face value) is paid.
3 The date marks the beginning of the period with the next Face value, for which the cashflow does not occur until later.

## Data Types: double | cell|char

## AdjustCashFlowsBasis - Adjusts cash flows according to accrual amount based on actual period day count

```
0 (false) (default) | nonnegative integer 0 or 1
```

Adjusts cash flows according to the accrual amount based on the actual period day count, specified as the comma-separated pair consisting of 'AdjustCashFlowsBasis' and a scalar or a NBONDS-by-1 vector.
Data Types: logical

## BusinessDayConvention - Business day conventions

'actual' (default)| character vector with values'actual', 'follow', 'modifiedfollow', 'previous'or 'modifiedprevious'

Business day conventions, specified as the comma-separated pair consisting of 'BusinessDayConvention' and a scalar or NBONDS-by-1 cell array of character vectors
of business day conventions to be used in computing payment dates. The selection for business day convention determines how nonbusiness days are treated. Nonbusiness days are defined as weekends plus any other date that businesses are not open (for example, statutory holidays). Values are:

- 'actual' - Nonbusiness days are effectively ignored. Cash flows that fall on nonbusiness days are assumed to be distributed on the actual date.
- 'follow' - Cash flows that fall on a nonbusiness day are assumed to be distributed on the following business day.
- 'modifiedfollow' - Cash flows that fall on a non-business day are assumed to be distributed on the following business day. However if the following business day is in a different month, the previous business day is adopted instead.
- 'previous ' - Cash flows that fall on a nonbusiness day are assumed to be distributed on the previous business day.
- 'modifiedprevious' - Cash flows that fall on a nonbusiness day are assumed to be distributed on the previous business day. However if the previous business day is in a different month, the following business day is adopted instead.


## Data Types: char | cell

## CompoundingFrequency - Compounding frequency for yield calculation

## SIA uses2, ICMA uses 1 (default) | integer with value of $1,2,3,4,6$, or 12

Compounding frequency for yield calculation, specified as the comma-separated pair consisting of 'CompoundingFrequency' and a scalar or a NBONDS-by-1 vector. Values are:

- 1 - Annual compounding
- 2 - Semiannual compounding
- 3-Compounding three times per year
- 4-Quarterly compounding
- 6 - Bimonthly compounding
- 12 - Monthly compounding

[^7]
## Data Types: double

## DiscountBasis - Basis used to compute the discount factors for computing the yield

SIA uses 0 (default) | integers of the set [0...13] | vector of integers of the set [0...13]

Basis used to compute the discount factors for computing the yield, specified as the comma-separated pair consisting of 'DiscountBasis ' and a scalar or a NBONDS-by-1 vector. Values are:

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- $2=$ actual/360
- $3=$ actual/365
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- $7=$ actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- $9=\mathrm{actual} / 360$ (ICMA)
- $10=$ actual/365 (ICMA)
- $11=30 / 360 \mathrm{E}$ (ICMA)
- $12=$ actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-0

Note If a SIA day-count basis is defined in the Basis input argument and there is no value assigned for DiscountBasis, the default behavior is for SIA bases to use the actual/actual day count to compute discount factors.

If an ICMA day-count basis or BUS/252 is defined in the Basis input argument and there is no value assigned for DiscountBasis, the specified bases from the Basis input argument are used.

## Data Types: double

## Holidays - Dates for holidays <br> holidays.m used (default)

Dates for holidays, specified as the comma-separated pair consisting of 'Holidays ' and a NHOLIDAYS-by-1 vector of MATLAB dates using serial date numbers, date character vectors, or datetime arrays. Holidays are used in computing business days.

Data Types: double | char | datetime
PrincipalType - Type of principal when a Face schedule is specified
sinking (default)|character vector with values 'sinking' or 'bullet'
Type of principal when a Face schedule, specified as the comma-separated pair consisting of 'PrincipalType' and a value of 'sinking' or 'bullet' using a scalar or a NBONDS-by-1 vector.

If 'sinking ', principal cash flows are returned throughout the life of the bond.
If 'bullet', principal cash flow is only returned at maturity.
Data Types: char |cell

## Output Arguments

CFlowAmounts - Cash flow amounts matrix

Cash flow amounts, returned as a NBONDS-by-NCFS (number of cash flows) matrix. The first entry in each row vector is the accrued interest due at settlement. This amount could be zero, positive or negative. If no accrued interest is due, the first column is zero. If the bond is trading ex-coupon then the accrued interest is negative.

## CFlowDates - Cash flow dates for a portfolio of bonds matrix

Cash flow dates for a portfolio of bonds, returned as a NBONDS-by-NCFS matrix. Each row represents a single bond in the portfolio. Each element in a column represents a cash flow date of that bond.

If all the above inputs (Settle, Maturity, IssueDate, FirstCouponDate, LastCouponDate, and StartDate) are either serial date numbers or date character
vectors, then CFlowDates is returned as a serial date number. If any of these inputs are datetime arrays, then CFlowDates is returned as a datetime array.

## TFactors - Matrix of time factors for a portfolio of bonds matrix

Matrix of time factors for a portfolio of bonds, returned as a NBONDS-by-NCFS matrix. Each row corresponds to the vector of time factors for each bond. Each element in a column corresponds to the specific time factor associated with each cash flow of a bond.

Time factors are for price/yield conversion and time factors are in units of whole semiannual coupon periods plus any fractional period using an actual day count. For more information on time factors, see "Time Factors" on page 19-533.

## CFlowFlags - Cash flow flags for a portfolio of bonds matrix

Cash flow flags for a portfolio of bonds, returned as a NBONDS-by-NCFS matrix. Each row corresponds to the vector of cash flow flags for each bond. Each element in a column corresponds to the specific flag associated with each cash flow of a bond. Flags identify the type of each cash flow (for example, nominal coupon cash flow, front, or end partial, or "stub" coupon, maturity cash flow).

| Flag | Cash Flow Type |
| :--- | :--- |
| 0 | Accrued interest due on a bond at settlement. |
| 1 | Initial cash flow amount smaller than normal due to a "stub" coupon <br> period. A stub period is created when the time from issue date to first <br> coupon date is shorter than normal. |
| 2 | Larger than normal initial cash flow amount because the first coupon <br> period is longer than normal. |
| 3 | Nominal coupon cash flow amount. |
| 4 | Normal maturity cash flow amount (face value plus the nominal coupon <br> amount). |
| 5 | End "stub" coupon amount (last coupon period is abnormally short and <br> actual maturity cash flow is smaller than normal). |
| 6 | Larger than normal maturity cash flow because the last coupon period <br> longer than normal. |


| Flag | Cash Flow Type |
| :--- | :--- |
| 7 | Maturity cash flow on a coupon bond when the bond has less than one <br> coupon period to maturity. |
| 8 | Smaller than normal maturity cash flow when the bond has less than <br> one coupon period to maturity. |
| 9 | Larger than normal maturity cash flow when the bond has less than one <br> coupon period to maturity. |
| 10 | Maturity cash flow on a zero coupon bond. |
| 11 | Sinking principal and initial cash flow amount smaller than normal due <br> to a "stub" coupon period. A stub period is created when the time from <br> issue date to first coupon date is shorter than normal. |
| 12 | Sinking principal and larger than normal initial cash flow amount <br> because the first coupon period is longer than normal. |
| 13 | Sinking principal and nominal coupon cash flow amount. |

CFPrincipal - Principal cash flows
matrix
Principal cash flows, returned as a NBONDS-by-NCFS matrix.
If PrincipalType is 'sinking', CFPrincipal output indicates when the principal is returned.

If PrincipalType is 'bullet', CFPrincipal is all zeros and, at Maturity, the appropriate Face value.

## Definitions

## Time Factors

Time factors help determine the present value of a stream of cash flows.
The term time factors refer to the exponent $T F$ in the discounting equation

$$
P V=\sum_{i=1}^{n}\left(\frac{C F}{\left(1+\frac{z}{f}\right)^{T F}}\right),
$$

where:

| $P V=$ | Present value of a cash flow. |
| :--- | :--- |
| $C F=$ | Cash flow amount. |
| $z=$ | Risk-adjusted annualized rate or yield corresponding to a given cash <br> flow. The yield is quoted on a semiannual basis. |
| $f=$ | Frequency of quotes for the yield. Default is 2 for Basis values 0 to 7 <br> and 13 and 1 for Basis values 8 to 12. The default can be overridden <br> by specifying the CompoundingFrequency name-value pair. |
| $T F=$ | Time factor for a given cash flow. The time factor is computed using the <br> compounding frequency and the discount basis. If these values are not <br> specified, then the defaults are as follows: CompoundingFrequency <br> default is 2 for Basis values 0 to 7 and 13 and 1 for Basis values 8 to <br> 12. <br> DiscountBasis is 0 for Basis values 0 to 7 and 13 and the value of <br> the input Basis for Basis values 8 to 12. |

Note The Basis is always used to compute accrued interest.

## References

[1] Krgin, D. Handbook of Global Fixed Income Calculations. Wiley, 2002.
[2] Mayle, J. "Standard Securities Calculations Methods: Fixed Income Securities Formulas for Analytic Measures." SIA, Vol 2, Jan 1994.
[3] Stigum, M., Robinson, F. Money Market and Bond Calculation. McGraw-Hill, 1996.
See Alsoaccrfrac|cfdates |cftimes | cpncount | cpndaten | cpndatenq| cpndatep|cpndatepq|cpndaysn|cpndaysp|datetime
Topics"Analyzing and Computing Cash Flows" on page 2-18
Introduced before R2006a

## cfconv

Cash flow convexity

## Syntax

CFlowConvexity $=$ cfconv(CashFlow,Yield)

## Arguments

| CashFlow | A vector of real numbers. |
| :--- | :--- |
| Yield | Periodic yield. A scalar. Enter as a decimal fraction. |

## Description

CFlowConvexity $=$ cfconv(CashFlow, Yield) returns the convexity of a cash flow in periods.

## Examples

## Compute the Convexity of a Cash Flow

This example shows how to return the convexity of a cash flow, given a cash flow of nine payments of $\$ 2.50$ and a final payment $\$ 102.50$, with a periodic yield of $2.5 \%$.

```
CashFlow = [2.5 2.5 2.5 2.5 2.5 2.5 2.5 2.5 2.5 102.5];
Convex = cfconv(CashFlow, 0.025)
Convex = 90.4493
```

See Also<br>bndconvp | bndconvy | bnddurp | bnddury | cfdur<br>\section*{Topics}<br>"Analyzing and Computing Cash Flows" on page 2-18<br>Introduced before R2006a

## cfdates

Cash flow dates for fixed-income security

## Syntax

CFlowDates = cfdates(Settle,Maturity,Period,Basis,EndMonthRule,IssueDate,FirstCouponDa

## Arguments

| Settle | Settlement date. A vector of serial date numbers, date <br> character vectors, or datetime arrays. Settle must be <br> earlier than Maturity. |
| :--- | :--- |
| Maturity | Maturity date. A vector of serial date numbers, date <br> character vectors, or datetime arrays. |
| Period | Optional) Coupons per year of the bond. A vector of <br> integers. Allowed values are 0, 1, 2 (default), 3, 4, 6, and 12. |


| Basis | (Optional) Day-count basis of the instrument. A vector of integers. <br> - $0=$ actual/actual (default) <br> - $1=30 / 360$ (SIA) <br> - 2 = actual/360 <br> - 3 = actual/ 365 <br> - $4=30 / 360$ (PSA) <br> - $5=30 / 360$ (ISDA) <br> - $6=30 / 360$ (European) <br> - 7 = actual/365 (Japanese) <br> - $8=$ actual/actual (ICMA) <br> - $9=\mathrm{actual} / 360$ (ICMA) <br> - $10=$ actual/365 (ICMA) <br> - $11=30 / 360 \mathrm{E}$ (ICMA) <br> - 12 = actual/365 (ISDA) <br> - 13 = BUS/252 <br> For more information, see basis on page Glossary-0 |
| :---: | :---: |
| EndMonthRule | (Optional) End-of-month rule. A vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days. $0=$ ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. $1=$ set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month. |
| IssueDate | (Optional) Date, specified as a serial date number, date character vector, or datetime array, when a bond was issued. |


| FirstCouponDate | (Optional) Date, specified as a serial date number, date <br> character vector, or datetime array, when a bond makes its <br> first coupon payment. FirstCouponDate is used when a <br> bond has an irregular first coupon period. When <br> FirstCouponDate and LastCouponDate are both <br> specified, FirstCouponDate takes precedence in <br> determining the coupon payment structure. If you do not <br> specify a FirstCouponDate, the cash flow payment dates <br> are determined from other inputs. |
| :--- | :--- |
| LastCouponDate | (Optional) Last coupon date of a bond before the maturity <br> date, specified as a serial date number, date character <br> vector, or datetime array. LastCouponDate is used when a <br> bond has an irregular last coupon period. In the absence of a <br> specified FirstCouponDate, a specified LastCouponDate <br> determines the coupon structure of the bond. The coupon <br> structure of a bond is truncated at the LastCouponDate, <br> regardless of where it falls, and is followed only by the <br> bond's maturity cash flow date. If you do not specify a <br> LastCouponDate, the cash flow payment dates are <br> determined from other inputs. |
| StartDate | (Optional) Date, specified as a serial date number, date <br> character vector, or datetime array, when a bond actually <br> starts (the date from which a bond cash flow is considered). <br> To make an instrument forward-starting, specify this date as <br> a future date. If you do not specify StartDate, the effective <br> start date is the Settle date. |

Required arguments must be number of bonds (NUMBONDS)-by-1 or 1-by-NUMBONDS conforming vectors or scalars. Optional arguments must be either NUMBONDS-by-1 or 1-by-NUMBONDS conforming vectors, scalars, or empty matrices.

Any input can contain multiple values, but if so, all other inputs must contain the same number of values or a single value that applies to all. For example, if Maturity contains N dates, then Settle must contain N dates or a single date.

## Description

CFlowDates =
cfdates(Settle, Maturity, Period, Basis,EndMonthRule, IssueDate,FirstCou ponDate, LastCouponDate, StartDate) returns a matrix of cash flow dates for a bond or set of bonds. cfdates determines all cash flow dates for a bond whether or not the coupon payment structure is normal or the first and/or last coupon period is long or short.

CFlowDates is an N-row matrix of serial date numbers, padded with NaNs as necessary to ensure that all rows have the same number of elements. Use the function datestr to convert serial date numbers to formatted date character vectors.

If all of the inputs for Settle, Maturity, IssueDate, FirstCouponDate, LastCouponDate, and StartDate are either serial date numbers or date character vectors, then CFlowDates is returned as a serial date number.

If any of the inputs for Settle, Maturity, IssueDate, FirstCouponDate, LastCouponDate, or StartDate are datetime arrays, then CFlowDates is returned as a datetime array.

Note The cash flow flags for a portfolio of bonds were formerly available as the cfdates second output argument, CFlowFlags. You can now use cfamounts to get these flags. If you specify a CFlowFlags argument, cfdates displays a message directing you to use cfamounts.

## Examples

```
CFlowDates = cfdates('14 Mar 1997', '30 Nov 1998', 2, 0, 1)
CFlowDates =
    729541 729724 729906 730089
datestr(CFlowDates)
ans =
31-May-1997
30-Nov-1997
31-May-1998
30-Nov-1998
```

If any of the inputs for Settle, Maturity, IssueDate, FirstCouponDate, LastCouponDate, or StartDate are datetime arrays, then CFlowDates is returned as a datetime array. For example:

```
CFlowDates = cfdates('14-Mar-1997', datetime('30-Nov-1998','Locale','en_US'), 2, 0, 1)
CFlowDates =
    31-May-1997 30-Nov-1997 31-May-1998 30-Nov-1998
```

Given three securities with different maturity dates and the same default arguments

```
Maturity = ['30-Sep-1997'; '31-Oct-1998'; '30-Nov-1998'];
CFlowDates = cfdates('14-Mar-1997', Maturity)
CFlowDates =
    729480 729663 NaN NaN
    729510 729694 729875 730059
    729541 729724 729906 730089
```

Look at the cash-flow dates for the last security.

```
datestr(CFlowDates(3,:))
```

ans $=$
31-May-1997
30-Nov-1997
31-May-1998
30-Nov-1998

## See Also

accrfrac|cfamounts |cftimes | cpncount | cpndaten |cpndatenq| cpndatep| cpndatepq|cpndaysn | cpndaysp|cpnpersz|datetime

## Topics

"Analyzing and Computing Cash Flows" on page 2-18

## Introduced before R2006a

## cfdatesq

Quasi-coupon dates for fixed-income security

## Syntax

QuasiCouponDates = cfdatesq(Settle,Maturity,Period,Basis,EndMonthRule,IssueDate,FirstC

## Arguments

| Settle | Settlement date. A vector of serial date numbers, date <br> character vectors, or datetime arrays. Settle must be <br> earlier than Maturity. |
| :--- | :--- |
| Maturity | Maturity date. A vector of serial date numbers, date <br> character vectors, or datetime arrays. |
| Period | (Optional) Coupons per year of the bond. A vector of <br> integers. Allowed values are 0, 1, 2 (default), 3, 4, 6, and 12. |


| Basis | (Optional) Day-count basis of the instrument. A vector of integers. <br> - $0=$ actual/actual (default) <br> - $1=30 / 360$ (SIA) <br> - 2 = actual/360 <br> - 3 = actual/365 <br> - $4=30 / 360$ (PSA) <br> - $5=30 / 360$ (ISDA) <br> - $6=30 / 360$ (European) <br> - 7 = actual/365 (Japanese) <br> - $8=$ actual/actual (ICMA) <br> - $9=$ actual/360 (ICMA) <br> - 10 = actual/365 (ICMA) <br> - $11=30 / 360 \mathrm{E}$ (ICMA) <br> - 12 = actual/365 (ISDA) <br> - 13 = BUS/252 <br> For more information, see basis on page Glossary-0 |
| :---: | :---: |
| EndMonthRule | (Optional) End-of-month rule. A vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days. $0=$ ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. $1=$ set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month. |
| IssueDate | (Optional) Date, specified as a serial date number, date character vector, or datetime array, when a bond was issued. |


| FirstCouponDate | (Optional) Date, specified as a serial date number, date <br> character vector, or datetime array, when a bond makes its <br> first coupon payment. FirstCouponDate is used when a <br> bond has an irregular first coupon period. When <br> FirstCouponDate and LastCouponDate are both <br> specified, FirstCouponDate takes precedence in <br> determining the coupon payment structure. If you do not <br> specify a FirstCouponDate, the cash flow payment dates <br> are determined from other inputs. |
| :--- | :--- |
| LastCouponDate | (Optional) Last coupon date of a bond before the maturity <br> date, specified as a serial date number, date character <br> vector, or datetime array. LastCouponDate is used when a <br> bond has an irregular last coupon period. In the absence of a <br> specified FirstCouponDate, a specified LastCouponDate <br> determines the coupon structure of the bond. The coupon <br> structure of a bond is truncated at the LastCouponDate, <br> regardless of where it falls, and is followed only by the <br> bond's maturity cash flow date. If you do not specify a <br> LastCouponDate, the cash flow payment dates are |
| determined from other inputs. |  |

Required arguments must be number of bonds (NUMBONDS)-by-1 or 1-by-NUMBONDS conforming vectors or scalars. Optional arguments must be either NUMBONDS-by-1 or 1-by-NUMBONDS conforming vectors, scalars, or empty matrices.

Any input can contain multiple values, but if so, all other inputs must contain the same number of values or a single value that applies to all. For example, if Maturity contains N dates, then Settle must contain N dates or a single date.

## Description

QuasiCouponDates = cfdatesq(Settle,Maturity, Period,Basis,EndMonthRule, IssueDate,FirstCo uponDate,LastCouponDate, PeriodsBeforeSettle,PeriodsAfterMaturity)
returns a matrix of quasi-coupon dates expressed in serial date format (default) or datetime format (if any inputs are in datetime format).

Successive quasi-coupon dates determine the length of the standard coupon period for the fixed-income security of interest, and do not necessarily coincide with actual coupon payment dates. Quasi-coupon dates are determined regardless of whether the first or last coupon periods are normal, long, or short.

QuasiCouponDates has NUMBONDS rows and the number of columns is determined by the maximum number of quasi-coupon dates required to hold the bond portfolio. NaNs are padded for bonds which have less than the maximum number quasi-coupon dates. By default, quasi- coupon dates after settlement and on or preceding maturity are returned. If settlement occurs on maturity, and maturity is a quasi-coupon date, then the maturity date is returned.

If the date inputs for Settle, Maturity, IssueDate, FirstCouponDate, and LastCouponDate are either serial date numbers or date character vectors, then QuasiCouponDates is returned as a serial date number.

If any of the date inputs for Settle, Maturity, IssueDate, FirstCouponDate, or LastCouponDate are datetime arrays, then QuasiCouponDates is returned as a datetime array.

## Examples

```
QuasiCouponDates = cfdatesq('14-Mar-1997', '30-Nov-1998', 2, 0, 1)
QuasiCouponDates =
    729541 729724 729906 730089
```

If any of the inputs for Settle, Maturity, IssueDate, FirstCouponDate, or LastCouponDate are datetime arrays, then CFlowDates is returned as a datetime array. For example:

QuasiCouponDates = cfdatesq('14-Mar-1997', datetime('30-Nov-1998','Locale','en_US'), 2, 0, 1)
QuasiCouponDates =
31-May-1997 30-Nov-1997 31-May-1998 30-Nov-1998
See Alsoaccrfrac|cfamounts |cftimes | cpncount | cpndaten | cpndatenq| cpndatep|cpndatepq| cpndaysn|cpndaysp|cpnpersz|datetime
Topics"Analyzing and Computing Cash Flows" on page 2-18
Introduced before R2006a

## cfdur

Cash-flow duration and modified duration

## Syntax

[Duration, ModDuration] = cfdur(CashFlow, Yield)

## Arguments

| CashFlow | A vector or matrix of real numbers. When using a matrix, each <br> column of the matrix is a separate CashFlow. |
| :--- | :--- |
| Yield | Periodic yield. A scalar or vector. Enter as a decimal fraction. |

## Description

[Duration, ModDuration] $=$ cfdur(CashFlow, Yield) calculates the duration and modified duration of a cash flow in periods.

## Examples

## Compute the Duration and Modified Duration of a Cash Flow

This example shows how to calculate the duration and modified duration of a cash flow, given a cash flow of nine payments of $\$ 2.50$ and a final payment $\$ 102.50$, with a periodic yield of $2.5 \%$.

```
CashFlow=[2.5 2.5 2.5 2.5 2.5 2.5 2.5 2.5 2.5 102.5];
```

[Duration, ModDuration] = cfdur(CashFlow, 0.025)

Duration $=8.9709$
ModDuration $=8.7521$
See Alsobndconvp | bndconvy | bnddurp | bnddury | cfconv
Topics
"Analyzing and Computing Cash Flows" on page 2-18
Introduced before R2006a

## cfplot

Visualize cash flows of financial instruments

## Syntax

cfplot(CFlowDates, CFlowAmounts)
cfplot( $\qquad$ , Name, Value)
$h=\operatorname{cfplot}(\ldots$, Name, Value) [h,axes_handle] $=\operatorname{cfplot}(\ldots \ldots$,Name, Value $)$

## Description

cfplot(CFlowDates, CFlowAmounts) plots a cash flow diagram for the specified cash flow amounts (CFlowAmounts) and dates (CFlowDates). The length and orientation of each arrow correspond to the cash flow amount.
cfplot( $\qquad$ , Name, Value) plots a cash flow diagram for the specified cash flow amounts ( $\overline{\mathrm{CFl}}$ owAmounts), dates (CFlowDates), and optional name-value pair arguments.
h = cfplot (__ , Name, Value) returns the handle to the line objects used in the cash flow diagram.
[h,axes_handle] = cfplot (__ Name, Value) returns the handles to the line objects and the axes using optional name-value pair arguments.

## Examples

## Plot Cash Flows

Define CFlowAmounts and CFlowDates using the cfamounts function.
CouponRate $=$ [0.06; 0.05; 0.03];
Settle = '03-Jun-1999';

```
Maturity = ['15-Aug-2000';'15-Dec-2000';'15-Jun-2000'];
Period = [1; 2; 2]; Basis = [1; 0; 0];
[CFlowAmounts, CFlowDates] = cfamounts(...
CouponRate, Settle, Maturity, Period, Basis)
CFlowAmounts = 3\times5
\begin{tabular}{rrrrr}
-4.8000 & 6.0000 & 106.0000 & NaN & NaN \\
-2.3352 & 2.5000 & 2.5000 & 2.5000 & 102.5000 \\
-1.4011 & 1.5000 & 1.5000 & 101.5000 & NaN
\end{tabular}
CFlowDates = 3×5
\begin{tabular}{rrrrr}
730274 & 730347 & 730713 & NaN & NaN \\
730274 & 730286 & 730469 & 730652 & 730835 \\
730274 & 730286 & 730469 & 730652 & NaN
\end{tabular}
```

Plot all cash flows on the same axes, and label the first two.
cfplot(CFlowDates, CFlowAmounts, 'ShowAmnt', [1 2])


Group the second and third cash flows.
figure;
cfplot(CFlowDates, CFlowAmounts, 'Groups', \{[2 3]\}, 'ShowAmnt', 1);


Format the date axis and place ticks on actual cash flow dates.
figure;
cfplot(CFlowDates, CFlowAmounts, 'Groups', \{[2 3]\}, 'ShowAmnt', 1, ... 'DateFormat', 6, 'DateSpacing', 100);


Stack the cash flow arrows occurring on the same dates.
figure; cfplot(CFlowDates, CFlowAmounts, 'Groups', \{[2 3]\}, 'ShowAmnt', 1, ... 'DateFormat', 6, 'DateSpacing', 100, 'Stacked', 1);


Form subplots of multiple groups and add titles using axes handles.

```
figure;
[h, axes_handle] = cfplot(CFlowDates, CFlowAmounts, ...
'Groups', {[1] [2 3]}, 'ShowAmnt', 1, 'Stacked', 2, ...
'DateSpacing', [1 60 2 100], 'DateFormat', [1 12 2 6]);
title(axes handle(1), 'Group 1', 'FontWeight', 'bold');
title(axes_handle(2), 'Group 2', 'FontWeight', 'bold');
```


## Group 1



Group 2


## Plot Cash Flows Using datetime Input for CFlowDates

Define CFlowDates using datetime input and plot the cash flow.

```
CouponRate = [0.06; 0.05; 0.03];
Settle = '03-Jun-1999';
Maturity = ['15-Aug-2000';'15-Dec-2000';'15-Jun-2000'];
Period = [1; 2; 2]; Basis = [1; 0; 0];
[CFlowAmounts, CFlowDates] = cfamounts(...
CouponRate, Settle, Maturity, Period, Basis);
cfplot(datetime(CFlowDates,'ConvertFrom','datenum','Locale','en_US'), CFlowAmounts,
```



## Plot Cash Flows for Swap

Define the swap using the swapbyzero function.

```
Settle = datenum('08-Jun-2010');
RateSpec = intenvset('Rates', [.005 .0075 .01 .014 .02 .025 .03]',...
'StartDates',Settle, 'EndDates',{'08-Dec-2010','08-Jun-2011',...
'08-Jun-2012','08-Jun-2013','08-Jun-2015','08-Jun-2017', ....
'08-Jun-2020'}');
Maturity = datenum('15-Sep-2020');
LegRate = [.025 50];
```

```
LegType = [1 0]; % fixed/floating
LatestFloatingRate = .005;
[Price, SwapRate, AI, RecCF, RecCFDates, PayCF,PayCFDates] = ...
swapbyzero(RateSpec, LegRate, Settle, Maturity,'LegType',LegType,...
'LatestFloatingRate',LatestFloatingRate)
Price = -6.7258
SwapRate = NaN
AI = 1.4575
RecCF = 1×12
    -1.8219 2.5000 2.5000 2.5000 2.5000 2.5000 2.5000 2.5000
RecCFDates = 1\times12
    734297 734396 
PayCF = 1\times12
    -0.3644 0.5000 1.4048 1.9823 2.8436 3.2842 3.8218
PayCFDates = 1\times12
    734297
        734396
        7 3 4 7 6 1
        7 3 5 1 2 7
        735492
        7 3 5 8 5 7
    736222
Define CFlowDates and CFlowAmounts for the swap and generate a cash flow plot using cfplot.
CFlowDates = [PayCFDates;RecCFDates];
CFlowAmounts = [-PayCF;RecCF];
cfplot(CFlowDates, CFlowAmounts, 'Groups', \{[1 2]\});
xlabel('Numeric Cash Flow Dates');
```



## Input Arguments

## CFlowDates - Matrix of serial date numbers for cash flows

vector
Matrix of serial date numbers or datetime arrays for cash flows, specified as a NINST-by(Number of cash flows) matrix of cash flow dates in date numbers, with empty entries padded with NaNs.

Each row of the CFlowDates matrix represents an instrument so that CFlowDates ( $k,:$ ) is the vector of cash flow dates for the kth instrument. Rows are padded with trailing NaNs if the number of cash flows is not the same for all instruments.
cfamounts can be used to generate CFlowDates.
Data Types: double
CFlowAmounts - Matrix of cash flow amounts
vector
Matrix of cash flow amounts, specified as a NINST-by-(Number of cash flows) matrix of cash flow amounts, with empty entries padded with NaNs. The CFlowAmounts matrix must be the same size as CFlowDates.
cfamounts can be used to generate CFlowAmounts.
Data Types: double

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Example: cfplot (CFlowDates, CFlowAmounts,'Groups',\{[2 3]\},'ShowAmnt', 1,'DateFormat',6,'DateSpacing',100)

## Groups - Group cash flows

'off' (default) | character vector with value 'off' or 'individual' | cell array of character vectors

Group cash flows, specified as the comma-separated pair consisting of 'Groups ' and the following values:

- 'off' - Show all instruments in one set of axes, arranged from top to bottom.
- 'individual' - Generate subplots and plot each instrument in its own axis.
- GRP - Cell array of instrument groups, \{Group1, Group2, .. \}. This generates subplots and plots each group in each axis. When specifying \{Group1, Group2, ... \}, each Group must be mutually exclusive vectors of INSTIndex. Unspecified instruments are not shown in the grouped plot.

Data Types: char | cell

## Stacked - Stack arrows if cash flows are in same direction on same day

ignored when 'Groups ' is 'off', otherwise 'off' (default)| character vector with values 'off', 'all', or 'GRPIndex'

Stack arrows if the cash flows are in the same direction on the same day, specified as the comma-separated pair consisting of 'Stacked ' and the following values:

- 'off' - For all groups, all arrows originate from the horizontal line.
- 'all' - For all groups, arrows are stacked if the cash flows are in the same direction on the same day.
- 'GRPIndex' - For specified groups, arrows are stacked if the cash flows are in the same direction on the same day.


## Data Types: char

## ShowAmnt - Show amount on arrows

'off' (default)| character vector with values 'off' or 'individual' | cell array of character vectors

Show amount on the arrows, specified as the comma-separated pair consisting of
'ShowAmnt' and the following values:

- 'off' - Hide cash flow amounts on arrows.
- 'all' - Show cash flow amounts on arrows.
- [INSTIndex or GRPIndex] - Show cash flow amounts for the specified vector of instruments (when 'Groups' is 'off') or groups.

Data Types: char | cell

## DateSpacing - Control for date axis tick spacing

## 'off' (default) | character vector with values 'off' or TickDateSpace| numeric value for TickDateSpace

Control for data spacing, specified as the comma-separated pair consisting of
'DateSpacing ' and the following values:

- 'off' - The date axis ticks are spaced regularly.
- TickDateSpace - The date axis ticks are placed on actual cash flow dates. The ticks skip some cash flows if they are less than TickDateSpace apart.


## Data Types: char | double

## DateFormat - Date format

## 'off' (default)| character vector with values 'off' or DateFormNum | numeric value for DateFormNum

Date format, specified as the comma-separated pair consisting of 'DateFormat ' and the following values:

- 'off' - The date axis tick labels are in date numbers.
- DateFormNum - The date format number ( $2=$ 'mm/dd/yy', $6={ }^{\prime} \mathrm{mm} / \mathrm{dd}{ }^{\prime}$, and $10=$ ' yyyy '). Additional values for DateFormNum are as follows:

| DateFormNum | Example |
| :--- | :--- |
| 2 | $03 / 01 / 00$ |
| 3 | Mar |
| 5 | 03 |
| 6 | $03 / 01$ |
| 7 | 01 |
| 8 | Wed |
| 9 | W |
| 10 | 2000 |
| 11 | 00 |
| 12 | Mar00 |
| 17 | Q1-00 |
| 18 | Q1 |
| 19 | $01 / 03$ |
| 20 | $01 / 03 / 00$ |
| 27 | Q1-2000 |
| 28 | Mar2000 |
| 29 | $2000-03-01$ |

Data Types: char | double

## Output Arguments

## h - Handles to line objects

## vector

Handles to line objects, returned as a NINST-by-3 matrix of handles to line objects, containing [hLines, hUArrowHead, hDArrowHead] where:

- hLines - Horizontal and vertical lines used in the cash flow diagram
- hUArrowHead - "Up" arrowheads
- hDArrowHead - "Down" arrowheads


## axes_handle - Handles to axes for plot or subplots

## vector

Handles to axes for the plot or subplots, returned as a (Number of axes)-by-1 vector of handles to axes.

See Also<br>cfamounts | cfdates | datetime | swapbyzero<br>\section*{Topics}<br>"Analyzing and Computing Cash Flows" on page 2-18<br>Introduced in R2013a

## cfport

Portfolio form of cash flow amounts

## Syntax

[CFBondDate,AllDates,AllTF,IndByBond] = cfport(CFlowAmounts,CFlowDates,TFactors)

## Arguments

| CFlowAmounts | Number of bonds (NUMBONDS) by number of cash flows <br> (NUMCFS) matrix with entries listing cash flow amounts <br> corresponding to each date in CFlowDates. |
| :--- | :--- |
| CFlowDates | NUMBONDS-by-NUMCFS matrix with rows listing cash flow <br> dates, specified as a serial date number, date character <br> vector, or datetime array, for each bond and padded with <br> NaNs. If CFlowDates is a serial date number or a date <br> character vector, AllDates is returned as an array of serial <br> date numbers. If CFlowDates is a datetime array, then <br> AllDates is returned as a datetime array. |
| TFactors | (Optional) NUMBONDS-by-NUMCFS matrix with entries listing <br> the time between settlement and the cash flow date <br> measured in semiannual coupon periods. |

## Description

[CFBondDate,AllDates,AllTF,IndByBond] =
cfport(CFlowAmounts, CFlowDates, TFactors) computes a vector of all cash flow dates of a bond portfolio, and a matrix mapping the cash flows of each bond to those dates. Use the matrix for pricing the bonds against a curve of discount factors.

CFBondDate is a NUMBONDS by number of dates (NUMDATES) matrix of cash flows indexed by bond and by date in AllDates. Each row contains a bond's cash flow values at the indices corresponding to entries in AllDates. Other indices in the row contain zeros.

AllDates is a NUMDATES-by- 1 list of all dates that have any cash flow from the bond portfolio.

AllTF is a NUMDATES-by-1 list of time factors corresponding to the dates in Alldates. If TFactors is not entered, AllTF contains the number of days from the first date in AllDates.

IndByBond is a NUMBONDS-by-NUMCFS matrix of indices. The $i$ th row contains a list of indices into AllDates where the $i$ th bond has cash flows. Since some bonds have more cash flows than others, the matrix is padded with NaNs.

## Examples

## Calculate the Cash Flow Amounts, Cash Flow Dates, and Time Factors for Each of Two Bonds

Use the function cfamounts to calculate the cash flow amounts, cash flow dates, and time factors for each of two bonds. Then use the function cfplot to plot the cash flow diagram.

```
Settle = '03-Aug-1999';
Maturity = ['15-Aug-2000';'15-Dec-2000'];
CouponRate= [0.06; 0.05];
Period = [3;2];
Basis = [1;0];
[CFlowAmounts, CFlowDates, TFactors] = cfamounts(CouponRate,...
Settle, Maturity, Period, Basis);
cfplot(CFlowDates,CFlowAmounts)
xlabel('Numeric Cash Flow Dates')
ylabel('Bonds')
title('Cash Flow Diagram')
```


## Cash Flow Diagram



Call the function cfport to map the cash flow amounts to the cash flow dates. Each row in the resultant CFBondDate matrix represents a bond. Each column represents a date on which one or more of the bonds has a cash flow. A 0 means the bond did not have a cash flow on that date. The dates associated with the columns are listed in AllDates. For example, the first bond had a cash flow of 2.000 on 730347 . The second bond had no cash flow on this date For each bond, IndByBond indicates the columns of CFBondDate, or dates in AllDates, for which a bond has a cash flow.
[CFBondDate, AllDates, AllTF, IndByBond] = ... cfport(CFlowAmounts, CFlowDates, TFactors)

CFBondDate $=2 \times 7$

| -1.8000 | 2.0000 | 2.0000 | 2.0000 | 0 | 102.0000 | 0 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |

```
    -0.6694 0 2.5000 0 0 2.5000 0
AllDates = 7×1
    7 3 0 3 3 5
    730347
    7 3 0 4 6 9
    7 3 0 5 9 1
    730652
    730713
    730835
AllTF = 7x1
    0
    0.0663
    0.7322
    1.3989
    1.7322
    2.0663
    2.7322
IndByBond = 2 5 5
    1 2 % 3 4 4 6
    1 3 5 5 7 NaN
```


## Calculate the Cash Flow Amounts, Cash Flow Dates Using a datetime Array, and Time Factors for Each of Two Bonds

Use the function cfamounts to calculate the cash flow amounts, cash flow dates, and time factors for each of two bonds.

```
Settle = datetime('03-Aug-1999','Locale','en_US');
Maturity = ['15-Aug-2000';'15-Dec-2000'];
CouponRate= [0.06; 0.05];
Period = [3;2];
Basis = [1;0];
```

```
[CFlowAmounts, CFlowDates, TFactors] = cfamounts(CouponRate,...
Settle, Maturity, Period, Basis);
```

Call the function cfport to map the cash flow amounts to the cash flow dates. Each row in the resultant CFBondDate matrix represents a bond. Each column represents a date on which one or more of the bonds has a cash flow. A 0 means the bond did not have a cash flow on that date. The dates associated with the columns are listed in AllDates returned as a datetime array.

```
[CFBondDate, AllDates, AllTF, IndByBond] = ...
cfport(CFlowAmounts, CFlowDates, TFactors)
CFBondDate = 2×7
```

| -1.8000 | 2.0000 | 2.0000 | 2.0000 | 0 | 102.0000 | 0 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| -0.6694 | 0 | 2.5000 | 0 | 2.5000 | 0 | 102.5000 |

AllDates = 7x1 datetime array
03-Aug-1999
15-Aug-1999
15-Dec-1999
15-Apr-2000
15-Jun-2000
15-Aug-2000
15-Dec-2000
AllTF $=7 \times 1$
0
0.0663
0.7322
1.3989
1.7322
2.0663
2.7322
IndByBond $=2 \times 5$
$\begin{array}{lllll}1 & 2 & 3 & 4 & 6\end{array}$
$\begin{array}{lllll}1 & 3 & 5 & 7 & \mathrm{NaN}\end{array}$
See Alsocfamounts|cfplot|datetime
Topics"Analyzing and Computing Cash Flows" on page 2-18
Introduced before R2006a

## cfprice

Compute price for cash flow given yield to maturity

## Syntax

Price $=$ cfprice(CFAmounts,CFDates, Yield, Settle)
Price $=$ cfprice(CFAmounts,CFDates, Yield, Settle, Name, Value)

## Description

Price = cfprice(CFAmounts,CFDates,Yield,Settle) computes a price given yield for a cash flow.

Price = cfprice(CFAmounts,CFDates,Yield,Settle,Name, Value) computes a price for a cash flow given yield to maturity with additional options specified by one or more Name, Value pair arguments.

## Input Arguments

## CFlowAmounts

NINST-by-MOSTCFS matrix of cash flow amounts. Each row is a list of cash flow values for one instrument. If an instrument has fewer than MOSTCFS cash flows, the end of the row is padded with NaNs.

## CFlowDates

NINST-by-MOSTCFS matrix of cash flow dates, specified as a serial date number, date character vector, or datetime array. Each entry contains the date of the corresponding cash flow in CFlowAmounts.

## Yield

NINST-by-1 vector of yields.

## Settle

Settlement date, specified as a serial date number, date character vector, or datetime array. Settlement date is the date on which the cash flows are priced.

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Note Any optional input of size N -by-1 is also acceptable as an array of size 1-by-N, or as a single value applicable to all contracts. Single values are internally expanded to an array of size N -by-1.

## Basis

N -by-1 vector of day-count basis:

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- $2=$ actual/360
- $3=$ actual $/ 365$
- $4=30 / 360$ (BMA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- $9=$ actual/360 (ICMA)
- $10=$ actual/365 (ICMA)
- $11=30 / 360 \mathrm{E}$ (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-0
Default: 0 (actual/actual)

## CompoundingFrequency

NINST-by-1 vector of Compounding Frequency. By default, SIA bases (0-7) and BUS/252 use a semiannual compounding convention and ICMA bases (8-12) use an annual compounding convention.

Default: 2

## Output Arguments

## Price

Price of cash flows.

## Examples

## Compute the Price for a Cash Flow Given Yield to Maturity

Use cfprice to compute the price for a cash flow given yield to maturity.
Define data for the yield curve.

```
Settle = datenum('01-Jul-2003');
Yield = .05;
CFAmounts = [30;40;30];
CFDates = datenum({'15-Jul-2004', '15-Jul-2005', '15-Jul-2006'});
Compute the Price.
```

```
Price = cfprice(CFAmounts, CFDates, Yield, Settle)
```

Price = cfprice(CFAmounts, CFDates, Yield, Settle)
Price = 3×1

```
    28.4999
    36.1689

\section*{Compute the Price for a Cash Flow Given Yield to Maturity Using datetime Inputs}

Use cfprice to compute the price for a cash flow given yield to maturity using datetime inputs.
```

Settle = datenum('01-Jul-2003');
Yield = .05;
CFAmounts = [30;40;30];
CFDates = datenum({'15-Jul-2004', '15-Jul-2005', '15-Jul-2006'});
CFDates = datetime(CFDates,'ConvertFrom','datenum','Locale','en US');
Settle = datetime(Settle,'ConvertFrom','datenum','Locale','en_US'');
Price = cfprice(CFAmounts, CFDates, Yield, Settle)
Price = 3×1

```
28.4999
36.1689
25.8195

\section*{See Also}
cfbyzero|cfspread|cfyield|datetime

\section*{Topics}
"Analyzing and Computing Cash Flows" on page 2-18

\section*{Introduced in R2012a}

\section*{cfspread}

Compute spread over yield curve for cash flow

\section*{Syntax}

Spread = cfspread(RateSpec,Price,CFAmounts,CFDates,Settle) Spread = cfspread(RateSpec,Price,CFAmounts,CFDates,Settle, Name, Value)

\section*{Description}

Spread \(=\) cfspread(RateSpec,Price,CFAmounts,CFDates,Settle) computes spread over a yield curve for a cash flow.

Spread = cfspread(RateSpec,Price,CFAmounts,CFDates,Settle, Name, Value) computes spread over a yield curve for a cash flow with additional options specified by one or more Name, Value pair arguments.

\section*{Input Arguments}

\section*{RateSpec}

Interest-rate specification for the initial risk free rate curve. See intenvset for information on declaring an interest-rate variable.

\section*{Price}

Price of cash flows.

\section*{CFlowAmounts}

NINST-by-MOSTCFS matrix of cash flow amounts. Each row is a list of cash flow values for one instrument. If an instrument has fewer than MOSTCFS cash flows, the end of the row is padded with NaNs.

\section*{CFlowDates}

NINST-by-MOSTCFS matrix of cash flow dates, specified as a serial date number, date character vector, or datetime array. Each entry contains the date of the corresponding cash flow in CFlowAmounts.

\section*{Settle}

Settlement date, specified as a serial date number, date character vector, or datetime array. Settlement date is the date on which the cash flows are priced.

\section*{Name-Value Pair Arguments}

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Note Any optional input of size N -by-1 is also acceptable as an array of size 1-by-N, or as a single value applicable to all contracts. Single values are internally expanded to an array of size N -by-1.

\section*{Basis}

N -by-1 vector of day-count basis:
- \(0=\) actual/actual
- \(1=30 / 360\) (SIA)
- 2 = actual/360
- 3 = actual/365
- \(4=30 / 360\) (BMA)
- \(5=30 / 360\) (ISDA)
- \(6=30 / 360\) (European)
- 7 = actual/365 (Japanese)
- \(8=\) actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- \(10=\) actual/365 (ICMA)
- \(11=30 / 360 \mathrm{E}\) (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-0
Default: 0 (actual/actual)

\section*{CompFreq}

Compounding frequency. By default, SIA bases (0-7) and BUS/252 use a semiannual compounding convention and ICMA bases (8-12) use an annual compounding convention.

Default: actual

\section*{Output Arguments}

\section*{Spread}

Spread of cash flows over a zero curve.

\section*{Examples}

\section*{Compute Spread Over a Yield Curve for a Cash Flow}

Use cfspread to compute the spread over a yield curve for a cash flow.
Define data for the yield curve.
```

Settle = datenum('01-Jul-2003');
CurveDates = daysadd(Settle,360*[.25 .5 1 2 3 5 7 10 20],1);
ZeroRates = [.0089 .0096 .0107 .0130 .0166 .0248 .0306 .0356 .0454]';
Compute the RateSpec.
RateSpec = intenvset('StartDates', Settle, 'EndDates', CurveDates,... 'Rates', ZeroRates)

```
```

RateSpec = struct with fields:
FinObj: 'RateSpec'
Compounding: 2
Disc: [9x1 double]
Rates: [9x1 double]
EndTimes: [9x1 double]
StartTimes: [9x1 double]
EndDates: [9x1 double]
StartDates: 731763
ValuationDate: 731763
Basis: 0
EndMonthRule: 1

```

Compute the spread.
Price = 98;
CFAmounts = [30;40;30];
CFDates = datenum(\{'15-Jul-2004', '15-Jul-2005', '15-Jul-2006'\});
Spread \(=\) cfspread(RateSpec, Price, CFAmounts, CFDates, Settle)
Spread = 3×1
\(10^{3} \times\)
-8.7956
\(-4.0774\)
\(-3.7073\)

\section*{Compute Spread Over a Yield Curve for a Cash Flow Using datetime Inputs}

Use cfspread to compute the spread over a yield curve for a cash flow using datetime inputs.

Settle = datenum('01-Jul-2003');
CurveDates = daysadd(Settle,360*[.25 .5 1235710 20],1);
ZeroRates = [.0089 . 0096 . 0107 . 0130 . 0166 . 0248 . 0306 . 0356 .0454]'; RateSpec = intenvset('StartDates', Settle, 'EndDates', CurveDates,...
'Rates', ZeroRates);
Price = 98;
CFAmounts = [30;40;30];
```

CFDates = datenum({'15-Jul-2004', '15-Jul-2005', '15-Jul-2006'});
CFDates = datetime(CFDates,'ConvertFrom','datenum','Locale','en US');
Settle = datetime(Settle,'ConvertFrom','datenum','Locale','en_US');
Spread = cfspread(RateSpec, Price, CFAmounts, CFDates, Settle)
Spread = 3\times1
103 x
-8.7956
-4.0774
$-3.7073$

```

\section*{See Also}
cfbyzero|cfprice|cfyield|datetime

\section*{Topics}
"Analyzing and Computing Cash Flows" on page 2-18

\section*{Introduced in R2012a}

\section*{cfyield}

Compute yield to maturity for cash flow given price

\section*{Syntax}
```

Yield = cfyield(CFAmounts,CFDates,Price,Settle)
Yield = cfyield(CFAmounts,CFDates,Price,Settle,Name,Value)

```

\section*{Description}

Yield = cfyield(CFAmounts,CFDates,Price,Settle) computes yield to maturity for a cash flow given price.

Yield = cfyield(CFAmounts,CFDates,Price,Settle,Name, Value) computes yield to maturity for a cash flow given price with additional options specified by one or more Name, Value pair arguments.

\section*{Input Arguments}

\section*{CFlowAmounts}

NINST-by-MOSTCFS matrix of cash flow amounts. Each row is a list of cash flow values for one instrument. If an instrument has fewer than MOSTCFS cash flows, the end of the row is padded with NaNs.

\section*{CFlowDates}

NINST-by-MOSTCFS matrix of cash flow dates, specified as a serial date number, date character vector, or datetime array. Each entry contains the serial date of the corresponding cash flow in CFlowAmounts.

\section*{Price}

Price.

\section*{Settle}

Settlement date, specified as a serial date number, date character vector, or datetime array. Settlement date is the date on which the cash flows are priced.

\section*{Name-Value Pair Arguments}

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Note Any optional input of size N -by-1 is also acceptable as an array of size 1-by-N, or as a single value applicable to all contracts. Single values are internally expanded to an array of size N-by-1.

\section*{Basis}

N -by-1 vector of day-count basis:
- \(0=\) actual/actual
- \(1=30 / 360\) (SIA)
- \(2=\) actual/360
- 3 = actual/365
- \(4=30 / 360\) (BMA)
- \(5=30 / 360\) (ISDA)
- \(6=30 / 360\) (European)
- 7 = actual/365 (Japanese)
- \(8=\) actual/actual (ICMA)
- \(9=\) actual/360 (ICMA)
- \(10=\) actual/365 (ICMA)
- \(11=30 / 360 \mathrm{E}\) (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-0
Default: 0 (actual/actual)

\section*{CompoundingFrequency}

Compounding frequency. By default, SIA bases (0-7) and BUS/252 use a semiannual compounding convention and ICMA bases (8-12) use an annual compounding convention.

Default: actual

\section*{Output Arguments}

\section*{Yield}

Yield for cash flows.

\section*{Examples}

\section*{Compute the Yield to Maturity for a Cash Flow When Given a Price}

Use cfyield to compute yield to maturity for a cash flow when given a price.
Define data for the yield curve and price.
```

Settle = datenum('01-Jul-2003');
Price = 98;
CFlowAmounts = [30 40 30];
CFlowDates = datenum({'15-Jul-2004', '15-Jul-2005', '15-Jul-2006'})';
Compute the Yield.

```
```

Yield = cfyield(CFlowAmounts, CFlowDates, Price, Settle)

```
Yield = cfyield(CFlowAmounts, CFlowDates, Price, Settle)
Yield = 0.0099
```

Yield = 0.0099

```

\section*{Compute the Yield to Maturity for a Cash Flow When Given a Price Using datetime Inputs}

Use cfyield to compute yield to maturity for a cash flow, when given a price using datetime inputs.
```

Settle = datenum('01-Jul-2003');
Price = 98;
CFlowAmounts = [30 40 30];
CFlowDates = datenum({'15-Jul-2004', '15-Jul-2005', '15-Jul-2006'})';
CFlowDates = datetime(CFlowDates,'ConvertFrom','datenum','Locale','en_US');
Settle = datetime(Settle,'ConvertFrom','datenum','Locale','en_US');
Yield = cfyield(CFlowAmounts, CFlowDates, Price, Settle)
Yield = 0.0099

```

\section*{See Also}
cfbyzero|cfprice|cfspread|datetime

\section*{Topics}
"Analyzing and Computing Cash Flows" on page 2-18

\section*{Introduced in R2012a}

\section*{cftimes}

Time factors corresponding to bond cash flow dates

\section*{Syntax}
```

[TFactors] = cftimes(Settle,Maturity)
[TFactors] =
cftimes(Settle,Maturity,Period,Basis,EndMonthRule,IssueDate,FirstCou
ponDate,LastCouponDate,StartDate)
[TFactors] =
cftimes(Settle,Maturity,'ParameterName',ParameterValue, ...)

```

\section*{Description}
[TFactors] = cftimes(Settle,Maturity) determines the time factors corresponding to the cash flows of a bond or set of bonds.
cftimes computes the time factor of a cash flow, which is the difference between the settlement date and the cash flow date, in units of semiannual coupon periods. In computing time factors, use SIA actual/actual day count conventions for all time factor calculations.
[TFactors] =
cftimes(Settle,Maturity, Period,Basis,EndMonthRule,IssueDate,FirstCou ponDate, LastCouponDate, StartDate) determines the time factors corresponding to the cash flows of a bond or set of bonds, including optional inputs.
[TFactors] =
cftimes(Settle,Maturity,'ParameterName',ParameterValue, ...) accepts optional inputs as one or more comma-separated parameter/value pairs.
'ParameterName' is the name of the parameter inside single quotes. ParameterValue is the value corresponding to 'ParameterName'. Specify parameter/value pairs in any order. Names are case-insensitive.

\section*{Input Arguments}

\section*{Settle}

Settlement date. A vector of serial date numbers, date character vectors, or datetime array. Settle must be earlier than Maturity.

\section*{Maturity}

Maturity date. A vector of serial date numbers, date character vectors, or datetime array.

\section*{Ordered Input or Parameter-Value Pairs}

Enter the following inputs using an ordered syntax or as parameter/value pairs. You cannot mix ordered syntax with parameter/value pairs.

\section*{Period}

Coupons per year of the bond. A vector of integers. Values are 0, 1, 2, 3, 4, 6, and 12.
Default: 2

\section*{Basis}

Day-count basis of the instrument. A vector of integers.
- \(0=\) actual/actual
- \(1=30 / 360\) (SIA)
- \(2=\) actual/360
- 3 = actual/365
- \(4=30 / 360\) (PSA)
- \(5=30 / 360\) (ISDA)
- \(6=30 / 360\) (European)
- 7 = actual/365 (Japanese)
- \(8=\) actual/actual (ICMA)
- \(9=\) actual/360 (ICMA)
- \(10=\) actual/365 (ICMA)
- \(11=30 / 360 E\) (ICMA)
- \(12=\) actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-0
Default: 0

\section*{EndMonthRule}

End-of-month rule. A vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days. \(0=\) ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month. \(1=\) set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

Default: 1

\section*{IssueDate}

Issue date, specified as a serial date number, date character vector, or datetime array, for a bond.

\section*{FirstCouponDate}

Date, specified as a serial date number, date character vector, or datetime array, when a bond makes its first coupon payment. FirstCouponDate is used when a bond has an irregular first coupon period. When FirstCouponDate and LastCouponDate are both specified, FirstCouponDate takes precedence in determining the coupon payment structure.

Default: If you do not specify a FirstCouponDate, the cash flow payment dates are determined from other inputs.

\section*{LastCouponDate}

Last coupon date of a bond before the maturity date, specified as a serial date number, date character vector, or datetime array. LastCouponDate is used when bond has an irregular last coupon period. In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated at the LastCouponDate, regardless of where it falls, and is followed only by the bond's maturity cash flow date.

Default: If you do not specify a LastCouponDate, the cash flow payment dates are determined from other inputs.

\section*{StartDate}

Date when a bond actually starts (the date from which a bond cash flow is considered), specified as a serial date number, date character vector, or datetime array. To make an instrument forward-starting, specify this date as a future date. If you do not specify StartDate, the effective start date is the Settle date.

\section*{Parameter-Value Pairs}

Enter the following inputs only as parameter/value pairs.

\section*{CompoundingFrequency}

Compounding frequency for yield calculation. By default, SIA bases (0-7) and BUS/252 use a semiannual compounding convention and ICMA bases (8-12) use an annual compounding convention.

\section*{DiscountBasis}

Basis used to compute the discount factors for computing the yield. The default behavior is for SIA bases to use the actual/actual day count to compute discount factors. If you use ICMA day counts and BUS/252, the specified bases are used.

\section*{Output Arguments}

\section*{TFactors}

TFactors has NUMBONDS rows and the number of columns is determined by the maximum number of cash flow payment dates required to hold the bond portfolio. NaNs are padded for bonds which have less than the maximum number of cash flow payment dates.

\section*{Examples}

\section*{Compute the Time Factor of a Cash Flow}

This example shows how to calculate a cash flow time factor.
```

Settle = '15-Mar-1997';
Maturity = '01-Sep-1999';
Period = 2;
TFactors = cftimes(Settle, Maturity, Period)
TFactors = 1×5

| 0.9239 | 1.9239 | 2.9239 | 3.9239 | 4.9239 |
| :--- | :--- | :--- | :--- | :--- |

```

\section*{References}

Krgin, Dragomir. Handbook of Global Fixed Income Calculations. John Wiley \& Sons, 2002.

Mayle, Jan. "Standard Securities Calculations Methods: Fixed Income Securities Formulas for Analytic Measures." SIA, Vol 2, Jan 1994.

Stigum, Marcia, and Franklin Robinson. Money Market and Bond Calculations. McGrawHill, 1996.

\section*{See Also}
accrfrac|cfamounts | cfdates | cpncount | cpndaten | cpndatenq| cpndatep| cpndatepq | cpndaysn | cpndaysp|date2time

\section*{Topics}
"Analyzing and Computing Cash Flows" on page 2-18

\section*{Introduced before R2006a}

\section*{chiakosc}

Chaikin oscillator

Note chaikosc has been partially removed and will no longer accept a fints object (tsobj) argument. Use a matrix, timetable, or table instead for financial time series.

Use fts2timetable to convert a fints object to a timetable object.

\section*{Syntax}
chosc \(=\) chaikocs(Data)

\section*{Description}
chosc \(=\) chaikocs(Data) calculates the Chaikin oscillator.

\section*{Examples}

\section*{Calculate the Chaikin Oscillator for a Stock}

Load the file SimulatedStock. mat, which provides a timetable (TMW) for financial data for TMW stock.
```

load SimulatedStock.mat
oscillator = chaikosc(TMW);
plot(oscillator.Time, oscillator.ChaikinOscillator)
title('Chaikin Oscillator for TMW')

```


\section*{Input Arguments}

\section*{Data - Data with high, low, open, close information}
matrix | table | timetable
Data with high, low, open, close information, specified as a matrix, table, or timetable. For matrix input, Data is an M-by-4 matrix of high, low, opening, and closing prices.
Timetables and tables with M rows must contain variables named 'High', 'Low', 'Open', and 'Close' (case insensitive).

Data Types: double| table| timetable

\section*{Output Arguments}
chosc - Chaikin oscillator
matrix | table | timetable
Chaikin oscillator, returned with the same number of rows (M) and type (matrix, table, or timetable) as the input Data.

\section*{References}
[1] Achelis, S. B. Technical Analysis from A to Z. Second Edition. McGraw-Hill, 1995, pp. 91-94.

\section*{See Also}
adline| chaikvolat|table|timetable

\section*{Topics}
"Using Timetables in Finance"
"Convert Financial Time Series Objects fints to Timetables" on page 12-2

\section*{Introduced before R2006a}

\section*{chaikvolat}

Chaikin volatility

Note chaikvolat has been partially removed and will no longer accept a fints object (tsobj) argument. Use a matrix, timetable, or table instead for financial time series.

Use fts2timetable to convert a fints object to a timetable object.

\section*{Syntax}
```

volatility = chaikvolat(Data)
volatility = chiakvolat(

```
\(\qquad\)
``` ,Name, Value)
```


## Description

volatility $=$ chaikvolat (Data) calculates the Chaikin volatility from a data series of high and low stock prices.
volatility = chiakvolat( ___ ,Name, Value) adds optional name-value pair arguments.

## Examples

## Calculate the Chaikin Volatility for a Stock

Load the file SimulatedStock. mat, which provides a timetable (TMW) for financial data for TMW stock.

Chaikin Volatility for TMW


## Input Arguments

## Data - Data with high, low, open, close information

matrix | table | timetable
Data with high, low, open, close information, specified as a vector, matrix, table, or timetable. For matrix input, Data is an M-by-2 matrix of high and low prices stored in the first and second columns. Timetables and tables with M rows must contain variables named 'High' and 'Low' (case insensitive).

Data Types: double | table | timetable

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Example: volatility = chaikvolat(TMW,'NumPeriods',10,'WindowSize',10)

## NumPeriods - Period difference

10 (default) | positive integer
Period difference, specified as the comma-separated pair consisting of 'NumPeriods ' and a scalar positive integer.

## Data Types: double

## WindowSize - Length of the exponential moving average in periods

 10 (default) | positive integerLength of the exponential moving average in periods, specified as the comma-separated pair consisting of 'WindowSize' and a scalar positive integer.

## Data Types: double

## Output Arguments

volatility - Chaikin volatility
matrix | table | timetable
Chaikin volatility, returned with the same number of rows $(\mathrm{M})$ and the same type (matrix, table, or timetable) as the input Data.

## Definitions

## Chaikin volatility

Chaikin volatility calculates the Chaikin's volatility from the series of high and low stock prices.

By default, Chaikin's volatility values are based on a 10-period exponential moving average and 10-period difference.

## References

[1] Achelis, S. B. Technical Analysis from A to Z. Second Edition. McGraw-Hill, 1995, pp. 304-305.

See Also<br>chaikosc|table|timetable<br>Topics<br>"Using Timetables in Finance"<br>"Convert Financial Time Series Objects fints to Timetables" on page 12-2<br>Introduced before R2006a

## chartfts

Interactive display

Note chartfts will be removed in a future release and will no longer accept a fints object.

Use fts2timetable to convert a fints object to a timetable object. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

chartfts(tsobj)

## Description

chartfts(tsobj) produces a figure window that contains one or more plots. You can use the mouse to observe the data at a particular time point of the plot.

Note Help menu item is not available for chartfts in MATLAB Online.

## Examples

Create a financial time series object from the supplied data file ibm9599.dat:

```
ibmfts = ascii2fts('ibm9599.dat', 1, 3, 2);
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In ascii2fts (line 64)
```

Chart the financial time series object ibmfts:

```
chartfts(ibmfts)
```

Warning: FINTS will be removed in a future release. Use TIMETABLE instead. > In fints/chartfts (line 24)

With the Zoom feature set off, a mouse click on the indicator line displays object data in a pop-up box.


With the Zoom feature set on, mouse clicks indicate the area of the chart to zoom.


You can find a tutorial on using chartfts in "Visualizing Financial Time Series Objects" on page 11-31. See "Zoom Tool" on page 11-34 for details on performing the zoom. Also see "Combine Axes Tool" on page 11-37 for information about combining axes for specified plots.

See Also<br>candle| highlow|plot

Introduced before R2006a

19-597

## checkFeasibility

Check feasibility of input portfolios against portfolio object
Use the checkFeasibility function with a Portfolio, PortfolioCVaR, or PortfolioMAD object to check the feasibility of input portfolios against a portfolio object.

For details on the respective workflows when using these different objects, see "Portfolio Object Workflow" on page 4-21, "PortfolioCVaR Object Workflow" on page 5-20, and "PortfolioMAD Object Workflow" on page 6-19.

## Syntax

status = checkFeasibility(obj,pwgt)

## Description

status = checkFeasibility(obj,pwgt) checks the feasibility of input portfolios against a portfolio object.

## Examples

## Determine if the Portfolio Is Feasible for a Portfolio Object

Given portfolio $p$, determine if $p$ is feasible.

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
p = Portfolio;
p = setAssetMoments(p, m, C);
```

```
p = setDefaultConstraints(p);
pwgt = estimateFrontier(p);
checkFeasibility(p, pwgt)
ans = lx10 logical array
    1
```


## Determine if the Portfolio Is Feasible for a PortfolioCVaR Object

Given portfolio $p$, determine if $p$ is feasible.

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;
AssetScenarios = mvnrnd(m, C, 20000);
p = PortfolioCVaR;
p = setScenarios(p, AssetScenarios);
p = setDefaultConstraints(p);
p = setProbabilityLevel(p, 0.95);
pwgt = estimateFrontier(p);
checkFeasibility(p, pwgt)
ans = 1x10 logical array
```

    \(\begin{array}{llllllllll}1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1\end{array}\)
    
## Determine if the Portfolio Is Feasible for a PortfolioMAD Object

Given portfolio $p$, determine if $p$ is feasible.

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;
```

AssetScenarios = mvnrnd(m, C, 20000);
p = PortfolioMAD;
p = setScenarios(p, AssetScenarios);
$p=$ setDefaultConstraints(p);
pwgt = estimateFrontier(p);
checkFeasibility(p, pwgt)
ans $=1 \times 10$ logical array

## Input Arguments

## obj - Object for portfolio

## object

Object for portfolio, specified using Portfolio, PortfolioCVaR, or PortfolioMAD object. For more information on creating a portfolio object, see

- Portfolio
- PortfolioCVaR
- PortfolioMAD


## pwgt - Portfolios to check

matrix

Portfolios to check, specified as a NumAssets-by-NumPorts matrix.

## Data Types: double

## Output Arguments

## status - Indicator if portfolio is feasible

row vector
Indicator if portfolio is feasible, returned as a row vector of NumPorts indicators that are true if portfolio is feasible and false otherwise.

Note By definition, any portfolio set must be nonempty and bounded. If the set is empty, no portfolios can be feasible. Use estimateBounds to test for nonempty and bounded sets.

Feasibility status is returned for Portfolio, PortfolioCVaR, or PortfolioMAD objects. For more information on creating a portfolio object, see

- Portfolio
- PortfolioCVaR
- PortfolioMAD


## Tips

- You can also use dot notation to check the feasibility of input portfolios against a portfolio object.

```
status = obj.checkFeasibility(pwgt);
```

- The constraint tolerance to assess whether a constraint is satisfied is obtained from the hidden property obj. defaultTolCon.


## See Also

estimateBounds

## Topics

"Validate the Portfolio Problem for Portfolio Object" on page 4-110
"Validate the CVaR Portfolio Problem" on page 5-96
"Validate the MAD Portfolio Problem" on page 6-91
"Portfolio Optimization Examples" on page 4-167
"Portfolio Optimization Theory" on page 4-3

## Introduced in R2011a

## chfield

Change data series name

Note chfield will be removed in a future release. Use timetable instead. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

```
newfts = chfield(oldfts,oldname,newname)
```


## Arguments

| oldfts | Name of an existing financial time series object. |
| :--- | :--- |
| oldname | Name of the existing component in oldfts. A MATLAB character <br> vector or column cell array. |
| newname | New name for the component in oldfts. A MATLAB character <br> vector or column cell array. |

## Description

newfts = chfield(oldfts,oldname, newname) changes the name of the financial time series object component from oldname to newname.

Set newfts = oldfts to change the name of an existing component without changing the name of the financial time series object.

To change the names of several components at once, specify the series of old and new component names in corresponding column cell arrays.

You cannot change the names of the object components desc, freq, and dates.

See Also<br>fieldnames |isfield|rmfield<br>\section*{Topics}<br>"What Is the Financial Time Series App?" on page 14-2<br>Introduced before R2006a

## convert2sur

Convert multivariate normal regression model to seemingly unrelated regression (SUR) model

## Syntax

DesignSUR = convert2sur(Design, Group)

## Arguments

| Design | A matrix or a cell array that depends on the number of data series <br> NUMSERIES. |
| :--- | :--- |
| GroupIf NUMSERIES $=1$, convert2 sur returns the Design matrix. <br> If NUMSERIES > 1, Design is a cell array with NUMSAMPLES <br> cells, where each cell contains a NUMSERIES-by-NUMPARAMS <br> matrix of known values. |  |
| Contains information about how data series are to be grouped, with <br> separate parameters for each group. Specify groups either by series <br> or by groups: |  |
| To identify groups by series, construct an index vector that has <br> NUMSERIES elements. Element i $=1, \ldots$, NUMSERIES in <br> the vector, and has the index $j=1, \ldots$, NUMGROUPS of the <br> group in which series $i$ is a member. <br> To identify groups by groups, construct a cell array with <br> NUMGROUPS elements. Each cell contains a vector with the <br> indexes of the series that populate a given group. |  |
| In either case, the number of series is NUMSERIES and the <br> number of groups is NUMGROUPS, with $1 \leq$ NUMGROUPS $\leq$ <br> NUMSERIES. |  |

## Description

DesignSUR = convert2sur(Design, Group) converts a multivariate normal regression model into a seemingly unrelated regression model with a specified grouping of the data series. DesignSUR is either a matrix or a cell array that depends on the value of NUMSERIES:

- If NUMSERIES = 1 , DesignSUR = Design, which is a NUMSAMPLES-by-NUMPARAMS matrix.
- If NUMSERIES > 1 and NUMGROUPS groups are to be formed, Design is a cell array with NUMSAMPLES cells, where each cell contains a NUMSERIES-by-(NUMGROUPS * NUMPARAMS) matrix of known values.

The original collection of parameters that are common to all series are replicated to form collections of parameters for each group.

## Examples

## Use convert2sur to Estimate Stock Alpha and Beta Values

This example shows a CAPM demonstration using 6 stocks and 60 months of simulated asset returns, where the model for each stock is AssetReturn = Alpha * 1 + CashReturn + Beta * (MarketReturn - CashReturn) + Noise and the parameters to estimate are Alpha and Beta.

Using simulated data, where the Alpha estimate(s) are displayed in the first row(s) and the Beta estimate(s) are display in the second row(s).

```
Market = (0.1 - 0.04) + 0.17*randn(60, 1);
Asset = (0.1 - 0.04) + 0.35*randn(60, 6);
Design = cell(60, 1);
for i = 1:60
    Design{i} = repmat([ 1, Market(i) ], 6, 1);
end
```

Obtain the aggregate estimates for all stocks.

```
[Param, Covar] = mvnrmle(Asset, Design);
disp({'All 6 Assets Combined'});
    'All 6 Assets Combined'
disp(Param);
    0.0233
    0.1050
```

Estimate parameters for individual stocks using convert2sur

```
Group = 1:6;
DesignSUR = convert2sur(Design, Group);
[Param, Covar] = mvnrmle(Asset, DesignSUR);
Param = reshape(Param, 2, 6);
disp({ 'A', 'B', 'C', 'D', 'E', 'F' });
    'A' 'B' 'C' 'D' 'E' 'F'
disp(Param);
\begin{tabular}{rrrrrr}
0.0144 & 0.0270 & 0.0046 & 0.0419 & 0.0376 & 0.0291 \\
0.3264 & -0.1716 & 0.3248 & -0.0630 & -0.0001 & 0.0637
\end{tabular}
```

Estimate parameters for pairs of stocks by forming groups.
disp(\{'A \& B', 'C \& D','E \& F'\});
'A \& B' 'C \& D' 'E \& F'
Group = \{ [1,2 ],[3,4],[5,6]\};
DesignSUR = convert2sur(Design, Group);
[Param, Covar] = mvnrmle(Asset, DesignSUR);
Param = reshape(Param, 2, 3);
disp(Param);

| 0.0186 | 0.0190 | 0.0334 |
| :--- | :--- | :--- |
| 0.0988 | 0.1757 | 0.0293 |

## See Also

ecmnfish|mvnrfish

## Topics

"Seemingly Unrelated Regression Without Missing Data" on page 9-21
"Multivariate Normal Linear Regression" on page 9-2

## Introduced in R2006a

## convertto

Convert to specified frequency

Note convertto will be removed in a future release. Use timetable instead. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

```
newfts = convertto(oldfts,newfreq)
newfts = convertto(oldfts,newfreq,'param1','value1','param2','value2', ... )
```


## Arguments

| oldfts | Name of an existing financial time series object. |
| :--- | :--- |
| newfreq | 1, DAILY, Daily, daily, D, d |
|  | 2, WEEKLY, Weekly, weekly, W, w |
|  | 3, MONTHLY, Monthly, monthly, M, m |
|  | 4, QUARTERLY, Quarterly, quarterly, Q, q |
|  | 5, SEMIANNUAL, Semiannual, semiannual, S, s |
|  | 6, ANNUAL, Annual, annual, A, a |

## Description

convertto converts a financial time series of any frequency to one of a specified frequency.
newfts $=$ convertto(oldfts, newfreq) converts the object oldfts to the new time series object newfts with the frequency newfreq.

Refer to the documentation for each frequency conversion function to determine the valid parameter/value pairs.

See Also<br>toannual| todaily|tomonthly|toquarterly|tosemi|toweekly<br>\section*{Topics}<br>"Data Transformation and Frequency Conversion" on page 13-13<br>Introduced before R2006a

## corrcoef

Correlation coefficients

Note corrcoef will be removed in a future release. Use timetable instead. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

$r=\operatorname{corrcoef}(X)$
$r=\operatorname{corrcoef}(X, Y)$

## Arguments

| X | Matrix where each row is an observation and each column is a <br> variable. |
| :--- | :--- |
| Y | Matrix where each row is an observation and each column is a <br> variable. |

## Description

corrcoef is based on the MATLAB corrcoef function. See corrcoef.
$r=\operatorname{corrcoef}(X)$ calculates a matrix $r$ of correlation coefficients for an array $X$, in which each row is an observation, and each column is a variable.
$r=\operatorname{corrcoef}(X, Y)$, where $X$ and $Y$ are column vectors, is the same as $r=\operatorname{corrcoef}$ ( $[X$ $\mathrm{Y}]$ ). corrcoef converts $X$ and $Y$ to column vectors if they are not; that is, $r=$ $\operatorname{corrcoef}(X, Y)$ is equivalent to $r=\operatorname{corrcoef}([X(:) Y(:)])$ in that case.

If $c$ is the covariance matrix, $c=\operatorname{cov}(X)$, then $\operatorname{corrcoef}(X)$ is the matrix whose $(i, j)$ 'th element is ci,j/sqrt(ci,i*c(j,j)).
$[r, p]=\operatorname{corrcoef}(\ldots)$ also returns $p$, a matrix of $p$-values for testing the hypothesis of no correlation. Each p-value is the probability of getting a correlation as large as the observed value by random chance, when the true correlation is zero. If $p(i, j)$ is less than 0.05 , then the correlation $r(i, j)$ is significant.
[r,p,rlo,rup]=corrcoef(...) also returns matrices rlo and rup, of the same size as $r$, containing lower and upper bounds for a $95 \%$ confidence interval for each coefficient.
[...]=corrcoef(...,'PARAM1', VAL1,'PARAM2',VAL2, ...) specifies additional parameters and their values. Valid parameters are:

- 'alpha' - A number from 0 through 1 to specify a confidence level of 100*(1ALPHA)\%. Default is 0.05 for $95 \%$ confidence intervals.
- 'rows' - Either 'all' (default) to use all rows, ' complete' to use rows with no NaN values, or 'pairwise' to compute r(i,j) using rows with no NaN values in column $i$ or $j$.

The p -value is computed by transforming the correlation to create a t-statistic having N 2 degrees of freedom, where $N$ is the number of rows of $X$. The confidence bounds are based on an asymptotic normal distribution of $0.5^{*} \log ((1+r) /(1-r))$, with an approximate variance equal to $1 /(\mathrm{N}-3)$. These bounds are accurate for large samples when X has a multivariate normal distribution. The 'pairwise' option can produce an $r$ matrix that is not positive definite.

## Examples

## Compute Correlation Coefficients

This example shows how to generate random data having correlation between column 4 and the other columns.

```
x = randn(30,4); % uncorrelated data
x(:,4) = sum(x,2); % introduce correlation
f = fints((today:today+29)', x); % create a fints object using x
```

Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in

```
[r,p] = corrcoef(x); % compute sample correlation and p-values
[i,j] = find(p<0.05); % find significant correlations
[i,j] % display their (row,col) indices
ans = 4×2
\begin{tabular}{ll}
4 & 1 \\
3 & 2 \\
2 & 3 \\
1 & 4
\end{tabular}
```

Class support for inputs $\mathrm{X}, \mathrm{Y}$ : float: double and single.

See Also<br>cov|std|var<br>Introduced before R2006a

## corr2cov

Convert standard deviation and correlation to covariance

## Syntax

ExpCovariance $=$ corr2cov(ExpSigma,ExpCorrC)

## Arguments

| ExpSigma | Vector of length $n$ with the standard deviations of each process. $n$ is <br> the number of random processes. |
| :--- | :--- |
| ExpCorrC | (Optional) $n$-by- $n$ correlation coefficient matrix. If ExpCorrC is not <br> specified, the processes are assumed to be uncorrelated, and the <br> identity matrix is used. |

## Description

corr2cov converts standard deviation and correlation to covariance.
ExpCovariance is an $n$-by- $n$ covariance matrix, where $n$ is the number of processes.

```
ExpCov(i,j)=ExpCorrC(i,j)*ExpSigma(i)*ExpSigma(j)
```


## Examples

## Convert Standard Deviation and Correlation to Covariance

This example shows how to convert standard deviation and correlation to covariance.

```
ExpSigma = [0.5 2.0];
ExpCorrC = [1.0 -0.5
    -0.5 1.0];
```

```
ExpCovariance = corr2cov(ExpSigma, ExpCorrC)
ExpCovariance = 2×2
    0.2500 -0.5000
    -0.5000 4.0000
```


## See Also

corrcoef|cov|cov2corr|ewstats|std

## Topics

"Data Transformation and Frequency Conversion" on page 13-13
Introduced before R2006a

## COV

Covariance matrix

Note cov will be removed in a future release. Use timetable instead. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

$\operatorname{cov}(X)$
$\operatorname{cov}(X, Y)$

## Arguments

| X | Financial times series object. |
| :--- | :--- |
| Y | Financial times series object. |

## Description

cov for financial time series objects is based on the MATLAB cov function. See cov.
If $X$ is a financial time series object with one series, $\operatorname{cov}(X)$ returns the variance. For a financial time series object containing multiple series, where each row is an observation, and each series a variable, $\operatorname{cov}(X)$ is the covariance matrix.
$\operatorname{diag}(\operatorname{cov}(X))$ is a vector of variances for each series and sqrt(diag( $\operatorname{cov}(X)))$ is a vector of standard deviations.
$\operatorname{cov}(X, Y)$, where $X$ and $Y$ are financial time series objects with the same number of elements, is equivalent to $\operatorname{cov}([\mathrm{X}(:) \mathrm{Y}(:)])$.
$\operatorname{cov}(X)$ or $\operatorname{cov}(X, Y)$ normalizes by $(N-1)$ if $N>1$, where $N$ is the number of observations. This makes cov $(X)$ the best unbiased estimate of the covariance matrix if the observations are from a normal distribution. For $\mathrm{N}=1$, cov normalizes by N .
$\operatorname{cov}(X, 1)$ or $\operatorname{cov}(X, Y, 1)$ normalizes by $N$ and produces the second moment matrix of the observations about their mean. $\operatorname{cov}(X, Y, 0)$ is the same as $\operatorname{cov}(X, Y)$ and $\operatorname{cov}(X, 0)$ is the same as $\operatorname{cov}(X)$. The mean is removed from each column before calculating the result.

## Examples

## Create a Covariance Matrix

This example shows how to create a covariance matrix for the following dates.

```
dates = {'01-Jan-2007';'02-Jan-2007';'03-Jan-2007'};
A = [-1 1 2 ; -2 3 1 ; 4 0 3];
f = fints(dates, A);
Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in
c = cov(f)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in
c = 3\times3
\begin{tabular}{rrr}
10.3333 & -4.1667 & 3.0000 \\
-4.1667 & 2.3333 & -1.5000 \\
3.0000 & -1.5000 & 1.0000
\end{tabular}
```


## See Also

corrcoef | cov \| mean | std | var

## Introduced before R2006a

## cov2corr

Convert covariance to standard deviation and correlation coefficient

## Syntax

[ExpSigma,ExpCorrC] = cov2corr(ExpCovariance)

## Arguments

| ExpCovariance | $n$-by- $n$ covariance matrix; for example, from cov or <br> ewstats. $n$ is the number of random processes. |
| :--- | :--- |

## Description

[ExpSigma,ExpCorrC] = cov2corr(ExpCovariance) converts covariance to standard deviations and correlation coefficients.

ExpSigma is a 1-by- $n$ vector with the standard deviation of each process.
ExpCorrC is an $n$-by- $n$ matrix of correlation coefficients.
$\operatorname{ExpSigma}(i)=\operatorname{sqrt}(E x p C o v a r i a n c e(i, i))$
$\operatorname{Exp} \operatorname{CorrC}(i, j)=\operatorname{ExpCovariance(i,j)/(ExpSigma(i)*ExpSigma(j))~}$

## Examples

## Convert Covariance to Standard Deviations and Correlation Coefficients

This example shows how to convert a covariance matrix to standard deviations and correlation coefficients.

```
ExpCovariance = [0.25 -0.5
    -0.5 4.0];
```

```
[ExpSigma, ExpCorrC] = cov2corr(ExpCovariance)
ExpSigma = 1\times2
    0.5000 2.0000
ExpCorrC = 2\times2
    1.0000 -0.5000
    -0.5000 1.0000
```


## See Also

corr2cov|corrcoef|cov|ewstats|std

## Topics

"Data Transformation and Frequency Conversion" on page 13-13

Introduced before R2006a

## Cpncount

Coupon payments remaining until maturity

## Syntax

```
NumCouponsRemaining = cpncount(Settle,Maturity)
NumCouponsRemaining = cpncount(___,Period,Basis,EndMonthRule,
IssueDate,FirstCouponDate, LastCouponDate)
```


## Description

NumCouponsRemaining = cpncount(Settle, Maturity) returns the whole number of coupon payments between the Settle and Maturity dates for a coupon bond or set of bonds. Coupons falling on or before Settle are not counted, except for the Maturity payment which is always counted.

Required input arguments must be number of bonds, NUMBONDS-by-1 or 1-by-NUMBONDS, conforming vectors or scalars.

NumCouponsRemaining = cpncount (__ , Period,Basis,EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate) returns the whole number of coupon payments between the Settle and Maturity dates for a coupon bond or set of bonds using optional input arguments.

Optional input arguments must be either NUMBONDS-by-1 or 1-by-NUMBONDS conforming vectors, scalars, or empty matrices.

## Examples

## Find Coupon Payments Remaining Until Maturity

This example shows how to find the coupon payments remaining until maturity.

NumCouponsRemaining = cpncount('14 Mar 1997', '30 Nov 2000',...
2, 0, 0)
NumCouponsRemaining = 8

## Find Coupon Payments Remaining Until Maturity for Different Maturity Dates

This example shows how to find the coupon payments remaining until maturity, given three coupon bonds with different maturity dates and the same default arguments.

```
Maturity = ['30 Sep 2000'; '31 Oct 2001'; '30 Nov 2002'];
NumCouponsRemaining = cpncount('14 Sep 1997', Maturity)
NumCouponsRemaining = 3\times1
```

    7
    9
    11
    
## Input Arguments

## Settle - Settlement date <br> serial date numbers | date character vector | datetime object

Settlement date, specified as a vector of serial date number, date character vector, or datetime array. Settle must be earlier than Maturity.

Data Types: double | char | datetime

## Maturity - Maturity date

serial date number | date character vector | datetime array
Maturity date, specified as a vector of serial date numbers, date character vectors, or datetime arrays.

Data Types: double | char | datetime

## Period - Coupons per year of the bond

2 (semiannual) (default) | vector of positive integers from the set [1, 2, 3, 4, 6, 12]

Coupons per year of the bond, specified as a vector of positive integers from the set [1,2,3,4,6,12].

## Data Types: single | double

## Basis - Day-count basis of the bond

## 0 (actual/actual) (default) | numeric with value 0 through 13 | vector of numerics with values 0 through 13

Day-count basis of the bond, specified as an integer with a value of 0 through 13 or an N -by-1 vector of integers with values of 0 through 13 .

- $0=$ actual/actual (default)
- $1=30 / 360$ (SIA)
- $2=$ actual/360
- $3=$ actual $/ 365$
- $4=30 / 360$ (BMA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- $9=\mathrm{actual} / 360$ (ICMA)
- $10=$ actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- $12=$ actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-0
Data Types: single | double

## EndMonthRule - End-of-month rule flag for month having $\mathbf{3 0}$ or fewer days

 1 (in effect) (default) | nonnegative integer [0,1]End-of-month rule flag for month having 30 or fewer days, specified as scalar nonnegative integer [ 0,1 ] or a using an N -by-1 vector of values. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days.

- $0=$ Ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month.
- $1=$ Set rule on, meaning that a bond's coupon payment date is always the last actual day of the month.

Data Types: logical

## IssueDate - Bond issue date <br> serial date number | date character vector | datetime array

Bond issue date, specified as a serial date number, date character vector, or datetime array.

Data Types: double | char | datetime

## FirstCouponDate - Date when bond makes first coupon payment serial date number | date character vector | datetime array

Date when a bond makes its first coupon payment, specified as a serial date number, date character vector, or datetime array.

FirstCouponDate is used when a bond has an irregular first coupon period. When FirstCouponDate and LastCouponDate are both specified, FirstCouponDate takes precedence in determining the coupon payment structure. If you do not specify a FirstCouponDate, the cash flow payment dates are determined from other inputs.

Data Types: double|char|datetime

## LastCouponDate - Last coupon date of bond before maturity date <br> serial date number | date character vector | datetime array

Last coupon date of a bond before maturity date, specified as a serial date number, date character vector, or datetime array.

LastCouponDate is used when a bond has an irregular last coupon period. In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated at the LastCouponDate, regardless of where it falls, and is followed only by the bond's maturity cash flow date. If you do not specify a LastCouponDate, the cash flow payment dates are determined from other inputs.

Data Types: double|char|datetime

## Output Arguments

## NumCouponsRemaining - Whole number of coupon payments between the settlement and maturity dates for a coupon bond or set of bonds

 vectorWhole number of coupon payments between the settlement and maturity dates for a coupon bond or set of bonds, returned as an NBONDS-by-1 vector.

Coupons falling on or before settlement are not counted, except for the maturity payment which is always counted.

## See Also

accrfrac|cfamounts |cfdates|cftimes | cpndaten | cpndatenq| cpndatep|
cpndatepq|cpndaysn|cpndaysp|cpnpersz|datetime

## Topics

"Pricing and Computing Yields for Fixed-Income Securities" on page 2-22

## Introduced before R2006a

## cpndaten

Next coupon date for fixed-income security

## Syntax

```
NextCouponDate = cpndaten(Settle,Maturity)
NextCouponDate = cpndaten(___ ,
FirstCouponDate,LastCouponDate)
```


## Description

NextCouponDate $=$ cpndaten(Settle,Maturity) returns the next coupon date after the Settle date. This function finds the next coupon date whether or not the coupon structure is synchronized with the Maturity date.

Required input arguments must be number of bonds, NUMBONDS-by-1 or 1-by-NUMBONDS, conforming vectors or scalars.

NextCouponDate $=$ cpndaten( $\qquad$ ,Period,Basis,EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate) returns the next coupon date after the Settle date using optional input arguments.

Optional input arguments must be either NUMBONDS-by-1 or 1-by-NUMBONDS conforming vectors, scalars, or empty matrices.

If all the inputs for Settle, Maturity, IssueDate, FirstCouponDate, and LastCouponDate are either serial date numbers or date character vectors, then NextCouponDate is returned as a serial date number. The function datestr converts a serial date number to a formatted date character vector.

If any of the inputs for Settle, Maturity, IssueDate, FirstCouponDate, and LastCouponDate are datetime arrays, then NextCouponDate is returned as a datetime array.

## Examples

## Calculate the Next Coupon Date After the Settlement Date

Determine the NextCouponDate when using character vectors for input arguments.

```
NextCouponDate = cpndaten('14-Mar-1997', '30-Nov-2000', 2, 0, 0);
datestr(NextCouponDate)
ans =
'30-May-1997'
```

Determine the NextCouponDate when using datetime arrays for input arguments.

```
NextCouponDate = cpndaten('14-Mar-1997', datetime('30-Nov-2000','Locale','en_US'),...
```

2, 0, 0)
NextCouponDate = datetime
30-May-1997

Determine the NextCouponDate when using character vectors for input arguments and the optional argument for EndMonthRule.

```
NextCouponDate = cpndaten('14-Mar-1997', '30-Nov-2000', 2, 0, 1);
datestr(NextCouponDate)
ans =
'31-May-1997'
```

Determine the NextCouponDate when using an input vector for Maturity.

```
Maturity = ['30-Sep-2000'; '31-Oct-2000'; '30-Nov-2000'];
NextCouponDate = cpndaten('14-Mar-1997', Maturity);
datestr(NextCouponDate)
ans = 3x11 char array
    '31-Mar-1997'
    '30-Apr-1997'
    '31-May-1997'
```


## Input Arguments

## Settle - Settlement date <br> serial date numbers | date character vector | datetime object

Settlement date, specified as a vector of serial date number, date character vector, or datetime array. Settle must be earlier than Maturity.

Data Types: double|char|datetime

## Maturity - Maturity date

serial date number | date character vector | datetime array
Maturity date, specified as a vector of serial date numbers, date character vectors, or datetime arrays.

Data Types: double | char | datetime

## Period - Coupons per year of the bond

2 (semiannual) (default) | vector of positive integers from the set [1, 2, 3, 4, 6, 12]
Coupons per year of the bond, specified as a vector of positive integers from the set [1,2,3,4,6,12].

## Data Types: single| double

## Basis - Day-count basis of the bond

0 (actual/actual) (default) | numeric with value 0 through $13 \mid$ vector of numerics with values 0 through 13

Day-count basis of the bond, specified as an integer with a value of 0 through 13 or an N -by-1 vector of integers with values of 0 through 13.

- $0=$ actual/actual (default)
- $1=30 / 360$ (SIA)
- 2 = actual/360
- 3 = actual/365
- $4=30 / 360$ (BMA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- $9=\mathrm{actual} / 360$ (ICMA)
- $10=$ actual/365 (ICMA)
- $11=30 / 360 E$ (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-0
Data Types: single | double

## EndMonthRule - End-of-month rule flag for month having 30 or fewer days 1 (in effect) (default) | nonnegative integer [0,1]

End-of-month rule flag for month having 30 or fewer days, specified as scalar nonnegative integer [0, 1] or a using an N -by- 1 vector of values. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days.

- $0=$ Ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month.
- $1=$ Set rule on, meaning that a bond's coupon payment date is always the last actual day of the month.


## Data Types: logical

## IssueDate - Bond issue date <br> serial date number | date character vector | datetime array

Bond issue date, specified as a serial date number, date character vector, or datetime array.
Data Types: double | char | datetime

## FirstCouponDate - Date when bond makes first coupon payment serial date number | date character vector | datetime array

Date when a bond makes its first coupon payment, specified as a serial date number, date character vector, or datetime array.

FirstCouponDate is used when a bond has an irregular first coupon period. When FirstCouponDate and LastCouponDate are both specified, FirstCouponDate takes
precedence in determining the coupon payment structure. If you do not specify a FirstCouponDate, the cash flow payment dates are determined from other inputs.

Data Types: double|char|datetime
LastCouponDate - Last coupon date of bond before maturity date
serial date number | date character vector | datetime array
Last coupon date of a bond before maturity date, specified as a serial date number, date character vector, or datetime array.

LastCouponDate is used when a bond has an irregular last coupon period. In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated at the LastCouponDate, regardless of where it falls, and is followed only by the bond's maturity cash flow date. If you do not specify a LastCouponDate, the cash flow payment dates are determined from other inputs.

Data Types: double | char | datetime

## Output Arguments

NextCouponDate - Next coupon date after the settlement date vector

Next coupon date after the settlement date, returned as an NUMBONDS-by-1 vector of next actual coupon dates after settlement. If settlement is a coupon date, this function never returns the settlement date. Instead, the actual coupon date strictly after settlement is returned, but not exceeding the maturity date. Thus, this function will always return the lesser of the actual maturity date and the next coupon payment date.

If all the inputs for Settle, Maturity, IssueDate, FirstCouponDate, and LastCouponDate are either serial date numbers or date character vectors, then NextCouponDate is returned as a serial date number. The function datestr converts a serial date number to a formatted date character vector.

If any of the inputs for Settle, Maturity, IssueDate, FirstCouponDate, and LastCouponDate are datetime arrays, then NextCouponDate is returned as a datetime array.
See Alsoaccrfrac|cfamounts |cfdates|cftimes | cpncount|cpndatenq| cpndatep|cpndatepq|cpndaysn|cpndaysp|cpnpersz|datetime
Topics"Pricing and Computing Yields for Fixed-Income Securities" on page 2-22
Introduced before R2006a

## cpndatenq

Next quasi-coupon date for fixed-income security

## Syntax

NextQuasiCouponDate = cpndatenq(Settle,Maturity) NextQuasiCouponDate = cpndatenq( __ , Period,Basis,EndMonthRule, IssueDate,FirstCouponDate,LastCouponDate)

## Description

NextQuasiCouponDate = cpndatenq(Settle,Maturity) determines the next quasi coupon date for a portfolio of NUMBONDS fixed income securities whether or not the first or last coupon is normal, short, or long. For zero coupon bonds, cpndatenq returns quasi coupon dates as if the bond had a semiannual coupon structure. Successive quasi coupon dates determine the length of the standard coupon period for the fixed income security of interest and do not necessarily coincide with actual coupon payment dates.

Required input arguments must be number of bonds, NUMBONDS-by-1 or 1-by-NUMBONDS, conforming vectors or scalars.

NextQuasiCouponDate = cpndatenq( $\qquad$ ,Period,Basis, EndMonthRule, IssueDate,FirstCouponDate, LastCouponDate) determines the next quasi coupon date for a portfolio of NUMBONDS fixed income securities whether or not the first or last coupon is normal, short, or long using optional input arguments.

Optional input arguments must be either NUMBONDS-by-1 or 1-by-NUMBONDS conforming vectors, scalars, or empty matrices.

If all the inputs for Settle, Maturity, IssueDate, FirstCouponDate, and LastCouponDate are either serial date numbers or date character vectors, then NextQuasiCouponDate is returned as a serial date number. The function datestr converts a serial date number to a formatted date character vector.

If any of the inputs for Settle, Maturity, IssueDate, FirstCouponDate, and LastCouponDate are datetime arrays, then NextQuasiCouponDate is returned as a datetime array.

## Examples

## Determine the Next Quasi Coupon Date for a Portfolio of Fixed-Income Securities

Given a pair of bonds with the following characteristics:

```
Settle = char('30-May-1997','10-Dec-1997');
Maturity = char('30-Nov-2002','10-Jun-2004');
```

Compute NextCouponDate for this pair of bonds.

```
NextCouponDate = cpndaten(Settle, Maturity);
datestr(NextCouponDate)
ans = 2x11 char array
    '31-May-1997'
    '10-Jun-1998'
```

Compute the next quasi coupon dates for these two bonds.

```
NextQuasiCouponDate = cpndatenq(Settle, Maturity);
datestr(NextQuasiCouponDate)
ans = 2x11 char array
    '31-May-1997'
    '10-Jun-1998'
```

Because no FirstCouponDate has been specified, the results are identical.
Now supply an explicit FirstCouponDate for each bond.
FirstCouponDate = char('30-Nov-1997','10-Dec-1998');
Compute the next coupon dates.

```
NextCouponDate = cpndaten(Settle, Maturity, 2, 0, 1, [],...
FirstCouponDate);
datestr(NextCouponDate)
ans = 2x11 char array
    '30-Nov-1997'
    '10-Dec-1998'
```

The next coupon dates are identical to the specified first coupon dates.
Now recompute the next quasi coupon dates.

```
NextQuasiCouponDate = cpndatenq(Settle, Maturity, 2, 0, 1, [],...
FirstCouponDate);
datestr(NextQuasiCouponDate)
ans = 2x11 char array
    '31-May-1997'
    '10-Jun-1998'
```

These results illustrate the distinction between actual coupon payment dates and quasi coupon dates. FirstCouponDate (and LastCouponDate, as well), when specified, is associated with an actual coupon payment and also serves as the synchronization date for determining all quasi coupon dates. Since each bond in this example pays semiannual coupons, and the first coupon date occurs more than six months after settlement, each will have an intermediate quasi coupon date before the actual first coupon payment occurs.

## Input Arguments

## Settle - Settlement date

serial date numbers | date character vector | datetime object
Settlement date, specified as a vector of serial date number, date character vector, or datetime array. Settle must be earlier than Maturity.
Data Types: double | char|datetime

## Maturity - Maturity date

serial date number | date character vector | datetime array

Maturity date, specified as a vector of serial date numbers, date character vectors, or datetime arrays.

## Data Types: double|char|datetime

## Period - Coupons per year of the bond

2 (semiannual) (default) | vector of positive integers from the set [1, 2, 3, 4, 6, 12]
Coupons per year of the bond, specified as a vector of positive integers from the set [1,2,3,4,6,12].
Data Types: single | double

## Basis - Day-count basis of the instrument

0 (actual/actual) (default) | numeric with value 0 through 13 | vector of numerics with values 0 through 13

Day-count basis of the instrument, specified as an integer with a value of 0 through 13 or an N -by- 1 vector of integers with values of 0 through 13.

- $0=$ actual/actual (default)
- $1=30 / 360$ (SIA)
- $2=$ actual/360
- $3=$ actual $/ 365$
- $4=30 / 360$ (BMA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- $9=$ actual/360 (ICMA)
- $10=$ actual/365 (ICMA)
- $11=30 / 360 \mathrm{E}$ (ICMA)
- $12=$ actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-0
Data Types: single | double

## EndMonthRule - End-of-month rule flag for month having 30 or fewer days 1 (in effect) (default) | nonnegative integer [0,1]

End-of-month rule flag for month having 30 or fewer days, specified as a nonnegative integer [0, 1] using an N -by- 1 vector of values. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days.

- $0=$ Ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a bond's coupon payment date is always the last actual day of the month.


## Data Types: logical

## IssueDate - Bond issue date <br> serial date number | date character vector | datetime array

Bond issue date, specified as a serial date number, date character vector, or datetime array.

Data Types: double|char|datetime

## FirstCouponDate - Date when bond makes first coupon payment

serial date number | date character vector | datetime array
Date when a bond makes its first coupon payment, specified as a serial date number, date character vector, or datetime array.

FirstCouponDate is used when a bond has an irregular first coupon period. When FirstCouponDate and LastCouponDate are both specified, FirstCouponDate takes precedence in determining the coupon payment structure. If you do not specify a FirstCouponDate, the cash flow payment dates are determined from other inputs.

Data Types: double | char |datetime

## LastCouponDate - Last coupon date of bond before maturity date serial date number | date character vector | datetime array

Last coupon date of a bond before maturity date, specified as a serial date number, date character vector, or datetime array.

LastCouponDate is used when a bond has an irregular last coupon period. In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the
coupon structure of the bond. The coupon structure of a bond is truncated at the LastCouponDate, regardless of where it falls, and is followed only by the bond's maturity cash flow date. If you do not specify a LastCouponDate, the cash flow payment dates are determined from other inputs.

Data Types: double | char |datetime

## Output Arguments

## NextQuasiCouponDate - Next quasi coupon date for portfolio of NUMBONDS fixed income securities

vector
Next quasi coupon date for a portfolio of NUMBONDS fixed income securities, whether or not the first or last coupon is normal, short, or long, returned as a NUMBONDS-by-1 vector.

For zero coupon bonds, cpndatenq returns quasi coupon dates as if the bond had a semiannual coupon structure. Successive quasi coupon dates determine the length of the standard coupon period for the fixed income security of interest and do not necessarily coincide with actual coupon payment dates.

If all of the inputs for Settle, Maturity, IssueDate, FirstCouponDate, and LastCouponDate are either serial date numbers or date character vectors, then NextQuasiCouponDate is returned as a serial date number. The function datestr converts a serial date number to a formatted date character vector.

If any of the inputs for Settle, Maturity, IssueDate, FirstCouponDate, and LastCouponDate are datetime arrays, then NextQuasiCouponDate is returned as a datetime array.

## See Also

accrfrac |cfamounts |cfdates|cftimes | cpncount|cpndaten|cpndatenq| cpndatep|cpndatepq|cpndaysn|cpndaysp|cpnpersz|datetime

## Topics

"Pricing and Computing Yields for Fixed-Income Securities" on page 2-22

Introduced before R2006a

## cpndatepq

Previous quasi-coupon date for fixed-income security

## Syntax

```
PreviousQuasiCouponDate = cpndatepq(Settle,Maturity)
PreviousQuasiCouponDate = cpndatepq(__ ,Period,Basis,EndMonthRule,
IssueDate,FirstCouponDate, LastCouponDate)
```


## Description

PreviousQuasiCouponDate $=$ cpndatepq(Settle, Maturity) determines the previous quasi-coupon date for a set of NUMBONDS fixed income securities. Prior quasicoupon dates determine the length of the standard coupon period for the fixed income security of interest, and do not necessarily coincide with actual coupon payment dates. This function finds the previous quasi-coupon date for bonds with a coupon structure whose first or last period is either normal, short, or long.

Required input arguments must be number of bonds, NUMBONDS-by-1 or 1-by-NUMBONDS, conforming vectors or scalars.

PreviousQuasiCouponDate $=$ cpndatepq(__ , Period,Basis,EndMonthRule, IssueDate,FirstCouponDate, LastCouponDate), using optional input arguments, determines the previous quasi-coupon date for a set of NUMBONDS fixed income securities.

Optional input arguments must be either NUMBONDS-by-1 or 1-by-NUMBONDS conforming vectors, scalars, or empty matrices.

If all the inputs for Settle, Maturity, IssueDate, FirstCouponDate, and LastCouponDate are either serial date numbers or date character vectors, then PreviousQuasiCouponDate is returned as a serial date number. The function datestr converts a serial date number to a formatted date character vector.

If any of the inputs for Settle, Maturity, IssueDate, FirstCouponDate, and LastCouponDate are datetime arrays, then PreviousQuasiCouponDate is returned as a datetime array.

## Examples

## Determine the Previous Quasi Coupon Date for a Portfolio of Fixed-Income Securities

Given a pair of bonds with the following characteristics:

```
Settle = char('30-May-1997','10-Dec-1997');
Maturity = char('30-Nov-2002','10-Jun-2004');
```

With no FirstCouponDate explicitly supplied, compute the PreviousCouponDate for this pair of bonds.

```
PreviousCouponDate = cpndatep(Settle, Maturity);
datestr(PreviousCouponDate)
ans = 2x11 char array
    '30-Nov-1996'
    '10-Dec-1997'
```

Note that since the settlement date for the second bond is also a coupon date, cpndatep returns this date as the previous coupon date.

Now establish a FirstCouponDate and IssueDate for this pair of bonds.

```
FirstCouponDate = char('30-Nov-1997','10-Dec-1998');
IssueDate = char('30-May-1996', '10-Dec-1996');
```

Recompute the PreviousCouponDate for this pair of bonds.

```
PreviousCouponDate = cpndatep(Settle, Maturity, 2, 0, 1, ...
IssueDate, FirstCouponDate);
datestr(PreviousCouponDate)
ans = 2x11 char array
    ' 30-May-1996'
    10-Dec-1996'
```

Since both of these bonds settled before the first coupon had been paid, cpndatep returns the IssueDate as the PreviousCouponDate.

Using the same data, compute PreviousQuasiCouponDate.

```
PreviousQuasiCouponDate = cpndatepq(Settle, Maturity, 2, 0, 1,...
IssueDate, FirstCouponDate);
datestr(PreviousQuasiCouponDate)
ans = 2x11 char array
    '30-Nov-1996'
    '10-Dec-1997'
```

For the first bond the settlement date is not a normal coupon date. The PreviousQuasiCouponDate is the coupon date before or on the settlement date. Since the coupon structure is synchronized to FirstCouponDate, the previous quasi coupon date is $30-$ Nov-1996. PreviousQuasiCouponDate disregards IssueDate and FirstCouponDate in this case. For the second bond the settlement date (10-Dec-1997) occurs on a date when a coupon would normally be paid in the absence of an explicit FirstCouponDate. cpndatepq returns this date as PreviousQuasiCouponDate.

## Input Arguments

## Settle - Settlement date <br> serial date numbers | date character vector | datetime object

Settlement date, specified as a vector of serial date number, date character vector, or datetime array. Settle must be earlier than Maturity.

Data Types: double | char | datetime

## Maturity - Maturity date

serial date number | date character vector | datetime array
Maturity date, specified as a vector of serial date numbers, date character vectors, or datetime arrays.
Data Types: double | char | datetime

## Period - Coupons per year of the bond

2 (semiannual) (default) | vector of positive integers from the set [1, 2, 3, 4, 6, 12]
Coupons per year of the bond, specified as a vector of positive integers from the set [1,2,3,4,6,12].

Data Types: single|double

## Basis - Day-count basis of the instrument

## 0 (actual/actual) (default) | numeric with value 0 through $13 \mid$ vector of numerics with values 0 through 13

Day-count basis of the instrument, specified as an integer with a value of 0 through 13 or an N -by- 1 vector of integers with values of 0 through 13.

- $0=$ actual/actual (default)
- $1=30 / 360$ (SIA)
- 2 = actual/360
- 3 = actual/365
- $4=30 / 360$ (BMA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- $7=$ actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- $11=30 / 360 E$ (ICMA)
- $12=$ actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-0
Data Types: single | double

## EndMonthRule - End-of-month rule flag for month having 30 or fewer days 1 (in effect) (default) | nonnegative integer [0,1]

End-of-month rule flag for month having 30 or fewer days, specified as a nonnegative integer [0, 1] using an N -by-1 vector of values. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days.

- $0=$ Ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month.
- $1=$ Set rule on, meaning that a bond's coupon payment date is always the last actual day of the month.


## Data Types: logical

## IssueDate - Bond issue date <br> serial date number | date character vector | datetime array

Bond issue date, specified as a serial date number, date character vector, or datetime array.
Data Types: double | char | datetime

## FirstCouponDate - Date when bond makes first coupon payment serial date number | date character vector | datetime array

Date when a bond makes its first coupon payment, specified as a serial date number, date character vector, or datetime array.

FirstCouponDate is used when a bond has an irregular first coupon period. When FirstCouponDate and LastCouponDate are both specified, FirstCouponDate takes precedence in determining the coupon payment structure. If you do not specify a FirstCouponDate, the cash flow payment dates are determined from other inputs.
Data Types: double|char|datetime
LastCouponDate - Last coupon date of bond before maturity date
serial date number | date character vector | datetime array
Last coupon date of a bond before maturity date, specified as a serial date number, date character vector, or datetime array.

LastCouponDate is used when a bond has an irregular last coupon period. In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated at the LastCouponDate, regardless of where it falls, and is followed only by the bond's maturity cash flow date. If you do not specify a LastCouponDate, the cash flow payment dates are determined from other inputs.

Data Types: double | char | datetime

## Output Arguments

## PreviousQuasiCouponDate - Previous quasi coupon date for portfolio of NUMBONDS fixed income securities

vector
Previous quasi coupon date for a portfolio of NUMBONDS fixed income securities, whether or not the first or last coupon is normal, short, or long, returned as a NUMBONDS-by-1 vector of previous quasi-coupon dates before settlement. If settlement is a coupon date, this function returns the settlement date.

If all the inputs for Settle, Maturity, IssueDate, FirstCouponDate, and LastCouponDate are either serial date numbers or date character vectors, then PreviousQuasiCouponDate is returned as a serial date number. The function datestr converts a serial date number to a formatted date character vector.

If any of the inputs for Settle, Maturity, IssueDate, FirstCouponDate, and LastCouponDate are datetime arrays, then PreviousQuasiCouponDate is returned as a datetime array.

## See Also

accrfrac|cfamounts |cfdates|cftimes | cpncount | cpndaten|cpndatenq| cpndatep|cpndaysn | cpndaysp|cpnpersz|datetime

## Topics

"Pricing and Computing Yields for Fixed-Income Securities" on page 2-22
Introduced before R2006a

## cpndatep

Previous coupon date for fixed-income security

## Syntax

PreviousCouponDate = cpndatep(Settle, Maturity)
PreviousCouponDate = cpndatep (__, Period,Basis,EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate)

## Description

PreviousCouponDate = cpndatep(Settle, Maturity) returns the previous coupon date on or before settlement for a portfolio of bonds. This function finds the previous coupon date whether or not the coupon structure is synchronized with the maturity date. For zero coupon bonds the previous coupon date is the issue date, if available. However, if the issue date is not supplied, the previous coupon date for zero coupon bonds is the previous quasi coupon date calculated as if the frequency is semiannual.

Required input arguments must be number of bonds, NUMBONDS-by-1 or 1-by-NUMBONDS, conforming vectors or scalars.

PreviousCouponDate $=$ cpndatep $($ $\qquad$ , Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, Last CouponDate) returns the previous coupon date on or before settlement for a portfolio of bonds.

Optional input arguments must be either NUMBONDS-by-1 or 1-by-NUMBONDS conforming vectors, scalars, or empty matrices.

If all the inputs for Settle, Maturity, IssueDate, FirstCouponDate, and LastCouponDate are either serial date numbers or date character vectors, then PreviousCouponDate is returned as a serial date number. The function datestr converts a serial date number to a formatted date character vector.

If any of the inputs for Settle, Maturity, IssueDate, FirstCouponDate, and LastCouponDate are datetime arrays, then PreviousCouponDate is returned as a datetime array.

## Examples

## Calculate the Previous Coupon Date on or Before Settlement

Determine the PreviousCouponDate when using character vectors for input arguments.

```
PreviousCouponDate = cpndatep('14-Mar-1997', '30-Jun-2000',...
```

2, 0, 0);
datestr(PreviousCouponDate)
ans =
'30-Dec-1996'

Determine the PreviousCouponDate when using datetime arrays for input arguments.

```
PreviousCouponDate = cpndatep(datetime('14-Mar-1997','Locale','en_US'), '30-Jun-2000',
2, 0, 0)
PreviousCouponDate = datetime
    30-Dec-1996
```

Determine the PreviousCouponDate when using character vectors for input arguments and the optional argument for EndMonthRule.

```
PreviousCouponDate = cpndatep('14-Mar-1997', '30-Jun-2000',...
2, 0, 1);
datestr(PreviousCouponDate)
ans =
'31-Dec-1996'
```

Determine the PreviousCouponDate when using an input vector for Maturity.

```
Maturity = ['30-Apr-2000'; '31-May-2000'; '30-Jun-2000'];
PreviousCouponDate = cpndatep('14-Mar-1997', Maturity);
datestr(PreviousCouponDate)
ans = 3x11 char array
    '31-0ct-1996'
    '30-Nov-1996'
    '31-Dec-1996'
```


## Input Arguments

## Settle - Settlement date serial date numbers | date character vector | datetime object

Settlement date, specified as a vector of serial date number, date character vector, or datetime array. Settle must be earlier than Maturity.

Data Types: double|char|datetime

## Maturity - Maturity date

serial date number | date character vector | datetime array
Maturity date, specified as a vector of serial date numbers, date character vectors, or datetime arrays.
Data Types: double | char | datetime

## Period - Coupons per year of the bond

2 (semiannual) (default) | vector of positive integers from the set [1, 2, 3, 4, 6, 12]
Coupons per year of the bond, specified as a vector of positive integers from the set [1, 2, 3, 4, 6, 12].

## Data Types: single| double

## Basis - Day-count basis of the instrument

0 (actual/actual) (default) | numeric with value 0 through $13 \mid$ vector of numerics with values 0 through 13

Day-count basis of the instrument, specified as an integer with a value of 0 through 13 or an N -by-1 vector of integers with values of 0 through 13.

- $0=$ actual/actual (default)
- $1=30 / 360$ (SIA)
- $2=$ actual/360
- 3 = actual/365
- $4=30 / 360$ (BMA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- $7=$ actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- $11=30 / 360 E$ (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-0
Data Types: single | double

## EndMonthRule - End-of-month rule flag for month having 30 or fewer days 1 (in effect) (default) | nonnegative integer [0,1]

End-of-month rule flag for month having 30 or fewer days, specified as a nonnegative integer [0, 1] using an N -by-1 vector of values. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days.

- $0=$ Ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a bond's coupon payment date is always the last actual day of the month.


## Data Types: logical

## IssueDate - Bond issue date <br> serial date number | date character vector | datetime array

Bond issue date, specified as a serial date number, date character vector, or datetime array.

Data Types: double|char|datetime

## FirstCouponDate - Date when bond makes first coupon payment serial date number | date character vector | datetime array

Date when a bond makes its first coupon payment, specified as a serial date number, date character vector, or datetime array.

FirstCouponDate is used when a bond has an irregular first coupon period. When FirstCouponDate and LastCouponDate are both specified, FirstCouponDate takes
precedence in determining the coupon payment structure. If you do not specify a FirstCouponDate, the cash flow payment dates are determined from other inputs.
Data Types: double|char|datetime
LastCouponDate - Last coupon date of bond before maturity date
serial date number | date character vector | datetime array
Last coupon date of a bond before maturity date, specified as a serial date number, date character vector, or datetime array.

LastCouponDate is used when a bond has an irregular last coupon period. In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated at the LastCouponDate, regardless of where it falls, and is followed only by the bond's maturity cash flow date. If you do not specify a LastCouponDate, the cash flow payment dates are determined from other inputs.

Data Types: double | char | datetime

## Output Arguments

## PreviousCouponDate - Previous coupon date on or before settlement for portfolio of bonds

vector
Previous coupon date on or before settlement for portfolio of bonds, returned as an NUMBONDS-by-1 vector. If settlement is a coupon date, this function returns the settlement date. The actual coupon date strictly on or before settlement is returned, but not exceeding the issue date, if available. Thus, this function always returns the lesser of the actual issue date and the previous coupon payment date with respect to the settlement date.

If all the inputs for Settle, Maturity, IssueDate, FirstCouponDate, and LastCouponDate are either serial date numbers or date character vectors, then PreviousCouponDate is returned as a serial date number. The function datestr converts a serial date number to a formatted date character vector.

If any of the inputs for Settle, Maturity, IssueDate, FirstCouponDate, and LastCouponDate are datetime arrays, then PreviousCouponDate is returned as a datetime array.

## See Also

accrfrac|cfamounts |cfdates|cftimes | cpncount|cpndaten|cpndatenq| cpndatepq|cpndaysn|cpndaysp|cpnpersz|datetime

## Topics

"Pricing and Computing Yields for Fixed-Income Securities" on page 2-22
Introduced before R2006a

## cpndaysn

Number of days to next coupon date

## Syntax

```
NumDaysNext = cpndaysn(Settle,Maturity)
NumDaysNext = cpndaysn(___,Period,Basis,EndMonthRule,IssueDate,
FirstCouponDate,LastCouponDate)
```


## Description

NumDaysNext $=$ cpndaysn(Settle, Maturity) returns the number of days from the settlement date to the next coupon date for a bond or set of bonds. For zero coupon bonds coupon dates are computed as if the bonds have a semiannual coupon structure. NumDaysNext returns a double for serial date number, date character vector, and datetime inputs.

Required input arguments must be number of bonds, NUMBONDS-by-1 or 1-by-NUMBONDS, conforming vectors or scalars.

NumDaysNext $=$ cpndaysn( __ , Period,Basis,EndMonthRule,IssueDate, FirstCouponDate, LastCouponDate) returns the number of days from the settlement date to the next coupon date for a bond or set of bonds using optional input arguments.

Optional input arguments must be either NUMBONDS-by-1 or 1-by-NUMBONDS conforming vectors, scalars, or empty matrices.

If all the inputs for Settle, Maturity, IssueDate, FirstCouponDate, and LastCouponDate are either serial date numbers or date character vectors, then NumDaysNext is returned as a serial date number. The function datestr converts a serial date number to a formatted date character vector.

If any of the inputs for Settle, Maturity, IssueDate, FirstCouponDate, and LastCouponDate are datetime arrays, then NumDaysNext is returned as a datetime array.

## Examples

## Calculate the Number of Days From Settlement Date to Next Coupon Date

Determine the NumDaysNext when using character vectors for input arguments.

```
NumDaysNext = cpndaysn('14-Sep-2000', '30-Jun-2001', 2, 0, 0)
NumDaysNext = 107
```

Determine the NumDaysNext when using datetime arrays for input arguments.
NumDaysNext = cpndaysn(datetime('14-Sep-2000','Locale','en_US'), '30-Jun-2001', 2, 0,
NumDaysNext = 107
Determine the NumDaysNext when using character vectors for input arguments and the optional argument for EndMonthRule.

```
NumDaysNext = cpndaysn('14-Sep-2000', '30-Jun-2001', 2, 0, 1)
NumDaysNext = 108
```

Determine the NumDaysNext when using an input vector for Maturity.

```
Maturity = ['30-Apr-2001'; '31-May-2001'; '30-Jun-2001'];
NumDaysNext = cpndaysn('14-Sep-2000', Maturity)
NumDaysNext = 3x1
```


## Input Arguments

## Settle - Settlement date

serial date numbers | date character vector | datetime object

Settlement date, specified as a vector of serial date number, date character vector, or datetime array. Settle must be earlier than Maturity.
Data Types: double | char | datetime

## Maturity - Maturity date

serial date number | date character vector | datetime array
Maturity date, specified as a vector of serial date numbers, date character vectors, or datetime arrays.

Data Types: double | char | datetime

## Period - Coupons per year of the bond

2 (semiannual) (default) | vector of positive integers from the set [1, 2, 3, 4, 6, 12]
Coupons per year of the bond, specified as a vector of positive integers from the set [1,2,3,4,6,12].

Data Types: single | double

## Basis - Day-count basis of the instrument

0 (actual/actual) (default) | numeric with value 0 through $13 \mid$ vector of numerics with values 0 through 13

Day-count basis of the instrument, specified as an integer with a value of 0 through 13 or an N -by- 1 vector of integers with values of 0 through 13.

- $0=$ actual/actual (default)
- $1=30 / 360$ (SIA)
- 2 = actual/360
- 3 = actual/365
- $4=30 / 360$ (BMA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- $9=$ actual/360 (ICMA)
- $10=$ actual/365 (ICMA)
- $11=30 / 360 \mathrm{E}$ (ICMA)
- $12=$ actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-0
Data Types: single | double

## EndMonthRule - End-of-month rule flag for month having 30 or fewer days

1 (in effect) (default) | nonnegative integer [0,1]
End-of-month rule flag for month having 30 or fewer days, specified as a nonnegative integer [ 0,1 ] using an N -by- 1 vector of values. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days.

- $0=$ Ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a bond's coupon payment date is always the last actual day of the month.


## Data Types: logical

## IssueDate - Bond issue date <br> serial date number | date character vector | datetime array

Bond issue date, specified as a serial date number, date character vector, or datetime array.

Data Types: double | char | datetime

## FirstCouponDate - Date when bond makes first coupon payment serial date number | date character vector | datetime array

Date when a bond makes its first coupon payment, specified as a serial date number, date character vector, or datetime array.

FirstCouponDate is used when a bond has an irregular first coupon period. When FirstCouponDate and LastCouponDate are both specified, FirstCouponDate takes precedence in determining the coupon payment structure. If you do not specify a FirstCouponDate, the cash flow payment dates are determined from other inputs.

Data Types: double | char |datetime

## LastCouponDate - Last coupon date of bond before maturity date serial date number | date character vector | datetime array

Last coupon date of a bond before maturity date, specified as a serial date number, date character vector, or datetime array.

LastCouponDate is used when a bond has an irregular last coupon period. In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated at the LastCouponDate, regardless of where it falls, and is followed only by the bond's maturity cash flow date. If you do not specify a LastCouponDate, the cash flow payment dates are determined from other inputs.
Data Types: double | char | datetime

## Output Arguments

## NumDaysNext - Number of days from settlement date to next coupon date vector

Number of days from settlement date to next coupon date, returned as an NUMBONDS-by-1 vector. For zero coupon bonds coupon dates are computed as if the bonds have a semiannual coupon structure. NumDaysNext returns a double for serial date number, date character vector, and datetime inputs.

If all the inputs for Settle, Maturity, IssueDate, FirstCouponDate, and LastCouponDate are either serial date numbers or date character vectors, then NumDaysNext is returned as a serial date number. The function datestr converts a serial date number to a formatted date character vector.

If any of the inputs for Settle, Maturity, IssueDate, FirstCouponDate, and LastCouponDate are datetime arrays, then NumDaysNext is returned as a datetime array.

## See Also

accrfrac|cfamounts |cfdates|cftimes | cpncount|cpndaten|cpndatenq| cpndatep|cpndatepq|cpndaysn|cpndaysp|cpnpersz|datetime

## Topics

"Pricing and Computing Yields for Fixed-Income Securities" on page 2-22

Introduced before R2006a

## cpndaysp

Number of days since previous coupon date

## Syntax

NumDaysPrevious = cpndaysp(Settle, Maturity)
NumDaysPrevious = cpndaysp(__, Period, Basis,EndMonthRule,IssueDate, FirstCouponDate,LastCouponDate)

## Description

NumDaysPrevious = cpndaysp(Settle, Maturity) returns the number of days between the previous coupon date and the settlement date for a bond or set of bonds. When the coupon frequency is 0 (a zero coupon bond), the previous coupon date is calculated as if the frequency were semiannual. NumDaysPrevious returns a double for serial date number, date character vector, and datetime inputs.

Required input arguments must be number of bonds, NUMBONDS-by-1 or 1-by-NUMBONDS, conforming vectors or scalars.

NumDaysPrevious = cpndaysp(__, Period,Basis,EndMonthRule,IssueDate, FirstCouponDate, LastCouponDate) returns the number of days between the previous coupon date and the settlement date for a bond or set of bonds using optional input arguments.

Optional input arguments must be either NUMBONDS-by-1 or 1-by-NUMBONDS conforming vectors, scalars, or empty matrices.

If all the inputs for Settle, Maturity, IssueDate, FirstCouponDate, and LastCouponDate are either serial date numbers or date character vectors, then NumDaysPrevious is returned as a serial date number. The function datestr converts a serial date number to a formatted date character vector.

If any of the inputs for Settle, Maturity, IssueDate, FirstCouponDate, and LastCouponDate are datetime arrays, then NumDaysPrevious is returned as a datetime array.

## Examples

## Calculate the Number of Days Between Previous Coupon Date and Settlement Date

Determine the NumDaysPrevious when using character vectors for input arguments.

```
NumDaysPrevious = cpndaysp('14-Mar-2000', '30-Jun-2001', 2, 0, 0)
NumDaysPrevious = 75
```

Determine the NumDaysPrevious when using a datetime array for an input argument.

```
NumDaysPrevious = cpndaysp(datetime('14-Mar-2000','Locale','en_US'), '30-Jun-2001', 2,
NumDaysPrevious = 75
```

Determine the NumDaysPrevious when using character vectors for input arguments and the optional argument for EndMonthRule.

```
NumDaysPrevious = cpndaysp('14-Mar-2000', '30-Jun-2001', 2, 0, 1)
NumDaysPrevious = 74
```

Determine the NumDaysPrevious when using an input vector for Maturity.

```
Maturity = ['30-Apr-2001'; '31-May-2001'; '30-Jun-2001'];
NumDaysPrevious = cpndaysp('14-Mar-2000', Maturity)
NumDaysPrevious = 3\times1
```

    135
    105
    74
    
## Input Arguments

## Settle - Settlement date

serial date numbers | date character vector | datetime object

Settlement date, specified as a vector of serial date number, date character vector, or datetime array. Settle must be earlier than Maturity.
Data Types: double | char | datetime

## Maturity - Maturity date

serial date number | date character vector | datetime array
Maturity date, specified as a vector of serial date numbers, date character vectors, or datetime arrays.

Data Types: double | char | datetime

## Period - Coupons per year of the bond

2 (semiannual) (default) | vector of positive integers from the set [1, 2, 3, 4, 6, 12]
Coupons per year of the bond, specified as a vector of positive integers from the set [1,2,3,4,6,12].
Data Types: single | double

## Basis - Day-count basis of the instrument

0 (actual/actual) (default) | numeric with value 0 through $13 \mid$ vector of numerics with values 0 through 13

Day-count basis of the instrument, specified as an integer with a with value 0 through 13 or an N -by- 1 vector of integers with values 0 through 13.

- $0=$ actual/actual (default)
- $1=30 / 360$ (SIA)
- 2 = actual/360
- 3 = actual/365
- $4=30 / 360$ (BMA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- $9=\mathrm{actual} / 360$ (ICMA)
- $10=$ actual/365 (ICMA)
- $11=30 / 360 E$ (ICMA)
- $12=$ actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-0
Data Types: single | double

## EndMonthRule - End-of-month rule flag for month having 30 or fewer days

1 (in effect) (default) | nonnegative integer [0,1]
End-of-month rule flag for month having 30 or fewer days, specified as a nonnegative integer [ 0,1 ] using an N -by- 1 vector of values. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days.

- $0=$ Ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a bond's coupon payment date is always the last actual day of the month.


## Data Types: logical

## IssueDate - Bond issue date <br> serial date number | date character vector | datetime array

Bond issue date, specified as a serial date number, date character vector, or datetime array.

Data Types: double | char | datetime

## FirstCouponDate - Date when bond makes first coupon payment serial date number | date character vector | datetime array

Date when a bond makes its first coupon payment, specified as a serial date number, date character vector, or datetime array.

FirstCouponDate is used when a bond has an irregular first coupon period. When FirstCouponDate and LastCouponDate are both specified, FirstCouponDate takes precedence in determining the coupon payment structure. If you do not specify a FirstCouponDate, the cash flow payment dates are determined from other inputs.

Data Types: double | char |datetime

## LastCouponDate - Last coupon date of bond before maturity date serial date number | date character vector | datetime array

Last coupon date of a bond before maturity date, specified as a serial date number, date character vector, or datetime array.

LastCouponDate is used when a bond has an irregular last coupon period. In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated at the LastCouponDate, regardless of where it falls, and is followed only by the bond's maturity cash flow date. If you do not specify a LastCouponDate, the cash flow payment dates are determined from other inputs.

Data Types: double | char|datetime

## Output Arguments

## NumDaysPrevious - Number of days between previous coupon date and settlement date <br> vector

Number of days between the previous coupon date and the settlement date, returned as an NUMBONDS-by-1 vector. If the settlement date is a coupon date, this function always returns the settlement date.

When the coupon frequency is 0 (a zero coupon bond), the previous coupon date is calculated as if the frequency were semiannual. NumDaysPrevious returns a double for serial date number, date character vector, and datetime inputs.

If all the inputs for Settle, Maturity, IssueDate, FirstCouponDate, and LastCouponDate are either serial date numbers or date character vectors, then NumDaysPrevious is returned as a serial date number. The function datestr converts a serial date number to a formatted date character vector.

If any of the inputs for Settle, Maturity, IssueDate, FirstCouponDate, and LastCouponDate are datetime arrays, then NumDaysPrevious is returned as a datetime array.
See Alsoaccrfrac|cfamounts|cfdates|cftimes | cpncount|cpndaten|cpndatenq|cpndatep|cpndatepq|cpndaysn|cpndaysp|cpnpersz|datetime
Topics
"Pricing and Computing Yields for Fixed-Income Securities" on page 2-22
Introduced before R2006a

## cpnpersz

Number of days in coupon period

## Syntax

NumDaysPeriod $=$ cpnpersz(Settle, Maturity)
NumDaysPeriod = cpnpersz( __,
FirstCouponDate, LastCouponDate)

## Description

NumDaysPeriod $=$ cpnpersz(Settle, Maturity) returns the number of days in the coupon period containing the settlement date. For zero coupon bonds, coupon dates are computed as if the bonds have a semiannual coupon structure. NumDaysPeriod returns a double for serial date number, date character vector, and datetime inputs.

Required input arguments must be number of bonds, NUMBONDS-by-1 or 1-by-NUMBONDS, conforming vectors or scalars.

NumDaysPeriod = cpnpersz(__, Period,Basis,EndMonthRule,IssueDate, FirstCouponDate, LastCouponDate) returns the number of days in the coupon period containing the settlement date using optional input arguments.

Optional input arguments must be either NUMBONDS-by-1 or 1-by-NUMBONDS conforming vectors, scalars, or empty matrices.

If all the inputs for Settle, Maturity, IssueDate, FirstCouponDate, and LastCouponDate are either serial date numbers or date character vectors, then NumDaysPeriod is returned as a serial date number. The function datestr converts a serial date number to a formatted date character vector.

If any of the inputs for Settle, Maturity, IssueDate, FirstCouponDate, and LastCouponDate are datetime arrays, then NumDaysPeriod is returned as a datetime array.

## Examples

## Calculate the Number of Days in the Coupon Period Containing Settlement Date

Determine the NumDaysPeriod when using character vectors for input arguments.

```
NumDaysPeriod = cpnpersz('14-Sep-2000', '30-Jun-2001', 2, 0, 0)
NumDaysPeriod = 183
```

Determine the NumDaysPeriod when using a datetime array for an input argument.

```
NumDaysPeriod = cpnpersz(datetime('14-Sep-2000','Locale','en_US'), '30-Jun-2001', 2, 0
NumDaysPeriod = 183
```

Determine the NumDaysPeriod when using character vectors for input arguments and the optional argument for EndMonthRule.

```
NumDaysPeriod = cpnpersz('14-Sep-2000', '30-Jun-2001', 2, 0, 1)
NumDaysPeriod = 184
```

Determine the NumDaysPeriod when using an input vector for Maturity.

```
Maturity = ['30-Apr-2001'; '31-May-2001'; '30-Jun-2001'];
NumDaysPeriod = cpnpersz('14-Sep-2000', Maturity)
NumDaysPeriod = 3×1
```

    184
    183
    184
    
## Input Arguments

## Settle - Settlement date

serial date numbers | date character vector | datetime object

Settlement date, specified as a vector of serial date number, date character vector, or datetime array. Settle must be earlier than Maturity.
Data Types: double | char \| datetime

## Maturity - Maturity date

serial date number | date character vector | datetime array
Maturity date, specified as a vector of serial date numbers, date character vectors, or datetime arrays.

Data Types: double | char | datetime

## Period - Coupons per year of the bond

2 (semiannual) (default) | vector of positive integers from the set [1, 2, 3, 4, 6, 12]
Coupons per year of the bond, specified as a vector of positive integers from the set [1,2,3,4,6,12].

Data Types: single | double

## Basis - Day-count basis of the instrument

0 (actual/actual) (default) | numeric with value 0 through $13 \mid$ vector of numerics with values 0 through 13

Day-count basis of the instrument, specified as an integer with a value of 0 through 13 or an N -by- 1 vector of integers with values of 0 through 13.

- $0=$ actual/actual (default)
- $1=30 / 360$ (SIA)
- 2 = actual/360
- 3 = actual/365
- $4=30 / 360$ (BMA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- $9=$ actual/360 (ICMA)
- $10=$ actual/365 (ICMA)
- $11=30 / 360 \mathrm{E}$ (ICMA)
- $12=$ actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-0
Data Types: single | double

## EndMonthRule - End-of-month rule flag for month having 30 or fewer days

1 (in effect) (default) | nonnegative integer [0,1]
End-of-month rule flag for month having 30 or fewer days, specified as a nonnegative integer [ 0,1 ] using an N -by- 1 vector of values. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days.

- $0=$ Ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a bond's coupon payment date is always the last actual day of the month.


## Data Types: logical

## IssueDate - Bond issue date <br> serial date number | date character vector | datetime array

Bond issue date, specified as a serial date number, date character vector, or datetime array.

Data Types: double | char | datetime

## FirstCouponDate - Date when bond makes first coupon payment serial date number | date character vector | datetime array

Date when a bond makes its first coupon payment, specified as a serial date number, date character vector, or datetime array.

FirstCouponDate is used when a bond has an irregular first coupon period. When FirstCouponDate and LastCouponDate are both specified, FirstCouponDate takes precedence in determining the coupon payment structure. If you do not specify a FirstCouponDate, the cash flow payment dates are determined from other inputs.

Data Types: double | char |datetime

## LastCouponDate - Last coupon date of bond before maturity date serial date number | date character vector | datetime array

Last coupon date of a bond before maturity date, specified as a serial date number, date character vector, or datetime array.

LastCouponDate is used when a bond has an irregular last coupon period. In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated at the LastCouponDate, regardless of where it falls, and is followed only by the bond's maturity cash flow date. If you do not specify a LastCouponDate, the cash flow payment dates are determined from other inputs.
Data Types: double | char | datetime

## Output Arguments

## NumDaysPeriod - Number of days in coupon period containing settlement date vector

Number of days in the coupon period containing the settlement date, returned as an NUMBONDS-by-1 vector. For zero coupon bonds, coupon dates are computed as if the bonds have a semiannual coupon structure.

If all the inputs for Settle, Maturity, IssueDate, FirstCouponDate, and LastCouponDate are either serial date numbers or date character vectors, then NumDaysPeriod is returned as a serial date number. The function datestr converts a serial date number to a formatted date character vector.

If any of the inputs for Settle, Maturity, IssueDate, FirstCouponDate, and LastCouponDate are datetime arrays, then NumDaysPeriod is returned as a datetime array.

## See Also

accrfrac|cfamounts|cfdates|cftimes | cpncount|cpndaten|cpndatenq| cpndatep|cpndatepq|cpndaysn|cpndaysp|datetime

## Topics

"Pricing and Computing Yields for Fixed-Income Securities" on page 2-22

## Introduced before R2006a

## createholidays

Create trading calendars

## Syntax

createholidays(Filename, Codefile,InfoFile,TargetDir, IncludeWkds, Wprompt, NoGUI)

## Description

createholidays(Filename, Codefile,InfoFile,TargetDir, IncludeWkds, Wprompt, NoGUI) programmatically generates the market-specific holidays.m files (from FinancialCalendar. com financial center holiday data) without displaying the interface.

Note To use createholidays, you must obtain data, codes, and info files from https://www.FinancialCalendar.com trading calendars. The data files must be in the required MATLAB format.

## Examples

## Create holidays.m Files Using createholidays

Use createholidays to create holidays*.m files from My_datafile.csv in the folder c: \work. Weekends are included in the holidays list based on the input flag INCLUDEWDKS = 1
createholidays('FinancialCalendar\My_datafile.csv',...
'FinancialCalendar\My_codesfile.csv',...
'FinancialCalendar\My_infofile.csv','C:\work',1,1,1)

## Input Arguments

## Filename - Data file name <br> character vector

Data file name, specified using a character vector.
Data Types: char

## Codefile - Code file name

character vector
Code file name, specified using a character vector.
Data Types: char

## InfoFile - Info file name

character vector
Info file name, specified using a character vector.
Data Types: char

## TargetDir - Target folder where to write new holidays.m files character vector

Target folder where to write the new holidays.m files, specified using a character vector.

## Data Types: char

## IncludeWkds - Option to include weekends in holiday list

numeric with value 0 or 1
Option to include weekends in the holiday list, specified using a numeric with value 0 or 1. Values are:

- 0 - Do not include weekends in the holiday list.
- 1 - Include weekends in the holiday list.


## Data Types: logical

## Wprompt - Option to prompt for file location for each holiday.m file that is created <br> numeric with value 0 or 1

Option to prompt for the file location for each holiday.m file that is created, specified using a numeric with value 0 or 1 . Values are:

- 0 - Do not prompt for the file location.
- 1 - Prompt for the file location.


## Data Types: logical

## NoGUI - Run createholidays without displaying Trading Calendars user interface

numeric with value 0 or 1
Run createholidays without displaying the Trading Calendars user interface, specified using a numeric with value 0 or 1 . Values are:

- 0 - Display the GUI.
- 1 - Do not display the GUI.

Data Types: logical

## See Also

holidays

## Topics

"Handle and Convert Dates" on page 2-2
"Trading Calendars User Interface" on page 16-2
"UICalendar User Interface" on page 16-4

Introduced in R2007b

## cumsum

Cumulative sum

Note cumsum will be removed in a future release. Use timetable instead. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

newfts $=$ cumsum(oldfts)

## Description

newfts $=$ cumsum(oldfts) calculates the cumulative sum of each individual time series data series in the financial time series object oldfts and returns the result in another financial time series object newfts. newfts contains the same data series names as oldfts.

## Examples

## Compute the Cumulative Sum

This example shows how to compute the cumulative sum for Disney stock and plot the results.
load disney.mat
Cs_dis = cumsum(fillts(dis));
Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in
Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in
plot(cs_dis)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in


## See Also

fints

## Topics

"Financial Time Series Operations" on page 13-7

Introduced before R2006a

## cur2frac

Decimal currency values to fractional values

## Syntax

Fraction = cur2frac(Decimal,Denominator)

## Description

Fraction = cur2frac(Decimal,Denominator) converts decimal currency values to fractional values. Fraction is returned as a character vector.

## Examples

## Convert Decimal Currency Values to Fractional Values

This example shows how to convert decimal currency values to fractional values.
Fraction $=$ cur2frac(12.125, 8)
Fraction =
'12.1'

## Input Arguments

## Decimal - Decimal currency value

numeric decimal
Decimal currency value, specified as a scalar or vector using numeric decimal values.
Data Types: double

## Denominator - Denominator of the fractions <br> numeric

Denominator of the fractions, specified as a scalar or vector using numeric values for the denominator.

## Data Types: double

## Output Arguments

## Fraction - Fractional values

character vector | cell array of character vectors
Fractional values, returned as a character vector or cell array of character vectors.

## See Also

cur2str|frac2cur

Introduced before R2006a

## cur2str

Bank-formatted text

## Syntax

BankText = cur2str(Value,Digits)

## Description

BankText $=$ cur2str(Value,Digits) returns the given value in bank format.
The output format for BankText is a numerical format with dollar sign prefix, two decimal places, and negative numbers in parentheses; for example, (\$123.45) and $\$ 6789.01$. The standard MATLAB bank format uses two decimal places, no dollar sign, and a minus sign for negative numbers; for example, -123.45 and 6789.01.

## Examples

## Return Bank Formatted Text

Return bank formatted text for a negative numeric value.

```
BankText = cur2str(-826444.4456,3)
BankText =
'($826444.446)'
\% Negative numbers are displayed in parentheses.
```


## Input Arguments

Value - Value to be formatted

numeric value

Value to be formatted, specified as a numeric value.

## Data Types: double

## Digits - Number of significant digits

2 (default) | integer
Number of significant digits, specified as an integer. A negative Digits rounds the value to the left of the decimal point.

## Data Types: double

## Output Arguments

## BankText - Bank-formatted text

character vector
Bank-formatted text (BankText) is returned as a character vector with a leading dollar sign (\$). Negative numbers are displayed in parentheses.

## See Also

cur2frac|frac2cur
Introduced before R2006a

## date2time

Time and frequency from dates

## Syntax

```
[TFactors,F] = date2time(Settle,Maturity)
[TFactors,F] = date2time( ___,Compounding,Basis,EndMonthRule)
```


## Description

[TFactors,F] = date2time(Settle, Maturity) computes time factors appropriate to compounded rate quotes between the Settle and Maturity dates. date2time is the inverse of time2date.
[TFactors,F] = date2time( $\qquad$ , Compounding, Basis, EndMonthRule) computes time factors appropriate to compounded rate quotes between the Settle and Maturity dates using optional input arguments for Compounding, Basis, and EndMonthRule. date2time is the inverse of time2date.

## Examples

## Compute date2time Using an actual/actual Basis

To get the date2time period between '31-Jul-2015' and '30-Sep-2015' using an actual/ actual basis:

```
date2time('31-Jul-2015', '30-Sep-2015', 2, 0, 1)
```

ans $=0.3333$
When using date2time quasi coupon, two quasi coupon dates are computed for a bond with a maturity corresponding to the Dates input. In this case, that would be "30-Sep-2015". Assuming that the compounding frequency is 2 , the other quasi coupon date is six months prior to this date. Assuming the end of month rule is in place, then the other
quasi coupon date is "31-Mar-2015". You can use these two dates to compute the total number of actual days in a period (which is 183). Given this, the fraction of time between the start and end date for the actual/actual basis is computed as follows.
(Actual Days between Start Date and End Date)/(Actual Number of Days between Quasi Coupon Dates)

There are 61 days between 31-Jul-2015 and 30-Sep-2015 and 183 days between the quasi coupon dates ("31-Mar-2015" and "30-Sep-2015") which leads to a final result of 61/183 or exactly $1 / 3$.

## Input Arguments

## Settle - Settlement date

serial date number | date character vector | datetime object
Settlement date, specified as a serial date number, date character vector, or datetime array.
Data Types: double | char | datetime

## Maturity - Maturity date <br> serial date number | date character vector

Maturity date, specified as a scalar or N -by-1 vector using serial date numbers, date character vectors, or datetime arrays.

Data Types: double | char | datetime
Compounding - Rate at which input zero rates are compounded when annualized
2 (Semiannual compounding) (default) | scalar with numeric values of $0,1,2,3,4,5,6$, 12, 365, -1

Rate at which input zero rates are compounded when annualized, specified as a scalar with numeric values of: $0,1,2,3,4,5,6,12,365$, or -1 . Allowed values are defined as:

- 0 - Simple interest (no compounding)
- 1 - Annual compounding
- 2 - Semiannual compounding (default)
- 3-Compounding three times per year
- 4 - Quarterly compounding
- 6 - Bimonthly compounding
- 12 - Monthly compounding
- 365 - Daily compounding
-     - 1 - Continuous compounding

The optional Compounding argument determines the formula for the discount factors (Disc):

- Compounding $=0$ for simple interest
- Disc $=1 /(1+Z * T)$, where $T$ is time in years and simple interest assumes annual times $\mathrm{F}=1$.
- Compounding $=1,2,3,4,6,12$
- Disc $=(1+Z / F)^{\wedge}(-T)$, where $F$ is the compounding frequency, $Z$ is the zero rate, and $T$ is the time in periodic units, for example, $T=F$ is one year.
- Compounding $=365$
- $\operatorname{Disc}=(1+Z / F)^{\wedge}(-T)$, where $F$ is the number of days in the basis year and $T$ is a number of days elapsed computed by basis.
- Compounding = - 1
- Disc $=\exp \left(-\mathrm{T}^{*} \mathrm{Z}\right)$, where T is time in years.


## Basis - Day-count basis

## 0 (actual/actual) (default) | numeric with value 0 through $13 \mid$ vector of numerics with values 0 through 13

Day-count basis, specified as an integer with a value 0 through 13 or a N -by- 1 vector of integers with values 0 through 13.

- $0=$ actual/actual (default)
- $1=30 / 360$ (SIA)
- 2 = actual/360
- 3 = actual/365
- $4=30 / 360$ (BMA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- $9=$ actual/360 (ICMA)
- $10=$ actual/365 (ICMA)
- $11=30 / 360 \mathrm{E}$ (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-0
Data Types: single | double

## EndMonthRule - End-of-month rule flag for month having $\mathbf{3 0}$ or fewer days 1 (in effect) (default) | nonnegative integer [0,1]

End-of-month rule flag for month having 30 or fewer days, specified as scalar nonnegative integer [0, 1] or a using a N-by-1 vector of values. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days.

- $0=$ Ignore rule, meaning that a payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a payment date is always the last actual day of the month.


## Data Types: logical

## Output Arguments

## TFactors - Time factors

vector
Time factors, appropriate to compounded rate quotes between Settle and Maturity dates, returned as a vector.

## F - Compounding frequencies

scalar

Compounding frequencies, returned as a scalar.

## Definitions

## Difference Between yearfrac and date2time

The difference between yearfrac and date2time is that date2time counts full periods as a whole integer, even if the number of actual days in the periods are different. yearfrac does not count full periods.

For example,
yearfrac('1/1/2000', '1/1/2001', 9)
ans =
1.0167
yearfrac for Basis 9 (ACT/360 ICMA) calculates $366 / 360=1.0167$. So, even if the dates have the same month and date, with a difference of 1 in the year, the returned value may not be exactly 1 . On the other hand, date2time calculates one full year period:

```
date2time('1/1/2000', '1/1/2001', 1, 9)
```

ans =

1

## See Also

cftimes | datetime | disc2rate | rate2disc | time2date | yearfrac

## Topics

"Handle and Convert Dates" on page 2-2

Introduced before R2006a

## dateaxis

Convert serial-date axis labels to calendar-date axis labels

## Syntax

dateaxis(Tickaxis, DateForm,StartDate)

## Arguments

| Tickaxis | (Optional) Determines which axis tick labels- $x, y$, or $z$-to replace. <br> Enter as a character vector. Default $=$$x^{\prime}$. |
| :--- | :--- |\(\left|\begin{array}{l}(Optional) Specifies which date format to use. Enter as an integer <br>

from 0 to 17. If no DateForm argument is entered, this function <br>
determines the date format based on the span of the axis limits. For <br>
example, if the difference between the axis minimum and maximum <br>
is less than 15, the tick labels are converted to three-letter day-of- <br>
the-week abbreviations (DateForm = 8). See DateForm on page <br>

19-682 format descriptions.\end{array}\right|\)| (Optional) Assigns the date to the first axis tick value, specified as a |
| :--- |
| serial date number, date character vector, or datetime array. The |
| tick values are treated as serial date numbers. The default |
| StartDate is the lower axis limit converted to the appropriate date |
| number. For example, a tick value of 1 is converted to the date 01- |
| San-0000. Entering StartDate as '06-apr-1999' assigns the |
| date April 6, 1999 to the first tick value and the axis tick labels are |
| set accordingly. |

## Description

dateaxis(Tickaxis, DateForm,StartDate) replaces axis tick labels with date labels on a graphic figure.

See the MATLAB set command for information on modifying the axis tick values and other axis parameters.

| DateForm | Format | Description |
| :--- | :--- | :--- |
| 0 | $01-$ Mar-1999 15:45:17 | day-month-year hour:minute:second |
| 1 | $01-$ mar-1999 | day-month-year |
| 2 | $03 / 01 / 99$ | month/day/year |
| 3 | Mar | month, three letters |
| 4 | M | month, single letter |
| 5 | 3 | month |
| 6 | $03 / 01$ | month/day |
| 7 | 1 | day of month |
| 8 | Wed | day of week, three letters |
| 9 | W | day of week, single letter |
| 10 | 1999 | year, four digits |
| 11 | 99 | year, two digits |
| 12 | Mar99 | month year |
| 13 | $15: 45: 17$ | hour:minute:second |
| 14 | $03: 45: 17$ PM | hour:minute:second AM or PM |
| 15 | $15: 45$ | hour:minute |
| 16 | $03: 45$ PM | hour:minute AM or PM |
| 17 | $95 / 03 / 01$ | year month day |

## Examples

```
dateaxis('x') or dateaxis
```

converts the $x$-axis labels to an automatically determined date format.
dateaxis('y', 6)
converts the $y$-axis labels to the month/day format.
dateaxis('x', 2, '03-Mar-1999')

```
or
dateaxis('x', 2, datetime('03-Mar-1999'))
converts the \(x\)-axis labels to the month/day/year format. The minimum \(x\)-tick value is treated as March 3, 1999.
```

```
See Also
bolling| candle|datenum| datestr| datetime| highlow|movavg|pointfig
```


## Introduced before R2006a

## datedisp

Display date entries

## Syntax

```
datedisp(NumMat)
datedisp( _ ,DateForm)
CharMat = datedisp(NumMat,DateForm)
```


## Description

datedisp(NumMat) displays a matrix with the serial dates formatted as date character vectors, using a matrix with mixed numeric entries and serial date number entries. Integers between datenum('01-Jan-1900') and datenum('01-Jan-2200') are assumed to be serial date numbers, while all other values are treated as numeric entries.
datedisp( __ ,DateForm), using the optional argument DateForm, displays a matrix with the serial dates formatted as date character vectors, using a matrix with mixed numeric entries and serial date number entries. Integers between datenum ( '01-Jan-1900') and datenum('01-Jan-2200') are assumed to be serial date numbers, while all other values are treated as numeric entries.

CharMat = datedisp(NumMat, DateForm) returns CharMat, character array representing NumMat. If no output variable is assigned, the function prints the array to the display.

## Examples

Display a Matrix with the Serial Dates Formatted as Date Character Vectors
This example shows how to display a matrix with the serial dates formatted as date character vectors.

```
NumMat = [730730, 0.03, 1200 730100;
    730731, 0.05, 1000 NaN];
datedisp(NumMat)
\begin{tabular}{cccc}
\(01-\) Sep-2000 & 0.03 & 1200 & \(11-\) Dec-1998 \\
\(02-\) Sep-2000 & 0.05 & 1000 & NaN
\end{tabular}
```


## Input Arguments

## NumMat - Numeric matrix to display

serial date numbers
Numeric matrix to display, specified as a serial date numbers.

## Data Types: double

## DateForm - Date format

scalar character vector to indicate format of text representing dates
Date format, specified as a scalar character vector to indicate the format of text representing dates. See datestr for available and default format flags.

## Data Types: char

## Output Arguments

## CharMat - Character array representing NumMat array of date character vectors

Character array representing NumMat, returned as an array of date character vectors. If no output variable is assigned, the function prints the array to the display.

## See Also

datenum | datestr

## Topics

"Handle and Convert Dates" on page 2-2

Introduced before R2006a

## datefind

Indices of dates in matrix

## Syntax

```
Indices = datefind(Subset,Superset)
Indices = datefind(
```

$\qquad$

``` ,Tolerance)
```


## Description

Indices = datefind(Subset, Superset) returns a vector of indices to the date numbers in Superset that are present in Subset. If no date numbers match, Indices = [].

Indices = datefind( $\qquad$ ,Tolerance) returns a vector of indices to the date numbers in Superset that are present in Subset, plus the optional argument for Tolerance. If no date numbers match, Indices = [].

## Examples

## Return a Vector of Indices to Date Numbers

This example shows how to return a vector of indices to date numbers.

```
Superset = datenum(1999, 7, 1:31);
Subset = [datenum(1999, 7, 10); datenum(1999, 7, 20)];
Indices = datefind(Subset, Superset, 1)
Indices = 6\times1
    9
    10
    1 1
    1 9
```


## Input Arguments

## Subset - Subset of dates to find matching dates matrix of nonnegative integers with values for serial date numbers or datetime arrays

Subset of dates to find matching dates in Superset, specified as a matrix of nonnegative integers with values for serial date numbers or datetime arrays.

Subset and Superset can be either serial date numbers or datetime arrays. These types do not have to match. datefind determines the underlying date to match dates of different data types.

Note The elements of Subset must be contained in Superset, without repetition. datefind works with non-repeating sequences of dates.

Example: Subset $=$ [datenum(1997,7,10); datenum(1997,7,20)];
Data Types: single | double

## Superset - Superset of dates

matrix of nonnegative integers with values for serial date numbers or datetime arrays
Superset of dates, specified as a matrix of serial date numbers or datetime arrays, whose elements are sought.

Subset and Superset can be either serial date numbers or datetime arrays. These types do not have to match. datefind determines the underlying date to match dates of different data types.

Note The elements of Subset must be contained in Superset, without repetition. datefind works with non-repeating sequences of dates.

```
Example:Superset = datenum(1997,7,1:31);
```


## Data Types: single | double

## Tolerance - Tolerance for matching dates in Superset 0 (default) | positive integer or duration object

Tolerance for matching dates (+/-) in Superset, specified as a positive integer or duration object.
Data Types: single | double

## Output Arguments

## Indices - Indices of dates in Superset that are present in Subset vector

Indices of dates in Superset that are present in Subset (plus or minus the tolerance if defined using the optional argument Tolerance), returned as a vector of indices to the date numbers or datetimes.

## See Also

datenum | datetime

## Topics

"Handle and Convert Dates" on page 2-2
Introduced before R2006a

## datemnth

Date of day in future or past month

## Syntax

```
TargetDate = datemnth(StartDate,NumberMonths)
TargetDate = datemnth( __ ,DayFlag,Basis,EndMonthRule)
```


## Description

TargetDate = datemnth(StartDate, NumberMonths) determines a date in a future or past month based on movement either forward or backward in time by a given number of months.

Any input can contain multiple values, but if so, all other inputs must contain the same number of values or a single value that applies to all. For example, if StartDate is an $n$ row character array of date character vectors, then NumberMonths must be an N-by-1 vector of integers or a single integer. TargetDate is then an N -by-1 vector of date numbers.

If StartDate is a serial date number or date character vector, TargetDate is returned as a serial date number. Use datestr to convert serial date numbers to formatted date character vectors.

If StartDate is a datetime array, then TargetDate is returned as a datetime array.
TargetDate = datemnth( $\qquad$ ,DayFlag,Basis,EndMonthRule) determines a date in a future or past month based on movement either forward or backward in time by a given number of months, using optional input arguments for DayFlag,Basis, and EndMonthRule.

## Examples

## Determine the Dates of Days in a Future Month

Determine the TargetDate in a future month using a date character vector for StartDate.

StartDate = '03-Jun-1997';
NumberMonths = 6;
DayFlag = 0;
Basis = 0;
EndMonthRule = 1;
TargetDate = datemnth(StartDate, NumberMonths, DayFlag,... Basis, EndMonthRule)

TargetDate = 729727
datestr(TargetDate)
ans =
'03-Dec-1997'
Determine the TargetDate in a future month using a datetime array for StartDate.
Day = datemnth(datetime('3-jun-2001','Locale','en_US'), 6, 0, 0, 0)
Day = datetime
03-Dec-2001

Determine the TargetDate in a future month using a vector for NumberMonths.

```
NumberMonths = [1; 3; 5; 7; 9];
TargetDate = datemnth('31-jan-2001', NumberMonths);
datestr(TargetDate)
```

```
ans = 5x11 char array
```

    '28-Feb-2001'
    '30-Apr-2001'
    '30-Jun-2001'
    '31-Aug-2001'
    '31-0ct-2001'
    
## Input Arguments

## StartDate - Start date <br> serial date number | date character vector | datetime object

Start date, specified as an N -by-1 or 1-by-N vector using serial date numbers, date character vectors, or datetime arrays.

Data Types: double | char | datetime

## NumberMonths - Number of months in future (positive) or past (negative) positive or negative integers

Number of months in future (positive) or past (negative), specified as an N-by-1 or 1-by-N vector containing positive or negative integers.

Data Types: double

## DayFlag - Flag for how actual day number for target date in future or past month is determined

0 (day number should be the day in the future or past month corresponding to the actual day number of the start date) (default) | numeric with values 0,1 , or 2

Flag for how the actual day number for the target date in future or past month is determined, specified as an N -by-1 or 1-by-N vector using a numeric with values0, 1, or 2.

Possible values are:

- 0 (default) = day number should be the day in the future or past month corresponding to the actual day number of the start date.
- 1 = day number should be the first day of the future or past month.
- 2 = day number should be the last day of the future or past month.

This flag has no effect if EndMonthRule is set to 1.

## Data Types: double

## Basis - Day-count basis of the instrument

[^8]Day-count basis to be used when determining the past or future date, specified as a scalar value with an integer with value of 0 through 13, or an N -by-1 or 1 -by-N vector of integers with values of 0 through 13 .

- $0=$ actual/actual (default)
- $1=30 / 360$ (SIA)
- $2=$ actual/360
- $3=$ actual/365
- $4=30 / 360$ (BMA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- $9=$ actual/360 (ICMA)
- $10=$ actual/365 (ICMA)
- $11=30 / 360 \mathrm{E}$ (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-0
Data Types: single | double

## EndMonthRule - End-of-month rule flag for month having $\mathbf{3 0}$ or fewer days

 0 (not in effect) (default) | nonnegative integer [0, 1]End-of-month rule flag for month having 30 or fewer days, specified as a scalar with a nonnegative integer 0 or 1 , or as an N -by- 1 or 1 -by- N vector of values 0 or 1 .

- $0=$ Ignore rule, meaning that rule is not in effect.
- $1=$ Set rule on, meaning that if you are beginning on the last day of a month, and the month has 30 or fewer days, you will end on the last actual day of the future or past month regardless of whether that month has $28,29,30$ or 31 days.


## Data Types: logical

## Output Arguments

## TargetDate - Target date in the future or past month <br> vector

Target date in the future or past month, returned as an N -by-1 or 1-by-N vector containing the serial date number (default) or datetime (if StartDate is a datetime array) of the target date.

## See Also

datestr|datetime|datevec|days360|days365|daysact|daysdif|wrkdydif

## Topics

"Handle and Convert Dates" on page 2-2
"Trading Calendars User Interface" on page 16-2
"UICalendar User Interface" on page 16-4

Introduced before R2006a

## datewrkdy

Date of future or past workday

## Syntax

EndDate = datewrkdy(StartDate,NumberWorkDays,NumberHolidays)

## Description

EndDate = datewrkdy(StartDate,NumberWorkDays,NumberHolidays) returns the serial number of the date a given number of workdays before or after the start date.

Any input can contain multiple values, but if so, all other inputs must contain the same number of values or a single value that applies to all.

For example, if StartDate is an n-row character array of date character vectors, then NumberWorkDays must be an N -by- 1 vector of integers or a single integer. EndDate is then an N -by-1 vector of date numbers.

If StartDate is a serial date number or date character vector, EndDate is returned as a date number. Use datestr to convert serial date numbers to formatted date character vectors.

If StartDate is a datetime array, then EndDate is returned as a datetime array.

## Examples

## Determine the Date for a Future Workday

Determine the EndDate for a future workday using a date character vector for StartDate.

StartDate = '20-Dec-1994';
NumberWorkDays = 16;

```
NumberHolidays = 2;
EndDate = datewrkdy(StartDate, NumberWorkDays, NumberHolidays)
EndDate = 728671
datestr(EndDate)
ans =
'12-Jan-1995'
```

Determine the EndDate for a future workday using a datetime array for StartDate.
EndDate = datewrkdy(datetime('12-dec-2000','Locale','en_US'), 16, 2)
EndDate = datetime
04-Jan-2001

Determine the EndDate for future workdays using a vector for NumberWorkDays.

```
NumberWorkDays = [16; 20; 44];
EndDate = datewrkdy('12-dec-2000', NumberWorkDays, 2);
datestr(EndDate)
ans = 3x11 char array
    '04-Jan-2001'
    '10-Jan-2001'
    '13-Feb-2001'
```


## Input Arguments

## StartDate - Start date

serial date number | date character vector | datetime object
Start date, specified as an N -by-1 or 1-by-N vector using serial date numbers, date character vectors, or datetime arrays.

Data Types: double | char | datetime

## NumberWorkDays - Number of work or business days in future (positive) or past (negative)

positive or negative integers
Number of work or business days in future (positive) or past (negative) that includes the starting date, specified as an N -by-1 or 1-by-N vector containing positive or negative integers.

NumberHolidays and NumberWorkDays must have the same sign.

## Data Types: double

## NumberHolidays - Number of holidays within NumberWorkDays

positive or negative integers
Number of holidays within NumberWorkDays, specified as positive or negative integers using an N -by-1 or 1 -by-N containing values for the number of days movement in terms of holidays into the future (if positive) or past (if negative).

NumberHolidays and NumberWorkDays must have the same sign.

## Data Types: double

## Output Arguments

## EndDate - Date of future or past workday vector

Date of future or past workday, returned as an N -by-1 or 1-by-N vector containing the serial date number (default) or the datetime (if StartDate is a datetime array) of the future or past date.

## See Also

busdate | datetime | holidays | isbusday | wrkdydif

## Topics

"Handle and Convert Dates" on page 2-2
"Trading Calendars User Interface" on page 16-2
"UICalendar User Interface" on page 16-4

## Introduced before R2006a

## day

Day of month

## Syntax

DayMonth = day(Date)
DayMonth = day( $\qquad$ , F)

## Description

DayMonth = day (Date) returns the day of the month given a serial date number or date character vector.

DayMonth = day( $\qquad$ ,F) returns the day of the month, given a serial date number, or character vector and an optional argument, F, defining the date format for Date.

## Examples

## Determine the Day of the Month for Various Date Formats

Find the day of the month using a serial date number.
DayMonth $=\operatorname{day}(730544)$
DayMonth $=28$
Find the day of the month using a date character vector.
DayMonth = day('2/28/00')
DayMonth $=28$
Find the day of the month using a date character vector with an optional argument F for a country-specific date format.

DayMonth $=$ day('28/02/00','dd/mm/yyyy')
DayMonth $=28$

## Input Arguments

## Date - Date to determine day of month

serial date number | date character vector | cell array of date character vectors
Date to determine day of month, specified as a serial date number, date character vector, or cell array of date character vectors.

All the date character vectors in Date must have same date character vector format. For more information on supported date character vector formats, see datestr.
Example: DayMonth = day(\{'2/28/00','3/10/06'\})
Data Types: single | double | char | cell

## F - Country-specific date format

character vector designating date format
Country-specific date format, specified as a character vector to designate the date format for the input argument Date. For more information on supported date character vector format symbols, see datestr. Note, formats with ' $Q$ ' are not accepted.
Example: DayMonth = day('28/02/00','dd/mm/yyyy')
Data Types: char

## Output Arguments

## DayMonth - Day of the month

nonnegative integer
Day of the month, returned as a nonnegative integer, given a serial date number, or date character vector.
See Also
datestr|datevec|eomday | month | year
Topics"Handle and Convert Dates" on page 2-2
Introduced before R2006a

## days252bus

Number of business days between dates

## Syntax

```
NumberDays = days252bus(StartDate,EndDate)
NumberDays = days252bus(StartDate,EndDate,HolidayVector)
```


## Arguments

| StartDate | N-by-1 or 1-by-N vector or scalar value, in serial date number, date <br> character vector, or datetime array form, representing the start <br> date. |
| :--- | :--- |
| EndDate | N-by-1 or 1-by-N vector or scalar value, in serial date number, date <br> character vector, or datetime array form, representing the end date. |
| HolidayVector | (Optional) N-by-1 or 1-by-N vector, in serial date number, date <br> character vector, or datetime array form, representing holidays. |

## Description

NumberDays = days252bus(StartDate,EndDate,HolidayVector) computes the number of business days (that is, non-holiday or non-weekend) between the two input dates. Note that a holiday vector may be optionally specified; if it is not, then the holidays.m file is used to determine the holidays.
days252bus returns NumberDays, a N-by-1 or 1-by-N vector or scalar value for the number of days between two dates. NumberDays returns as a double for serial date number, date character vector, and datetime inputs.

## Examples

## Computes the Number of Business Days Between Two Input Dates

This example shows how to compute the number of business days (i.e. non-holiday or nonweekend) between two dates using the days252bus convention.

```
NumberDays = days252bus('1/1/2009', '8/1/2009')
NumberDays = 146
```


## Computes the Number of Business Days Between Two Input Dates Using a datetime Array

This example shows how to compute the number of business days (i.e. non-holiday or nonweekend) between two dates, specified as a datetime array, using the days252bus convention.

```
NumberDays = days252bus(datetime('1-Jan-2009','Locale','en_US'), '8/1/2009')
NumberDays = 146
```


## See Also

datetime | days360psa|days365 | daysact | daysdif

## Topics

"Handle and Convert Dates" on page 2-2
Introduced before R2006a

## days360

Days between dates based on 360-day year

## Syntax

NumDays $=$ days360(StartDate,EndDate)

## Description

NumDays $=$ days360(StartDate, EndDate) returns the number of days between StartDate and EndDate based on a 360-day year (that is, all months contain 30 days). If EndDate is earlier than StartDate, NumDays is negative.

Either input argument can contain multiple values, but if so, the other must contain the same number of values or a single value that applies to all. For example, if StartDate is an $n$-row character array of date character vectors, then EndDate must be an N -by- 1 vector of integers or a single integer. NumDays is then an N -by- 1 vector of date numbers.

## Examples

Determine the Number of Days Between Two Dates Based on a 360-Day Year
Determine the NumDays using date character vectors for StartDate and EndDate.
NumDays = days360('15-jan-2000', '15-mar-2000')
NumDays $=60$
Determine the NumDays using a datetime array for StartDate.

```
NumDays = days360(datetime('15-jan-2000','Locale','en_US'), '15-mar-2000')
NumDays = 60
```

Determine the NumDays using a vector for EndDate.

```
MoreDays = ['15-mar-2000'; '15-apr-2000'; '15-jun-2000'];
NumDays = days360('15-jan-2000', MoreDays)
NumDays = 3x1
    6 0
    90
    1 5 0
```


## Input Arguments

## StartDate - Start date <br> serial date number | date character vector | datetime object

Start date, specified as a scalar or an N -by-1 or 1-by-N vector using serial date numbers, date character vectors, or datetime arrays.
Data Types: double | char | datetime

## EndDate - End date

serial date number | date character vector | datetime object
End date, specified as a scalar or an N-by-1 or 1-by-N vector using serial date numbers, date character vectors, or datetime arrays.
Data Types: double | char | datetime

## Output Arguments

## NumDays - Number of days between two dates

vector
Number of days between two dates, returned as a scalar or an N -by-1 or 1-by-N vector containing the number of days.

NumDays returns as a double for serial date number, date character vector, or datetime inputs for StartDate and EndDate.

## References

[1] Addendum to Securities Industry Association, Standard Securities Calculation Methods: Fixed Income Securities Formulas for Analytic Measures. Vol. 2, Spring 1995.

## See Also

datetime | days365|daysact|daysdif|wrkdydif|yearfrac

## Topics

"Handle and Convert Dates" on page 2-2

## Introduced before R2006a

## days360e

Days between dates based on 360-day year (European)

## Syntax

NumDays $=$ days360e(StartDate,EndDate)

## Description

NumDays = days360e(StartDate,EndDate) returns the number of days between StartDate and EndDate based on a 360-day year (that is, all months contain 30 days). If EndDate is earlier than StartDate, NumDays is negative. This day count convention is used primarily in Europe. Under this convention, all months contain 30 days.

Either input argument can contain multiple values, but if so, the other must contain the same number of values or a single value that applies to all. For example, if StartDate is an $n$-row character array of date character vectors, then EndDate must be an N -by- 1 vector of integers or a single integer. "Determine the Number of Days Between Two Dates Given a Basis of 30/360 Based on European Convention" on page 19-708NumDays is then an N -by-1 vector of date numbers.

## Examples

## Determine the Number of Days Between Two Dates Given a Basis of 30/360 Based on European Convention

Determine the NumDays using date character vectors for StartDate and EndDate for the month of January.

```
StartDate = '1-Jan-2002';
EndDate = '1-Feb-2002';
NumDays = days360e(StartDate, EndDate)
NumDays = 30
```

Determine the NumDays in the month of January using a datetime array for StartDate.

```
NumDays = days360e(datetime('1-Jan-2002','Locale','en_US'), '1-Feb-2002')
NumDays = 30
```

Determine the NumDays using a vector for EndDate.

```
MoreDays = ['15-mar-2000'; '15-apr-2000'; '15-jun-2000'];
NumDays = days360e('15-jan-2000', MoreDays)
NumDays = 3\times1
```

    60
    90
    150
    
## Input Arguments

## StartDate - Start date <br> serial date number | date character vector | datetime object

Start date, specified as a scalar or an N -by-1 or 1-by-N vector using serial date numbers, date character vectors, or datetime arrays.

Data Types: double | char | datetime

## EndDate - End date

serial date number | date character vector | datetime object
End date, specified as a scalar or an N -by-1 or 1-by-N vector using serial date numbers, date character vectors, or datetime arrays.

Data Types: double| char|datetime

## Output Arguments

## NumDays - Number of days between two dates given a basis of 30/360 based on European convention

vector

Number of days between two dates given a basis of 30/360 based on European convention, returned as a scalar or an N -by-1 or 1-by- N vector containing the number of days.

NumDays returns as a double for serial date number, date character vector, or datetime inputs for StartDate and EndDate.

## References

[1] Addendum to Securities Industry Association, Standard Securities Calculation Methods: Fixed Income Securities Formulas for Analytic Measures. Vol. 2, Spring 1995.

## See Also

datetime | days360|days360isda|days360psa

## Topics

"Handle and Convert Dates" on page 2-2

## Introduced before R2006a

## days360isda

Days between dates based on 360-day year (International Swap Dealer Association (ISDA) compliant)

## Syntax

NumDays = days360isda(StartDate,EndDate)

## Description

NumDays = days360isda(StartDate,EndDate) returns the number of days between StartDate and EndDate based on a 360-day year (that is, all months contain 30 days) and is International Swap Dealer Association (ISDA) compliant. If EndDate is earlier than StartDate, NumDays is negative. Under this convention, all months contain 30 days.

Either input argument can contain multiple values, but if so, the other must contain the same number of values or a single value that applies to all. For example, if StartDate is an $n$-row character array of date character vectors, then EndDate must be an N -by- 1 vector of integers or a single integer. NumDays is then an N -by- 1 vector of date numbers.

## Examples

## Determine the Number of Days Between Two Dates Given a Basis of 30/360 Based on ISDA Compliance

Determine the NumDays using date character vectors for StartDate and EndDate for the month of January.

```
StartDate = '1-Jan-2002';
EndDate = '1-Feb-2002';
NumDays = days360isda(StartDate, EndDate)
NumDays = 30
```

Determine the NumDays in the month of January using a datetime array for StartDate.

```
NumDays = days360isda(datetime('1-Jan-2002','Locale','en_US'), '1-Feb-2002')
NumDays = 30
```

Determine the NumDays using a vector for EndDate.

```
MoreDays = ['15-mar-2000'; '15-apr-2000'; '15-jun-2000'];
NumDays = days360isda('15-jan-2000', MoreDays)
NumDays = 3\times1
```

    60
    90
    150
    
## Input Arguments

## StartDate - Start date <br> serial date number | date character vector | datetime object

Start date, specified as a scalar or an N-by-1 or 1-by-N vector using serial date numbers, date character vectors, or datetime arrays.
Data Types: double | char \| datetime

## EndDate - End date

serial date number | date character vector | datetime object
End date, specified as a scalar or an N -by-1 or 1-by-N vector using serial date numbers, date character vectors, or datetime arrays.

Data Types: double | char | datetime

## Output Arguments

## NumDays - Number of days between two dates given a basis of 30/360 based on European convention

vector

Number of days between two dates given a basis of 30/360 based on International Swap Dealer Association (ISDA) compliance, returned as a scalar or an N-by-1 or 1-by-N vector containing the number of days.

NumDays returns as a double for serial date number, date character vector, or datetime inputs for StartDate and EndDate.

## References

1] Addendum to Securities Industry Association, Standard Securities Calculation Methods: Fixed Income Securities Formulas for Analytic Measures. Vol. 2, Spring 1995.

## See Also

datetime | days360|days360e

## Topics

"Handle and Convert Dates" on page 2-2

## Introduced before R2006a

## days360psa

Days between dates based on 360-day year (Public Securities Association (PSA) compliant)

## Syntax

NumDays $=$ days360psa(StartDate,EndDate)

## Description

NumDays = days360psa(StartDate,EndDate) returns the number of days between StartDate and EndDate based on a 360-day year (that is, all months contain 30 days) and is Public Securities Association (PSA) compliant. If EndDate is earlier than StartDate, NumDays is negative. Under this convention, all months contain 30 days.

Either input argument can contain multiple values, but if so, the other must contain the same number of values or a single value that applies to all. For example, if StartDate is an $n$-row character array of date character vectors, then EndDate must be an N -by- 1 vector of integers or a single integer. NumDays is then an N -by- 1 vector of date numbers.

## Examples

## Determine the Number of Days Between Two Dates Given a Basis of 30/360 Based on PSA Compliance

Determine the NumDays using date character vectors for StartDate and EndDate for the month of January.

```
StartDate = '1-Jan-2002';
EndDate = '1-Feb-2002';
NumDays = days360psa(StartDate, EndDate)
NumDays = 30
```

Determine the NumDays in the month of January using a datetime array for StartDate.

```
NumDays = days360psa(datetime('1-Jan-2002','Locale','en_US'), '1-Feb-2002')
NumDays = 30
```

Determine the NumDays using a vector for EndDate.

```
MoreDays = ['15-mar-2000'; '15-apr-2000'; '15-jun-2000'];
NumDays = days360psa('15-jan-2000', MoreDays)
NumDays = 3\times1
```

    60
    90
    150
    
## Input Arguments

## StartDate - Start date <br> serial date number | date character vector | datetime object

Start date, specified as a scalar or an N -by-1 or 1-by-N vector using serial date numbers, date character vectors, or datetime arrays.
Data Types: double | char | datetime

## EndDate - End date

serial date number | date character vector | datetime object
End date, specified as a scalar or an $\mathrm{N}-$ by-1 or 1-by-N vector using serial date numbers, date character vectors, or datetime arrays.

Data Types: double | char |datetime

## Output Arguments

## NumDays - Number of days between two dates given a basis of 30/360 based on European convention

vector

Number of days between two dates given a basis of 30/360 based on Public Securities Association (PSA) compliance, returned as a scalar or an N -by-1 or 1-by-N vector containing the number of days.

NumDays returns as a double for serial date number, date character vector, or datetime inputs for StartDate and EndDate.

## References

[1] Addendum to Securities Industry Association, Standard Securities Calculation Methods: Fixed Income Securities Formulas for Analytic Measures. Vol. 2, Spring 1995.

## See Also

datetime | days360|days360e | days360isda

## Topics

"Handle and Convert Dates" on page 2-2
Introduced before R2006a

## days365

Days between dates based on 365-day year

## Syntax

```
NumDays = days365(StartDate,EndDate)
```


## Description

NumDays = days365(StartDate,EndDate) returns the number of days between StartDate and EndDate based on a 365-day year. All months contain their actual number of days. February always contains 28 days.

If EndDate is earlier than StartDate, NumDays is negative. Under this convention, all months contain 30 days.

Either input argument can contain multiple values, but if so, the other must contain the same number of values or a single value that applies to all. For example, if StartDate is an $n$-row character array of date character vectors, then EndDate must be an N -by- 1 vector of integers or a single integer. NumDays is then an N -by- 1 vector of date numbers.

## Examples

Determine the Number of Days Between Two Dates Based on a 365-Day Year
Determine the NumDays using date character vectors for StartDate and EndDate.
NumDays = days365('15-jan-2000', '15-mar-2000')
NumDays $=59$
Determine the NumDays using a datetime array for StartDate.
NumDays = days365(datetime('15-jan-2000','Locale','en_US'), '15-mar-2000')

```
NumDays = 59
```

Determine the NumDays using a vector for EndDate.

```
MoreDays = ['15-mar-2000'; '15-apr-2000'; '15-jun-2000'];
NumDays = days365('15-jan-2000', MoreDays)
NumDays = 3\times1
```

    59
    90
    151
    
## Input Arguments

## StartDate - Start date <br> serial date number | date character vector | datetime object

Start date, specified as a scalar or an N-by-1 or 1-by-N vector using serial date numbers, date character vectors, or datetime arrays.
Data Types: double | char \| datetime

## EndDate - End date

serial date number | date character vector | datetime object
End date, specified as a scalar or an N -by-1 or 1-by-N vector using serial date numbers, date character vectors, or datetime arrays.
Data Types: double | char | datetime

## Output Arguments

## NumDays - Number of days between two dates based on 365-day year vector

Number of days between two dates based on a 365-day year, returned as a scalar or an N -by-1 or 1-by-N vector containing the number of days.

NumDays returns as a double for serial date number, date character vector, or datetime inputs for StartDate and EndDate.

## References

[1] Addendum to Securities Industry Association, Standard Securities Calculation Methods: Fixed Income Securities Formulas for Analytic Measures. Vol. 2, Spring 1995.

See Also<br>datetime | days360|daysact|daysdif|wrkdydif|yearfrac

## Topics

"Handle and Convert Dates" on page 2-2

Introduced before R2006a

## daysact

Actual number of days between dates

## Syntax

```
NumDays = daysact(StartDate)
NumDays = daysact(
```

$\qquad$

``` ,EndDate)
```


## Description

NumDays $=$ daysact(StartDate) returns the actual number of days between the MATLAB base date and StartDate. In MATLAB, the base date 1 is 1 -Jan-0000 A.D. See datenum for a similar function.

NumDays = daysact ( $\qquad$ , EndDate) returns the actual number of days between StartDate and the optional argument EndDate.

If EndDate is earlier than StartDate, NumDays is negative. Under this convention, all months contain 30 days.

Either input argument can contain multiple values, but if so, the other must contain the same number of values or a single value that applies to all. For example, if StartDate is an $n$-row character array of date character vectors, then EndDate must be an N -by-1 vector of integers or a single integer. NumDays is then an N -by- 1 vector of date numbers.

## Examples

## Determine the Number of Days Between Two Dates Based the Actual Number of Days

Determine the NumDays using date character vectors for StartDate and EndDate.
NumDays = daysact('7-sep-2002', '25-dec-2002')

NumDays $=109$
Determine the NumDays using a datetime array for StartDate.

```
NumDays = daysact(datetime('7-sep-2002','Locale','en_US'), '25-dec-2002')
NumDays = 109
```

Determine the NumDays using a vector for EndDate.

```
MoreDays = ['09/07/2002'; '10/22/2002'; '11/05/2002'];
NumDays = daysact(MoreDays, '12/25/2002')
NumDays = 3x1
```

    109
    64
    50
    
## Input Arguments

## StartDate - Start date

serial date number | date character vector | datetime object
Start date, specified as a scalar or an N-by-1 or 1-by-N vector using serial date numbers, date character vectors, or datetime arrays.

Data Types: double | char | datetime

## EndDate - End date

serial date number | date character vector | datetime object
End date, specified as a scalar or an N -by-1 or 1-by- N vector using serial date numbers, date character vectors, or datetime arrays.

Data Types: double | char \| datetime

## Output Arguments

## NumDays - Number of days between two dates based actual number of days vector

Number of days between two dates based on the actual number of days, returned as a scalar or an N -by-1 or 1-by-N vector containing the number of days.

NumDays returns as a double for serial date number, date character vector, or datetime inputs for StartDate and EndDate.

## References

[1] Addendum to Securities Industry Association, Standard Securities Calculation Methods: Fixed Income Securities Formulas for Analytic Measures. Vol. 2, Spring 1995.

## See Also

datenum | datetime | datevec | days360|days365 | daysdif

## Topics

"Handle and Convert Dates" on page 2-2
"Trading Calendars User Interface" on page 16-2
"UICalendar User Interface" on page 16-4
Introduced before R2006a

## daysadd

Date away from starting date for any day-count basis

## Syntax

```
NewDate = daysadd(StartDate,NumDays)
NewDate = daysadd(
```

$\qquad$

``` ,Basis)
```


## Description

NewDate = daysadd(StartDate,NumDays) returns a date NewDate number of days away from StartDate.

If StartDate is a serial date number or date character vector, NewDate is returned as a date number.

If StartDate is a datetime array, then NewDate is returned as a datetime array.
NewDate $=$ daysadd (__ , Basis) returns a date NewDate number of days away from StartDate, using the optional argument Basis for day-count.

If StartDate is a serial date number or date character vector, NewDate is returned as a date number.

If StartDate is a datetime array, then NewDate is returned as a datetime array.

## Examples

Determine the Date for Given Number of Days Away From StartDate
Determine the NewDate using a date character vector for StartDate.

```
NewDate = daysadd('01-Feb-2004', 31)
```

NewDate = 732009

```
datestr(NewDate)
ans =
'03-Mar-2004
```

Determine the NewDate using a datetime array for StartDate.

```
NewDate = daysadd(datetime('01-Feb-2004','Locale','en_US'), 31)
NewDate = datetime
    03-Mar-2004
```

Determine the NewDate using a vector for StartDate.

```
MoreDays = ['09/07/2002'; '10/22/2002'; '11/05/2002'];
NewDate = daysadd(MoreDays, 31 ,2)
NewDate = 3x1
    731497
    731542
    731556
datestr(NewDate)
ans = 3x11 char array
    '08-Oct-2002'
    '22-Nov-2002'
    '06-Dec-2002'
```


## Input Arguments

## StartDate - Start date

serial date number | date character vector | datetime object
Start date, specified as a scalar or an N-by-1 or 1-by-N vector using serial date numbers, date character vectors, or datetime arrays.

Data Types: double | char|datetime

## NumDays - Number of days from StartDate

## positive or negative integer

Number of days from StartDate, specified an N-by-1 or 1-by-N vector using positive or negative integers. Enter a negative integer for dates before start date.

## Data Types: double

## Basis - Day-count basis of the instrument

## 0 (actual/actual) (default) | numeric with value 0 through $13 \mid$ vector of numerics with values 0 through 13

Day-count basis of the instrument, specified as an integer with a value of 0 through 13 or a N -by-1 vector of integers with values of 0 through 13.

- $0=$ actual/actual (default)
- $1=30 / 360$ (SIA)
- 2 = actual/360
- 3 = actual/365
- $4=30 / 360$ (BMA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- $9=$ actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- $12=$ actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-0

Note When using the 30/360 day-count basis, it is not always possible to find the exact date NewDate number of days away because of a known discontinuity in the method of counting days. A warning is displayed if this occurs.

## Data Types: single | double

## Output Arguments

NewDate - Date for given number of days away from StartDate vector

Date for given number of days away from StartDate, returned as a scalar or an N -by-1 vector containing dates.

If StartDate is a serial date number or date character vector, NewDate is returned as a date number.

If StartDate is a datetime array, then NewDate is returned as a datetime array.

## References

[1] Stigum, Marcia L. and Franklin Robinson. Money Market and Bond Calculations. Richard D. Irwin, 1996, ISBN 1-55623-476-7

## See Also

datetime|daysdif

## Topics

"Handle and Convert Dates" on page 2-2
"Trading Calendars User Interface" on page 16-2
"UICalendar User Interface" on page 16-4

Introduced before R2006a

## daysdif

Days between dates for any for any day-count basis

## Syntax

```
NumDays = daysdif(StartDate,EndDate)
NumDays = daysdif(
```

$\qquad$

``` ,Basis)
```


## Description

NumDays = daysdif(StartDate,EndDate) returns the number of days between dates StartDate and EndDate. The first date for StartDate is not included when determining the number of days between first and last date.

Any input argument can contain multiple values, but if so, the other inputs must contain the same number of values or a single value that applies to all. For example, if StartDate is an n-row array of character vector dates, thenEndDate must be an n-row array of character vector dates or a single date. NumDays is then an N -by-1 vector of numbers.

NumDays = daysdif $\qquad$ ,Basis) returns the number of days between dates StartDate and EndDate using the optional argument Basis for day-count. The first date for StartDate is not included when determining the number of days between first and last date.

Any input argument can contain multiple values, but if so, the other inputs must contain the same number of values or a single value that applies to all. For example, if StartDate is an n-row array of character vector dates, then EndDate must be an n-row array of character vector dates or a single date. NumDays is then an N -by-1 vector of numbers.

## Examples

## Determine the Number of Days Between StartDate and EndDate

Determine the NumDays using date character vectors for StartDate and EndDate.

```
NumDays = daysdif('3/1/99', '3/1/00', 1)
NumDays = 360
```

Determine the NumDays using a datetime array for StartDate.

```
NumDays = daysdif(datetime('1-Mar-1999','Locale','en_US'), '3/1/00', 1)
NumDays = 360
```

Determine the NumDays using a vector for EndDate.

```
MoreDays = ['3/1/2001'; '3/1/2002'; '3/1/2003'];
NumDays = daysdif('3/1/98', MoreDays)
NumDays = 3\times1
```


## Input Arguments

## StartDate - Start date <br> serial date number | date character vector | datetime object

Start date, specified as a scalar or an N -by-1 or 1-by-N vector using serial date numbers, date character vectors, or datetime arrays.

Data Types: double | char | datetime

## EndDate - End date

serial date number | date character vector | datetime object
End date, specified as a scalar or an N -by-1 or 1-by-N vector using serial date numbers, date character vectors, or datetime arrays.

Data Types: double|char|datetime

## Basis - Day-count basis of the instrument

## 0 (actual/actual) (default) | numeric with value 0 through $13 \mid$ vector of numerics with values 0 through 13

Day-count basis of the instrument, specified as an integer with a value of 0 through 13 or a N -by-1 vector of integers with values of 0 through 13.

- $0=$ actual/actual (default)
- $1=30 / 360$ (SIA)
- 2 = actual/360
- 3 = actual/365
- $4=30 / 360$ (BMA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- $9=$ actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- $11=30 / 360 E$ (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-0
Data Types: single | double

## Output Arguments

## NumDays - Number of days between dates StartDate and EndDate <br> integer

Number of days between the StartDate and EndDate. NumDays returns as a double for serial date number, date character vector, and datetime inputs.

The first date for StartDate is not included when determining the number of days between first and last date.

## References

[1] Stigum, Marcia L. and Franklin Robinson. Money Market and Bond Calculations. Richard D. Irwin, 1996, ISBN 1-55623-476-7

## See Also

datetime|dec2thirtytwo

## Topics

"Handle and Convert Dates" on page 2-2

Introduced before R2006a

## dec2thirtytwo

Decimal to thirty-second quotation

## Syntax

[OutNumber,Fractions] = dec2thirtytwo(InNumber,Accuracy)

## Description

[OutNumber,Fractions] = dec2thirtytwo(InNumber,Accuracy) changes a decimal price quotation for a bond or bond future to a fraction with a denominator of 32 .

## Examples

## Convert Decimal to Thirty-Second Quotation

This example shows two bonds that are quoted with decimal prices of 101.78 and 102.96. These prices are converted to fractions with a denominator of 32 .

InNumber $=$ [101.78; 102.96];
[OutNumber, Fractions] = dec2thirtytwo(InNumber)
OutNumber $=2 \times 1$
101
102

Fractions = $2 \times 1$
25
31

## Input Arguments

## InNumber - Input number

numeric decimal fraction
Input number, specified as an N -by-1 vector of numeric decimal fractions.

## Data Types: double

## Accuracy - Rounding

1 (round down to nearest thirty second) (default) | numeric with values $1,2,4$ or 10
Rounding, specified as an N -by-1 vector of accuracy desired with numeric values of 1, 2, 4 or 10. The values are: 1, round down to nearest thirty second, 2 (nearest half), 4 (nearest quarter), or 10 (nearest decile).

Data Types: double

## Output Arguments

## OutNumber - Output number which is InNumber rounded downward to closest integer

numeric
Output number which is InNumber rounded downward to closest integer, returned as a numeric value.

Fractions - Fractional part in units of thirty-second
numeric
Fractional part in units of thirty-second, returned as a numeric value. The Fractions output conforms to accuracy as prescribed by the input Accuracy.

## See Also

thirtytwo2dec

Introduced before R2006a

## depfixdb

Fixed declining-balance depreciation schedule

## Syntax

Depreciation = depfixdb(Cost,Salvage,Life,Period,Month)

## Arguments

| Cost | Scalar for the initial value of the asset. |
| :--- | :--- |
| Salvage | Scalar for the salvage value of the asset. |
| Life | Scalar value for the life of the asset in years. |
| Period | Scalar integer for the number of years to calculate. |
| Month | (Optional) Scalar value for the number of months in the first year of <br> asset life. Default $=12$. |

## Description

Depreciation $=$ depfixdb(Cost,Salvage,Life, Period,Month) calculates the fixed declining-balance depreciation for each period.

## Examples

## Compute the Fixed Declining-Balance Depreciation

This example shows how to calculate the depreciation for the first five years for a car is purchased for $\$ 11,000$, with a salvage value of $\$ 1500$, and a lifetime of eight years.

Depreciation $=$ depfixdb(11000, 1500, 8, 5)

```
Depreciation = 1\times5
103 x
\(2.4251 \quad 1.8904 \quad 1.4737 \quad 1.1488 \quad 0.8955\)
```


## See Also

depgendb|deprdv|depsoyd|depstln

## Topics

"Analyzing and Computing Cash Flows" on page 2-18

## Introduced before R2006a

## depgendb

General declining-balance depreciation schedule

## Syntax

Depreciation $=$ depgendb(Cost,Salvage, Life,Factor)

## Arguments

| Cost | Cost of the asset. |
| :--- | :--- |
| Salvage | Estimated salvage value of the asset. |
| Life | Number of periods over which the asset is depreciated. |
| Factor | Depreciation factor. Factor $=2$ uses the double-declining-balance <br> method. |

## Description

Depreciation $=$ depgendb(Cost,Salvage,Life,Factor) calculates the decliningbalance depreciation for each period.

## Examples

## Calculate the Declining-Balance Depreciation

A car is purchased for $\$ 10,000$ and is to be depreciated over five years. The estimated salvage value is $\$ 1000$. Using the double-declining-balance method, the function calculates the depreciation for each year and returns the remaining depreciable value at the end of the life of the car.

Define the depreciation.

Life = 5;
Salvage = 0;
Cost $=10000$;
Factor=2;
Use depgendb to calculate the depreciation.
Depreciation $=$ depgendb $(10000,1000,5,2)$
Depreciation $=1 \times 5$
$10^{3} \times$
$\begin{array}{lllll}4.0000 & 2.4000 & 1.4400 & 0.8640 & 0.2960\end{array}$

The large value returned at the final year is the sum of the depreciation over the life time and is equal to the difference between the Cost and Salvage. The value of the asset in the final year is computed as (Cost-Salvage) $=$ Sum_Depreciation_Upto_Final_Year.

## See Also

depfixdb|deprdv|depsoyd|depstln

## Topics

"Analyzing and Computing Cash Flows" on page 2-18

Introduced before R2006a

## deprdv

Remaining depreciable value

## Syntax

Value = deprdv(Cost,Salvage,Accum)

## Arguments

| Cost | Cost of the asset. |
| :--- | :--- |
| Salvage | Salvage value of the asset. |
| Accum | Accumulated depreciation of the asset for prior periods. |

## Description

Value = deprdv(Cost,Salvage,Accum) returns the remaining depreciable value for an asset.

## Examples

## Compute the Remaining Depreciable Value for an Asset

This example shows how to find the remaining depreciable value after six years for the cost of an asset for $\$ 13,000$ with a life of 10 years. The salvage value is $\$ 1000$.

Accum $=$ depstln(13000, 1000, 10) * 6
Accum $=7200$
Value = deprdv(13000, 1000, 7200)
Value $=4800$

See Also<br>depfixdb|depgendb|depsoyd|depstln<br>\section*{Topics}<br>"Analyzing and Computing Cash Flows" on page 2-18<br>Introduced before R2006a

## depsoyd

Sum of years' digits depreciation

## Syntax

```
Sum = depsoyd(Cost,Salvage,Life)
```


## Arguments

| Cost | Cost of the asset. |
| :--- | :--- |
| Salvage | Salvage value of the asset. |
| Life | Depreciable life of the asset in years. |

## Description

Sum = depsoyd(Cost,Salvage,Life) calculates the depreciation for an asset using the sum of years' digits method. Sum is a 1-by-Life vector of depreciation values with each element corresponding to a year of the asset's life.

## Examples

## Compute the Depreciation for an Asset Using the Sum of Years' Digits Method

This example shows how to calculate the depreciation for an asset using the sum of years' digits method where the asset is $\$ 13,000$ with a life of 10 years. The salvage value of the asset is $\$ 1000$.

```
Sum = depsoyd(13000, 1000, 10)'
Sum = 10\times1
103 x
```

2.1818
1.9636
1.7455
1.5273
1.3091
1.0909
0.8727
0.6545
0.4364
0.2182

## See Also

depfixdb|depgendb|deprdv|depstln

## Topics

"Analyzing and Computing Cash Flows" on page 2-18
Introduced before R2006a

## depstln

Straight-line depreciation schedule

## Syntax

```
Depreciation = depstln(Cost,Salvage,Life)
```


## Arguments

| Cost | Cost of the asset. |
| :--- | :--- |
| Salvage | Salvage value of the asset. |
| Life | Depreciable life of the asset in years. |

## Description

Depreciation = depstln(Cost,Salvage,Life) calculates straight-line depreciation for an asset.

## Examples

## Compute the Straight-Line Depreciation for an Asset

This example shows how to calculate the straight-line depreciation for an asset that costs $\$ 13,000$ with a life of 10 years. The salvage value of the asset is $\$ 1000$.

Depreciation $=$ depstln(13000, 1000, 10)
Depreciation = 1200

See Also<br>depfixdb|depgendb|deprdv|depsoyd<br>Topics<br>"Analyzing and Computing Cash Flows" on page 2-18<br>\section*{Introduced before R2006a}

## diff

Differencing

Note diff will be removed in a future release. Use timetable instead. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

```
newfts = diff(oldfts)
```


## Description

diff computes the differences of the data series in a financial time series object. It returns another time series object containing the difference.
newfts $=$ diff(oldfts) computes the difference of all the data in the data series of the object oldfts and returns the result in the object newfts. newfts is a financial time series object containing the same data series (names) as the input oldfts.

## See Also

diff

## Topics

"Data Transformation and Frequency Conversion" on page 13-13

Introduced before R2006a

## disc2zero

Zero curve given discount curve

Note In R2017b, the specification of optional input arguments has changed. While the previous ordered inputs syntax is still supported, it may no longer be supported in a future release. Use the optional name-value pair inputs: Compounding and Basis.

## Syntax

[ZeroRates,CurveDates] = disc2zero(DiscRates,CurveDates, Settle) [ZeroRates,CurveDates] = disc2zero( __ , Name, Value)

## Description

[ZeroRates,CurveDates] = disc2zero(DiscRates,CurveDates,Settle) returns a zero curve given a discount curve and its maturity dates. If either inputs for CurveDates or Settle are datetime arrays, the output CurveDates is returned as datetime arrays.
[ZeroRates,CurveDates] = disc2zero( $\qquad$ , Name, Value) adds optional namevalue pair arguments

## Examples

## Determine the Zero Curve Given a Discount Curve and Maturity Dates

Given the following discount factors DiscRates over a set of maturity dates CurveDates, and a settlement date Settle:

DiscRates $=[0.9996$
0.9947
0.9896
0.9866

```
    0.9826
    0.9786
    0.9745
    0.9665
    0.9552
    0.9466];
CurveDates = [datenum('06-Nov-2000')
    datenum('11-Dec-2000')
    datenum('15-Jan-2001')
    datenum('05-Feb-2001')
    datenum('04-Mar-2001')
    datenum('02-Apr-2001')
    datenum('30-Apr-2001')
    datenum('25-Jun-2001')
    datenum('04-Sep-2001')
    datenum('12-Nov-2001')];
```

Settle = datenum('03-Nov-2000');

Set daily compounding for the output zero curve, on an actual/365 basis.
Compounding = 365;
Basis = 3;
Execute the function disc2zero which returns the zero curve ZeroRates at the maturity dates CurveDates.
[ZeroRates, CurveDates] = disc2zero(DiscRates, CurveDates,... Settle, Compounding, Basis)

ZeroRates $=10 \times 1$
0.0487
0.0510
0.0523
0.0524
0.0530
0.0526
0.0530
0.0532
0.0549
0.0536

## CurveDates $=10 \times 1$

730796
730831
730866
730887
730914
730943
730971
731027
731098
731167

For readability, DiscRates and ZeroRates are shown here only to the basis point. However, MATLAB® software computed them at full precision. If you enter DiscRates as shown, ZeroRates may differ due to rounding.

## Determine the Zero Curve Given a Discount Curve and Maturity Dates Using datetime Inputs

Given the following discount factors, DiscRates, over a set of maturity dates, CurveDates, and a settlement date, Settle, use datetime inputs to return the zero curve, ZeroRates, at the maturity dates, CurveDates.

```
DiscRates = [0.9996
    0.9947
    0.9896
    0.9866
    0.9826
    0.9786
    0.9745
    0.9665
    0.9552
    0.9466];
CurveDates = [datenum('06-Nov-2000')
    datenum('11-Dec-2000')
    datenum('15-Jan-2001')
    datenum('05-Feb-2001')
    datenum('04-Mar-2001')
```

```
    datenum('02-Apr-2001')
    datenum('30-Apr-2001')
    datenum('25-Jun-2001')
    datenum('04-Sep-2001')
    datenum('12-Nov-2001')];
Settle = datenum('03-Nov-2000');
Compounding = 365;
Basis = 3;
CurveDates = datetime(CurveDates,'ConvertFrom','datenum','Locale','en US');
Settle = datetime(Settle,'ConvertFrom','datenum','Locale','en US');
[ZeroRates, CurveDates] = disc2zero(DiscRates, CurveDates,...
Settle, Compounding, Basis)
ZeroRates = 10\times1
    0.0487
    0.0510
    0.0523
    0.0524
    0.0530
    0.0526
    0.0530
    0.0532
    0.0549
    0.0536
CurveDates = 10x1 datetime array
    06-Nov-2000 00:00:00
    11-Dec-2000 00:00:00
    15-Jan-2001 00:00:00
    05-Feb-2001 00:00:00
    04-Mar-2001 00:00:00
    02-Apr-2001 00:00:00
    30-Apr-2001 00:00:00
    25-Jun-2001 00:00:00
    04-Sep-2001 00:00:00
    12-Nov-2001 00:00:00
```


## Input Arguments

## DiscRates - Discount factors

decimal fraction
Discount factors, specified as a column vector of decimal fractions. In aggregate, the factors in DiscRates constitute a discount curve for the investment horizon represented by CurveDates.

## Data Types: double

```
CurveDates - Maturity dates
serial date number | date character vector | datetime
```

Maturity dates that correspond to the discount factors in DiscRates, specified as a column vector using serial date numbers, date character vectors, or datetime arrays.
Data Types: double | datetime | char

## Settle - Common settlement date for discount rates in DiscRates serial date number | date character vector | datetime

Common settlement date for the discount rates in DiscRates, specified as serial date numbers, date character vectors, or datetime arrays.
Data Types: double | datetime | char

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, ... , NameN, ValueN.

Example: [ZeroRates,CurveDates] = disc2zero(DiscRates,CurveDates,Settle,'Compounding', 6,'Basis', 9)

## Compounding - Rate at which output zero rates are compounded when annualized

2 (default) | numeric values: $0,1,2,3,4,6,12,365,-1$
Rate at which the output zero rates are compounded when annualized, specified as a numeric value. Allowed values are:

- 0 - Simple interest (no compounding)
- 1 - Annual compounding
- 2 - Semiannual compounding (default)
- 3 - Compounding three times per year
- 4 - Quarterly compounding
- 6 - Bimonthly compounding
- 12 - Monthly compounding
- 365 - Daily compounding
-     - 1 - Continuous compounding


## Data Types: double

## Basis - Day-count basis used for annualizing output zero rates

0 (default) | numeric values: $0,1,2,3,4,6,7,8,9,10,11,12,13$
Day-count basis used for annualizing the output zero rates, specified as a numeric value. Allowed values are:

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- 2 = actual/360
- 3 = actual/365
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- $7=$ actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- $11=30 / 360 \mathrm{E}$ (ICMA)
- $12=$ actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-0

## Output Arguments

## ZeroRates - Zero curve for investment horizon represented by CurveDates

numeric
Zero curve for the investment horizon represented by CurveDates, returned as a column vector of decimal fractions. The zero rates are the yields to maturity on theoretical zerocoupon bonds.

## CurveDates - Maturity dates that correspond to ZeroRates <br> serial date number | date character vector | datetime

Maturity dates that correspond to the ZeroRates, returned as a column vector. This vector is the same as the input vector CurveDates, but the output is sorted by ascending maturity. If either inputs for CurveDates or Settle are datetime arrays, the output CurveDates is returned as datetime arrays.

## See Also

datetime | fwd2zero| prbyzero| pyld2zero|zbtprice|zbtyield|zero2disc| zero2disc|zero2fwd |zero2pyld

## Topics

"Term Structure of Interest Rates" on page 2-42
"Fixed-Income Terminology" on page 2-22

Introduced before R2006a

## discrate

Bank discount rate of security

## Syntax

DiscRate = discrate(Settle,Maturity,Face,Price)
DiscRate = discrate( __ ,Basis)

## Description

DiscRate = discrate(Settle, Maturity, Face, Price) computes the bank discount rate of a security. The bank discount rate normalizes by the face value of the security (for example, U. S. Treasury Bills) and understates the true yield earned by investors.

DiscRate = discrate( $\qquad$ ,Basis) adds an optional argument for Basis.

## Examples

## Compute the Bank Discount Rate of a Security

This example shows how to find the bank discount rate of a security.
DiscRate = discrate('12-jan-2000', '25-jun-2000', 100, 97.74, 0)
DiscRate = 0.0501

## Compute the Bank Discount Rate of a Security Using datetime Inputs

This example shows how to use datetime inputs to find the bank discount rate of a security.

```
DiscRate = discrate(datetime('12-jan-2000','Locale','en US'), datetime('25-jun-2000',
```


## Input Arguments

## Settle - Settlement date

serial date number | date character vector | datetime
Settlement date, specified as serial date numbers, date character vectors, or datetime arrays.
Data Types: double \| datetime | char

## Maturity - Maturity date

serial date number | date character vector | datetime
Maturity date, specified as serial date number, date character vector, or datetime array.
Data Types: double | char \| datetime

## Face - Redemption (par, face) value

## numeric

Redemption (par, face) value, specified as a numeric value.

## Data Types: double

## Price - Security price

numeric
Security price, specified as a numeric value.
Data Types: double

## Basis - Day-count basis of the instrument

0 (actual/actual) (default) | integer with value of 0 to 13
(Optional) Day-count basis of the instrument, specified as a vector of integers.

- $0=$ actual/actual (default)
- $1=30 / 360$ (SIA)
- 2 = actual/360
- $3=$ actual/365
- $4=30 / 360$ (BMA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- $9=$ actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-0
Data Types: double

## Output Arguments

DiscRate - Discount rate of security
decimal
Discount rate of security, returned as a decimal.

## References

[1] Mayle. "Standard Securities Calculation Methods." Volumes I-II, 3rd edition. Formula 2.

## See Also

acrudisc|datetime|fvdisc|prdisc|ylddisc

## Topics

"Term Structure of Interest Rates" on page 2-42
"Fixed-Income Terminology" on page 2-22

Introduced before R2006a

## ecmlsrmle

Least-squares regression with missing data

## Syntax

[Parameters,Covariance,Resid,Info] = ecmlsrmle(Data,Design,MaxIterations,TolParam,Tol0

## Arguments

| Data | NUMSAMPLES-by-NUMSERIES matrix with NUMSAMPLES <br> samples of a NUMSERIES-dimensional random vector. <br> Missing values are represented as NaNs. Only samples that <br> are entirely NaNs are ignored. (To ignore samples with at <br> least one NaN, use mvnrmle.) |
| :--- | :--- |
| Design | A matrix or a cell array that handles two model structures: <br> - If NUMSERIES = 1, Design is a NUMSAMPLES-by- <br> NUMPARAMS matrix with known values. This structure is <br> the standard form for regression on a single series. <br> If NUMSERIES $\geq 1$, Design is a cell array. The cell array <br> contains either one or NUMSAMPLES cells. Each cell <br> contains a NUMSERIES-by-NUMPARAMS matrix of known <br> values. |
| If Design has a single cell, it is assumed to have the |  |
| same Design matrix for each sample. If Design has |  |
| more than one cell, each cell contains a Design matrix |  |
| for each sample. |  |


| TolParam | (Optional) Convergence tolerance for estimation algorithm based on changes in model parameter estimates. Default value is sqrt (eps) which is about $1.0 \mathrm{e}-8$ for double precision. The convergence test for changes in model parameters is |
| :---: | :---: |
|  | $\\|$ Param $_{k}-$ Param $_{k-1} \\|<$ TolParam $\times\left(1+\\|\right.$ Param $\left._{k} \\|\right)$ |
|  | where Param represents the output Parameters, and iteration $k=2,3, \ldots$. Convergence is assumed when both the TolParam and TolObj conditions are satisfied. If both TolParam $\leq 0$ and Tol0bj $\leq 0$, do the maximum number of iterations (MaxIterations), whatever the results of the convergence tests. |
| Tol0bj | (Optional) Convergence tolerance for estimation algorithm based on changes in the objective function. Default value is eps $\wedge 3 / 4$ which is about $1.0 \mathrm{e}-12$ for double precision. The convergence test for changes in the objective function is $\mid O b j_{k}-\text { Obj }_{k-1} \mid<T o l O b j \times\left(1+\left\|\mathrm{Obj}_{k}\right\|\right)$ <br> for iteration $k=2,3, \ldots$. Convergence is assumed when both the TolParam and TolObj conditions are satisfied. If both TolParam $\leq 0$ and TolObj $\leq 0$, do the maximum number of iterations (MaxIterations), whatever the results of the convergence tests. |
| Param0 | (Optional) NUMPARAMS-by-1 column vector that contains a user-supplied initial estimate for the parameters of the regression model. Default is a zero vector. |


| Covar0 | (Optional) NUMSERIES-by-NUMSERIES matrix that contains a <br> user-supplied initial or known estimate for the covariance <br> matrix of the regression residuals. Default is an identity <br> matrix. |
| :--- | :--- |
| For covariance-weighted least-squares calculations, this |  |
| matrix corresponds with weights for each series in the |  |
| regression. The matrix also serves as an initial guess for the |  |
| residual covariance in the expectation conditional |  |
| maximization (ECM) algorithm. |  |

## Description

[Parameters, Covariance, Resid, Info] = ecmlsrmle(Data, Design, MaxIterations, TolParam, TolObj, Param0, Covar0, CovarFormat) estimates a least-squares regression model with missing data. The model has the form

$$
\text { Data }_{k} \sim N\left(\text { Design }_{k} \times \text { Parameters, Covariance }\right)
$$

for samples $k=1, \ldots$, NUMSAMPLES.
ecmlsrmle estimates a NUMPARAMS-by-1 column vector of model parameters called Parameters, and a NUMSERIES-by-NUMSERIES matrix of covariance parameters called Covariance.
ecmlsrmle(Data, Design) with no output arguments plots the log-likelihood function for each iteration of the algorithm.

To summarize the outputs of ecmlsrmle:

- Parameters is a NUMPARAMS-by-1 column vector of estimates for the parameters of the regression model.
- Covariance is a NUMSERIES-by-NUMSERIES matrix of estimates for the covariance of the regression model's residuals. For least-squares models, this estimate may not be a maximum likelihood estimate except under special circumstances.
- Resid is a NUMSAMPLES-by-NUMSERIES matrix of residuals from the regression.

Another output, Info, is a structure that contains additional information from the regression. The structure has these fields:

- Info. Obj - A variable-extent column vector, with no more than MaxIterations elements, that contain each value of the objective function at each iteration of the estimation algorithm. The last value in this vector, Obj (end), is the terminal estimate of the objective function. If you do least-squares, the objective function is the leastsquares objective function.
- Info. PrevParameters - NUMPARAMS-by-1 column vector of estimates for the model parameters from the iteration just prior to the terminal iteration.
- Info. PrevCovariance - NUMSERIES-by-NUMSERIES matrix of estimates for the covariance parameters from the iteration just prior to the terminal iteration.


## Notes

If doing covariance-weighted least-squares, Covar0 should usually be a diagonal matrix. Series with greater influence should have smaller diagonal elements in Covar0 and series with lesser influence should have larger diagonal elements. Note that if doing CWLS, Covar0 do not need to be a diagonal matrix even if CovarFormat = 'diagonal'.

You can configure Design as a matrix if NUMSERIES $=1$ or as a cell array if NUMSERIES $\geq 1$.

- If Design is a cell array and NUMSERIES $=1$, each cell contains a NUMPARAMS row vector.
- If Design is a cell array and NUMSERIES > 1, each cell contains a NUMSERIES-byNUMPARAMS matrix.

These points concern how Design handles missing data:

- Although Design should not have NaN values, ignored samples due to NaN values in Data are also ignored in the corresponding Design array.
- If Design is a 1-by-1 cell array, which has a single Design matrix for each sample, no NaN values are permitted in the array. A model with this structure must have NUMSERIES $\geq$ NUMPARAMS with rank(Design\{1\}) = NUMPARAMS.
- ecmlsrmle is more strict than mvnrmle about the presence of NaN values in the Design array.

Use the estimates in the optional output structure Info for diagnostic purposes.

## Examples

See "Multivariate Normal Regression" on page 9-17, "Least-Squares Regression" on page 9-17, "Covariance-Weighted Least Squares" on page 9-18, "Feasible Generalized Least Squares" on page 9-19, and "Seemingly Unrelated Regression" on page 9-20.

## References

Roderick J. A. Little and Donald B. Rubin. Statistical Analysis with Missing Data. 2nd Edition. John Wiley \& Sons, Inc., 2002.

Xiao-Li Meng and Donald B. Rubin. "Maximum Likelihood Estimation via the ECM Algorithm." Biometrika. Vol. 80, No. 2, 1993, pp. 267-278.

Joe Sexton and Anders Rygh Swensen. "ECM Algorithms that Converge at the Rate of EM." Biometrika. Vol. 87, No. 3, 2000, pp. 651-662.
A. P. Dempster, N.M. Laird, and D. B. Rubin. "Maximum Likelihood from Incomplete Data via the EM Algorithm." Journal of the Royal Statistical Society. Series B, Vol. 39, No. 1, 1977, pp. 1-37.

## See Also

ecmlsrobj | ecmmvnrmle | ecmmvnrmle

## Topics

"Least-Squares Regression Without Missing Data" on page 9-18
"Covariance-Weighted Least Squares Without Missing Data" on page 9-19

## Introduced in R2006a

## ecmlsrobj

Log-likelihood function for least-squares regression with missing data

## Syntax

Objective = ecmlsrobj(Data,Design,Parameters,Covariance)

## Arguments

| Data | NUMSAMPLES-by-NUMSERIES matrix with NUMSAMPLES samples of a <br> NUMSERIES-dimensional random vector. Missing values are <br> represented as NaNs. Only samples that are entirely NaNs are <br> ignored. (To ignore samples with at least one NaN, use mvn rmle.) |
| :--- | :--- |
| Design | A matrix or a cell array that handles two model structures: <br> If NUMSERIES $=1$, Design is a NUMSAMPLES-by-NUMPARAMS <br> matrix with known values. This structure is the standard form <br> for regression on a single series. <br> If NUMSERIES $\geq 1$, Design is a cell array. The cell array contains <br> either one or NUMSAMPLES cells. Each cell contains a <br> NUMSERIES-by-NUMPARAMS matrix of known values. |
| If Design has a single cell, it is assumed to have the same |  |
| Design matrix for each sample. If Design has more than one |  |
| cell, each cell contains a Design matrix for each sample. |  |$|$| NUMPARAMS-by-1 column vector of estimates for the parameters of |
| :--- |
| the regression model. |

## Description

Objective = ecmlsrobj(Data,Design, Parameters,Covariance) computes a least-squares objective function based on current parameter estimates with missing data. Objective is a scalar that contains the least-squares objective function.

## Notes

ecmlsrobj requires that Covariance be positive-definite.
Note that
ecmlsrobj(Data, Design, Parameters) = ecmmvnrobj(Data, ...
Design, Parameters, IdentityMatrix)
where IdentityMatrix is a NUMSERIES-by-NUMSERIES identity matrix.
You can configure Design as a matrix if NUMSERIES $=1$ or as a cell array if NUMSERIES $\geq 1$.

- If Design is a cell array and NUMSERIES = 1, each cell contains a NUMPARAMS row vector.
- If Design is a cell array and NUMSERIES > 1, each cell contains a NUMSERIES-byNUMPARAMS matrix.


## Examples

See "Multivariate Normal Regression" on page 9-17, "Least-Squares Regression" on page 9-17, "Covariance-Weighted Least Squares" on page 9-18, "Feasible Generalized Least Squares" on page 9-19, and "Seemingly Unrelated Regression" on page 9-20.

## See Also

ecmlsrmle|mvnrmle

## Topics

"Least-Squares Regression With Missing Data" on page 9-18

## Introduced in R2006a

## ecmmvnrfish

Fisher information matrix for multivariate normal regression model

## Syntax

Fisher = ecmmvnrfish(Data,Design,Covariance,Method,MatrixFormat,CovarFormat)

## Arguments

| Data | NUMSAMPLES-by-NUMSERIES matrix with NUMSAMPLES <br> samples of a NUMSERIES-dimensional random vector. <br> Missing values are represented as NaNs. Only samples that <br> are entirely NaNs are ignored. (To ignore samples with at <br> least one NaN, use mvnrfish.) |
| :--- | :--- |
| Design | A matrix or a cell array that handles two model structures: <br> - If NUMSERIES = 1, Design is a NUMSAMPLES-by- <br> NUMPARAMS matrix with known values. This structure is <br> the standard form for regression on a single series. <br> If NUMSERIES $\geq 1$, Design is a cell array. The cell array <br> contains either one or NUMSAMPLES cells. Each cell <br> contains a NUMSERIES-by-NUMPARAMS matrix of known <br> values. |
| If Design has a single cell, it is assumed to have the |  |
| same Design matrix for each sample. If Design has |  |
| more than one cell, each cell contains a Design matrix |  |
| for each sample. |  |

$\left.\begin{array}{|l|l|}\hline \text { Method } & \begin{array}{l}\text { (Optional) Character vector that identifies method of } \\ \text { calculation for the information matrix: }\end{array} \\ \text { - hessian - Default method. Use the expected Hessian } \\ \text { matrix of the observed log-likelihood function. This } \\ \text { method is recommended since the resultant standard } \\ \text { errors incorporate the increased uncertainties due to } \\ \text { missing data. } \\ \text { - fisher - Use the Fisher information matrix. }\end{array}\right\}$

## Description

Fisher =
ecmmvnrfish(Data,Design, Covariance, Method, MatrixFormat, CovarFormat) computes a Fisher information matrix based on current maximum likelihood or leastsquares parameter estimates that account for missing data.

Fisher is a NUMPARAMS-by-NUMPARAMS Fisher information matrix or Hessian matrix. The size of NUMPARAMS depends on MatrixFormat and on current parameter estimates. If MatrixFormat = 'full',

NUMPARAMS $=$ NUMSERIES $*($ NUMSERIES +3$) / 2$

```
If MatrixFormat = 'paramonly',
NUMPARAMS = NUMSERIES
```

Note ecmmvnrfish operates slowly if you calculate the full Fisher information matrix.

## Examples

See "Multivariate Normal Regression" on page 9-17, "Least-Squares Regression" on page 9-17, "Covariance-Weighted Least Squares" on page 9-18, "Feasible Generalized Least Squares" on page 9-19, and "Seemingly Unrelated Regression" on page 9-20.

## See Also

ecmnmle | ecmnstd

## Topics

"Multivariate Normal Regression Without Missing Data" on page 9-17
"Fisher Information" on page 9-6
"Multivariate Normal Linear Regression" on page 9-2
Introduced in R2006a

## ecmmvnrmle

Multivariate normal regression with missing data

## Syntax

[Parameters,Covariance,Resid,Info] = ecmmvnrmle(Data,Design,MaxIterations,TolParam,Tol

## Arguments

| Data | NUMSAMPLES-by-NUMSERIES matrix with NUMSAMPLES <br> samples of a NUMSERIES-dimensional random vector. Missing <br> values are represented as NaNs. Only samples that are <br> entirely NaNs are ignored. (To ignore samples with at least <br> one NaN, use mvn rmle.) |
| :--- | :--- |
| Design | A matrix or a cell array that handles two model structures: <br> - If NUMSERIES = 1, Design is a NUMSAMPLES-by- <br> NUMPARAMS matrix with known values. This structure is <br> the standard form for regression on a single series. <br> If NUMSERIES $\geq 1$, Design is a cell array. The cell array <br> contains either one or NUMSAMPLES cells. Each cell <br> contains a NUMSERIES-by-NUMPARAMS matrix of known <br> values. |
| If Design has a single cell, it is assumed to have the same <br> Design matrix for each sample. If Design has more than <br> one cell, each cell contains a Design matrix for each <br> sample. |  |
| MaxIterations | (Optional) Maximum number of iterations for the estimation <br> algorithm. Default value is 100. |


| TolParam | (Optional) Convergence tolerance for estimation algorithm based on changes in model parameter estimates. Default value is sqrt (eps) which is about $1.0 \mathrm{e}-8$ for double precision. The convergence test for changes in model parameters is $\left\\|\operatorname{Param}_{k}-\operatorname{Param}_{k-1}\right\\|<\text { TolParam } \times\left(1+\\| \text { Param }_{k} \\|\right)$ <br> where Param represents the output Parameters, and iteration $k=2,3, \ldots$. Convergence is assumed when both the TolParam and TolObj conditions are satisfied. If both TolParam $\leq 0$ and TolObj $\leq 0$, do the maximum number of iterations (MaxIterations), whatever the results of the convergence tests. |
| :---: | :---: |
| Tol0bj | (Optional) Convergence tolerance for estimation algorithm based on changes in the objective function. Default value is eps $\wedge 3 / 4$ which is about $1.0 \mathrm{e}-12$ for double precision. The convergence test for changes in the objective function is $\left\|O b j_{k}-O b j_{k-1}\right\|<T o l O b j \times\left(1+\left\|O b j_{k}\right\|\right)$ <br> for iteration $k=2,3, \ldots$. Convergence is assumed when both the TolParam and TolObj conditions are satisfied. If both TolParam $\leq 0$ and TolObj $\leq 0$, do the maximum number of iterations (MaxIterations), whatever the results of the convergence tests. |
| Param0 | (Optional) NUMPARAMS-by-1 column vector that contains a user-supplied initial estimate for the parameters of the regression model. |
| Covar0 | (Optional) NUMSERIES-by-NUMSERIES matrix that contains a user-supplied initial or known estimate for the covariance matrix of the regression residuals. |


| CovarFormat | (Optional) Character vector that specifies the format for the <br> covariance matrix. The choices are: <br> - full' - Default method. Compute the full covariance <br> matrix. <br> 'diagonal' - Force the covariance matrix to be a <br> diagonal matrix. |
| :--- | :--- |

## Description

[Parameters,Covariance,Resid,Info] =
ecmmvnrmle(Data,Design, MaxIterations,TolParam, Tol0bj, Param0, Covar0, C ovarFormat) estimates a multivariate normal regression model with missing data. The model has the form

$$
\text { Data }_{k} \sim N\left(\text { Design }_{k} \times \text { Parameters, Covariance }\right)
$$

for samples $k=1, \ldots$, NUMSAMPLES.
ecmmvnrmle estimates a NUMPARAMS-by-1 column vector of model parameters called Parameters, and a NUMSERIES-by-NUMSERIES matrix of covariance parameters called Covariance.
ecmmvnrmle(Data, Design) with no output arguments plots the log-likelihood function for each iteration of the algorithm.

To summarize the outputs of ecmmvnrmle:

- Parameters is a NUMPARAMS-by-1 column vector of estimates for the parameters of the regression model.
- Covariance is a NUMSERIES-by-NUMSERIES matrix of estimates for the covariance of the regression model's residuals.
- Resid is a NUMSAMPLES-by-NUMSERIES matrix of residuals from the regression. For any missing values in Data, the corresponding residual is the difference between the conditionally imputed value for Data and the model, that is, the imputed residual.

Note The covariance estimate Covariance cannot be derived from the residuals.

Another output, Info, is a structure that contains additional information from the regression. The structure has these fields:

- Info. Obj - A variable-extent column vector, with no more than MaxIterations elements, that contain each value of the objective function at each iteration of the estimation algorithm. The last value in this vector, 0 bj (end), is the terminal estimate of the objective function. If you do maximum likelihood estimation, the objective function is the log-likelihood function.
- Info. PrevParameters - NUMPARAMS-by-1 column vector of estimates for the model parameters from the iteration just prior to the terminal iteration.nfo. PrevCovariance - NUMSERIES-by-NUMSERIES matrix of estimates for the covariance parameters from the iteration just prior to the terminal iteration.


## Notes

ecmmvnrmle does not accept an initial parameter vector, since the parameters are estimated directly from the first iteration onward.

You can configure Design as a matrix if NUMSERIES = 1 or as a cell array if NUMSERIES $\geq 1$.

- If Design is a cell array and NUMSERIES = 1, each cell contains a NUMPARAMS row vector.
- If Design is a cell array and NUMSERIES > 1, each cell contains a NUMSERIES-byNUMPARAMS matrix.

These points concern how Design handles missing data:

- Although Design should not have NaN values, ignored samples due to NaN values in Data are also ignored in the corresponding Design array.
- If Design is a 1-by-1 cell array, which has a single Design matrix for each sample, no NaN values are permitted in the array. A model with this structure must have NUMSERIES $\geq$ NUMPARAMS with rank(Design\{1\}) $=$ NUMPARAMS.
- ecmmvnrmle is more strict than mvnrmle about the presence of NaN values in the Design array.

Use the estimates in the optional output structure Info for diagnostic purposes.

## Examples

See "Multivariate Normal Regression" on page 9-17, "Least-Squares Regression" on page 9-17, "Covariance-Weighted Least Squares" on page 9-18, "Feasible Generalized Least Squares" on page 9-19, and "Seemingly Unrelated Regression" on page 9-20.

## References

Roderick J. A. Little and Donald B. Rubin. Statistical Analysis with Missing Data. 2nd Edition. John Wiley \& Sons, Inc., 2002.

Xiao-Li Meng and Donald B. Rubin. "Maximum Likelihood Estimation via the ECM Algorithm." Biometrika. Vol. 80, No. 2, 1993, pp. 267-278.

Joe Sexton and Anders Rygh Swensen. "ECM Algorithms that Converge at the Rate of EM." Biometrika. Vol. 87, No. 3, 2000, pp. 651-662.
A. P. Dempster, N.M. Laird, and D. B. Rubin. "Maximum Likelihood from Incomplete Data via the EM Algorithm." Journal of the Royal Statistical Society. Series B, Vol. 39, No. 1, 1977, pp. 1-37.

## See Also

ecmmvnrobj|mvnrmle

## Topics

"Multivariate Normal Regression With Missing Data" on page 9-17
"Multivariate Normal Linear Regression" on page 9-2

## Introduced in R2006a

## ecmmvnrobj

Log-likelihood function for multivariate normal regression with missing data

## Syntax

Objective = ecmmvnrobj(Data,Design,Parameters,Covariance,CovarFormat)

## Arguments

| Data | NUMSAMPLES-by-NUMSERIES matrix with NUMSAMPLES samples of a <br> NUMSERIES-dimensional random vector. Missing values are <br> represented as NaNs. Only samples that are entirely NaNs are <br> ignored. (To ignore samples with at least one NaN, use mvnrmle.) |
| :--- | :--- |
| Design | A matrix or a cell array that handles two model structures: <br> - If NUMSERIES $=1$, Design is a NUMSAMPLES-by-NUMPARAMS <br> matrix with known values. This structure is the standard form <br> for regression on a single series. <br> If NUMSERIES $\geq 1$, Design is a cell array. The cell array <br> contains either one or NUMSAMPLES cells. Each cell contains a <br> NUMSERIES-by-NUMPARAMS matrix of known values. <br> If Design has a single cell, it is assumed to have the same <br> Design matrix for each sample. If Design has more than one <br> cell, each cell contains a Design matrix for each sample. |
| Parameters | NUMPARAMS-by-1 column vector of estimates for the parameters of <br> the regression model. |
| Covariance | NUMSERIES-by-NUMSERIES matrix of estimates for the covariance of <br> the residuals of the regression. |


| CovarFormat | (Optional) Character vector that specifies the format for the <br> covariance matrix. The choices are: <br> -'full' - Default method. The covariance matrix is a full <br> matrix. <br> - 'diagonal' - The covariance matrix is a diagonal matrix. |
| :--- | :--- |

## Description

Objective =
ecmmvnrobj(Data,Design, Parameters,Covariance, CovarFormat) computes a log-likelihood function based on current maximum likelihood parameter estimates with missing data. Objective is a scalar that contains the least-squares objective function.

## Notes

You can configure Design as a matrix if NUMSERIES = 1 or as a cell array if NUMSERIES $\geq 1$.

- If Design is a cell array and NUMSERIES = 1, each cell contains a NUMPARAMS row vector.
- If Design is a cell array and NUMSERIES > 1, each cell contains a NUMSERIES-byNUMPARAMS matrix.


## Examples

See "Multivariate Normal Regression" on page 9-17, "Least-Squares Regression" on page 9-17, "Covariance-Weighted Least Squares" on page 9-18, "Feasible Generalized Least Squares" on page 9-19, and "Seemingly Unrelated Regression" on page 9-20.

## See Also

ecmmvnrmle|mvnrmle|mvnrobj

## Topics

"Multivariate Normal Regression With Missing Data" on page 9-17
"Portfolios with Missing Data" on page 9-27
"Multivariate Normal Linear Regression" on page 9-2
Introduced in R2006a

## ecmmvnrstd

Evaluate standard errors for multivariate normal regression model

## Syntax

[StdParameters, StdCovariance] = ecmmvnrstd(Data,Design,Covariance,Method,CovarFormat)

## Arguments

| Data | NUMSAMPLES-by-NUMSERIES matrix with NUMSAMPLES samples of a <br> NUMSERIES-dimensional random vector. Missing values are <br> represented as NaNs. Only samples that are entirely NaNs are <br> ignored. (To ignore samples with at least one NaN, use mvnrstd.) |
| :--- | :--- |
| Design | A matrix or a cell array that handles two model structures: <br> - If NUMSERIES $=1$, Design is a NUMSAMPLES-by-NUMPARAMS <br> matrix with known values. This structure is the standard form <br> for regression on a single series. <br> If NUMSERIES $\geq 1$, Design is a cell array. The cell array contains <br> either one or NUMSAMPLES cells. Each cell contains a <br> NUMSERIES-by-NUMPARAMS matrix of known values. |
| If Design has a single cell, it is assumed to have the same |  |
| Design matrix for each sample. If Design has more than one |  |
| cell, each cell contains a Design matrix for each sample. |  |


| Method | (Optional) Character vector that identifies method of calculation for <br> the information matrix: <br> - <br> hessian - Default method. Use the expected Hessian matrix of <br> the observed log-likelihood function. This method is <br> recommended since the resultant standard errors incorporate <br> the increased uncertainties due to missing data. <br> fisher - Use the Fisher information matrix. |
| :--- | :--- |
| CovarFormat | (Optional) Character vector that specifies the format for the <br> covariance matrix. The choices are: |
| - full' - Default method. The covariance matrix is a full |  |
| matrix. |  |
| 'diagonal' - The covariance matrix is a diagonal matrix. |  |

## Description

[StdParameters,StdCovariance] =
ecmmvnrstd(Data,Design, Covariance, Method, CovarFormat) evaluates standard errors for a multivariate normal regression model with missing data. The model has the form

$$
\text { Data }_{k} \sim N\left(\text { Design }_{k} \times \text { Parameters, Covariance }\right)
$$

for samples $k=1, \ldots$, NUMSAMPLES.
ecmmvnrstd computes two outputs:

- StdParameters is a NUMPARAMS-by-1 column vector of standard errors for each element of Parameters, the vector of estimated model parameters.
- StdCovariance is a NUMSERIES-by-NUMSERIES matrix of standard errors for each element of Covariance, the matrix of estimated covariance parameters.

Note ecmmvnrstd operates slowly when you calculate the standard errors associated with the covariance matrix Covariance.

## Notes

You can configure Design as a matrix if NUMSERIES $=1$ or as a cell array if NUMSERIES $\geq 1$.

- If Design is a cell array and NUMSERIES = 1, each cell contains a NUMPARAMS row vector.
- If Design is a cell array and NUMSERIES > 1, each cell contains a NUMSERIES-byNUMPARAMS matrix.


## Examples

See "Multivariate Normal Regression" on page 9-17, "Least-Squares Regression" on page 9-17, "Covariance-Weighted Least Squares" on page 9-18, "Feasible Generalized Least Squares" on page 9-19, and "Seemingly Unrelated Regression" on page 9-20.

## References

Roderick J. A. Little and Donald B. Rubin. Statistical Analysis with Missing Data. 2nd edition, John Wiley \& Sons, Inc., 2002.

## See Also

ecmmvnrmle |ecmmvnrstd

## Topics

"Multivariate Normal Regression With Missing Data" on page 9-17
"Multivariate Normal Linear Regression" on page 9-2

## Introduced in R2006a

## ecmnfish

Fisher information matrix

## Syntax

Fisher = ecmnfish(Data,Covariance,InvCovariance,MatrixFormat)

## Arguments

| Data | NUMSAMPLES-by-NUMSERIES matrix of observed multivariate <br> normal data |
| :--- | :--- |
| Covariance | NUMSERIES-by-NUMSERIES matrix with covariance estimate <br> of Data |
| InvCovariance | (Optional) Inverse of covariance matrix: inv (Covariance) |
| MatrixFormat | (Optional) Character vector that identifies parameters <br> included in the Fisher information matrix. If MatrixFormat <br> $=[$ [ or ' ' , the default method full is used. The parameter <br> choices are |
| - full - (Default) Compute full Fisher information matrix. <br> - meanonly - Compute only components of the Fisher <br> information matrix associated with the mean. |  |

## Description

Fisher = ecmnfish(Data,Covariance,InvCovariance,MatrixFormat)
computes a NUMPARAMS-by-NUMPARAMS Fisher information matrix based on current parameter estimates, where

NUMPARAMS $=$ NUMSERIES*(NUMSERIES +3 ) $/ 2$
if MatrixFormat = 'full' and

```
NUMPARAMS = NUMSERIES
```

```
if MatrixFormat \(=\) 'meanonly'
```

The data matrix has NaNs for missing observations. The multivariate normal model has

```
NUMPARAMS = NUMSERIES + NUMSERIES*(NUMSERIES + 1)/2
```

distinct parameters. Therefore, the full Fisher information matrix is of size NUMPARAMS-by-NUMPARAMS. The first NUMSERIES parameters are estimates for the mean of the data in Mean and the remaining NUMSERIES* (NUMSERIES +1 )/2 parameters are estimates for the lower-triangular portion of the covariance of the data in Covariance, in rowmajor order.

If MatrixFormat $=$ 'meanonly', the number of parameters is reduced to NUMPARAMS $=$ NUMSERIES, where the Fisher information matrix is computed for the mean parameters only. In this format, the routine executes fastest.

This routine expects the inverse of the covariance matrix as an input. If you do not pass in the inverse, the routine computes it. You can obtain an approximation for the lower-bound standard errors of estimation of the parameters from

```
Stderr = (1.0/sqrt(NumSamples)) .* sqrt(diag(inv(Fisher)));
```

Because of missing information, these standard errors can be smaller than the estimated standard errors derived from the expected Hessian matrix. To see the difference, compare to standard errors calculated with ecmnhess.

## See Also

ecmnhess | ecmnmle

## Topics

"Multivariate Normal Regression With Missing Data" on page 9-17
"Fisher Information" on page 9-6

Introduced before R2006a

## ecmnhess

Hessian of negative log-likelihood function

## Syntax

Hessian = ecmnhess(Data,Covariance,InvCovariance, MatrixFormat)

## Arguments

| Data | NUMSAMPLES-by-NUMSERIES matrix of observed multivariate <br> normal data |
| :--- | :--- |
| Covariance | NUMSERIES-by-NUMSERIES matrix with covariance estimate <br> of Data |
| InvCovariance | (Optional) Inverse of covariance matrix: inv (Covariance) |
| MatrixFormat | (Optional) Character vector that identifies parameters <br> included in the Hessian matrix. If Mat rixFormat $=[$ ] or <br> ' 1, the default method full is used. The parameter choices <br> are: |
| - full - (Default) Compute full Hessian matrix. |  |
| - meanonly - Compute only components of the Hessian |  |
| matrix associated with the mean. |  |

## Description

Hessian = ecmnhess(Data,Covariance,InvCovariance,MatrixFormat) computes a NUMPARAMS-by-NUMPARAMS Hessian matrix of the observed negative loglikelihood function based on current parameter estimates, where

```
NUMPARAMS = NUMSERIES*(NUMSERIES + 3)/2
if MatrixFormat = 'full' and
```

NUMPARAMS = NUMSERIES
if MatrixFormat $=$ 'meanonly'.
This routine is slow for NUMSERIES $>10$ or NUMSAMPLES $>1000$.
The data matrix has NaNs for missing observations. The multivariate normal model has
NUMPARAMS $=$ NUMSERIES + NUMSERIES*(NUMSERIES + 1)/2
distinct parameters. Therefore, the full Hessian is a NUMPARAMS-by-NUMPARAMS matrix.

The first NUMSERIES parameters are estimates for the mean of the data in Mean and the remaining NUMSERIES* (NUMSERIES +1 )/2 parameters are estimates for the lowertriangular portion of the covariance of the data in Covariance, in row-major order.

If MatrixFormat = 'meanonly', the number of parameters is reduced to NUMPARAMS $=$ NUMSERIES, where the Hessian is computed for the mean parameters only. In this format, the routine executes fastest.

This routine expects the inverse of the covariance matrix as an input. If you do not pass in the inverse, the routine computes it.

The equation

```
Stderr = (1.0/sqrt(NumSamples)) .* sqrt(diag(inv(Hessian)));
```

provides an approximation for the observed standard errors of estimation of the parameters.

Because of the additional uncertainties introduced by missing information, these standard errors can be larger than the estimated standard errors derived from the Fisher information matrix. To see the difference, compare to standard errors calculated from ecmnfish.

## See Also

ecmnfish | ecmnmle

## Topics

"Maximum Likelihood Estimation" on page 9-3

Introduced before R2006a

## ecmninit

Initial mean and covariance

## Syntax

[Mean,Covariance] = ecmninit(Data,InitMethod)

## Arguments

| Data | NUMSAMPLES-by-NUMSERIES matrix with NUMSAMPLES samples of a <br> NUMSERIES-dimensional random vector. Missing values are <br> indicated by NaNs. |
| :--- | :--- |
| InitMethod | (Optional) Character vector that identifies one of three defined <br> initialization methods to compute initial estimates for the mean and <br> covariance of the data. If InitMethod $=[$ ] or ' ' , the default <br> method nanskip is used. The initialization methods are |
| - nanskip - (Default) Skip all records with NaNs. |  |
| - twostage - Estimate mean. Fill NaNs with the mean. Then |  |
| estimate the covariance. |  |
| - diagonal - Form a diagonal covariance. |  |

## Description

[Mean,Covariance] = ecmninit(Data,InitMethod) creates initial mean and covariance estimates for the function ecmnmle. Mean is a NUMSERIES-by-1 column vector estimate for the mean of Data. Covariance is a NUMSERIES-by-NUMSERIES matrix estimate for the covariance of Data.

## Algorithms

## Model

The general model is

$$
Z \sim N(\text { Mean, Covariance }),
$$

where each row of Data is an observation of $Z$.
Each observation of $Z$ is assumed to be iid (independent, identically distributed) multivariate normal, and missing values are assumed to be missing at random (MAR).

## Initialization Methods

This routine has three initialization methods that cover most cases, each with its advantages and disadvantages.

## nanskip

The nanskip method works well with small problems (fewer than 10 series or with monotone missing data patterns). It skips over any records with NaNs and estimates initial values from complete-data records only. This initialization method tends to yield fastest convergence of the ECM algorithm. This routine switches to the twostage method if it determines that significant numbers of records contain NaN.

## twostage

The twostage method is the best choice for large problems (more than 10 series). It estimates the mean for each series using all available data for each series. It then estimates the covariance matrix with missing values treated as equal to the mean rather than as NaNs. This initialization method is robust but tends to result in slower convergence of the ECM algorithm.

## diagonal

The diagonal method is a worst-case approach that deals with problematic data, such as disjoint series and excessive missing data (more than 33\% missing data). Of the three initialization methods, this method causes the slowest convergence of the ECM algorithm.

## See Also

ecmnmle

## Topics

"Mean and Covariance Estimation" on page 9-5

Introduced before R2006a

## ecmnmle

Mean and covariance of incomplete multivariate normal data

## Syntax

[Mean,Covariance] = ecmnmle(Data,InitMethod,MaxIterations,Tolerance,Mean0,Covar0)

## Arguments

\(\left.$$
\begin{array}{|l|l|}\hline \text { Data } & \begin{array}{l}\text { NUMSAMPLES-by-NUMSERIES matrix with NUMSAMPLES } \\
\text { samples of a NUMSERIES-dimensional random vector. } \\
\text { Missing values are indicated by NaNs. A sample is also called } \\
\text { an observation or a record. }\end{array} \\
\hline \text { InitMethod } & \begin{array}{l}\text { (Optional) Character vector that identifies one of three } \\
\text { defined initialization methods to compute initial estimates } \\
\text { for the mean and covariance of the data. If InitMethod }= \\
\text { [ ] or ' ' , the default method nanskip is used. The } \\
\text { initialization methods are: }\end{array}
$$ <br>
- nanskip - (Default) Skip all records with NaNs. <br>
- twostage - Estimate mean. Fill NaNs with mean. Then <br>
estimate covariance. <br>

diagonal - Form a diagonal covariance.\end{array}\right\}\)| Note If you supply Mean0 and Covar0, InitMethod is |
| :--- |
| not executed. |


| Tolerance | (Optional) Convergence tolerance for the ECM algorithm <br> (Default = 1.0e-8.) If Tolerance $\leq 0$, perform maximum <br> iterations specified by MaxIterations and do not evaluate <br> the objective function at each step unless in display mode, as <br> described below. |
| :--- | :--- |
| Mean0 | (Optional) Initial NUMSERIES-by-1 column vector estimate <br> for the mean. If you leave Mean0 unspecified ([ ]), the <br> method specified by InitMethod is used. If you specify <br> Mean0, you must also specify Covar0. |
| Covar0 | (Optional) Initial NUMSERIES-by-NUMSERIES matrix estimate <br> for the covariance, where the input matrix must be positive- <br> definite. If you leave Covar0 unspecified ([ ]), the method <br> specified by InitMethod is used. If you specify Covar0, you <br> must also specify Mean0. |

## Description

[Mean,Covariance] = ecmnmle(Data, InitMethod, MaxIterations, Tolerance, Mean0, Covar0) estimates the mean and covariance of a data set. If the data set has missing values, this routine implements the ECM algorithm of Meng and Rubin [2] with enhancements by Sexton and Swensen [3]. ECM stands for expectation conditional maximization, a conditional maximization form of the EM algorithm of Dempster, Laird, and Rubin [4].

This routine has two operational modes.

## Display Mode

With no output arguments, this mode displays the convergence of the ECM algorithm. It estimates and plots objective function values for each iteration of the ECM algorithm until termination, as shown in the following plot.


Display mode can determine MaxIter and Tolerance values or serve as a diagnostic tool. The objective function is the negative log-likelihood function of the observed data and convergence to a maximum likelihood estimate corresponds with minimization of the objective.

## Estimation Mode

With output arguments, this mode estimates the mean and covariance via the ECM algorithm.

## Examples

To see an example of how to use ecmnmle, run the program ecmguidemo.

## Algorithms

## Model

The general model is

$$
Z \sim N(\text { Mean, Covariance }),
$$

where each row of Data is an observation of $Z$.
Each observation of $Z$ is assumed to be iid (independent, identically distributed) multivariate normal, and missing values are assumed to be missing at random (MAR). See Little and Rubin [1] for a precise definition of MAR.

This routine estimates the mean and covariance from given data. If data values are missing, the routine implements the ECM algorithm of Meng and Rubin [2] with enhancements by Sexton and Swensen [3].

If a record is empty (every value in a sample is NaN ), this routine ignores the record because it contributes no information. If such records exist in the data, the number of nonempty samples used in the estimation is $\leq$ NumSamples.

The estimate for the covariance is a biased maximum likelihood estimate (MLE). To convert to an unbiased estimate, multiply the covariance by Count/(Count - 1), where Count is the number of nonempty samples used in the estimation.

## Requirements

This routine requires consistent values for NUMSAMPLES and NUMSERIES with NUMSAMPLES > NUMSERIES. It must have enough nonmissing values to converge. Finally, it must have a positive-definite covariance matrix. Although the references provide some necessary and sufficient conditions, general conditions for existence and uniqueness of solutions in the missing-data case, do not exist. The main failure mode is an illconditioned covariance matrix estimate. Nonetheless, this routine works for most cases that have less than $15 \%$ missing data (a typical upper bound for financial data).

## Initialization Methods

This routine has three initialization methods that cover most cases, each with its advantages and disadvantages. The ECM algorithm always converges to a minimum of
the observed negative log-likelihood function. If you override the initialization methods, you must ensure that the initial estimate for the covariance matrix is positive-definite.

The following is a guide to the supported initialization methods.

## nanskip

The nanskip method works well with small problems (fewer than 10 series or with monotone missing data patterns). It skips over any records with NaNs and estimates initial values from complete-data records only. This initialization method tends to yield fastest convergence of the ECM algorithm. This routine switches to the twostage method if it determines that significant numbers of records contain NaN.

## twostage

The twostage method is the best choice for large problems (more than 10 series). It estimates the mean for each series using all available data for each series. It then estimates the covariance matrix with missing values treated as equal to the mean rather than as NaNs. This initialization method is robust but tends to result in slower convergence of the ECM algorithm.

## diagonal

The diagonal method is a worst-case approach that deals with problematic data, such as disjoint series and excessive missing data (more than $33 \%$ of data missing). Of the three initialization methods, this method causes the slowest convergence of the ECM algorithm. If problems occur with this method, use display mode to examine convergence and modify either MaxIterations or Tolerance, or try alternative initial estimates with Mean0 and Covar0. If all else fails, try

Mean0 = zeros(NumSeries);
Covar0 = eye(NumSeries,NumSeries);
Given estimates for mean and covariance from this routine, you can estimate standard errors with the companion routine ecmnstd.

## Convergence

The ECM algorithm does not work for all patterns of missing values. Although it works in most cases, it can fail to converge if the covariance becomes singular. If this occurs, plots
of the log-likelihood function tend to have a constant upward slope over many iterations as the log of the negative determinant of the covariance goes to zero. In some cases, the objective fails to converge due to machine precision errors. No general theory of missing data patterns exists to determine these cases. An example of a known failure occurs when two time series are proportional wherever both series contain nonmissing values.

## References

[1] Little, Roderick J. A. and Donald B. Rubin. Statistical Analysis with Missing Data. 2nd Edition. John Wiley \& Sons, Inc., 2002.
[2] Meng, Xiao-Li and Donald B. Rubin. "Maximum Likelihood Estimation via the ECM Algorithm." Biometrika. Vol. 80, No. 2, 1993, pp. 267-278.
[3] Sexton, Joe and Anders Rygh Swensen. "ECM Algorithms that Converge at the Rate of EM." Biometrika. Vol. 87, No. 3, 2000, pp. 651-662.
[4] Dempster, A. P., N. M. Laird, and Donald B. Rubin. "Maximum Likelihood from Incomplete Data via the EM Algorithm." Journal of the Royal Statistical Society. Series B, Vol. 39, No. 1, 1977, pp. 1-37.

## See Also

ecmnfish|ecmnhess |ecmninit|ecmnobj|ecmnstd

## Topics

"Multivariate Normal Regression With Missing Data" on page 9-17
"Mean and Covariance Estimation" on page 9-5

Introduced before R2006a

## ecmnobj

Multivariate normal negative log-likelihood function

## Syntax

Objective = ecmnobj(Data,Mean,Covariance,CholCovariance)

## Arguments

| Data | NUMSAMPLES-by-NUMSERIES matrix of observed multivariate <br> normal data |
| :--- | :--- |
| Mean | NUMSERIES-by-1 column vector with mean estimate of Data |
| Covariance | NUMSERIES-by-NUMSERIES matrix with covariance estimate <br> of Data |
| CholCovariance | (Optional) Cholesky decomposition of covariance matrix: <br> chol(Covariance) |

## Description

Objective = ecmnobj (Data, Mean, Covariance, CholCovariance) computes the value of the observed negative log-likelihood function over the data given current estimates for the mean and covariance of the data.

The data matrix has NaNs for missing observations. The inputs Mean and Covariance are current estimates for model parameters.

This routine expects the Cholesky decomposition of the covariance matrix as an input. The routine computes the Cholesky decomposition if you do not explicitly specify it.

## See Also

chol | ecmnmle

## Topics

"Multivariate Normal Regression Without Missing Data" on page 9-17
"Multivariate Normal Regression With Missing Data" on page 9-17

## Introduced before R2006a

## ecmnstd

Standard errors for mean and covariance of incomplete data

## Syntax

[StdMean,StdCovariance] = ecmnstd(Data, Mean, Covariance, Method)

## Arguments

| Data | NUMSAMPLES-by-NUMSERIES matrix with NUMSAMPLES samples of a <br> NUMSERIES-dimensional random vector. Missing values are <br> indicated by NaNs. |
| :--- | :--- |
| Mean | NUMSERIES-by-1 column vector of maximum-likelihood parameter <br> estimates for the mean of Data using the expectation conditional <br> maximization (ECM) algorithm |
| Covariance | NUMSERIES-by-NUMSERIES matrix of maximum-likelihood <br> covariance estimates for the covariance of Data using the ECM <br> algorithm |
| Method | (Optional) Character vector indicating method of estimation for <br> standard error calculations. The methods are: <br> - hessian - (Default) Hessian of the observed negative log- <br> likelihood function. <br> fisher - Fisher information matrix. |

## Description

[StdMean, StdCovariance] = ecmnstd(Data,Mean,Covariance,Method) computes standard errors for mean and covariance of incomplete data.

StdMean is a NUMSERIES-by-1 column vector of standard errors of estimates for each element of the mean vector Mean.

StdCovariance is a NUMSERIES-by-NUMSERIES matrix of standard errors of estimates for each element of the covariance matrix Covariance.

Use this routine after estimating the mean and covariance of Data with ecmnmle. If the mean and distinct covariance elements are treated as the parameter $\theta$ in a complete-data maximum-likelihood estimation, then as the number of samples increases, $\theta$ attains asymptotic normality such that

$$
\theta-E[\theta] \sim N\left(0, I^{-1}(\theta)\right),
$$

where $E[\theta]$ is the mean and $I(\theta)$ is the Fisher information matrix.
With missing data, the Hessian $H(\theta)$ is a good approximation for the Fisher information (which can only be approximated when data is missing).

It is usually advisable to use the default Method since the resultant standard errors incorporate the increased uncertainty due to missing data. In particular, standard errors calculated with the Hessian are generally larger than standard errors calculated with the Fisher information matrix.

Note This routine is slow for NUMSERIES > 10 or NUMSAMPLES > 1000.

## See Also

ecmnmle

## Topics

"Mean and Covariance Estimation" on page 9-5

## Introduced before R2006a

## effrr

Effective rate of return

## Syntax

Return $=$ effrr(Rate,NumPeriods)

## Arguments

| Rate | Annual percentage rate. Enter as a decimal fraction. |
| :--- | :--- |
| NumPeriods | Number of compounding periods per year, an integer. |

## Description

Return $=$ effrr(Rate, NumPeriods) calculates the annual effective rate of return. Compounding continuously returns Return equivalent to ( $e^{\wedge}$ Rate-1).

## Examples

## Compute the Annual Effective Rate of Return

This example shows how to find the effective annual rate of return based on an annual percentage rate of $9 \%$ compounded monthly.

Return $=\operatorname{effrr}(0.09,12)$
Return $=0.0938$

## See Also

nomrr

[^9]
## elpm

Compute expected lower partial moments for normal asset returns

## Syntax

elpm(Mean, Sigma)
elpm(Mean,Sigma,MAR)
elpm(Mean,Sigma,MAR,Order)
Moment $=$ elpm(Mean,Sigma,MAR,Order)

## Arguments

| Mean | NUMSERIES vector with mean returns for a collection of NUMSERIES <br> assets. |
| :--- | :--- |
| Sigma | NUMSERIES vector with standard deviation of returns for a <br> collection of NUMSERIES assets. |
| MAR | (Optional) Scalar minimum acceptable return (default MAR $=0$ ). <br> This is a cutoff level of return such that all returns above MAR <br> contribute nothing to the lower partial moment. |
| Order | (Optional) Either a scalar or a NUMORDERS vector of nonnegative <br> integer moment orders. If no order specified, default Order $=0$, <br> which is the shortfall probability. This function will not work for <br> negative or noninteger orders. |

## Description

Given NUMSERIES asset returns with a vector of mean returns in a NUMSERIES vector Mean, a vector of standard deviations of returns in a NUMSERIES vector Sigma, a scalar minimum acceptable return MAR, and one or more nonnegative integer moment orders in a NUMORDERS vector Order, compute expected lower partial moments (elpm) relative to MAR for each asset in a NUMORDERS-by-NUMSERIES matrix Moment.

The output, Moment, is a NUMORDERS-by-NUMSERIES matrix of expected lower partial moments with NUMORDERS Orders and NUMSERIES series, that is, each row contains expected lower partial moments for a given order.

Note To compute upper partial moments, reverse the signs of both the input Mean and MAR (do not reverse the signs of either Sigma or the output). This function computes expected lower partial moments with the mean and standard deviation of normally distributed asset returns. To compute sample lower partial moments from asset returns which have no distributional assumptions, use lpm.

## Examples

See "Expected Lower Partial Moments" on page 7-17.

## References

Vijay S. Bawa. "Safety-First, Stochastic Dominance, and Optimal Portfolio Choice." Journal of Financial and Quantitative Analysis. Vol. 13, No. 2, June 1978, pp. 255-271.
W. V. Harlow. "Asset Allocation in a Downside-Risk Framework." Financial Analysts Journal. Vol. 47, No. 5, September/October 1991, pp. 28-40.
W. V. Harlow and K. S. Rao. "Asset Pricing in a Generalized Mean-Lower Partial Moment Framework: Theory and Evidence." Journal of Financial and Quantitative Analysis. Vol. 24, No. 3, September 1989, pp. 285-311.

Frank A. Sortino and Robert van der Meer. "Downside Risk." Journal of Portfolio Management. Vol. 17, No. 5, Spring 1991, pp. 27-31.

## See Also

lpm

## Topics

"Expected Lower Partial Moments" on page 7-17
"Performance Metrics Overview" on page 7-2

## Introduced in R2006b

## emaxdrawdown

Compute expected maximum drawdown for Brownian motion

## Syntax

EDD = emaxdrawdown(Mu,Sigma,T)

## Arguments

| Mu | Scalar. Drift term of a Brownian motion with drift. |
| :--- | :--- |
| Sigma | Scalar. Diffusion term of a Brownian motion with drift. |
| T | A time period of interest or a vector of times. |

## Description

EDD = emaxdrawdown(Mu,Sigma,T) computes the expected maximum drawdown for a Brownian motion for each time period in T using the following equation:

$$
d X(t)=\mu d t+\sigma d W(t)
$$

If the Brownian motion is geometric with the stochastic differential equation

$$
d S(t)=\mu_{0} S(t) d t+\sigma_{0} S(t) d W(t)
$$

then use Ito's lemma with $X(t)=\log (S(t))$ such that

$$
\begin{aligned}
\mu & =\mu_{0}-0.5 \sigma_{0}{ }^{2}, \\
\sigma & =\sigma_{0}
\end{aligned}
$$

converts it to the form used here.

The output argument ExpDrawdown is computed using an interpolation method. Values are accurate to a fraction of a basis point. Maximum drawdown is nonnegative since it is the change from a peak to a trough.

Note To compare the actual results from maxdrawdown with the expected results of emaxdrawdown, set the Format input argument of maxdrawdown to either of the nondefault values ('arithmetic' or 'geometric'). These are the only two formats emaxdrawdown supports.

## Examples

See "Expected Maximum Drawdown" on page 7-22.

## References

Malik Magdon-Ismail, Amir F. Atiya, Amrit Pratap, and Yaser S. Abu-Mostafa. "On the Maximum Drawdown of a Brownian Motion." Journal of Applied Probability. Vol. 41, Number 1, March 2004, pp. 147-161.

See Also

maxdrawdown

## Topics

"Expected Maximum Drawdown" on page 7-22
"Performance Metrics Overview" on page 7-2

## Introduced in R2006b

## end

Last date entry

Note end will be removed in a future release. Use timetable instead. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

## end

## Description

end returns the index to the last date entry in a financial time series object.

## Examples

Consider a financial time series object called MyFts:

```
dates = ['01-Jan-2001';'01-Jan-2001'; '02-Jan-2001'; ...
    '02-Jan-2001'; '03-Jan-2001';'03-Jan-2001'];
times = ['11:00';'12:00';'11:00';'12:00';'11:00';'12:00'];
dates_times = cellstr([dates, repmat(' ',size(dates,1),1),...
times]);
myFts = fints(dates_times,(1:6)',{'Datal'},1,'My first FINTS')
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints (line 165)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/display (line 66)
myFts =
    desc: My first FINTS
    freq: Daily (1)
```

| dates: (6)' | 'times: | (6) ${ }^{\prime}$ | 'Datal: (6)' |
| :---: | :---: | :---: | :---: |
| '01-Jan-2001' | '11:00' |  | $1]$ |
| , | '12:00' |  | 2] |
| '02-Jan-2001' | '11:00' |  | $3]$ |
| , " , | '12:00' |  | 4] |
| '03-Jan-2001' | '11:00' |  | 5] |
|  | '12:00' |  | 6] |

Use end to return the last date entry in the financial time series object myFts.

```
myFts(end)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/end (line 57)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/subsref (line 106)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/display (line 66)
ans =
    desc: My first FINTS
    freq: Daily (1)
    'dates: (1)' 'times: (1)' 'Datal: (1)
    '03-Jan-2001' '12:00' [ 6]
```


## See Also

subsasgn | subsref

## Topics

"Financial Time Series Operations" on page 13-7

Introduced before R2006a

## eomdate

Last date of month

## Syntax

DayMonth = eomdate(Date)
DayMonth = eomdate(Year,Month,outputType)

## Description

DayMonth = eomdate(Date) returns the serial date number of the last date of the month for the given Date.

DayMonth = eomdate(Year,Month, outputType) returns the serial date number of the last date of the month for the given year and month. However, if outputType is 'datetime', then DayMonth is a datetime array. By default, outputType is 'datenum'.

## Examples

## Determine the Last Day of the Month for Various Dates

Find the last day of the month using Year and Month.

```
DayMonth = eomdate(2001, 2)
DayMonth = 730910
datestr(DayMonth)
ans =
'28-Feb-2001'
```

Find the last day of the month using multiples values for Year and a single Month.

```
Year = [2002 2003 2004 2005];
DayMonth = eomdate(Year, 2);
datestr(DayMonth)
ans = 4x11 char array
    '28-Feb-2002'
    '28-Feb-2003'
    '29-Feb-2004'
    '28-Feb-2005'
```

Find the last day of the month using a datetime array for Date.

```
DayMonth = eomdate(datetime('1-Jan-2015','Locale','en_US'))
DayMonth = datetime
    31-Jan-2015
```

Find the last day of the month using an outputType for 'datetime'.

```
DayMonth = eomdate(2001, 2,'datetime')
```

DayMonth = datetime
28-Feb-2001

## Input Arguments

## Date - Date to determine last day of month

serial date number | date character vector | datetime array
Date to determine last day of month, specified as a serial date number, date character vector, or datetime array.

If Date is a serial date number or a date character vector, DayMonth is returned as a serial date number. If Date is a datetime array, then DayMonth is returned as a datetime array.

Use the function datestr to convert serial date numbers to formatted date character vectors or datenum to convert date and time to a serial date number.

Data Types: single|double|char|datetime

## Year - Year to determine last date of month <br> four-digit nonnegative integer | vector of four-digit nonnegative integers

Year to determine last date of month, specified as a four-digit nonnegative integer.
Either input argument for Year or Month can contain multiple values, but if so, the other input must contain the same number of values or a single value that applies to all. For example, if Year is a 1-by- $n$ vector of integers, then Month must be a 1-by- $n$ vector of integers or a single integer. DayMonth output is then a 1-by-n vector of date numbers.

## Data Types: single | double

## Month - Month to determine last date of month integer from 1 through $12 \mid$ vector of integers from 1 through 12

Month to determine last date of month, specified as an integer from 1 through 12.
Either input argument for Year or Month can contain multiple values, but if so, the other input must contain the same number of values or a single value that applies to all. For example, if Year is a 1-by- $n$ vector of integers, then Month must be a 1-by- $n$ vector of integers or a single integer. DayMonth output is then a 1-by-n vector of date numbers.

Data Types: single | double

## outputType - Output date format

'datenum' (default)|character vector with values 'datenum' or 'datetime'
Output date format, specified as a character vector with values 'datenum' or 'datetime'. If outputType is 'datenum', then DayMonth is a serial date number. However, if outputType is 'datetime', then DayMonth is a datetime array.
Data Types: char

## Output Arguments

## DayMonth - Last day of month

serial date number | datetime array
Last day of the month, returned as a serial date number or datetime array.
See Alsodatetime | day | eomday | lbusdate | month | year
Topics"Financial Time Series Operations" on page 13-7
Introduced before R2006a

## eq (fts)

Multiple financial times series object equality

Note eq (fts) will be removed in a future release. Use timetable instead. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

```
tsobj_1 == tsobj_2
iseq = eq(tsobj_1,tsobj_2)
```


## Arguments

| tsobj_1 | Financial time series object. |
| :--- | :--- |
| tsobj_2 | Financial time series object. |

## Description

tsobj_1 == tsobj_2 returns True (1) if both financial time series objects have the same dates, frequencies, data series names, and data values. Otherwise, eq returns False (0).

Note The data series names are case-sensitive, but do not have to be in the same order within each object.

## Examples

## Determine Multiple Financial Times Series Object Equality

This example shows how to determine if multiple financial times series objects are equal.

```
load disney
```

dis == dis
ans $=$ logical
1

## See Also

isequal

## Topics

"What Is the Financial Time Series App?" on page 14-2
Introduced before R2006a

## estimateAssetMoments

Estimate mean and covariance of asset returns from data

Note estimateAssetMoments has been partially removed and will no longer accept a fints object for the AssetReturns argument. Use timetable instead for financial time series

Use fts2timetable to convert a fints object to a timetable object.

## Syntax

```
obj = estimateAssetMoments(obj,AssetReturns)
obj = estimateAssetMoments(
```

$\qquad$

``` Name, Value)
```


## Description

obj = estimateAssetMoments(obj,AssetReturns) estimates mean and covariance of asset returns from data for a Portfolio object. For details on the workflow, see "Portfolio Object Workflow" on page 4-21.
obj = estimateAssetMoments( __ ,Name,Value) estimates mean and covariance of asset returns from data for a Portfolio object with additional options for one or more Name, Value pair arguments.

## Examples

## Estimate Mean and Covariance of Asset Returns from Data for a Portfolio Object

To illustrate using the estimateAssetMoments function, generate random samples of 120 observations of asset returns for four assets from the mean and covariance of asset returns in the variables m and C with the portsim function. The default behavior portsim creates simulated data with estimated mean and covariance identical to the
input moments $m$ and $C$. In addition to a return series created by the portsim function in the variable $X$, a price series is created in the variable $Y$ :

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;
X = portsim(m', C, 120);
Y = ret2tick(X);
```

Given asset returns and prices in the variables $X$ and $Y$ from above, the following examples demonstrate equivalent ways to estimate asset moments for the Portfolio object. A Portfolio object is created in $p$ with the moments of asset returns set directly in the Portfolio object and a second Portfolio object is created in q to obtain the mean and covariance of asset returns from asset return data in X using the estimateAssetMoments function.

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;
X = portsim(m', C, 120);
p = Portfolio('mean',m,'covar',C);
q = Portfolio;
q = estimateAssetMoments(q, X);
[passetmean, passetcovar] = getAssetMoments(p)
passetmean = 4×1
    0.0042
    0.0083
    0.0100
    0.0150
passetcovar = 4×4
```

```
    0.0005 0.0003 0.0002 0
    0.0003 0.0024 0.0017 0.0010
    0.0002 0.0017 0.0048 0.0028
    0.0010 0.0028 0.0102
[qassetmean, qassetcovar] = getAssetMoments(q)
qassetmean = 4×1
    0.0042
    0.0083
    0.0100
    0.0150
qassetcovar = 4×4
\begin{tabular}{rrrr}
0.0005 & 0.0003 & 0.0002 & -0.0000 \\
0.0003 & 0.0024 & 0.0017 & 0.0010 \\
0.0002 & 0.0017 & 0.0048 & 0.0028 \\
0.0000 & 0.0010 & 0.0028 & 0.0102
\end{tabular}
```

Notice how either approach yields the same moments. The default behavior of the estimateAssetMoments function is to work with asset returns. If, instead, you have asset prices, such as in the variable $Y$, the estimateAssetMoments function accepts a parameter name 'DataFormat' with a corresponding value set to 'prices' to indicate that the input to the method is in the form of asset prices and not returns (the default parameter value for 'DataFormat' is 'returns'). The following example compares direct assignment of moments in the Portfolio object $p$ with estimated moments from asset price data in $Y$ in the Portfolio object q:

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;
X = portsim(m', C, 120);
Y = ret2tick(X);
p = Portfolio('mean',m,'covar',C);
```

```
q = Portfolio;
q = estimateAssetMoments(q, Y, 'dataformat', 'prices');
[passetmean, passetcovar] = getAssetMoments(p)
passetmean = 4×1
    0.0042
    0.0083
    0.0100
    0.0150
passetcovar = 4×4
    0.0005 0.0003 0.0002 0
    0.0003 0.0024 0.0017 0.0010
    0.0002 0.0017 0.0048 0.0028
    0.0010 0.0028 0.0102
[qassetmean, qassetcovar] = getAssetMoments(q)
qassetmean = 4×1
    0.0042
    0.0083
    0.0100
    0.0150
qassetcovar = 4×4
\begin{tabular}{rrrr}
0.0005 & 0.0003 & 0.0002 & -0.0000 \\
0.0003 & 0.0024 & 0.0017 & 0.0010 \\
0.0002 & 0.0017 & 0.0048 & 0.0028 \\
-0.0000 & 0.0010 & 0.0028 & 0.0102
\end{tabular}
```


## Estimate Mean and Covariance of Asset Returns from Timetable Data for a Portfolio Object

To illustrate using the estimateAssetMoments function with AssetReturns data continued in a timetable object, use the CAPMuniverse.mat which contains a timetable object (AssetTimeTable) for returns data.
load CAPMuniverse
AssetsTimeTable.Properties; head(AssetsTimeTable,5)


Notice that G00G has missing data (NaN), because it was not listed before Aug 2004. The estimateAssetMoments function has a name-value pair argument 'MissingData' that indicates with a Boolean value whether to use the missing data capabilities of Financial Toolbox ${ }^{\mathrm{TM}}$ software. The default value for 'MissingData' is false which removes all samples with NaN values. If, however, 'MissingData' is set to true, estimateAssetMoments uses the ECM algorithm to estimate asset moments.
$r=$ Portfolio;
r = estimateAssetMoments(r,AssetsTimeTable,'dataformat','returns','missingdata',true);
In addition, the estimateAssetMoments function also extracts asset names or identifiers from a timetable object when the name-value argument 'GetAssetList' set to true (its default value is false). If the 'GetAssetList' value is true, the timetable column identifiers are used to set the AssetList property of the Portfolio object. To show this, the formation of the Portfolio object $r$ is repeated with the 'GetAssetList' flag set to true.

```
r = estimateAssetMoments(r,AssetsTimeTable,'GetAssetList',true);
disp(r.AssetList')
```

' AAPL'
'AMZN '

```
'CSCO'
'DELL'
'EBAY'
'GOOG '
'HPQ'
'IBM'
'INTC'
'MSFT'
'ORCL'
'YHOO'
'MARKET
'CASH'
```


## Input Arguments

## obj - Object for portfolio

## object

Object for portfolio, specified using a Portfolio object. For more information on creating a portfolio object, see

- Portfolio


## Data Types: object

## AssetReturns - Matrix, table, or timetable that contains asset price data that can be converted to asset returns

matrix | table | table
Matrix, table, or timetable that contains asset price data that can be converted to asset returns, specified by a NumSamples-by-NumAssets matrix.

AssetReturns data can be:

- NumSamples-by-NumAssets matrix.
- Table of NumSamples prices or returns at a given periodicity for a collection of NumAssets assets
- Timetable object with NumSamples observations and NumAssets time series

Use the optional DataFormat argument to convert AssetReturns input data that is asset prices into asset returns. Be careful when using asset price data because portfolio optimization usually requires total returns and not simply price returns.

## Data Types: double|table|timetable

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Example: $p$ = estimateAssetMoments( $\mathrm{p}, \mathrm{Y}$, 'dataformat','prices')

## DataFormat - Flag to convert input data as prices into returns

'Returns' (default)|character vector with values 'Returns' or 'Prices'
Flag to convert input data as prices into returns, specified as the comma-separated pair consisting of 'DataFormat ' and a character vector with the values:

- 'Returns' - Data in AssetReturns contains asset total returns.
- 'Prices' - Data in AssetReturns contains asset total return prices.

Data Types: char

## MissingData - Flag indicating whether to use ECM algorithm or exclude samples with NaN values

false (default) | logical with value true or false
Flag indicating whether to use ECM algorithm or excludes samples with NaN values, specified as the comma-separated pair consisting of 'MissingData' and a logical with a value of true or false.

To handle time series with missing data (indicated with NaN values), the MissingData flag either uses the ECM algorithm to obtain maximum likelihood estimates in the presences of NaN values or excludes samples with NaN values. Since the default is false, it is necessary to specify MissingData as true to use the ECM algorithm.

Acceptable values for MissingData are:

- false - Do not use ECM algorithm to handle NaN values (exclude NaN values).
- true - Use ECM algorithm to handle NaN values.

For more information on the ECM algorithm, see ecmnmle and "Multivariate Normal Regression" on page 9-2.

## Data Types: logical

## GetAssetList - Flag indicating which asset names to use for asset list

false (default) | logical with value true or false
Flag indicating which asset names to use for the asset list, specified as the commaseparated pair consisting of 'GetAssetList' and a logical with a value of true or false. Acceptable values for GetAssetList are:

- false - Do not extract or create asset names.
- true - Extract or create asset names from a table or timetable object.

If a table or timetable is passed into this function using the AssetReturns argument and the GetAssetList flag is true, the column names from the table or timetable object are used as asset names in obj. AssetList.

If a matrix is passed and the GetAssetList flag is true, default asset names are created based on the AbstractPortfolio property defaultforAssetList, which is 'Asset'.

If the GetAssetList flag is false, no action occurs, which is the default behavior.
Data Types: logical

## Output Arguments

## obj - Updated portfolio object

object for portfolio
Updated portfolio object, returned as a Portfolio object. For more information on creating a portfolio object, see

- Portfolio


## Tips

You can also use dot notation to estimate the mean and covariance of asset returns from data.
obj = obj.estimateAssetMoments(AssetReturns);

## See Also

Portfolio|estimateBounds | portsim

## Topics

"Asset Returns and Moments of Asset Returns Using Portfolio Object" on page 4-48
"Portfolio Optimization Examples" on page 4-167
"Portfolio Optimization Theory" on page 4-3
Introduced in R2011a
$19-819$

## estimateBounds

Estimate global lower and upper bounds for set of portfolios

## Syntax

[glb,gub,isbounded] = estimateBounds(obj)
[glb,gub,isbounded] = estimateBounds(obj,obtainExactBounds)

## Description

[glb,gub,isbounded] = estimateBounds(obj) estimates global lower and upper bounds for set of portfolios for Portfolio, PortfolioCVaR, or PortfolioMAD objects. For details on the respective workflows when using these different objects, see "Portfolio Object Workflow" on page 4-21, "PortfolioCVaR Object Workflow" on page 5-20, and "PortfolioMAD Object Workflow" on page 6-19.

Note The estimateBounds function does not consider cardinality or semicontinuous constraints. For more information, see "Working with 'Conditional' BoundType, MinNumAssets, and MaxNumAssets Constraints Using Portfolio Object" on page 4-94.
[glb,gub,isbounded] = estimateBounds(obj,obtainExactBounds) estimates global lower and upper bounds for set of portfolios with an additional option specified for obtainExactBounds.

## Examples

## Create an Unbounded Portfolio for a Portfolio Object

Create an unbounded portfolio set.

```
p = Portfolio('AInequality', [1 -1; 1 1 ], 'bInequality', 0);
[lb, ub, isbounded] = estimateBounds(p)
```

```
lb}=2\times
    - Inf
    -Inf
ub = 2x1
10-8}
    -0.3712
        Inf
isbounded = logical
    0
```

The estimateBounds function returns (possibly infinite) bounds and sets the isbounded flag to false. The result shows which assets are unbounded so that you can apply bound constraints as necessary.

## Create an Unbounded Portfolio for a PortfolioCVaR Object

Create an unbounded portfolio set.

```
p = PortfolioCVaR('AInequality', [1 -1; 1 1 ], 'bInequality', 0);
[lb, ub, isbounded] = estimateBounds(p)
lb = 2x1
    -Inf
    -Inf
ub = 2×1
10-8}
    -0.3712
        Inf
```

```
isbounded = logical
    0
```

The estimateBounds function returns (possibly infinite) bounds and sets the isbounded flag to false. The result shows which assets are unbounded so that you can apply bound constraints as necessary.

## Create an Unbounded Portfolio for a PortfolioMAD Object

Create an unbounded portfolio set.

```
p = PortfolioMAD('AInequality', [1 -1; 1 1 ], 'bInequality', 0);
[lb, ub, isbounded] = estimateBounds(p)
lb = 2×1
    -Inf
    -Inf
ub = 2×1
10-8}
    -0.3712
        Inf
isbounded = logical
    0
```

The estimateBounds function returns (possibly infinite) bounds and sets the isbounded flag to false. The result shows which assets are unbounded so that you can apply bound constraints as necessary.

## Input Arguments

## obj - Object for portfolio

object

Object for portfolio, specified using Portfolio, PortfolioCVaR, or PortfolioMAD object. For more information on creating a portfolio object, see

- Portfolio
- PortfolioCVaR
- PortfolioMAD


## Data Types: object

## obtainExactBounds - Flag to specify whether to solve for all bounds or to accept specified bounds whenever available <br> true (default) | logical

Flag to specify whether to solve for all bounds or to accept specified bounds whenever available, specified as a logical with values of true or false. If bounds are known, set obtainExactBounds to false to accept known bounds. The default for obtainExactBounds is true.

Data Types: logical

## Output Arguments

## glb - Global lower bounds for portfolio set

vector
Global lower bounds for portfolio set, returned as vector for a Portfolio, PortfolioCVaR, or PortfolioMAD input object (obj).

## gub - Global upper bounds for portfolio set

vector
Global upper bounds for portfolio set, returned as vector for a Portfolio, PortfolioCVaR, or PortfolioMAD input object (obj).
isbounded - Indicator for whether portfolio set is empty, bounded, or unbounded
logical
Indicator for whether portfolio set is empty ([ ]), bounded (true), or unbounded (false), returned as a logical.

Note By definition, any portfolio set must be nonempty and bounded:

- If the set is empty, isbounded $=[$ ].
- If the set is nonempty and unbounded, isbounded = false.
- If the set is nonempty and bounded, isbounded = true.
- If the set is empty, glb and gub are set to NaN vectors.

An isbounded value is returned for Portfolio, PortfolioCVaR, or PortfolioMAD input object (obj).

## Tips

- You can also use dot notation to estimate the global lower and upper bounds for a given set of portfolios.
[glb, gub, isbounded] = obj.estimateBounds;
- Estimated bounds are accurate in most cases to within 1.0e-8. If you intend to use these bounds directly in a portfolio object, ensure that if you impose such bound constraints, a lower bound of 0 is probably preferable to a lower bound of, for example, 1.0e-10 for portfolio weights.


## See Also

checkFeasibility

## Topics

"Validate the Portfolio Problem for Portfolio Object" on page 4-110
"Validate the CVaR Portfolio Problem" on page 5-96
"Validate the MAD Portfolio Problem" on page 6-91
"Portfolio Optimization Examples" on page 4-167
"Portfolio Optimization Theory" on page 4-3

## Introduced in R2011a

## estimateFrontier

Estimate specified number of optimal portfolios on the efficient frontier

## Syntax

[pwgt, pbuy, psell] = estimateFrontier(obj)
[pwgt, pbuy,psell] = estimateFrontier(obj,NumPorts)

## Description

[pwgt, pbuy, psell] = estimateFrontier(obj) estimates the specified number of optimal portfolios on the efficient frontier for Portfolio, PortfolioCVaR, or PortfolioMAD objects. For details on the respective workflows when using these different objects, see "Portfolio Object Workflow" on page 4-21, "PortfolioCVaR Object Workflow" on page 5-20, and "PortfolioMAD Object Workflow" on page 6-19.
[pwgt, pbuy, psell] = estimateFrontier(obj,NumPorts) estimates the specified number of optimal portfolios on the efficient frontier with an additional option specified for NumPorts.

## Examples

## Create a Portfolio Object and Determine Efficient Portfolios

Create efficient portfolios:
load CAPMuniverse

```
p = Portfolio('AssetList',Assets(1:12));
p = estimateAssetMoments(p, Data(:,1:12),'missingdata',true);
p = setDefaultConstraints(p);
plotFrontier(p);
```



```
pwgt = estimateFrontier(p, 5);
pnames = cell(1,5);
for i = 1:5
    pnames{i} = sprintf('Port%d',i);
end
Blotter = dataset([{pwgt},pnames],'obsnames',p.AssetList);
disp(Blotter);
\begin{tabular}{llllll} 
& Port1 & Port2 & Port3 & Port4 & Port5 \\
AAPL & 0.017926 & 0.058247 & 0.097816 & 0.12955 & 0
\end{tabular}
```

| AMZN | 0 | 0 | 0 | 0 | 0 |
| :--- | ---: | ---: | ---: | ---: | ---: |
| CSCO | 0 | 0 | 0 | 0 | 0 |
| DELL | 0.0041906 | 0 | 0 | 0 | 0 |
| EBAY | 0.16144 | 0.35678 | 0.55228 | 0.75116 | 0 |
| GOOG | 0.052566 | 0.032302 | 0.011186 | 0 | 0 |
| HPQ | 0.46422 | 0.36045 | 0.25577 | 0.11928 | 0 |
| IBM | 0 | 0 | 0 | 0 | 0 |
| INTC | 0.29966 | 0.19222 | 0.082949 | 0 | 0 |
| MSFT | 0 | 0 | 0 | 0 | 0 |
| ORCL | 0 | 0 | 0 | 0 | 0 |

## Create a Portfolio Object with BoundType and MaxNumAssets Constraints and Determine Efficient Portfolios

Create a Portfolio object for 12 stocks based on CAPMuniverse.mat.

```
load CAPMuniverse
p0 = Portfolio('AssetList',Assets(1:12));
p0 = estimateAssetMoments(p0, Data(:,1:12),'missingdata',true);
p0 = setDefaultConstraints(p0);
```

Use setMinMaxNumAssets to define a maximum number of 3 assets.
p1 = setMinMaxNumAssets(p0, [], 3);
Use setBounds to define a lower and upper bound and a BoundType of 'Conditional'.

```
p1 = setBounds(p1, 0.1, 0.5,'BoundType', 'Conditional');
pwgt = estimateFrontier(p1, 5);
```

The following table shows that the optimized allocations only have maximum 3 assets invested, and small positions less than 0.1 are avoided.

```
result = table(p0.AssetList', pwgt)
result=12\times2 table
    Var1
        pwgt
    'AAPL' 0 0 0 0 0.14301 
```

| 'AMZN ' | 0 | 0 | 0 | 0 | 0 |
| :--- | ---: | ---: | ---: | ---: | ---: |
| 'CSCO' | 0 | 0 | 0 | 0 | 0 |
| 'DELL' | 0 | 0 | 0 | 0 | 0 |
| 'EBAY' | 0 | 0 | 0 | 0 | 0.5 |
| 'GOOG' | 0.16981 | 0.29588 | 0.42214 | 0.49999 | 0.5 |
| 'HPQ' | 0 | 0 | 0 | 0 | 0 |
| 'IBM' | 0.49592 | 0.43629 | 0.37308 | 0.35699 | 0 |
| 'INTC' | 0 | 0 | 0 | 0 | 0 |
| 'MSFT' | 0.33426 | 0.26783 | 0.20478 | 0 | 0 |
| 'ORCL' | 0 | 0 | 0 | 0 | 0 |
| 'YHOO' | 0 | 0 | 0 | 0 | 0 |

The estimateFrontier function uses the MINLP solver to solve this problem. Use the setSolverMINLP function to configure the SolverType and options.
p1.solverTypeMINLP
ans =
'OuterApproximation'
p1.solverOptionsMINLP
ans $=$ struct with fields:
MaxIterations: 1000
AbsoluteGapTolerance: 1.0000e-07
RelativeGapTolerance: 1.0000e-05
NonlinearScalingFactor: 1000
ObjectiveScalingFactor: 1000
IntMasterSolverOptions: [1x1 optim.options.Intlinprog]
Display: 'off'
CutGeneration: 'basic'
MaxIterationsInactiveCut: 30
ActiveCutTolerance: 1.0000e-07
NumIterationsEarlyIntegerConvergence: 30

## Construct a PortfolioCVaR Object and Determine Efficient Portfolios

Create efficient portfolios:
load CAPMuniverse

```
p = PortfolioCVaR('AssetList',Assets(1:12));
p = simulateNormalScenariosByData(p, Data(:,1:12), 20000 ,'missingdata',true);
p = setDefaultConstraints(p);
p = setProbabilityLevel(p, 0.95);
plotFrontier(p);
```


pwgt = estimateFrontier(p, 5);
pnames $=\operatorname{cell}(1,5)$;
for $\mathrm{i}=1: 5$
pnames\{i\} $=$ sprintf('Port\%d',i);
end

```
Blotter = dataset([{pwgt},pnames],'obsnames',p.AssetList);
disp(Blotter);
\begin{tabular}{lrrrrr} 
& Port1 & \multicolumn{1}{l}{ Port2 } & Port3 & Port4 & Port5 \\
AAPL & 0.010984 & 0.073246 & 0.11933 & 0.13068 & \(1.5092 \mathrm{e}-14\) \\
AMZN & 0 & 0 & \(1.1755 \mathrm{e}-38\) & 0 & \(2.8997 \mathrm{e}-14\) \\
CSCO & \(5.8775 \mathrm{e}-39\) & 0 & \(5.8775 \mathrm{e}-39\) & \(1.4106 \mathrm{e}-37\) & \(4.1869 \mathrm{e}-14\) \\
DELL & 0.022454 & \(9.484 \mathrm{e}-33\) & 0 & \(3.7616 \mathrm{e}-37\) & \(3.9048 \mathrm{e}-14\) \\
EBAY & 0 & 0 & \(1.782 \mathrm{e}-38\) & \(2.351 \mathrm{e}-37\) & \(1.3394 \mathrm{e}-15\) \\
GOOG & 0.20335 & 0.38055 & 0.56242 & 0.75932 & 1 \\
HPQ & 0.041724 & 0.0099223 & 0 & \(1.822 \mathrm{e}-37\) & \(3.8894 \mathrm{e}-14\) \\
IBM & 0.44482 & 0.36453 & 0.26282 & 0.11 & \(3.7902 \mathrm{e}-14\) \\
INTC & 0 & 0 & \(4.1142 \mathrm{e}-38\) & 0 & \(3.8264 \mathrm{e}-14\) \\
MSFT & 0.27667 & 0.17175 & 0.055435 & \(2.9387 \mathrm{e}-37\) & \(4.0873 \mathrm{e}-14\) \\
ORCL & 0 & 0 & \(7.053 \mathrm{e}-38\) & 0 & \(3.7811 \mathrm{e}-14\) \\
YHOO & 0 & 0 & 0 & 0 & \(3.535 \mathrm{e}-14\)
\end{tabular}
```


## Create a PortfolioMAD Object and Determine Efficient Portfolios

Create efficient portfolios:
load CAPMuniverse

```
p = PortfolioMAD('AssetList',Assets(1:12));
```

p = simulateNormalScenariosByData(p, Data(:,1:12), 20000 ,'missingdata',true);
$\mathrm{p}=$ setDefaultConstraints(p);
plotFrontier(p);


```
pwgt = estimateFrontier(p, 5);
pnames = cell(1,5);
for i = 1:5
    pnames{i} = sprintf('Port%d',i);
end
Blotter = dataset([{pwgt},pnames],'obsnames',p.AssetList);
disp(Blotter);
```

```
AAPL
Port1
0.030236
```

Port2
0.075387
Port3
0.11278

```
Port4
0.13456
\begin{tabular}{lrrrrr} 
AMZN & \(2.0475 \mathrm{e}-21\) & 0 & \(7.7965 \mathrm{e}-23\) & 0 & \(2.8997 \mathrm{e}-14\) \\
CSCO & \(9.6659 \mathrm{e}-22\) & \(3.4382 \mathrm{e}-23\) & \(4.0358 \mathrm{e}-23\) & \(3.7616 \mathrm{e}-37\) & \(4.1869 \mathrm{e}-14\) \\
DELL & 0.0089659 & 0 & 0 & 0 & \(3.9048 \mathrm{e}-14\) \\
EBAY & \(1.3835 \mathrm{e}-21\) & \(2.9706 \mathrm{e}-23\) & \(2.9385 \mathrm{e}-23\) & \(4.9619 \mathrm{e}-37\) & \(1.3394 \mathrm{e}-15\) \\
GOOG & 0.16117 & 0.35201 & 0.54486 & 0.74888 & 1 \\
HPQ & 0.056551 & 0.024037 & 0 & \(3.7616 \mathrm{e}-36\) & \(3.8894 \mathrm{e}-14\) \\
IBM & 0.45905 & 0.37891 & 0.29383 & 0.11656 & \(3.7902 \mathrm{e}-14\) \\
INTC & 0.28403 & \(7.0923 \mathrm{e}-22\) & \(3.7644 \mathrm{e}-23\) & \(3.0093 \mathrm{e}-36\) & \(3.8264 \mathrm{e}-14\) \\
MSFT & 0.16966 & 0.048527 & \(2.8212 \mathrm{e}-36\) & \(4.0873 \mathrm{e}-14\) \\
ORCL & \(3.9607 \mathrm{e}-22\) & 0 & \(5.6691 \mathrm{e}-23\) & \(2.6449 \mathrm{e}-37\) & \(3.7811 \mathrm{e}-14\) \\
YHOO & \(8.27 \mathrm{e}-22\) & \(3.6285 \mathrm{e}-23\) & \(7.3462 \mathrm{e}-24\) & 0 & \(3.535 \mathrm{e}-14\)
\end{tabular}

\section*{Obtain the Default Number of Efficient Portfolios for a Portfolio Object}

Obtain the default number of efficient portfolios over the entire range of the efficient frontier.
```

m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
0.00408 0.0289 0.0204 0.0119;
0.00192 0.0204 0.0576 0.0336;
0 0.0119 0.0336 0.1225 ];
p = Portfolio;
p = setAssetMoments(p, m, C);
p = setDefaultConstraints(p);
pwgt = estimateFrontier(p);
disp(pwgt);

```

Columns 1 through 7
\begin{tabular}{lrrrrrr}
0.8891 & 0.7215 & 0.5540 & 0.3865 & 0.2190 & 0.0515 & 0 \\
0.0369 & 0.1289 & 0.2209 & 0.3129 & 0.4049 & 0.4969 & 0.4049 \\
0.0404 & 0.0567 & 0.0730 & 0.0893 & 0.1056 & 0.1219 & 0.1320 \\
0.0336 & 0.0929 & 0.1521 & 0.2113 & 0.2705 & 0.3297 & 0.4630
\end{tabular}

Columns 8 through 10
\begin{tabular}{rrr}
0 & 0 & 0 \\
0.2314 & 0.0579 & 0 \\
0.1394 & 0.1468 & 0 \\
0.6292 & 0.7953 & 1.0000
\end{tabular}

\section*{Obtain Purchases and Sales for Portfolios on the Efficient Frontier for a Portfolio Object}

Starting from the initial portfolio, the estimateFrontier function returns purchases and sales to get from your initial portfolio to each efficient portfolio on the efficient frontier. Given an initial portfolio in pwgt0, you can obtain purchases and sales.
```

m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
0.00408 0.0289 0.0204 0.0119;
0.00192 0.0204 0.0576 0.0336;
0 0.0119 0.0336 0.1225 ];
p = Portfolio;
p = setAssetMoments(p, m, C);
p = setDefaultConstraints(p);
pwgt0 = [ 0.3; 0.3; 0.2; 0.1 ];
p = setInitPort(p, pwgt0);
[pwgt, pbuy, psell] = estimateFrontier(p);
display(pwgt);
pwgt = 4×10

```
\begin{tabular}{rrrrrrrr}
0.8891 & 0.7215 & 0.5540 & 0.3865 & 0.2190 & 0.0515 & 0 & 0 \\
0.0369 & 0.1289 & 0.2209 & 0.3129 & 0.4049 & 0.4969 & 0.4049 & 0.2314 \\
0.0404 & 0.0567 & 0.0730 & 0.0893 & 0.1056 & 0.1219 & 0.1320 & 0.1394 \\
0.0336 & 0.0929 & 0.1521 & 0.2113 & 0.2705 & 0.3297 & 0.4630 & 0.6292
\end{tabular}
display(pbuy);
pbuy \(=4 \times 10\)
\begin{tabular}{rrrrrrrr}
0.5891 & 0.4215 & 0.2540 & 0.0865 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.0129 & 0.1049 & 0.1969 & 0.1049 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.0521 & 0.1113 & 0.1705 & 0.2297 & 0.3630 & 0.5292
\end{tabular}
display(psell);
```

psell = 4×10

```
\begin{tabular}{rrrrrrrr}
0 & 0 & 0 & 0 & 0.0810 & 0.2485 & 0.3000 & 0.3000 \\
0.2631 & 0.1711 & 0.0791 & 0 & 0 & 0 & 0 & 0.0686 \\
0.1596 & 0.1433 & 0.1270 & 0.1107 & 0.0944 & 0.0781 & 0.0680 & 0.0606 \\
0.0664 & 0.0071 & 0 & 0 & 0 & 0 & 0 & 0
\end{tabular}

\section*{Obtain the Default Number of Efficient Portfolios for a PortfolioCVaR Object}

Obtain the default number of efficient portfolios over the entire range of the efficient frontier.
```

m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
0.00408 0.0289 0.0204 0.0119;
0.00192 0.0204 0.0576 0.0336;
0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;

```
rng(11);
AssetScenarios \(=\) mvnrnd(m, C, 20000);
p = PortfolioCVaR;
\(\mathrm{p}=\) setScenarios(p, AssetScenarios);
p = setDefaultConstraints(p);
p = setProbabilityLevel(p, 0.95);
pwgt = estimateFrontier(p);
disp(pwgt);

Columns 1 through 7
\begin{tabular}{lllllll}
0.8454 & 0.6847 & 0.5151 & 0.3541 & 0.1902 & 0.0314 & 0.0000 \\
0.0599 & 0.1427 & 0.2302 & 0.3165 & 0.3980 & 0.4733 & 0.3513 \\
0.0462 & 0.0639 & 0.0945 & 0.1079 & 0.1345 & 0.1583 & 0.1756 \\
0.0485 & 0.1087 & 0.1602 & 0.2215 & 0.2773 & 0.3371 & 0.4731
\end{tabular}

\section*{Columns 8 through 10}
\begin{tabular}{rrr}
0 & 0.0000 & 0.0000 \\
0.1806 & 0 & 0.0000 \\
0.1916 & 0.2212 & 0.0000 \\
0.6278 & 0.7788 & 1.0000
\end{tabular}

The function \(\mathrm{rng}\left({ }^{\text {seed }}\right)\) resets the random number generator to produce the documented results. It is not necessary to reset the random number generator to simulate scenarios.

\section*{Obtain Purchases and Sales for Portfolios on the Efficient Frontier for a PortfolioCVaR Object}

Starting from the initial portfolio, the estimateFrontier function returns purchases and sales to get from your initial portfolio to each efficient portfolio on the efficient frontier. Given an initial portfolio in pwgt0, you can obtain purchases and sales.
```

m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
0.00408 0.0289 0.0204 0.0119;
0.00192 0.0204 0.0576 0.0336;
0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;
rng(11);
AssetScenarios = mvnrnd(m, C, 20000);
p = PortfolioCVaR;
p = setScenarios(p, AssetScenarios);
p = setDefaultConstraints(p);
p = setProbabilityLevel(p, 0.95);
pwgt0 = [ 0.3; 0.3; 0.2; 0.1 ];
p = setInitPort(p, pwgt0);
[pwgt, pbuy, psell] = estimateFrontier(p);
display(pwgt);
pwgt = 4×10

```
\(0.8454 \quad 0.6847 \quad 0.5151 \quad 0.3541 \quad 0.1902 \quad 0.0314 \quad 0.0000\)

19-835
\begin{tabular}{llllllll}
0.0599 & 0.1427 & 0.2302 & 0.3165 & 0.3980 & 0.4733 & 0.3513 & 0.1806 \\
0.0462 & 0.0639 & 0.0945 & 0.1079 & 0.1345 & 0.1583 & 0.1756 & 0.1916 \\
0.0485 & 0.1087 & 0.1602 & 0.2215 & 0.2773 & 0.3371 & 0.4731 & 0.6278
\end{tabular}
```

display(pbuy);

```
pbuy \(=4 \times 10\)
\begin{tabular}{rrrrrrrr}
0.5454 & 0.3847 & 0.2151 & 0.0541 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.0165 & 0.0980 & 0.1733 & 0.0513 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.0087 & 0.0602 & 0.1215 & 0.1773 & 0.2371 & 0.3731 & 0.5278
\end{tabular}
display(psell);
psell \(=4 \times 10\)
\begin{tabular}{rrrrrrrr}
0 & 0 & 0 & 0 & 0.1098 & 0.2686 & 0.3000 & 0.3000 \\
0.2401 & 0.1573 & 0.0698 & 0 & 0 & 0 & 0 & 0.1194 \\
0.1538 & 0.1361 & 0.1055 & 0.0921 & 0.0655 & 0.0417 & 0.0244 & 0.0084 \\
0.0515 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{tabular}

The function \(\mathrm{rng}\left({ }^{\text {seed }}\right)\) resets the random number generator to produce the documented results. It is not necessary to reset the random number generator to simulate scenarios.

\section*{Obtain the Default Number of Efficient Portfolios for a PortfolioMAD Object}

Obtain the default number of efficient portfolios over the entire range of the efficient frontier.
```

m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
0.00408 0.0289 0.0204 0.0119;
0.00192 0.0204 0.0576 0.0336;
0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;
rng(11);

```
```

AssetScenarios = mvnrnd(m, C, 20000);
p = PortfolioMAD;
p = setScenarios(p, AssetScenarios);
p = setDefaultConstraints(p);
pwgt = estimateFrontier(p);
disp(pwgt);
Columns 1 through 7

```
\begin{tabular}{lllllll}
0.8815 & 0.7151 & 0.5487 & 0.3817 & 0.2170 & 0.0499 & 0.0000 \\
0.0431 & 0.1282 & 0.2128 & 0.2981 & 0.3825 & 0.4662 & 0.3609 \\
0.0389 & 0.0605 & 0.0826 & 0.1053 & 0.1241 & 0.1492 & 0.1786 \\
0.0365 & 0.0963 & 0.1559 & 0.2149 & 0.2764 & 0.3348 & 0.4605
\end{tabular}

Columns 8 through 10
\begin{tabular}{rrr}
0.0000 & 0.0000 & 0.0000 \\
0.1755 & 0 & 0.0000 \\
0.2095 & 0.2266 & 0.0000 \\
0.6150 & 0.7734 & 1.0000
\end{tabular}

The function \(\mathrm{rng}\left({ }^{\text {seed }}\right)\) resets the random number generator to produce the documented results. It is not necessary to reset the random number generator to simulate scenarios.

\section*{Obtain Purchases and Sales for Portfolios on the Efficient Frontier for a PortfolioMAD Object}

Starting from the initial portfolio, the estimateFrontier function returns purchases and sales to get from your initial portfolio to each efficient portfolio on the efficient frontier. Given an initial portfolio in pwgt0, you can obtain purchases and sales.
```

m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
0.00408 0.0289 0.0204 0.0119;
0.00192 0.0204 0.0576 0.0336;
0 0.0119 0.0336 0.1225 ];
m = m/12;

```
```

C = C/12;
rng(11);
AssetScenarios = mvnrnd(m, C, 20000);
p = PortfolioMAD;
p = setScenarios(p, AssetScenarios);
p = setDefaultConstraints(p);
pwgt0 = [ 0.3; 0.3; 0.2; 0.1 ];
p = setInitPort(p, pwgt0);
[pwgt, pbuy, psell] = estimateFrontier(p);
display(pwgt);
pwgt = 4 }\times1

| 0.8815 | 0.7151 | 0.5487 | 0.3817 | 0.2170 | 0.0499 | 0.0000 | 0.0000 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.0431 | 0.1282 | 0.2128 | 0.2981 | 0.3825 | 0.4662 | 0.3609 | 0.1755 |
| 0.0389 | 0.0605 | 0.0826 | 0.1053 | 0.1241 | 0.1492 | 0.1786 | 0.2095 |
| 0.0365 | 0.0963 | 0.1559 | 0.2149 | 0.2764 | 0.3348 | 0.4605 | 0.6150 |

display(pbuy);
pbuy = 4×10

| 0.5815 | 0.4151 | 0.2487 | 0.0817 | 0 | 0 | 0 | 0 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 0 | 0 | 0 | 0 | 0.0825 | 0.1662 | 0.0609 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.0095 |
| 0 | 0 | 0.0559 | 0.1149 | 0.1764 | 0.2348 | 0.3605 | 0.5150 |

display(psell);
psell = 4×10

| 0 | 0 | 0 | 0 | 0.0830 | 0.2501 | 0.3000 | 0.3000 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 0.2569 | 0.1718 | 0.0872 | 0.0019 | 0 | 0 | 0 | 0.1245 |
| 0.1611 | 0.1395 | 0.1174 | 0.0947 | 0.0759 | 0.0508 | 0.0214 | 0 |
| 0.0635 | 0.0037 | 0 | 0 | 0 | 0 | 0 | 0 |

```

The function \(\mathrm{rng}\left({ }^{\text {seed }}\right)\) resets the random number generator to produce the documented results. It is not necessary to reset the random number generator to simulate scenarios.

\section*{Input Arguments}

\section*{obj - Object for portfolio}
object
Object for portfolio, specified using Portfolio, PortfolioCVaR, or PortfolioMAD object. For more information on creating a portfolio object, see
- Portfolio
- PortfolioCVaR
- PortfolioMAD

Data Types: object

\section*{NumPorts - Number of points to obtain on efficient frontier \\ value from hidden property defaultNumPorts (default value is 10) (default) | scalar integer}

Number of points to obtain on the efficient frontier, specified as a scalar integer.

Note If no value is specified for NumPorts, the default value is obtained from the hidden property defaultNumPorts (default value is 10). If NumPorts \(=1\), this function returns the portfolio specified by the hidden property defaultFrontierLimit (current default value is 'min').

\section*{Data Types: double}

\section*{Output Arguments}
pwgt - Optimal portfolios on efficient frontier with specified number of portfolios spaced equally from minimum to maximum portfolio return matrix

Optimal portfolios on the efficient frontier with specified number of portfolios spaced equally from minimum to maximum portfolio return, returned as a NumAssets-by-

NumPorts matrix. pwgt is returned for a Portfolio, PortfolioCVaR, or PortfolioMAD input object (obj).

\section*{pbuy - Purchases relative to initial portfolio for optimal portfolios on efficient frontier}
matrix
Purchases relative to an initial portfolio for optimal portfolios on the efficient frontier, returned as NumAssets-by-NumPorts matrix.

Note If no initial portfolio is specified in obj. InitPort, that value is assumed to be 0 such that pbuy \(=\max (0, \mathrm{pwgt})\) and psell \(=\max (0,-\mathrm{pwgt})\).
pbuy is returned for a Portfolio, PortfolioCVaR, or PortfolioMAD input object (obj).

\section*{psell - Sales relative to initial portfolio for optimal portfolios on efficient frontier}
matrix
Sales relative to an initial portfolio for optimal portfolios on the efficient frontier, returned as a NumAssets-by-NumPorts matrix.

Note If no initial portfolio is specified in obj. InitPort, that value is assumed to be 0 such that pbuy \(=\max (0, \mathrm{pwgt})\) and psell \(=\max (0,-\mathrm{pwg})\).
psell is returned for Portfolio, PortfolioCVaR, or PortfolioMAD input object (obj).

\section*{Tips}
- You can also use dot notation to estimate the specified number of optimal portfolios over the entire efficient frontier.
```

[pwgt, pbuy, psell] = obj.estimateFrontier(NumPorts);

```
- When introducing transaction costs and turnover constraints to the Portfolio, PortfolioCVaR, or PortfolioMAD object, the portfolio optimization objective
contains a term with an absolute value. For more information on how Financial Toolbox handles such cases algorithmically, see "References" on page 19-841.

\section*{References}
[1] Cornuejols, G., and R. Tutuncu. Optimization Methods in Finance. Cambridge University Press, 2007.

\section*{See Also}
estimateFrontierByReturn|estimateFrontierByRisk|
estimateFrontierLimits | setBounds | setMinMaxNumAssets

\section*{Topics}
"Estimate Efficient Portfolios for Entire Efficient Frontier for Portfolio Object" on page 4115
"Estimate Efficient Frontiers for Portfolio Object" on page 4-140
"Estimate Efficient Portfolios for Entire Frontier for PortfolioCVaR Object" on page 5-101
"Estimate Efficient Frontiers for PortfolioCVaR Object" on page 5-118
"Estimate Efficient Portfolios Along the Entire Frontier for PortfolioMAD Object" on page 6-96
"Estimate Efficient Frontiers for PortfolioMAD Object" on page 6-111
"Portfolio Optimization Examples" on page 4-167
"Portfolio Optimization Theory" on page 4-3

\section*{Introduced in R2011a}

\section*{estimateFrontierByReturn}

Estimate optimal portfolios with targeted portfolio returns

\section*{Syntax}
```

[pwgt,pbuy,psell] = estimateFrontierByReturn(obj,TargetReturn)

```

\section*{Description}
[pwgt,pbuy,psell] = estimateFrontierByReturn(obj,TargetReturn) estimates optimal portfolios with targeted portfolio returns for Portfolio, PortfolioCVaR, or PortfolioMAD objects. For details on the respective workflows when using these different objects, see "Portfolio Object Workflow" on page 4-21, "PortfolioCVaR Object Workflow" on page 5-20, and "PortfolioMAD Object Workflow" on page 6-19.

\section*{Examples}

\section*{Obtain the Portfolio for Targeted Portfolio Returns for a Portfolio Object}

To obtain efficient portfolios that have targeted portfolio returns, the estimateFrontierByReturn function accepts one or more target portfolio returns and obtains efficient portfolios with the specified returns. Assume you have a universe of four assets where you want to obtain efficient portfolios with target portfolio returns of \(6 \%\), \(9 \%\), and \(12 \%\).
```

m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
0.00408 0.0289 0.0204 0.0119;
0.00192 0.0204 0.0576 0.0336;
0 0.0119 0.0336 0.1225 ];
p = Portfolio;
p = setAssetMoments(p, m, C);

```
```

p = setDefaultConstraints(p);
pwgt = estimateFrontierByReturn(p, [0.06, 0.09, 0.12]);
display(pwgt);
pwgt = 4×3

| 0.8772 | 0.5032 | 0.1293 |
| :--- | :--- | :--- |
| 0.0434 | 0.2488 | 0.4541 |
| 0.0416 | 0.0780 | 0.1143 |
| 0.0378 | 0.1700 | 0.3022 |

```

\section*{Obtain Portfolios with Targeted Portfolio Returns for a Portfolio Object with BoundType, MinNumAsset, and MaxNumAsset Constraints}

When any one, or any combination of the constraints from 'Conditional' BoundType, MinNumAssets, and MaxNumAssets are active, the portfolio problem is formulated as mixed integer programming problem and the MINLP solver is used.

Create a Portfolio object for three assets.
```

AssetMean = [ 0.0101110; 0.0043532; 0.0137058 ];
AssetCovar = [ 0.00324625 0.00022983 0.00420395;
0.00022983 0.00049937 0.00019247;
0.00420395 0.00019247 0.00764097 ];
p = Portfolio('AssetMean', AssetMean, 'AssetCovar', AssetCovar);
p = setDefaultConstraints(p);

```

Use setBounds with semicontinuous constraints to set \(x i=0\) or \(0.02<=x i<=0.5\) for all \(i=1, \ldots\) NumAssets.
```

p = setBounds(p, 0.02, 0.7,'BoundType', 'Conditional', 'NumAssets', 3);

```

When working with a Portfolio object, the setMinMaxNumAssets function enables you to set up the limits on the number of assets invested (as known as cardinality) constraints. This sets the total number of allocated assets satisfying the Bound constraints that are between MinNumAssets and MaxNumAssets. By setting MinNumAssets \(=\) MaxNumAssets \(=2\), only two of the three assets are invested in the portfolio.
```

p = setMinMaxNumAssets(p, 2, 2);

```

Use estimateFrontierByReturn to estimate optimal portfolios with targeted portfolio returns.
```

[pwgt, pbuy, psell] = estimateFrontierByReturn(p,[ 0.0072321, 0.0119084 ])
pwgt = 3\times2

| 0 | 0.5000 |
| ---: | ---: |
| 0.6922 | 0 |
| 0.3078 | 0.5000 |

pbuy = 3\times2
$0 \quad 0.5000$
0.6922 0
0.3078 0.5000
psell = 3\times2
0
0}
0

```

The estimateFrontierByReturn function uses the MINLP solver to solve this problem. Use the setSolverMINLP function to configure the SolverType and options.
```

p.solverTypeMINLP
ans =
'OuterApproximation'
p.solverOptionsMINLP
ans = struct with fields:
MaxIterations: 1000
AbsoluteGapTolerance: 1.0000e-07
RelativeGapTolerance: 1.0000e-05
NonlinearScalingFactor: 1000
ObjectiveScalingFactor: 1000
IntMasterSolverOptions: [1x1 optim.options.Intlinprog]
Display: 'off'
CutGeneration: 'basic'
MaxIterationsInactiveCut: 30

```

ActiveCutTolerance: 1.0000e-07
NumIterationsEarlyIntegerConvergence: 30

\section*{Obtain the Portfolio for Targeted Portfolio Returns for a PortfolioCVaR Object}

To obtain efficient portfolios that have targeted portfolio returns, the estimateFrontierByReturn function accepts one or more target portfolio returns and obtains efficient portfolios with the specified returns. Assume you have a universe of four assets where you want to obtain efficient portfolios with target portfolio returns of 7\%, \(10 \%\), and \(13 \%\).
```

m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
0.00408 0.0289 0.0204 0.0119;
0.00192 0.0204 0.0576 0.0336;
0 0.0119 0.0336 0.1225 ];
rng(11);
p = PortfolioCVaR;
p = simulateNormalScenariosByMoments(p, m, C, 2000);
p = setDefaultConstraints(p);
p = setProbabilityLevel(p, 0.95);
pwgt = estimateFrontierByReturn(p, [0.07 0.10, 0.13]);
display(pwgt);
pwgt = 4×3

| 0.7371 | 0.3071 | 0 |
| :--- | ---: | ---: |
| 0.1504 | 0.3919 | 0.4396 |
| 0.0286 | 0.1011 | 0.1360 |
| 0.0839 | 0.1999 | 0.4244 |

```

The function \(\mathrm{rng}\left({ }^{\text {seed }}\right)\) is used to reset the random number generator to produce the documented results. It is not necessary to reset the random number generator to simulate scenarios.

\section*{Obtain the Portfolio for Targeted Portfolio Returns for a PortfolioMAD Object}

To obtain efficient portfolios that have targeted portfolio returns, the estimateFrontierByReturn function accepts one or more target portfolio returns and obtains efficient portfolios with the specified returns. Assume you have a universe of four assets where you want to obtain efficient portfolios with target portfolio returns of 7\%, \(10 \%\), and \(13 \%\).
m = [ 0.05; 0.1; 0.12; 0.18];
\(C=[0.00640 .004080 .001920 ;\)
0.004080 .02890 .02040 .0119 ;
\(0.001920 .02040 .05760 .0336 ;\)
00.01190 .03360 .1225 ];
rng(11);
p = PortfolioMAD;
p = simulateNormalScenariosByMoments(p, m, C, 2000);
\(\mathrm{p}=\) setDefaultConstraints(p);
pwgt = estimateFrontierByReturn(p, [0.07 0.10, 0.13]);
display(pwgt);
pwgt \(=4 \times 3\)
\begin{tabular}{lll}
0.7436 & 0.3147 & 0.0000 \\
0.1357 & 0.3835 & 0.4422 \\
0.0328 & 0.0939 & 0.1324 \\
0.0879 & 0.2079 & 0.4254
\end{tabular}

The function \(\mathrm{rng}\left({ }^{\text {seed }}\right)\) is used to reset the random number generator to produce the documented results. It is not necessary to reset the random number generator to simulate scenarios.

\section*{Input Arguments}

\section*{obj - Object for portfolio \\ object}

Object for portfolio, specified using Portfolio, PortfolioCVaR, or PortfolioMAD object. For more information on creating a portfolio object, see
- Portfolio
- PortfolioCVaR
- PortfolioMAD

\section*{Data Types: object}

\section*{TargetReturn - Target values for portfolio return \\ vector}

Target values for portfolio return, specified as a NumPorts vector.

Note TargetReturn specifies target returns for portfolios on the efficient frontier. If any TargetReturn values are outside the range of returns for efficient portfolios, the TargetReturn is replaced with the minimum or maximum efficient portfolio return, depending upon whether the target return is below or above the range of efficient portfolio returns.

Data Types: double

\section*{Output Arguments}
pwgt - Optimal portfolios on efficient frontier with specified target returns matrix

Optimal portfolios on the efficient frontier with specified target returns from TargetReturn, returned as a NumAssets-by-NumPorts matrix. pwgt is returned for a Portfolio, PortfolioCVaR, or PortfolioMAD input object (obj).

\section*{pbuy - Purchases relative to initial portfolio for optimal portfolios on efficient frontier}
matrix
Purchases relative to an initial portfolio for optimal portfolios on the efficient frontier, returned as NumAssets-by-NumPorts matrix.

Note If no initial portfolio is specified in obj. InitPort, that value is assumed to be 0 such that pbuy \(=\max (0, \mathrm{pwgt})\) and psell \(=\max (0,-\mathrm{pwg})\).
pbuy is returned for a Portfolio, PortfolioCVaR, or PortfolioMAD input object (obj).

\section*{psell - Sales relative to initial portfolio for optimal portfolios on efficient frontier}
matrix
Sales relative to an initial portfolio for optimal portfolios on the efficient frontier, returned as a NumAssets-by-NumPorts matrix.

Note If no initial portfolio is specified in obj. InitPort, that value is assumed to be 0 such that pbuy \(=\max (0, \mathrm{pwgt})\) and psell \(=\max (0,-\) pwgt \()\).
psell is returned for Portfolio, PortfolioCVaR, or PortfolioMAD input object (obj).

\section*{Tips}

You can also use dot notation to estimate optimal portfolios with targeted portfolio returns.
```

[pwgt, pbuy, psell] = obj.estimateFrontierByReturn(TargetReturn);

```

\section*{See Also}
estimateFrontier|estimateFrontierByRisk|estimateFrontierLimits | setBounds| setMinMaxNumAssets
Topics"Estimate Efficient Portfolios for Entire Efficient Frontier for Portfolio Object" on page 4-115
"Estimate Efficient Frontiers for Portfolio Object" on page 4-140
"Estimate Efficient Portfolios for Entire Frontier for PortfolioCVaR Object" on page 5-101
"Estimate Efficient Frontiers for PortfolioCVaR Object" on page 5-118"Estimate Efficient Portfolios Along the Entire Frontier for PortfolioMAD Object" on page6-96
"Estimate Efficient Frontiers for PortfolioMAD Object" on page 6-111
"Portfolio Optimization Examples" on page 4-167
"Portfolio Optimization Theory" on page 4-3
Introduced in R2011a

\section*{estimateFrontierByRisk}

Estimate optimal portfolios with targeted portfolio risks

\section*{Syntax}
[pwgt, pbuy, psell] = estimateFrontierByRisk(obj,TargetRisk)
[pwgt,pbuy,psell] = estimateFrontierByRisk(___, Name,Value)

\section*{Description}
[pwgt,pbuy,psell] = estimateFrontierByRisk(obj,TargetRisk) estimates optimal portfolios with targeted portfolio risks for Portfolio, PortfolioCVaR, or PortfolioMAD objects. For details on the respective workflows when using these different objects, see "Portfolio Object Workflow" on page 4-21, "PortfolioCVaR Object Workflow" on page 5-20, and "PortfolioMAD Object Workflow" on page 6-19.
[pwgt,pbuy, psell] = estimateFrontierByRisk( __ ,Name,Value) adds nameoptional name-value pair arguments for Portfolio or PortfolioMAD objects.

\section*{Examples}

\section*{Obtain Portfolios with Targeted Portfolio Risks for a Portfolio Object}

To obtain efficient portfolios that have targeted portfolio risks, the estimateFrontierByRisk function accepts one or more target portfolio risks and obtains efficient portfolios with the specified risks. Assume you have a universe of four assets where you want to obtain efficient portfolios with target portfolio risks of \(12 \%\), \(14 \%\), and \(16 \%\). This example uses the default 'direct ' method to estimate the optimal portfolios with targeted portfolio risks.
```

m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
0.00408 0.0289 0.0204 0.0119;

```
```

    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
    p = Portfolio;
p = setAssetMoments(p, m, C);
p = setDefaultConstraints(p);
pwgt = estimateFrontierByRisk(p, [0.12, 0.14, 0.16]);
display(pwgt);
pwgt = 4×3

```
\begin{tabular}{lll}
0.3984 & 0.2659 & 0.1416 \\
0.3064 & 0.3791 & 0.4474 \\
0.0882 & 0.1010 & 0.1131 \\
0.2071 & 0.2540 & 0.2979
\end{tabular}

\section*{Obtain Portfolios with Targeted Portfolio Risks for a Portfolio Object with BoundType, MinNumAsset, and MaxNumAsset Constraints}

When any one, or any combination of the constraints from 'Conditional ' BoundType, MinNumAssets, and MaxNumAssets are active, the portfolio problem is formulated as mixed integer programming problem and the MINLP solver is used.

Create a Portfolio object for three assets.
```

AssetMean = [ 0.0101110; 0.0043532; 0.0137058 ];
AssetCovar = [ 0.00324625 0.00022983 0.00420395;
0.00022983 0.00049937 0.00019247;
0.00420395 0.00019247 0.00764097 ];
p = Portfolio('AssetMean', AssetMean, 'AssetCovar', AssetCovar);
p = setDefaultConstraints(p);

```

Use setBounds with semicontinuous constraints to set \(x i=0\) or \(0.02<=x i<=0.5\) for all \(i=1, \ldots\) NumAssets.
```

p = setBounds(p, 0.02, 0.7,'BoundType', 'Conditional', 'NumAssets', 3);

```

When working with a Portfolio object, the setMinMaxNumAssets function enables you to set up the limits on the number of assets invested (as known as cardinality) constraints. This sets the total number of allocated assets satisfying the Bound
constraints that are between MinNumAssets and MaxNumAssets. By setting MinNumAssets \(=\) MaxNumAssets \(=2\), only two of the three assets are invested in the portfolio.
```

p = setMinMaxNumAssets(p, 2, 2);

```

Use estimateFrontierByRisk to estimate optimal portfolios with targeted portfolio risks.
```

[pwgt, pbuy, psell] = estimateFrontierByRisk(p,[0.0324241, 0.0694534 ])

```
pwgt \(=3 \times 2\)
    \(0.0000 \quad 0.5000\)
    0.6907
        0
        \(0.3093 \quad 0.5000\)
pbuy \(=3 \times 2\)
    \(0.0000 \quad 0.5000\)
    \(0.6907 \quad 0\)
    \(0.3093 \quad 0.5000\)
psell \(=3 \times 2\)
    \(0 \quad 0\)
    \(0 \quad 0\)
    \(0 \quad 0\)

The estimateFrontierByRisk function uses the MINLP solver to solve this problem. Use the setSolverMINLP function to configure the SolverType and options.
p.solverTypeMINLP
ans =
'OuterApproximation'
p.solverOptionsMINLP
ans \(=\) struct with fields:
MaxIterations: 1000
AbsoluteGapTolerance: 1.0000e-07
RelativeGapTolerance: 1.0000e-05
```

    NonlinearScalingFactor: 1000
    ObjectiveScalingFactor: 1000
    IntMasterSolverOptions: [1x1 optim.options.Intlinprog]
    Display: 'off'
    CutGeneration: 'basic'
    MaxIterationsInactiveCut: 30
    ActiveCutTolerance: 1.0000e-07
    NumIterationsEarlyIntegerConvergence: 30

```

\section*{Obtain Portfolios with Targeted Portfolio Risks for a Portfolio Object Using the Direct Method and Solver Options}

To obtain efficient portfolios that have targeted portfolio risks, the estimateFrontierByRisk function accepts one or more target portfolio risks and obtains efficient portfolios with the specified risks. Assume you have a universe of four assets where you want to obtain efficient portfolios with target portfolio risks of \(12 \%\), \(14 \%\), and \(16 \%\). This example uses the default 'direct ' method to estimate the optimal portfolios with targeted portfolio risks. The 'direct' method uses fmincon to solve the optimization problem that maximizes portfolio return, subject to the target risk as the quadratic nonlinear constraint. setSolver specifies the solverType and SolverOptions for fmincon.
```

m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
0.00408 0.0289 0.0204 0.0119;
0.00192 0.0204 0.0576 0.0336;
0 0.0119 0.0336 0.1225 ];
p = Portfolio;
p = setAssetMoments(p, m, C);
p = setDefaultConstraints(p);
p = setSolver(p, 'fmincon', 'Display', 'off', 'Algorithm', 'sqp', ...
'SpecifyObjectiveGradient', true, 'SpecifyConstraintGradient', true, ...
'ConstraintTolerance', 1.0e-8, 'OptimalityTolerance', 1.0e-8, 'StepTolerance',
pwgt = estimateFrontierByRisk(p, [0.12, 0.14, 0.16]);
display(pwgt);

```
```

pwgt = 4×3

```
\begin{tabular}{lll}
0.3984 & 0.2659 & 0.1416 \\
0.3064 & 0.3791 & 0.4474 \\
0.0882 & 0.1010 & 0.1131 \\
0.2071 & 0.2540 & 0.2979
\end{tabular}

\section*{Obtain Portfolios with Targeted Portfolio Risks for a PortfolioCVaR Object}

To obtain efficient portfolios that have targeted portfolio risks, the estimateFrontierByRisk function accepts one or more target portfolio risks and obtains efficient portfolios with the specified risks. Assume you have a universe of four assets where you want to obtain efficient portfolios with target portfolio risks of \(12 \%\), \(20 \%\), and \(30 \%\).
```

m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
0.00408 0.0289 0.0204 0.0119;
0.00192 0.0204 0.0576 0.0336;
0 0.0119 0.0336 0.1225 ];
rng(11);
p = PortfolioCVaR;
p = simulateNormalScenariosByMoments(p, m, C, 2000);
p = setDefaultConstraints(p);
p = setProbabilityLevel(p, 0.95);
pwgt = estimateFrontierByRisk(p, [0.12, 0.20, 0.30]);
display(pwgt);
pwgt = 4×3
0.5363 0.1388 0
0.2655 0.4990 0.3830
0.0568 0.1239 0.1461
0.1413 0.2383 0.4709

```

The function rng(seed \()\) resets the random number generator to produce the documented results. It is not necessary to reset the random number generator to simulate scenarios.

\section*{Obtain Portfolios with Targeted Portfolio Risks for a PortfolioMAD Object}

To obtain efficient portfolios that have targeted portfolio risks, the estimateFrontierByRisk function accepts one or more target portfolio risks and obtains efficient portfolios with the specified risks. Assume you have a universe of four assets where you want to obtain efficient portfolios with target portfolio risks of \(12 \%\), \(20 \%\), and \(25 \%\). This example uses the default ' direct' method to estimate the optimal portfolios with targeted portfolio risks.
```

m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
0.00408 0.0289 0.0204 0.0119;
0.00192 0.0204 0.0576 0.0336;
0 0.0119 0.0336 0.1225 ];
rng(11);
p = PortfolioMAD;
p = simulateNormalScenariosByMoments(p, m, C, 2000);
p = setDefaultConstraints(p);
pwgt = estimateFrontierByRisk(p, [0.12, 0.20, 0.25]);
display(pwgt);
pwgt = 4×3

| 0.1613 | 0.0000 | 0.0000 |
| :--- | :--- | :--- |
| 0.4777 | 0.2139 | 0.0037 |
| 0.1118 | 0.1381 | 0.1214 |
| 0.2492 | 0.6480 | 0.8749 |

```

The function \(\mathrm{rng}\left({ }^{\text {seed }}\right)\) resets the random number generator to produce the documented results. It is not necessary to reset the random number generator to simulate scenarios.

\section*{Obtain Portfolios with Targeted Portfolio Risks for a PortfolioMAD Object Using the Direct Method and Solver Options}

To obtain efficient portfolios that have targeted portfolio risks, the estimateFrontierByRisk function accepts one or more target portfolio risks and obtains efficient portfolios with the specified risks. Assume you have a universe of four assets where you want to obtain efficient portfolios with target portfolio risks of \(12 \%\), \(20 \%\), and \(25 \%\). This example uses the default ' direct ' method to estimate the optimal portfolios with targeted portfolio risks. The 'direct' method uses fmincon to solve the optimization problem that maximizes portfolio return, subject to the target risk as the quadratic nonlinear constraint. setSolver specifies the solverType and SolverOptions for fmincon.
```

m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
0.00408 0.0289 0.0204 0.0119;
0.00192 0.0204 0.0576 0.0336;
0 0.0119 0.0336 0.1225 ];
rng(11);
p = PortfolioMAD;
p = simulateNormalScenariosByMoments(p, m, C, 2000);
p = setDefaultConstraints(p);
p = setSolver(p, 'fmincon', 'Display', 'off', 'Algorithm', 'sqp', ...
'SpecifyObjectiveGradient', true, 'SpecifyConstraintGradient', true, ...
'ConstraintTolerance', 1.0e-8, 'OptimalityTolerance', 1.0e-8,''StepTolerance',
plotFrontier(p);

```

pwgt = estimateFrontierByRisk(p, [0.12 0.20, 0.25]);
display(pwgt);
pwgt \(=4 \times 3\)
\begin{tabular}{lrr}
0.1613 & 0 & 0 \\
0.4777 & 0.2139 & 0.0037 \\
0.1118 & 0.1381 & 0.1214 \\
0.2492 & 0.6480 & 0.8749
\end{tabular}

\section*{Input Arguments}

\section*{obj - Object for portfolio}
object
Object for portfolio, specified using Portfolio, PortfolioCVaR, or PortfolioMAD object. For more information on creating a portfolio object, see
- Portfolio
- PortfolioCVaR
- PortfolioMAD

Note If no initial portfolio is specified in obj. InitPort, it is assumed to be 0 so that pbuy \(=\max (0, \mathrm{pwgt})\) and psell \(=\max (0,-\mathrm{pwgt})\). For more information on setting an initial portfolio, see setInitPort.

\section*{Data Types: object}

\section*{TargetRisk - Target values for portfolio risk \\ vector}

Target values for portfolio risk, specified as a NumPorts vector.

Note If any TargetRisk values are outside the range of risks for efficient portfolios, the target risk is replaced with the minimum or maximum efficient portfolio risk, depending on whether the target risk is below or above the range of efficient portfolio risks.

\section*{Data Types: double}

\section*{Name-Value Pair Arguments}

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Example: [pwgt,pbuy,psell] = estimateFrontierByRisk(p,'method’, 'direct')

\section*{Method - Method to estimate frontier by risk for Portfolio or PortfolioMAD objects \\ ```
'direct' (default)| character vector with value 'direct' or 'iterative'
```}

Method to estimate frontier by risk for Portfolio or PortfolioMAD objects, specified as the comma-separated pair consisting of 'Method ' and a character vector with one of the following values:
- 'direct ' - Construct one optimization problem to maximize the portfolio return with target risk as the nonlinear constraint, and solve it directly using fmincon, instead of iteratively exploring the efficient frontier. For an example of using the 'direct' option, see "Obtain Portfolios with Targeted Portfolio Risks for a Portfolio Object Using the Direct Method and Solver Options" on page 19-853 and "Obtain Portfolios with Targeted Portfolio Risks for a PortfolioMAD Object Using the Direct Method and Solver Options" on page 19-855.
- 'iterative' - One-dimensional optimization using fminbnd to find the portfolio return between min and max return on the frontier that minimizes the difference between the actual risk and target risk. Then the portfolio weights are obtained by solving a frontier by return problem. Consequently, the 'iterative' method is slower than the 'direct'.

\section*{Data Types: char}

\section*{Output Arguments}

\section*{pwgt - Optimal portfolios on efficient frontier with specified target risks matrix}

Optimal portfolios on the efficient frontier with specified target returns from TargetRisk, returned as a NumAssets-by-NumPorts matrix. pwgt is returned for a Portfolio, PortfolioCVaR, or PortfolioMAD input object (obj).

\section*{pbuy - Purchases relative to initial portfolio for optimal portfolios on efficient frontier}
matrix
Purchases relative to an initial portfolio for optimal portfolios on the efficient frontier, returned as NumAssets-by-NumPorts matrix.

Note If no initial portfolio is specified in obj. InitPort, that value is assumed to be 0 such that pbuy \(=\max (0, \mathrm{pwgt})\) and psell \(=\max (0,-\mathrm{pwgt})\).
pbuy is returned for a Portfolio, PortfolioCVaR, or PortfolioMAD input object (obj).

\section*{psell - Sales relative to initial portfolio for optimal portfolios on efficient frontier}
matrix
Sales relative to an initial portfolio for optimal portfolios on the efficient frontier, returned as a NumAssets-by-NumPorts matrix.

Note If no initial portfolio is specified in obj. InitPort, that value is assumed to be 0 such that pbuy \(=\max (0, \mathrm{pwgt})\) and psell \(=\max (0,-\mathrm{pwg} \mathrm{t})\).
psell is returned for Portfolio, PortfolioCVaR, or PortfolioMAD input object (obj).

\section*{Tips}

You can also use dot notation to estimate optimal portfolios with targeted portfolio risks.
```

[pwgt,pbuy,psell] = obj.estimateFrontierByRisk(TargetRisk);

```
or
```

[pwgt,pbuy,psell] = obj.estimateFrontierByRisk(TargetRisk,Name,Value);

```

\section*{See Also}
estimateFrontier|estimateFrontierByReturn|estimateFrontierLimits | rng| setBounds| setInitPort| setMinMaxNumAssets

\section*{Topics}
"Estimate Efficient Portfolios for Entire Efficient Frontier for Portfolio Object" on page 4115
"Estimate Efficient Frontiers for Portfolio Object" on page 4-140
"Estimate Efficient Portfolios for Entire Frontier for PortfolioCVaR Object" on page 5-101
"Estimate Efficient Frontiers for PortfolioCVaR Object" on page 5-118
"Estimate Efficient Portfolios Along the Entire Frontier for PortfolioMAD Object" on page 6-96
"Estimate Efficient Frontiers for PortfolioMAD Object" on page 6-111
"Portfolio Optimization Examples" on page 4-167
"Portfolio Optimization Theory" on page 4-3

\section*{Introduced in R2011a}

\section*{estimateFrontierLimits}

Estimate optimal portfolios at endpoints of efficient frontier

\section*{Syntax}
[pwgt,pbuy,psell] = estimateFrontierLimits(obj)
[pwgt, pbuy,psell] = estimateFrontierLimits(obj,Choice)

\section*{Description}
[pwgt,pbuy,psell] = estimateFrontierLimits(obj) estimates optimal portfolios at endpoints of efficient frontier for Portfolio, PortfolioCVaR, or PortfolioMAD objects. For details on the respective workflows when using these different objects, see "Portfolio Object Workflow" on page 4-21, "PortfolioCVaR Object Workflow" on page 5-20, and "PortfolioMAD Object Workflow" on page 6-19.
[pwgt, pbuy, psell] = estimateFrontierLimits(obj,Choice) estimates optimal portfolios at endpoints of efficient frontier with an additional option specified for the Choice argument.

\section*{Examples}

\section*{Obtain Endpoint Portfolios for a Portfolio Object}

Given portfolio \(p\), the estimateFrontierLimits function obtains the endpoint portfolios.
```

m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
0.00408 0.0289 0.0204 0.0119;
0.00192 0.0204 0.0576 0.0336;
0 0.0119 0.0336 0.1225 ];
p = Portfolio;

```
```

p = setAssetMoments(p, m, C);
p = setDefaultConstraints(p);
pwgt = estimateFrontierLimits(p);
disp(pwgt);
0.8891 0
0.0369 0
0.0404 0
0.0336 1.0000

```

\section*{Obtain Endpoint Portfolios for a Portfolio Object with BoundType, MinNumAsset, and MaxNumAsset Constraints}

When any one, or any combination of the constraints from 'Conditional ' BoundType, MinNumAssets, and MaxNumAssets are active, the portfolio problem is formulated as mixed integer programming problem and the MINLP solver is used.

Create a Portfolio object for three assets.
```

AssetMean = [ 0.0101110; 0.0043532; 0.0137058 ];
AssetCovar = [ 0.00324625 0.00022983 0.00420395;
0.00022983 0.00049937 0.00019247;
0.00420395 0.00019247 0.00764097 ];
p = Portfolio('AssetMean', AssetMean, 'AssetCovar', AssetCovar);
p = setDefaultConstraints(p);

```

Use setBounds with semicontinuous constraints to set xi=0 or \(0.02<=x i<=0.5\) for all \(i=1\),...NumAssets.
p = setBounds(p, 0.02, 0.7,'BoundType', 'Conditional', 'NumAssets', 3);
When working with a Portfolio object, the setMinMaxNumAssets function enables you to set up the limits on the number of assets invested (as known as cardinality) constraints. This sets the total number of allocated assets satisfying the Bound constraints that are between MinNumAssets and MaxNumAssets. By setting MinNumAssets \(=\) MaxNumAssets \(=2\), only two of the three assets are invested in the portfolio.
```

p = setMinMaxNumAssets(p, 2, 2);

```

Use estimateFrontierLimits to estimate the optimal portfolios at endpoints of the efficient frontier.
```

[pwgt, pbuy, psell] = estimateFrontierLimits(p,'Both')
pwgt = 3\times2
0.3000 0.3000
0.7000 0
0.7000
pbuy = 3\times2
0.3000 0.3000
0.7000 0
0.7000
psell = 3\times2
0
0}
0}

```

The estimateFrontierLimits function uses the MINLP solver to solve this problem. Use the setSolverMINLP function to configure the SolverType and options.
```

p.solverTypeMINLP
ans =
'OuterApproximation'
p.solverOptionsMINLP
ans = struct with fields:
MaxIterations: 1000
AbsoluteGapTolerance: 1.0000e-07
RelativeGapTolerance: 1.0000e-05
NonlinearScalingFactor: 1000
ObjectiveScalingFactor: 1000
IntMasterSolverOptions: [1x1 optim.options.Intlinprog]
Display: 'off'
CutGeneration: 'basic'
MaxIterationsInactiveCut: 30

```

ActiveCutTolerance: 1.0000e-07
NumIterationsEarlyIntegerConvergence: 30

\section*{Obtain Endpoint Portfolios for a PortfolioCVaR Object}

Given portfolio \(p\), the estimateFrontierLimits function obtains the endpoint portfolios.
```

m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
0.00408 0.0289 0.0204 0.0119;
0.00192 0.0204 0.0576 0.0336;
0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;

```
rng(11);
AssetScenarios = mvnrnd(m, C, 20000);
p = PortfolioCVaR;
\(\mathrm{p}=\) setScenarios(p, AssetScenarios);
\(p=\) setDefaultConstraints(p);
p = setProbabilityLevel(p, 0.95);
pwgt = estimateFrontierLimits(p);
disp(pwgt);
\begin{tabular}{ll}
0.8454 & 0.0000 \\
0.0599 & 0.0000 \\
0.0462 & 0.0000 \\
0.0485 & 1.0000
\end{tabular}

The function rng(seed \()\) resets the random number generator to produce the documented results. It is not necessary to reset the random number generator to simulate scenarios.

\section*{Obtain Endpoint Portfolios for a PortfolioMAD Object}

Given portfolio \(p\), the estimateFrontierLimits function obtains the endpoint portfolios.
```

m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
0.00408 0.0289 0.0204 0.0119;
0.00192 0.0204 0.0576 0.0336;
0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;

```
rng(11);
AssetScenarios \(=\) mvnrnd(m, C, 20000);
p = PortfolioMAD;
\(\mathrm{p}=\) setScenarios( p , AssetScenarios);
\(p\) = setDefaultConstraints(p);
pwgt = estimateFrontierLimits(p);
disp(pwgt);
    0.88150 .0000
    \(0.0431 \quad 0.0000\)
    \(0.0389 \quad 0.0000\)
    \(0.0365 \quad 1.0000\)

The function rng \(\left({ }^{\text {seed }}\right)\) resets the random number generator to produce the documented results. It is not necessary to reset the random number generator to simulate scenarios.

\section*{Input Arguments}

\section*{obj - Object for portfolio}
object
Object for portfolio, specified using Portfolio, PortfolioCVaR, or PortfolioMAD object. For more information on creating a portfolio object, see
- Portfolio
- PortfolioCVaR
- PortfolioMAD

Data Types: object

\section*{Choice - Indicator for which portfolios to obtain at extreme ends of efficient frontier}
[] (default) | character vector with values 'Both', 'Min', 'Max' | string with values "Both", "Min", "Max"
(Optional) Indicator which portfolios to obtain at the extreme ends of the efficient frontier, specified as a character vector with values 'Both' or "Both", 'Min' or "Min", or 'Max' or "Max". The options for a Choice action are:
- [] - Compute both minimum-risk and maximum-return portfolios.
- 'Both' or "Both" - Compute both minimum-risk and maximum-return portfolios.
- 'Min' or "Min" - Compute minimum-risk portfolio only.
- 'Max' or "Max" - Compute maximum-return portfolio only.

Data Types: char|string

\section*{Output Arguments}

\section*{pwgt - Optimal portfolios at endpoints of efficient frontier \\ matrix}

Optimal portfolios at the endpoints of the efficient frontier TargetReturn, returned as a NumAssets-by-NumPorts matrix. pwgt is returned for a Portfolio, PortfolioCVaR, or PortfolioMAD input object (obj).

\section*{pbuy - Purchases relative to an initial portfolio for optimal portfolios at endpoints of efficient frontier}
matrix
Purchases relative to an initial portfolio for optimal portfolios at the endpoints of the efficient frontier, returned as NumAssets-by-NumPorts matrix.

Note If no initial portfolio is specified in obj. InitPort, that value is assumed to be 0 such that pbuy \(=\max (0, \mathrm{pwgt})\) and psell \(=\max (0,-\mathrm{pwg})\).
pbuy is returned for a Portfolio, PortfolioCVaR, or PortfolioMAD input object (obj).

\section*{psell - Sales relative to an initial portfolio for optimal portfolios at endpoints of efficient frontier}

\section*{matrix}

Sales relative to an initial portfolio for optimal portfolios on the efficient frontier, returned as a NumAssets-by-NumPorts matrix.

Note If no initial portfolio is specified in obj. InitPort, that value is assumed to be 0 \(\underline{\text { such that pbuy }=\max (0, ~ p w g t) ~ a n d ~ p s e l l ~}=\max (0,-\mathrm{pwgt})\).
psell is returned for Portfolio, PortfolioCVaR, or PortfolioMAD input object (obj).

\section*{Tips}

You can also use dot notation to estimate the optimal portfolios at the endpoints of the efficient frontier.
```

[pwgt, pbuy, psell] = obj.estimateFrontierLimits(Choice);

```

\section*{See Also}
estimateFrontier|estimateFrontierByReturn|estimateFrontierByRisk| rng|setBounds| setMinMaxNumAssets

\section*{Topics}
"Estimate Efficient Portfolios for Entire Efficient Frontier for Portfolio Object" on page 4115
"Estimate Efficient Frontiers for Portfolio Object" on page 4-140
"Estimate Efficient Portfolios for Entire Frontier for PortfolioCVaR Object" on page 5-101

\footnotetext{
"Estimate Efficient Frontiers for PortfolioCVaR Object" on page 5-118
"Estimate Efficient Portfolios Along the Entire Frontier for PortfolioMAD Object" on page 6-96
"Estimate Efficient Frontiers for PortfolioMAD Object" on page 6-111
"Portfolio Optimization Examples" on page 4-167
"Portfolio Optimization Theory" on page 4-3

Introduced in R2011a
}

\section*{estimateMaxSharpeRatio}

Estimate efficient portfolio to maximize Sharpe ratio for Portfolio object

\section*{Syntax}
[pwgt,pbuy,psell] = estimateMaxSharpeRatio(obj)
[pwgt,pbuy,psell] = estimateMaxSharpeRatio( \(\qquad\) ,Name, Value)

\section*{Description}
[pwgt,pbuy,psell] = estimateMaxSharpeRatio(obj) estimates efficient portfolio to maximize Sharpe ratio for Portfolio object. For details on the workflow, see "Portfolio Object Workflow" on page 4-21.
[pwgt,pbuy,psell] = estimateMaxSharpeRatio( \(\qquad\) ,Name, Value) adds optional name-value pair arguments.

\section*{Examples}

\section*{Estimate Efficient Portfolio that Maximizes the Sharpe Ratio for a Portfolio Object}

Estimate the efficient portfolio that maximizes the Sharpe ratio. The estimateMaxSharpeRatio function maximizes the Sharpe ratio among portfolios on the efficient frontier. This example uses the default 'direct ' method to estimate the maximum Sharpe ratio. For more information on the 'direct' method, see "Algorithms" on page 19-880.
```

p = Portfolio('AssetMean',[0.3, 0.1, 0.5], 'AssetCovar',...
[0.01, -0.010, 0.004; -0.010, 0.040, -0.002; 0.004, -0.002, 0.023]);
p = setDefaultConstraints(p);
plotFrontier(p, 20);
weights = estimateMaxSharpeRatio(p);
[risk, ret] = estimatePortMoments(p, weights);

```
hold on
plot(risk,ret,'*r');


\section*{Estimate Efficient Portfolio that Maximizes the Sharpe Ratio for a Portfolio Object Using Solver Options with the Direct Method}

Estimate the efficient portfolio that maximizes the Sharpe ratio. The estimateMaxSharpeRatio function maximizes the Sharpe ratio among portfolios on the efficient frontier. This example uses the 'direct ' method for a Portfolio object (p) that does not specify a tracking error and only uses linear constraints. The setSolver function is used to control the SolverType and SolverOptions. In this case, the

SolverType is quadprog. For more information on the 'direct' method, see "Algorithms" on page 19-880.
```

p = Portfolio('AssetMean',[0.3, 0.1, 0.5], 'AssetCovar',...
[0.01, -0.010, 0.004; -0.010, 0.040, -0.002; 0.004, -0.002, 0.023]);
p = setDefaultConstraints(p);
plotFrontier(p, 20);
p = setSolver(p,'quadprog','Display','off','ConstraintTolerance',1.0e-8,'0ptimalityTole
weights = estimateMaxSharpeRatio(p);
[risk, ret] = estimatePortMoments(p, weights);
hold on
plot(risk,ret,'*r');

```

Efficient Frontier


\section*{Estimate Efficient Portfolio that Maximizes the Sharpe Ratio for a Portfolio Object With Tracking Error Using the Direct Method and Solver Options}

Estimate the efficient portfolio that maximizes the Sharpe ratio. The estimateMaxSharpeRatio function maximizes the Sharpe ratio among portfolios on the efficient frontier. This example uses the 'direct' method for a Portfolio object (p) that specifies a tracking error uses nonlinear constraints. The setSolver function is used to control the SolverType and SolverOptions. In this case fmincon is the SolverType.
```

p = Portfolio('AssetMean',[0.3, 0.1, 0.5], 'AssetCovar',...
[0.01, -0.010, 0.004; -0.010, 0.040, -0.002; 0.004, -0.002, 0.023],'lb', 0,'budget
plotFrontier(p, 20);
p = setSolver(p, 'fmincon', 'Display', 'off', 'Algorithm', 'sqp', ...
'SpecifyObjectiveGradient', true, 'SpecifyConstraintGradient', true, ...
'ConstraintTolerance', 1.0e-8, 'OptimalityTolerance', 1.0e-8, 'StepTolerance',
weights = estimateMaxSharpeRatio(p);
te = 0.08;
p = setTrackingError(p,te,weights);
[risk, ret] = estimatePortMoments(p,weights);
hold on
plot(risk,ret,'*r');

```


\section*{Estimate Efficient Portfolio that Maximizes the Sharpe Ratio for a Portfolio Object With a Risk-Free Asset Using the Direct and Iterative Method}

The estimateMaxSharpeRatio function maximizes the Sharpe ratio among portfolios on the efficient frontier. In the case of Portfolio with a risk-free asset, there are multiple efficient portfolios that maximize the Sharpe ratio on the capital asset line. Because of the nature of 'direct' and 'iterative' methods, the portfolio weights (pwgts) output from each of these methods might be different, but the Sharpe ratio is the same. This example demonstrates the scenario where the pwgts are different and the Sharpe ratio is the same.
```

load BlueChipStockMoments
mret = MarketMean;
mrsk = sqrt(MarketVar);
cret = CashMean;
crsk = sqrt(CashVar);
p = Portfolio('AssetList', AssetList, 'RiskFreeRate', CashMean);
p = setAssetMoments(p, AssetMean, AssetCovar);
p = setInitPort(p, 1/p.NumAssets);
[ersk, eret] = estimatePortMoments(p, p.InitPort);
p = setDefaultConstraints(p);
pwgt = estimateFrontier(p, 20);
[prsk, pret] = estimatePortMoments(p, pwgt);
pwgtshpr_fully = estimateMaxSharpeRatio(p,'Method','direct');
[riskshpr_fully, retshpr_fully] = estimatePortMoments(p,pwgtshpr_fully);
q = setBudget(p, 0, 1);
qwgt = estimateFrontier(q, 20);
[qrsk, qret] = estimatePortMoments(q, qwgt);

```

Plot the efficient frontier with a tangent line (0 to 1 cash).
```

pwgtshpr_direct = estimateMaxSharpeRatio(q,'Method','direct');
pwgtshpr iter = estimateMaxSharpeRatio(q,'Method','iterative');
[riskshpr diret, retshpr diret] = estimatePortMoments(q,pwgtshpr direct);
[riskshpr_iter, retshpr_íter] = estimatePortMoments(q,pwgtshpr_i\overline{ter);}
clf;
portfolioexamples_plot('Efficient Frontier with Capital Allocation Line', ...
{'line', prsk, pret, {'EF'}, '-r', 2}, ...
{'line', qrsk, qret, {'EF with riskfree'}, '-b', 1}, ...
{'scatter', [mrsk, crsk, ersk, riskshpr_fully, riskshpr_diret, riskshp।
[mret, cret, eret, retshpr_fully , retshpr_diret, rētshpr_iter], {'Market', 'Cash'
{'scatter', sq\overline{r}t(diag(p.AssetCōvar)), p.AssetM

```


When a risk-free asset is not available to the portfolio, or in other words, the portfolio is fully invested, the efficient frontier is curved, corresponding to the red line in the above figure. Therefore, there is a unique (risk, return) point that maximizes the Sharpe ratio, which the 'iterative' and 'direct' methods will both find. If the portfolio is allowed to invest in a risk-free asset, part of the red efficient frontier line is replaced by the capital allocation line, resulting in the efficient frontier of a portfolio with a risk-free investment (blue line). All the (risk, return) points on the straight blue line share the same Sharpe ratio. Also, it is likely that the 'iterative' and 'direct' methods end up with different points, therefore there are different portfolio allocations.

\section*{Estimate Efficient Portfolio that Maximizes the Sharpe Ratio for a Portfolio Object with Semicontinuous and Cardinality Constraints}

Create a Portfolio object for three assets.
```

AssetMean = [ 0.0101110; 0.0043532; 0.0137058 ];
AssetCovar = [ 0.00324625 0.00022983 0.00420395;
0.00022983 0.00049937 0.00019247;
0.00420395 0.00019247 0.00764097 ];
p = Portfolio('AssetMean', AssetMean, 'AssetCovar', AssetCovar);
p = setDefaultConstraints(p);

```

Use setBounds with semicontinuous constraints to set \(x i=0\) or \(0.02<=x i<=0.5\) for all \(i=1, .\). NumAssets.
```

p = setBounds(p, 0.02, 0.5,'BoundType', 'Conditional', 'NumAssets', 3);

```

When working with a Portfolio object, the setMinMaxNumAssets function enables you to set up cardinality constraints for a long-only portfolio. This sets the cardinality constraints for the Portfolio object, where the total number of allocated assets satisfying the nonzero semi-continuous constraints are between MinNumAssets and MaxNumAssets. By setting MinNumAssets = MaxNumAssets = 2, only two of the three assets are invested in the portfolio.
```

p = setMinMaxNumAssets(p, 2, 2);

```

Use estimateMaxSharpeRatio to estimate efficient portfolio to maximize Sharpe ratio.
```

weights = estimateMaxSharpeRatio(p,'Method','iterative')
weights = 3x1

```
0.0000
0.5000
0.5000

The estimateMaxSharpeRatio function uses the MINLP solver to solve this problem. Use the setSolverMINLP function to configure the SolverType and options.
p.solverOptionsMINLP
ans \(=\) struct with fields:
MaxIterations: 1000
AbsoluteGapTolerance: 1.0000e-07
```

    RelativeGapTolerance: 1.0000e-05
    NonlinearScalingFactor: 1000
    ObjectiveScalingFactor: 1000
    IntMasterSolverOptions: [1x1 optim.options.Intlinprog]
            Display: 'off'
            CutGeneration: 'basic'
    MaxIterationsInactiveCut: 30
ActiveCutTolerance: 1.0000e-07
NumIterationsEarlyIntegerConvergence: 30

```

\section*{Input Arguments}

\section*{obj - Object for portfolio}
object
Object for portfolio, specified using a Portfolio object.

Note The risk-free rate is obtained from the property RiskFreeRate in the Portfolio object. If you leave the RiskFreeRate unset, it is assumed to be 0. If the max return of portfolio is less than the RiskFreeRate, the solution is set as pwgt at max return and the resulting Sharpe ratio will be negative.

For more information on creating a portfolio object, see
- Portfolio

Data Types: object

\section*{Name-Value Pair Arguments}

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Example: [pwgt,pbuy,psell] = estimateMaxSharpeRatio(p,'Method','iterative')

\section*{Method - Method to estimate Sharpe ratio}
'direct' (default) | character vector with value 'direct' or 'iterative'
Method to estimate Sharpe ratio, specified as the comma-separated pair consisting of 'Method ' and a character vector with one of the following values:
- 'direct' - Transform the Sharpe ratio function into a quadratic one and solve one optimization problem directly, instead of iteratively exploring the efficient frontier. The 'direct ' option either uses the solver quadprog (for a problem with linear constraints) or fmincon (for a problem with nonlinear constraints). For an example of using the 'direct ' option, see "Estimate Efficient Portfolio that Maximizes the Sharpe Ratio for a Portfolio Object Using Solver Options with the Direct Method" on page 19-871and "Estimate Efficient Portfolio that Maximizes the Sharpe Ratio for a Portfolio Object With Tracking Error Using the Direct Method and Solver Options" on page 19-873.
- 'iterative' - One-dimensional optimization using fminbnd to find the portfolio that maximizes the Sharpe ratio by iteratively exploring the efficient frontier.

Note If you are using estimateMaxSharpeRatio with a Portfolio object with semicontinuous and cardinality constraints specified by setBounds and setMinMaxNumAssets, you can only use the 'iterative' method.

\section*{Data Types: char}

\section*{Output Arguments}

\section*{pwgt - Portfolio on efficient frontier with maximum Sharpe ratio} vector

Portfolio on the efficient frontier with a maximum Sharpe ratio, returned as a NumAssets vector.

\section*{pbuy - Purchases relative to initial portfolio for portfolio on efficient frontier with maximum Sharpe ratio \\ vector}

Purchases relative to an initial portfolio for a portfolio on the efficient frontier with a maximum Sharpe ratio, returned as a NumAssets vector.
pbuy is returned for a Portfolio input object (obj).

\section*{psell - Sales relative to initial portfolio for portfolio on efficient frontier with maximum Sharpe ratio}
vector
Sales relative to an initial portfolio for a portfolio on the efficient frontier with maximum Sharpe ratio, returned as a NumAssets vector.
psell is returned for a Portfolio input object (obj).

\section*{Definitions}

\section*{Sharpe Ratio}

The Sharpe ratio is the ratio of the difference between the mean of portfolio returns and the risk-free rate divided by the standard deviation of portfolio returns.

The estimateMaxSharpeRation function maximizes the Sharpe ratio among portfolios on the efficient frontier.

\section*{Tips}

You can also use dot notation to estimate an efficient portfolio that maximizes the Sharpe ratio.
```

[pwgt,pbuy,psell] = obj.estimateMaxSharpeRatio;

```

\section*{Algorithms}

The maximization of the Sharpe ratio is accomplished by either using the 'direct ' or 'iterative' method. For the 'direct' method, consider the following scenario. To maximize the Sharpe ratio is to:
\[
\operatorname{Maximize} \frac{\mu^{T} x-r_{f}}{\sqrt{x^{T} C x}} \text {,s.t. } \sum x_{i}=1, \quad 0 \leq x_{i} \leq 1,
\]
where \(\mu\) and \(C\) are the mean and covariance matrix, and \(r_{f}\) is the risk-free rate.
If \(\mu^{T} \chi-r_{f} \leq 0\) for all \(x\) the portfolio that maximizes the Sharpe ratio is the one with maximum return.

If \(\mu^{T} x-r_{f}>0\), let \(t=\frac{1}{\mu^{T} x-r_{f}}\)
and \(y=t x\) (Cornuejols [1] section 8.2). Then after some substitutions, we can transform the original problem into the following form,
\[
\text { Minimize } y^{T} C y \text {, s.t. } \sum y_{i}=t, t>0,0 \leq y_{i} \leq \mathrm{t}, \mu^{T} y-r_{f} \mathrm{t}=1 \text {. }
\]

Only one optimization needs to be solved, hence the name "direct". The portfolio weights can be recovered by \(x^{*}=y^{*} / t^{*}\).

For the 'iterative' method, the idea is to iteratively explore the portfolios at different return levels on the efficient frontier and locate the one with maximum Sharpe ratio. Therefore, multiple optimization problems are solved during the process, instead of only one in the 'direct' method. Consequently, the 'iterative' method is slow compared to 'direct' method.

\section*{References}
[1] Cornuejols, G. and Reha Tütüncü. Optimization Methods in Finance. Cambridge University Press, 2007.

\section*{See Also}
estimateFrontier|estimateFrontierByReturn|estimateFrontierByRisk| estimatePortSharpeRatio| setBounds| setMinMaxNumAssets

\section*{Topics}
"Estimate Efficient Portfolios for Entire Efficient Frontier for Portfolio Object" on page 4115
"Working with 'Conditional' BoundType, MinNumAssets, and MaxNumAssets Constraints Using Portfolio Object" on page 4-94
"Portfolio Optimization Examples" on page 4-167
"Portfolio Optimization Theory" on page 4-3

\section*{Introduced in R2011a}

\section*{estimatePortSharpeRatio}

Estimate Sharpe ratio of given portfolio weights for Portfolio object

\section*{Syntax}
```

psharpe = estimatePortSharpeRatio(obj,pwgt)

```

\section*{Description}
psharpe = estimatePortSharpeRatio(obj, pwgt) estimates the Sharpe ratio of given portfolio weights for a Portfolio object. For details on the workflow, see "Portfolio Object Workflow" on page 4-21.

\section*{Examples}

\section*{Estimate Sharpe Ratios of the Given Portfolio Weights}

This example shows how to find efficient portfolios that satisfy the target returns and then find the Sharpe ratios corresponding to each of the portfolios.

To obtain efficient portfolios that have targeted portfolio returns, the estimateFrontierByReturn function accepts one or more target portfolio returns and obtains efficient portfolios with the specified returns. Assume you have a universe of four assets where you want to obtain efficient portfolios with target portfolio returns of \(6 \%\), \(9 \%\), and \(12 \%\). Use estimatePortSharpeRatio to obtain the Sharpe ratio for the collection of portfolios (pwgt).
```

m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
0.00408 0.0289 0.0204 0.0119;
0.00192 0.0204 0.0576 0.0336;
0 0.0119 0.0336 0.1225 ];
p = Portfolio;

```
```

p = setAssetMoments(p, m, C);
p = setDefaultConstraints(p);
pwgt = estimateFrontierByReturn(p, [0.06, 0.09, 0.12]);
display(pwgt);
pwgt = 4×3
0.8772 0.5032 0.1293
0.0434 0.2488 0.4541
0.0416 0.0780 0.1143
0.0378 0.1700 0.3022

```
pwgt is a NumAssets-by-NumPorts matrix where NumAssets is the number of asset in the universe and NumPorts is the number of portfolios in the collection of portfolios.
```

psharpe = estimatePortSharpeRatio(p,pwgt)
psharpe = 3×1
0.7796
0.8519
0.7406

```
psharpe is a NumPorts-by-1 vector, where NumPorts is the number of portfolios in the collection of portfolios.

\section*{Input Arguments}

\section*{obj - Object for portfolio}
object
Object for portfolio, specified using a Portfolio object.

Note The risk-free rate is obtained from the property RiskFreeRate in the Portfolio object. If you leave the RiskFreeRate unset, it is assumed to be 0 . For more information on creating a portfolio object, see Portfolio.

\section*{Data Types: object}

\section*{pwgt - Collection of portfolios}

\section*{matrix}

Collection of portfolios, specified as a NumAssets-by- NumPorts matrix where NumAssets is the number of assets in the universe and NumPorts is the number of portfolios in the collection of portfolios.
Data Types: double

\section*{Output Arguments}
psharpe - Sharpe ratios of the given portfolios
vector
Sharpe ratios of the given portfolios, returned as a NumPorts-by-1 vector.

\section*{Definitions}

\section*{Sharpe Ratio}

The Sharpe ratio is the ratio of the difference between the mean of portfolio returns and the risk-free rate divided by the standard deviation of portfolio returns for each portfolio in pwgt.
estimatePortSharpeRatio computes the Sharpe ratio with mean and standard deviation (which is the square-root of variance) of portfolio returns.

\section*{Tips}

You can also use dot notation to estimate the Sharpe ratio of given portfolio weights.
psharpe = obj.estimatePortSharpeRatio(pwgt);

\section*{See Also}
estimateFrontier|estimateFrontierByReturn|estimateFrontierByRisk| estimateMaxSharpeRatio

\section*{Topics}
"Estimate Efficient Portfolios for Entire Efficient Frontier for Portfolio Object" on page 4115
"Portfolio Optimization Examples" on page 4-167
"Portfolio Optimization Theory" on page 4-3

Introduced in R2018a

\section*{estimatePortMoments}

Estimate moments of portfolio returns for Portfolio object

\section*{Syntax}
```

[prsk,pret] = estimatePortMoments(obj,pwgt)

```

\section*{Description}
[prsk, pret] = estimatePortMoments(obj,pwgt) estimate moments of portfolio returns for a Portfolio object. For details on the workflow, see "Portfolio Object Workflow" on page 4-21.

The estimate of port moments is specific to mean-variance portfolio optimization and computes the mean and standard deviation (which is the square-root of variance) of portfolio returns.

\section*{Examples}

\section*{Identify the Range of Risks and Returns for Efficient Portfolios for a Portfolio Object}

Given portfolio \(p\), use the estimatePortMoments function to show the range of risks and returns for efficient portfolios.
```

m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
0.00408 0.0289 0.0204 0.0119;
0.00192 0.0204 0.0576 0.0336;
0 0.0119 0.0336 0.1225 ];
p = Portfolio;
p = setAssetMoments(p, m, C);
p = setDefaultConstraints(p);

```
```

pwgt = estimateFrontierLimits(p);
[prsk, pret] = estimatePortMoments(p, pwgt);
disp([prsk, pret]);
0.0769 0.0590
0.3500 0.1800

```

\section*{Input Arguments}

\section*{obj - Object for portfolio object}

Object for portfolio, specified using a Portfolio object. For more information on creating a portfolio object, see
- Portfolio

Data Types: object

\section*{pwgt - Collection of portfolios \\ matrix}

Collection of portfolios, specified as a NumAssets-by-NumPorts matrix where NumAssets is the number of assets in the universe and NumPorts is the number of portfolios in the collection of portfolios.

\section*{Data Types: double}

\section*{Output Arguments}
prsk - Estimates for standard deviations of portfolio returns for each portfolio in pwgt
vector
Estimates for standard deviations of portfolio returns for each portfolio in pwgt, returned as a NumPorts vector.
prsk is returned for a Portfolio input object (obj).

\section*{pret - Estimates for means of portfolio returns for each portfolio in pwgt vector}

Estimates for means of portfolio returns for each portfolio in pwgt, returned as a NumPorts vector.
pret is returned for a Portfolio input object (obj).

\section*{Tips}

\section*{You can also use dot notation to estimate the moments of portfolio returns.}
[prsk, pret] = obj.estimatePortMoments(pwgt);

\section*{See Also}
estimatePortReturn|estimatePortRisk

\section*{Topics}
"Estimate Efficient Portfolios for Entire Efficient Frontier for Portfolio Object" on page 4115
"Portfolio Optimization Examples" on page 4-167
"Portfolio Optimization Theory" on page 4-3

\section*{Introduced in R2011a}

\section*{estimatePortReturn}

Estimate mean of portfolio returns

\section*{Syntax}
pret \(=\) estimatePortReturn(obj, pwgt)

\section*{Description}
pret \(=\) estimatePortReturn(obj, pwgt) estimates the mean of portfolio returns (as the proxy for portfolio return) for Portfolio, PortfolioCVaR, or PortfolioMAD objects. For details on the respective workflows when using these different objects, see "Portfolio Object Workflow" on page 4-21, "PortfolioCVaR Object Workflow" on page 5-20, and "PortfolioMAD Object Workflow" on page 6-19.

\section*{Examples}

\section*{Estimate the Mean of Portfolio Returns for a Portfolio Object}

Given portfolio \(p\), use the estimatePortReturn function to estimate the mean of portfolio returns.
```

m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
0.00408 0.0289 0.0204 0.0119;
0.00192 0.0204 0.0576 0.0336;
0 0.0119 0.0336 0.1225 ];
p = Portfolio;
p = setAssetMoments(p, m, C);
p = setDefaultConstraints(p);
pwgt = estimateFrontierLimits(p);
pret = estimatePortReturn(p, pwgt);
disp(pret)

```

\section*{Estimate the Mean of Portfolio Returns for a PortfolioCVaR Object}

Given portfolio \(p\), use the estimatePortReturn function to estimate the mean of portfolio returns.
```

m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
0.00408 0.0289 0.0204 0.0119;
0.00192 0.0204 0.0576 0.0336;
0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;
rng(11);
AssetScenarios = mvnrnd(m, C, 20000);
p = PortfolioCVaR;
p = setScenarios(p, AssetScenarios);
p = setDefaultConstraints(p);
p = setProbabilityLevel(p, 0.95);
pwgt = estimateFrontierLimits(p);
pret = estimatePortReturn(p, pwgt);
disp(pret)
0.0050
0.0154

```

The function \(\mathrm{rng}\left({ }^{\text {seed }}\right)\) resets the random number generator to produce the documented results. It is not necessary to reset the random number generator to simulate scenarios.

\section*{Estimate the Mean of Portfolio Returns for a PortfolioMAD Object}

Given portfolio p, use the estimatePortReturn function to estimate the mean of portfolio returns.
```

m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
0.00408 0.0289 0.0204 0.0119;
0.00192 0.0204 0.0576 0.0336;
0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;
rng(11);
AssetScenarios = mvnrnd(m, C, 20000);
p = PortfolioMAD;
p = setScenarios(p, AssetScenarios);
p = setDefaultConstraints(p);
pwgt = estimateFrontierLimits(p);
pret = estimatePortReturn(p, pwgt);
disp(pret)
0.0048
0.0154

```

The function \(\mathrm{rng}\left({ }^{\text {seed }}\right)\) resets the random number generator to produce the documented results. It is not necessary to reset the random number generator to simulate scenarios.

\section*{Input Arguments}

\section*{obj - Object for portfolio \\ object}

Object for portfolio, specified using Portfolio, PortfolioCVaR, or PortfolioMAD object. For more information on creating a portfolio object, see
- Portfolio
- PortfolioCVaR
- PortfolioMAD

Data Types: object

\section*{pwgt - Collection of portfolios}

\section*{matrix}

Collection of portfolios, specified as a NumAssets-by-NumPorts matrix, where NumAssets is the number of assets in the universe and NumPorts is the number of portfolios in the collection of portfolios.

\section*{Data Types: double}

\section*{Output Arguments}
pret - Estimates for means of portfolio returns for each portfolio in pwgt vector

Estimates for means of portfolio returns for each portfolio in pwgt, returned as a NumPorts vector.
pret is returned for a Portfolio, PortfolioCVaR, or PortfolioMAD input object (obj).

Note Depending on whether costs have been set, the portfolio return is either gross or net portfolio returns. For information on setting costs, see setCosts.

\section*{Tips}

You can also use dot notation to estimate the mean of portfolio returns (as the proxy for portfolio return).
```

pret = obj.estimatePortReturn(pwgt);

```

\section*{See Also}
estimateFrontierByReturn|estimateFrontierByRisk|estimatePortRisk| rng

\section*{Topics}
"Estimate Efficient Portfolios for Entire Efficient Frontier for Portfolio Object" on page 4115
"Estimate Efficient Frontiers for Portfolio Object" on page 4-140
"Estimate Efficient Portfolios for Entire Frontier for PortfolioCVaR Object" on page 5-101
"Estimate Efficient Frontiers for PortfolioCVaR Object" on page 5-118
"Estimate Efficient Portfolios Along the Entire Frontier for PortfolioMAD Object" on page 6-96
"Estimate Efficient Frontiers for PortfolioMAD Object" on page 6-111
"Portfolio Optimization Examples" on page 4-167
"Portfolio Optimization Theory" on page 4-3

\section*{Introduced in R2011a}

\section*{estimatePortRisk}

Estimate portfolio risk according to risk proxy associated with corresponding object

\section*{Syntax}
```

prsk = estimatePortRisk(obj,pwgt)

```

\section*{Description}
prsk = estimatePortRisk(obj,pwgt) estimates portfolio risk according to the risk proxy associated with the corresponding object (obj) for Portfolio, PortfolioCVaR, or PortfolioMAD objects. For details on the respective workflows when using these different objects, see "Portfolio Object Workflow" on page 4-21, "PortfolioCVaR Object Workflow" on page 5-20, and "PortfolioMAD Object Workflow" on page 6-19.

\section*{Examples}

\section*{Standard Deviation of Portfolio Returns as the Proxy for Portfolio Risk for a Portfolio Object}

Given portfolio \(p\), use the estimatePortRisk function to show the standard deviation of portfolio returns for each portfolio in pwgt.
```

m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
0.00408 0.0289 0.0204 0.0119;
0.00192 0.0204 0.0576 0.0336;
0 0.0119 0.0336 0.1225 ];
p = Portfolio;
p = setAssetMoments(p, m, C);
p = setDefaultConstraints(p);
pwgt = estimateFrontierLimits(p);
prsk = estimatePortRisk(p, pwgt);
disp(prsk)

```
0.0769
0.3500

\section*{Conditional Value-at-Risk of Portfolio Returns as the Proxy for Portfolio Risk for a PortfolioCVaR Object}

Given a portfolio pwgt, use the estimatePortRisk function to show the conditional value-at-risk (CVaR) of portfolio returns for each portfolio.
```

m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
0.00408 0.0289 0.0204 0.0119;
0.00192 0.0204 0.0576 0.0336;
0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;
rng(11);
AssetScenarios = mvnrnd(m, C, 20000);
p = PortfolioCVaR;
p = setScenarios(p, AssetScenarios);
p = setDefaultConstraints(p);
p = setProbabilityLevel(p, 0.95);
pwgt = estimateFrontierLimits(p);
prsk = estimatePortRisk(p, pwgt);
disp(prsk)
0.0407
0.1911

```

The function \(\mathrm{rng}\left({ }^{\text {seed }}\right)\) resets the random number generator to produce the documented results. It is not necessary to reset the random number generator to simulate scenarios.

\section*{Mean-Absolute Deviation Returns as the Proxy for Portfolio Risk for a PortfolioMAD Object}

Given a portfolio pwgt, use the estimatePortRisk function to show the mean-absolute deviation of portfolio returns for each portfolio.
```

m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
0.00408 0.0289 0.0204 0.0119;
0.00192 0.0204 0.0576 0.0336;
0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;
rng(11);
AssetScenarios = mvnrnd(m, C, 20000);
p = PortfolioMAD;
p = setScenarios(p, AssetScenarios);
p = setDefaultConstraints(p);
pwgt = estimateFrontierLimits(p);
prsk = estimatePortRisk(p, pwgt);
disp(prsk)

```
0.0177
0.0809

The function \(\mathrm{rng}\left({ }^{\text {seed }}\right)\) resets the random number generator to produce the documented results. It is not necessary to reset the random number generator to simulate scenarios.

\section*{Input Arguments}

\section*{obj - Object for portfolio}

\section*{object}

Object for portfolio, specified using Portfolio, PortfolioCVaR, or PortfolioMAD object. For more information on creating a portfolio object, see
- Portfolio
- PortfolioCVaR
- PortfolioMAD

\section*{Data Types: object}

\section*{pwgt - Collection of portfolios}
matrix
Collection of portfolios, specified as a NumAssets-by-NumPorts matrix, where NumAssets is the number of assets in the universe and NumPorts is the number of portfolios in the collection of portfolios.
Data Types: double

\section*{Output Arguments}
prsk - Estimates for portfolio risk according to the risk proxy associated with the corresponding object (obj) for each portfolio in pwgt
vector
Estimates for portfolio risk according to the risk proxy associated with the corresponding object (obj) for each portfolio in pwgt, returned as a NumPorts vector.
prsk is returned for a Portfolio, PortfolioCVaR, or PortfolioMAD input object (obj).

\section*{Tips}

You can also use dot notation to estimate portfolio risk according to the risk proxy associated with the corresponding object (obj).
prsk = obj.estimatePortRisk(pwgt);

\section*{See Also}
estimateFrontierByReturn|estimateFrontierByRisk|estimatePortStd| estimatePortVaR|rng
Topics"Estimate Efficient Portfolios for Entire Efficient Frontier for Portfolio Object" on page 4-115
"Estimate Efficient Frontiers for Portfolio Object" on page 4-140
"Estimate Efficient Portfolios for Entire Frontier for PortfolioCVaR Object" on page 5-101
"Estimate Efficient Frontiers for PortfolioCVaR Object" on page 5-118"Estimate Efficient Portfolios Along the Entire Frontier for PortfolioMAD Object" on page6-96
"Estimate Efficient Frontiers for PortfolioMAD Object" on page 6-111
"Portfolio Optimization Examples" on page 4-167
"Portfolio Optimization Theory" on page 4-3
Introduced in R2011a

\section*{estimatePortStd}

Estimate standard deviation of portfolio returns

\section*{Syntax}
```

pstd = estimatePortStd(obj,pwgt)

```

\section*{Description}
pstd = estimatePortStd(obj, pwgt) estimate standard deviation of portfolio returns for PortfolioCVaR or PortfolioMAD objects. For details on the workflows, see "PortfolioCVaR Object Workflow" on page 5-20 and "PortfolioMAD Object Workflow" on page 6-19.

\section*{Examples}

\section*{Estimate Standard Deviations for Portfolio Returns for a PortfolioCVaR Object}

Given a portfolio pwgt, use the estimatePortStd function to show the standard deviation of portfolio returns.
```

m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
0.00408 0.0289 0.0204 0.0119;
0.00192 0.0204 0.0576 0.0336;
0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;
rng(11);
AssetScenarios = mvnrnd(m, C, 20000);
p = PortfolioCVaR;

```
```

p = setScenarios(p, AssetScenarios);
p = setDefaultConstraints(p);
p = setProbabilityLevel(p, 0.95);
pwgt = estimateFrontierLimits(p);
pstd = estimatePortStd(p, pwgt);
disp(pstd)

```
0.0223
0.1010

The function rng(seed \()\) resets the random number generator to produce the documented results. It is not necessary to reset the random number generator to simulate scenarios.

\section*{Estimate Standard Deviations for Portfolio Returns for a PortfolioMAD Object}

Given a portfolio pwgt, use the estimatePortStd function to show the standard deviation of portfolio returns.
```

m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
0.00408 0.0289 0.0204 0.0119;
0.00192 0.0204 0.0576 0.0336;
0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;
rng(11);
AssetScenarios = mvnrnd(m, C, 20000);
p = PortfolioMAD;
p = setScenarios(p, AssetScenarios);
p = setDefaultConstraints(p);
pwgt = estimateFrontierLimits(p);
pstd = estimatePortStd(p, pwgt);
disp(pstd)

```
0.0222
0.1010

The function \(\mathrm{rng}\left({ }^{\text {seed }}\right)\) resets the random number generator to produce the documented results. It is not necessary to reset the random number generator to simulate scenarios.

\section*{Input Arguments}

\section*{obj - Object for portfolio}
object
Object for portfolio, specified using a PortfolioCVaR or PortfolioMADobject.
For more information on creating a PortfolioCVaR or PortfolioMAD object, see
- PortfolioCVaR
- PortfolioMAD

Data Types: object

\section*{pwgt - Collection of portfolios}

\section*{matrix}

Collection of portfolios, specified as a NumAssets-by-NumPorts matrix, where NumAssets is the number of assets in the universe and NumPorts is the number of portfolios in the collection of portfolios.

Data Types: double

\section*{Output Arguments}

\section*{pstd - Estimates for standard deviations of portfolio returns for each portfolio in pwgt}
vector
Estimates for standard deviations of portfolio returns for each portfolio in pwgt, returned as a NumPorts vector.

\section*{Tips}

You can also use dot notation to estimate the standard deviation of portfolio returns.
pstd = obj.estimatePortStd(pwgt);

\section*{See Also}
estimateFrontierByReturn|estimateFrontierByRisk|estimatePortReturn | estimatePortVaR|rng

\section*{Topics}
"Estimate Efficient Frontiers for PortfolioCVaR Object" on page 5-118
"Estimate Efficient Frontiers for PortfolioMAD Object" on page 6-111
"Portfolio Optimization Theory" on page 4-3

\section*{External Websites}

Analyzing Investment Strategies with CVaR Portfolio Optimization in MATLAB (50 min 42 sec )

Introduced in R2012b

\section*{estimatePortVaR}

Estimate value-at-risk for PortfolioCVaR object

\section*{Syntax}
```

pvar = estimatePortVaR(obj,pwgt)

```

\section*{Description}
pvar = estimatePortVaR(obj, pwgt) estimates value-at-risk for a PortfolioCVaR object where the probability level used is from the PortfolioCVaR property ProbabilityLevel. For details on the workflow, see "PortfolioCVaR Object Workflow" on page 5-20.

\section*{Examples}

\section*{Estimate Value-at-Risk for a PortfolioCVaR Object}

Given a portfolio pwgt, use the estimatePortVaR function to estimate the value-at-risk of portfolio.
```

m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
0.00408 0.0289 0.0204 0.0119;
0.00192 0.0204 0.0576 0.0336;
0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;
rng(11);
AssetScenarios = mvnrnd(m, C, 20000);
p = PortfolioCVaR;

```
```

p = setScenarios(p, AssetScenarios);
p = setDefaultConstraints(p);
p = setProbabilityLevel(p, 0.95);
pwgt = estimateFrontierLimits(p);
pvar = estimatePortVaR(p, pwgt);
disp(pvar)
0.0314
0.1483

```

The function \(\mathrm{rng}\left({ }^{\text {seed }}\right)\) resets the random number generator to produce the documented results. It is not necessary to reset the random number generator to simulate scenarios.

\section*{Input Arguments}

\section*{obj - Object for portfolio}
object
Object for portfolio, specified using a PortfolioCVaR object.
For more information on creating a PortfolioCVaR object, see
- PortfolioCVaR

\section*{Data Types: object}

\section*{pwgt - Collection of portfolios}

\section*{matrix}

Collection of portfolios, specified as a NumAssets-by-NumPorts matrix, where NumAssets is the number of assets in the universe and NumPorts is the number of portfolios in the collection of portfolios.

\section*{Data Types: double}

\section*{Output Arguments}
pvar - Estimates for value-at-risk of portfolio returns for each portfolio in pwgt vector

Estimates for value-at-risk of portfolio returns for each portfolio in pwgt, returned as a NumPorts vector.

\section*{Tips}

You can also use dot notation to estimate the value-at-risk of PortfolioCVaR object.
pvar = obj.estimatePortVaR(pwgt);

\section*{See Also}
estimatePortStd|rng|setProbabilityLevel

\section*{Topics}
"Estimate Efficient Frontiers for PortfolioCVaR Object" on page 5-118
"Conditional Value-at-Risk" on page 5-6

\section*{External Websites}

Analyzing Investment Strategies with CVaR Portfolio Optimization in MATLAB (50 min 42 sec )

Introduced in R2012b

\section*{estimateScenarioMoments}

Estimate mean and covariance of asset return scenarios

\section*{Syntax}
[ScenarioMean,ScenarioCovar] = estimateScenarioMoments(obj)

\section*{Description}
[ScenarioMean,ScenarioCovar] = estimateScenarioMoments(obj) estimates mean and covariance of asset return scenarios for PortfolioCVaR or PortfolioMAD objects. For details on the workflows, see "PortfolioCVaR Object Workflow" on page 5-20, and "PortfolioMAD Object Workflow" on page 6-19.

\section*{Examples}

\section*{Estimate Mean and Covariance of Asset Return Scenarios for a PortfolioCVaR Object}

Given PortfolioCVaR object p, use the estimatePortRisk function to estimate mean and covariance of asset return scenarios.
```

m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
0.00408 0.0289 0.0204 0.0119;
0.00192 0.0204 0.0576 0.0336;
0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;
rng(11);
AssetScenarios = mvnrnd(m, C, 20000);

```
```

p = PortfolioCVaR;
p = setScenarios(p, AssetScenarios);
p = setDefaultConstraints(p);
p = setProbabilityLevel(p, 0.95);
[ScenarioMean, ScenarioCovar] = estimateScenarioMoments(p)
ScenarioMean = 4×I
0.0039
0.0082
0.0102
0.0154
ScenarioCovar = 4×4

| 0.0005 | 0.0003 | 0.0001 | -0.0001 |
| ---: | ---: | ---: | ---: |
| 0.0003 | 0.0024 | 0.0017 | 0.0010 |
| 0.0001 | 0.0017 | 0.0048 | 0.0028 |
| -0.0001 | 0.0010 | 0.0028 | 0.0102 |

```

The function rng( \(\left.{ }^{\text {seed }}\right)\) resets the random number generator to produce the documented results. It is not necessary to reset the random number generator to simulate scenarios.

\section*{Estimate Mean and Covariance of Asset Return Scenarios for a PortfolioMAD Object}

Given PortfolioMAD object p, use the estimatePortRisk function to estimate mean and covariance of asset return scenarios.
```

m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
0.00408 0.0289 0.0204 0.0119;
0.00192 0.0204 0.0576 0.0336;
0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;
rng(11);

```
```

AssetScenarios = mvnrnd(m, C, 20000);
p = PortfolioMAD;
p = setScenarios(p, AssetScenarios);
p = setDefaultConstraints(p);
[ScenarioMean, ScenarioCovar] = estimateScenarioMoments(p)
ScenarioMean = 4×1
0.0039
0.0082
0.0102
0.0154
ScenarioCovar = 4×4

| 0.0005 | 0.0003 | 0.0001 | -0.0001 |
| ---: | ---: | ---: | ---: |
| 0.0003 | 0.0024 | 0.0017 | 0.0010 |
| 0.0001 | 0.0017 | 0.0048 | 0.0028 |
| -0.0001 | 0.0010 | 0.0028 | 0.0102 |

```

The function \(\mathrm{rng}\left({ }^{\text {seed }}\right)\) resets the random number generator to produce the documented results. It is not necessary to reset the random number generator to simulate scenarios.

\section*{Input Arguments}

\section*{obj - Object for portfolio}
object
Object for portfolio, specified using a PortfolioCVaR or PortfolioMAD object.
For more information on creating a PortfolioCVaR or PortfolioMAD object, see
- PortfolioCVaR
- PortfolioMAD

Data Types: object

\section*{Output Arguments}

\section*{ScenarioMean - Estimate for mean of scenarios}
[] (default) | vector
Estimate for mean of scenarios, returned as a NumPorts vector or [].

Note If no scenarios are associated with the specified object, both ScenarioMean and ScenarioCovar are set to empty [].

\section*{ScenarioCovar - Estimate for covariance of scenarios}

\section*{[] (default) | matrix}

Estimate for covariance of scenarios, returned as a NumAssets-by-NumAssets matrix or [].

Note If no scenarios are associated with the specified object, both ScenarioMean and ScenarioCovar are set to empty [].

\section*{Tips}

You can also use dot notation to estimate the mean and covariance of asset return scenarios for a portfolio.
[ScenarioMean, ScenarioCovar] = obj.estimateScenarioMoments

\section*{See Also}
estimatePortRisk|rng|setScenarios|simulateNormalScenariosByMoments

\section*{Topics}
"Asset Returns and Scenarios Using PortfolioCVaR Object" on page 5-44
"Asset Returns and Scenarios Using PortfolioMAD Object" on page 6-42

\section*{External Websites}

Analyzing Investment Strategies with CVaR Portfolio Optimization in MATLAB (50 min 42 sec)

Introduced in R2012b

\section*{ewstats}

Expected return and covariance from return time series

\section*{Syntax}
```

[ExpReturn,ExpCovariance,NumEffObs] = ewstats(RetSeries)
[ExpReturn,ExpCovariance,NumEffObs] = ewstats(

```
\(\qquad\)
``` ,DecayFactor, WindowLength)
```


## Description

[ExpReturn,ExpCovariance,NumEffObs] = ewstats(RetSeries) computes estimated expected returns (ExpReturn), estimated covariance matrix (ExpCovariance), and the number of effective observations (NumEffObs). These outputs are maximum likelihood estimates which are biased.
[ExpReturn,ExpCovariance,NumEffObs] = ewstats( $\qquad$ ,DecayFactor, WindowLength) adds optional input arguments for DecayFactor and WindowLength.

## Examples

## Compute Estimated Expected Returns and Estimated Covariance Matrix

This example shows how to compute the estimated expected returns and the estimated covariance matrix.

```
RetSeries = [ 0.24 0.08
    0.15 0.13
    0.27 0.06
    0.14 0.13 ];
DecayFactor = 0.98;
[ExpReturn, ExpCovariance] = ewstats(RetSeries, DecayFactor)
```


## ExpReturn = 1×2

### 0.19950 .1002

ExpCovariance $=2 \times 2$

| 0.0032 | -0.0017 |
| ---: | ---: |
| -0.0017 | 0.0010 |

## Input Arguments

## RetSeries - Return series <br> matrix

Return series, specified the number of observations (NUMOBS) by number of assets (NASSETS) matrix of equally spaced incremental return observations. The first row is the oldest observation, and the last row is the most recent.

## Data Types: double

## DecayFactor - Controls how much less each observation is weighted than its successor

1 (default) | numeric
(Optional) Controls how much less each observation is weighted than its successor, specified as a numeric value. The $k$ th observation back in time has weight DecayFactork. DecayFactor must lie in the range: $0<$ DecayFactor $<=1$.

The default value of 1 is the equally weighted linear moving average model (BIS).

## Data Types: double

## WindowLength - Number of recent observations in computation

NUMOBS (default) | numeric
(Optional) Number of recent observations in the computation, specified as a numeric value.

Data Types: double

## Output Arguments

## ExpReturn - Estimated expected returns

## vector

Estimated expected returns, returned as a 1-by-NASSETS vector.

## ExpCovariance - Estimated covariance matrix <br> matrix

Estimated covariance matrix, returned as a NASSETS-by-NASSETS matrix.
The standard deviations of the asset return processes are defined as

$$
\text { STDVec }=\text { sqrt(diag(ExpCovariance)) }
$$

The correlation matrix is
CorrMat = ExpCovariance./( STDVec*STDVec' )

## NumEffObs - Number of effective observations

## numeric

NumEffObs is the number of effective observations where

$$
\text { NumEffObs }=\frac{1-\text { DecayFactor }^{\text {WindowLength }}}{1-\text { DecayFactor }}
$$

A smaller DecayFactor or WindowLength emphasizes recent data more strongly but uses less of the available data set.

## Algorithms

For a return series $r(1), \ldots, r(n)$, where $(n)$ is the most recent observation, and $w$ is the decay factor, the expected returns (ExpReturn) are calculated by

$$
E(r)=\frac{\left(r(n)+w r(n-1)+w^{2} r(n-2)+\ldots+w^{n-1} r(1)\right)}{N u m E f f O b s}
$$

where the number of effective observations NumEff0bs is defined as

$$
\text { NumEffObs }=1+w+w^{2}+\ldots+w^{n-1}=\frac{1-w^{n}}{1-w}
$$

$E(r)$ is the weighed average of $r(n), \ldots, r(1)$. The unnormalized weights are $w, w^{2}, \ldots, w^{(n-1)}$. The unnormalized weights do not sum up to 1 , so NumEff0bs rescales the unnormalized weights. After rescaling, the normalized weights (which sum up to 1 ) are used for averaging. When $w=1$, then NumEffObs $=n$, which is the number of observations. When $w<1$, NumEffObs is still interpreted as the sample size, but it is less than $n$ due to the down-weight on the observations of the remote past.

Note There is no relationship between ewstats function and the RiskMetrics ${ }^{\circledR}$ approach for determining the expected return and covariance from a return time series.

## See Also

cov | cov2corr|mean

## Topics

"Portfolio Optimization Functions" on page 3-4

## Introduced before R2006a

## exp

Exponential values

Note exp will be removed in a future release. Use timetable instead. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

```
newfts = exp(tsobj)
```


## Description

newfts $=\exp (t s o b j)$ calculates the natural exponential (base e) of all the data in the data series of the financial time series object tsobj and returns the result in the object newfts.

## See Also

$\log |\log 10| \log 2$

## Topics

"Financial Time Series Operations" on page 13-7

Introduced before R2006a

## extfield

Data series extraction

Note extfield will be removed in a future release. Use timetable instead. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

ftse = extfield(tsobj,fieldnames)

## Arguments

| tsobj | Financial time series object |
| :--- | :--- |
| fieldnames | Data series to be extracted. A cell array of character vectors if a list <br> of data series names (fieldnames) is supplied. A character vector <br> if only one is wanted. |

## Description

ftse = extfield(tsobj,fieldnames) extracts from tsobj the dates and data series specified by fieldnames into a new financial time series object ftse. ftse has all the dates in tsobj but contains a smaller number of data series.

## Examples

extfield is identical to referencing a field in the object. For example,

```
dates = ['01-Jan-2001';'01-Jan-2001'; '02-Jan-2001'; ...
    02-Jan-2001'; '03-Jan-2001';'03-Jan-2001'];
times = ['11:00';'12:00';'11:00';'12:00';'11:00';'12:00'];
dates times = cellstr([dates, repmat(' ',size(dates,1),1),...
times]);
```

```
myFts = fints(dates_times,(1:6)',{'Data1'},1,'My first FINTS');
ftse = extfield(myF\overline{ts},'Datal')
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints (line 165)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/extfield (line 26)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/display (line 66)
ftse =
    desc: My first FINTS
    freq: Daily (1)
\begin{tabular}{|c|c|c|}
\hline dates: (6)' & 'times: (6)' & 'Datal: (6)' \\
\hline '01-Jan-2001' & '11:00' & \(1]\) \\
\hline " , & '12:00' & 2] \\
\hline '02-Jan-2001' & '11:00' & \(3]\) \\
\hline ' ' & '12:00' & 4] \\
\hline '03-Jan-2001' & '11:00' & \(5]\) \\
\hline & '12:00' & ] \\
\hline
\end{tabular}
```

is the same as

## ftse = ftse.Data1

Warning: FINTS will be removed in a future release. Use TIMETABLE instead. > In fints/subsref (line 106)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead. > In fints/display (line 66)

```
ftse =
```

| 'dates: (6)' | 'times: | (6) ${ }^{\prime}$ | 'Datal: | (6) ' |
| :---: | :---: | :---: | :---: | :---: |
| '01-Jan-2001' | '11:00' |  | [ | $1]$ |
| ' " ' | '12:00' |  | [ | $2]$ |
| '02-Jan-2001' | '11:00' |  | [ | 3] |
| ' " ' | '12:00' |  | [ | 4] |
| '03-Jan-2001' | '11:00' |  | [ | 5] |
|  | '12:00' |  | [ | $6]$ |

This function is the complement of the function rmfield.

See Also<br>rmfield<br>\section*{Topics}<br>"What Is the Financial Time Series App?" on page 14-2<br>\section*{Introduced before R2006a}

## fanplot

Plot combined historical and forecast data to visualize possible outcomes

## Syntax

```
fanplot(historical,forecast)
fanplot(
```

$\qquad$

``` , Name, Value)
h = fanplot(historical,forecast)
h = fanplot(
```

$\qquad$

``` , Name, Value)
```


## Description

fanplot (historical, forecast) generates a fan chart. In time series analysis, a fan chart is a chart that joins a simple line chart for observed past data with ranges for possible values of future data. The historical data and possible future data are joined with a line showing a central estimate or most likely value for the future outcomes.
fanplot supports three plotting scenarios:

- Matching - This scenario occurs when the time period perfectly matches for historical and forecast data.
- Backtest - This scenario occurs when there are overlaps between historical and forecast data.
- Gap - This scenario occurs when there are NaN values in the historical or forecast data.
fanplot( $\qquad$ , Name, Value) generates a fan chart using optional name-value pair arguments.
$h=$ fanplot(historical,forecast) generates a fan chart and returns the figure handle h . In time series analysis, a fan chart is a chart that joins a simple line chart for observed past data with ranges for possible values of future data. The historical data and possible future data are joined with a line showing a central estimate or most likely value for the future outcomes.
fanplot supports three plotting scenarios:
- Matching - This scenario occurs when the time period perfectly matches for historical and forecast data.
- Backtest - This scenario occurs when there are overlaps between historical and forecast data.
- Gap - This scenario occurs when there are NaN values in the historical or forecast data.
h = fanplot ( $\qquad$ ,Name, Value) generates a fan chart and returns the figure handle h using optional name-value pair arguments.


## Examples

## Create a Fan Plot Using Cell Array Data

Define the data inputs for historical as a 5-by-2 cell array and forecast as a 5-by-21 cell array with 20 observations.
historical \(\left.\left.=\begin{array}{cl}\{[2006] \& {[-0.0110]} <br>
{[2007]} \& {[0.0120]} <br>
{[2008]} \& {[0.0090]} <br>

\& {[2009]}\end{array}\right][0.0120]\right\}\) [2010] | $[0.0150]\} ;$ |
| :---: | :---: |

| forecast $=$ | $\{[2011]$ | $[0.0203]$ | $[-0.0155]$ | $[0.0311]$ | $[-0.0026]$ | $[0.0035]$ |
| ---: | :---: | :---: | :--- | :--- | :--- | :--- |
|  | $[2012]$ | $[0.0533]$ | $[0.0430]$ | $[-0.0094]$ | $[0.0037]$ | $[0.0587]$ |
|  | $[0.0141]$ | $[0.0337]$ | $[0.0187]$ | $[0.0095]$ | $[-0.0291]$ | $[0.0185]$ |
|  | $[2013]$ | $[0.0518]$ | $[-0.0116]$ | $[0.0708]$ | $[0.0112]$ | $[-0.0292]$ |
|  |  | $[0.0168]$ | $[0.0405]$ | $[0.0224]$ | $[0.0157]$ | $[-0.0351]$ |
|  | $[2014]$ | $[0.0546]$ | $[-0.0171]$ | $[0.0762]$ | $[0.0088]$ | $[0.0210]$ |
|  |  | $[0.0151]$ | $[0.0419]$ | $[0.0214]$ | $[0.0139]$ | $[-0.0442]$ |
|  | $[2015]$ | $[0.0565]$ | $[-0.0207]$ | $[0.0797]$ | $[0.0072]$ | $[0.0203]$ |
|  |  | $[0.0139]$ | $[0.0428]$ | $[0.0207]$ | $[0.0126]$ | $[-0.049]$ |

Generate the fan plot.
fanplot (historical, forecast);

Financial Forecast Chart


The dotted points are the historical lines and the filled lines indicate the mean for the forecasts. This fanplot represents a matching scenario where the time period perfectly matches for the historical and forecast data.

## Create a Fan Plot Using Matrix Data

Define the data inputs for historical as a 5-by-2 matrix and forecast as a 5-by-21 matrix with 20 observations.

$$
\text { historical }=\begin{array}{rr}
{\left[\begin{array}{l}
1.0000 \\
2.0000
\end{array}\right.} & 2.8046 \\
\text {; }
\end{array}
$$

| 3.0000 | $6.7292 ;$ |
| ---: | ---: |
| 4.0000 | 8.6486 |
| 5.0000 | 10.4747 |$;$


| forecast | [ 3.0000 | $\begin{aligned} & 28.9874 \\ & 20.8557 \end{aligned}$ | $\begin{aligned} & 18.3958 \\ & 27.0691 \end{aligned}$ | $\begin{aligned} & 19.6376 \\ & 23.0803 \end{aligned}$ | $\begin{aligned} & 29.5627 \\ & 20.7885 \end{aligned}$ | $\begin{array}{r} 8.3462 \\ 18.0205 \end{array}$ | $\begin{array}{r} 7.1502 \\ 17.2294 \end{array}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 4.0000 | 4.8933 | 27.2659 | 7.2206 | 24.4703 | 10.5895 | 15.0212 |
|  |  | 8.0007 | 18.7114 | 19.1691 | 24.5963 | 4.2835 | 4.0676 |
|  | 5.0000 | 20.9732 | 19.7069 | 11.6862 | 25.7018 | 31.8940 | 7.2664 |
|  |  | 31.7996 | 3.6419 | 3.2695 | 27.1422 | 10.5487 | 32.6529 |
|  | 6.0000 | 11.0069 | 29.1965 | 4.5551 | 20.2627 | 10.9209 | 15.2675 |
|  |  | 23.9532 | 18.4804 | 25.5484 | 4.8747 | 8.0036 | 11.5329 |
|  | 7.0000 | 5.9699 | 11.1486 | 26.0449 | 13.4619 | 21.1196 | 28.8068 |
|  |  | 21.2390 | 29.2396 | 18.4828 | 28.3945 | 21.9342 | 14.4642 |

Generate the fan plot and return the figure handle.
h = fanplot(historical, forecast)

Financial Forecast Chart

h =
Figure (1) with properties:
Number: 1
Name: ''
Color: [0.9400 0.9400 0.9400]
Position: [360 502560 420]
Units: 'pixels'
Show all properties

The dotted points are the historical lines and the filled lines indicate the mean for the forecasts. This fanplot represents a backtest scenario where there is an overlap between the historical and forecast data.

## Create a Fan Plot Using Cell Array Data and Customize the Plot With Name-Value Pair Arguments

Define the data inputs for historical as a 5-by-2 cell array and forecast as a 5-by-21 cell array with 20 observations.

```
historical = {[2006] [-0.0110]
    [2007] [ 0.0120]
    [2008] [ 0.0090]
    [2009] [ 0.0120]
    [2010] [ 0.0150]};
```

| forecast = | \{[2011] | [0.0203] | [-0.0155] | [0.0311] | [-0.0026] | [0.0035] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | [0.0533] | [0.0139] | [0.0037] | [-0.0727] | [-0.0291] |
|  | [2012] | [0.0430] | [-0.0094] | [0.0587] | [ 0.0095] | [0.0185] |
|  |  | [0.0141] | [0.0337] | [0.0187] | [0.0132] | [-0.0292] |
|  | [2013] | [0.0518] | [-0.0116] | [0.0708] | [0.0112] | [0.0221] |
|  |  | [0.0168] | [0.0405] | [0.0224] | [0.0157] | [-0.0356] |
|  | [2014] | [0.0546] | [-0.0171] | [0.0762] | [0.0088] | [0.0210] |
|  |  | [0.0151] | [0.0419] | [0.0214] | [0.0139] | [-0.0442] |
|  | [2015] | [0.0565] | [-0.0207] | [0.0797] | [0.0072] | [0.0203] |
|  |  | [0.0139] | [0.0428] | [0.0207] | [0.0126] | [-0.0499] |

Generate the fan plot using name-value pair arguments to customize the presentation.
fanplot(historical,forecast,'FanFaceColor',[1 1 1;1 0 0],'FanLineStyle','--','Forecast)

Financial Forecast Chart


## Create a Fan Plot Using Table Data

Create table of historical dates and data.
historicalDates = datetime(2006:2010,1,1)';
historicalData $=[-0.0110 ; 0.0120 ; 0.0090 ; 0.0120 ; 0.0150]$;
historical = table(historicalDates,historicalData,'VariableNames',\{'Dates','Data'\});
Create table of forecast dates and data.

```
forecastDates = datetime(2011:2015,1,1)';
forecastData = }\begin{array}{llllll}{0.0203 -0.0155 0.0311 }&{-0.0026}&{0.0035}&{0.0049}
```

| 0.0533 | 0.0139 | 0.0037 | -0.0727 | -0.0291 | -0.0058 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 0.0430 | -0.0094 | 0.0587 | 0.0095 | 0.0185 | 0.0205 |
| 0.0141 | 0.0337 | 0.0187 | 0.0132 | -0.0292 | 0.0048 |
| 0.0518 | -0.0116 | 0.0708 | 0.0112 | 0.0221 | 0.0246 |
| 0.0168 | 0.0405 | 0.0224 | 0.0157 | -0.0356 | 0.0056 |
| 0.0546 | -0.0171 | 0.0762 | 0.0088 | 0.0210 | 0.0239 |
| 0.0151 | 0.0419 | 0.0214 | 0.0139 | -0.0442 | 0.0024 |
| 0.0565 | -0.0207 | 0.0797 | 0.0072 | 0.0203 | 0.0234 |
| 0.0139 | 0.0428 | 0.0207 | 0.0126 | -0.0499 | 0.0026 |

forecast = [table(forecastDates,'VariableName',\{'Dates'\}),array2table(forecastData)];
Plot the data using fanplot.
fanplot(historical,forecast);

Financial Forecast Chart


# Input Arguments 

historical - Historical dates and data<br>matrix | cell array | table

Historical dates and data, specified as an N-by-2 matrix, cell array, or table, where the first column is the date, and the second column is the data associated for that date. N indicates the number of dates. By using the cell array format for the input, you can make the first column datetime and produce the same plot as would serial date numbers or date character vectors. For example:

```
historical(:,1) = num2cell(datetime(2006:2010,1,1));
forecast(:,1) = num2cell(datetime(2011:2015,1,1));
fanplot (historical, forecast);
Data Types: cell|double|table
```

forecast - Forecast dates and data<br>matrix | cell array of character vectors | table

Forecast dates and data, specified as an N-by-M matrix, cell array, or table, where the first column is the date, and the second to the last columns are the data observations. N indicates the number of the dates and ( $\mathrm{M}-1$ ) is the number for data observations. By using the cell array format for the input, you can make the first column datetime and produce the same plot as would serial date numbers or date character vectors. For example:
historical(:,1) = num2cell(datetime(2006:2010,1,1));
forecast(:,1) = num2cell(datetime(2011:2015,1,1));
fanplot (historical, forecast);
Data Types: cell|double | table

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Example: fanplot(historical,forecast, 'NumQuantiles', 14,'FanLineColor','blue','HistoricalLineWidth', 1.8,'ForecastLineColor','red')

## NumQuantiles - Number of quantiles to display

20 (default) | positive integer
Number of quantiles to display in fan chart, specified as a positive integer.
Data Types: double

## FanLineStyle - Style of the lines separating fans

'none ' (default)| character vector
Style of the lines separating fans, specified as a character vector. For more information on supported character vectors for line styles, see Primitive Line.

## Data Types: char

## FanLineColor - Color of lines separating fans

'black' (default) | character vector for color or RGB triplet
Color of lines separating fans, specified as a character vector for color or an RGB triplet. For more information on supported color character vectors, see Primitive Line.

Data Types: double | char

## FanFaceColor - Color of each fan

## [1 1 0;1 0 0] (yellow to red) (default)| matrix

Color of each fan, specified as an N-by-3 matrix controlling the color of each fan, where each row is an RGB triplet. There are three possible values of $N$ :

- When $N=$ NumQuantiles, the color of each fan is specified by the corresponding row in the matrix.
- When $N=$ ceil(NumQuantiles/2), the specified colors represent the bottom half of the fans. The colors of the top half are determined by reversing the order of these colors. For more information, see ceil.
- When $N=2$, the colors in the bottom half of the fan are a linear interpolation between the two specified colors. The pattern is reversed for the top half.


## Data Types: double

## HistoricalMarker - Marker symbol of historical line ' o' (default) | character vector

Marker symbol of historical line, specified as a character vector. For more information on supported character vectors for markers, see Primitive Line.

## Data Types: char

## HistoricalMarkerSize - Marker size of historical line

5 (default) | positive value in point units
Marker size of historical line, specified as a positive value in point units.
Data Types: double | char

## HistoricalMarkerFaceColor - Marker fill color of historical line

## 'blue' (default)|character vector with a value of 'none', 'auto', color identifier, or RGB triplet

Marker fill color of historical line, specified as a character vector with a value of ' none', 'auto ', a character vector for color, or an RGB triplet. For more information on supported character vectors for color, see Primitive Line.

Data Types: double | char

## HistoricalMarkerEdgeColor - Marker outline color of historical line

'blue' (default) | character vector with a value of 'none', 'auto', color identifier, or RGB triplet

Marker outline color of historical line, specified as a character vector with a value of ' none', 'auto', a character vector for color, or an RGB triplet. For more information on supported character vectors for color, see Primitive Line.

## Data Types: double | char

## HistoricalLineColor - Color of historical line

'black' (default)| character vector with a value of ' none', color identifier, or RGB triplet

Color of historical line, specified as a character vector with a value of 'none', a character vector for color, or an RGB triplet. For more information on supported character vectors for color, see Primitive Line.

Data Types: double | char

## HistoricalLineStyle - Style of historical line

## ' - - ' (default)| character vector

Style of historical line, specified as a character vector. For more information on supported character vectors for line styles, see Primitive Line.

Data Types: char

## HistoricalLineWidth - Width of historical line

1.5 (default) | positive value in point units

Width of historical line, specified as a positive value in point units.

## Data Types: double

## ForcecastMarker - Marker symbol of forecast line

' none' (default)| character vector
Marker symbol of forecast line, specified as a character vector. For more information on supported character vectors for marker symbols, see Primitive Line.

Data Types: char

## ForecastMarkerSize - Marker size of forecast line

5 (default) | positive value in point units
Marker size of forecast line, specified as a positive value in point units.
Data Types: double

## ForecastMarkerFaceColor - Marker fill color of forecast line

'none' (default)|character vector with a value of 'none', 'auto', color identifier, or RGB triplet

Marker fill color of forecast line, specified as a character vector with a value of ' none ', ' auto ' , a character vector for color, or an RGB triplet. For more information on supported character vectors for color, see Primitive Line.

## Data Types: double | char

## ForecastMarkerEdgeColor - Marker outline color of forecast line

'auto ' (default)| character vector with a value of 'none', 'auto', color identifier, or RGB triplet

Marker outline color of forecast line, specified as a character vector with a value of 'none', 'auto', character vector for color, or an RGB triplet. For more information on supported character vectors for color, see Primitive Line.

## Data Types: double | char

## ForecastLineColor - Color of forecast line

'black' (default) | character vector with a value of ' none', color identifier, or RGB triplet

Color of forecast line, specified as a character vector with a value of ' none', a character vector for color, or an RGB triplet. For more information on supported character vectors for color, see Primitive Line.

Data Types: double | char

## ForecastLineStyle - Style of forecast line

' - ' (default) | character vector
Style of forecast line, specified as a character vector. For more information on supported character vectors for line styles, see Primitive Line.

Data Types: char

## ForecastLineWidth - Width of forecast line

2 (default) | positive value in point units
Width of forecast line, specified as a positive value in point units.
Data Types: double

## Output Arguments

## h - Figure handle for fanplot

handle object
Figure handle for the fanplot, returned as handle object.

## See Also

bolling| candle |ceil|datetime |highlow|linebreak|movavg|pointfig| priceandvol|renko|volarea

## Introduced in R2014b

## fbusdate

First business date of month

## Syntax

Date = fbusdate(Year, Month)
Date $=$ fbusdate( $\qquad$ ,Holiday, Weekend, outputType)

## Description

Date $=$ fbusdate (Year, Month) returns the serial date number for the first business date of the given year and month.

Year and Month can contain multiple values. If one contains multiple values, the other must contain the same number of values or a single value that applies to all. For example, ifYear is a 1-by-N vector of integers, then Month must be a 1-by-N vector of integers or a single integer. Date is then a 1-by-N vector of date numbers.

Use the function datestr to convert serial date numbers to formatted date character vectors.

Date $=$ fbusdate( $\qquad$ ,Holiday, Weekend, outputType) returns the serial date number for the first business date of the given year and month using optional input arguments. The optional argument Holiday specifies nontrading days.

If neither Holiday nor outputType are specified, Date is returned as a serial date number. If Holiday is specified, but not outputType, then the type of the holiday variable controls the type of date. If Holiday is a serial date number or date character vector, then Date is returned as a serial date number.

## Examples

## Return a Serial Date Number for the First Business Date

This example shows how to return serial date numbers for the first business date, given year and month.

```
Date = fbusdate(2001, 11)
Date = 731156
datestr(Date)
ans =
'01-Nov-2001'
Year = [2002 2003 2004];
Date = fbusdate(Year, 11)
Date = 1×3
```

    \(731521 \quad 731888 \quad 732252\)
    datestr(Date)
ans $=3 x 11$ char array
'01-Nov-2002'
'03-Nov-2003'
'01-Nov-2004'

## Return a Serial Date Number for the First Business Date Using the Weekend Argument

This example shows how to return serial date numbers for the first business date, given year and month, and also indicate that Saturday is a business day by setting the Weekend argument. March 1, 2003, is a Saturday. Use fbusdate to check that this Saturday is actually the first business day of the month.

```
Weekend = [1 0 0 0 0 0 0];
Date = datestr(fbusdate(2003, 3, [], Weekend))
```


## Date =

```
'01-Mar-2003'
```


## Return a datetime array for Date for the First Business Date Using the outputType Argument

This example shows how to return a datetime array for Date using an outputType of 'datetime'.

Date = fbusdate(2001, 11,[],[],'datetime')
Date = datetime
01-Nov-2001

## Input Arguments

## Year - Year to determine occurrence of weekday

4-digit integer | vector of 4-digit integers
Year to determine occurrence of weekday, specified as a 4-digit integer or vector of 4-digit integers.
Data Types: single | double

## Month - Month to determine occurrence of weekday

integer with value 1 through $12 \mid$ vector of integers with values 1 through 12
Month to determine occurrence of weekday, specified as an integer or vector of integers with values 1 through 12.

## Data Types: single|double

## Holiday - Holidays and nontrading-day dates

non-trading day vector is determined by the routine holidays (default) | serial date number | date character vector | datetime array

Holidays and nontrading-day dates, specified as vector.

All dates in Holiday must be the same format: either serial date numbers, or date character vectors, or datetime arrays. (Using serial date numbers improves performance.) The holidays function supplies the default vector.

If Holiday is a datetime array, then Date is returned as a datetime array. If outputType is specified, then its value determines the output type of Date. This overrides any influence of Holiday.

Data Types: double | char | datetime

## Weekend - Weekend days

[100000 1] (Saturday and Sunday form the weekend) (default) | vector of length 7 , containing 0 and 1 , where 1 indicates weekend days

Weekend days, specified as a vector of length 7 , containing 0 and 1 , where 1 indicates weekend days and the first element of this vector corresponds to Sunday.

Data Types: double

## outputType - Year to determine days

'datenum' (default)| character vector with values 'datenum' or 'datetime'
A character vector specified as either 'datenum ' or 'datetime'. The output Date is in serial date format if 'datenum' is specified, or datetime format if 'datetime' is specified. By default the output Date is in serial date format, or match the format of Holiday, if specified.

Data Types: char

## Output Arguments

## Date - Date for the first business date of given year and month

serial date number | date character vector | datetime array
Date for the first business date of a given year and month, returned as a serial date number, date character vector, or datetime array.

If neither Holiday nor outputType are specified, Date is returned as a serial date number. If Holiday is specified, but not outputType, then the type of the holiday variable controls the type of date:

- If Holiday is a serial date number or date character vector, then Date is returned as a serial date number
- If Holiday is a datetime array, then Date is returned as a datetime array.


## See Also

busdate | datetime | eomdate | holidays | isbusday | lbusdate

## Topics

"Handle and Convert Dates" on page 2-2
"Trading Calendars User Interface" on page 16-2
"UICalendar User Interface" on page 16-4

Introduced before R2006a

## fetch

Data from financial time series object

Note fetch will be removed in a future release. Use timetable instead. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

newfts = fetch(oldfts,StartDate,StartTime,EndDate,EndTime,delta,dmy_specifier,time_ref

## Arguments

| oldfts | Existing financial time series object. |
| :--- | :--- |
| StartDate | First date in the range from which data is to be extracted. |
| StartTime | Beginning time on each day. If you do not require specific <br> times or oldfts does not contain time information, use [ ]. <br> If you specify StartTime, you must also specify EndTime. |
| EndDate | Last date in the range from which data is to be extracted. |
| EndTime | Ending time on each day. If you do not require specific times <br> or oldfts does not contain time information, use [ ]. If you <br> specify EndTime, you must also specify StartTime. |
| delta | Skip interval. Can be any positive integer. Units for the skip <br> interval specified by dmy_specifier. |
| dmy_specifier | Specifies the units for delta. Can be <br> - D, d (Days) <br> - M, m (Months) <br> - Y, y (Years) |


| time_ref | Time reference intervals or specific times. Valid time <br> reference intervals are 1, 5, 15, or 60 minutes. Enter specific <br> times as 'hh:mm'. |
| :--- | :--- |

## Description

## newfts =

fetch(oldfts,StartDate,StartTime,EndDate,EndTime,delta,dmy_specifier ,time_ref) requests data from a financial time series object beginning from the start date and/or start time to the end date and/or end time, skipping a specified number of days, months, or years.

Note If time information is present in oldfts, using [ ] for start or end times results in fetch returning all instances of a specific date.

## Examples

Example 1. Create a financial time series object containing both dates and times:

```
dates = ['01-Jan-2001';'01-Jan-2001'; '02-Jan-2001'; ...
    '02-Jan-2001'; '03-Jan-2001';'03-Jan-2001'];
times = ['11:00';'12:00';'11:00';'12:00';'11:00';'12:00'];
dates_times = cellstr([dates, repmat(' ',size(dates,1),1),...
times]);
myFts = fints(dates_times,(1:6)',{'Datal'},1,'My first FINTS')
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints (line 165)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/display (line 66)
myFts =
    desc: My first FINTS
    freq: Daily (1)
    'dates: (6)' 'times: (6)' 'Data1: (6)
    '01-Jan-2001' '11:00' [ 1]
    '" ' '12:00' [ 2]
    '02-Jan-2001' '11:00' [ 3]
```

| ' " ' | ' $12: 00{ }^{\prime}$ | 4] |
| :---: | :---: | :---: |
| '03-Jan-2001' | '11:00' | [ 5] |
| ' " ' | '12:00' | $6]$ |

To fetch all dates and times from this financial time series, enter

```
fetch(myFts,'01-Jan-2001',[],'03-Jan-2001',[],1,'d')
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/fetch (line 126)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead,
> In fints/display (line 66)
ans =
    desc: My first FINTS
    freq: Daily (1)
    'dates: (6)' 'times: (6)' 'Datal: (6)
    '01-Jan-2001' '11:00' [ 1]
    ' " ' '12:00' [ 2]
    '02-Jan-2001' '11:00' [ 3]
    ' " ' '12:00' [ 4]
    '03-Jan-2001' '11:00' [ 5]
    ' " ' '12:00' [ 6]
or
fetch(myFts,'01-Jan-2001','11:00','03-Jan-2001','12:00',1,'d')
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/fetch (line 126)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/display (line 66)
ans =
    desc: My first FINTS
    freq: Daily (1)
    'dates: (6)' 'times: (6)' 'Data1: (6)'
    '01-Jan-2001' '11:00' [ 1]
    ' " ' '12:00' [ 2]
    '02-Jan-2001' '11:00' [ 3]
    ' " ' '12:00' [ 4]
```



These commands reproduce the entire time series shown above.
To fetch every other day's data, enter

```
fetch(myFts,'01-Jan-2001',[],'03-Jan-2001',[],2,'d')
```


## This returns:

```
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/fetch (line 126)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/display (line 66)
ans =
    desc: My first FINTS
    freq: Daily (1)
\begin{tabular}{|c|c|c|c|}
\hline 'dates: (4)' & 'times: & (4) \({ }^{\prime}\) & 'Datal: (4) \\
\hline '01-Jan-2001' & '11:00' & & \(1]\) \\
\hline " ' & '12:00' & & \(2]\) \\
\hline '03-Jan-2001' & '11:00' & & 5] \\
\hline " ' & '12:00' & & \(6]\) \\
\hline
\end{tabular}
```

Example 2. Create a financial time series object with time intervals of less than 1 hour:

```
dates2 = ['01-Jan-2001';'01-Jan-2001'; '01-Jan-2001';...
'02-Jan-2001'; '02-Jan-2001';'02-Jan-2001'];
times2 = ['11:00';'11:05';'11:06';'12:00';'12:05';'12:06'];
dates_times2 = cellstr([dates2, repmat(' ',size(dates2,1),1),...
times2]);
myFts2 = fints(dates_times2,(1:6)',{'Data1'},1,'My second FINTS')
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints (line 165)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/display (line 66)
myFts2 =
    desc: My second FINTS
    freq: Daily (1)
    'dates: (6)' 'times: (6)' 'Datal: (6)
```

| '01-Jan-2001' | '11:00' | $1]$ |
| :---: | :---: | :---: |
| ' " ' | '11:05' | $2]$ |
| ' " ' | '11:06' | [ 3] |
| ' $02-$ Jan-2001' | '12:00' | [ 4] |
| " | ' $12: 05{ }^{\prime}$ | 5] |
| ' " ' | ' 12:06' | $6]$ |

Use fetch to extract data from this time series object at 5-minute intervals for each day starting at 11:00 o'clock on January 1, 2001.

```
fetch(myFts2,'01-Jan-2001',[],'02-Jan-2001',[],1,'d',5)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/fetch (line 126)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/display (line 66)
ans =
    desc: My second FINTS
    freq: Daily (1)
    'dates: (4)' 'times: (4)' 'Datal: (4)
    '01-Jan-2001' '11:00' [ 1]
    ' ' '11:05' [ 2]
    '02-Jan-2001' '12:00' [ 4]
```

You can use this version of fetch to extract data at specific times. For example, to fetch data only at 11:06 and 12:06 from myFts2, enter

```
fetch(myFts2,'01-Jan-2001',[],'02-Jan-2001',[],1,''d',...
{'11:06';'12:06'})
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/fetch (line 126)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/display (line 66)
ans =
    desc: My second FINTS
    freq: Daily (1)
    'dates: (2)' 'times: (2)' 'Datal: (2)'
```

'01-Jan-2001' '11:06'
3]
'02-Jan-2001' '12:06
6]

## See Also

extfield|ftsbound|getfield|subsref

## Topics

"What Is the Financial Time Series App?" on page 14-2

Introduced before R2006a

## fieldnames

Get names of fields

Note fieldname will be removed in a future release. Use timetable instead. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

```
fnames = fieldnames(tsobj)
fnames = fieldnames(tsobj,srsnameonly)
```


## Arguments

| tsobj | Financial time series object |
| :--- | :--- |
| srsnameonly | Field names returned: |
|  | $0=$ All field names (default). |
|  | $1=$ Data series names only. |

## Description

fieldnames gets field names in a financial time series object.
fnames = fieldnames(tsobj) returns the field names associated with the financial time series object tsobj as a cell array of character vectors, including the common fields: desc, freq, dates (and times if present).
fnames = fieldnames(tsobj,srsnameonly) returns field names depending upon the setting of srsnameonly. If srsnameonly is 0 , the function returns all field names, including the common fields: desc, freq, dates, and times. If srsnameonly is set to 1 , fieldnames returns only the data series in fnames.

See Also<br>chfield|getfield|isfield|rmfield|setfield<br>Topics<br>"What Is the Financial Time Series App?" on page 14-2<br>Introduced before R2006a

## fillts

Fill missing values in time series

Note fillts will be removed in a future release. Use timetable instead. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

```
newfts = fillts(oldfts,fill_method)
newfts = fillts(oldfts,fill_method,newdates)
newfts = fillts(oldfts,fill_method,newdates,{'T1','T2',...})
newfts = fillts(oldfts,fill_method,newdates,'SPAN',{'TS','TE'},delta)
newfts = fillts(... sortmode)
```


## Arguments

| oldfts | Financial time series object. |
| :--- | :--- |


| fill_method | (Optional) Replaces missing values (NaN) in oldfts using an interpolation process, a constant, or a zero-order hold. <br> Valid fill methods (interpolation methods) are: <br> - linear-'linear ' - 'l' (default) <br> - linear with extrapolation - 'linearExtrap' - 'le' <br> - cubic-'cubic' - 'c' <br> - cubic with extrapolation - 'cubicExtrap' - 'ce' <br> - spline - 'spline' - 's' <br> - spline with extrapolation - 'splineExtrap' - 'se' <br> - nearest - 'nearest' - 'n' <br> - nearest with extrapolation - 'nearestExtrap' - 'ne' <br> - pchip - 'pchip' - 'p' <br> - pchip with extrapolation - 'pchipExtrap' - 'pe' <br> (See interp1 for a discussion of extrapolation.) <br> To fill with a constant, enter that constant. <br> A zero-order hold ('zero') fills a missing value with the value immediately preceding it. If the first value in the time series is missing, it remains a NaN . |
| :---: | :---: |
| newdates | (Optional) Column vector of serial dates, a date character vector, or a column cell array of character vector dates. If oldfts contains time of day information, newdates must be accompanied by a time vector (newtimes). Otherwise, newdates is assumed to have times of '00:00'. |
| T1, T2, TS, TE | First time, second time, start time, end time |
| delta | Time interval in minutes to span between the start time and end time |
| sortmode | (Optional) Default = 0 (unsorted). 1 = sorted. |

## Description

newfts = fillts(oldfts,fill_method) replaces missing values (represented by NaN ) in the financial time series object oldfts with real values, using either a constant or the interpolation process indicated by fill_method.
newfts $=$ fillts(oldfts,fill method, newdates) replaces all the missing values on the specified dates newdates added to the financial time series oldfts with new values. The values can be a single constant or values obtained through the interpolation process designated by fill_method. If any of the dates in newdates exists in oldfts, the existing one has precedence.
newfts $=$ fillts(oldfts,fill_method, newdates,\{'T1','T2',...\}) additionally allows the designation of specific times of day for addition or replacement of data.
newfts $=$ fillts(oldfts,fill_method, newdates,'SPAN', \{'TS', 'TE'\}, delta) is similar to the previous format except that you designate only a start time and an end time. You follow these times with a spanning time interval, delta.

If you specify only one date for newdates, specifying a start and end time generates only times for that specific date.
newfts $=$ fillts (... sortmode) additionally denotes whether you want the order of the dates in the output object to stay the same as in the input object or to be sorted chronologically.
sortmode $=0$ (unsorted) appends any new dates to the end. The interpolation and zeroorder processes that calculate the values for the new dates work on a sorted object. Upon completion, the existing dates are reordered as they were originally, and the new dates are appended to the end.
sortmode $=1$ sorts the output. After interpolation, no reordering of the date sequence occurs.

## Examples

Example 1. Create a financial time series object with missing data in the fourth and fifth rows.

```
dates = ['01-Jan-2001';'01-Jan-2001'; '02-Jan-2001';...
    '02-Jan-2001'; '03-Jan-2001';'03-Jan-2001'];
times = ['11:00';'12:00';'11:00';'12:00';'11:00';'12:00'];
dates_times = cellstr([dates, repmat(' ',size(dates,1),1),...
    times]);
OpenFts = fints(dates times,[(1:3)'; nan; nan; 6],{'Datal'},1,...
'Open Financial Time S̄eries')
```


## OpenFts looks like this:

Warning: FINTS will be removed in a future release. Use TIMETABLE instead. > In fints (line 165)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead. > In fints/display (line 66)

OpenFts =
desc: Open Financial Time Series
freq: Daily (1)

Example 2. Fill the missing data in OpenFts using cubic interpolation.
FilledFts = fillts(OpenFts,'cubic')
Warning: FINTS will be removed in a future release. Use TIMETABLE instead. > In fints/fillts (line 213)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead. > In fints/display (line 66)

FilledFts =
desc: Filled Open Financial Time Series
freq: Unknown (0)
'dates: (6)' 'times: (6)' 'Data1: (6)'
'01-Jan-2001' '11:00' [ 1]
' " ' '12:00' [ 2]
'02-Jan-2001' '11:00' [ 3]


Example 3. Fill the missing data in OpenFts with a constant value.

```
FilledFts = fillts(OpenFts,0.3)
```

Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/fillts (line 213)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/display (line 66)
FilledFts =
desc: Filled Open Financial Time Series
freq: Unknown (0)

| 'dates: (6)' | 'times: | (6) ' | 'Data1: (6) |
| :---: | :---: | :---: | :---: |
| '01-Jan-2001' | '11:00' |  | 1 |
| ' " ' | '12:00' |  | 2] |
| '02-Jan-2001' | '11:00' |  | $3]$ |
| , " , | '12:00' |  | [ 0.3000] |
| '03-Jan-2001' | '11:00' |  | 0.3000] |
|  | '12:00' |  | 61 |

Example 4. You can use fillts to identify a specific time on a specific day for the replacement of missing data. This example shows how to replace missing data at 12:00 on January 2 and 11:00 on January 3.

FilltimeFts = fillts(OpenFts,'c',...
\{'02-Jan-2001';'03-Jan-2001'\}, \{'12:00';'11:00'\},0)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead. > In fints/fillts (line 213)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead. > In fints/display (line 66)

FilltimeFts =

```
desc: Filled Open Financial Time Series
freq: Unknown (0)
'dates: (6)' 'times: (6)' 'Data1: (6)'
'01-Jan-2001' '11:00' [ 1]
```

| ' " | '12:00' | $2]$ |
| :---: | :---: | :---: |
| '02-Jan-2001' | '11:00' | 3] |
| ' " , | '12:00' | $3.0663]$ |
| '03-Jan-2001' | '11:00' | $5.8411]$ |
| ' " ' | '12:00' | $6.0000]$ |

Example 5. Use a spanning time interval to add an additional day to OpenFts.

```
SpanFts = fillts(OpenFts,'c','04-Jan-2001','span',...
    {'11:00';'12:00'},60,0)
```

Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/fillts (line 213)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/display (line 66)

## SpanFts =

```
    desc: Filled Open Financial Time Series
```

    freq: Unknown (0)
    | dates: (8)' | 'times: | (8) ${ }^{\prime}$ | 'Datal: (8)' |
| :---: | :---: | :---: | :---: |
| '01-Jan-2001' | '11:00' |  | [ 1] |
| ' " , | '12:00' |  | [ 2] |
| '02-Jan-2001' | '11:00' |  | [ 3] |
| ' " ' | '12:00' |  | [ 3.0663] |
| '03-Jan-2001' | '11:00' |  | [ 5.8411] |
| ' " ' | '12:00' |  | [ 6.0000] |
| '04-Jan-2001' | '11:00' |  | [ 9.8404] |
| ' " ' | '12:00' |  | $9.9994]$ |

## See Also

## interp1

## Topics

"Data Transformation and Frequency Conversion" on page 13-13
"What Is the Financial Time Series App?" on page 14-2

## Introduced before R2006a

## filter

Linear filtering

Note filter will be removed in a future release. Use timetable instead. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

```
newfts = filter(B,A,oldfts)
```


## Description

filter filters an entire financial time series object with certain filter specifications. The filter is specified in a transfer function expression.
newfts $=$ filter $(B, A, o l d f t s)$ filters the data in the financial time series object oldfts with the filter described by vectors $A$ and $B$ to create the new financial time series object newfts. The filter is a "Direct Form II Transposed" implementation of the standard difference equation. newfts is a financial time series object containing the same data series (names) as the input oldfts.

## See Also

filter2

## Topics

"Data Transformation and Frequency Conversion" on page 13-13
Introduced before R2006a

## fints

Construct financial time series object

Note fints will be removed in a future release. Use timetable instead. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

```
tsobj = fints(dates_and_data)
tsobj = fints(dates,data)
tsobj = fints(dates,data,datanames)
tsobj = fints(dates,data,datanames,freq)
tsobj = fints(dates,data,datanames,freq,desc)
```


## Arguments

| dates_and_data | Column-oriented matrix containing one column of dates and <br> a single column for each series of data. In this format, dates <br> must be entered in serial date number format. If the input <br> serial date numbers encode time-of-day information, the <br> output object contains a column labeled 'dates ' containing <br> the date information and another labeled 'times ' <br> containing the time information. <br> You can use the MATLAB function today to enter date <br> information or the MATLAB function now to enter date with <br> time information. |
| :--- | :--- |


| dates | Column vector of dates. Dates can be date character vectors <br> or serial date numbers and can include time of day <br> information. When entering time-of-day information as serial <br> date numbers, the entry must be a column-oriented matrix <br> when multiple entries are present. If the time-of-day <br> information is in character vector format, the entry must be <br> a column-oriented cell array of character vector dates and <br> times when multiple entries are present. <br> Valid date and time character vector formats are: |
| :--- | :--- |
|  | - 'ddmmmyy hh: mm' or 'ddmmmyyyy hh:mm' <br> $-\quad$ 'mm/dd/yy hh:mm' or 'mm/dd/yyyy hh:mm' <br> - 'dd-mmm-yy hh:mm' or 'dd-mmm-yyyy hh:mm' <br> - 'mmm. dd, yy hh:mm' or 'mmm. dd, yyyy hh:mm' <br> Dates and times can initially be separate column-oriented <br> vectors, but they must be concatenated into a single column- <br> oriented matrix before being passed to fints. You can use <br> the MATLAB functions today and now to help with entering <br> date and time information. |
| data | Column-oriented matrix containing a column for each series <br> of data. The number of values in each data series must <br> match the number of dates. If a mismatch occurs, MATLAB <br> does not generate the financial time series object, and you <br> receive an error message. |


| datanames | Cell array of data series names. Overrides the default data <br> series names. Default data series names are series1, <br> series2, and so on. |
| :--- | :--- |
| Note Not all character vectors are accepted as datanames <br> parameters. Supported data series names cannot start with a <br> number and must contain only these characters: |  |
| -Lowercase Latin alphabet, a to z |  |
| - Uppercase Latin alphabet, A to Z |  |
| - Underscore, _- |  |

## Description

fints constructs a financial time series object. A financial time series object is a MATLAB object that contains a series of dates and one or more series of data. Before you perform an operation on the data, you must set the frequency indicator (freq). You can optionally provide a description (desc) for the time series.
tsobj = fints(dates_and_data) creates a financial time series object containing the dates and data from the matrix dates_and_data. If the dates contain time-of-day information, the object contains an additional series of times. The date series and each data series must each be a column in the input matrix. The names of the data series default to series1, ..., seriesn. The desc and freq fields are set to their defaults.
tsobj = fints(dates,data) generates a financial time series object containing dates from the dates column vector of dates and data from the matrix data. If the dates contain time-of-day information, the object contains an additional series of times. The data matrix must be column-oriented, that is, each column in the matrix is a data series. The names of the series default to series1, ..., seriesn, where $n$ is the total number of columns in data. The desc and freq fields are set to their defaults.
tsobj $=$ fints(dates,data,datanames) also allows you to rename the data series. The names are specified in the datanames cell array. The number of character vectors in datanames must correspond to the number of columns in data. The desc and freq fields are set to their defaults.
tsobj $=$ fints(dates, data, datanames,freq) also sets the frequency when you create the object. The desc field is set to its default ' '.
tsobj $=$ fints(dates,data,datanames,freq, desc) provides a description (desc) specified as a character vector for the financial time series object.

Note fints only supports hourly and minute time series. Seconds are not supported and will be disregarded when the fints object is created (that is, 01 -jan-2001 12:00:01 will be considered as 01 -jan-2001 12:00). If there are duplicate dates and times, the fints constructor sorts the dates and times and chooses the first instance of the duplicate dates and times. The other duplicate dates and times are removed from the object along with their corresponding data.

## Examples

## Create a Financial Time Series Object Containing Days and Data

Define the data:

```
data = [1:6]'
```

```
data = 6x1
    1
    2
    3
    4
    5
    6
```

Define the dates:

```
dates = [today:today+5]'
dates = 6×1
    737299
    7 3 7 3 0 0
    7 3 7 3 0 1
    7 3 7 3 0 2
    7 3 7 3 0 3
    737304
```

Create the financial times series object:
tsobjkt = fints(dates, data)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in

```
tsobjkt =
    desc: (none)
    freq: Unknown (0)
    'dates: (6)' 'series1: (6)
    '27-Aug-2018' [ 1]
    '28-Aug-2018' [ 2]
    '29-Aug-2018' [ 3]
    '30-Aug-2018' [ 4]
    '31-Aug-2018' [ 5]
    '01-Sep-2018' [ 6]
```


## Create a Financial Time Series Object Containing Days, Time of Day, and Data

Define the data:

```
data = [1:6]'
data = 6x1
    1
    2
    3
    4
    5
    6
```

Define the dates:

```
dates = [now:now+5]'
dates = 6×1
105 x
7.3730
7.3730
7.3730
7.3730
7.3730
7.3730
```

Create the financial times series object:
tsobjkt = fints(dates, data)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in

```
tsobjkt =
    desc: (none)
    freq: Unknown (0)
    'dates: (6)' 'times: (6)' 'series1: (6)
    '27-Aug-2018' '15:34' [ 1]
    '28-Aug-2018' '15:34' [ 2]
```

| '29-Aug-2018' | '15:34' | $[$ | $3]$ |
| :--- | :--- | :--- | :--- |
| '30-Aug-2018' | '15:34' | $[$ | $4]$ |
| '31-Aug-2018' | '15:34' | $[$ | $5]$ |
| $01-$ Sep-2018' | '15:34' | $[$ | $6]$ |

## Create a Financial Time Series Object From a Single Input for Dates and Times

Define the dates and times:

```
dates_and_times = (now:now+5)'
dates and times = 6\times1
105 x
    7.3730
    7.3730
    7.3730
    7.3730
    7.3730
    7.3730
```

Create the financial times series object:

```
f = fints(dates and times, randn(6,1))
```

Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in
$f=$
desc: (none)
freq: Unknown (0)

| 'dates: (6)' | 'times: | (6)' | 'series1: (6)' |
| :--- | :--- | :--- | :--- |
| '27-Aug-2018' | '15:36' |  | $[$ |
| '28-Aug-2018' | '15:36' | $0.5377]$ |  |
| '29-Aug-2018' | '15:36' | $[$ | $1.8339]$ |
| '30-Aug-2018' | '15:36' | $[$ | $-2.2588]$ |
| '31-Aug-2018' | '15:36' | $[$ | $0.8622]$ |
| '01-Sep-2018' | '15:36' | $[$ | $0.3188]$ |

This generates a financial time series object, f , and obtains the dates and data from the matrix dates_and_times. The dates and times in the input matrix must be oriented
column-wise (i.e. the date series and each time series are columns in the input matrix). In addition, the dates entered must be in the serial date format (i.e. 01-Jan-2001 is 730852). You can also use the function now to enter in date information. The names of the series will default to 'series1', ..., '|seriesN'| where N is the total number of columns in dates_and_times less 1 (that is the number of data columns). The default contents of the desc and freq fields are|"| and | 'Unknown'| (0), respectively.

## See Also

datenum|datestr|ftsgui|ftstool

## Topics

"Data Transformation and Frequency Conversion" on page 13-13
"What Is the Financial Time Series App?" on page 14-2

## Introduced before R2006a

## floatdiscmargin

Discount margin for floating-rate bond

## Syntax

```
Margin = floatdiscmargin(Price,SpreadSettle,Maturity,RateInfo,
LatestFloatingRate)
Margin = floatdiscmargin(
```

$\qquad$

``` , Name, Value)
```


## Description

Margin = floatdiscmargin(Price,SpreadSettle, Maturity, RateInfo, LatestFloatingRate) calculates the discount margin or zero discount margin for a floating-rate bond.

The input RateInfo determines whether the discount margin or the zero discount margin is calculated. Principal schedules are supported using Principal.

Margin = floatdiscmargin( $\qquad$ , Name, Value) adds optional name-value pair arguments.

## Examples

## Compute the Zero Discount Margin Using a Yield Curve

Use floatdiscmargin to compute the discount margin and zero discount margin for a floating-rate note.

Define data for the floating-rate note.
Price = 99.99;
Spread = 50;
Settle = '20-Jan-2011';
Maturity = '15-Jan-2012';

```
LatestFloatingRate = 0.05;
StubRate = 0.049;
SpotRate = 0.05;
Reset = 4;
Basis = 2;
```

Compute the discount margin.

```
dMargin = floatdiscmargin(Price, Spread, Settle, Maturity, ...
[StubRate, SpotRate], LatestFloatingRate,'Reset', Reset, 'Basis', Basis, ...
'AdjustCashFlowsBasis', true)
```

dMargin $=48.4810$

Usually you want to set AdjustCashFlowsBasis to true, so cash flows are calculated with adjustments on accrual amounts.

Create an annualized zero-rate term structure to calculate the zero discount margin.

```
Rates = [0.0500;
    0.0505;
    0.0510;
    0.0520];
StartDates = ['20-Jan-2011';
    '15-Apr-2011';
    '15-Jul-2011';
    '15-0ct-2011'];
EndDates = ['15-Apr-2011';
    '15-Jul-2011';
    '15-0ct-2011';
    '15-Jan-2012'];
ValuationDate = '20-Jan-2011';
RateSpec = intenvset('Compounding', Reset, 'Rates', Rates,...
'StartDates', StartDates, 'EndDates', EndDates,...
'ValuationDate', ValuationDate, 'Basis', Basis);
```

Calculate the zero discount margin using the previous yield curve.

```
dMargin = floatdiscmargin(Price, Spread, Settle, Maturity, ...
RateSpec, LatestFloatingRate,'Reset', Reset, 'Basis', Basis, ...
'AdjustCashFlowsBasis', true)
```

dMargin = 46.0688

## Compute the Zero Discount Margin Using a Yield Curve With datetime Inputs

Use floatdiscmargin to compute the discount margin and zero discount margin for a floating-rate note using datetime inputs.

```
Price = 99.99;
Spread = 50;
Settle = '20-Jan-2011';
Maturity = '15-Jan-2012';
LatestFloatingRate = 0.05;
StubRate = 0.049;
SpotRate = 0.05;
Reset = 4;
Basis = 2;
Settle = datetime(Settle,'Locale','en_US');
Maturity = datetime(Maturity,'Locale','en_US');
dMargin = floatdiscmargin(Price, Spread, Settle, Maturity, ...
[StubRate, SpotRate], LatestFloatingRate,'Reset', Reset, 'Basis', Basis, ...
'AdjustCashFlowsBasis', true)
dMargin = 48.4810
```


## Input Arguments

## Price - Bond prices where discount margin is to be computed matrix

Bond prices where discount margin is to be computed, specified as a NINST-by-1 matrix.

Note The spread is calculated against the clean price (the function internally does not add the accrued interest to the price specified by the Price input). If the spread is required against the dirty price, you must supply the dirty price for the Price input.

Data Types: double

## Spread - Number of basis points over the reference rate numeric

Number of basis points over the reference rate, specified as a NINST-by-1 matrix.

## Data Types: double

## Settle - Settlement date of the floating-rate bonds serial date number | date character vector | datetime

Settlement date of the floating-rate bonds, specified as serial date number, date character vector, or datetime array. If supplied as a NINST-by-1 vector of dates, all settlement dates must be the same (only a single settlement date is supported)

Data Types: double | char|datetime

## Maturity - Maturity date of the floating-rate bond serial date number | date character vector | datetime

Maturity date of the floating-rate bond, specified as serial date number, date character vector, or datetime array.

Data Types: double | char|datetime

## RateInfo - Interest-rate information

numeric
interest-rate information, specified as NINST-by-2 vector where the:

- First column is the stub rate between the settlement date and the first coupon rate.
- Second column is the reference rate for the term of the floating coupons (for example, the 3-month LIBOR from settlement date for a bond with a Reset of 4).

Note If the RateInfo argument is an annualized zero-rate term structure created by intenvset, the zero discount margin is calculated.

## Data Types: double

## LatestFloatingRate - Rate for next floating payment set at last reset date numeric

Rate for the next floating payment set at the last reset date, specified as NINST-by-1 vector.

Data Types: double

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.
Example: Margin = floatdiscmargin(Price,Spread, Settle, Maturity, RateInfo, LatestFloating Rate,'Reset',2,'Basis',5)

## Reset - Frequency of payments per year

1 (default) | numeric
Frequency of payments per year, specified as NINST-by-1 vector.
Data Types: double

## Basis - Day-count basis used for time factor calculations

0 (actual/actual) (default) | integers of the set [0...13] | vector of integers of the set [0...13]

Day-count basis used for time factor calculations, specified as a NINST-by-1 vector. Values are:

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- $2=$ actual/360
- $3=$ actual $/ 365$
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- $7=$ actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- $9=$ actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- $11=30 / 360 E$ (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-0
Data Types: double

## Principal - Notional principal amounts

100 (default) | numeric
Notional principal amounts, specified as a NINST-by-1 vector or a NINST-by-1 cell array where each element is a NUMDATES-by-2 cell array where the first column is dates and the second column is the associated principal amount. The date indicates the last day that the principal value is valid.
Data Types: double | cell

## EndMonthRule - End-of-month rule flag

1 (in effect) (default) | nonnegative integer 0 or 1
End-of-month rule flag, specified as a NINST-by-1 vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days.

- $0=$ Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- $1=$ Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.


## Data Types: logical

## AdjustCashFlowsBasis - Adjust cash flows according to accrual amount

 0 (not in effect) (default) | nonnegative integer 0 or 1Adjusts cash flows according to the accrual amount, specified as a NINST-by-1 vector of logicals.

Note Usually you want to set AdjustCashFlowsBasis to 1, so cash flows are calculated with adjustments on accrual amounts. The default is set to 0 to be consistent with floatbyzero.

## Holidays - Dates for holidays

holidays.m used (default)
Dates for holidays, specified as NHOLIDAYS-by-1 vector of MATLAB dates using serial date numbers, date character vectors, or datetime arrays. Holidays are used in computing business days.

Data Types: double | char|datetime

## BusinessDayConvention - Business day conventions

'actual' (default)| character vector with values 'actual', 'follow', 'modifiedfollow', 'previous'or 'modifiedprevious'

Business day conventions, specified as a NINST-by-1 cell array of character vectors of business day conventions to be used in computing payment dates. The selection for business day convention determines how nonbusiness days are treated. Nonbusiness days are defined as weekends plus any other date that businesses are not open (for example, statutory holidays). Values are:

- 'actual' - Nonbusiness days are effectively ignored. Cash flows that fall on nonbusiness days are assumed to be distributed on the actual date.
- 'follow' - Cash flows that fall on a nonbusiness day are assumed to be distributed on the following business day.
- 'modifiedfollow' - Cash flows that fall on a non-business day are assumed to be distributed on the following business day. However if the following business day is in a different month, the previous business day is adopted instead.
- 'previous ' - Cash flows that fall on a nonbusiness day are assumed to be distributed on the previous business day.
- 'modifiedprevious' - Cash flows that fall on a nonbusiness day are assumed to be distributed on the previous business day. However if the previous business day is in a different month, the following business day is adopted instead.

Data Types: char | cell

## Output Arguments

Margin - Discount margin<br>numeric

Discount margin, returned as a NINST-by-1 vector of the discount margin if RateInfo is specified as a NINST-by-2 vector of stub and spot rates.

If RateInfo is specified as an annualized zero rate term structure created by intenvset, Margin is returned as a NINST-by-NCURVES matrix of the zero discount margin.

## References

[1] Fabozzi, Frank J., Mann, Steven V. Floating-Rate Securities. John Wiley and Sons, New York, 2000.
[2] Fabozzi, Frank J., Mann, Steven V. Introduction to Fixed Income Analytics: Relative Value Analysis, Risk Measures and Valuation. John Wiley and Sons, New York, 2010.
[3] O'Kane, Dominic, Sen, Saurav. "Credit Spreads Explained." Lehman Brothers Fixed Income Quantitative Research, March 2004.

## See Also

bndspread | datetime | floatbyzero|floatmargin|intenvset

## Topics

"Fixed-Income Terminology" on page 2-22

Introduced in R2012b

## floatmargin

Margin measures for floating-rate bond

## Syntax

[Margin,AdjPrice] = floatmargin(Price,SpreadSettle,Maturity) [Margin,AdjPrice] = floatmargin( $\qquad$ ,Name, Value)

## Description

[Margin,AdjPrice] = floatmargin(Price,SpreadSettle,Maturity) calculates margin measures for a floating-rate bond.

Use floatmargin to calculate the following types of margin measures for a floating-rate bond:

- Spread for life
- Adjusted simple margin
- Adjusted total margin

To calculate the discount margin or zero discount margin, see floatdiscmargin.
[Margin,AdjPrice] = floatmargin( $\qquad$ ,Name, Value) adds optional name-value pair arguments.

## Examples

## Compute Margin Measures for a Floating-Rate Bond

Use floatmargin to compute margin measures for spreadforlife, adjustedsimple, and adjustedtotal for a floating-rate note.

Define data for the floating-rate note.

```
Price = 99.99;
Spread = 50;
Settle = '20-Jan-2011';
Maturity = '15-Jan-2012';
LatestFloatingRate = 0.05;
StubRate = 0.049;
SpotRate = 0.05;
Reset = 4;
Basis = 2;
```

Calculate spreadforlife.

```
Margin = floatmargin(Price, Spread, Settle, Maturity, 'Reset', ...
```

Reset, 'Basis', Basis)
Margin $=51.0051$

Calculate adjustedsimple margin.

```
[Margin, AdjPrice] = floatmargin(Price, Spread, Settle, Maturity, ...
SpreadType', 'adjustedsimple', 'RateInfo', [StubRate, SpotRate], ...
'LatestFloatingRate', LatestFloatingRate, 'Reset', Reset, 'Basis', Basis)
Margin = 53.2830
AdjPrice = 99.9673
```

Calculate adjustedtotal margin.
[Margin, AdjPrice] = floatmargin(Price, Spread, Settle, Maturity, ... 'SpreadType', 'adjustedtotal', 'RateInfo', [StubRate, SpotRate], ...
'LatestFloatingRate', LatestFloatingRate, 'Reset', Reset, 'Basis', Basis)
Margin = 53.4463
AdjPrice $=99.9673$

## Compute Margin Measures for a Floating-Rate Bond Using datetime Inputs

Use floatmargin to calculate margin measures for spreadforlife, adjustedsimple, and adjustedtotal for a floating-rate note using datetime inputs.

Price = 99.99;
Spread = 50;

```
Settle = '20-Jan-2011';
Maturity = '15-Jan-2012';
LatestFloatingRate = 0.05;
StubRate = 0.049;
SpotRate = 0.05;
Reset = 4;
Basis = 2;
Settle = datetime(Settle,'Locale','en_US');
Maturity = datetime(Maturity,'Locale','en_US');
[Margin, AdjPrice] = floatmargin(Price, Spread, Settle, Maturity, ...
'SpreadType', 'adjustedsimple', 'RateInfo', [StubRate, SpotRate], ...
'LatestFloatingRate', LatestFloatingRate, 'Reset', Reset, 'Basis', Basis)
Margin = 53.2830
AdjPrice = 99.9673
```


## Input Arguments

## Price - Bond prices where spreads are to be computed matrix

Bond prices where spreads are to be computed, specified as a NINST-by-1 matrix.
Data Types: double

## Spread - Number of basis points over the reference rate numeric

Number of basis points over the reference rate, specified as a NINST-by-1 matrix.
Data Types: double
Settle - Settlement date of the floating-rate bonds
serial date number | date character vector | datetime
Settlement date of the floating-rate bonds, specified as serial date number, date character vector, or datetime array. If supplied as a NINST-by-1 vector of dates, all settlement dates must be the same (only a single settlement date is supported)

Data Types: double | char|datetime

## Maturity - Maturity date of the floating-rate bond <br> serial date number | date character vector | datetime

Maturity date of the floating-rate bond, specified as serial date number, date character vector, or datetime array.
Data Types: double | char | datetime

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Example: [Margin,AdjPrice] = floatmargin(Price,Spread,Settle,Maturity, 'SpreadType','adjustedtotal','RateInfo',
[StubRate, SpotRate],'LatestFloatingRate', .0445,'Reset', 2,'Basis', 5)

## SpreadType - Type of spread to calculate

spreadforlife (default)| adjustedsimpleadjustedtotal
Type of spread to calculates, specified by type, specified as spreadforlife,adjustedsimple, or adjustedtotal.

Note If the SpreadType is spreadforlife (default), then the name-value arguments LatestFloatingRate and RateInfo are not used. If the SpreadType is adjustedsimple or adjustedtotal, then the name-value arguments LatestFloatingRate and RateInfo must be specified.

Data Types: double
LatestFloatingRate - Rate for next floating payment set at last reset date numeric

Rate for the next floating payment set at the last reset date, specified as NINST-by-1 vector.

Note This rate must be specified for a SpreadType of adjustedsimple and adjustedtotal.

## Data Types: double

## RateInfo - Interest-rate information

numeric
interest-rate information, specified as NINST-by-2 vector where the:

- First column is the stub rate between the settlement date and the first coupon rate.
- Second column is the reference rate for the term of the floating coupons (for example, the 3-month LIBOR from settlement date for a bond with a Reset of 4).

Note The RateInfo must be specified for SpreadType of adjustedsimple and adjustedtotal.

## Data Types: double

## Reset - Frequency of payments per year

## 1 (default) | numeric

Frequency of payments per year, specified as NINST-by-1 vector.

## Data Types: double

## Basis - Day-count basis used for time factor calculations

0 (actual/actual) (default) | integers of the set [0...13] | vector of integers of the set [0...13]

Day-count basis used for time factor calculations, specified as a NINST-by-1 vector. Values are:

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- 2 = actual/360
- $3=$ actual $/ 365$
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- $9=$ actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- $11=30 / 360 \mathrm{E}$ (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-0
Data Types: double

## Principal - Notional principal amounts

100 (default) | numeric
Notional principal amounts, specified as NINST-by-1 vector.
Data Types: double

## EndMonthRule - End-of-month rule flag

1 (in effect) (default) | nonnegative integer 0 or 1
End-of-month rule flag, specified as a NINST-by-1 vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days.

- $0=$ Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- 1 = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.


## Data Types: logical

## Holidays - Dates for holidays

holidays.m used (default)
Dates for holidays, specified as NHOLIDAYS-by-1 vector of MATLAB dates using serial date numbers, date character vectors, or datetime arrays. Holidays are used in computing business days.

Data Types: double|char|datetime

## BusinessDayConvention - Business day conventions

'actual' (default)|character vector with values'actual', 'follow', 'modifiedfollow', 'previous'or 'modifiedprevious'

Business day conventions, specified as a NINST-by-1 cell array of character vectors of business day conventions to be used in computing payment dates. The selection for business day convention determines how nonbusiness days are treated. Nonbusiness days are defined as weekends plus any other date that businesses are not open (for example, statutory holidays). Values are:

- 'actual' - Nonbusiness days are effectively ignored. Cash flows that fall on nonbusiness days are assumed to be distributed on the actual date.
- 'follow' - Cash flows that fall on a nonbusiness day are assumed to be distributed on the following business day.
- 'modifiedfollow' - Cash flows that fall on a non-business day are assumed to be distributed on the following business day. However if the following business day is in a different month, the previous business day is adopted instead.
- 'previous ' - Cash flows that fall on a nonbusiness day are assumed to be distributed on the previous business day.
- 'modifiedprevious' - Cash flows that fall on a nonbusiness day are assumed to be distributed on the previous business day. However if the previous business day is in a different month, the following business day is adopted instead.

Data Types: char|cell

## Output Arguments

## Margin - Spreads for floating-rate bond <br> numeric

Spreads for the floating-rate bond, returned as a NINST-by-1 vector.

## AdjPrice - Adjusted price used to calculate spreads for SpreadType of adjustedsimple and adjustedtotal

numeric
Adjusted price used to calculate spreads for SpreadType of adjustedsimple and adjustedtotal, returned as a NINST-by-1 vector.

## References

[1] Fabozzi, Frank J., Mann, Steven V. Floating-Rate Securities. John Wiley and Sons, New York, 2000.
[2] Fabozzi, Frank J., Mann, Steven V. Introduction to Fixed Income Analytics: Relative Value Analysis, Risk Measures and Valuation. John Wiley and Sons, New York, 2010.

## See Also

bndspread|datetime|floatbyzero|floatdiscmargin

## Topics

"Fixed-Income Terminology" on page 2-22

Introduced in R2012b

## fpctkd

Fast stochastics

Note fpctkd will be removed in a future release. Use stochosc instead.

## Syntax

```
[pctk,pctd] = fpctkd(highp,lowp,closep)
[pctk,pctd] = fpctkd([highp lowp closep])
[pctk,pctd] = fpctkd(highp,lowp,closep,kperiods,dperiods, dmamethod)
[pctk,pctd] = fpctkd([highp lowp closep],kperiods,dperiods,dmamethod)
pkdts = fpctkd(tsobj,kperiods,dperiods,dmamethod)
pkdts = fpctkd(tsobj,kperiods,dperiods,dmamethod,'ParameterName',ParameterValue, ...)
```


## Arguments

| highp | High price (vector). |
| :--- | :--- |
| lowp | Low price (vector). |
| closep | Closing price (vector). |
| kperiods | (Optional) \%K periods. Default $=10$. |
| dperiods | (Optional) \%D periods. Default $=3$. |
| damethod | (Optional) \%D moving average method. Default = ' e ' (exponential). |
| tsobj | Financial time series object. |


| 'ParameterName' | Valid parameter names are: <br>  <br>  <br>  <br> - HighName: high prices series name <br> - LowName: low prices series name |
| :--- | :--- |
| ParameterValue | Parameter values are the character vectors that represent the valid <br> parameter names. |

## Description

fpctkd calculates the stochastic oscillator.
[pctk,pctd] = fpctkd(highp,lowp,closep) calculates the fast stochastics F\%K and F\%D from the stock price data highp (high prices), lowp (low prices), and closep (closing prices).
[pctk,pctd] = fpctkd([highp lowp closep]) accepts a three-column matrix of high (highp), low (lowp), and closing prices (closep), in that order.
[pctk,pctd] = fpctkd(highp,lowp,closep,kperiods,dperiods,dmamethod) calculates the fast stochastics F\%K and F\%D from the stock price data highp (high prices), lowp (low prices), and closep (closing prices). kperiods sets the \%K period. dperiods sets the \%D period.
damethod specifies the \%D moving average method. Valid moving average methods for $\%$ D are Exponential ('e') and Triangular ('t'). See tsmovavg for explanations of these methods.
[pctk,pctd]= fpctkd([highp lowp closep],kperiods,dperiods,dmamethod) accepts a three-column matrix of high (highp), low (lowp), and closing prices (closep), in that order.
pkdts = fpctkd(tsobj,kperiods,dperiods,dmamethod) calculates the fast stochastics F\%K and F\%D from the stock price data in the financial time series object tsobj. tsobj must minimally contain the series High (high prices), Low (low prices), and Close (closing prices). pkdts is a financial time series object with similar dates to tsobj and two data series named PercentK and PercentD.
pkdts =
fpctkd(tsobj,kperiods,dperiods,dmamethod,'ParameterName',ParameterVa
lue, ....) accepts parameter name/parameter value pairs as input. These pairs specify the name(s) for the required data series if it is different from the expected default name(s). Parameter values are the character vectors that represent the valid parameter names.

## Examples

## Compute the Stochastic Oscillator

This example shows how to compute the stochastic oscillator for Disney stock and plot the results.
load disney.mat
Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in dis_FastStoc = fpctkd(dis);

Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in plot(dis_FastStoc)

Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in title('Stochastic Oscillator for Disney')


## References

Achelis, Steven B. Technical Analysis from A to Z. Second Edition. McGraw-Hill, 1995, pp. 268-271.

See Also<br>spctkd|stochosc|tsmovavg

Introduced before R2006a

## frac2cur

Fractional currency value to decimal value

## Syntax

```
Decimal = frac2cur(Fraction,Denominator)
```


## Description

Decimal $=$ frac2cur(Fraction, Denominator) converts a fractional currency value to a decimal value. Fraction is the fractional currency value input as a character vector, and Denominator is the denominator of the fraction.

## Examples

## Convert a Fractional Currency Value to a Decimal Value

This example shows how to convert a fractional currency value to a decimal value.

```
Decimal = frac2cur('12.1', 8)
Decimal = 12.1250
```


## Input Arguments

## Fraction - Fractional currency values

character vector | cell array of character vectors
Fractional currency values, specified as a character vector or cell array of character vectors.

Data Types: char | cell

## Denominator - Denominator of the fractions <br> numeric

Denominator of the fractions, specified as a scalar or vector using numeric values for the denominator.

## Data Types: double

## Output Arguments

## Decimal - Decimal currency value

numeric decimal
Decimal currency value, returned as a scalar or vector with numeric decimal values.
Data Types: double

## See Also

cur2frac|cur2str

## Introduced before R2006a

## freqnum

Convert character vector frequency indicator to numeric frequency indicator

Note freqnum will be removed in a future release. Use timetable instead. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

```
nfreq = freqnum(sfreq)
```


## Arguments

| sfreq | UNKNOWN, Unknown, unknown, U, u |
| :--- | :--- |
|  | DAILY, Daily, daily, D, d |
| WEEKLY, Weekly, weekly, W, w |  |
|  | MONTHLY, Monthly, monthly, M, m |
| QUARTERLY, Quarterly, quarterly, Q, q |  |
|  | SEMIANNUAL, Semiannual, semiannual, S, s |
|  | ANNUAL, Annual, annual, A, a |

## Description

$\mathrm{nfreq}=\mathrm{fr}$ reqnum(sfreq) converts a character vector frequency indicator into a numeric value.

| Character Vector Frequency Indicator | Numeric Representation |
| :--- | :--- |
| UNKNOWN, Unknown, unknown, $\mathrm{U}, \mathrm{u}$ | 0 |

Character Vector Frequency Indicator Numeric RepresentationDAILY, Daily, daily, D, d1
WEEKLY, Weekly, weekly, W, w ..... 2
MONTHLY, Monthly, monthly, M, m ..... 3
QUARTERLY, ly, quarterly, Q, q ..... 4
SEMIANNUAL, Semiannual, semiannual, ..... 5
S, s
ANNUAL, Annual, annual, A, a ..... 6
See Also
freqstr
Topics
"What Is the Financial Time Series App?" on page 14-2
Introduced before R2006a

## freqstr

Convert numeric frequency indicator to character vector representation

Note freqstr will be removed in a future release. Use timetable instead. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

sfreq = freqstr(nfreq)

## Arguments

| nfreq | 0 |
| :--- | :--- |
|  | 1 |
|  | 2 |
| 3 |  |
|  | 4 |
| 5 |  |
|  | 6 |

## Description

sfreq $=$ freqstr(nfreq) converts a numeric frequency indicator into a character vector representation.

| Numeric Frequency <br> Indicator | Character Vector Representation |
| :--- | :--- |
| 0 | Unknown |
| 1 | Daily |
| 2 | Weekly |
| 3 | Monthly |
| 4 | Quarterly |
| 5 | Semiannual |
| 6 | Annual |

## See Also

freqnum

## Topics

"What Is the Financial Time Series App?" on page 14-2

Introduced before R2006a

## frontier

Rolling efficient frontier

## Syntax

[PortWts,AllMean,AllCovariance] = frontier(Universe,Window,Offset,NumPorts,ActiveMap,C

## Arguments

| Universe | Number of observations (NUMOBS) by number of assets plus one <br> (NASSETS + 1) time series array containing total return data for a <br> group of securities. Each row represents an observation. Column 1 <br> contains MATLAB serial date numbers. The remaining columns <br> contain the total return data for each security. |
| :--- | :--- |
| Window | Number of data periods used to calculate each frontier. |
| Offset | Increment in number of periods between each frontier. |
| NumPorts | Number of portfolios to calculate on each frontier. |
| ActiveMap | (Optional) Number of observations (NUMOBS) by number of assets <br> (NASSETS) matrix with Boolean elements corresponding to the <br> Universe. Each element indicates if the asset is part of the <br> Universe on the corresponding date. Default = NUMOBS-by- <br> NASSETS matrix of 1's (all assets active on all dates). |
| Conset | (Optional) Constraint matrix for a portfolio of asset investments, <br> created using portcons with the ' Default constraint type. This <br> single constraint matrix is applied to each frontier. |
| NumNonNan | (Optional) Minimum number of non-NaN points for each active asset <br> in each window of data needed to perform the optimization. The <br> default value is Window - NASSETS. |

## Description

[PortWts,AllMean,AllCovariance] = frontier(Universe, Window, Offset, NumPorts, ActiveMap, ConSet, NumNonNan) generates a surface of efficient frontiers showing how asset allocation influences risk and return over time.

PortWts is a number of curves (NCURVES)-by-1 cell array, where each element is a NPORTS-by-NASSETS matrix of weights allocated to each asset.

AllMean is a NCURVES-by-1 cell array, where each element is a 1-by-NASSETS vector of the expected asset returns used to generate each curve on the surface.

AllCovariance is a NCURVES-by-1 cell array, where each element is a NASSETS-byNASSETS vector of the covariance matrix used to generate each curve on the surface.

## See Also

portcons|portopt|portstats

## Topics

"Portfolio Construction Examples" on page 3-7
"Portfolio Selection and Risk Aversion" on page 3-9
"Active Returns and Tracking Error Efficient Frontier" on page 3-43
"Analyzing Portfolios" on page 3-2
"Portfolio Optimization Functions" on page 3-4
Introduced before R2006a

## fts2ascii

Write elements of time series data into ASCII file

Note fts2ascii will be removed in a future release. Use timetable instead. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

```
stat = fts2ascii(filename,tsobj,exttext)
stat = fts2ascii(filename,dates,data,colheads,desc,exttext)
```


## Arguments

| filename | Name of an ASCII file |
| :--- | :--- |
| tsobj | Financial time series object |
| exttext | (Optional) Extra text written after the description line (line 2 in the <br> file). |
| dates | Column vector containing dates. Dates must be in serial date <br> number format and can specify time of day. |
| data | Column-oriented matrix. Each column is a series. |
| colheads | (Optional) Cell array of column headers (names); first cell must <br> always be the one for the dates column. colheads is written to the <br> file just before the data. |
| desc | (Optional) Description text, which is the first line in the file. |

## Description

stat $=$ fts2ascii(filename,tsobj,exttext) writes the financial time series object tsobj into an ASCII file filename. The data in the file is tab delimited.
stat $=$ fts2ascii(filename,dates,data, colheads,desc,exttext) writes into an ASCII file filename the dates, times, and data contained in the column vector dates and the column-oriented matrix data. The first column in filename contains the dates, followed by times (if specified). Subsequent columns contain the data. The data in the file is tab delimited.
stat indicates whether file creation is successful (1) or not (0).

## Examples

## Use fts2ascii to Write a Time Series to an ASCII File

Create a data file with time information.

```
dates = ['01-Jan-2001';'01-Jan-2001'; '02-Jan-2001'; ...
'02-Jan-2001'; '03-Jan-2001';'03-Jan-2001'];
times = ['11:00';'12:00';'11:00';'12:00';'11:00';'12:00'];
serial_dates_times = [datenum(dates), datenum(times)];
data = round(10*rand (6,2));
```

Use fts2ascii to write the time series to an ascii file.

```
stat = fts2ascii('myfts_file2.txt',serial_dates_times,data, ...
```

\{'dates';'times';'Data1';'Data2'\},'My FTS with Time')

Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in
stat $=1$

Read the data file back and create a financial time series object using ascii2fts.

```
MyFts = ascii2fts('myfts_file2.txt','t',1,2,1)
```

Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in
MyFts =
desc: My FTS with Time
freq: Unknown (0)
'dates: (6)' 'times: (6)' 'Data1: (6)' 'Data2: (6)'

| $' 01-J a n-200 '^{\prime}$ | $' 11: 00 '$ | $[$ | $8]$ | $[$ |
| :---: | :---: | :---: | :---: | :---: |

## See Also

ascii2fts

## Topics

"Working with Financial Time Series Objects" on page 13-2 "Creating a Financial Time Series Object" on page 14-11 "What Is the Financial Time Series App?" on page 14-2

## Introduced before R2006a

## fts2timetable

Convert financial time series objects (fints) to timetable object

## Syntax

TT $=$ fts2timetable(FTS)

## Description

TT = fts2timetable(FTS) converts a M-by-N fints object to a M-by-N MATLAB timetable object (TT). The date and time information in the fints object becomes the TT time vector, while the remaining variables in the fints object become variables in TT. The fints description property (DESC) is mapped to TT. Properties. Description. The fints frequency indicator property (FREQ) is unused and removed from the output TT. For more information, see "Convert Financial Time Series Objects fints to Timetables" on page 12-2.

## Examples

## Convert fints Object to a timetable Object and Plot the Financial Data

Load the financial data.
load SimulatedStockValues
Convert a fints object to a timetable object.
TMW = fts2timetable(TMW_fts);
TMW.Properties
ans =
TimetableProperties with properties:
Description: 'Simulated stock OHLCV data.'
UserData: []

```
    DimensionNames: {'Time' 'Variables'}
    VariableNames: {'Open' 'High' 'Low' 'Close' 'Volume'}
VariableDescriptions: {}
            VariableUnits: {}
VariableContinuity: []
            RowTimes: [1000x1 datetime]
            StartTime: 31-Aug-2012
            SampleRate: NaN
            TimeStep: NaN
    CustomProperties: No custom properties are set.
Use addprop and rmprop to modify CustomProperties.
```

Visualize the financial data.

```
mal = movavg(TMW,'exponential',14);
ma2 = movavg(TMW,'exponential',26);
selctRange = 70:120;
figure;
plot(mal.Time(selctRange), mal.Close(selctRange), ...
    ma2.Time(selctRange), ma2.Close(selctRange));
ax = gca;
hold on
% Plot on a target axis
candle(ax, TMW(selctRange,:));
legend(ax, '14-day MA', '26-day MA', 'Price')
ylabel(ax, 'Price')
xlabel(ax, 'Date')
title(ax, 'TMW Simulated Stock Prices')
hold off
```



## Input Arguments

## FTS - Financial time series object

object
Financial time series object, specified by using a M-by-N fints object.
Data Types: object

## Output Arguments

TT - Timetable<br>object<br>Timetable, returned as a M-by-N object.

See Also<br>timetable<br>\section*{Topics}<br>"Convert Financial Time Series Objects fints to Timetables" on page 12-2<br>Introduced in R2018a

## fts2mat

Convert to matrix

Note fts2mat will be removed in a future release. Use timetable instead. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

```
tsmat = fts2mat(tsobj)
tsmat = fts2mat(tsobj,datesflag)
tsmat = fts2mat(tsobj,seriesnames)
tsmat = fts2mat(tsobj,datesflag,seriesnames)
```


## Arguments

| tsobj | Financial time series object |
| :--- | :--- |
| datesflag | (Optional) Specifies inclusion of dates vector: <br> datesflag $=0$ (default) excludes dates. <br> datesflag $=1$ includes dates vector. |
| seriesnames | (Optional) Specifies the data series to be included in the matrix. Can <br> be a cell array of character vectors. |

## Description

tsmat $=$ fts2mat(tsobj) takes the data series in the financial time series object tsobj and puts them into the matrix tsmat as columns. The order of the columns is the same as the order of the data series in the object tsobj.
tsmat $=$ fts2mat(tsobj, datesflag) specifies whether you want the dates vector included. The dates vector is the first column. The dates are represented as serial date numbers. Dates can include time-of-day information.
tsmat $=$ fts2mat(tsobj, seriesnames) extracts the data series named in seriesnames and puts its values into tsmat. The seriesnames argument can be a cell array of character vectors.
tsmat $=$ fts2mat(tsobj, datesflag, seriesnames) puts into tsmat the specific data series named in seriesnames. The datesflag argument must be specified. If datesflag is set to 1 , the dates vector is included. If you specify an empty matrix ([ ]) for datesflag, the default behavior is adopted.

## See Also

subsref

## Topics

"Working with Financial Time Series Objects" on page 13-2
"Creating a Financial Time Series Object" on page 14-11
"What Is the Financial Time Series App?" on page 14-2

## Introduced before R2006a

## ftsbound

Start and end dates

Note ftsbound will be removed in a future release. Use timetable instead. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

```
datesbound = ftsbound(tsobj)
datesbound = ftsbound(tsobj,dateform)
```


## Arguments

| tsobj | Financial time series object |
| :--- | :--- |
| dateform | dateform is an integer representing the format of a date character <br> vector. See datestr for a description of these formats. |

## Description

ftsbound returns the start and end dates of a financial time series object. If the object contains time-of-day data, ftsbound also returns the starting time on the first date and the ending time on the last date.
datesbound $=$ ftsbound(tsobj) returns the start and end dates contained in tsobj as serial dates in the column matrix datesbound. The first row in datesbound corresponds to the start date, and the second corresponds to the end date.
datesbound $=$ ftsbound(tsobj,dateform) returns the starting and ending dates contained in the object, tsobj, as date character vectors in the column matrix, datesbound. The first row in datesbound corresponds to the start date, and the second corresponds to the end date. The dateform argument controls the format of the output dates.
See Also
datestr
Topics"What Is the Financial Time Series App?" on page 14-2
Introduced before R2006a

## ftsgui

Financial time series GUI

Note ftsgui will be removed in a future release and will no longer accept a fints object.

Use fts2timetable to convert a fints object to a timetable object. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

ftsgui

## Description

ftsgui displays the financial time series graphical user interface (GUI) main window.
The use of the financial time series GUI is described in "Using the Financial Time Series GUI" on page 15-7.

Note Help menu item is not available for ftsgui in MATLAB Online.

## Examples

ftsgui


## See Also

ftstool

## Topics

"Using the Financial Time Series App" on page 14-11
"Using the Financial Time Series App with GUIs" on page 14-19
"What Is the Financial Time Series App?" on page 14-2
Introduced before R2006a

## ftsinfo

Financial time series object information

Note ftsinfo will be removed in a future release. Use timetable instead. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

ftsinfo(tsobj)
infofts = ftsinfo(tsobj)

## Arguments

| tsobj | Financial time series object. |
| :--- | :--- |

## Description

ftsinfo(tsobj) displays information about the financial time series object tsobj.
infofts $=$ ftsinfo(tsobj) stores information about the financial time series object tsobj in the structure infofts.
infofts has these fields.

| Field | Contents |
| :--- | :--- |
| version | Financial time series object version. |
| desc | Description of the time series object (tsobj. desc). |
| freq | Numeric representation of the time series data frequency <br> (tsobj. freq). See freqstr for list of numeric frequencies and <br> what they represent. |

## Field

startdate enddate seriesnames ndata nseries

## Contents

Earliest date in the time series.
Latest date in the time series.
Cell array containing the time series data column names.
Number of data points in the time series.
Number of columns of time series data.

## Examples

Convert the supplied file disney. dat into a financial time series object named dis:

```
dis = ascii2fts('disney.dat', 1, 3);
```

Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In ascii2fts (line 64)

Now use ftsinfo to obtain information about dis:

```
ftsinfo(dis)
```

    Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
    > In fints/ftsinfo (line 64)
FINTS version: 2.0 or higher
Description: Walt Disney Company (DIS)
Frequency: Unknown
Start date: 29-Mar-1996
End date: 29-Mar-1999
Series names: OPEN
HIGH
LOW
CLOSE
VOLUME
\# of data: 782
\# of series: 5

Then, executing
infodis $=$ ftsinfo(dis)
creates the structure infodis containing the values

```
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/ftsinfo (line 64)
infodis =
    struct with fields:
            ver: '2.0 or higher'
            desc: 'Walt Disney Company (DIS)
            freq: 0
        startdate: '29-Mar-1996'
        enddate: '29-Mar-1999'
    seriesnames: {5×1 cell}
            ndata: 782
        nseries: 5
```


## See Also

fints|freqnum|freqstr|ftsbound

## Topics

"Using the Financial Time Series App" on page 14-11
"Using the Financial Time Series App with GUIs" on page 14-19
"What Is the Financial Time Series App?" on page 14-2

Introduced before R2006a

## ftstool

Financial Time Series app

Note ftstool will be removed in a future release and will no longer accept a fints object.

Use fts2timetable to convert a fints object to a timetable object. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

ftstool

## Description

ftstool creates and manages Financial Time Series objects. ftstool allows the creation and management of Financial Time Series objects via the Financial Time Series app. ftstool can interface with ftsgui, meaning Line Plots generated with ftstool can be analyzed with ftsgui. However, ftsgui must be running prior to the generation of any Line Plots.

The use of the Financial Time Series app is described in "Getting Started with the Financial Time Series App" on page 14-4.

Note Help menu item is not available for ftstool in MATLAB Online.

## Examples

ftstool


Alternatively, on the MATLAB desktop toolstrip, click the Apps tab and in the apps gallery, under Computational Finance, click Financial Time Series. The Financial Time Series app opens. For an overview of the Financial Time Series app, see "What Is the Financial Time Series App?" on page 14-2.

## See Also

ftsgui

## Topics

"Using the Financial Time Series App" on page 14-11
"Using the Financial Time Series App with GUIs" on page 14-19
"What Is the Financial Time Series App?" on page 14-2

Introduced in R2006b

## ftsuniq

Determine uniqueness

Note ftsuniq will be removed in a future release. Use timetable instead. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

```
uniq = ftsuniq(dates_and_times)
[uniq,dup] = ftsuniq(dates_and_times)
```


## Arguments

| dates_and_times | A single column vector of serial date numbers. The serial <br> date numbers can include time-of-day information. |
| :--- | :--- |

## Description

uniq $=$ ftsuniq(dates_and_times) returns 1 if the dates and times within the financial time series object are unique and 0 if duplicates exist.
[uniq,dup] = ftsuniq(dates_and_times) additionally returns a structure dup. In the structure

- dup. DT contains the character vectors of the duplicate dates and times and their locations in the object.
- dup.intIdx contains the integer indices of duplicate dates and times in the object.


## See Also

fints

## Topics

"What Is the Financial Time Series App?" on page 14-2

Introduced before R2006a

## fvdisc

Future value of discounted security

## Syntax

FutureVal = fvdisc(Settle,Maturity,Price,Discount,Basis)

## Arguments

| Settle | Settlement date. Enter as serial date number, date character vector, <br> or datetime array. Settle must be earlier than Maturity. |
| :--- | :--- |
| Maturity | Maturity date. Enter as serial date number, date character vector, or <br> datetime array. |
| Price | Price (present value) of the security. |
| Discount | Bank discount rate of the security. Enter as decimal fraction. |


| Basis | (Optional) Day-count basis of the instrument. A vector of integers. <br> - $0=$ actual/actual (default) <br> - $1=30 / 360$ (SIA) <br> - $2=$ actual/360 <br> - 3 = actual/365 <br> - $4=30 / 360$ (PSA) <br> - $5=30 / 360$ (ISDA) <br> - $6=30 / 360$ (European) <br> - 7 = actual/365 (Japanese) <br> - $8=$ actual/actual (ICMA) <br> - $9=$ actual/360 (ICMA) <br> - $10=$ actual/365 (ICMA) <br> - $11=30 / 360 E$ (ICMA) <br> - $12=$ actual/365 (ISDA) <br> - 13 = BUS/252 <br> For more information, see basis on page Glossary-0 |
| :---: | :---: |

## Description

FutureVal = fvdisc(Settle,Maturity,Price,Discount,Basis) finds the amount received at maturity for a fully vested security.

## Examples

## Find the Amount Received at Maturity for a Fully-Vested Security

This example shows how to find the amount received at maturity for a fully-vested security, using the following data.

Settle = '02/15/2001';
Maturity = '05/15/2001';

```
Price = 100;
Discount = 0.0575;
Basis = 2;
```

FutureVal = fvdisc(Settle, Maturity, Price, Discount, Basis)
FutureVal = 101.4420

## Find the Amount Received at Maturity for a Fully-Vested Security Using datetime Inputs

This example shows how to use datetime inputs to find the amount received at maturity for a fully-vested security, using the following data.

```
Settle = datetime('02/15/2001','Locale','en_US');
Maturity = datetime('05/15/2001','Locale','en_US');
Price = 100;
Discount = 0.0575;
Basis = 2;
FutureVal = fvdisc(Settle, Maturity, Price, Discount, Basis)
FutureVal = 101.4420
```


## References

Mayle. Standard Securities Calculation Methods. Volumes I-II, 3rd edition.

## See Also

acrudisc|datetime|discrate|prdisc|ylddisc

## Topics

"Analyzing and Computing Cash Flows" on page 2-18

Introduced before R2006a

## fvfix

Future value with fixed periodic payments

## Syntax

```
FutureVal = fvfix(Rate,NumPeriods,Payment,PresentVal,Due)
```


## Arguments

| Rate | Periodic interest rate, as a decimal fraction. |
| :--- | :--- |
| NumPeriods | Number of periods. |
| Payment | Periodic payment. |
| PresentVal | (Optional) Initial value. Default $=0$. |
| Due | (Optional) When payments are due or made: $0=$ end of period <br> (default), or $1=$ beginning of period. |

## Description

FutureVal = fvfix(Rate,NumPeriods,Payment,PresentVal,Due) returns the future value of a series of equal payments.

## Examples

## Return the Future Value of a Series of Equal Payments

This example shows how to compute the future value of a series of equal payments using a savings account that has a starting balance of $\$ 1500$. $\$ 200$ is added at the end of each month for 10 years and the account pays $9 \%$ interest compounded monthly.

FutureVal = fvfix(0.09/12, 12*10, 200, 1500, 0)

```
FutureVal = 4.2380e+04
```


## See Also

fvvar|pvfix|pvvar

## Topics

"Analyzing and Computing Cash Flows" on page 2-18
Introduced before R2006a

## fvvar

Future value of varying cash flow

## Syntax

FutureVal = fvvar(CashFlow, Rate, CFDates)

## Arguments

| CashFlow | A vector of varying cash flows. Include the initial investment as the <br> initial cash flow value (a negative number). |
| :--- | :--- |
| Rate | Periodic interest rate. Enter as a decimal fraction. |
| CFDates | (Optional) For irregular (nonperiodic) cash flows, a vector of dates <br> on which the cash flows occur. Enter dates as serial date numbers, <br> date character vectors, or datetime arrays. Default assumes <br> CashFlow contains regular (periodic) cash flows. |

## Description

FutureVal = fvvar(CashFlow,Rate,CFDates) returns the future value of a varying cash flow.

## Examples

This cash flow represents the yearly income from an initial investment of $\$ 10,000$. The annual interest rate is $8 \%$.

| Year 1 | $\$ 2000$ |
| :--- | :--- |
| Year 2 | $\$ 1500$ |
| Year 3 | $\$ 3000$ |


| Year 4 | $\$ 3800$ |
| :--- | :--- |
| Year 5 | $\$ 5000$ |

For the future value of this regular (periodic) cash flow
FutureVal = fvvar([-10000 2000150030003800 5000], 0.08)
returns
FutureVal =
2520.47

An investment of $\$ 10,000$ returns this irregular cash flow. The original investment and its date are included. The periodic interest rate is $9 \%$.

| Cash Flow | Dates |
| :--- | :--- |
| $(\$ 10000)$ | January 12, 2000 |
| $\$ 2500$ | February 14, 2001 |
| $\$ 2000$ | March 3, 2001 |
| $\$ 3000$ | June 14, 2001 |
| $\$ 4000$ | December 1, 2001 |

To calculate the future value of this irregular (nonperiodic) cash flow
CashFlow = [-10000, 2500, 2000, 3000, 4000];
CFDates = ['01/12/2000'
'02/14/2001'
'03/03/2001'
'06/14/2001'
'12/01/2001'];
FutureVal = fvvar(CashFlow, 0.09, CFDates)
returns
FutureVal =
170.66

See Also<br>datetime|fvfix|irr|payuni|pvfix|pvvar<br>Topics<br>"Analyzing and Computing Cash Flows" on page 2-18<br>Introduced before R2006a

## fwd2zero

Zero curve given forward curve

Note In R2017b, the specification of optional input arguments has changed. While the previous ordered inputs syntax is still supported, it may no longer be supported in a future release. Use the new optional name-value pair inputs: InputCompounding, InputBasis, OutputCompounding, and OutputBasis.

## Syntax

[ZeroRates,CurveDates] = fwd2zero(ForwardRates,CurveDates,Settle) [ZeroRates,CurveDates] = fwd2zero( $\qquad$ , Name, Value)

## Description

[ZeroRates,CurveDates] = fwd2zero(ForwardRates,CurveDates,Settle) returns a zero curve given an implied forward rate curve and its maturity dates. If both inputs for CurveDates and Settle are serial date numbers or date character vectors, CurveDates is returned as serial date numbers. However, if either of the inputs for CurveDates and Settle are a datetime array, CurveDates is returned as a datetime array.
[ZeroRates,CurveDates] = fwd2zero( $\qquad$ , Name, Value) adds optional namevalue pair arguments

## Examples

## Compute the Zero Curve Given the Forward Curve

This example shows how to compute the zero curve, given an implied forward rate curve over a set of maturity dates, a settlement date, and a compounding rate.

ForwardRates = [0.0469
0.0519

```
    0.0549
    0.0535
    0.0558
    0.0508
    0.0560
    0.0545
    0.0615
    0.0486];
CurveDates = [datenum('06-Nov-2000')
        datenum('11-Dec-2000')
        datenum('15-Jan-2001')
        datenum('05-Feb-2001')
        datenum('04-Mar-2001')
        datenum('02-Apr-2001')
        datenum('30-Apr-2001')
        datenum('25-Jun-2001')
        datenum('04-Sep-2001')
        datenum('12-Nov-2001')];
Settle = datenum('03-Nov-2000');
InputCompounding = 1;
InputBasis = 2;
OutputCompounding = 1;
OutputBasis = 2;
```

Execute the function fwd2zero to return the zero-rate curve ZeroRates at the maturity dates CurveDates.
[ZeroRates, CurveDates] = fwd2zero(ForwardRates, CurveDates,...
Settle,'InputCompounding',1,'InputBasis',2,'OutputCompounding',1,'OutputBasis',2)
ZeroRates = 10×1
0.0469
0.0515
0.0531
0.0532
0.0538
0.0532
0.0536
0.0539
0.0556
0.0543

```
CurveDates = 10×1
```

730796
730831
730866
730887
730914
730943
730971
731027
731098
731167

## Compute the Zero Curve Given the Forward Curve Using datetime Inputs

This example shows how to use datetime inputs compute the zero curve, given an implied forward rate curve over a set of maturity dates, a settlement date, and a compounding rate.

```
ForwardRates = [0.0469
    0.0519
    0.0549
    0.0535
    0.0558
    0.0508
    0.0560
    0.0545
    0.0615
    0.0486];
```

CurveDates = [datenum('06-Nov-2000')
datenum('11-Dec-2000')
datenum('15-Jan-2001')
datenum('05-Feb-2001')
datenum('04-Mar-2001')
datenum('02-Apr-2001')
datenum('30-Apr-2001')
datenum('25-Jun-2001')

```
    datenum('04-Sep-2001')
datenum('12-Nov-2001')];
Settle = datenum('03-Nov-2000');
InputCompounding = 1;
InputBasis = 2;
OutputCompounding = 1;
OutputBasis = 2;CurveDates = datetime(CurveDates,'ConvertFrom','datenum','Locale','en
Settle = datetime(Settle,'ConvertFrom','datenum','Locale','en US');
[ZeroRates, CurveDates] = fwd2zero(ForwardRates, CurveDates
Settle,'InputCompounding',1,'InputBasis',2,'OutputCompounding',1,'OutputBasis',2)
ZeroRates = 10×1
    0.0469
    0.0515
    0.0531
    0.0532
    0.0538
    0.0532
    0.0536
    0.0539
    0.0556
    0.0543
CurveDates = 10x1 datetime array
    06-Nov-2000 00:00:00
    11-Dec-2000 00:00:00
    15-Jan-2001 00:00:00
    05-Feb-2001 00:00:00
    04-Mar-2001 00:00:00
    02-Apr-2001 00:00:00
    30-Apr-2001 00:00:00
    25-Jun-2001 00:00:00
    04-Sep-2001 00:00:00
    12-Nov-2001 00:00:00
```


## Input Arguments

## ForwardRates - Annualized implied forward rates decimal fraction

Annualized implied forward rates, specified as a (NUMBONDS)-by-1 vector using decimal fractions. In aggregate, the rates in ForwardRates constitute an implied forward curve for the investment horizon represented by CurveDates. The first element pertains to forward rates from the settlement date to the first curve date.

## Data Types: double

## CurveDates - Maturity dates <br> serial date number | date character vector | datetime

Maturity dates, specified as a NUMBONDS-by-1 vector using serial date numbers, date character vectors, or datetime arrays, that correspond to the ForwardRates.
Data Types: double \| datetime | char

## Settle - Common settlement date for ForwardRates <br> serial date number | date character vector | datetime

Common settlement date for ForwardRates, specified as serial date numbers, date character vectors, or datetime arrays.
Data Types: double \| datetime | char

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1,Value1, ... , NameN, ValueN.

Example: [ZeroRates,CurveDates] = fwd2zero(ForwardRates,CurveDates, Settle,' InputCompounding', 3,'InputBasis',5,'OutputCompounding',4,'OutputBasis',5)

## InputCompounding - Compounding frequency of input forward rates 2 (default) | numeric values: $0,1,2,3,4,6,12,365,-1$

Compounding frequency of input forward rates, specified with allowed values:

- 0 - Simple interest (no compounding)
- 1 - Annual compounding
- 2 - Semiannual compounding (default)
- 3 - Compounding three times per year
- 4 - Quarterly compounding
- 6 - Bimonthly compounding
- 12 - Monthly compounding
- 365 - Daily compounding
-     - 1 - Continuous compounding

Note If InputCompounding is not specified, then InputCompounding is assigned the value specified for OutputCompounding. If either InputCompounding or OutputCompounding are not specified, the default is 2

## Data Types: double

## InputBasis - Day-count basis of input forward rates

## 0 (default) | numeric values: $0,1,2,3,4,6,7,8,9,10,11,12,13$

Day count basis of input forward rates, specified as a numeric value. Allowed values are:

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- 2 = actual/360
- 3 = actual/365
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- $9=$ actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- $12=$ actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-0

Note If InputBasis is not specified, then InputBasis is assigned the value specified for OutputBasis. If either InputBasis or Outputbasis are not specified, the default is 0 (actual/actual) for both.

## Data Types: double

## OutputCompounding - Compounding frequency of output zero rates

2 (default) | numeric values: $0,1,2,3,4,6,12,365,-1$
Compounding frequency of output zero rates, specified with the allowed values:

- 0 - Simple interest (no compounding)
- 1 - Annual compounding
- 2 - Semiannual compounding (default)
- 3-Compounding three times per year
- 4-Quarterly compounding
- 6-Bimonthly compounding
- 12 - Monthly compounding
- 365 - Daily compounding
-     - 1 - Continuous compounding

Note If OutputCompounding is not specified, then OutputCompounding is assigned the value specified for InputCompounding. If either InputCompounding or OutputCompounding are not specified, the default is 2 (semiannual) for both.

## Data Types: double

## OutputBasis - Day-count basis of output zero rates

```
0 (default) | numeric values: \(0,1,2,3,4,6,7,8,9,10,11,12,13\)
```

Day count basis of output zero rates, specified as a numeric value. Allowed values are:

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- 2 = actual/360
- 3 = actual/365
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-0

Note If OutputBasis is not specified, then OutputBasis is assigned the value specified for InputBasis. If either InputBasis or OutputBasis are not specified, the default is 0 (actual/actual) for both.

## Data Types: double

## Output Arguments

## ZeroRates - Zero curve for investment horizon represented by CurveDates numeric

Zero curve for the investment horizon represented by CurveDates, returned as a NUMBONDS-by-1 vector of decimal fractions. In aggregate, the rates in ZeroRates constitute a zero curve for the investment horizon represented by CurveDates.

## CurveDates - Maturity dates that correspond to ZeroRates

serial date number | date character vector | datetime

Maturity dates that correspond to the ZeroRates, returned as a NUMBONDS-by-1 vector of maturity dates that correspond to the zero rates in ZeroRates. This vector is the same as the input vector CurveDates, but is sorted by ascending maturity.

If both inputs for CurveDates and Settle are serial date numbers or date character vectors, CurveDates is returned as serial date numbers. However, if either of the inputs for CurveDates and Settle are a datetime array, CurveDates is returned as a datetime array.

```
See Also
prbyzero|pyld2zero|zbtprice|zbtyield|zero2disc|zero2fwd|zero2fwd|
zero2pyld
```


## Topics

"Term Structure of Interest Rates" on page 2-42
"Fixed-Income Terminology" on page 2-22

## Introduced before R2006a

## geom2arith

Geometric to arithmetic moments of asset returns

## Syntax

```
[ma,Ca] = geom2arith(mg,Cg)
[ma,Ca] = geom2arith(mg,Cg,t)
```


## Arguments

| mg | Continuously compounded or "geometric" mean of asset returns <br> (positive n-vector). |
| :--- | :--- |
| Cg | Continuously compounded or "geometric" covariance of asset <br> returns, a n-by-n symmetric, positive-semidefinite matrix. |
| t | (Optional) Target period of arithmetic moments in terms of <br> periodicity of geometric moments with default value 1 (positive <br> scalar). |

## Description

geom2arith transforms moments associated with a continuously compounded geometric Brownian motion into equivalent moments associated with a simple Brownian motion with a possible change in periodicity.
[ma,Ca] = geom2arith(mg, Cg,t) returns ma, arithmetic mean of asset returns over the target period ( n -vector), and Ca, which is an arithmetic covariance of asset returns over the target period (n-by-n matrix).

Geometric returns over period $t_{G}$ are modeled as multivariate lognormal random variables with moments

$$
E[\mathrm{Y}]=1+\mathrm{m}_{G}
$$

and

$$
\operatorname{cov}(\mathrm{Y})=\mathrm{C}_{G}
$$

Arithmetic returns over period $t_{A}$ are modeled as multivariate normal random variables with moments

$$
\begin{aligned}
& E[\mathrm{X}]=\mathrm{m}_{A} \\
& \operatorname{cov}(\mathrm{X})=\mathrm{C}_{A}
\end{aligned}
$$

Given $t=t_{A} / t_{G}$, the transformation from geometric to arithmetic moments is

$$
\begin{aligned}
& \mathrm{C}_{A_{i j}}=t \log \left(1+\frac{\mathrm{C}_{G_{i j}}}{\left(1+\mathrm{m}_{G_{i}}\right)\left(1+\mathrm{m}_{G_{j}}\right)}\right) \\
& \mathrm{m}_{A_{i}}=t \log \left(1+\mathrm{m}_{G_{i}}\right)-\frac{1}{2} \mathrm{C}_{A_{i i}}
\end{aligned}
$$

For $i, j=1, \ldots, n$.

Note If $t=1$, then $\mathbf{X}=\log (\mathbf{Y})$.

This function requires that the input mean must satisfy $1+m g>0$ and that the input covariance Cg must be a symmetric, positive, semidefinite matrix.

The functions geom2arith and arith2geom are complementary so that, given $\mathrm{m}, \mathrm{C}$, and $t$, the sequence

```
[ma,Ca] = geom2arith(m,C,t);
[mg,Cg] = arith2geom(ma,Ca,1/t);
yields mg=m and Cg=C.
```


## Examples

Example 1. Given geometric mean $m$ and covariance $C$ of monthly total returns, obtain annual arithmetic mean ma and covariance Ca. In this case, the output period (1 year) is 12 times the input period ( 1 month) so that $\mathrm{t}=12$ with
[ma, Ca] = geom2arith(m, C, 12);
Example 2. Given annual geometric mean $m$ and covariance $C$ of asset returns, obtain monthly arithmetic mean ma and covariance Ca. In this case, the output period (1 month) is $1 / 12$ times the input period ( 1 year) so that $\mathrm{t}=1 / 12$ with

```
[ma, Ca] = geom2arith(m, C, 1/12);
```

Example 3. Given geometric means $m$ and standard deviations $s$ of daily total returns (derived from 260 business days per year), obtain annualized arithmetic mean ma and standard deviations sa with

```
[ma, Ca] = geom2arith(m, diag(s .^2), 260);
sa = sqrt(diag(Ca));
```

Example 4. Given geometric mean $m$ and covariance $C$ of monthly total returns, obtain quarterly arithmetic return moments. In this case, the output is 3 of the input periods so that $t=3$ with

```
[ma, Ca] = geom2arith(m, C, 3);
```

Example 5. Given geometric mean $m$ and covariance C of 1254 observations of daily total returns over a 5 -year period, obtain annualized arithmetic return moments. Since the periodicity of the geometric data is based on 1254 observations for a 5 -year period, a 1year period for arithmetic returns implies a target period of $t=1254 / 5$ so that

```
[ma, Ca] = geom2arith(m, C, 1254/5);
```


## See Also

arith2geom

## Topics

"Data Transformation and Frequency Conversion" on page 13-13

Introduced before R2006a

## getAssetMoments

Obtain mean and covariance of asset returns from Portfolio object
Use the getAssetMoments function with a Portfolio object to obtain mean and covariance of asset returns.

For details on the workflow, see "Portfolio Object Workflow" on page 4-21.

## Syntax

[AssetMean,AssetCovar] = getAssetMoments(obj)

## Description

[AssetMean,AssetCovar] = getAssetMoments(obj) obtains mean and covariance of asset returns for a Portfolio object.

## Examples

## Obtain Asset Moment Properties for a Portfolio Object

Given the mean and covariance of asset returns in the variables $m$ and $C$, the asset moment properties can be set and then obtained using the getAssetMoments function:

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;
p = Portfolio;
p = setAssetMoments(p, m, C);
[assetmean, assetcovar] = getAssetMoments(p)
```

```
assetmean = 4×1
    0.0042
    0.0083
    0.0100
    0.0150
assetcovar = 4×4
\begin{tabular}{rrrr}
0.0005 & 0.0003 & 0.0002 & 0 \\
0.0003 & 0.0024 & 0.0017 & 0.0010 \\
0.0002 & 0.0017 & 0.0048 & 0.0028 \\
0 & 0.0010 & 0.0028 & 0.0102
\end{tabular}
```


## Input Arguments

## obj - Object for portfolio

object
Object for portfolio, specified using a Portfolio object. For more information on creating a portfolio object, see

- Portfolio

Data Types: object

## Output Arguments

## AssetMean - Mean of asset returns

vector

Mean of asset returns, returned as a vector.

## AssetCovar - Covariance of asset returns

matrix
Covariance of asset returns, returned as a matrix.

## Tips

You can also use dot notation to obtain the mean and covariance of asset returns from a Portfolio object:
[AssetMean, AssetCovar] = obj.getAssetMoments;

## See Also

setAssetMoments

## Topics

"Asset Returns and Moments of Asset Returns Using Portfolio Object" on page 4-48
"Portfolio Optimization Examples" on page 4-167
"Portfolio Optimization Theory" on page 4-3

## Introduced in R2011a

## getBounds

Obtain bounds for portfolio weights from portfolio object
Use the getBounds function with a Portfolio, PortfolioCVaR, or PortfolioMAD object to obtain bounds for portfolio weights from portfolio objects.

For details on the respective workflows when using these different objects, see "Portfolio Object Workflow" on page 4-21, "PortfolioCVaR Object Workflow" on page 5-20, and "PortfolioMAD Object Workflow" on page 6-19.

## Syntax

[LowerBound,UpperBound] = getBounds(obj)

## Description

[LowerBound,UpperBound] = getBounds(obj) obtains bounds for portfolio weights from portfolio objects.

## Examples

## Obtain Values for Lower and Upper Bounds for a Portfolio Object

Given portfolio $p$ with the default constraints set, obtain the values for LowerBound and UpperBound.

```
p = Portfolio;
p = setDefaultConstraints(p, 5);
[LowerBound, UpperBound] = getBounds(p)
LowerBound = 5×1
```

    0
    0
    ```
    0
0
0
UpperBound =
[]
```


## Obtain Values for Lower and Upper Bounds for a PortfolioCVaR Object

Given a PortfolioCVaR object $p$ with the default constraints set, obtain the values for LowerBound and UpperBound.
p = PortfolioCVaR;
$\mathrm{p}=$ setDefaultConstraints(p, 5);
[LowerBound, UpperBound] = getBounds(p)
LowerBound $=5 \times 1$
0
0
0
0
0

UpperBound =
[]

## Obtain Values for Lower and Upper Bounds for a PortfolioMAD Object

Given a PortfolioMAD object $p$ with the default constraints set, obtain the values for LowerBound and UpperBound.
p = PortfolioMAD;
$\mathrm{p}=$ setDefaultConstraints(p, 5);
[LowerBound, UpperBound] = getBounds(p)

LowerBound $=5 \times 1$
0
0
0
0
0

## UpperBound =

[]

## Input Arguments

## obj - Object for portfolio

## object

Object for portfolio, specified using Portfolio, PortfolioCVaR, or PortfolioMAD object. For more information on creating a portfolio object, see

- Portfolio
- PortfolioCVaR
- PortfolioMAD

Data Types: object

## Output Arguments

## LowerBound - Lower-bound weight for each asset <br> vector

Lower-bound weight for each asset, returned as a vector for a Portfolio,
PortfolioCVaR, or PortfolioMAD input object (obj). For more information on creating a portfolio object, see

- Portfolio
- PortfolioCVaR
- PortfolioMAD


## UpperBound - Upper-bound weight for each asset vector

Upper-bound weight for each asset, returned as a vector for a Portfolio, PortfolioCVaR, or PortfolioMAD input object (obj). For more information on creating a portfolio object, see

- Portfolio
- PortfolioCVaR
- PortfolioMAD


## Tips

You can also use dot notation to obtain bounds for portfolio weights from portfolio objects.
[LowerBound, UpperBound] = obj.getBounds;

## See Also

## setBounds

## Topics

"Working with 'Simple' Bound Constraints Using Portfolio Object" on page 4-73
"Working with Bound Constraints Using PortfolioCVaR Object" on page 5-68
"Working with Bound Constraints Using PortfolioMAD Object" on page 6-66
"Portfolio Optimization Examples" on page 4-167
"Portfolio Set for Optimization Using Portfolio Object" on page 4-10
"Portfolio Set for Optimization Using PortfolioCVaR Object" on page 5-10
"Portfolio Set for Optimization Using PortfolioMAD Object" on page 6-10

## Introduced in R2011a

## getBudget

Obtain budget constraint bounds from portfolio object
Use the getBudget function with a Portfolio, PortfolioCVaR, or PortfolioMAD object to obtain budget constraint bounds from portfolio objects.

For details on the respective workflows when using these different objects, see "Portfolio Object Workflow" on page 4-21, "PortfolioCVaR Object Workflow" on page 5-20, and "PortfolioMAD Object Workflow" on page 6-19.

## Syntax

[LowerBudget,UpperBudget] = getBudget(obj)

## Description

[LowerBudget,UpperBudget] = getBudget(obj) obtains budget constraint bounds from portfolio objects.

## Examples

## Obtain Values for Lower and Upper Budgets for a Portfolio Object

Given portfolio $p$ with the default constraints set, obtain the values for LowerBudget and UpperBudget.

```
p = Portfolio;
p = setDefaultConstraints(p, 5);
[LowerBudget, UpperBudget] = getBudget(p)
LowerBudget = 1
UpperBudget = 1
```


## Obtain Values for Lower and Upper Budgets for a PortfolioCVaR Object

Given a PortfolioCVaR object $p$ with the default constraints set, obtain the values for LowerBudget and UpperBudget.
p = PortfolioCVaR;
p = setDefaultConstraints(p, 5);
[LowerBudget, UpperBudget] = getBudget(p)
LowerBudget = 1
UpperBudget = 1

## Obtain Values for Lower and Upper Budgets for a PortfolioMAD Object

Given a PortfolioMAD object $p$ with the default constraints set, obtain the values for LowerBudget and UpperBudget.
p = PortfolioMAD;
$\mathrm{p}=$ setDefaultConstraints(p, 5);
[LowerBudget, UpperBudget] = getBudget(p)
LowerBudget = 1
UpperBudget = 1

## Input Arguments

## obj - Object for portfolio <br> object

Object for portfolio, specified using Portfolio, PortfolioCVaR, or PortfolioMAD object. For more information on creating a portfolio object, see

- Portfolio
- PortfolioCVaR
- PortfolioMAD

Data Types: object

## Output Arguments

## LowerBudget - Lower-bound weight for each asset

scalar
Lower bound for budget constraint, returned as a scalar for a Portfolio, PortfolioCVaR, or PortfolioMAD input object (obj).

## UpperBudget - Upper bound for budget constraint

scalar
Upper bound for budget constraint, returned as a scalar for a Portfolio, PortfolioCVaR, or PortfolioMAD input object (obj).

## Tips

You can also use dot notation to obtain the budget constraint bounds from portfolio objects.
[LowerBudget, UpperBudget] = obj.getBudget;

## See Also

setBudget

## Topics

"Working with Budget Constraints Using Portfolio Object" on page 4-77
"Working with Budget Constraints Using PortfolioCVaR Object" on page 5-71
"Working with Budget Constraints Using PortfolioMAD Object" on page 6-69
"Portfolio Optimization Examples" on page 4-167
"Portfolio Set for Optimization Using Portfolio Object" on page 4-10
"Portfolio Set for Optimization Using PortfolioCVaR Object" on page 5-10
"Portfolio Set for Optimization Using PortfolioMAD Object" on page 6-10

## Introduced in R2011a

## getCosts

Obtain buy and sell transaction costs from portfolio object
Use the getCosts function with a Portfolio, PortfolioCVaR, or PortfolioMAD object to obtain buy and sell transaction costs from portfolio objects.

For details on the respective workflows when using these different objects, see "Portfolio Object Workflow" on page 4-21, "PortfolioCVaR Object Workflow" on page 5-20, and "PortfolioMAD Object Workflow" on page 6-19.

## Syntax

[BuyCost,SellCost] = getCosts(obj)

## Description

[BuyCost, SellCost] = getCosts(obj) obtains buy and sell transaction costs from portfolio objects.

## Examples

## Obtain Buy and Sell Costs for a Portfolio Object

Given portfolio $p$ with the costs set, obtain the values for BuyCost and SellCost.

```
p = Portfolio;
p = setCosts(p, 0.001, 0.001, 5);
[BuyCost, SellCost] = getCosts(p)
BuyCost = 1.0000e-03
SellCost = 1.0000e-03
```


## Obtain Buy and Sell Costs for a PortfolioCVaR Object

Given a PortfolioCVaR object $p$ with the costs set, obtain the values for BuyCost and SellCost.

```
p = PortfolioCVaR;
p = setCosts(p, 0.001, 0.001, 5);
[BuyCost, SellCost] = getCosts(p)
BuyCost = 1.0000e-03
SellCost = 1.0000e-03
```


## Obtain Buy and Sell Costs for a PortfolioMAD Object

Given a PortfolioMAD object p with the costs set, obtain the values for BuyCost and SellCost.
p = PortfolioMAD;
$\mathrm{p}=$ setCosts(p, 0.001, 0.001, 5);
[BuyCost, SellCost] = getCosts(p)
BuyCost $=1.0000 \mathrm{e}-03$
SellCost $=1.0000 \mathrm{e}-03$

## Input Arguments

## obj - Object for portfolio <br> object

Object for portfolio, specified using Portfolio, PortfolioCVaR, or PortfolioMAD object. For more information on creating a portfolio object, see

- Portfolio
- PortfolioCVaR
- PortfolioMAD

Data Types: object

## Output Arguments

## BuyCost - Proportional transaction cost to purchase each asset vector

Proportional transaction cost to purchase each asset, returned as a vector for a Portfolio, PortfolioCVaR, or PortfolioMAD input object (obj).

## SellCost - Proportional transaction cost to sell each asset

vector
Proportional transaction cost to sell each asset, returned as a vector for a Portfolio, PortfolioCVaR, or PortfolioMAD input object (obj).

## Tips

You can also use dot notation to obtain the buy and sell transaction costs from portfolio objects.

```
[BuyCost, SellCost] = obj.getCosts;
```


## See Also

setCosts

## Topics

"Working with Transaction Costs" on page 4-63
"Working with Transaction Costs" on page 5-58
"Working with Transaction Costs" on page 6-56
"Portfolio Optimization Examples" on page 4-167
"Portfolio Set for Optimization Using Portfolio Object" on page 4-10
"Portfolio Set for Optimization Using PortfolioCVaR Object" on page 5-10
"Portfolio Set for Optimization Using PortfolioMAD Object" on page 6-10

## Introduced in R2011a

## getEquality

Obtain equality constraint arrays from portfolio object
Use the getEquality function with a Portfolio, PortfolioCVaR, or PortfolioMAD object to obtain equality constraint arrays from portfolio objects.

For details on the respective workflows when using these different objects, see "Portfolio Object Workflow" on page 4-21, "PortfolioCVaR Object Workflow" on page 5-20, and "PortfolioMAD Object Workflow" on page 6-19.

## Syntax

[AEquality,bEquality] = getEquality(obj)

## Description

[AEquality,bEquality] = getEquality(obj) obtains equality constraint arrays from portfolio objects.

## Examples

## Obtain Equality Constraints for a Portfolio Object

Suppose you have a portfolio of five assets and you want to ensure that the first three assets are exactly $50 \%$ of your portfolio. Given a Portfolio object p, set the linear equality constraints and obtain the values for AEquality and bEquality:
$A=\left[\begin{array}{llllll}1 & 1 & 1 & 0 & 0\end{array}\right] ;$
b $=0.5$;
p = Portfolio;
p = setEquality(p, A, b);
[AEquality, bEquality] = getEquality(p)
AEquality $=1 \times 5$

```
1 1 1 1 1 0
bEquality = 0.5000
```


## Obtain Equality Constraints for a PortfolioCVaR Object

Suppose you have a portfolio of five assets and you want to ensure that the first three assets are $50 \%$ of your portfolio. Given a PortfolioCVaR object p, set the linear equality constraints and obtain the values for AEquality and bEquality:

A = [ 1111000$] ;$
b = 0.5;
p = PortfolioCVaR;
p = setEquality(p, A, b);
[AEquality, bEquality] = getEquality(p)
AEquality = $1 \times 5$
$\begin{array}{lllll}1 & 1 & 1 & 0 & 0\end{array}$
bEquality $=0.5000$

## Obtain Equality Constraints for a PortfolioMAD Object

Suppose you have a portfolio of five assets and you want to ensure that the first three assets are $50 \%$ of your portfolio. Given a PortfolioMAD object p, set the linear equality constraints and obtain the values for AEquality and bEquality:

```
A = [ 1 1 1 0 0 ];
b = 0.5;
p = PortfolioMAD;
p = setEquality(p, A, b);
[AEquality, bEquality] = getEquality(p)
AEquality = 1×5
    1 
```


## Input Arguments

## obj - Object for portfolio <br> object

Object for portfolio, specified using Portfolio, PortfolioCVaR, or PortfolioMAD object. For more information on creating a portfolio object, see

- Portfolio
- PortfolioCVaR
- PortfolioMAD

Data Types: object

## Output Arguments

## AEquality - Matrix to form linear equality constraints matrix

Matrix to form linear equality constraints, returned as a matrix for a Portfolio, PortfolioCVaR, or PortfolioMAD input object (obj).

## bEquality - Vector to form linear equality constraints

vector
Vector to form linear equality constraints, returned as a vector for a Portfolio, PortfolioCVaR, or PortfolioMAD input object (obj).

## Tips

You can also use dot notation to obtain the equality constraint arrays from portfolio objects.

```
[AEquality, bEquality] = obj.getEquality;
```

See AlsosetEquality
Topics"Working with Linear Equality Constraints Using Portfolio Object" on page 4-88
"Working with Linear Equality Constraints Using PortfolioCVaR Object" on page 5-82
"Working with Linear Equality Constraints Using PortfolioMAD Object" on page 6-79
"Portfolio Optimization Examples" on page 4-167
"Portfolio Set for Optimization Using Portfolio Object" on page 4-10
"Portfolio Set for Optimization Using PortfolioCVaR Object" on page 5-10
"Portfolio Set for Optimization Using PortfolioMAD Object" on page 6-10
Introduced in R2011a

## getGroupRatio

Obtain group ratio constraint arrays from portfolio object
Use the getGroupRatio function with a Portfolio, PortfolioCVaR, or PortfolioMAD object to obtain group ratio constraint arrays from portfolio objects.

For details on the respective workflows when using these different objects, see "Portfolio Object Workflow" on page 4-21, "PortfolioCVaR Object Workflow" on page 5-20, and "PortfolioMAD Object Workflow" on page 6-19.

## Syntax

[GroupA,GroupB, LowerRatio,UpperRatio] = getGroupRatio(obj)

## Description

[GroupA, GroupB, LowerRatio,UpperRatio] = getGroupRatio(obj) obtains equality constraint arrays from portfolio objects.

## Examples

## Obtain Group Ratio Constraints for a Portfolio Object

Suppose you want to make sure that the ratio of financial to nonfinancial companies in your portfolios never goes above $50 \%$. Assume you have 6 assets with 3 financial companies (assets 1-3) and 3 nonfinancial companies (assets 4-6). After setting group ratio constraints, obtain the values for GroupA, GroupB, LowerRatio, and UpperRatio.

```
GA = [ true true true false false false ]; % financial companies
GB = [ false false false true true true ]; % nonfinancial companies
p = Portfolio;
p = setGroupRatio(p, GA, GB, [], 0.5);
[GroupA, GroupB, LowerRatio, UpperRatio] = getGroupRatio(p)
```

```
GroupA = 1\times6
    1
GroupB = 1\times6
    0
LowerRatio =
    []
UpperRatio = 0.5000
```


## Obtain Group Ratio Constraints for a PortfolioCVaR Object

Suppose you want to ensure that the ratio of financial to nonfinancial companies in your portfolios never exceeds $50 \%$. Assume you have six assets with three financial companies (assets 1-3) and three nonfinancial companies (assets 4-6). After setting group ratio constraints, obtain the values for GroupA, GroupB, LowerRatio, and UpperRatio.
$G A=[$ true true true false false false ]; \% financial companies
GB = [ false false false true true true ]; \% nonfinancial companies
$\mathrm{p}=$ PortfolioCVaR;
$\mathrm{p}=$ setGroupRatio(p, GA, GB, [], 0.5);
[GroupA, GroupB, LowerRatio, UpperRatio] = getGroupRatio(p)
GroupA $=1 \times 6$
$\begin{array}{llllll}1 & 1 & 1 & 0 & 0 & 0\end{array}$

GroupB $=1 \times 6$
$\begin{array}{llllll}0 & 0 & 0 & 1 & 1 & 1\end{array}$

LowerRatio =

## Obtain Group Ratio Constraints for a PortfolioMAD Object

Suppose you want to ensure that the ratio of financial to nonfinancial companies in your portfolios never exceeds 50\%. Assume you have six assets with three financial companies (assets 1-3) and three nonfinancial companies (assets 4-6). After setting group ratio constraints, obtain the values for GroupA, GroupB, LowerRatio, and UpperRatio.

```
GA = [ true true true false false false ]; % financial companies
GB = [ false false false true true true ]; % nonfinancial companies
p = PortfolioMAD;
p = setGroupRatio(p, GA, GB, [], 0.5);
[GroupA, GroupB, LowerRatio, UpperRatio] = getGroupRatio(p)
```

GroupA $=1 \times 6$
$\begin{array}{llllll}1 & 1 & 1 & 0 & 0 & 0\end{array}$
GroupB = 1×6
$\begin{array}{llllll}0 & 0 & 0 & 1 & 1 & 1\end{array}$
LowerRatio =
[]
UpperRatio = 0.5000

## Input Arguments

## obj - Object for portfolio

## object

Object for portfolio, specified using Portfolio, PortfolioCVaR, or PortfolioMAD object. For more information on creating a portfolio object, see

- Portfolio
- PortfolioCVaR
- PortfolioMAD

Data Types: object

## Output Arguments

## GroupA - Matrix that forms base groups for comparison matrix

Matrix that forms base groups for comparison, returned as a matrix for a Portfolio, PortfolioCVaR, or PortfolioMAD input object (obj).

## GroupB - Matrix that forms comparison groups

## matrix

Matrix that forms comparison groups, returned as a matrix Portfolio, PortfolioCVaR, or PortfolioMAD input object (obj).

LowerRatio - Lower bound for ratio of GroupB groups to GroupA groups vector

Lower bound for ratio of GroupB groups to GroupA groups, returned as a vector for a Portfolio, PortfolioCVaR, or PortfolioMAD input object (obj).

## UpperRatio - Upper bound for ratio of GroupB groups to GroupA groups vector

Upper bound for ratio of GroupB groups to GroupA groups, returned as a vector for a Portfolio, PortfolioCVaR, or PortfolioMAD input object (obj).

## Tips

You can also use dot notation to obtain group ratio constraint arrays from portfolio objects.
[GroupA, GroupB, LowerRatio, UpperRatio] = obj.getGroupRatio;

## See Also

setGroupRatio

## Topics

"Working with Group Ratio Constraints Using Portfolio Object" on page 4-84
"Working with Group Constraints Using PortfolioCVaR Object" on page 5-74
"Working with Group Constraints Using PortfolioMAD Object" on page 6-71
"Portfolio Optimization Examples" on page 4-167
"Portfolio Set for Optimization Using Portfolio Object" on page 4-10
"Portfolio Set for Optimization Using PortfolioCVaR Object" on page 5-10
"Portfolio Set for Optimization Using PortfolioMAD Object" on page 6-10

## Introduced in R2011a

## getGroups

Obtain group constraint arrays from portfolio object
Use the getGroups function with a Portfolio, PortfolioCVaR, or PortfolioMAD object to obtain group constraint arrays from portfolio objects.

For details on the respective workflows when using these different objects, see "Portfolio Object Workflow" on page 4-21, "PortfolioCVaR Object Workflow" on page 5-20, and "PortfolioMAD Object Workflow" on page 6-19.

## Syntax

[GroupMatrix,LowerGroup,UpperGroup] = getGroups(obj)

## Description

[GroupMatrix,LowerGroup,UpperGroup] = getGroups(obj) obtains group constraint arrays from portfolio objects.

## Examples

## Obtain Group Constraints for a Portfolio Object

Suppose you have a portfolio of five assets and you want to ensure that the first three assets constitute no more than $30 \%$ of your portfolio. Given a Portfolio object $p$ with the group constraints set, obtain the values for GroupMatrix, LowerGroup, and UpperGroup.

```
G = [ true true true false false ];
p = Portfolio;
p = setGroups(p, G, [], 0.3);
[GroupMatrix, LowerGroup, UpperGroup] = getGroups(p)
```

GroupMatrix = $1 \times 5$

```
    1 1 1 1 0
LowerGroup =
    [ ]
UpperGroup = 0.3000
```


## Obtain Group Constraints for a PortfolioCVaR Object

Suppose you have a portfolio of five assets and you want to ensure that the first three assets constitute at most $30 \%$ of your portfolio. Given a PortfolioCVaR object p with the group constraints set, obtain the values for GroupMatrix, LowerGroup, and UpperGroup.

```
G = [ true true true false false ];
p = PortfolioCVaR;
p = setGroups(p, G, [], 0.3);
[GroupMatrix, LowerGroup, UpperGroup] = getGroups(p)
```

GroupMatrix = $1 \times 5$
$\begin{array}{lllll}1 & 1 & 1 & 0 & 0\end{array}$
LowerGroup =
[]
UpperGroup = 0.3000

## Obtain Group Constraints for a PortfolioMAD Object

Suppose you have a portfolio of five assets and you want to ensure that the first three assets constitute at most $30 \%$ of your portfolio. Given a PortfolioMAD object p with the group constraints set, obtain the values for GroupMatrix, LowerGroup, and UpperGroup.

```
G = [ true true true false false ];
p = PortfolioMAD;
p = setGroups(p, G, [], 0.3);
[GroupMatrix, LowerGroup, UpperGroup] = getGroups(p)
GroupMatrix = 1×5
    1 
LowerGroup =
    []
UpperGroup = 0.3000
```


## Input Arguments

## obj - Object for portfolio

## object

Object for portfolio, specified using Portfolio, PortfolioCVaR, or PortfolioMAD object. For more information on creating a portfolio object, see

- Portfolio
- PortfolioCVaR
- PortfolioMAD


## Data Types: object

## Output Arguments

## GroupMatrix - Group constraint matrix <br> matrix

Group constraint matrix, returned as a matrix for a Portfolio, PortfolioCVaR, or PortfolioMAD input object (obj).

## LowerGroup - Lower bound for group constraints

vector

Lower bound for group constraints, returned as a vector for a Portfolio, PortfolioCVaR, or PortfolioMAD input object (obj).

## UpperGroup - Upper bound for group constraints

vector
Upper bound for group constraints, returned as a vector for a Portfolio, PortfolioCVaR, or PortfolioMAD input object (obj).

## Tips

You can also use dot notation to obtain the group constraint arrays from portfolio objects.
[GroupMatrix, LowerGroup, UpperGroup] = obj.getGroups;

## See Also

setGroups

## Topics

"Working with Group Constraints Using Portfolio Object" on page 4-80
"Working with Group Constraints Using PortfolioCVaR Object" on page 5-74
"Working with Group Constraints Using PortfolioMAD Object" on page 6-71
"Portfolio Optimization Examples" on page 4-167
"Portfolio Set for Optimization Using Portfolio Object" on page 4-10
"Portfolio Set for Optimization Using PortfolioCVaR Object" on page 5-10
"Portfolio Set for Optimization Using PortfolioMAD Object" on page 6-10

## Introduced in R2011a

## getScenarios

Obtain scenarios from portfolio object
Use the getScenarios function with a PortfolioCVaR or PortfolioMAD objects to obtain scenarios.

For details on the workflows, see "PortfolioCVaR Object Workflow" on page 5-20, and "PortfolioMAD Object Workflow" on page 6-19.

## Syntax

Y = getScenarios(obj)

## Description

$Y=$ getScenarios(obj) obtains scenarios for PortfolioCVaR or PortfolioMAD objects.

## Examples

## Obtain Scenarios for a CVaR Portfolio Object

For a given PortfolioCVaR object p, display the defined scenarios.

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;
rng(11);
```

```
AssetScenarios = mvnrnd(m, C, 10);
p = PortfolioCVaR;
p = setScenarios(p, AssetScenarios);
p = setDefaultConstraints(p);
p = setProbabilityLevel(p, 0.95);
Y = getScenarios(p)
Y = 10\times4
\begin{tabular}{rrrr}
-0.0056 & 0.0440 & 0.1186 & 0.0488 \\
-0.0368 & -0.0753 & 0.0087 & 0.1124 \\
0.0025 & 0.0856 & 0.0484 & 0.1404 \\
0.0318 & 0.0826 & 0.0377 & 0.0404 \\
0.0013 & -0.0561 & -0.1466 & -0.0621 \\
0.0035 & 0.0310 & -0.0183 & 0.1225 \\
-0.0519 & -0.1634 & -0.0526 & 0.1528 \\
0.0029 & -0.1163 & -0.0627 & -0.0760 \\
0.0192 & -0.0182 & -0.1243 & -0.1346 \\
0.0440 & 0.0189 & 0.0098 & 0.0821
\end{tabular}
```

The function $\mathrm{rng}\left({ }^{\text {seed }}\right)$ resets the random number generator to produce the documented results. It is not necessary to reset the random number generator to simulate scenarios.

## Obtain Scenarios for a MAD Portfolio Object

For a given PortfolioMAD object p, display the defined scenarios.

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;
```

rng(11);

```
AssetScenarios = mvnrnd(m, C, 10);
p = PortfolioMAD;
p = setScenarios(p, AssetScenarios);
p = setDefaultConstraints(p);
Y = getScenarios(p)
Y = 10\times4
\begin{tabular}{rrrr}
-0.0056 & 0.0440 & 0.1186 & 0.0488 \\
-0.0368 & -0.0753 & 0.0087 & 0.1124 \\
0.0025 & 0.0856 & 0.0484 & 0.1404 \\
0.0318 & 0.0826 & 0.0377 & 0.0404 \\
0.0013 & -0.0561 & -0.1466 & -0.0621 \\
0.0035 & 0.0310 & -0.0183 & 0.1225 \\
-0.0519 & -0.1634 & -0.0526 & 0.1528 \\
0.0029 & -0.1163 & -0.0627 & -0.0760 \\
0.0192 & -0.0182 & -0.1243 & -0.1346 \\
0.0440 & 0.0189 & 0.0098 & 0.0821
\end{tabular}
```

The function $\mathrm{rng}\left({ }^{\text {seed }}\right)$ resets the random number generator to produce the documented results. It is not necessary to reset the random number generator to simulate scenarios.

## Input Arguments

## obj - Object for portfolio

object
Object for portfolio, specified using a PortfolioCVaR or PortfolioMAD object.
For more information on creating a PortfolioCVaR or PortfolioMAD object, see

- PortfolioCVaR
- PortfolioMAD

Data Types: object

## Output Arguments

## Y - Scenarios matrix <br> matrix

Scenarios matrix, returned as a NumScenarios-by-NumAssets matrix for a PortfolioCVaR or PortfolioMAD object.

## Tips

You can also use dot notation to obtain scenarios from a PortfolioCVaR or PortfolioMAD object.

Y = obj.getScenarios;

## See Also

rng| setScenarios

## Topics

"Asset Returns and Scenarios Using PortfolioCVaR Object" on page 5-44
"Asset Returns and Scenarios Using PortfolioMAD Object" on page 6-42

## External Websites

Analyzing Investment Strategies with CVaR Portfolio Optimization in MATLAB (50 min 42 sec )

Introduced in R2012b

## getfield

Content of specific field

Note getfield will be removed in a future release. Use timetable instead. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

fieldval = getfield(tsobj,field)
fieldval = getfield(tsobj,field,\{dates\})

## Arguments

| tsobj | Financial time series object. |
| :--- | :--- |
| field | Field name within tsobj. |
| dates | Date range. Dates can be expanded to include time-of-day <br> information. |

## Description

getfield treats the contents of a financial times series object tsobj as fields in a structure.
fieldval $=$ getfield(tsobj,field) returns the contents of the specified field. This is equivalent to the syntax fieldval $=$ tsobj field.
fieldval $=$ getfield(tsobj, field, \{dates\}) returns the contents of the specified field for the specified dates. dates can be individual cells of date character vectors or a cell of a date character vector range using the : : operator, such as '03/01/99::03/31/99'.

## Examples

Create a financial time series object containing both date and time-of-day information:

```
dates = ['01-Jan-2001';'01-Jan-2001'; '02-Jan-2001'; ...
    '02-Jan-2001'; '03-Jan-2001';'03-Jan-2001'];
times = ['11:00';'12:00';'11:00';'12:00';'11:00';'12:00'];
dates_times = cellstr([dates, repmat(' ',size(dates,1),1),...
times]);
AnFts = fints(dates_times,[(1:4)'; nan; 6],{'Datal'},1,...
                            'Yet Ānother Financial Time Series')
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints (line 165)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/display (line 66)
AnFts =
    desc: Yet Another Financial Time Series
    freq: Daily (1)
\begin{tabular}{|c|c|c|c|}
\hline 'dates: (6)' & 'times: & (6) ' & 'Datal: (6)' \\
\hline '01-Jan-2001' & '11:00' & & \(1]\) \\
\hline , " ' & '12:00' & & \(2]\) \\
\hline '02-Jan-2001' & '11:00' & & \(3]\) \\
\hline ' \({ }^{\text {' }}\) & '12:00' & & 4] \\
\hline '03-Jan-2001' & '11:00' & & \(\mathrm{NaN}]\) \\
\hline & '12:00' & & 61 \\
\hline
\end{tabular}
```

Example 1. Get the contents of the times field in AnFts:

```
F = datestr(getfield(AnFts, 'times'))
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/getfield (line 67)
F =
    6\times8 char array
    '11:00 AM'
    '12:00 PM'
    '11:00 AM'
    '12:00 PM'
    '11:00 AM'
    '12:00 PM'
```

```
Example 2. Extract the contents of specific data fields within AnFts:
FF = getfield(AnFts,'Data1',...
    '01-Jan-2001 12:00::02-Jan-2001 12:00')
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/getfield (line 67)
FF =
    2
    3
    4
```


## See Also

chfield|fieldnames |isfield|rmfield|setfield

## Topics

"What Is the Financial Time Series App?" on page 14-2

## Introduced before R2006a

## getInequality

Obtain inequality constraint arrays from portfolio object
Use the getInequality function with a Portfolio, PortfolioCVaR, or PortfolioMAD object to obtain inequality constraint arrays from portfolio objects.

For details on the respective workflows when using these different objects, see "Portfolio Object Workflow" on page 4-21, "PortfolioCVaR Object Workflow" on page 5-20, and "PortfolioMAD Object Workflow" on page 6-19.

## Syntax

[AInequality,bInequality] = getInequality(obj)

## Description

[AInequality,bInequality] = getInequality(obj) obtains equality constraint arrays from portfolio objects.

## Examples

## Obtain Inequality Constraints for a Portfolio Object

Suppose you have a portfolio of five assets and you want to ensure that the first three assets are no more than $50 \%$ of your portfolio. Given a Portfolio object p, set the linear inequality constraints and then obtain values for AInequality and bInequality.

```
A = [ 1 1 1 0 0 ];
b = 0.5;
p = Portfolio;
p = setInequality(p, A, b);
[AInequality, bInequality] = getInequality(p)
```

AInequality = $1 \times 5$

```
    1 1 1 1 1 0 0
bInequality = 0.5000
```


## Obtain Inequality Constraints for a PortfolioCVaR Object

Suppose you have a portfolio of five assets and you want to ensure that the first three assets constitute at most $50 \%$ of your portfolio. Given a PortfolioCVaR object p, set the linear inequality constraints and then obtain values for AInequality and bInequality.

```
A = [ 1 1 1 0 0 ];
b = 0.5;
p = PortfolioCVaR;
p = setInequality(p, A, b);
[AInequality, bInequality] = getInequality(p)
AInequality = 1×5
    1 1 1 1 0 0
bInequality = 0.5000
```


## Obtain Inequality Constraints for a PortfolioMAD Object

Suppose you have a portfolio of five assets and you want to ensure that the first three assets constitute at most $50 \%$ of your portfolio. Given a PortfolioMAD object p, set the linear inequality constraints and then obtain values for AInequality and bInequality.

```
A = [ 1 1 1 0 0 ];
b = 0.5;
p = PortfolioMAD;
p = setInequality(p, A, b);
[AInequality, bInequality] = getInequality(p)
```

AInequality $=1 \times 5$
$\begin{array}{lllll}1 & 1 & 1 & 0 & 0\end{array}$

## Input Arguments

## obj - Object for portfolio <br> object

Object for portfolio, specified using Portfolio, PortfolioCVaR, or PortfolioMAD object. For more information on creating a portfolio object, see

- Portfolio
- PortfolioCVaR
- PortfolioMAD

Data Types: object

## Output Arguments

## AInequality - Matrix to form linear inequality constraints matrix

Matrix to form linear inequality constraints, returned as a matrix for a Portfolio, PortfolioCVaR, or PortfolioMAD input object (obj).

## bInequality - Vector to form linear inequality constraints

vector
Vector to form linear inequality constraints, returned as a vector for a Portfolio, PortfolioCVaR, or PortfolioMAD input object (obj).

## Tips

You can also use dot notation to obtain the inequality constraint arrays from portfolio objects.

```
[AInequality, bInequality] = obj.getInequality;
```

See Also
setInequality
Topics"Working with Linear Inequality Constraints Using Portfolio Object" on page 4-91"Working with Linear Inequality Constraints Using PortfolioCVaR Object" on page 5-85
"Working with Linear Inequality Constraints Using PortfolioMAD Object" on page 6-82
"Portfolio Optimization Examples" on page 4-167
"Portfolio Set for Optimization Using Portfolio Object" on page 4-10
"Portfolio Set for Optimization Using PortfolioCVaR Object" on page 5-10
"Portfolio Set for Optimization Using PortfolioMAD Object" on page 6-10
Introduced in R2011a

## getOneWayTurnover

Obtain one-way turnover constraints from portfolio object
Use the getOneWayTirnover function with a Portfolio, PortfolioCVaR, or PortfolioMAD object to obtain one-way turnover constraints from portfolio objects.

For details on the respective workflows when using these different objects, see "Portfolio Object Workflow" on page 4-21, "PortfolioCVaR Object Workflow" on page 5-20, and "PortfolioMAD Object Workflow" on page 6-19.

## Syntax

[BuyTurnover,SellTurnover] = getOneWayTurnover(obj)

## Description

[BuyTurnover,SellTurnover] = getOneWayTurnover(obj) obtain one-way turnover constraints from portfolio objects.

## Examples

## Obtain One-Way Turnover Costs for a Portfolio Object

Set one-way turnover costs.

```
p = Portfolio('AssetMean',[0.1, 0.2, 0.15], 'AssetCovar',...
[ 0.005, -0.010, 0.004; -0.010, 0.040, -0.002; 0.004, -0.002, 0.023]);
p = setBudget(p, 1, 1);
p = setOneWayTurnover(p, 1.3, 0.3, 0); %130-30 portfolio
plotFrontier(p);
```



Obtain one-way turnover costs.
[BuyTurnover,SellTurnover] = getOneWayTurnover(p)
BuyTurnover = 1.3000
SellTurnover = 0.3000

## Obtain One-Way Turnover Costs for a PortfolioCVaR Object

Set one-way turnover costs and obtain the buy and sell turnover values.

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;
AssetScenarios = mvnrnd(m, C, 20000);
p = PortfolioCVaR;
p = setScenarios(p, AssetScenarios);
p = setDefaultConstraints(p);
p = setProbabilityLevel(p, 0.95);
p = setBudget(p, 1, 1);
p = setOneWayTurnover(p, 1.3, 0.3, 0); %130-30 portfolio
```

[BuyTurnover,SellTurnover] = getOneWayTurnover(p)
BuyTurnover = 1.3000
SellTurnover = 0.3000

## Obtain One-Way Turnover Costs for a PortfolioMAD Object

Set one-way turnover costs and obtain the buy and sell turnover values.

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;
AssetScenarios = mvnrnd(m, C, 20000);
p = PortfolioMAD;
p = setScenarios(p, AssetScenarios);
p = setDefaultConstraints(p);
```

$\mathrm{p}=\operatorname{set} \mathrm{Budget}(\mathrm{p}, 1,1)$;
$\mathrm{p}=\operatorname{setOneWayTurnover}(\mathrm{p}, 1.3,0.3,0)$; \%130-30 portfolio
[BuyTurnover,SellTurnover] = getOneWayTurnover(p)
BuyTurnover = 1.3000
SellTurnover $=0.3000$

## Input Arguments

## obj - Object for portfolio object

Object for portfolio, specified using Portfolio, PortfolioCVaR, or PortfolioMAD object. For more information on creating a portfolio object, see

- Portfolio
- PortfolioCVaR
- PortfolioMAD

Data Types: object

## Output Arguments

## BuyTurnover - Turnover constraint on purchases

scalar
Turnover constraint on purchases, returned as a scalar for a Portfolio, PortfolioCVaR, or PortfolioMAD input object (obj).

## SellTurnover - Turnover constraint on sales <br> scalar

Turnover constraint on sales, returned as a scalar for a Portfolio, PortfolioCVaR, or PortfolioMAD input object (obj).

## Definitions

## One-way Turnover Constraint

One-way turnover constraints ensure that estimated optimal portfolios differ from an initial portfolio by no more than specified amounts according to whether the differences are purchases or sales.

The constraints take the form

$$
\begin{aligned}
& 1^{T} \max \left\{0, x-x_{0}\right\} \leq \tau_{B} \\
& 1^{T} \max \left\{0, x_{0}-x\right\} \leq \tau_{S}
\end{aligned}
$$

with

- x - The portfolio (NumAssets vector)
- $\mathrm{X}_{0}$ - Initial portfolio (NumAssets vector)
- $\tau_{B}$ - Upper-bound for turnover constraint on purchases (scalar)
- $\tau_{S}$ - Upper-bound for turnover constraint on sales (scalar)

Specify one-way turnover constraints using the following properties in a supported portfolio object: BuyTurnover for $\tau_{B}$, SellTurnover for $\tau_{S}$, and InitPort for $x_{0}$.

Note The average turnover constraint (which is set using setTurnover) is not just the combination of the one-way turnover constraints with the same value for the constraint.

## Tips

You can also use dot notation to get the one-way turnover constraint for portfolio objects.
[BuyTurnover,SellTurnover] = obj.getOneWayTurnover

## See Also

setOneWayTurnover| setTurnover

## Topics

"Working with One-Way Turnover Constraints Using Portfolio Object" on page 4-102
"Working with One-Way Turnover Constraints Using PortfolioCVaR Object" on page 5-92
"Working with One-Way Turnover Constraints Using PortfolioMAD Object" on page 6-88
"Portfolio Optimization Examples" on page 4-167
"Portfolio Set for Optimization Using Portfolio Object" on page 4-10
"Portfolio Set for Optimization Using PortfolioCVaR Object" on page 5-10
"Portfolio Set for Optimization Using PortfolioMAD Object" on page 6-10

## Introduced in R2011a

## getnameidx

Find name in list

Note getnameidx will be removed in a future release. Use contains instead. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

```
nameidx = getnameidx(list,name)
```


## Arguments

| list | Cell array of name character vectors. |
| :--- | :--- |
| name | Character vector or cell array of name character vectors. |

## Description

nameidx = getnameidx(list, name) finds the occurrence of a name or set of names in a list. It returns an index (order number) indicating where the specified names are located within the list. If name is not found, nameidx returns 0 .

If name is a cell array of names, getnameidx returns a vector containing the indices (order number) of the name character vectors within list. If none of the names in the name cell array is in list, it returns zero. If some of the names in name are not found, the indices for these names are zeros.
getnameidx finds only the first occurrence of the name in the list of names. This function is meant to be used on a list of unique names (character vectors) only. It does not find multiple occurrences of a name or a list of names within list.

## Examples

Given

```
poultry = {'duck', 'chicken'}
animals = {'duck', 'cow', 'sheep', 'horse', 'chicken'}
nameidx = getnameidx(animals, poultry)
ans =
    1 5
```

Given
poultry = \{'duck', 'goose', 'chicken'\}
animals = \{'duck', 'cow', 'sheep', 'horse', 'chicken'\}
nameidx = getnameidx(animals, poultry)
ans =
105

## See Also

strcmp|strfind

## Topics

"What Is the Financial Time Series App?" on page 14-2

## Introduced before R2006a

## hhigh

Highest high

Note hhigh has been partially removed and will no longer accept a fints object (tsobj) argument. Use a matrix, timetable, or table instead for financial time series.

Use fts2timetable to convert a fints object to a timetable object.

## Syntax

```
values = hhigh(Data)
values = hhigh(
,Name,Value)
```


## Description

values $=$ hhigh(Data) generates a vector of highest high values from the series of high prices for the past $n$ periods.
values = hhigh( $\qquad$ , Name, Value) adds optional name-value pair arguments.

## Examples

## Calculate the Highest High for a Data Series for a Stock

Load the file SimulatedStock. mat, which provides a timetable (TMW) for financial data for TMW stock.

```
load SimulatedStock.mat
values = hhigh(TMW);
plot(values.Time,values.HighestHigh)
title('Highest High for TMW')
```

Highest High for TMW


## Input Arguments

## Data - Data for high prices

matrix | table | timetable
Data for high prices, specified as a matrix, table, or timetable. Timetables and tables with M rows must contain a variable named 'High' (case insensitive).
Data Types: double| table| timetable

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Example: values = hhigh(TMW_HIGH,'NumPeriods',10)

## NumPeriods - Moving window for the highest high calculation

 14 (default) | positive integerMoving window for the highest high calculation, specified as the comma-separated pair consisting of 'NumPeriods ' and a scalar positive integer.

Data Types: double

## Output Arguments

## values - Highest high series

matrix | table | timetable
Highest high series, returned with the same number of rows (M) and the same type (matrix, table, or timetable) as the input Data.

## References

[1] Achelis, S. B. Technical Analysis from A to Z. Second Edition. McGraw-Hill, 1995.

## See Also

llow|table|timetable

## Topics

"Using Timetables in Finance"
"Convert Financial Time Series Objects fints to Timetables" on page 12-2

## Introduced before R2006a

## highlow (fts)

Time series High-Low plot

Note highlow (fts) will be removed in a future release. Use highlow instead.

## Syntax

```
highlow(tsobj)
highlow(tsobj,color)
highlow(tsobj,color,dateform)
highlow(tsobj,color,dateform,'ParameterName',ParameterValue, ...)
hll = highlow(tsobj,color,dateform,'ParameterName',ParameterValue, ...)
```


## Arguments

| tsobj | Financial time series object. |
| :---: | :---: |
| color | (Optional) A three-element row vector representing RGB or a color identifier. (See plot.) |
| dateform | (Optional) Date format used as the $x$-axis tick labels. (See datetick.) You can specify a dateform only when tsobj does not contain time-of-day data. If tsobj contains time-ofday data, dateform is restricted to 'dd-mmm-yyyy HH:MM'. |
| 'ParameterName' | ' ParameterName' can be: <br> - HighName: high prices series name <br> - LowName: low prices series name <br> - OpenName: open prices series name <br> - CloseName: closing prices series name |


| ParameterValue | The parameter value is a character vector that represents the <br> valid parameter name. |
| :--- | :--- |

## Description

highlow(tsobj) generates a High-Low plot of the data in the financial time series object tsobj. tsobj must contain at least four data series representing the high, low, open, and closing prices. These series must have the names High, Low, Open, and Close (case-insensitive).
highlow(tsobj, color) additionally specifies the color of the plot.
highlow(tsobj, color, dateform) additionally specifies the date format used as the $x$ axis tick labels. See datestr for a list of date formats.
highlow(tsobj, color,dateform,'ParameterName',ParameterValue, ...) indicates the actual names of the required data series if the data series do not have the default names.

You can specify open prices as optional by providing the parameter name 'OpenName' and the parameter value ' ' .
highlow(tsobj, color, dateform, 'OpenName', '')
hhll =
highlow(tsobj, color, dateform,'ParameterName', ParameterValue, ...) returns the handle to the line object that makes up the High-Low plot.

## Examples

## Generate a High-Low Plot

This example shows how to generate a High-Low plot for Disney stock for the dates May 28 to June 18, 1998.
load disney.mat
Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in
highlow(dis('28-May-1998::18-Jun-1998'))
Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in title('High-Low Plot for Disney')

High-Low Plot for Disney


## See Also <br> candle <br> Introduced before R2006a

## highlow

High, low, open, close chart

Note highlow is updated to accept data input as a matrix, timetable, or table.

## Syntax

```
highlow(Data)
highlow(Data,Color)
h = highlow(ax,
```

$\qquad$

## Description

highlow (Data) displays a highlow chart from a series of opening, high, low, and closing prices of a security. The plots are vertical lines whose top is the high, bottom is the low, open is a left tick, and close is a right tick.
highlow(Data,Color) adds an optional argument for Color.
h = highlow(ax, ___ ) adds an optional argument for ax.

## Examples

## Generate a Highlow Chart for a Data Series for a Stock

Load the file SimulatedStock.mat, which provides a timetable (TMW) for financial data for TMW stock. The highlow chart plots the price data using blue lines.

```
load SimulatedStock.mat
range = 1:25;
highlow(TMW(range,:),'b');
title('High, Low, Open, Close Chart for TMW')
```



## Input Arguments

Data - Data for opening, high, low, and closing prices

## matrix | table | timetable

Data for opening, high, low, and closing prices, specified as a matrix, table, or timetable.
For matrix input, Data is an M-by-4 matrix of opening, high, low, and closing prices stored in the corresponding columns. Timetables and tables with $M$ rows must contain variables named 'Open', 'High', 'Low', and 'Close' (case insensitive).

Data Types: double| table| timetable

## Color - Three element color vector

background color of figure window (default) | color vector [R G B] | string
(Optional) Three element color vector, specified as a [R G B] color vector or a string specifying the color name. The default color differs depending on the background color of the figure window. For more information, see colorspec.

## Data Types: double|string

## ax - Valid axis object

current axes (ax = gca) (default) | axes object
(Optional) Valid axis object, specified as an axes object. The highlow plot is created in the axes specified by ax instead of in the current axes ( $a x=\mathrm{gca}$ ). The option ax can precede any of the input argument combinations.

## Data Types: object

## Output Arguments

## h - Graphic handle of the figure <br> handle object

Graphic handle of the figure, returned as a handle object.

## See Also

candle|kagi|linebreak|movavg|pointfig|priceandvol|renko|table| timetable|volarea

## Topics

"Using Timetables in Finance"
"Convert Financial Time Series Objects fints to Timetables" on page 12-2

## Introduced before R2006a

## hist

Histogram

Note hist will be removed in a future release. Use timetable instead. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

hist(tsobj, numbins)
ftshist = hist(tsobj, numbins)
[ftshist, binpos] = hist(tsobj, numbins)

## Arguments

| tsobj | Financial time series object. |
| :--- | :--- |
| numbins | (Optional) Number of histogram bins. Default $=10$. |

## Description

hist(tsobj, numbins) calculates and displays the histogram of the data series contained in the financial time series object tsobj.
ftshist = hist(tsobj, numbins) calculates, but does not display, the histogram of the data series contained in the financial time series object tsobj. The output ftshist is a structure with field names similar to the data series names of tsobj.
[ftshist,binpos] = hist(tsobj, numbins) additionally returns the bin positions binpos. The positions are the centers of each bin. binpos is a column vector.

## Examples

## Create a Histogram

This example shows how to generate a histogram of Disney open, high, low, and close prices.
load disney.mat
dis = rmfield(dis,'VOLUME'); \% Remove VOLUME field hist(dis)

Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in title('Disney Histogram')


## See Also

histogram|mean|std

## Topics

"Financial Time Series Operations" on page 13-7

Introduced before R2006a

## holdings2weights

Portfolio holdings into weights

## Syntax

```
Weights = holdings2weights(Holdings,Prices,Budget)
```


## Arguments

| Holdings | Number of portfolios (NPORTS) by number of assets (NASSETS) <br> matrix with the holdings of NPORTS portfolios containing NASSETS <br> assets. |
| :--- | :--- |
| Prices | NASSETS vector of asset prices. |
| Budget | (Optional) Scalar or NPORTS vector of nonzero budget constraints. <br> Default $=1$. |

## Description

Weights $=$ holdings2weights(Holdings,Prices, Budget) converts portfolio holdings into portfolio weights. The weights must satisfy a budget constraint such that the weights sum to Budget for each portfolio.

Weights is a NPORTS by NASSETS matrix containing the normalized weights of NPORTS portfolios containing NASSETS assets.

## Notes

- Holdings may be negative to indicate a short position, but the overall portfolio weights must satisfy a nonzero budget constraint.
- The weights in each portfolio sum to the Budget value (which is 1 if Budget is unspecified.)
See Alsoweights2holdings
Topics"Data Transformation and Frequency Conversion" on page 13-13
Introduced before R2006a


## holidays

Holidays and nontrading days

## Syntax

```
H = holidays
H = holidays(StartDate,EndDate)
H = holidays(
```

$\qquad$

``` , AltHolidays)
```


## Description

$\mathrm{H}=$ holidays returns a vector of serial date numbers corresponding to all holidays and nontrading days.
$H=$ holidays(StartDate, EndDate) returns a vector of serial date numbers corresponding to the holidays and nontrading days betweenStartDate and EndDate, inclusive.

H = holidays( $\qquad$ ,AltHolidays) returns a vector of serial date numbers corresponding to the alternate list of holidays and nontrading days.

## Examples

## Determine Holidays for a Given StartDate and EndDate

Create a vector of serial date numbers corresponding to all holidays and nontrading dates between a specified StartDate and EndDate:

H = holidays('jan 1 2001', 'jun 23 2001')
$H=5 \times 1$
730852
730866

```
datestr(H)
ans = 5x11 char array
    '01-Jan-2001'
    '15-Jan-2001'
    '19-Feb-2001'
    '13-Apr-2001'
    '28-May-2001'
```

Alternatively, using a datetime array for StartDate and EndDate returns a datetime array for H .

```
H = holidays(datetime('1-Jan-2001','Locale','en_US'),...
datetime('23-Jun-2001','Locale','en_US'))
H = 5x1 datetime array
    01-Jan-2001
    15-Jan-2001
    19-Feb-2001
    13-Apr-2001
    28-May-2001
```


## Input Arguments

## StartDate - Start date

serial date number | date character vector | datetime object
Start date, specified using a serial date number, date character vector, or datetime array.
Data Types: double | char|datetime

## EndDate - End date

serial date number | date character vector | datetime object
End date, specified using a serial date number, date character vector, or datetime array.

Data Types: double | char | datetime

## AltHolidays - Alternate list of holidays and nontrading days

serial date number | date character vector | datetime object
Alternate list of holidays and nontrading days, specified using a serial date number, date character vector, or datetime array.

Data Types: double|char|datetime

## Output Arguments

## H - Dates corresponding to all holidays and nontrading days

 vectorDates corresponding to all holidays and nontrading days, returned as a vector of dates. If StartDate, EndDate, and AltHolidays are all either serial date numbers or date character vectors, H is returned as serial date numbers. If either StartDate, EndDate, or AltHolidays are datetime arrays, H is returned as a datetime array.

## Definitions

## holidays

The holidays function is based on a modern 5-day workweek.
This function contains all holidays and special nontrading days for the New York Stock Exchange from January 1, 1885 to December 31, 2070.

Since the New York Stock Exchange was open on Saturdays before September 29, 1952, exact closures from 1885 to 2070 should include Saturday trading days. To capture these dates, use the function nyseclosures. The results from holidays and nyseclosures are identical if the WorkWeekFormat in nyseclosures is 'Modern'.

## See Also

busdate|createholidays|datetime |isbusday|lbusdate|nyseclosures

## Topics

"Handle and Convert Dates" on page 2-2
"Trading Calendars User Interface" on page 16-2
"UICalendar User Interface" on page 16-4

## Introduced before R2006a

## horzcat

Concatenate financial time series objects horizontally

Note horzcat will be removed in a future release. Use timetable instead. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

horzcat

## Description

horzcat implements horizontal concatenation of financial time series objects. horzcat essentially merges the data columns of the financial time series objects. The time series objects must contain the exact same dates and times.

When multiple instances of a data series name occur, concatenation adds a suffix to the current names of the data series. The suffix has the format objectname<n>, where $n$ is a number indicating the position of the time series, from left to right, in the concatenation command. The n part of the suffix appears only when there is more than one instance of a particular data series name.

The description fields are concatenated as well. They are separated by two forward slashes (//).

## Examples

Construct three financial time series, each containing a data series named DataSeries:

```
firstfts = fints((today:today+4)', (1:5)','DataSeries','d')
secondfts = fints((today:today+4)', (11:15)','DataSeries','d')
thirdfts = fints((today:today+4)', (21:25)','DataSeries','d')
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints (line 165)
```

```
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/display (line 66)
firstfts =
    desc: (none)
    freq: Daily (1)
    'dates: (5)' 'DataSeries: (5)'
    '02-0ct-2017' [ 1]
    '03-0ct-2017' [ 2]
    '04-Oct-2017' [ 3]
    '05-0ct-2017' [ 4]
    '06-Oct-2017' [ 5]
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints (line 165)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/display (line 66)
secondfts =
desc: (none)
freq: Daily (1)
'dates: (5)' 'DataSeries: (5)'
'02-0ct-2017' [ 11]
'03-0ct-2017' [ 12]
'04-Oct-2017' [ 13]
'05-0ct-2017' [ 14]
'06-0ct-2017' [ 15]
Warning: FINTS will be removed in a future release. Use TIMETABLE instead. > In fints (line 165)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead. > In fints/display (line 66)
```

```
thirdfts =
```

thirdfts =
desc: (none)
freq: Daily (1)
'dates: (5)' 'DataSeries: (5)'
'02-Oct-2017' [ 21]

```
\begin{tabular}{lll} 
'03-0ct-2017' & {\([\)} & \(22]\) \\
'04-0ct-2017' & {\([\)} & \(23]\) \\
'05-0ct-2017' & {\([\)} & \(24]\) \\
\hline \(06-0 c t-2017 '\) & {\([\)} & \(25]\)
\end{tabular}

Concatenate the time series horizontally into a new financial time series newfts:
```

newfts = [firstfts secondfts thirdfts secondfts]

```
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/horzcat (line 56)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/display (line 66)
newfts =
desc: // // //
freq: Daily (1)
\begin{tabular}{|c|c|c|c|c|}
\hline 'dates: (5) & & 'DataSeries_firstfts' & 'DataSeries_secondft' & 'DataSe \\
\hline & & ': (5) & 's2: (5) ' & : (5) \\
\hline '02-0ct-2017' & & [ 1] & [ 11] & [ \\
\hline '03-0ct-2017' & & [ 2] & [ 12] & [ \\
\hline '04-0ct-2017' & & [ 3] & [ 13] & [ \\
\hline ' 05-0ct-2017' & & [ 4] & [ 14] & [ \\
\hline '06-0ct-2017' & & 5] & [ 15] & [ \\
\hline
\end{tabular}

The resulting object newfts has data series names DataSeries firstfts, DataSeries_secondfts2, DataSeries_thirdfts, and DataSeries_secondfts4.

Verify this with the command
```

fieldnames(newfts)

```
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/fieldnames (line 21)
ans \(=\)
    'desc'
    'freq'
    'dates'
    'DataSeries firstfts'
    'DataSeries_secondfts2'
    'DataSeries_thirdfts'

\section*{'DataSeries secondfts4' \\ 'times'}

Use chfield to change the data series names.

Note If all input objects have the same frequency, the new object has that frequency as well. However, if one of the objects concatenated has a different frequency from the others, the frequency indicator of the resulting object is set to Unknown (0).

\author{
See Also \\ busdate|createholidays|isbusday|lbusdate|nyseclosures
}

\section*{Topics}
"Merge Financial Time Series Objects" on page 14-12

Introduced before R2006a

\section*{hour}

Hour of date or time

\section*{Syntax}
```

Hour = hour(Date)
Hour = hour(
,F)

```

\section*{Description}

Hour \(=\) hour(Date) returns the hour of the day given a serial date number or a date character vector.

Hour \(=\) hour (__, F) returns the hour of one or more date character vectors using format defined by the optional input F. Date can be an array of date character vectors, where each row corresponds to one date character vector, or a one-dimensional cell array of character vectors. All the character vectors in Date must have the same format F. F must designate a supported date format symbol. For more information on supported date formats, see datestr.

\section*{Examples}

\section*{Determine the Hour of the Day for Various Dates}

Find the hour of the day for Date using a serial date number.
```

Hour = hour(730473.5584278936)
Hour = 13

```

Find the hour of the day for Date using a date character vector format.
```

Hour = hour('19-dec-1999, 13:24:08.17')
Hour = 13

```

\section*{Input Arguments}

\section*{Date - Date to determine hour}
serial date number | date character vector | cell array of date character vectors
Date to determine hour, specified as a serial date number or date character vector.
Date can be an array of date character vectors, where each row corresponds to one date character vector, or a one-dimensional cell array of character vectors. All of the character vectors in Date must have the same format F. F must designate a supported date format symbol. For more information on supported date formats, see datestr
Data Types: single \| double \| char \| cell

\section*{F - Date format symbol}
character vector designating date format
Date format symbol, specified as a character vector to designate the date format symbol for input argument Date. For more information on supported date character vector formats, see datestr. Note, formats with ' \(Q\) ' are not accepted.
Data Types: char

\section*{Output Arguments}

\section*{Hour - Hour of day}
serial date number | datetime array
Hour of the day, returned as a serial date number or date character vector.

\section*{See Also}
datevec|minute | second

\section*{Topics}
"Handle and Convert Dates" on page 2-2

\section*{Introduced before R2006a}

\section*{inforatio}

Calculate information ratio for one or more assets

\section*{Syntax}
inforatio(Asset, Benchmark)
Ratio = inforatio(Asset,Benchmark)
[Ratio,TE] = inforatio(Asset,Benchmark)

\section*{Arguments}
\begin{tabular}{|l|l|}
\hline Asset & \begin{tabular}{l} 
NUMSAMPLES-by-NUMSERIES matrix with NUMSAMPLES observations \\
of asset returns for NUMSERIES asset return series.
\end{tabular} \\
\hline Benchmark & \begin{tabular}{l} 
NUMSAMPLES vector of returns for a benchmark asset. The \\
periodicity must be the same as the periodicity of Asset. For \\
example, if Asset is monthly data, then Benchmark must be \\
monthly returns.
\end{tabular} \\
\hline
\end{tabular}

\section*{Description}

Given NUMSERIES assets with NUMSAMPLES returns for each asset in a NUMSAMPLES x NUMSERIES matrix Asset and given a NUMSAMPLES vector of benchmark returns in Benchmark, inforatio computes the information ratio and tracking error for each asset relative to the Benchmark.

To summarize the outputs of inforatio:
- Ratio is a \(1 \times\) NUMSERIES row vector of information ratios for each series in Asset. Any series in Asset with a tracking error of 0 has a NaN value for its information ratio.
- TE is a \(1 \times\) NUMSERIES row vector of tracking errors, that is, the standard deviation of Asset relative to Benchmark returns, for each series.

Note NaN values in the data are ignored. If the Asset and Benchmark series are identical, the information ratio is NaN since the tracking error is 0 . The information ratio and the Sharpe ratio of an Asset versus a riskless Benchmark (a Benchmark with standard deviation of returns equal to 0 ) are equivalent. This equivalence is not necessarily true if the Benchmark is risky.

\section*{Examples}

See "Information Ratio" on page 7-9.

\section*{References}

Richard C. Grinold and Ronald N. Kahn. Active Portfolio Management. 2nd. Edition. McGraw-Hill, 2000.

Jack Treynor and Fischer Black. "How to Use Security Analysis to Improve Portfolio Selection." Journal of Business. Vol. 46, No. 1, January 1973, pp. 66-86.

\section*{See Also}
portalpha|sharpe

\section*{Topics}
"Information Ratio" on page 7-9
"Performance Metrics Overview" on page 7-2

\section*{Introduced in R2006b}

\section*{irr}

Internal rate of return

\section*{Syntax}

Return = irr(CashFlow) [Return,AllRates] = irr( \(\qquad\)

\section*{Description}

Return \(=\operatorname{irr}(\) CashFlow \()\) calculates the internal rate of return for a series of periodic cash flows.
irr uses the following conventions:
- If one or more internal rates of returns (warning if multiple) are strictly positive rates, Return sets to the minimum.
- If there is no strictly positive rate of returns, but one or multiple (warning if multiple) returns are nonpositive rates, Return sets to the maximum.
- If no real-valued rates exist, Return sets to NaN (no warnings).
[Return, AllRates] = irr (__ ) calculates the internal rate of return and a vector of all internal rates for a series of periodic cash flows.

\section*{Examples}

\section*{Find a Single Internal Rate of Return}

Find the internal rate of return for a simple investment with a unique positive rate of return. The initial investment is \(\$ 100,000\) and the following cash flows represent the yearly income from the investment.
- Year 1 - \$10,000
- Year \(2-\$ 20,000\)
- Year 3 - \$30,000
- Year 4 - \$40,000
- Year 5 - \$50,000

Calculate the internal rate of return on the investment:
Return = irr([-100000 10000 200003000040000 50000])
This returns:

\section*{Return =}
0.1201

If the cash flow payments were monthly, then the resulting rate of return is multiplied by 12 for the annual rate of return.

\section*{Find Multiple Internal Rates of Return}

Find the internal rate of return for multiple rates of return. The project has the following cash flows and a market rate of \(10 \%\).
```

CashFlow = [-1000 6000 -10900 5800]

```

Use irr with a single output argument:
Return \(=\) irr(CashFlow)
A warning appears and irr returns a \(100 \%\) rate of return. The \(100 \%\) rate on the project looks attractive:
```

Warning: Multiple rates of return
> In irr at 166
Return =
1.0000

```

Use irr with two output arguments:
[Return, AllRates] = irr(CashFlow)

This returns:
```

>> [Return, AllRates] = irr(CashFlow)
Return =
1.0000
AllRates =
-0.0488
1.0000
2.0488

```

The rates of return in AllRates are \(-4.88 \%, 100 \%\), and \(204.88 \%\). Though some rates are lower and some higher than the market rate, based on the work of Hazen, any rate gives a consistent recommendation on the project. However, you can use a present value analysis in these kinds of situations. To check the present value of the project, use pvvar:
```

PV = pvvar(CashFlow,0.10)

```

This returns:
PV =
-196.0932
The second argument is the \(10 \%\) market rate. The present value is -196.0932 , negative, so the project is undesirable.

\section*{Input Arguments}

\section*{CashFlow - Stream of periodic cash flows}

\section*{vector | matrix}

Stream of periodic cash flows, specified as a vector or matrix. The first entry in CashFlow is the initial investment. If CashFlow is a matrix, irr handles each column of CashFlow as a separate cash-flow stream.
Data Types: double

\section*{Output Arguments}

\section*{Return - Internal rate of return}
vector
Internal rate of return associated to CashFlow, returned as a vector whose entry \(j\) is an internal rate of return for column \(j\) in CashFlow

\section*{AllRates - All the internal rates of return}
serial date number | matrix
All the internal rates of return associated with CashFlow, returned as a matrix with the same number of columns as CashFlow and one less row. Also, column jin AllRates contains all the rates of return associated to column \(j\) in CashFlow (including complexvalued rates).

\section*{References}
[1] Brealey and Myers. Principles of Corporate Finance. McGraw-Hill Higher Education, Chapter 5, 2003.
[2] Hazen G. "A New Perspective on Multiple Internal Rates of Return." The Engineering Economist. Vol. 48-1, 2003, pp. 31-51.

\section*{See Also}
effrr|mirr|nomrr|pvvar|xirr

\section*{Topics}
"Interest Rates/Rates of Return" on page 2-18
"Analyzing and Computing Cash Flows" on page 2-18
"Performance Metrics Overview" on page 7-2

Introduced before R2006a

\section*{isbusday}

True for dates that are business days

\section*{Syntax}
```

Busday = isbusday(Date)
Busday = isbusday(

```
\(\qquad\)
``` ,Holiday, Weekend)
```


## Description

Busday $=$ isbusday (Date) returns logical true (1) if Date is a business day and logical false (0) otherwise.

Busday = isbusday( $\qquad$ , Holiday, Weekend), using optional input arguments, returns logical true (1) if Date is a business day, and logical false (0) otherwise.

## Examples

## Determine If a Given Date Is a Business Day

Determine if Date is a business day.

```
Busday = isbusday('16 jun 2001')
```

Busday = logical
0

Determine if a Date vector are business days.

```
Date = ['15 feb 2001'; '16 feb 2001'; '17 feb 2001'];
Busday = isbusday(Date)
Busday = 3x1 logical array
```

1
1
0

Determine if a Date vector are business days using a datetime array.

```
Date = ['15-feb-2001'; '16-feb-2001'; '17-feb-2001'];
Busday = isbusday(datetime(Date,'Locale','en_US'))
Busday = 3x1 logical array
```

    1
    1
    0
    Set June 21, 2003 (a Saturday) as a business day.

```
Weekend = [1 0 0 0 0 0 0];
```

isbusday('June 21, 2003', [], Weekend)

```
ans = logical
```

    1
    If the second argument, Holiday, is empty ([ ]), the default Holidays vector (generated with holidays and then associated to the NYSE calendar) is used.

## Input Arguments

## Date - Date being checked

serial date number | date character vector | datetime array
Date being checked, specified as a serial date number, date character vector, or datetime array. Date can contain multiple dates, but they must all be in the same format. Dates are assumed to be whole date numbers or date stamps with no fractional or time values.

Data Types: double | char | datetime

## Holiday - Holidays and nontrading-day dates

non-trading day vector is determined by the routine holidays (default) | serial date
number | date character vector | datetime array

Holidays and nontrading-day dates, specified as vector.
All dates in Holiday must be the same format: either serial date numbers, or date character vectors, or datetime arrays. (Using serial date numbers improves performance.) The holidays function supplies the default vector.

## Data Types: double | char | datetime

## Weekend - Weekend days

[1000000cll (Saturday and Sunday form the weekend) (default) | vector of length 7 , containing 0 and 1 , where 1 indicates weekend days

Weekend days, specified as a vector of length 7 , containing 0 and 1 , where 1 indicates weekend days and the first element of this vector corresponds to Sunday.

## Data Types: double

## Output Arguments

## Busday - Logical true if a business day

logical 0 or 1
Logical true if a business day, returned as a logical true (1) if Date is a business day and logical false (0) otherwise.

## See Also

busdate|datetime | fbusdate| holidays | lbusdate

## Topics

"Handle and Convert Dates" on page 2-2
"Trading Calendars User Interface" on page 16-2
"UICalendar User Interface" on page 16-4
Introduced before R2006a

## iscompatible

Structural equality

Note iscompatible will be removed in a future release. Use timetable instead. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

iscomp = iscompatible(tsobj_1,tsobj_2)

## Arguments

tsobj_1, tsobj_2 $\quad$ A pair of financial time series objects.

## Description

iscomp = iscompatible(tsobj_1,tsobj_2) returns 1 if both financial time series objects tsobj_1 and tsobj_2 have the same dates and data series names. It returns 0 if any component is different.
iscomp = 1 indicates that the two objects contain the same number of data points and equal number of data series. However, the values contained in the data series can be different.

Note Data series names are case-sensitive.

## See Also

isequal

## Topics

"What Is the Financial Time Series App?" on page 14-2

Introduced before R2006a

## isequal

Multiple object equality

Note isequal will be removed in a future release. Use timetable instead. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

```
iseq = isequal(tsobj_1,tsobj_2, ...)
```


## Arguments

| tsobj_1 ... | A list of financial time series objects. |
| :--- | :--- |

## Description

iseq $=$ isequal (tsobj_1,tsobj_2, ...) returns 1 if all listed financial time series objects have the same dates, data series names, and values contained in the data series. It returns 0 if any of those components is different.

Note Data series names are case-sensitive.
iseq $=1$ implies that each object contains the same number of dates and the same data. Only the descriptions can differ.

## See Also

eq | iscompatible

## Topics

"What Is the Financial Time Series App?" on page 14-2

Introduced before R2006a

## isempty

True for empty financial time series objects

Note isempty will be removed in a future release. Use timetable instead. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

```
tf = isempty(fts)
```


## Arguments

fts $\quad$ Financial time series object.

## Description

isempty for financial times series objects is based on the MATLAB isempty function. See isempty.
$t f=$ isempty (fts) returns true (1) if fts is an empty financial time series object and false (0) otherwise. An empty financial times series object has no elements, that is, length(fts) $=0$.

## See Also

nanmax | nanmean | nanmedian | nanmin | nanstd | nanvar

## Topics

"What Is the Financial Time Series App?" on page 14-2

Introduced before R2006a

## isfield

Check whether character vector is field name

Note isfield will be removed in a future release. Use timetable instead. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

$F=i s f i e l d(t s o b j$, name $)$

## Description

$\mathrm{F}=$ isfield(tsobj, name) returns true (1) if name is the name of a data series in tsobj. Otherwise, isfield returns false (0).

## See Also

fieldnames | getfield|setfield

## Topics

"What Is the Financial Time Series App?" on page 14-2

Introduced before R2006a

## issorted

Check whether dates and times are monotonically increasing

Note issorted will be removed in a future release. Use timetable instead. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

```
monod = issorted(tsobj)
```


## Arguments

```
tsobj Financial time series object
```


## Description

## monod = issorted(tsobj) returns 1 if the dates and times in tsobj are monotonically increasing or 0 if they are not.

## See Also

sortfts

## Topics

"What Is the Financial Time Series App?" on page 14-2

Introduced before R2006a

## kagi

Kagi chart

Note kagi is updated to accept data input as a matrix, timetable, or table.
The syntax for kagi has changed. Previously, when using table input, the first column of dates could be serial date numbers, date character vectors, or datetime arrays, and you were required to have specific number of columns.

When using table input, the new syntax for kagi supports:

- No need for time information. If you want to pass in date information, use timetable input.
- No requirement of specific number of columns. However, you must provide valid column names. kagi must contain a column named 'price' (case insensitive).


## Syntax

kagi(Data)
h = kagi(ax,Data)

## Description

kagi (Data) plots a Kagi chart from a series of prices.
$\mathrm{h}=$ kagi(ax, Data) adds an optional argument for ax .

## Examples

## Generate a Kagi Chart for a Data Series for a Stock

Load the file SimulatedStock. mat, which provides a timetable (TMW) for financial data for TMW stock. This Kagi chart is for closing prices of the stock TMW for the most recent

21 days. Note that the variable name of asset price is be renamed to 'Price' (case insensitive).
load SimulatedStock.mat
TMW.Properties.VariableNames\{'Close'\} = 'Price';
kagi(TMW(end-20:end,:))
title('Kagi Chart for TMW')
Kagi Chart for TMW


2016

## Input Arguments

## Data - Data for a series of prices

matrix | table | timetable

Data for a series of prices, specified as a matrix, table, or timetable. Timetables and tables with M rows must contain a variable named 'Price' (case insensitive).

## Data Types: double | table | timetable

## ax - Valid axis object

current axes ( $\mathrm{ax}=\mathrm{gca}$ ) (default) | axes object
(Optional) Valid axis object, specified as an axes object. The kagi plot is created in the axes specified by ax instead of in the current axes ( $a x=g c a$ ). The option ax can precede any of the input argument combinations.
Data Types: object

## Output Arguments

## h - Graphic handle of the figure <br> handle object

Graphic handle of the figure, returned as a handle object.

## See Also

candle| highlow| linebreak|movavg|pointfig|priceandvol|renko|table| timetable|volarea

## Topics

"Using Timetables in Finance"
"Convert Financial Time Series Objects fints to Timetables" on page 12-2

## Introduced in R2008a

## lagts

Lag time series object

Note lagts will be removed in a future release. Use timetable instead. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

```
newfts = lagts(oldfts)
newfts = lagts(oldfts,lagperiod)
newfts = lagts(oldfts,lagperiod,padmode)
```


## Arguments

| oldfts | Financial time series object |
| :--- | :--- |
| lagperiod | Number of lag periods expressed in the frequency of the time series <br> object |
| padmode | Data padding value |

## Description

lagts delays a financial time series object by a specified time step.
newfts = lagts(oldfts) delays the data series in oldfts by one time series date entry and returns the result in the object newfts. The end is padded with zeros, by default.
newfts = lagts(oldfts,lagperiod) shifts time series values to the right on an increasing time scale. lagts delays the data series to happen later. lagperiod is the number of lag periods expressed in the frequency of the time series object oldfts. For example, if oldfts is a daily time series, lagperiod is specified in days. lagts pads the data with zeros (default).
newfts $=$ lagts(oldfts, lagperiod, padmode) lets you pad the data with an arbitrary value, NaN, or Inf rather than zeros by setting padmode to the desired value.

## See Also

leadts

Topics<br>"Data Transformation and Frequency Conversion" on page 13-13

Introduced before R2006a

## lbusdate

Last business date of month

## Syntax

Date = lbusdate(Year,Month)
Date = lbusdate( $\qquad$ ,Holiday, Weekend, outputType)

## Description

Date = lbusdate(Year,Month) returns the serial date number for the last business date of the given year and month.

Year and Month can contain multiple values. If one contains multiple values, the other must contain the same number of values or a single value that applies to all. For example, if Year is a 1-by-N vector of integers, then Month must be a 1-by-N vector of integers or a single integer. Date is then a 1-by-N vector of date numbers.

Use the function datestr to convert serial date numbers to formatted date character vectors.

Date = lbusdate( $\qquad$ ,Holiday, Weekend, outputType) returns the serial date number for the last business date of the given year and month using optional input arguments. The optional argument Holiday specifies nontrading days.

If neither Holiday nor outputType are specified, Date is returned as a serial date number. If Holiday is specified, but not outputType, then the type of the holiday variable controls the type of date. If Holiday is a serial date number or date character vector, then Date is returned as a serial date number.

## Examples

## Determine the Last Business Date of a Given Year and Month

Determine the Date using an input argument for Year and Month.

```
Date = lbusdate(2001, 5)
Date = 731002
datestr(Date)
ans =
'31-May-2001'
```

Determine the Date using the optional input argument for outputType.

```
Date = lbusdate(2001, 11,[],[],'datetime')
```

Date = datetime
30-Nov-2001

Indicate that Saturday is a business day by appropriately setting the Weekend argument. May 31, 2003, is a Saturday. Use lbusdate to check that this Saturday is actually the last business day of the month.

```
Weekend = [1 0 0 0 0 0 0];
Date = datestr(lbusdate(2003, 5, [], Weekend))
Date =
'31-May-2003'
```


## Input Arguments

## Year - Year to determine occurrence of weekday <br> 4-digit integer | vector of 4-digit integers

Year to determine occurrence of weekday, specified as a 4-digit integer or vector of 4-digit integers.
Data Types: single | double

## Month - Month to determine occurrence of weekday

integer with value 1 through 12 | vector of integers with values 1 through 12

Month to determine occurrence of weekday, specified as an integer or vector of integers with values 1 through 12.

## Data Types: single | double

## Holiday - Holidays and nontrading-day dates

non-trading day vector is determined by the routine holidays (default) | serial date number | date character vector | datetime array

Holidays and nontrading-day dates, specified as vector.
All dates in Holiday must be the same format: either serial date numbers, or date character vectors, or datetime arrays. (Using serial date numbers improves performance.) The holidays function supplies the default vector.

If Holiday is a datetime array, then Date is returned as a datetime array. If outputType is specified, then its value determines the output type of Date. This overrides any influence of Holiday.

Data Types: double | char | datetime

## Weekend - Weekend days

## [1 0 0 0 0 0 1] (Saturday and Sunday form the weekend) (default) | vector of length 7 , containing 0 and 1 , where 1 indicates weekend days

Weekend days, specified as a vector of length 7 , containing 0 and 1 , where 1 indicates weekend days and the first element of this vector corresponds to Sunday.

## Data Types: double

## outputType - Year to determine days

'datenum' (default)| character vector with values 'datenum' or 'datetime'
A character vector specified as either 'datenum' or 'datetime'. The output Date is in serial date format if 'datenum' is specified, or datetime format if 'datetime' is specified. By default the output Date is in serial date format, or match the format of Holiday, if specified.
Data Types: char

## Output Arguments

## Date - Date for the last business date of given year and month

serial date number | date character vector | datetime array
Date for the last business date of a given year and month, returned as a serial date number, date character vector, or datetime array.

If neither Holiday nor outputType are specified, Date is returned as a serial date number. If Holiday is specified, but not outputType, then the type of the holiday variable controls the type of date:

- If Holiday is a serial date number or date character vector, then Date is returned as a serial date number
- If Holiday is a datetime array, then Date is returned as a datetime array.


## See Also

busdate|datetime|eomdate|fbusdate|holidays|isbusday

## Topics

"Handle and Convert Dates" on page 2-2
"Trading Calendars User Interface" on page 16-2
"UICalendar User Interface" on page 16-4

## Introduced before R2006a

## leadts

Lead time series object

Note leadts will be removed in a future release. Use timetable instead. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

```
newfts = leadts(oldfts)
newfts = leadts(oldfts,leadperiod)
newfts = leadts(oldfts,leadperiod,padmode)
```


## Arguments

| oldfts | Financial time series object. |
| :--- | :--- |
| leadperiod | Number of lead periods expressed in the frequency of the time <br> series object. |
| padmode | Data padding value. |

## Description

leadts advances a financial time series object by a specified time step.
newfts = leadts(oldfts) advances the data series in oldfts by one time series date entry and returns the result in the object newfts. The end will be padded with zeros, by default.
newfts $=$ leadts(oldfts,leadperiod) shifts time series values to the left on an increasing time scale. leadts advances the data series to happen at an earlier time. leadperiod is the number of lead periods expressed in the frequency of the time series object oldfts. For example, if oldfts is a daily time series, leadperiod is specified in days. leadts pads the data with zeros (default).
newfts = leadts(oldfts,leadperiod, padmode) lets you pad the data with an arbitrary value, NaN, or Inf rather than zeros by setting padmode to the desired value.

## See Also

lagts

Topics<br>"Data Transformation and Frequency Conversion" on page 13-13

## Introduced before R2006a

## length

Get number of dates (rows)

Note length will be removed in a future release. Use timetable instead. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

lenfts = length(tsobj)

## Description

lenfts $=$ length(tsobj) returns the number of dates (rows) in the financial time series object tsobj. This is the same as issuing lenfts = size(tsobj, 1).

## See Also

length|size

## Topics

"Financial Time Series Operations" on page 13-7

Introduced before R2006a

## lifetableconv

Convert life table series into life tables with forced termination

## Syntax

```
[qx,lx,dx] = lifetableconv(x0,lx0)
[qx,lx,dx] = lifetableconv(x0,y0,y0type)
```


## Description

$[q x, l x, d x]=$ lifetableconv(x0,lx0) converts life table with ages $x 0$ and survival counts lx0 into life tables with termination.
[ $q x, l x, d x]=$ lifetableconv(x0,y0,y0type) converts life table with ages $x 0$ and series y0, specified by the optional argument y0type, into life tables with termination.

## Examples

## Convert Life Table Series into Life Tables with Forced Termination

Load the life table data file.
load us_lifetable_2009
Convert life table series into life tables with forced termination.

```
[qx,lx,dx] = lifetableconv(x,lx);
display(qx(1:20,:))
\begin{tabular}{lll}
0.0064 & 0.0070 & 0.0057 \\
0.0004 & 0.0004 & 0.0004 \\
0.0003 & 0.0003 & 0.0002 \\
0.0002 & 0.0002 & 0.0002 \\
0.0002 & 0.0002 & 0.0001
\end{tabular}
```

| 0.0001 | 0.0002 | 0.0001 |
| :--- | :--- | :--- |
| 0.0001 | 0.0001 | 0.0001 |
| 0.0001 | 0.0001 | 0.0001 |
| 0.0001 | 0.0001 | 0.0001 |
| 0.0001 | 0.0001 | 0.0001 |
| 0.0001 | 0.0001 | 0.0001 |
| 0.0001 | 0.0001 | 0.0001 |
| 0.0001 | 0.0001 | 0.0001 |
| 0.0002 | 0.0002 | 0.0002 |
| 0.0003 | 0.0004 | 0.0002 |
| 0.0004 | 0.0005 | 0.0002 |
| 0.0005 | 0.0006 | 0.0003 |
| 0.0005 | 0.0007 | 0.0003 |
| 0.0006 | 0.0009 | 0.0004 |
| 0.0007 | 0.0010 | 0.0004 |

## display(lx(1:20,:))

1.0e+05 *

| 1.0000 | 1.0000 | 1.0000 |
| :--- | :--- | :--- |
| 0.9936 | 0.9930 | 0.9943 |
| 0.9932 | 0.9926 | 0.9939 |
| 0.9930 | 0.9923 | 0.9937 |
| 0.9927 | 0.9920 | 0.9935 |
| 0.9926 | 0.9919 | 0.9933 |
| 0.9924 | 0.9917 | 0.9932 |
| 0.9923 | 0.9916 | 0.9931 |
| 0.9922 | 0.9914 | 0.9930 |
| 0.9921 | 0.9913 | 0.9929 |
| 0.9920 | 0.9912 | 0.9928 |
| 0.9919 | 0.9911 | 0.9927 |
| 0.9918 | 0.9910 | 0.9926 |
| 0.9917 | 0.9909 | 0.9925 |
| 0.9915 | 0.9907 | 0.9923 |
| 0.9912 | 0.9903 | 0.9921 |
| 0.9908 | 0.9898 | 0.9919 |
| 0.9904 | 0.9892 | 0.9916 |
| 0.9899 | 0.9885 | 0.9913 |
| 0.9892 | 0.9876 | 0.9909 |

## display(dx(1:20,:))

| 637.2266 | 698.8750 | 572.6328 |
| ---: | ---: | ---: |
| 40.4063 | 43.9297 | 36.7188 |

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| 27.1875 | 30.0938 | 24.1406 |
| ---: | ---: | ---: |
| 20.7656 | 23.0781 | 18.3359 |
| 15.9141 | 17.2109 | 14.5625 |
| 14.8672 | 16.3125 | 13.3516 |
| 13.3672 | 14.7891 | 11.8750 |
| 12.1328 | 13.3828 | 10.8203 |
| 10.8125 | 11.6094 | 9.9844 |
| 9.4609 | 9.5781 | 9.3438 |
| 8.6172 | 8.1328 | 9.1172 |
| 9.2656 | 8.8359 | 9.7188 |
| 12.5938 | 13.5078 | 11.6328 |
| 19.1016 | 22.9844 | 15.0234 |
| 27.6719 | 35.5938 | 19.3516 |
| 36.6328 | 48.5703 | 24.0547 |
| 45.0156 | 60.7109 | 28.4844 |
| 53.1406 | 72.8906 | 32.2813 |
| 60.8984 | 85.1172 | 35.2578 |
| 68.3438 | 97.2266 | 37.6875 |

Plot the $q \times$ series and display the legend. The series $q x$ is the conditional probability that a person at age $x$ will die between age $x$ and the next age in the series.

```
plot(x,log(qx))
legend(series)
```



## Convert the Life Table dx Series After Fitting and Generating the Life Table Series

Load the life table data file.
load us_lifetable_2009
Calibrate life table from survival data with the default Heligman-Pollard parametric model.
a = lifetablefit(x,lx)

```
a = 8\times3
\begin{tabular}{rrr}
0.0005 & 0.0006 & 0.0004 \\
0.0592 & 0.0819 & 0.0192 \\
0.1452 & 0.1626 & 0.1048 \\
0.0007 & 0.0011 & 0.0007 \\
6.2840 & 6.7639 & 1.1040 \\
24.1387 & 24.2894 & 53.1672 \\
0.0000 & 0.0000 & 0.0000 \\
1.0971 & 1.0987 & 1.1100
\end{tabular}
```

Generate life table series from the calibrated mortality model.


Convert life table series into life tables with forced termination.

```
[~,~,dx] = lifetableconv((0:120),qx,'qx');
```

display(dx(1:20,:))

| 630.9953 | 686.9417 | 571.6095 |
| ---: | ---: | ---: |
| 48.7927 | 55.1040 | 40.9872 |


| 24.8017 | 26.3779 | 23.6167 |
| ---: | ---: | ---: |
| 17.0832 | 17.5877 | 17.0317 |
| 13.6181 | 13.8187 | 13.6143 |
| 11.8661 | 12.0075 | 11.6314 |
| 10.9781 | 11.1572 | 10.4905 |
| 10.5995 | 10.8604 | 9.9488 |
| 10.5757 | 10.9395 | 9.8951 |
| 10.8789 | 11.3612 | 10.2717 |
| 11.6084 | 12.2508 | 11.0417 |
| 12.9922 | 13.9271 | 12.1763 |
| 15.3480 | 16.8834 | 13.6482 |
| 18.9940 | 21.6791 | 15.4301 |
| 24.1395 | 28.7663 | 17.4945 |
| 30.8009 | 38.3212 | 19.8136 |
| 38.7717 | 50.1487 | 22.3607 |
| 47.6536 | 63.6907 | 25.1104 |
| 56.9302 | 78.1270 | 28.0392 |
| 66.0576 | 92.5260 | 31.1265 |

Plot the $d x$ series and display the legend. The series $d x$ is the number of people who die out of 100,000 alive at birth between age $x$ and the next age in the series.

```
plot((0:119),dx(1:end-1,:));
legend(series, 'location', 'northwest');
title('\bfLife Table Yearly Decrements');
xlabel('Age');
ylabel('Number Dying within a Given Year');
```



## Input Arguments

## x0 - Increasing ages for raw data

vector of nonnegative integer values
Increasing ages for raw data, specified as nonnegative integer values in an N0 vector.
The vector of ages $x$ must contain nonnegative integer values. If the input series is the discrete survival function $l x$, then the starting age need only be nonnegative. Otherwise, the starting age must be 0 .

Data Types: double

## lx0 - Collection of num standardized survivor series matrix

Collection of num standardized survivor series, specified as an N0-by-num matrix. The input $l x 0$ series is the number of people alive at age $x$, given 100,000 alive at birth. Values of 0 or NaN in the input table lx0 are ignored.

## Data Types: double

## y0 - Collection of num life table series to be converted

 matrixCollection of num life table series to be converted, specified as an N0-by-num matrix. The default y0 series is $l x 0$.

## Data Types: double

## y0type - Type of mortality series for input y0 with default ' $\mathbf{l x}$ '

'lx' (default)| character vector with values 'qx', 'lx', 'dx'
(Optional) Type of mortality series for input y0, specified as a character vector with one of the following values:

- ' $q x$ ' - Input is a table of discrete hazards (qx).
- ' lx' - Input is a table of discrete survival counts (lx).
- ' dx ' - Input is a table of discrete decrements ( dx ).

Whereas the output series have forced termination, the input series (y0) can have one of several types of termination:

- Natural termination runs out to the last person so that $l x$ goes to $0, q \times$ goes to 1 , and dx goes to 0 . For more information, see "Natural Termination" on page 19-1141.
- Truncated termination stops at a terminal age so that $l x$ is positive, $q x$ is less than 1 , and $d x$ is positive. Any ages after the terminal age are NaN values. For more information, see "Truncated Termination" on page 19-1142.


## Data Types: char

## Output Arguments

## $q x$ - Discrete hazard function matrix

Discrete hazard function, returned as an N0-by-num matrix with forced termination. For more information, see "Forced Termination" on page 19-1140.

The series qx is the conditional probability that a person at age x will die between age x and the next age in the series.

## lx - Discrete survival function <br> matrix

Discrete survival function, returned as an N0-by-num matrix with forced termination. For more information, see "Forced Termination" on page 19-1140.

The series $l x$ is the number of people alive at age $x$, given 100,000 alive at birth.

## $d x$ - Discrete decrements function

matrix
Discrete decrements function, returned as an N0-by-num matrix with forced termination. For more information, see "Forced Termination" on page 19-1140.

The series dx is the number of people who die out of 100,000 alive at birth, between age $x$ and the next age in the series.

## Definitions

## Forced Termination

Most modern life tables have "forced" termination. Forced termination means that the last row of the life table applies for all persons with ages on or after the last age in the life table.

This sample illustrates forced termination.

## United States Life Tables 2009

|  | $1 \times \mathrm{WM}$ | $l_{x}$ WF | $1 \times \mathrm{BM}$ | $I_{x} \mathrm{BF}$ | $q_{\times}$WM | $q_{\times}$WF | $q_{\times} \mathrm{BM}$ | $q_{x} \mathrm{BF}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 93 | 9533 | 18368 | 6615 | 15685 | 0.219553 | 0.177156 | 0.200605 | 0.157858 |
| 94 | 7440 | 15114 | 5288 | 13209 | 0.23871 | 0.194852 | 0.214448 | 0.170868 |
| 95 | 5664 | 12169 | 4154 | 10952 | 0.258475 | 0.213411 | 0.229177 | 0.184624 |
| 96 | 4200 | 9572 | 3202 | 8930 | 0.27881 | 0.232971 | 0.24391 | 0.198992 |
| 97 | 3029 | 7342 | 2421 | 7153 | 0.299439 | 0.253337 | 0.259397 | 0.214036 |
| 98 | 2122 | 5482 | 1793 | 5622 | 0.320452 | 0.27417 | 0.274958 | 0.229634 |
| 99 | 1442 | 3979 | 1300 | 4331 | 0.341886 | 0.295803 | 0.291538 | 0.245671 |
| 100+ | 949 | 2802 | 921 | 3267 | 1 | 1 | 1 |  |

In this case, the last row of the life table applies for all persons aged 100 or older. Specifically, $q_{x}$ probabilities are ${ }_{1} q_{x}$ for ages less than 100 and, technically, ${ }_{\infty} q_{x}$ for age 100.

Forced termination has terminal age values that apply to all ages after the terminal age so that $l \mathrm{x}$ is positive, qx is 1 , and dx is positive. Ages after the terminal age are NaN values, although $l x$ and $d x$ can be 0 and $q x$ can be 1 for input series. Forced termination is triggered by a naturally terminating series, the last age in a truncated series, or the first NaN value in a series.

## Natural Termination

Before 1970, life tables were often published with data that included all ages for which persons associated with a given series were still alive. Tables in this form have "natural" termination. In natural termination, the last row of the life table for each series counts the deaths or probabilities of deaths of the last remaining person at the corresponding age. Tables in this form can be problematic due to the granularity of the data and the fact that groups of series can terminate at distinct ages. Natural termination is illustrated in the following sample of the last few years of a life table.

United States Life Tables 1940

| $x$ | $l_{x} \mathrm{WM}$ | $l_{x} \mathrm{WF}$ | $l_{x} \mathrm{BM}$ | $l_{x} \mathrm{BF}$ | $q_{x} \mathrm{WM}$ | $q_{x} \mathrm{WF}$ | $q_{x} \mathrm{BM}$ | $q_{x} \mathrm{BF}$ |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 103 | 14 | 29 | 59 | 234 | 0.428571 | 0.413793 | 0.389831 | 0.333333 |
| 104 | 8 | 17 | 36 | 156 | 0.5 | 0.470588 | 0.416667 | 0.358974 |
| 105 | 4 | 9 | 21 | 100 | 0.5 | 0.444444 | 0.47619 | 0.38 |
| 106 | 2 | 5 | 11 | 62 | 0.5 | 0.4 | 0.454545 | 0.403226 |
| 107 | 1 | 3 | 6 | 37 | 0 | 0.666667 | 0.5 | 0.432432 |
| 108 | 1 | 1 | 3 | 21 | 1 | 0 | 0.666667 | 0.47619 |
| 109 |  | 1 | 1 | 11 |  | 1 | 1 | 0.454545 |
| 110 |  |  |  | 6 |  |  |  | 0.5 |
| 111 |  |  |  | 3 |  |  |  | 0.666667 |
| 112 |  |  |  | 1 |  |  |  | 0 |
| 113 |  |  |  | 1 |  |  |  | 1 |
| 114 |  |  |  |  |  |  |  |  |

This form for life tables poses a number of issues that go beyond the obvious statistical issues. First, the $l_{x}$ table on the left terminates at ages 108, 109, 109, and 113 for the four series in the table. Technically, the numbers after these ages are 0, but can also be NaN values because no person is alive after these terminating ages. Second, the probabilities $q_{x}$ on the right terminate with fluctuating probabilities that go from 0 to 1 in some cases. In this case, however, all probabilities are ${ }_{1} q_{x}$ probabilities (unlike the forced termination probabilities). You can argue that the probabilities after the ages of termination can be 1 (anyone alive at this age is expected to die in the next year), 0 (the age lies outside the support of the probability distribution), or NaN values.

## Truncated Termination

Truncated termination occurs with truncation of life tables at an arbitrary age. For example, from 1970-1990, United States life tables truncated at age 85. This format is problematic because life table probabilities must either terminate with probability 1 (forced termination) or discard data that exceeds the terminating age. This sample of the last few years of a life table illustrates truncated termination. The raw data for this table is the $l_{x}$ series. The $q_{x}$ series is derived from this series.

## United States Life Tables 1980

| $x$ | $l_{x} \mathrm{WM}$ | $l_{x} \mathrm{WF}$ | $l_{x} \mathrm{BM}$ | $l_{x} \mathrm{BF}$ | $q_{x} \mathrm{WM}$ | $q_{x} \mathrm{WF}$ | $q_{x} \mathrm{BM}$ | $q_{x} \mathrm{BF}$ |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 78 | 38558 | 61100 | 25481 | 45776 | 0.078064 | 0.045221 | 0.082689 | 0.054789 |
| 79 | 35548 | 58337 | 23374 | 43268 | 0.084477 | 0.050037 | 0.088303 | 0.059351 |
| 80 | 32545 | 55418 | 21310 | 40700 | 0.091504 | 0.055542 | 0.094603 | 0.064619 |
| 81 | 29567 | 52340 | 19294 | 38070 | 0.099165 | 0.061922 | 0.101741 | 0.070607 |
| 82 | 26635 | 49099 | 17331 | 35382 | 0.107528 | 0.06935 | 0.109746 | 0.077356 |
| 83 | 23771 | 45694 | 15429 | 32645 | 0.116613 | 0.078194 | 0.118608 | 0.084822 |
| 84 | 20999 | 42121 | 13599 | 29876 | 0.126339 | 0.081788 | 0.128392 | 0.093018 |
| 85 | 18346 | 38676 | 11853 | 27097 |  |  |  |  |

This life table format poses problems for termination because, for example, over $27 \%$ of the population for the fourth $l_{x}$ series is still alive at age 85 . To claim that the probability of dying for all ages after age 85 is $100 \%$ might be true but is uninformative. Notwithstanding the statistical issues, however, tables in this form are completed by forced termination.

## References

[1] Arias, E. "United States Life Tables." National Vital Statistics Reports, U.S. Department of Health and Human Services. Vol. 62, No. 7, 2009.

## See Also

lifetablefit|lifetablegen

## Topics

"Case Study for Life Tables Analysis" on page 2-53
"About Life Tables" on page 2-50

Introduced in R2015a

## lifetablefit

Calibrate life table from survival data with parametric models

## Syntax

```
[a,elx] = lifetablefit(x,lx)
[a,elx] = lifetablefit( ___,lifemodel,objtype,interpmethod,a0)
```


## Description

[a,elx] = lifetablefit(x,lx) calibrates a life table, $x$, from survival data, lx, using parametric models.
[a,elx] = lifetablefit( $\qquad$ ,lifemodel,objtype,interpmethod,a0) calibrates a life table, $x$, from survival data, $l x$, using parametric models using optional arguments for lifemodel, objtype, interpmethod, and a0.

## Examples

Calibrate Life Table from Survival Data Using a Heligman-Pollard Parametric Model

Load the life table data file.
load us_lifetable_2009
Calibrate the life table from survival data using the default heligman-pollard parametric model.

```
[a,elx] = lifetablefit(x,lx);
display(a)
a = 8\times3
```

| 0.0005 | 0.0006 | 0.0004 |
| :---: | :---: | :---: |
| 0.0592 | 0.0819 | 0.0192 |
| 0.1452 | 0.1626 | 0.1048 |
| 0.0007 | 0.0011 | 0.0007 |
| 6.2840 | 6.7639 | 1.1040 |
| 24.1387 | 24.2894 | 53.1672 |
| 0.0000 | 0.0000 | 0.0000 |
| 1.0971 | 1.0987 | 1.1100 |
| display(elx(1:20, :) ) |  |  |
| 1.0e+05 * |  |  |
| 1.0000 | 1.0000 | 1.0000 |
| 0.9937 | 0.9931 | 0.9943 |
| 0.9932 | 0.9926 | 0.9939 |
| 0.9930 | 0.9923 | 0.9936 |
| 0.9928 | 0.9921 | 0.9935 |
| 0.9926 | 0.9920 | 0.9933 |
| 0.9925 | 0.9919 | 0.9932 |
| 0.9924 | 0.9918 | 0.9931 |
| 0.9923 | 0.9917 | 0.9930 |
| 0.9922 | 0.9916 | 0.9929 |
| 0.9921 | 0.9914 | 0.9928 |
| 0.9920 | 0.9913 | 0.9927 |
| 0.9919 | 0.9912 | 0.9926 |
| 0.9917 | 0.9910 | 0.9924 |
| 0.9915 | 0.9908 | 0.9923 |
| 0.9913 | 0.9905 | 0.9921 |
| 0.9910 | 0.9901 | 0.9919 |
| 0.9906 | 0.9896 | 0.9917 |
| 0.9901 | 0.9890 | 0.9914 |
| 0.9895 | 0.9882 | 0.9912 |

Plot the qx series and display the legend. The series qx is the conditional probability that a person at age $x$ will die between age $x$ and the next age in the series

```
plot(x,log(qx))
legend(series)
```



## Input Arguments

## x - Increasing ages for raw data

vector of nonnegative integers
Increasing ages for raw data, specified as a $\mathbf{N}$ vector for nonnegative integers.
Data Types: double

## lx - Collection of num discrete survival counts matrix

Collection of num discrete survival counts, specified as an $N$-by-num matrix. The input $l x$ series is the number of people alive at age $x$, given 100,000 alive at birth. Values of 0 or NaN in the input table lx are ignored.

## Data Types: double

## lifemodel - Parametric mortality model type

'heligman-pollard' (default)| character vector with values 'heligman-pollard', 'heligman-pollard-2''heligman-pollard-3', 'gompertz', 'makeham', 'siler'
(Optional) Parametric mortality model type, specified as a character vector with one of the following values:

- 'heligman-pollard' - Eight-parameter Heligman-Pollard model (version 1), specified in terms of the discrete hazard function:

$$
\frac{q(x)}{1-q(x)}=A^{(x+B)^{c}}+D \exp \left(-E\left(\log \frac{x}{F}\right)^{2}\right)+G H^{X}
$$

for ages $x \geq 0$, with parameters $A, B, C, D, E, F, G, H \geq 0$.

- 'heligman-pollard-2' - Eight-parameter Heligman-Pollard model (version 2), specified in terms of the discrete hazard function:

$$
\frac{q(x)}{1-q(x)}=A^{(x+B)^{C}}+D \exp \left(-E\left(\log \frac{x}{F}\right)^{2}\right)+\frac{G H^{X}}{1+G H^{X}}
$$

for ages $x \geq 0$, with parameters $A, B, C, D, E, F, G, H \geq 0$.

- 'heligman-pollard-3' - Eight-parameter Heligman-Pollard model (version 3), specified in terms of the discrete hazard function:

$$
q(x)=A^{(x+B)^{c}}+D \exp \left(-E\left(\log \frac{x}{F}\right)^{2}\right)+G H^{X}
$$

for ages $x \geq 0$, with parameters $A, B, C, D, E, F, G, H \geq 0$.

- 'gompertz' - Two-parameter Gompertz model, specified in terms of the continuous hazard function:
$h(x)=A \exp (B x)$
for ages $x \geq 0$, with parameters $A, B \geq 0$.
- 'makeham ' - Three-parameter Gompertz-Makeham model, specified in terms of the continuous hazard function:
$h(x)=A \exp (B x)+C$
for ages $x \geq 0$, with parameters $A, B, C \geq 0$.
- 'siler' - Five-parameter Siler model, specified in terms of the continuous hazard function:
$h(x)=A \exp (B x)+C+D \exp (-E x)$
for ages $x \geq 0$, with parameters $A, B, C, D, E \geq 0$.


## Data Types: char

## objtype - Objective for nonlinear least-squares estimation

'ratio' (default)| character vector with values 'ratio' ' logratio'
(Optional) Objective for nonlinear least-squares estimation, specified as a character vector with the following values:

- 'ratio' - Given raw data $q_{\mathrm{x}}$ and model estimates $\hat{q}_{x}$ for $x=1, \ldots, N$, the first objective (which is the preferred objective) has the form

$$
\Phi=\sum_{x=1}^{N}\left(1-\frac{\hat{q}_{x}}{q_{x}}\right)^{2}
$$

- ' logratio' - Given raw data $q_{\mathrm{x}}$ and model estimates $\hat{q}_{x}$ for $x=1, \ldots, N$, the second objective has the form

$$
\Phi=\sum_{x=1}^{N}\left(\log \left(q_{x}\right)-\log \left(q_{x}\right)\right)^{2}
$$

## Data Types: char

interpmethod - Interpolation method to use for abridged life table inputs
'cubic' (default)| character vector with values 'cubic', 'linear', 'none'
(Optional) Interpolation method to use for abridged life table inputs, specified as a character vector with the following values:

- 'cubic' - Cubic interpolation that uses 'pchip' method in interp1.
- 'linear' - Linear interpolation.
- 'none' - No interpolation.

Note If the ages in $x$ are not consecutive years and interpolation is set to 'none', then the estimates for the parameters are suitable only for the age vector $x$.

If you use the parameter estimates to compute life table values for arbitrary years, interpolate using the default ' cubic' method.

Interpolation with abridged life tables forms internal interpolated full life tables, which usually improves model fits.

## Data Types: char

## a0 - Initial parameter estimate to be applied to all series

## vector

(Optional) Initial parameter estimate to be applied to all series, specified as a numparam vector. This vector must conform to the number of parameters in the model specified using the lifemodel argument.
Data Types: double

## Output Arguments

a - Parameter estimates for each num series
matrix

Parameter estimates for each num series, returned as a numparam-by-num matrix.

## elx - Estimated collection of num standardized survivor series

matrix

Estimated collection of num standardized survivor series, returned as an N -by-num matrix. The elx output series is the number of people alive at age $x$, given 100,000 alive at birth. Values of 0 or NaN in the input table lx are ignored.

## References

[1] Arias, E. "United States Life Tables." National Vital Statistics Reports, U.S. Department of Health and Human Services. Vol. 62, No. 7, 2009.
[2] Carriere, F. "Parametric Models for Life Tables." Transactions of the Society of Actuaries. Vol. 44, 1992, pp. 77-99.
[3] Gompertz, B. "On the Nature of the Function Expressive of the Law of Human Mortality, and on a New Mode of Determining the Value of Life Contingencies." Philosophical Transactions of the Royal Society. Vol. 115, 1825, pp. 513-582.
[4] Heligman, L. M. A., and J. H. Pollard. "The Age Pattern of Mortality." Journal of the Institute of Actuaries Vol. 107, Pt. 1, 1980, pp. 49-80.
[5] Makeham, W. M. "On the Law of Mortality and the Construction of Annuity Tables." Journal of the Institute of Actuaries Vol. 8, 1860, pp. 301-310.
[6] Siler, W. "A Competing-Risk Model for Animal Mortality." Ecology Vol. 60, pp. 750-757, 1979.
[7] Siler, W. "Parameters of Mortality in Human Populations with Widely Varying Life Spans." Statistics in Medicine Vol. 2, 1983, pp. 373-380.

## See Also

lifetableconv|lifetablegen

## Topics

"Case Study for Life Tables Analysis" on page 2-53
"About Life Tables" on page 2-50

Introduced in R2015a

## lifetablegen

Generate life table series from calibrated mortality model

## Syntax

```
[qx,lx,dx] = lifetablegen(x,a)
[qx,lx,dx] = lifetablegen(x,a,lifemodel)
```


## Description

$[q x, l x, d x]=$ lifetablegen $(x, a)$ generates a life table series from a calibrated mortality model.
[ $q x, l x, d x$ ] = lifetablegen( $x, a, l i f e m o d e l)$ generates a life table series from a calibrated mortality model using the optional argument for lifemodel.

## Examples

## Generate Life Table Series from a Calibrated Mortality Model for HeligmanPollard

Load the life table data file.
load us_lifetable_2009
Calibrate the life table from survival data using the default heligman-pollard parametric model.

```
a = lifetablefit(x, lx)
a = 8×3
\begin{tabular}{lll}
0.0005 & 0.0006 & 0.0004 \\
0.0592 & 0.0819 & 0.0192 \\
0.1452 & 0.1626 & 0.1048
\end{tabular}
```

| 0.0007 | 0.0011 | 0.0007 |
| ---: | ---: | ---: |
| 6.2840 | 6.7639 | 1.1040 |
| 24.1387 | 24.2894 | 53.1672 |
| 0.0000 | 0.0000 | 0.0000 |
| 1.0971 | 1.0987 | 1.1100 |

Generate a life table series from the calibrated mortality model.

```
qx = lifetablegen(x,a);
display(qx(1:20,:))
\begin{tabular}{lll}
0.0063 & 0.0069 & 0.0057 \\
0.0005 & 0.0006 & 0.0004 \\
0.0002 & 0.0003 & 0.0002 \\
0.0002 & 0.0002 & 0.0002 \\
0.0001 & 0.0001 & 0.0001 \\
0.0001 & 0.0001 & 0.0001 \\
0.0001 & 0.0001 & 0.0001 \\
0.0001 & 0.0001 & 0.0001 \\
0.0001 & 0.0001 & 0.0001 \\
0.0001 & 0.0001 & 0.0001 \\
0.0001 & 0.0001 & 0.0001 \\
0.0001 & 0.0001 & 0.0001 \\
0.0002 & 0.0002 & 0.0001 \\
0.0002 & 0.0002 & 0.0002 \\
0.0002 & 0.0003 & 0.0002 \\
0.0003 & 0.0004 & 0.0002 \\
0.0004 & 0.0005 & 0.0002 \\
0.0005 & 0.0006 & 0.0003 \\
0.0006 & 0.0008 & 0.0003 \\
0.0007 & 0.0009 & 0.0003
\end{tabular}
```

Plot the qx series and display the legend. The series qx is the conditional probability that a person at age $x$ will die between age $x$ and the next age in the series.

```
plot(x,log(qx))
legend(series)
```



## Prepare Life Table Data Using the qx Series and Generate the qx Life Table

Load the life table data file.
load us lifetable 2009
Convert the life table series into life tables with forced termination.
[~, lx] = lifetableconv(x, qx, 'qx');
Calibrate the life table from survival data using the default heligman-pollard parametric model.

```
a = lifetablefit(x, lx)
a = 8×3
\begin{tabular}{rrr}
0.0005 & 0.0006 & 0.0004 \\
0.0592 & 0.0819 & 0.0192 \\
0.1452 & 0.1626 & 0.1048 \\
0.0007 & 0.0011 & 0.0007 \\
6.2854 & 6.7638 & 1.1156 \\
24.1384 & 24.2895 & 52.6365 \\
0.0000 & 0.0000 & 0.0000 \\
1.0971 & 1.0987 & 1.1099
\end{tabular}
```

Generate a life table series from the calibrated mortality model.

```
qx = lifetablegen((0:100), a)
qx = 101×3
\begin{tabular}{lll}
0.0063 & 0.0069 & 0.0057 \\
0.0005 & 0.0006 & 0.0004 \\
0.0002 & 0.0003 & 0.0002 \\
0.0002 & 0.0002 & 0.0002 \\
0.0001 & 0.0001 & 0.0001 \\
0.0001 & 0.0001 & 0.0001 \\
0.0001 & 0.0001 & 0.0001 \\
0.0001 & 0.0001 & 0.0001 \\
0.0001 & 0.0001 & 0.0001 \\
0.0001 & 0.0001 & 0.0001
\end{tabular}
```

Plot the qx series and display the legend. The series qx is the conditional probability that a person at age $x$ will die between age $x$ and the next age in the series.

```
plot((0:100), log(qx));
legend(series, 'location', 'southeast');
title('Conditional Probability of Dying within One Year of Current Age');
xlabel('Age');
ylabel('Log Probability');
```



## Input Arguments

## x - Increasing ages for raw data

vector of nonnegative integers
Increasing ages for raw data, specified as a $\mathbf{N}$ vector of nonnegative integer values. The ages must start at 0 (birth).

Data Types: double
a - Model parameters for num models
matrix

Model parameters for num models, specified as a numparam-by-num matrix, where the number of parameters (numparam) depends on the model specified using the lifemodel argument.

## Data Types: double

## lifemodel - Parametric mortality model type

'heligman-pollard' (default)| character vector with values 'heligman-pollard', 'heligman-pollard-2''heligman-pollard-3', 'gompertz','makeham', 'siler'
(Optional) Parametric mortality model type, specified as a character vector with one of the following values:

- 'heligman-pollard' - Eight-parameter Heligman-Pollard model (version 1), specified in terms of the discrete hazard function:

$$
\frac{q(x)}{1-q(x)}=A^{(x+B)^{c}}+D \exp \left(-E\left(\log \frac{x}{F}\right)^{2}\right)+G H^{X}
$$

for ages $x \geq 0$, with parameters $A, B, C, D, E, F, G, H \geq 0$.

- 'heligman-pollard-2' - Eight-parameter Heligman-Pollard model (version 2), specified in terms of the discrete hazard function:

$$
\frac{q(x)}{1-q(x)}=A^{(x+B)^{C}}+D \exp \left(-E\left(\log \frac{x}{F}\right)^{2}\right)+\frac{G H^{X}}{1+G H^{X}}
$$

for ages $x \geq 0$, with parameters $A, B, C, D, E, F, G, H \geq 0$.

- 'heligman-pollard-3' - Eight-parameter Heligman-Pollard model (version 3), specified in terms of the discrete hazard function:

$$
q(x)=A^{(x+B)^{c}}+D \exp \left(-E\left(\log \frac{x}{F}\right)^{2}\right)+G H^{X}
$$

for ages $x \geq 0$, with parameters $A, B, C, D, E, F, G, H \geq 0$.

- 'gompertz' - Two-parameter Gompertz model, specified in terms of the continuous hazard function:
$h(x)=A \exp (B x)$
for ages $x \geq 0$, with parameters $A, B \geq 0$.
- 'makeham ' - Three-parameter Gompertz-Makeham model, specified in terms of the continuous hazard function:
$h(x)=A \exp (B x)+C$
for ages $x \geq 0$, with parameters $A, B, C \geq 0$.
- 'siler' - Five-parameter Siler model, specified in terms of the continuous hazard function:
$h(x)=A \exp (B x)+C+D \exp (-E x)$
for ages $x \geq 0$, with parameters $A, B, C, D, E \geq 0$.
Data Types: char


## Output Arguments

## $q x$ - Conditional probabilities of dying for $\mathbf{N}$ ages and num series <br> matrix

Conditional probabilities of dying for N ages and num series, returned as an N -by-num matrix. The series $q x$ is the conditional probability that a person at age $x$ will die between age $x$ and the next age in the series. For the last age, qx represents probabilities or counts for all ages after the last age.

The last row of the N -by-num output for qx is the values for all ages on or after the last age in $x$ (due to "Forced Termination" on page 19-1158). Therefore, the last row of qx contains 1 ( $100 \%$ probability of dying on or after the last age).

## lx - Survival counts for N ages and num series <br> matrix

Survival counts for N ages and num series, returned as an N -by-num matrix. The series lx is the number of people alive at age $x$, given 100,000 alive at birth.

## dx - Decrement counts for N ages and num series <br> matrix

Decrement counts for N ages and num series, returned as an N -by-num matrix. The series dx is the number of people out of 100,000 alive at birth who die between age $x$ and the
next age in the series. For the last age, dx represent probabilities or counts for all ages after the last age.

The last row of the N -by-num output for dx are values for all ages on or after the last age in $x$ (due to "Forced Termination" on page 19-1158). Therefore, the last row of dx contains the remaining count of 100,000 people alive at birth who have not died by the last age.

## Definitions

## Forced Termination

Most modern life tables have "forced" termination. Forced termination means that the last row of the life table applies for all persons with ages on or after the last age in the life table.

This sample illustrates forced termination.

## United States Life Tables 2009



In this case, the last row of the life table applies for all persons aged 100 or older. Specifically, $q_{x}$ probabilities are ${ }_{1} q_{x}$ for ages less than 100 and, technically, ${ }_{\infty} q_{x}$ for age 100.

Forced termination has terminal age values that apply to all ages after the terminal age so that $l \mathrm{x}$ is positive, qx is 1 , and dx is positive. Ages after the terminal age are NaN values, although $l x$ and $d x$ can be 0 and $q x$ can be 1 for input series. Forced termination
is triggered by a naturally terminating series, the last age in a truncated series, or the first NaN value in a series.

## References

[1] Arias, E. "United States Life Tables." National Vital Statistics Reports, U.S. Department of Health and Human Services. Vol. 62, No. 7, 2009.
[2] Carriere, F. "Parametric Models for Life Tables." Transactions of the Society of Actuaries. Vol. 44, 1992, pp. 77-99.
[3] Gompertz, B. "On the Nature of the Function Expressive of the Law of Human Mortality, and on a New Mode of Determining the Value of Life Contingencies." Philosophical Transactions of the Royal Society. Vol. 115, 1825, pp. 513-582.
[4] Heligman, L. M. A., and J. H. Pollard. "The Age Pattern of Mortality." Journal of the Institute of Actuaries Vol. 107, Pt. 1, 1980, pp. 49-80.
[5] Makeham, W. M. "On the Law of Mortality and the Construction of Annuity Tables." Journal of the Institute of Actuaries Vol. 8, 1860, pp. 301-310.
[6] Siler, W. "A Competing-Risk Model for Animal Mortality." Ecology Vol. 60, pp. 750-757, 1979.
[7] Siler, W. "Parameters of Mortality in Human Populations with Widely Varying Life Spans." Statistics in Medicine Vol. 2, 1983, pp. 373-380.

## See Also

lifetableconv|lifetablefit

## Topics

"Case Study for Life Tables Analysis" on page 2-53
"About Life Tables" on page 2-50

## Introduced in R2015a

## linebreak

Line break chart

Note linebreak is updated to accept data input as a matrix, timetable, or table.
The syntax for linebreak has changed. Previously, when using table input, the first column of dates could be serial date numbers, date character vectors, or datetime arrays, and you were required to have specific number of columns.

When using table input, the new syntax for linebreak supports:

- No need for time information. If you want to pass in date information, use timetable input.
- No requirement of specific number of columns. However, you must provide valid column names. linebreak must contain a column named 'price' (case insensitive).


## Syntax

linebreak(Data)
h = linebreak(ax, Data)

## Description

linebreak(Data) plots the asset data, in a line break chart.
$\mathrm{h}=$ linebreak(ax, Data) adds an optional argument for ax.

## Examples

## Generate a Line Break Chart for a Data Series for a Stock

Load the file SimulatedStock. mat, which provides a timetable (TMW) for financial data for TMW stock. This Linebreak chart is for closing prices of the stock TMW for the most
recent 21 days. Note that the variable name of asset price is be renamed to 'Price' (case insensitive).
load SimulatedStock.mat
TMW.Properties.VariableNames\{'Close'\} = 'Price';
linebreak(TMW(end-20:end,:))
title('Line Break Chart for TMW')


Jul 27 Jul 30 Aug 02Aug 05 Aug 08 Aug 11 Aug 14 Aug 17 Aug 20 Aug 23 Aug 26 2016

## Input Arguments

## Data - Data for a series of prices

matrix | table | timetable

Data for a series of prices, specified as a matrix, table, or timetable. Timetables and tables with M rows must contain a variable named 'Price' (case insensitive).

## Data Types: double | table | timetable

## ax - Valid axis object

current axes ( $\mathrm{ax}=\mathrm{gca}$ ) (default) | axes object
(Optional) Valid axis object, specified as an axes object. The linebreak plot is created in the axes specified by ax instead of in the current axes ( $a x=g c a$ ). The option ax can precede any of the input argument combinations.

Data Types: object

## Output Arguments

## h - Graphic handle of the figure <br> handle object

Graphic handle of the figure, returned as a handle object.

## See Also

candle | highlow|kagi|movavg|pointfig|priceandvol|renko|table| timetable|volarea

## Topics

"Using Timetables in Finance"
"Convert Financial Time Series Objects fints to Timetables" on page 12-2

## Introduced in R2008a

## Ilow

Lowest low

Note llow has been partially removed and will no longer accept a fints object (tsobj) argument. Use a matrix, timetable, or table instead for financial time series.

Use fts2timetable to convert a fints object to a timetable object.

## Syntax

```
values = llow(Data)
values = llow(
```

$\qquad$

``` ,Name, Value)
```


## Description

values = llow(Data) generates a vector of lowest low values from the series of low prices for the past $n$ periods.
values = llow( $\qquad$ , Name, Value) adds optional name-value pair arguments.

## Examples

## Calculate the Lowest Low for a Data Series for a Stock

Load the file SimulatedStock. mat, which provides a timetable (TMW) for financial data for TMW stock.
load SimulatedStock.mat values = llow(TMW); plot(values.Time, values.LowestLow) title('Lowest Low for TMW')

Lowest Low for TMW


## Input Arguments

## Data - Data for low prices

matrix | table | timetable
Data for low prices, specified as a matrix, table, or timetable. Timetables and tables with M rows must contain a variable named 'Low' (case insensitive).
Data Types: double|table|timetable

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Example: values = llow(TMW_LOW,'NumPeriods',10)

## NumPeriods - Moving window for the lowest low calculation 14 (default) | positive integer

Moving window for the lowest low calculation, specified as the comma-separated pair consisting of 'NumPeriods ' and a scalar positive integer.

Data Types: double

## Output Arguments

## values - Lowest low series

matrix | table | timetable
Lowest low series, returned with the same number of rows (M) and the same type (matrix, table, or timetable) as the input Data.

## References

[1] Achelis, S. B. Technical Analysis from A to Z. Second Edition. McGraw-Hill, 1995.

## See Also

hhigh | table|timetable

## Topics

"Using Timetables in Finance"
"Convert Financial Time Series Objects fints to Timetables" on page 12-2

Introduced before R2006a

## $\log$

Natural logarithm

Note log will be removed in a future release. Use timetable instead. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

```
newfts = log(tsobj)
```


## Description

newfts $=\log (\mathrm{tsobj})$ calculates the natural logarithm (log base e) of the data series in a financial time series object tsobj. It returns another time series object, newfts, containing the natural logarithms.

## See Also

$\exp |\log 10| \log 2$

## Topics

"Financial Time Series Operations" on page 13-7

Introduced before R2006a

## $\log 10$

Common logarithm

Note log10 will be removed in a future release. Use timetable instead. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

```
newfts = log10(tsobj)
```


## Description

newfts $=$ log10(tsobj) calculates the common logarithm (base 10) of all the data in the data series of the financial time series object tsobj and returns the result in the object newfts.

## See Also

$\exp |\log | \log 2$

## Topics

"Financial Time Series Operations" on page 13-7

Introduced before R2006a

## $\log 2$

Base 2 logarithm

Note log2 will be removed in a future release. Use timetable instead. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

```
newfts = log2(tsobj)
```


## Description

newfts $=\log 2(t s o b j)$ calculates the base 2 logarithm of the data series in a financial time series object tsobj. It returns another time series object newfts containing the logarithms.

## See Also

$\exp |\log | \log 10$

## Topics

"Financial Time Series Operations" on page 13-7

Introduced before R2006a

## Ipm

Compute sample lower partial moments of data

## Syntax

```
lpm(Data)
lpm(Data,MAR)
lpm(Data,MAR,Order)
Moment = lpm(Data,MAR,Order)
```


## Arguments

| Data | NUMSAMPLES-by-NUMSERIES matrix with NUMSAMPLES observations <br> of NUMSERIES asset returns. |
| :--- | :--- |
| MAR | (Optional) Scalar minimum acceptable return (default MAR $=0$ ). <br> This is a cutoff level of return such that all returns above MAR <br> contribute nothing to the lower partial moment. |
| Order | (Optional) Either a scalar or a NUMORDERS vector of nonnegative <br> integer moment orders. If no order specified, default Order $=0$, <br> which is the shortfall probability. Although this function works for <br> noninteger orders and, in some cases, for negative orders, this falls <br> outside customary usage. |

## Description

Given NUMSERIES assets with NUMSAMPLES returns in a NUMSAMPLES-by-NUMSERIES matrix Data, a scalar minimum acceptable return MAR, and one or more nonnegative moment orders in a NUMORDERS vector Order, lpm computes lower partial moments relative to MAR for each asset in a NUMORDERS x NUMSERIES matrix Moment.

The output Moment is a NUMORDERS x NUMSERIES matrix of lower partial moments with NUMORDERS Orders and NUMSERIES series, that is, each row contains lower partial moments for a given order.

Note To compute upper partial moments, reverse the signs of both Data and MAR (do not reverse the sign of the output). This function computes sample lower partial moments from data. To compute expected lower partial moments for multivariate normal asset returns with a specified mean and covariance, use elpm. With lpm, you can compute various investment ratios such as Omega ratio, Sortino ratio, and Upside Potential ratio, where:

- Omega = lpm(-Data, -MAR, 1) / lpm(Data, MAR, 1)
- Sortino $=($ mean (Data) - MAR) $/ \operatorname{sqrt(lpm(Data,~MAR,~2))~}$
- Upside = lpm(-Data, -MAR, 1) / sqrt(lpm(Data, MAR, 2))


## Examples

See "Sample Lower Partial Moments" on page 7-16.

## References

Vijay S. Bawa. "Safety-First, Stochastic Dominance, and Optimal Portfolio Choice." Journal of Financial and Quantitative Analysis. Vol. 13, No. 2, June 1978, pp. 255-271.
W. V. Harlow. "Asset Allocation in a Downside-Risk Framework." Financial Analysts Journal. Vol. 47, No. 5, September/October 1991, pp. 28-40.
W. V. Harlow and K. S. Rao. "Asset Pricing in a Generalized Mean-Lower Partial Moment Framework: Theory and Evidence." Journal of Financial and Quantitative Analysis. Vol. 24, No. 3, September 1989, pp. 285-311.

Frank A. Sortino and Robert van der Meer. "Downside Risk." Journal of Portfolio Management. Vol. 17, No. 5, Spring 1991, pp. 27-31.

## See Also

elpm

## Topics

"Sample Lower Partial Moments" on page 7-16
"Performance Metrics Overview" on page 7-2
Introduced in R2006b

## Iweekdate

Date of last occurrence of weekday in month

## Syntax

LastDate = lweekdate(Weekday,Year,Month)
LastDate = lweekdate( ,NextDay, outputType)

## Description

LastDate = lweekdate(Weekday,Year,Month) returns the date number for the last occurrence of Weekday in the given year and month.

Any input can contain multiple values, but if so, all other inputs must contain the same number of values or a single value that applies to all. For example, if Year is a 1-by-n vector of integers, then Month must be a 1-by-n vector of integers or a single integer. LastDate is then a 1-by-n vector of date numbers.

LastDate = lweekdate( $\qquad$ ,NextDay, outputType) returns the date of last occurrence of weekday in month using the optional arguments for NextDay and outputType.

The type of the output for LastDate depends on the input outputType. If this variable is 'datenum', LastDate is a serial date number. If outputType is 'datetime', then LastDate is a datetime array. By default, outputType is set to 'datenum'.

Use the function datestr to convert serial date numbers to formatted date character vectors.

## Examples

## Determine the Date of Last Occurrence of Weekday in a Month

Determine the last Monday in June 2001.

```
LastDate = lweekdate(2, 2001, 6); datestr(LastDate)
ans =
'25-Jun-2001
```

Determine the last Monday in a week that also contains a Friday in June 2001 returned as a datetime array.

```
LastDate = lweekdate(2, 2001, 6,[],'datetime')
LastDate = datetime
    25-Jun-2001
```

Determine the last Monday in a week that also contains a Friday in June 2001:

```
LastDate = lweekdate(2, 2001, 6, 6); datestr(LastDate)
```

ans =
'25-Jun-2001

Determine the last Monday in May for 2001, 2002, and 2003:

```
Year = [2001:2003];
LastDate = lweekdate(2, Year, 5);
datestr(LastDate)
ans = 3x11 char array
    '28-May-2001'
    ' 27-May-2002'
    '26-May-2003'
```


## Input Arguments

## Weekday - Weekday whose date you seek

integer with value 1 through 7 | vector of integers with values 1 through 7
Weekday whose date you seek, specified as an integer or a vector of integers from 1 through 7.

- 1 - Sunday
- 2 - Monday
- 3-Tuesday
- 4 - Wednesday
- 5 - Thursday
- 6 - Friday
- 7 - Saturday

Data Types: single | double

## Year - Year to determine occurrence of weekday <br> 4 -digit integer | vector of 4-digit integers

Year to determine occurrence of weekday, specified as a 4-digit integer or vector of 4-digit integers.

## Data Types: single | double

## Month - Month to determine occurrence of weekday

integer with value 1 through 12 | vector of integers with values 1 through 12
Month to determine occurrence of weekday, specified as an integer or vector of integers with values 1 through 12.

## Data Types: single | double

## NextDay - Weekday that must occur after Weekday in same week

$0=$ ignore (default) | integer with value 0 through $7 \mid$ vector of integers with values 0 through 7

Weekday that must occur after Weekday in same week, specified as an integer or a vector of integers from 0 through 7 , where $0=$ ignore (default) and 1 through 7 are the same as for Weekday.

## Data Types: single | double

## outputType - Year to determine days

'datenum' (default) | character vector with values 'datenum' or 'datetime'
A character vector specified as either 'datenum' or 'datetime'. The output LastDate is in serial date format if 'datenum ' is specified, or datetime format if 'datetime' is specified. By default the output LastDate is in serial date format.

## Data Types: char

## Output Arguments

## LastDate - Date for last occurrence of Weekday in given year and month serial date number | date character vector

Date for last occurrence of Weekday in given year and month, returned as a serial date number or date character vector.

The type of the output for LastDate depends on the optional input argument outputType. If this variable is 'datenum', LastDate is a serial date number. If outputType is 'datetime', then LastDate is a datetime array. By default, outputType is set to 'datenum'.

## See Also

datetime | eomdate | lbusdate | nweekdate

## Topics

"Handle and Convert Dates" on page 2-2

Introduced before R2006a

## m2xdate

MATLAB date to Excel serial date number

## Syntax

DateNum = m2xdate(MATLABDateNumber,Convention)

## Description

DateNum = m2xdate(MATLABDateNumber, Convention) converts MATLAB serial date numbers, date character vectors, or datetime arrays to Excel serial date numbers. MATLAB date numbers start with $1=$ January 1, 0000 A.D., hence there is a difference of 693960 relative to the 1900 date system, or 695422 relative to the 1904 date system. This function is useful with Spreadsheet Link ${ }^{\text {TM }}$ software.

## Examples

## Convert MATLAB Serial Date Numbers Using 1900 Date System

This example shows how to convert MATLAB serial date numbers using the 1900 date system. Given MATLAB date numbers for Christmas 2001 through 2004, convert them to Excel date numbers in the 1900 system.

```
DateNum = datenum(2001:2004, 12, 25);
ExDate = m2xdate(DateNum)
ExDate = 1\times4
```

    \(\begin{array}{llll}37250 & 37615 & 37980 & 38346\end{array}\)
    
## Convert MATLAB Serial Date Numbers Using 1900 Date System With a datetime Array

This example shows how to convert MATLAB® date numbers using a datetime array with the 1900 date system. Given MATLAB date numbers for Christmas 2001 through 2004, convert them to Excel date numbers in the 1904 system.

```
DateNum = datetime(2001:2004, 12, 25,'Locale','en_US');
ExDate = m2xdate(DateNum)
ExDate = 1×4
```

| 37250 | 37615 | 37980 | 38346 |
| :--- | :--- | :--- | :--- |

## Convert MATLAB Serial Date Numbers Using 1904 Date System

This example shows how to convert MATLAB serial date numbers using the 1904 date system. Given MATLAB date numbers for Christmas 2001 through 2004, convert them to Excel date numbers in the 1904 system.

DateNum = datenum(2001:2004, 12, 25);
ExDate = m2xdate(DateNum, 1)
ExDate $=1 \times 4$
$3578836153 \quad 36518 \quad 36884$

## Input Arguments

## MATLABDateNumber - MATLAB dates <br> serial date number | date character vector | datetime array

MATLAB dates, specified as a scalar or vector of MATLAB serial date numbers, date character vectors, or datetime arrays.

Data Types: double | char|datetime

## Convention - Flag for Excel date system

0 (Excel 1900 date system is in effect) (default) | numeric with value 0 or 1
Flag for Excel date system, specified as a scalar or vector as a numeric with a value 0 or 1.

When Convention $=0$ (default), the Excel 1900 date system is in effect. When Convention = 1, the Excel 1904 date system in used.

In the Excel 1900 date system, the Excel serial date number 1 corresponds to January 1, 1900 A.D. In the Excel 1904 date system, date number 0 is January 1, 1904 A.D.

Due to a software limitation in Excel software, the year 1900 is considered a leap year. As a result, all DATEVALUE's reported by Excel software between Jan. 1, 1900 and Feb. 28, 1900 (inclusive) differs from the values reported by 1. For example:

- In Excel software, Jan. 1, $1900=1$
- In MATLAB, Jan. 1, 1900-693960 (for 1900 date system) $=2$
datenum('Jan 1, 1900') - 693960
ans $=$
2
Data Types: logical


## Output Arguments

DateNum - Excel serial date number
array of serial date numbers
Excel serial date number, returned as an array of serial date numbers in Excel serial date number form.

See Also<br>datenum | datestr|datetime | x2mdate

## Topics

"Handle and Convert Dates" on page 2-2
Introduced before R2006a

## macd

Moving Average Convergence/Divergence (MACD)

Note macd has been partially removed and will no longer accept a fints object (tsobj) argument. Use a matrix, timetable, or table instead for financial time series.

Use fts2timetable to convert a fints object to a timetable object.

## Syntax

[MACDLine,SignalLine] $=$ macd(Data)

## Description

[MACDLine,SignalLine] = macd(Data) calculates the Moving Average Convergence/Divergence (MACD) line from the series of data and the nine-period exponential moving average from the MACDLine.

## Examples

## Calculate the Moving Average Convergence/Divergence for a Stock

Load the file SimulatedStock. mat, which provides a timetable (TMW) for financial data for TMW stock.
load SimulatedStock.mat
[MACDLine, signalLine]= macd(TMW);
plot (MACDLine.Time, MACDLine.Close, signalLine.Time, signalLine. Close);
legend('MACDLine', 'NinePerMA')
title('MACD for TMW')


## Input Arguments

## Data - Data with high, low, open, close information

matrix | table | timetable
Data with high, low, open, close information, specified as a matrix, table, or timetable. For matrix input, Data is an M-by-4 matrix of high, low, opening, and closing prices.
Timetables and tables with M rows must contain variables named 'High', 'Low', 'Open', and 'Close' (case insensitive).

Data Types: double| table| timetable

## Output Arguments

## MACDLine - MACD series

matrix | table | timetable
MACD series, returned with the same number of rows ( M ) and type (matrix, table, or timetable) as the input Data. The MACDLine is calculated by subtracting the 26 -period (7.5\%) exponential.

## SignalLine - Nine-period exponential series matrix | table | timetable

Nine-period exponential series, returned with the same number of rows (M) and type (matrix, table, or timetable) as the input Data. The nine-period (20\%) exponential moving average of the MACDLine is used as the SignalLine.

## Definitions

## MACD

The MACD is calculated by subtracting the 26 -period (7.5\%) exponential moving average from the 12-period (15\%) moving average.

The nine-period (20\%) exponential moving average of the MACD line is used as the "signal" line. When the two lines are plotted, they can give you indications on when to buy or sell a stock, when overbought or oversold is occurring, and when the end of trend may occur. For example, when the MACD and the 20-day moving average line have crossed and the MACD line becomes below the other line, it is time to sell.

## References

[1] Achelis, S. B. Technical Analysis from A to Z. Second Edition. McGraw-Hill, 1995, pp. 166-168.

See Also<br>adline|table|timetable|willad

## Topics

"Using Timetables in Finance"
"Convert Financial Time Series Objects fints to Timetables" on page 12-2

## Introduced before R2006a

## max

Maximum value

Note max will be removed in a future release. Use timetable instead. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

```
tsmax = max(tsobj)
```


## Description

tsmax $=\max (\mathrm{tsobj})$ finds the maximum value in each data series in the financial time series object (tsobj) and returns it in a structure tsmax. The tsmax structure contains field name(s) identical to the data series name(s).

Note tsmax returns only the values and does not return the dates associated with the values. The maximum values are not necessarily from the same date.

## See Also

min

## Topics

"Financial Time Series Operations" on page 13-7

Introduced before R2006a

## maxdrawdown

Compute maximum drawdown for one or more price series

## Syntax

```
MaxDD = maxdrawdown(Data)
MaxDD = maxdrawdown(
```

$\qquad$

```
,Format)
```

[MaxDD,MaxDDIndex] = maxdrawdown( $\qquad$ )

## Description

MaxDD = maxdrawdown(Data) computes maximum drawdown for each series in an N vector MaxDD and identifies start and end indexes of maximum drawdown periods for each series in a 2-by-N matrix MaxDDIndex.

MaxDD = maxdrawdown( $\qquad$ ,Format) adds an optional argument for Format.
[MaxDD,MaxDDIndex] = maxdrawdown( $\qquad$ ) adds an optional output for MaxDDIndex.

## Examples

## Calculate Maximum Drawdown

Calculate the maximum drawdown (MaxDD) using example data with a fund, market, and cash series:

```
load FundMarketCash
MaxDD = maxdrawdown(TestData)
MaxDD = 1×3
```

$$
\begin{array}{lll}
0.1658 & 0.3381 & 0
\end{array}
$$

The maximum drop in the given time period was $16.58 \%$ for the fund series, and $33.81 \%$ for the market series. There was no decline in the cash series, as expected, because the cash account never loses value.

## Input Arguments

## Data - Total return price series <br> matrix

Total return price series, specified as a T-by-N matrix with T samples of N total return price series.

## Data Types: double

## Format - (Optional) Format of Data

character vector with value of 'return', 'arithmetic', or 'geometric'
Format of Data, specified as character vector with the following possible values:

- 'return ' (default) - Maximum drawdown in terms of maximum percentage drop from a peak.
- 'arithmetic'
- Maximum drawdown of an arithmetic Brownian motion with drift (differences of data from peak to trough) using the equation

$$
d X(t)=\mu d t+\sigma d W(t)
$$

- 'geometric'
- Maximum drawdown of a geometric Brownian motion with drift (differences of log of data from peak to trough) using the equation

$$
d S(t)=\mu_{0} S(t) d t+\sigma_{0} S(t) d W(t)
$$

## Data Types: char

## Output Arguments

## MaxDD - Maximum drawdown

vector
Maximum drawdown, returned as a 1-by-N vector with maximum drawdown for each of N time series.

## Note

- Drawdown is the percentage drop in total returns from the start to the end of a period. If the total equity time series is increasing over an entire period, drawdown is 0 . Otherwise, it is a positive number. Maximum drawdown is an ex-ante proxy for downside risk that computes the largest drawdown over all intervals of time that can be formed within a specified interval of time.
- Maximum drawdown is sensitive to quantization error.


## MaxDDIndex - Start and end indexes for each maximum drawdown period for each total equity time series

vector
Start and end indexes for each maximum drawdown period for each total equity time series, returned as a 2 -by- N vector of start and end indexes. The first row of the vector contains the start indexes and the second row contains the end indexes of each maximum drawdown period.

## References

[1] Christian S. Pederson and Ted Rudholm-Alfvin. "Selecting a Risk-Adjusted Shareholder Performance Measure." Journal of Asset Management. Vol. 4, No. 3, 2003, pp. 152-172.

## See Also

emaxdrawdown

## Topics

"Maximum Drawdown" on page 7-19
"Performance Metrics Overview" on page 7-2

## Introduced in R2006b

## mean

Arithmetic average

Note mean will be removed in a future release. Use timetable instead. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

tsmean $=$ mean(tsobj)

## Description

tsmean $=$ mean(tsobj) computes the arithmetic mean of all data in all series in the financial time series object (tsobj) and returns it in a structure tsmean. The tsmean structure contains field name(s) identical to the data series name(s).

## See Also

peravg|tsmovavg

## Topics

"Financial Time Series Operations" on page 13-7

Introduced before R2006a

## medprice

Median price

Note medprice has been partially removed and will no longer accept a fints object (tsobj) argument. Use a matrix, timetable, or table instead for financial time series.

Use fts2timetable to convert a fints object to a timetable object.

## Syntax

MedianPrice $=$ medprice $($ Data $)$

## Description

MedianPrice $=$ medprice(Data) calculates the median prices from the series of high and low prices. The median price is the average of the high and low prices for each period.

## Examples

## Calculate the Median Prices for a Data Series for a Stock

Load the file SimulatedStock.mat, which provides a timetable (TMW) for financial data for TMW stock.
load SimulatedStock.mat
MedianPrice = medprice(TMW);
plot(MedianPrice.Time, MedianPrice.MedianPrice)
title('Median Price for TMW')


## Input Arguments

## Data - Data for high, low, and close prices

matrix | table | timetable
Data for high, low, and close prices, specified as a matrix, table, or timetable. For matrix input, Data is an M-by-3 matrix of high, low, and close prices stored in the corresponding columns. Timetables and tables with M rows must contain a variable named 'High ', 'Low', 'Close' (case insensitive).

Data Types: double| table | timetable

## Output Arguments

## MedianPrice - Median price series <br> matrix | table | timetable

Median price series, returned with the same number of rows ( $M$ ) and the same type (matrix, table, or timetable) as the input Data.

## References

[1] Achelis, S. B. Technical Analysis from A to Z. Second Edition. McGraw-Hill, 1995, pp. 177-178.

See Also<br>table|timetable|typprice|wclose

## Topics

"Using Timetables in Finance"
"Convert Financial Time Series Objects fints to Timetables" on page 12-2
Introduced before R2006a

## merge

Merge multiple financial time series objects

Note merge will be removed in a future release. Use timetable instead. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

```
newfts = merge(fts1,fts2)
newfts = merge(fts1,fts2, ..., ftsx)
newfts = merge(fts1,fts2, ..., ftsx,'PARAM1',VALUE1,'PARAM2',VALUE2, ...)
```


## Arguments

| fts1, fts2, ... | Comma-separated list of financial time series objects to merge. <br>  <br> 'DateSetMethod ' <br> Note Multiple Financial Time Series objects can be merged at <br> once. The merged objects must appear in a comma separated <br> list before the optional inputs. The order of the inputs is <br> significant. <br> (Optional) Merge method. Valid merge values are: <br> ' union ' - (Default) Returns the combined values of all merged <br> objects. <br> 'intersection ' - Returns the values common to all merged <br> objects. <br> Ref0bj - Maps all values to a reference time contained in a <br> Financial Time Series object (RefObj) or vector of date <br> numbers. |
| :--- | :--- |

\(\left.$$
\begin{array}{|l|l|}\hline \text { 'DataSetMethod ' } & \begin{array}{l}\text { (Optional) Merge method. Valid merge values are: } \\
\text { 'closest' }- \text { (Default) Returns data based on the order of the } \\
\text { inputs. However, the first missing data point (NaN value) of a } \\
\text { date will be replaced by the closest non-NaN data point that } \\
\text { appears on the same date of subsequent merged objects. } \\
\text { 'order' } ~-~ R e t u r n s ~ d a t a ~ b a s e d ~ s t r i c t l y ~ o n ~ t h e ~ o r d e r ~ o f ~ t h e ~\end{array}
$$ <br>

inputs.\end{array}\right\}\)| (Optional) Sorts columns. Valid merge values are: |
| :--- |
| True/1 - (Default) Sorts the columns based on the headers |
| (series names). The headers are sorted in alphabetical order. |
| False/0 - Columns are not sorted. |

## Description

```
newfts = merge(fts1,fts2, ...,
```

ftsx,'PARAM1', VALUE1,'PARAM2', VALUE2' , ...) merges multiple financial time series objects. The optional parameter and value pair argument specifies the values contained in the output financial time series object ftsout.

## Examples

## Create Three Financial Time Series Objects and Merge into a Single Object

Create three financial time series objects and merged into a new time series object t123.

```
dates = {'jan-01-2001'; 'jan-02-2001'; 'jan-03-2001'; ...
    'jan-04-2001'; 'jan-06-2001'};
data = [1; 1; 1; 1; 1];
t1 = fints(dates, data);
Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in
dates = {'jan-02-2001'; 'jan-03-2001'; 'jan-04-2001';
```

```
    'jan-05-2001'};
data = [2; 2; 2; 2];
t2 = fints(dates, data);
```

Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in

```
dates = {'jan-03-2001'; 'jan-04-2001'; 'jan-05-2001';
    jan-06-2001'};
data = [3; 3; 3; 3];
t3 = fints(dates, data);
```

Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in

```
t123 = merge(t1, t2, t3)
```

Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in

```
t123 =
    desc: || ||
    freq: Unknown (0)
    'dates: (6)' 'series1: (6)'
    '01-Jan-2001' [ 1]
    '02-Jan-2001' [ 1]
    '03-Jan-2001' [ 1]
    '04-Jan-2001' [ 1]
    '05-Jan-2001' [ 2]
    '06-Jan-2001' [ 1]
```

If you change the order of input time series, the output may contain different data when duplicate dates exist. Here, for example, is the result of using the same three time series defined above but with the order changed.

```
merge(t3, t2, t1)
```

Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in
ans =
desc: || ||
freq: Unknown (0)

19-1195

| 'dates: (6)' | 'series1: | (6)' |
| :--- | :--- | :--- |
| '01-Jan-2001' | $[$ | $1]$ |
| '02-Jan-2001' | $[$ | $2]$ |
| '03-Jan-2001' | $[$ | $3]$ |
| '04-Jan-2001' | $[$ | $3]$ |
| '05-Jan-2001' | $[$ | $3]$ |
| '06-Jan-2001' | $[$ | $3]$ |

t123 contains all 1's except on '05-Jan-2001' because t1 appears first in the list of inputs and takes precedence. The same logic can be applied to t321. By changing the order of inputs, you can overwrite old financial time series data with new data by placing the new time series ahead of the old one in the list of inputs to the merge function.

## Merge Financial Time Series Objects with Different Headers (Series Names)

Merge time series objects with different headers into a new time series object t45.

```
dates = {'jan-01-2001'; 'jan-02-2001'; 'jan-03-2001'; ...
'jan-04-2001'; 'jan-06-2001'};
data = [1; 1; 1; 1; 1];
t4 = fints(dates, data, 'ts4');
Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in
dates = {'jan-02-2001'; 'jan-03-2001'; 'jan-04-2001'; 'jan-05-2001'};
data = [2; 2; 2; 2];
t5 = fints(dates, data, 'ts5');
Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in
t45 = merge(t4, t5)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in
t45 =
    desc: ||
    freq: Unknown (0)
```

| 'dates: (6)' | $' t s 4:$ | $(6) '$ | $' t s 5:$ | $(6) '$ |
| :--- | :--- | :--- | :--- | :--- |
| '01-Jan-2001' | $[$ | $1]$ | $[$ | NaN $]$ |
| '02-Jan-2001' | $[$ | $1]$ | $[$ | $2]$ |
| '03-Jan-2001' | $[$ | $1]$ | $[$ | $2]$ |
| '04-Jan-2001' | $[$ | $1]$ | $[$ | $2]$ |
| '05-Jan-2001' | $[$ | $\mathrm{NaN}]$ | $[$ | $2]$ |
| '06-Jan-2001' | $[$ | $1]$ | $[$ | $\mathrm{NaN}]$ |

## Merge Two Financial Index Series and Keep Intersecting Dates

Merge two index series into the final merged object (t12) and keep the intersecting dates.

```
dates = {'jan-01-2001'; 'jan-02-2001'; 'jan-03-2001'; 'jan-04-2001'; 'jan-06-2001'};
data = [1; 1; 1; 1; 1];
t1 = fints(dates, data,'A')
Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in
t1 =
    desc: (none)
    freq: Unknown (0)
    'dates: (5)' 'A: (5)
    '01-Jan-2001' [ 1]
    '02-Jan-2001' [ 1]
    '03-Jan-2001' [ 1]
    '04-Jan-2001' [ 1]
    '06-Jan-2001' [ 1]
```

dates $=$ \{'jan-02-2001'; 'jan-03-2001'; 'jan-04-2001'; 'jan-05-2001'\};
data $=$ [2; 2; 2; 2];
t2 = fints(dates, data,'B')
Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in
t2 =
desc: (none)
freq: Unknown (0)

| tes: (4)' | 'B: (4)' |
| :---: | :---: |
| '02-Jan-2001' | $2]$ |
| 03-Jan-2001' | $2]$ |
| '04-Jan-2001' | $2]$ |
| 05-Jan-2001' | $2]$ |

```
t12 = merge(t1, t2,'DateSetMethod','Intersection')
```

Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in
t12 =
desc: ||
freq: Unknown (0)
'dates: (3)' 'A: (3)' 'B: (3)'
'02-Jan-2001' [ 1] [ 2]
'03-Jan-2001' [ 1] [ 2]
'04-Jan-2001' [ 1] [ 2]

## See Also

horzcat|vertcat

## Topics

"Merge Financial Time Series Objects" on page 14-12

## Introduced before R2006a

## min

Minimum value

Note MIN will be removed in a future release. Use timetable instead. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

tsmin $=\min (t s o b j)$

## Description

tsmin $=\min (t s o b j)$ finds the minimum value in each data series in the financial time series object (tsobj) and returns it in the structure tsmin. The tsmin structure contains field name(s) identical to the data series name(s).

Note tsmin returns only the values and does not return the dates associated with the values. The minimum values are not necessarily from the same date.

## See Also

max

## Topics

"Financial Time Series Operations" on page 13-7

Introduced before R2006a

## minus

Financial time series subtraction

Note minus will be removed in a future release. Use timetable instead. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

```
newfts = tsobj_1 - tsobj_2
newfts = tsobj - array
newfts = array - tsobj
```


## Arguments

| tsobj_1, tsobj_2 | A pair of financial time series objects . |
| :--- | :--- |
| array | A scalar value or array with the number of rows equal to the <br> number of dates in tsobj and the number of columns equal <br> to the number of data series in tsobj. |

## Description

minus is an element-by-element subtraction of the components.
newfts = tsobj_1 - tsobj_2 subtracts financial time series objects. If an object is to be subtracted from another object, both objects must have the same dates and data series names, although the order need not be the same. The order of the data series, when one financial time series object is subtracted from another, follows the order of the first object.
newfts = tsobj - array subtracts an array element by element from a financial time series object.
newfts = array - tsobj subtracts a financial time series object element by element from an array.

See Also<br>plus|rdivide|times<br>\section*{Topics}<br>"Financial Time Series Operations" on page 13-7<br>Introduced before R2006a

## minute

Minute of date or time

## Syntax

```
Minute = minute(Date)
Minute = minute(
,F)
```


## Description

Minute $=$ minute (Date) returns the minute of date or time given a serial date number or a date character vector.

> Minute $=$ minute $(\ldots, F)$ returns the minute of date or time given a serial date number or a date character vector, Date, using format defined by the optional input F. Date can be a character array where each row corresponds to one date character vector, or a one-dimensional cell array of character vectors. All the character vectors in Date must have the same format F. F must designate a supported date format symbol. For more information on supported date formats, see datestr.

## Examples

## Determine the Minutes of the Date for Various Dates

Find the minutes of the day for Date using a serial date number.

```
Minute = minute(731204.5591223380)
Minute = 25
```

Find the minutes of the day for Date using a date character vector format.

```
Minute = minute('19-dec-2001, 13:25:08.17')
Minute = 25
```


## Input Arguments

## Date - Date to determine minute <br> serial date number | date character vector | cell array of date character vectors

Date to determine minute, specified as a serial date number or date character vector.
Date can be an array of date character vectors, where each row corresponds to one date character vector, or a one-dimensional cell array of character vectors. All the character vectors in Date must have the same format F. F must designate a supported date format symbol. For more information on supported date formats, see datestr

## Data Types: single | double | char | cell

## F - Date format symbol

character vector designating date format
Date format symbol, specified as a character vector to designate the date format symbol for input argument Date. For more information on supported date character vector formats, see datestr. Note, formats with ' $Q$ ' are not accepted.
Data Types: char

## Output Arguments

Minute - Minute of date or time
serial date number | datetime array
Minute of date or time, returned as a serial date number or date character vector.

## See Also

datevec | hour | second

## Topics

"Handle and Convert Dates" on page 2-2

Introduced before R2006a

## mirr

Modified internal rate of return

## Syntax

Return = mirr(CashFlow,FinRate,Reinvest)

## Arguments

| CashFlow | Vector of cash flows. The first entry is the initial investment. |
| :--- | :--- |
| FinRate | Finance rate for negative cash flow values. Enter as a decimal <br> fraction. |
| Reinvest | Reinvestment rate for positive cash flow values, as a decimal <br> fraction. |

## Description

Return $=$ mirr(CashFlow, FinRate, Reinvest) calculates the modified internal rate of return for a series of periodic cash flows. This function calculates only positive rates of return; for nonpositive rates of return, Return $=0$.

## Examples

This cash flow represents the yearly income from an initial investment of $\$ 100,000$. The finance rate is $9 \%$ and the reinvestment rate is $12 \%$.

| Year 1 | $\$ 20,000$ |
| :--- | :--- |
| Year 2 | $(\$ 10,000)$ |
| Year 3 | $\$ 30,000$ |
| Year 4 | $\$ 38,000$ |

## Year $5 \quad \$ 50,000$

To calculate the modified internal rate of return on the investment Return $=\operatorname{mirr}([-10000020000-100003000038000$ 50000], 0.09,... 0.12 )
returns

```
Return =
    0.0832 (8.32%)
```


## References

Brealey and Myers. Principles of Corporate Finance. Chapter 5.

## See Also

annurate|effrr|irr|nomrr|pvvar|xirr

## Topics

"Analyzing and Computing Cash Flows" on page 2-18

## Introduced before R2006a

## month

Month of date

## Syntax

[MonthNum, MonthString] = month(Date)
[MonthNum, MonthString] = month( $\qquad$ , F)

## Description

[MonthNum, MonthString] = month(Date) returns the month of date given a serial date number or a date character vector.
[MonthNum,MonthString] = month( $\qquad$ ,F) returns the month of date given a serial date number or a date character vector, Date, using format defined by the optional input F. Date can be a character array where each row corresponds to one date character vector, or a one-dimensional cell array of character vectors. All the character vectors in Date must have the same format F. F must designate a supported date format symbol. For more information on supported date formats, see datestr.

## Examples

## Determine the Month for Various Dates

Find the month for Date using a serial date number.
[MonthNum, MonthString] = month(730368)
MonthNum = 9
MonthString =
'Sep'
Find the month for Date using a date character vector format.

```
[MonthNum, MonthString] = month('05-Sep-1999')
MonthNum = 9
MonthString =
'Sep'
```

Use the optional F argument to designate a country-specific date format for a given Date.

```
[MonthNum, MonthString] = month('1999/05/09','yyyy/dd/mm')
```

```
MonthNum = 9
```

MonthString =
'Sep'

## Input Arguments

## Date - Date to determine month

serial date number | date character vector | cell array of date character vectors
Date to determine month, specified as a serial date number or date character vector.
Date can be an array of date character vectors, where each row corresponds to one date character vector, or a one-dimensional cell array of character vectors. All the character vectors in Date must have the same format F. F must designate a supported date format symbol. For more information on supported date formats, see datestr
Data Types: single | double | char | cell

## F - Date format symbol

character vector designating date format
Date format symbol, specified as a character vector to designate the date format symbol for input argument Date. For more information on supported date character vector formats, see datestr. Note, formats with ' Q' are not accepted.
Data Types: char

## Output Arguments

## MonthNum - Numeric representation of the month <br> nonnegative integer <br> Month of date, returned as a nonnegative integer. <br> MonthString - Three letter abbreviation for month <br> character vector

Three letter abbreviation for month, returned as a character vector.

See Also<br>datevec | day | year<br>\section*{Topics}<br>"Handle and Convert Dates" on page 2-2<br>Introduced before R2006a

## months

Number of whole months between dates

## Syntax

MyMonths $=$ months(StartDate, EndDate)
MyMonths = months( $\qquad$ ,EndMonthFlag)

## Description

MyMonths = months(StartDate, EndDate) returns the number of whole months between StartDate and EndDate. If EndDate is earlier than StartDate, MyMonths is negative.

Any input argument can contain multiple values, but if so, all other inputs must contain the same number of values or a single value that applies to all. For example, if StartDate is an n-row character array of character vector dates, then EndDate must be an $n$-row character array of character vector dates or a single date. MyMonths is then an N -by-1 vector of numbers.

MyMonths = months (__ ,EndMonthFlag) returns the number of whole months between StartDate and EndDate using an optional argument for EndMonthFlag. If EndDate is earlier than StartDate, MyMonths is negative.

## Examples

## Determine the Number of Whole Months Between Dates

Find the number of whole months using date character vectors.
MyMonths = months('may $312000 ', \quad$ jun $302000 ', 1)$
MyMonths = 1

```
Find the number of whole months using date character vectors when the optional EndMonthFlag \(=0\).
```

```
MyMonths = months('may 31 2000','jun 30 2000', 0)
MyMonths = 0
```

Find the number of whole months using a cell array of date character vectors.

```
Dates = ['mar 31 2002'; 'apr 30 2002'; 'may 31 2002'];
MyMonths = months(Dates, 'jun 30 2002')
MyMonths = 3\times1
```

    3
    2
    1
    
## Input Arguments

## StartDate - Starting date for number of whole months between dates serial date number | date character vector | cell array of date character vectors

Starting date for number of whole months between dates, specified as a serial date number or date character vector.

Any input argument can contain multiple values, but if so, all other inputs must contain the same number of values or a single value that applies to all. For example, if StartDate is an n-row character array of character vector dates, then EndDate must be an n-row character array of character vector dates or a single date. MyMonths is then an $n$-by-1 vector of numbers.

## Data Types: single| double | char | cell

## EndDate - Ending date for number of whole months between dates serial date number | date character vector | cell array of date character vectors

Ending date for number of whole months between dates, specified as a serial date number or date character vector.

Any input argument can contain multiple values, but if so, all other inputs must contain the same number of values or a single value that applies to all. For example, if

StartDate is an $n$-row character array of character vector dates, then EndDate must be an $n$-row character array of character vector dates or a single date. MyMonths is then an $n$-by- 1 vector of numbers.

## Data Types: single | double | char | cell

## EndMonthFlag - Flag for end-of-month rule

1 (default) | nonnegative integer with values 0 or 1
Flag for end-of-month rule, specified as a nonnegative integer with values 0 or 1 .
If StartDate and EndDate are end-of-month dates and EndDate has fewer days than StartDate, EndMonthFlag = 1. In this case, EndDate is treated as the end of a whole month, while EndMonthFlag $=0$ does not.
Data Types: logical

## Output Arguments

## MyMonths - Number of whole months between dates

nonnegative integer
Number of whole months between dates, returned as a nonnegative integer.

See Also<br>yearfrac<br>\section*{Topics}<br>"Handle and Convert Dates" on page 2-2<br>Introduced before R2006a

## movavg

Moving average of a financial time series

Note movavg is updated to accept data input as a matrix, table, or timetable.
The syntax for movavg has changed. There is no longer support for the input arguments Lead and Lag, only a single windowSize is supported, and there is only one output argument (ma). If you want to compute the leading and lagging moving averages, you need to run movavg twice and adjust the windowSize.

## Syntax

```
ma = movavg(Data,type,windowSize)
ma = movavg(
```

$\qquad$

``` , Initialpoints)
ma = movavg(Data,type,weights)
ma = movavg(
```

$\qquad$

``` , Initialpoints)
```


## Description

ma = movavg(Data,type, windowSize) computes the moving average (MA) of a financial time series.
ma = movavg( __, Initialpoints) adds an optional argument for Initialpoints.
ma = movavg(Data,type, weights) computes the moving average (MA) of a financial time series using a 'custom' type and weights.
ma $=$ movavg ( _ _ , Initialpoints) adds an optional argument for Initialpoints.

## Examples

## Calculate the Moving Average for a Data Series

Load the file SimulatedStock.mat, which provides a timetable (TMW) for financial data.

```
load SimulatedStock.mat
type = 'linear';
windowSize = 14;
ma = movavg(TMW_CLOSE,type,windowSize)
ma = 1000×1
```

    100.2500
    100.3433
    100.8700
    100.4916
        99.9937
        99.3603
        98.8769
        98.6364
        98.4348
        97.8491
    
## Input Arguments

## Data - Data for a financial series

matrix | table | timetable

Data for a financial series, specified as a column-oriented matrix, table, or timetable. Timetables and tables must contain variables with only a numeric type.

Data Types: double | table | timetable

## type - Type of moving average to compute

character vector with value of 'simple', 'square-root', 'linear', 'square', 'exponential', 'triangular', 'modified', or 'custom' | string with value of "simple", "square-root", "linear", "square", "exponential", "triangular", "modified", or "custom"

Type of moving average to compute, specified as a character vector or string with an associated value.

## Data Types: char|string

## windowSize - Number of observations of the input series to include in moving average

## positive integer

Number of observations of the input series to include in moving average, specified as a scalar positive integer. The observations include (windowSize - 1) previous data points and the current data point.

Note The windowSize argument applies only to moving averages whose type is 'simple', 'square-root', 'linear', 'square', 'exponential','triangular', or 'modified'.

## Data Types: double

## weights - Custom weights used to compute moving average vector

Custom weights used to compute the moving average, specified as a vector.

Note The length of weights $(N)$ determines the size of the moving average window (windowSize). The weights argument applies only to a 'custom' type of moving average.

To compute moving average with custom weights, the weights (w) are first normalized such that they sum to one:
$W(i)=w(i) / s u m(w)$, for $i=1,2, \ldots, N$
The normalized weights (W) are then used to form the $N$-point weighted moving average (y) of the input Data (x):
$y(t)=W(1) * x(t)+W(2) * x(t-1)+\ldots+W(N) * x(t-N)$
The initial moving average values within the window size are then adjusted according to the method specified in the name-value pair argument Initialpoints.

Data Types: double

## Initialpoints - Indicates how moving average is calculated at initial points 'shrink' (default)| character vector with values of 'shrink', 'fill', or 'zero' | string with values of "shrink", "fill", or "zero"

(Optional) Indicates how the moving average is calculated at initial points (before there is enough data to fill the window), specified as a character vector or string using one of the following values:

- 'shrink' - Initializes the moving average such that the initial points include only observed data
- 'zero' - Initializes the initial points with 0
- 'fill' - Fills initial points with NaNs

Note The Initialpoints argument applies to all type specifications except for the 'exponential' and 'modified' options.

Data Types: char|string

## Output Arguments

ma - Moving average series
matrix | table | timetable
Moving average series, returned with the same number of rows ( $M$ ) and the same type (matrix, table, or timetable) as the input Data.

## References

[1] Achelis, S. B. Technical Analysis from A to Z. Second Edition. McGraw-Hill, 1995, pp. 184-192.

## See Also

bolling|candle|dateaxis|highlow|pointfig|table|timetable

## Topics

"Using Timetables in Finance"
"Convert Financial Time Series Objects fints to Timetables" on page 12-2

## Introduced before R2006a

## mrdivide

Financial time series matrix division

Note mrdivide will be removed in a future release. Use timetable instead. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

```
newfts = tsobj_1 / tsobj_2
newfts = tsobj / array
newfts = array / tsobj
```


## Arguments

| tsobj_1, tsobj_2 | A pair of financial time series objects. |
| :--- | :--- |
| array | A scalar value or array with number of rows equal to the <br> number of dates in tsobj and number of columns equal to <br> the number of data series in tsobj. |

## Description

The mrdivide method divides element by element the components of one financial time series object (tsobj) by the components of the other. You can also divide the whole object by an array or divide a financial time series object into an array.

If an object is to be divided by another object, both objects must have the same dates and data series names, although the order need not be the same. The order of the data series, when an object is divided by another object, follows the order of the first object.
newfts = tsobj_1 / tsobj_2 divides financial time series objects element by element.

```
newfts \(=\) tsobj \(/\) array divides a financial time series object element by element by
an array.
```

newfts = array / tsobj divides an array element by element by a financial time series object.

For financial time series objects, the mrdivide operation is identical to the rdivide operation.

See Also<br>minus | plus | rdivide | times<br>\section*{Topics}<br>"Financial Time Series Operations" on page 13-7<br>Introduced before R2006a

## mtimes

Financial time series matrix multiplication

Note mtimes will be removed in a future release. Use timetable instead. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

```
newfts = tsobj_1 * tsobj_2
newfts = tsobj * array
newfts = array * tsobj
```


## Arguments

| tsobj_1, tsobj_2 | A pair of financial time series objects. |
| :--- | :--- |
| array | A scalar value or array with number of rows equal to the <br> number of dates in tsobj and number of columns equal to <br> the number of data series in tsobj. |

## Description

The mtimes method multiplies element by element the components of one financial time series object (tsobj) by the components of the other. You can also multiply the entire object by an array.

If an object is to be multiplied by another object, both objects must have the same dates and data series names, although the order need not be the same. The order of the data series, when an object is multiplied by another object, follows the order of the first object.
newfts = tsobj_1 * tsobj_2 multiplies financial time series objects element by element.

# newfts $=$ tsobj $*$ array multiplies a financial time series object element by element by an array. <br> newfts $=$ array $*$ tsobj and newfts $=$ array $/$ tsobj multiplies an array element by element by a financial time series object. <br> For financial time series objects, the mtimes operation is identical to the times operation. 

See Also<br>minus|mrdivide|plus|times<br>\section*{Topics}<br>"Financial Time Series Operations" on page 13-7<br>Introduced before R2006a

## mvnrfish

Fisher information matrix for multivariate normal or least-squares regression

## Syntax

Fisher = mvnrfish(Data,Design,Covariance,MatrixFormat,CovarFormat)

## Arguments

| Data | NUMSAMPLES-by-NUMSERIES matrix with NUMSAMPLES <br> samples of a NUMSERIES-dimensional random vector. If a <br> data sample has missing values, represented as NaNs, the <br> sample is ignored. |
| :--- | :--- |
| Design | A matrix or a cell array that handles two model structures: <br> - If NUMSERIES $=1$, Design is a NUMSAMPLES-by- <br> NUMPARAMS matrix with known values. This structure is <br> the standard form for regression on a single series. <br> If NUMSERIES $\geq 1$, Design is a cell array. The cell array <br> contains either one or NUMSAMPLES cells. Each cell <br> contains a NUMSERIES-by-NUMPARAMS matrix of known <br> values. |
| If Design has a single cell, it is assumed to have the |  |
| same Design matrix for each sample. If Design has |  |
| more than one cell, each cell contains a Design matrix |  |
| for each sample. |  |


| Mat rixFormat | (Optional) Character vector that identifies parameters to be <br> included in the Fisher information matrix: <br> - full - Default format. Compute the full Fisher <br> information matrix for both model and covariance <br> parameter estimates. <br> paramonly - Compute only components of the Fisher <br> information matrix associated with the model parameter <br> estimates. |
| :--- | :--- |
| CovarFormat | Optional) Character vector that specifies the format for the <br> covariance matrix. The choices are: <br> - full' - Default method. The covariance matrix is a full <br> matrix. |
| 'diagonal' - The covariance matrix is a diagonal <br> matrix. |  |

## Description

Fisher = mvnrfish(Data,Design,Covariance, MatrixFormat,CovarFormat) computes a Fisher information matrix based on current maximum likelihood or leastsquares parameter estimates.

Fisher is a TOTALPARAMS-by-TOTALPARAMS Fisher information matrix. The size of TOTALPARAMS depends on MatrixFormat and on current parameter estimates. If MatrixFormat = 'full',

```
TOTALPARAMS = NUMPARAMS + NUMSERIES * (NUMSERIES + 1)/2
```

If MatrixFormat = 'paramonly',
TOTALPARAMS = NUMPARAMS

Note mvnrfish operates slowly if you calculate the full Fisher information matrix.

## Examples

See "Multivariate Normal Linear Regression" on page 9-2.
See Also
mvnrmle|mvnrstd
Topics"Multivariate Normal Regression With Missing Data" on page 9-17
"Multivariate Normal Regression Without Missing Data" on page 9-17
"Least-Squares Regression With Missing Data" on page 9-18
"Least-Squares Regression Without Missing Data" on page 9-18
"Fisher Information" on page 9-6
"Multivariate Normal Linear Regression" on page 9-2
"Least-Squares Regression" on page 9-5
Introduced in R2006a

## mvnrmle

Multivariate normal regression (ignore missing data)

## Syntax

[Parameters,Covariance,Resid,Info] = mvnrmle(Data,Design,MaxIterations,TolParam,TolObj

## Arguments

| Data | NUMSAMPLES-by-NUMSERIES matrix with NUMSAMPLES <br> samples of a NUMSERIES-dimensional random vector. If a <br> data sample has missing values, represented as NaNs, the <br> sample is ignored. (Use ecmmvn rmle to handle missing <br> data.) |
| :--- | :--- |
| Design | Matrix or a cell array that handles two model structures: <br> If NUMSERIES $=1$, Design is a NUMSAMPLES-by- <br> NUMPARAMS matrix with known values. This structure is <br> the standard form for regression on a single series. <br> If NUMSERIES $\geq 1$, Design is a cell array. The cell array <br> contains either one or NUMSAMPLES cells. Each cell <br> contains a NUMSERIES-by-NUMPARAMS matrix of known <br> values. |
| If Design has a single cell, it is assumed to have the |  |
| same Design matrix for each sample. If Design has |  |
| more than one cell, each cell contains a Design matrix |  |
| for each sample. |  |

$\left.\left.\begin{array}{|l|l|}\hline \text { TolParam } & \begin{array}{l}\text { (Optional) Convergence tolerance for estimation algorithm } \\ \text { based on changes in model parameter estimates. Default } \\ \text { value is sqrt (eps) which is about 1.0e-8 for double } \\ \text { precision. The convergence test for changes in model } \\ \text { parameters is }\end{array} \\ \hline & \| \text { Param }_{k}-\text { Param }_{k-1} \|<\text { TolParam } \times\left(1+\| \text { Param }_{k} \|\right)\end{array} \right\rvert\, \begin{array}{l}\text { where Param represents the output Parameters, and } \\ \text { iteration } k=2,3, \ldots \text {. Convergence is assumed when both } \\ \text { the TolParam and Tol0bj conditions are satisfied. If both } \\ \text { TolParam } \leq 0 \text { and Tolobj } \leq 0, \text { do the maximum number of } \\ \text { iterations (MaxIterations), whatever the results of the } \\ \text { convergence tests. }\end{array}\right\}$

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## Description

[Parameters,Covariance,Resid,Info] = mvnrmle(Data, Design, MaxIterations,TolParam,TolObj, Covar0,CovarFormat ) estimates a multivariate normal regression model without missing data. The model has the form

Data $_{k} \sim N\left(\right.$ Design $_{k} \times$ Parameters, Covariance $)$
for samples $k=1, \ldots$, NUMSAMPLES.
mvnrmle estimates a NUMPARAMS-by-1 column vector of model parameters called Parameters, and a NUMSERIES-by-NUMSERIES matrix of covariance parameters called Covariance.
mvnrmle(Data, Design) with no output arguments plots the log-likelihood function for each iteration of the algorithm.

To summarize the outputs of mvnrmle:

- Parameters is a NUMPARAMS-by-1 column vector of estimates for the parameters of the regression model.
- Covariance is a NUMSERIES-by-NUMSERIES matrix of estimates for the covariance of the regression model's residuals.
- Resid is a NUMSAMPLES-by-NUMSERIES matrix of residuals from the regression. For any row with missing values in Data, the corresponding row of residuals is represented as all NaN missing values, since this routine ignores rows with NaN values.

Another output, Info, is a structure that contains additional information from the regression. The structure has these fields:

- Info. Obj - A variable-extent column vector, with no more than MaxIterations elements, that contain each value of the objective function at each iteration of the estimation algorithm. The last value in this vector, 0 bj (end), is the terminal estimate of the objective function. If you do maximum likelihood estimation, the objective function is the log-likelihood function.
- Info. PrevParameters - NUMPARAMS-by-1 column vector of estimates for the model parameters from the iteration just before the terminal iteration.
- Info. PrevCovariance - NUMSERIES-by-NUMSERIES matrix of estimates for the covariance parameters from the iteration just before the terminal iteration.


## Notes

mvnrmle does not accept an initial parameter vector, because the parameters are estimated directly from the first iteration onward.

You can configure Design as a matrix if NUMSERIES $=1$ or as a cell array if NUMSERIES $\geq 1$.

- If Design is a cell array and NUMSERIES $=1$, each cell contains a NUMPARAMS row vector.
- If Design is a cell array and NUMSERIES > 1, each cell contains a NUMSERIES-byNUMPARAMS matrix.

These points concern how Design handles missing data:

- Although Design should not have NaN values, ignored samples due to NaN values in Data are also ignored in the corresponding Design array.
- If Design is a 1-by-1 cell array, which has a single Design matrix for each sample, no NaN values are permitted in the array. A model with this structure must have NUMSERIES $\geq$ NUMPARAMS with rank(Design\{1\}) $=$ NUMPARAMS.
- Two functions for handling missing data, ecmmvnrmle and ecmlsrmle, are stricter about the presence of NaN values in Design.

Use the estimates in the optional output structure Info for diagnostic purposes.

## Examples

See "Multivariate Normal Regression" on page 9-17, "Least-Squares Regression" on page 9-17, "Covariance-Weighted Least Squares" on page 9-18, "Feasible Generalized Least Squares" on page 9-19, and "Seemingly Unrelated Regression" on page 9-20.

## References

Roderick J. A. Little and Donald B. Rubin. Statistical Analysis with Missing Data., 2nd Edition. John Wiley \& Sons, Inc., 2002.

Xiao-Li Meng and Donald B. Rubin. "Maximum Likelihood Estimation via the ECM Algorithm." Biometrika. Vol. 80, No. 2, 1993, pp. 267-278.
See Alsoecmmvnrmle |mvnrobj|mvnrstd|mvregress
Topics"Multivariate Normal Regression With Missing Data" on page 9-17"Multivariate Normal Regression Without Missing Data" on page 9-17
"Capital Asset Pricing Model with Missing Data"
"Multivariate Normal Linear Regression" on page 9-2
Introduced in R2006a

## mvnrobj

Log-likelihood function for multivariate normal regression without missing data

## Syntax

Objective = mvnrobj(Data, Design,Parameters,Covariance,CovarFormat)

## Arguments

| Data | NUMSAMPLES-by-NUMSERIES matrix with NUMSAMPLES samples of a <br> NUMSERIES-dimensional random vector. If a data sample has <br> missing values, represented as NaNs, the sample is ignored. (Use <br> ecmmvn rmle to handle missing data.) |
| :--- | :--- |
| Design | A matrix or a cell array that handles two model structures: <br> If NUMSERIES $=1$, Design is a NUMSAMPLES-by-NUMPARAMS <br> matrix with known values. This structure is the standard form <br> for regression on a single series. <br> If NUMSERIES $\geq 1$, Design is a cell array. The cell array contains <br> either one or NUMSAMPLES cells. Each cell contains a <br> NUMSERIES-by-NUMPARAMS matrix of known values. <br> If Design has a single cell, it is assumed to have the same <br> Design matrix for each sample. If Design has more than one <br> cell, each cell contains a Design matrix for each sample. |
| Parameters | NUMPARAMS-by-1 column vector of estimates for the parameters of <br> the regression model. |
| Covariance | NUMSERIES-by-NUMSERIES matrix of estimates for the covariance of <br> the residuals of the regression. |


| CovarFormat | (Optional) Character vector that specifies the format for the <br> covariance matrix. The choices are: <br> -full' - Default method. The covariance matrix is a full <br> matrix. <br> - 'diagonal' - The covariance matrix is a diagonal matrix. |
| :--- | :--- |

## Description

Objective = mvnrobj(Data,Design,Parameters,Covariance,CovarFormat) computes the log-likelihood function based on current maximum likelihood parameter estimates without missing data. Objective is a scalar that contains the log-likelihood function.

## Notes

You can configure Design as a matrix if NUMSERIES $=1$ or as a cell array if NUMSERIES $\geq 1$.

- If Design is a cell array and NUMSERIES = 1, each cell contains a NUMPARAMS row vector.
- If Design is a cell array and NUMSERIES > 1, each cell contains a NUMSERIES-byNUMPARAMS matrix.

Although Design should not have NaN values, ignored samples due to NaN values in Data are also ignored in the corresponding Design array.

## Examples

See "Multivariate Normal Regression" on page 9-17, "Least-Squares Regression" on page 9-17, "Covariance-Weighted Least Squares" on page 9-18, "Feasible Generalized Least Squares" on page 9-19, and "Seemingly Unrelated Regression" on page 9-20.

## See Also

ecmmvnrmle | ecmmvnrobj|mvnrmle

## Topics

"Multivariate Normal Regression Without Missing Data" on page 9-17
"Multivariate Normal Linear Regression" on page 9-2
Introduced in R2006a

19-1232

## mvnrstd

Evaluate standard errors for multivariate normal regression model

## Syntax

[StdParameters,StdCovariance] = mvnrstd(Data,Design,Covariance,CovarFormat)

## Arguments

| Data | NUMSAMPLES-by-NUMSERIES matrix with NUMSAMPLES samples of a <br> NUMSERIES-dimensional random vector. If a data sample has <br> missing values, represented as NaNs, the sample is ignored. (Use <br> ecmmvnrmle to handle missing data.) |
| :--- | :--- |
| Design | A matrix or a cell array that handles two model structures: <br> - If NUMSERIES $=1$, Design is a NUMSAMPLES-by-NUMPARAMS <br> matrix with known values. This structure is the standard form <br> for regression on a single series. <br> If NUMSERIES $\geq$ 1, Design is a cell array. The cell array contains <br> either one or NUMSAMPLES cells. Each cell contains a <br> NUMSERIES-by-NUMPARAMS matrix of known values. |
| Covariance | If Design has a single cell, it is assumed to have the same <br> Design matrix for each sample. If Design has more than one <br> cell, each cell contains a Design matrix for each sample. |
| CovarFormat | NUMSERIES-by-NUMSERIES matrix of estimates for the covariance of <br> the regression residuals. |
| (Optional) Character vector that specifies the format for the <br> covariance matrix. The choices are: <br> - ' full' - Default method. The covariance matrix is a full <br> matrix. <br> - diagonal ' - The covariance matrix is a diagonal matrix. |  |

## Description

[StdParameters,StdCovariance] = mvnrstd(Data, Design, Covariance, CovarFormat) evaluates standard errors for a multivariate normal regression model without missing data. The model has the form

Data $_{k} \sim N\left(\right.$ Design $_{k} \times$ Parameters, Covariance $)$
for samples $k=1, \ldots$, NUMSAMPLES.
mvnrstd computes two outputs:

- StdParameters is a NUMPARAMS-by-1 column vector of standard errors for each element of Parameters, the vector of estimated model parameters.
- StdCovariance is a NUMSERIES-by-NUMSERIES matrix of standard errors for each element of Covariance, the matrix of estimated covariance parameters.

Note mvnrstd operates slowly when you calculate the standard errors associated with the covariance matrix Covariance.

## Notes

You can configure Design as a matrix if NUMSERIES = 1 or as a cell array if NUMSERIES $\geq 1$.

- If Design is a cell array and NUMSERIES = 1, each cell contains a NUMPARAMS row vector.
- If Design is a cell array and NUMSERIES > 1, each cell contains a NUMSERIES-byNUMPARAMS matrix.


## Examples

See "Multivariate Normal Regression" on page 9-17, "Least-Squares Regression" on page 9-17, "Covariance-Weighted Least Squares" on page 9-18, "Feasible Generalized Least Squares" on page 9-19, and "Seemingly Unrelated Regression" on page 9-20.

## References

Roderick J. A. Little and Donald B. Rubin. Statistical Analysis with Missing Data. 2nd Edition. John Wiley \& Sons, Inc., 2002.

## See Also

ecmmvnrmle|ecmmvnrstd |mvnrmle

Topics<br>"Multivariate Normal Regression Without Missing Data" on page 9-17<br>"Multivariate Normal Regression Without Missing Data" on page 9-17<br>"Multivariate Normal Linear Regression" on page 9-2

## Introduced in R2006a

## nancov

## Covariance ignoring NaNs

Note nancov will be removed in a future release. Use timetable instead. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

$c=\operatorname{nancov}(X)$
c = nancov(...,'pairwise')

## Arguments

| X | Financial times series object. |
| :--- | :--- |
| Y | Financial times series object. |

## Description

nancov for financial times series objects is based on the Statistics and Machine Learning Toolbox function nancov. See nancov.
$c=\operatorname{nancov}(X)$, if $X$ is a financial time series object with one series and returns the sample variance of the values in $X$, treating NaNs as missing values. For a financial time series object containing more than one series, where each row is an observation and each series a variable, nancov $(X)$ is the covariance matrix computing using rows of $X$ that do not contain any NaN values. nancov $(X, Y)$, where $X$ and $Y$ are financial time series objects with the same number of elements, is equivalent to nancov([X(:) Y(:)]).
nancov $(X)$ or nancov $(X, Y)$ normalizes by $(N-1)$ if $N>1$, where $N$ is the number of observations after removing missing values. This makes nancov the best unbiased estimate of the covariance matrix if the observations are from a normal distribution. For N $=1$, cov normalizes by N .
nancov $(X, 1)$ or nancov $(X, Y, 1)$ normalizes by $N$ and produces the second moment matrix of the observations about their mean. nancov $(X, Y, 0)$ is the same as $\operatorname{nancov}(X, Y)$, and $\operatorname{nancov}(X, 0)$ is the same as nancov $(X)$.
$c=$ nancov(...,'pairwise') computes $c(i, j)$ using rows with no NaN values in columns ior $j$. The result may not be a positive definite matrix. $c=$ nancov(..., ' complete') is the default, and it omits rows with any NaN values, even if they are not in column $i$ or $j$. The mean is removed from each column before calculating the result.

## Examples

To generate random data having nonzero covariance between column 4 and the other columns:

```
x = randn(30, 4); % uncorrelated data
x(:, 4) = sum(x, 2); % introduce correlation
x(2, 3) = NaN; % introduce one missing value
f = fints((today:today+29)', x); % create a fints object using x
c = nancov(f) % compute sample covariance
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints (line 165)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/nancov (line 43)
C =
\begin{tabular}{rrrr}
1.6898 & -0.0005 & 0.3612 & 1.9143 \\
-0.0005 & 1.0833 & -0.5513 & 0.6059 \\
0.3612 & -0.5513 & 1.0369 & 0.7570 \\
1.9143 & 0.6059 & 0.7570 & 4.4895
\end{tabular}
```


## See Also

cov | nanvar|var

```
Introduced before R2006a
```


## nanmax

Maximum ignoring NaNs

Note nanmax will be removed in a future release. Use timetable instead. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

```
m = nanmax(X)
[m,ndx] = nanmax(X)
m = nanmax(X,Y)
[m,ndx] = nanmax(X,[],DIM)
```


## Arguments

| X | Financial times series object. |
| :--- | :--- |
| Y | Financial times series object or scalar. |
| DIM | Dimension of X. |

## Description

nanmax for financial times series objects is based on the Statistics and Machine Learning Toolbox function nanmax. See nanmax.
$m=$ nanmax $(X)$ returns the maximum of a financial time series object $X$ with NaNs treated as missing. $m$ is the largest non-NaN element in $X$.
$[m, n d x]=$ nanmax $(X)$ returns the indices of the maximum values in $X$. If the values along the first nonsingleton dimension contain multiple maximal elements, the index of the first one is returned.
$m=$ nanmax $(X, Y)$ returns an array the same size as $X$ and $Y$ with the largest elements taken from X or Y . Only Y can be a scalar double.
[m,ndx] = nanmax(X,[],DIM) operates along the dimension DIM.

## Examples

To compute nanmax for the following dates:

```
dates = {'01-Jan-2007';'02-Jan-2007';'03-Jan-2007'};
f = fints(dates, magic(3));
f.seriesl(1) = nan;
f.series2(3) = nan;
f.series3(2) = nan;
[nmax, maxidx] = nanmax(f)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints (line 165)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/subsasgn (line 118)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/subsasgn (line 118)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/subsasgn (line 118)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/nanmax (line 20)
nmax =
    4 5 6
maxidx =
    3 2 1
```


## See Also

max | nanmean | nanmedian | nanmin | nanstd| nanvar

## Introduced before R2006a

## nanmean

Mean ignoring NaNs

Note nanmean will be removed in a future release. Use timetable instead. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

$\mathrm{m}=$ nanmean $(\mathrm{X})$
m = nanmean(X,DIM)

## Arguments

| $X$ | Financial times series object. |
| :--- | :--- |
| DIM | Dimension along which the operation is conducted. |

## Description

nanmean for financial times series objects is based on the Statistics and Machine Learning Toolbox function nanmean. See nanmean.
$m=$ nanmean $(X)$ returns the sample mean of a financial time series object $X$, treating NaNs as missing values. m is a row vector containing the mean value of the non- NaN elements in each series.
$m=$ nanmean (X,DIM) takes the mean along dimension DIM of $X$.

## Examples

To compute nanmean for the following dates:

```
dates = {'01-Jan-2007';'02-Jan-2007';'03-Jan-2007'};
f = fints(dates, magic(3));
f.seriesl(1) = nan;
f.series2(3) = nan;
f.series3(2) = nan;
nmean = nanmean(f)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints (line 165)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/subsasgn (line 118)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/subsasgn (line 118)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/subsasgn (line 118)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/nanmean (line 14)
nmean =
    3.5000 3.0000 4.0000
```


## See Also

mean | nanmax | nanmin | nanstd | nansum | nanvar

## Introduced before R2006a

## nanmedian

Median ignoring NaNs

Note nanmedian will be removed in a future release. Use timetable instead. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

$\mathrm{m}=$ nanmedian $(\mathrm{X})$
m = nanmedian(X,DIM)

## Arguments

| $X$ | Financial time series object. |
| :--- | :--- |
| DIM | Dimension along which the operation is conducted. |

## Description

nanmedian for financial time series objects is based on the Statistics and Machine Learning Toolbox function nanmedian. See nanmedian.
$m=$ nanmedian $(X)$ returns the sample median of a financial time series object $X$, treating NaNs as missing values. m is a row vector containing the median value of non- NaN elements in each column.
$m=$ nanmedian (X,DIM) takes the median along the dimension DIM of $X$.

## Examples

To compute nanmedian for the following dates:

```
dates = {'01-Jan-2007';'02-Jan-2007';'03-Jan-2007';'04-Jan-2007'};
f = fints(dates, magic(4));
f.series1(1) = nan;
f.series2(2) = nan;
f.series3([1 3]) = nan;
nmedian = nanmedian(f)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints (line 165)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/subsasgn (line 118)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/subsasgn (line 118)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/subsasgn (line 118)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/nanmedian (line 14)
nmedian =
    5.0000 7.0000 12.5000 10.0000
```


## See Also

mean | nanmax | nanmin | nanstd | nansum | nanvar

## Introduced before R2006a

## nanmin

Minimum ignoring NaNs

Note nanmin will be removed in a future release. Use timetable instead. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

```
m = nanmin(X)
[m,ndx] = nanmin(X)
m = nanmin(X,Y)
[m,ndx] = nanmin(X,[],DIM)
```


## Arguments

| X | Financial times series object. |
| :--- | :--- |
| Y | Financial times series object or scalar. |
| DIM | Dimension along which the operation is conducted. |

## Description

nanmin for financial times series objects is based on the Statistics and Machine Learning Toolbox function nanmin. See nanmin.
$m=$ nanmin $(X)$ returns the minimum of a financial time series object $X$ with NaNs treated as missing. $m$ is the smallest non-NaN element in $X$.
$[m, n d x]=$ nanmin $(X)$ returns the indices of the minimum values in $X$. If the values along the first nonsingleton dimension contain multiple elements, the index of the first one is returned.
$m=$ nanmin $(X, Y)$ returns an array the same size as $X$ and $Y$ with the smallest elements taken from $X$ or $Y$. Only $Y$ can be a scalar double.
[m,ndx] = nanmin(X, [], DIM) operates along the dimension DIM.

## Examples

To compute nanmin for the following dates:

```
dates = {'01-Jan-2007';'02-Jan-2007';'03-Jan-2007'};
f = fints(dates, magic(3));
f.series1(1) = nan;
f.series2(3) = nan;
f.series3(2) = nan;
[nmin, minidx] = nanmin(f)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints (line 165)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/subsasgn (line 118)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/subsasgn (line 118)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/subsasgn (line 118)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/nanmin (line 20)
nmin =
    3 1 2
minidx =
    2 1 3
```


## See Also

mean | nanmax | nanstd | nanvar

## Introduced before R2006a

## nanstd

Standard deviation ignoring NaNs

Note nanstd will be removed in a future release. Use timetable instead. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

```
y = nanstd(X)
y = nanstd(X,1)
y = nanstd(X,FLAG,DIM)
```


## Arguments

| $X$ | Financial times series object. |
| :--- | :--- |
| FLAG | Normalization flag. |
| DIM | Dimension along which the operation is conducted. |

## Description

nanstd for financial times series objects is based on the Statistics and Machine Learning Toolbox function nanstd. See nanstd.
$y=$ nanstd $(X)$ returns the sample standard deviation of the values in a financial time series object $X$, treating NaNs as missing values. $y$ is the standard deviation of the nonNaN elements of X .
nanstd normalizes $y$ by $(N-1)$, where $N$ is the sample size. This is the square root of an unbiased estimator of the variance of the population from which $X$ is drawn, as long as $X$ consists of independent, identically distributed samples and data are missing at random.
$y=$ nanstd $(X, 1)$ normalizes by $N$ and produces the square root of the second moment of the sample about its mean. nanstd $(X, 0)$ is the same as nanstd $(X)$.
$\mathrm{y}=$ nanstd(X,flag,dim) takes the standard deviation along the dimension dim of X . Set the value of flag to 0 to normalize the result by $n-1$; set the value of flag to 1 to normalize the result by n .

## Examples

To compute nanstd for the following dates:

```
dates = {'01-Jan-2007';'02-Jan-2007';'03-Jan-2007'};
f = fints(dates, magic(3));
f.series1(1) = nan;
f.series2(3) = nan;
f.series3(2) = nan;
nstd = nanstd(f)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints (line 165)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/subsasgn (line 118)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/subsasgn (line 118)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/subsasgn (line 118)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/nanstd (line 25)
nstd =
```

0.71
2.83
2.83

## See Also

nanmax | nanmean | nanmedian | nanmin| nanvar | std

## Introduced before R2006a

## nansum

Sum ignoring NaNs

Note nansum will be removed in a future release. Use timetable instead. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

$y=\operatorname{nansum}(X)$
$y=n a n s u m(X, D I M)$

## Arguments

| $X$ | Financial time series object. |
| :--- | :--- |
| DIM | Dimension along which the operation is conducted. |

## Description

nansum for financial times series objects is based on the Statistics and Machine Learning Toolbox function nansum. See nansum.
$y=$ nansum $(X)$ returns the sum of a financial time series object $X$, treating $N a N s$ as missing values. y is the sum of the non-NaN elements in X .
$y=$ nansum ( $X$, DIM $)$ takes the sum along dimension DIM of $X$.

## Examples

To compute nansum for the following dates:

```
dates = {'01-Jan-2007';'02-Jan-2007';'03-Jan-2007'};
f = fints(dates, magic(3));
```

```
f.series1(1) = nan;
f.series2(3) = nan;
f.series3(2) = nan;
nsum = nansum(f)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints (line 165)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/subsasgn (line 118)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/subsasgn (line 118)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/subsasgn (line 118)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/nansum (line 13)
nsum =
    7 6 8
```


## See Also

nanmax | nanmean | nanmedian | nanmin | nanstd|nanvar

## Introduced before R2006a

## nanvar

Variance ignoring NaNs

Note nanvar will be removed in a future release. Use timetable instead. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

```
y = nanvar(X)
y = nanvar(X,1)
y = nanvar(X,W)
y = nanvar(X,W,DIM)
```


## Arguments

| $X$ | Financial times series object. |
| :--- | :--- |
| $W$ | Weight vector. |
| DIM | Dimension along which the operation is conducted. |

## Description

nanvar for financial times series objects is based on the Statistics and Machine Learning Toolbox function nanvar. See nanvar.
$y=$ nanvar $(X)$ returns the sample variance of the values in a financial time series object $X$, treating NaNs as missing values. $y$ is the variance of the non-NaN elements of each series in $X$.
nanvar normalizes y by $N-1$ if $N>1$, where $N$ is the sample size of the non- NaN elements. This is an unbiased estimator of the variance of the population from which X is
drawn, as long as $X$ consists of independent, identically distributed samples, and data are missing at random. For $N=1, y$ is normalized by $N$.
$\mathrm{y}=$ nanvar( $\mathrm{X}, 1$ ) normalizes by N and produces the second moment of the sample about its mean. nanvar $(X, 0)$ is the same as nanvar $(X)$.
$y=$ nanvar $(X, W)$ computes the variance using the weight vector $W$. The length of $W$ must equal the length of the dimension over which nanvar operates, and its non-NaN elements must be nonnegative. Elements of $X$ corresponding to $N a N$ elements of Ware ignored.
$\mathrm{y}=$ nanvar( $\mathrm{X}, \mathrm{W}, \mathrm{DIM})$ takes the variance along dimension DIM of X.

## Examples

To compute nanvar:

```
f = fints((today:today+1)', [4 -2 1; 9 5 7])
f.seriesl(1) = nan;
f.series3(2) = nan;
nvar = nanvar(f)
```

```
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
```

Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/subsasgn (line 118)
> In fints/subsasgn (line 118)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/subsasgn (line 118)
> In fints/subsasgn (line 118)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/nanvar (line 28)

```
> In fints/nanvar (line 28)
```

nvar =
$0 \quad 24.5000 \quad 0$

## See Also

nanmax | nanmean | nanmedian | nanmin | nanstd | var

## Introduced before R2006a

## negvolidx

Negative volume index

Note negvolidx has been partially removed and will no longer accept a fints object (tsobj) argument. Use a matrix, timetable, or table instead for financial time series.

Use fts2timetable to convert a fints object to a timetable object.

## Syntax

volume $=$ negvolidx(Data)
volume = negvolidx( $\qquad$ ,Name, Value)

## Description

volume $=$ negvolidx(Data) calculates the negative volume index from the series of closing stock prices and trade volume.
volume $=$ negvolidx( $\qquad$ , Name, Value) adds optional name-value pair arguments.

## Examples

## Calculate the Negative Volume Index for a Data Series for a Stock

Load the file SimulatedStock.mat, which provides a timetable (TMW) for financial data for TMW stock.
load SimulatedStock.mat
volume $=$ negvolidx(TMW);
plot(volume.Time, volume.NegativeVolume)
title('Negative Volume Index for TMW')


## Input Arguments

## Data - Data with closing prices and trade volume

matrix | table | timetable
Data with closing prices and trade volume, specified as a matrix, table, or timetable. For matrix input, Data is an M-by-2 with closing prices and trade volume stored in the first and second columns. Timetables and tables with M rows must contain variables named 'Close' and 'Volume' (case insensitive).

Data Types: double|table|timetable

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.
Example: volume = negvolidx(TMW,'InitialValue',500)

## InitialValue - Initial value for negative volume index <br> 100 (default) | positive integer

Initial value for negative volume index, specified as the comma-separated pair consisting of 'InitialValue' and a scalar positive integer.
Data Types: double

## Output Arguments

volume - Negative volume index
matrix | table | timetable
Negative volume index, returned with the same number of rows ( $M$ ) and the same type (matrix, table, or timetable) as the input Data.

## Definitions

## Negative Volume Index

Negative volume index shows the days when the trading volume of a particular security is substantially lower than other days.

## References

[1] Achelis, S. B. Technical Analysis from A to Z. Second Edition. McGraw-Hill, 1995, pp. 193-194.
See Alsoonbalvol| posvolidx|table|timetable
Topics
"Using Timetables in Finance"
"Convert Financial Time Series Objects fints to Timetables" on page 12-2
Introduced before R2006a

## nomrr

Nominal rate of return

## Syntax

Return $=$ nomrr(Rate,NumPeriods)

## Arguments

| Rate | Effective annual percentage rate. Enter as a decimal fraction. |
| :--- | :--- |
| NumPeriods | Number of compounding periods per year, an integer. |

## Description

Return $=$ nomrr(Rate, NumPeriods) calculates the nominal rate of return.

## Examples

## Calculate the Nominal Rate of Return

This example shows how to calculate the nominal rate of return based on an effective annual percentage rate of $9.38 \%$ compounded monthly.

Return $=$ nomrr $(0.0938$, 12)
Return $=0.0900$

## See Also

effrr|irr|mirr|taxedrr|xirr

## Topics

"Analyzing and Computing Cash Flows" on page 2-18

Introduced before R2006a

## nweekdate

Date of specific occurrence of weekday in month

## Syntax

Date $=$ nweekdate( $n$, Weekday, Year, Month $)$
Date $=$ nweekdate( $\qquad$ , Same, outputType)

## Description

Date $=$ nweekdate $(\mathrm{n}$, Weekday, Year, Month) returns the date number for the specific occurrence of the weekday in the given Year and Month.

Any input argument can contain multiple values, but if so, all other input arguments must contain the same number of values or a single value that applies to all. For example, if Year is a 1-by-n vector of integers, then Month must be a 1-by-n vector of integers or a single integer. Date is then a 1-by-n vector of date numbers.

Use the function datestr to convert serial date numbers to formatted date character vectors.

Date = nweekdate( ___ Same, outputType) returns the date number for the specific occurrence of the weekday in the given Year and Month and also contains the optional arguments for weekday Same and outputType.

## Examples

Determine the Date of a Specific Occurrence of a Weekday in a Month
Find the first Thursday in May 2001.
Date = nweekdate(1, 5, 2001, 5); datestr(Date)

```
ans =
' 03-May-2001
```

Find the first Thursday in May 2001 returned as a datetime array.

```
Date = nweekdate(1, 5, 2001, 5,[],'datetime')
Date = datetime
    03-May-2001
```

Find the first Thursday in a week that also contains a Wednesday in May 2001.

```
Date = nweekdate(2, 5, 2001, 5, 4);
datestr(Date)
ans =
' 10-May-2001
```

Find the third Monday in February for 2001, 2002, and 2003.

```
Year = [2001:2003];
Date = nweekdate(3, 2, Year, 2);
datestr(Date)
ans = 3x11 char array
    '19-Feb-2001'
    '18-Feb-2002'
    '17-Feb-2003'
```


## Input Arguments

## n - Nth occurrence of weekday in a month

integer with value 1 through 5 | vector of integers with values 1 through 5
Nth occurrence of the weekday in a month, specified as an integer or a vector of integers from 1 through 5.

If n is larger than the last occurrence of Weekday, the output Date $=0$.
Data Types: single | double

## Weekday - Weekday whose date you seek <br> integer with value 1 through 7 | vector of integers with values 1 through 7

Weekday whose date you seek, specified as an integer or a vector of integers from 1 through 7.

- 1 - Sunday
- 2 - Monday
- 3 - Tuesday
- 4 - Wednesday
- 5 - Thursday
- 6 - Friday
- 7 - Saturday

Data Types: single | double

## Year - Year to determine occurrence of weekday

4-digit integer | vector of 4-digit integers
Year to determine occurrence of weekday, specified as a 4-digit integer or vector of 4-digit integers.
Data Types: single | double

## Month - Month to determine occurrence of weekday

integer with value 1 through 12 | vector of integers with values 1 through 12
Month to determine occurrence of weekday, specified as an integer or vector of integers with values 1 through 12.

## Data Types: single | double

## Same - Weekday that must occur in same week with Weekday

$0=$ ignore (default) | integer with value 0 through $7 \mid$ vector of integers with values 0 through 7

Weekday that must occur in the same week with Weekday, specified as an integer or a vector of integers from 0 through 7 , where $0=$ ignore (default) and 1 through 7 are as for Weekday.

Data Types: single | double

## outputType - Year to determine days

## 'datenum' (default)| character vector with values 'datenum' or 'datetime'

A character vector specified as either 'datenum ' or 'datetime'. The output Date is in serial date format if 'datenum' is specified, or datetime format if 'datetime' is specified. By default the output Date is in serial date format.

## Data Types: single | double

## Output Arguments

## Date - Date of specific occurrence of weekday in month <br> serial date number | date character vector

Date of specific occurrence of weekday in month, returned as a serial date number or date character vector.

The type of the output for Date depends on the input outputType. If this variable is 'datenum', Date is a serial date number. If outputType is 'datetime', then Date is a datetime array. By default, outputType is set to 'datenum'.

## See Also

datetime| fbusdate|lbusdate|lweekdate

## Topics

"Handle and Convert Dates" on page 2-2

Introduced before R2006a

## nyseclosures

New York Stock Exchange closures from 1885 to 2070

## Syntax

Closures = nyseclosures [Closures,SatTransition] = nyseclosures(StartDate,EndDate, WorkWeekFormat)

## Description

Closures $=$ nyseclosures returns a vector of serial date numbers for all known or anticipated closures from January 1, 1885 to December 31, 2070.

Since the New York Stock Exchange was open on Saturdays before September 29, 1952, exact closures from 1885 to 1952 are based on a 6-day workweek. nyseclosures contains all holiday and special non-trading days for the New York Stock Exchange from 1885 through 2070 based on a six-day work week (always closed on Sundays).
[Closures,SatTransition] = nyseclosures(StartDate,EndDate, WorkWeekFormat), using optional input arguments, returns a vector of serial date numbers corresponding to market closures between StartDate and EndDate, inclusive.

Since the New York Stock Exchange was open on Saturdays before September 29, 1952, exact closures from 1885 to 1952 are based on a 6-day workweek. nyseclosures contains all holiday and special non-trading days for the New York Stock Exchange from 1885 through 2070 based on a six-day work week (always closed on Sundays). Use WorkWeekFormat to modify the list of dates.

## Examples

## Find NYSE Closures

Find the NYSE closures for 1899:

```
datestr(nyseclosures('1-jan-1899','31-dec-1899'),'dd-mmm-yyyy ddd')
ans = 16x15 char array
    '02-Jan-1899 Mon'
    '11-Feb-1899 Sat'
    '13-Feb-1899 Mon'
    '22-Feb-1899 Wed'
    '31-Mar-1899 Fri'
    '29-May-1899 Mon'
    '30-May-1899 Tue'
    '03-Jul-1899 Mon'
    '04-Jul-1899 Tue'
    '04-Sep-1899 Mon'
    '29-Sep-1899 Fri'
    '30-Sep-1899 Sat'
    '07-Nov-1899 Tue'
    '25-Nov-1899 Sat'
    '30-Nov-1899 Thu'
    '25-Dec-1899 Mon'
```

Find the NYSE closures for 1899 using a datetime array:
[Closures,SatTransition] = nyseclosures(datetime('1-jan-1899','Locale','en_US'),'30-Jur
Closures = 7x1 datetime array
02-Jan-1899
11-Feb-1899
13-Feb-1899
22-Feb-1899
31-Mar-1899
29-May-1899
30-May-1899
SatTransition = datetime
29-Sep-1952

Find the NYSE closure dates using the 'Archaic' value for WorkWeekFormat:

```
datestr(nyseclosures('1-sep-1952','31-oct-1952','a'),1)
```

ans $=10 \times 11$ char array
'01-Sep-1952'
'06-Sep-1952'

```
'13-Sep-1952'
'20-Sep-1952'
'27-Sep-1952'
'04-0ct-1952'
'11-0ct-1952'
'13-0ct-1952'
'18-0ct-1952'
'25-0ct-1952'
```

The exchange was closed on Saturdays for much of 1952 before the official transition to a 5-day workweek.

## Input Arguments

## StartDate - Start date

start of default date range, January 1, 1885 (default) | serial date number | date character vector | datetime object

Start date, specified using a serial date number, date character vector, or datetime array.
Data Types: double| char|datetime

## EndDate - End date

end of default date range, December 31, 2070 (default) | serial date number | date character vector | datetime object

End date, specified using a serial date number, date character vector, or datetime array. Data Types: double | char | datetime

## WorkWeekFormat - Method to handle the workweek

'Implicit' (default) | date character vector with values 'Modern', 'Implicit', or 'Archaic'

Method to handle the workweek, specified using a date character vector with values 'Modern', 'Implicit', or 'Archaic'. This function accepts the first letter for each method as input and is not case-sensitive. Acceptable values are:

- 'Modern' - 5-day workweek with all Saturday trading days removed.
- 'Implicit' - 6-day workweek until 1952 and 5-day week afterward (no need to exclude Saturdays).
- 'Archaic' - 6-day workweek throughout and Saturdays treated as closures after 1952.


## Data Types: char

## Output Arguments

## Closures - Market closures between StartDate and EndDate, inclusive vector

Market closures between the StartDate and EndDate, inclusive, returned as a vector of dates.

If StartDate or EndDate are all either serial date numbers or date character vectors, both Closures and SatTransition are returned as serial date numbers. If either StartDate or EndDate are datetime arrays, both Closures and SatTransition are returned as datetime arrays.

If both StartDate and EndDate are not specified or are empty, Closures contains all known or anticipated closures from January 1, 1885 to December 31, 2070 based on a WorkWeekFormat of 'implicit'.

## SatTransition - Date of transition for New York Stock Exchange from 6-day workweek to 5-day workweek <br> serial date number | datetime array

Date of transition for the New York Stock Exchange from a 6-day workweek to a 5-day workweek, returned as the date September 29, 1952 (serial date number 713226).

If StartDate or EndDate are all either serial date numbers or date character vectors, both Closures and SatTransition are returned as serial date numbers. If either StartDate or EndDate are datetime arrays, both Closures and SatTransition are returned as datetime arrays.

## Definitions

## holidays

The holidays function is based on a modern 5-day workweek.
This function contains all holidays and special nontrading days for the New York Stock Exchange from January 1, 1885 to December 31, 2070.

Since the New York Stock Exchange was open on Saturdays before September 29, 1952, exact closures from 1885 to 2070 should include Saturday trading days. To capture these dates, use nyseclosures. The results from holidays and nyseclosures are identical if the WorkWeekFormat in nyseclosures is 'Modern'.

## See Also

busdate | createholidays | datetime|fbusdate|holidays|isbusday| lbusdate

## Topics

"Handle and Convert Dates" on page 2-2
"Trading Calendars User Interface" on page 16-2
"UICalendar User Interface" on page 16-4

## Introduced before R2006a

## onbalvol

On-Balance Volume (OBV)

Note onbalvol has been partially removed and will no longer accept a fints object (tsobj) argument. Use a matrix, timetable, or table instead for financial time series.

Use fts2timetable to convert a fints object to a timetable object.

## Syntax

volume = onbalvol(Data)

## Description

volume $=$ onbalvol(Data) calculates the On-Balance Volume from the series of closing stock prices and trade volume.

## Examples

## Calculate the On-Balance Volume for a Data Series for a Stock

Load the file SimulatedStock. mat, which provides a timetable (TMW) for financial data for TMW stock.

```
load SimulatedStock.mat
volume = onbalvol(TMW);
plot(volume.Time,volume.OnBalanceVolume)
title('On-Balance Volume for TMW')
```



## Input Arguments

## Data - Data for closing prices and trade volume

matrix | table | timetable
Data for closing prices and trade volume, specified as a matrix, table, or timetable. For matrix input, Data is an M-by-2 matrix of closing prices and trade volume stored in the first and second columns. Timetables and tables with M rows must contain variables named 'Close' and 'Volume' (case insensitive).

Data Types: double | table | timetable

## Output Arguments

## volume - On-Balance Volume <br> matrix | table | timetable

On-Balance Volume, returned with the same number of rows $(\mathrm{M})$ and the same type (matrix, table, or timetable) as the input Data.

## References

[1] Achelis, S. B. Technical Analysis from A to Z. Second Edition. McGraw-Hill, 1995, pp. 207-209.

## See Also

negvolidx| posvolidx|table|timetable

## Topics

"Using Timetables in Finance"
"Convert Financial Time Series Objects fints to Timetables" on page 12-2

## Introduced before R2006a

## opprofit

Option profit

## Syntax

```
Profit = opprofit(AssetPrice,Strike,Cost,PosFlag,OptType)
```


## Description

Profit $=$ opprofit(AssetPrice, Strike, Cost, PosFlag,OptType) returns the profit of an option.

## Examples

## Calculate the Profit of an Option

This example shows how to return the profit of an option. For example, consider buying (going long on) a call option with a strike price of $\$ 90$ on an underlying asset with a current price of $\$ 100$ for a cost of $\$ 4$.

```
Profit = opprofit(100, 90, 4, 0, 0)
```

Profit $=6$

## Input Arguments

## AssetPrice - Asset price

numeric
Asset price, specified as a scalar or a NINST-by-1 vector.
Data Types: double

## Strike - Strike or exercise price <br> numeric

Strike or exercise price, specified as a scalar or a NINST-by-1 vector.

## Data Types: double

## Cost - Cost of option

numeric
Cost of the option, specified as a scalar or a NINST-by-1 vector.
Data Types: double

## PosFlag - Option position

0 = long | 1 = short
Option position, specified as a scalar or a NINST-by-1 vector using the values 0 (long) or 1 (short).

Data Types: logical

## OptType - Option type

$0=$ call option | 1 = put option
Option type, specified as a scalar or a NINST-by-1 vector using the values 0 (call option) or 1 (put option).

Data Types: logical

## Output Arguments

## Profit - Option profit

vector
Option profit, returned as a scalar or a NINST-by-1 vector.

See Also<br>binprice|blsprice

## Topics

"Pricing and Analyzing Equity Derivatives" on page 2-45
"Greek-Neutral Portfolios of European Stock Options" on page 10-18
"Plotting Sensitivities of an Option" on page 10-30
"Plotting Sensitivities of a Portfolio of Options" on page 10-33

## Introduced before R2006a

## payadv

Periodic payment given number of advance payments

## Syntax

Payment = payadv(Rate,NumPeriods,PresentValue,FutureValue,Advance)

## Arguments

| Rate | Lending or borrowing rate per period. Enter as a decimal <br> fraction. Must be greater than or equal to 0. |
| :--- | :--- |
| NumPeriods | Number of periods in the life of the instrument. |
| PresentValue | Present value of the instrument. |
| FutureValue | Future value or target value to be attained after <br> NumPeriods periods. |
| Advance | Number of advance payments. If the payments are made at <br> the beginning of the period, add 1 to Advance. |

## Description

Payment = payadv(Rate,NumPeriods,PresentValue,FutureValue,Advance) returns the periodic payment given a number of advance payments.

## Examples

## Compute the Periodic Payment

This example shows how to compute the periodic payment, given a number of advance payments. For example, the present value of a loan is $\$ 1000.00$ and it will be paid in full
in 12 months. The annual interest rate is $10 \%$ and three payments are made at closing time.

```
Payment = payadv(0.1/12, 12, 1000, 0, 3)
Payment = 85.9389
```


## See Also

amortize| payodd | payper

## Topics

"Analyzing and Computing Cash Flows" on page 2-18

## Introduced before R2006a

## payodd

Payment of loan or annuity with odd first period

## Syntax

Payment = payodd(Rate,NumPeriods,PresentValue,FutureValue,Days)

## Arguments

| rate | Interest rate per period. Enter as a decimal fraction. |
| :--- | :--- |
| NumPeriods | Number of periods in the life of the instrument. |
| PresentValue | Present value of the instrument. |
| FutureValue | Future value or target value to be attained after <br> NumPeriods periods. |
| Days | Actual number of days until the first payment is made. |

## Description

Payment = payodd(Rate,NumPeriods,PresentValue,FutureValue,Days) returns the payment for a loan or annuity with an odd first period.

## Examples

## Compute the Payment for a Loan or Annuity With an Odd First Period

This example shows how to return the payment for a loan or annuity with an odd first period. For example, consider a two-year loan for $\$ 4000$ that has an annual interest rate of $11 \%$ and the first payment will be made in 36 days.

```
Payment = payodd(0.11/12, 24, 4000, 0, 36)
```


## See Also

amortize | payadv | payper

## Topics

"Analyzing and Computing Cash Flows" on page 2-18
Introduced before R2006a

## payper

Periodic payment of loan or annuity

## Syntax

Payment = payper(Rate,NumPeriods,PresentValue,FutureValue,Due)

## Arguments

| Rate | Interest rate per period. Enter as a decimal fraction. |
| :--- | :--- |
| NumPeriods | Number of payment periods in the life of the instrument. |
| PresentValue | Present value of the instrument. |
| FutureValue | (Optional) Future value or target value to be attained after <br> NumPeriods periods. Default $=0$. |
| Due | (Optional) When payments are due: $0=$ end of period <br> (default), or $1=$ beginning of period. |

## Description

Payment = payper(Rate,NumPeriods,PresentValue,FutureValue,Due) returns the periodic payment of a loan or annuity.

## Examples

## Compute the Periodic Payment of a Loan or Annuity

This example shows how to find the monthly payment for a three-year loan of $\$ 9000$ with an annual interest rate of $11.75 \%$.

Payment = payper(0.1175/12, 36, 9000, 0, 0)

```
Payment = 297.8553
```


## See Also

amortize|fvfix|payadv|payodd|pvfix

## Topics

"Analyzing and Computing Cash Flows" on page 2-18

Introduced before R2006a

## payuni

Uniform payment equal to varying cash flow

## Syntax

```
Series = payuni(CashFlow,Rate)
```


## Arguments

| CashFlow | A vector of varying cash flows. Include the initial investment as the <br> initial cash flow value (a negative number). |
| :--- | :--- |
| Rate | Periodic interest rate. Enter as a decimal fraction. |

## Description

Series = payuni(CashFlow, Rate) returns the uniform series value of a varying cash flow.

## Examples

This cash flow represents the yearly income from an initial investment of $\$ 10,000$. The annual interest rate is $8 \%$.

| Year 1 | $\$ 2000$ |
| :--- | :--- |
| Year 2 | $\$ 1500$ |
| Year 3 | $\$ 3000$ |
| Year 4 | $\$ 3800$ |
| Year 5 | $\$ 5000$ |

To calculate the uniform series value

```
Series = payuni([-10000 2000 1500 3000 3800 5000], 0.08)
returns
Series =
    429.63
```


## See Also

fvfix|fvvar|irr|pvfix|pvvar

## Topics

"Analyzing and Computing Cash Flows" on page 2-18

## Introduced before R2006a

## pcalims

Linear inequalities for individual asset allocation
As an alternative to pcalims, use the Portfolio object (Portfolio) for mean-variance portfolio optimization. This object supports gross or net portfolio returns as the return proxy, the variance of portfolio returns as the risk proxy, and a portfolio set that is any combination of the specified constraints to form a portfolio set. For information on the workflow when using Portfolio objects, see "Portfolio Object Workflow" on page 4-21.

## Syntax

[A,b] = pcalims(AssetMin,AssetMax,NumAssets)

## Arguments

| AssetMin | Scalar or NASSETS vector of minimum allocations in each asset. NaN <br> indicates no constraint. |
| :--- | :--- |
| AssetMax | Scalar or NASSETS vector of maximum allocations in each asset. <br> NaN indicates no constraint. |
| NumAssets | (Optional) Number of assets. Default = length of AssetMin or <br> AssetMax. |

## Description

$[A, b]=p c a l i m s(A s s e t M i n, A s s e t M a x, N u m A s s e t s)$ specifies the lower and upper bounds of portfolio allocations in each of NumAssets available asset investments.
$A$ is a matrix and $b$ is a vector such that $A * P o r t W t s{ }^{\prime}<=b$, where PortWts is a 1-byNASSETS vector of asset allocations.

If pcalims is called with fewer than two output arguments, the function returns $A$ concatenated with $b[A, b]$.

## Examples

Set the minimum weight in every asset to 0 (no short-selling), and set the maximum weight of IBM stock to 0.5 and CSCO to 0.8 , while letting the maximum weight in INTC float.


Portfolio weights of $50 \%$ in IBM and $50 \%$ in INTC satisfy the constraints.
Set the minimum weight in every asset to 0 and the maximum weight to 1 .

| Asset | IBM | INTC | CSCO |
| :--- | :--- | :--- | :--- |
| Minimum Weight | 0 | 0 | 0 |
| Maximum Weight | 1 | 1 | 1 |

AssetMin $=0$
AssetMax = 1
NumAssets $=3$
[A,b] = pcalims(AssetMin, AssetMax, NumAssets)

19-1282

| 1 | 0 | 0 |
| ---: | ---: | ---: |
| 0 | 1 | 0 |
| 0 | 0 | 1 |
| -1 | 0 | 0 |
| 0 | -1 | 0 |
| 0 | 0 | -1 |

b =

Portfolio weights of $50 \%$ in IBM and $50 \%$ in INTC satisfy the constraints.

## See Also

Portfolio| pcgcomp|pcglims|pcpval|portcons|portopt|portstats

## Topics

"Portfolio Construction Examples" on page 3-7
"Portfolio Selection and Risk Aversion" on page 3-9
"Active Returns and Tracking Error Efficient Frontier" on page 3-43
"Analyzing Portfolios" on page 3-2
"Portfolio Optimization Functions" on page 3-4

Introduced before R2006a

## pcgcomp

Linear inequalities for asset group comparison constraints
As an alternative to pcgcomp, use the Portfolio object (Portfolio) for mean-variance portfolio optimization. This object supports gross or net portfolio returns as the return proxy, the variance of portfolio returns as the risk proxy, and a portfolio set that is any combination of the specified constraints to form a portfolio set. For information on the workflow when using Portfolio objects, see "Portfolio Object Workflow" on page 4-21.

## Syntax

[A,b] = pcgcomp(GroupA,AtoBmin,AtoBmax,GroupB)

## Arguments

| GroupA | Number of groups (NGROUPS) by number of assets (NASSETS) <br> specifications of groups to compare. Each row specifies a group. For <br> a specific group, Group $(i, j)=1$ if the group contains asset $j ;$ <br> otherwise, Group $(i, j)=0$. |
| :--- | :--- |
| AroupB | Scalar or NGROUPS-long vectors of minimum and maximum ratios of <br> allocations in GroupA to allocations in GroupB. NaN indicates no <br> constraint between the two groups. Scalar bounds are applied to all <br> group pairs. The total number of assets allocated to GroupA divided <br> by the total number of assets allocated to GroupB is $>=$ AtoBmin <br> and $<=$ AtoBmax. |

## Description

$[A, b]=p c g c o m p(G r o u p A, A t o B m i n, A t o B m a x, G r o u p B)$ specifies that the ratio of allocations in one group to allocations in another group is at least AtoBmin to 1 and at most AtoBmax to 1 . Comparisons can be made between an arbitrary number of group pairs NGROUPS comprising subsets of NASSETS available investments.
$A$ is a matrix and $b$ a vector such that $A * P o r t W t s{ }^{\prime}<=b$, where PortWts is a 1-byNASSETS vector of asset allocations.

If pcgcomp is called with fewer than two output arguments, the function returns $A$ concatenated with $b[A, b]$.

## Examples

| Asset | INTC | XOM | RD |
| :--- | :--- | :--- | :--- |
| Region | North America | North America | Europe |
| Sector | Technology | Energy | Energy |


| Group | Min. Exposure | Max. Exposure |
| :--- | :--- | :--- |
| North America | 0.30 | 0.75 |
| Europe | 0.10 | 0.55 |
| Technology | 0.20 | 0.50 |
| Energy | 0.20 | 0.80 |

Make the North American energy sector compose exactly 20\% of the North American investment.

```
% INTC XOM RD
GroupA = [ 0 1 0 ]; % North American Energy
GroupB = [ 1 1 1 0 ]; % North America
AtoBmin = 0.20;
AtoBmax = 0.20;
[A,b] = pcgcomp(GroupA, AtoBmin, AtoBmax, GroupB)
A =
    0.2000 -0.8000 0
b =
    0
    0
```

Portfolio weights of $40 \%$ for INTC, $10 \%$ for XOM, and $50 \%$ for RD satisfy the constraints.

See Also<br>Portfolio|pcalims | pcglims| pcpval| portcons| portopt<br>\section*{Topics}<br>"Portfolio Construction Examples" on page 3-7<br>"Portfolio Selection and Risk Aversion" on page 3-9<br>"Active Returns and Tracking Error Efficient Frontier" on page 3-43<br>"Analyzing Portfolios" on page 3-2<br>"Portfolio Optimization Functions" on page 3-4<br>Introduced before R2006a

## pcglims

Linear inequalities for asset group minimum and maximum allocation
As an alternative to pcglims, use the Portfolio object (Portfolio) for mean-variance portfolio optimization. This object supports gross or net portfolio returns as the return proxy, the variance of portfolio returns as the risk proxy, and a portfolio set that is any combination of the specified constraints to form a portfolio set. For information on the workflow when using Portfolio objects, see "Portfolio Object Workflow" on page 4-21.

## Syntax

[A,b] = pcglims(Groups,GroupMin,GroupMax)

## Arguments

| Groups | Number of groups (NGROUPS) by number of assets (NASSETS) <br> specification of which assets belong to which group. Each row <br> specifies a group. For a specific group, Group $(i, j)=1$ if the <br> group contains asset j; otherwise, Group $(i, j)=0$. |
| :--- | :--- |
| GroupMin | Scalar or NGROUPS-long vectors of minimum and maximum <br> combined allocations in each group. NaN indicates no constraint. <br> Scalar bounds are applied to all groups. |
| GroupMax |  |

## Description

[A,b] = pcglims(Groups, GroupMin, GroupMax) specifies minimum and maximum allocations to groups of assets. An arbitrary number of groups, NGROUPS, comprising subsets of NASSETS investments, is allowed.
$A$ is a matrix and $b$ a vector such that $A * P o r t W t s s^{\prime}<=b$, where PortWts is a 1-byNASSETS vector of asset allocations.

If pcglims is called with fewer than two output arguments, the function returns A concatenated with b [A, b].

## Examples

| Asset | INTC | XOM | RD |
| :--- | :--- | :--- | :--- |
| Region | North America | North America | Europe |
| Sector | Technology | Energy | Energy |
|  |  | Min. Exposure | Max. Exposure |
| Group | 0.30 | 0.75 |  |
| North America | 0.10 | 0.55 |  |
| Europe | 0.20 | 0.50 |  |
| Technology | 0.50 | 0.50 |  |
| Energy |  |  |  |

Set the minimum and maximum investment in various groups.

```
% INTC XOM RD
Groups = [ [ 1 1 1 0 ; % North America
    0 0 1 ; % Europe
    1 0 0 ; % Technology
    0 1 1 ]; % Energy
GroupMin = [0.30
    0.10
    0.20
    0.50];
GroupMax = [0.75
    0.55
    0.50
    0.50];
[A,b] = pcglims(Groups, GroupMin, GroupMax)
A =
\begin{tabular}{rrr}
-1 & -1 & 0 \\
0 & 0 & -1 \\
-1 & 0 & 0 \\
0 & -1 & -1 \\
1 & 1 & 0 \\
0 & 0 & 1 \\
1 & 0 & 0
\end{tabular}
```

```
            0 1
=
\(-0.3000\)
-0.1000
-0. 2000
-0.5000
0.7500
0.5500
0.5000
0.5000
```

Portfolio weights of 50\% in INTC, $25 \%$ in XOM, and $25 \%$ in RD satisfy the constraints.

## See Also

Portfolio| pcalims | pcgcomp| pcpval | portcons | portopt

## Topics

"Portfolio Construction Examples" on page 3-7
"Portfolio Selection and Risk Aversion" on page 3-9
"Active Returns and Tracking Error Efficient Frontier" on page 3-43
"Analyzing Portfolios" on page 3-2
"Portfolio Optimization Functions" on page 3-4

Introduced before R2006a

## pcpval

Linear inequalities for fixing total portfolio value
As an alternative to pcpval, use the Portfolio object (Portfolio) for mean-variance portfolio optimization. This object supports gross or net portfolio returns as the return proxy, the variance of portfolio returns as the risk proxy, and a portfolio set that is any combination of the specified constraints to form a portfolio set. For information on the workflow when using Portfolio objects, see "Portfolio Object Workflow" on page 4-21.

## Syntax

$[\mathrm{A}, \mathrm{b}]=$ pcpval(PortValue,NumAssets)

## Arguments

| PortValue | Scalar total value of asset portfolio (sum of the allocations in <br> all assets). PortValue = 1 specifies weights as fractions of <br> the portfolio and return and risk numbers as rates instead of <br> value. |
| :--- | :--- |
| NumAssets | Number of available asset investments. |

## Description

$[A, b]=p c p v a l(P o r t V a l u e, N u m A s s e t s)$ scales the total value of a portfolio of NumAssets assets to PortValue. All portfolio weights, bounds, return, and risk values except ExpReturn and ExpCovariance (see portopt) are in terms of PortValue.
$A$ is a matrix and $b$ a vector such that $A * P o r t W t s^{\prime}<=b$, where PortWts is a 1-byNASSETS vector of asset allocations.

If pcpval is called with fewer than two output arguments, the function returns $A$ concatenated with $b[A, b]$.

## Examples

Scale the value of a portfolio of three assets $=1$, so all return values are rates and all weight values are in fractions of the portfolio.

```
PortValue = 1;
NumAssets = 3;
[A,b] = pcpval(PortValue, NumAssets)
A =
    1 1 1
    -1 -1 -1
b =
    1
-1
```

Portfolio weights of $40 \%, 10 \%$, and $50 \%$ in the three assets satisfy the constraints.

## See Also

Portfolio|pcalims | pcgcomp| pcglims | portcons | portopt

## Topics

"Portfolio Construction Examples" on page 3-7
"Portfolio Selection and Risk Aversion" on page 3-9
"Active Returns and Tracking Error Efficient Frontier" on page 3-43
"Analyzing Portfolios" on page 3-2
"Portfolio Optimization Functions" on page 3-4

## Introduced before R2006a

## peravg

Periodic average of FINTS object

Note peravg will be removed in a future release. Use timetable instead. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

```
avgfts = peravg(tsobj)
avgfts = peravg(tsobj,numperiod)
avgfts = peravg(tsobj,daterange)
```


## Arguments

| tsobj | Financial time series object |
| :--- | :--- |
| numperiod | (Optional) Integer specifying the number of data points over which <br> each periodic average should be averaged |
| daterange | (Optional) Time period over which the data is averaged |

## Description

peravg calculates periodic averages of a financial time series object. Periodic averages are calculated from the values per period defined. If the period supplied is a character vector, it is assumed as a range of date character vector. If the period is entered as numeric, the number represents the number of data points (financial time series periods) to be included in a period for the calculation. For example, if you enter '01/01/98: :01/01/99' as the period input argument, peravg returns the average of the time series between those dates, inclusive. However, if you enter the number 5 as the period input, peravg returns a series of averages from the time series data taken 5 date points (financial time series periods) at a time.
avgfts $=$ peravg(tsobj, numperiod) returns a structure avgfts that contains the periodic (per numperiod periods) average of the financial time series object. avgfts has field names identical to the data series names of tsobj.
avgfts = peravg(tsobj, daterange) returns a structure avgfts that contains the periodic (as specified by daterange) average of the financial time series object. avgfts has field names identical to the data series names of tsobj.

Note peravg calculates periodic averages of a FINTS object. Periodic averages are calculated from the values per period defined. If the period supplied is a character vector, it is assumed as a range of date character vectors. If the period is entered as numeric, the number represents the number of data points to be included in a period for the calculation.

## Examples

If you enter 01-Jan-2001: :03-Jan-2001 as the period input argument, peravg returns the average of the time series between those dates, inclusive. However, if you enter the number 5 as the period input, peravg returns a series of averages from the time series data, taken 5 date points at a time.

```
%% Create the FINTS object %%
dates = ['01-Jan-2001';'01-Jan-2001'; '02-Jan-2001'; ...
'02-Jan-2001'; '03-Jan-2001';'03-Jan-2001'];
times = ['11:00';'12:00';'11:00';'12:00';'11:00';'12:00'];
dates_times = cellstr([dates, repmat(' ', size(dates, 1), 1), times]);
data = [(1:6)', 2*(1:6)'];
myFts = fints(dates_times, data, {'Data1', 'Data2'}, 1, 'My first FINTS')
%% Create the FINTS object %%
[p, pFts] = peravg(myFts, 3)
```

Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints (line 165)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/display (line 66)
myFts =
desc: My first FINTS
freq: Daily (1)
'dates: (6)' 'times: (6)' 'Data1: (6)' 'Data2: (6)'

| '01-Jan-2001' | '11:00' | [ | 1] | [ | 2] |
| :---: | :---: | :---: | :---: | :---: | :---: |
| ' " ' | '12:00' | [ | 2] | [ | 4] |
| '02-Jan-2001' | '11:00' | [ | $3]$ | [ | $6]$ |
|  | '12:00' | [ | 4] | [ | 8] |
| '03-Jan-2001' | '11:00' | [ | 5] | [ | 10] |
|  | '12:00' |  | $6]$ | [ | $12]$ |

Warning: FINTS will be removed in a future release. Use TIMETABLE instead. > In fints/peravg (line 85)
$p=$
struct with fields:
Datal: [2 5]
Data2: [4 10]
Warning: FINTS will be removed in a future release. Use TIMETABLE instead. > In fints/display (line 66)
pFts =
desc: My first FINTS
freq: Daily (1)
'dates: (2)' 'times: (2)' 'Data1: (2)' 'Data2: (2)'
'02-Jan-2001' '11:00' [ 2] [ $\quad$ [
'03-Jan-2001' '12:00' [ 5] [ 10]
[p, pFts] = peravg(myFts,'01-Jan-2001 12:00::03-Jan-2001 11:00')
Warning: FINTS will be removed in a future release. Use TIMETABLE instead. > In fints/peravg (line 85)
$p=$
struct with fields:
Datal: 3.5000
Data2: 7
Warning: FINTS will be removed in a future release. Use TIMETABLE instead. > In fints/display (line 66)
pFts =

```
desc: My first FINTS
freq: Daily (1)
\begin{tabular}{|c|c|c|c|}
\hline (1) ' & 'times: (1)' & 'Datal: (1)' & 'Data2: (1)' \\
\hline 03-Jan-2001' & '11:00' & 3.5000] & [ 7] \\
\hline
\end{tabular}
```


## See Also

mean |tsmovavg

## Topics

"Data Transformation and Frequency Conversion" on page 13-13

## Introduced before R2006a

## periodicreturns

Periodic total returns from total return prices

## Syntax

TotalReturn = periodicreturns(TotalReturnPrices)
TotalReturn = periodicreturns(TotalReturnPrices,Period)

## Arguments

| TotalReturnPrices | TotalReturnPrices can be the number of observations <br> (NUMOBS) by number of assets (NASSETS + 1) matrix of <br> total return prices for a given security. Column 1 contains <br> MATLAB serial date numbers. The remaining columns <br> contain total return price data. In addition, <br> TotalReturnPrices can also be a table where the first <br> column of the table represents the dates (as either serial <br> date numbers, date character vectors, or datetime arrays) <br> while the other columns represent the returns data. If a <br> table is used, TotalReturn is returned as a table. |
| :--- | :--- |
| Period | Optional) Periodicity flag used to compute total returns: <br> 'd = daily values (default) <br> ' $w$ ' = weekly values <br> ' $m$ ' = monthly values <br> $n=$ rolling return periodic values, where $n$ is an integer |

## Description

TotalReturn = periodicreturns(TotalReturnPrices) calculates the daily total returns from a daily total return price series.

TotalReturn = periodicreturns(TotalReturnPrices,Period) calculates the total returns for a periodicity that you specify from a daily total return price series.

If TotalReturnPrices input is a matrix, TotalReturn is a NUMOBS-by-NASSETS + 1 matrix containing month-end dates and return values. Each row represents an observation. Column 1 contains month-end dates in MATLAB serial date number format. The remaining columns contain monthly return values.

Note Although input returns can have dates in either ascending or descending order, output total returns in TotalReturn have dates in ascending order, with the earliest date in the first row TotalReturn, and the most recent date in the last row of TotalReturn.

If TotalReturnPrices input is a table where the first column of the table represents the dates (as either serial date numbers, date character vectors, or datetime arrays) while the other columns represent the returns data, TotalReturn is returned as a table.

## Examples

## Compute TotalReturn Using datetime Input for TotalReturnPrices

Compute TotalReturn returned as a table using datetime input in a table for TotalReturnPrices.

```
Dates = datetime(2015,1,1:10,'Locale','en US')';
Prices = [0.01 0.03 0.1 -0.05 0.02 0.07-0.03 -0.01 -0.02 0.01]';
TotalReturnPrices = table(Dates,Prices);
TotalReturn = periodicreturns(TotalReturnPrices)
TotalReturn=9\times2 table
    Dates Prices
    02-Jan-2015 2
    03-Jan-2015 2.3333
    04-Jan-2015 -1.5
    05-Jan-2015 -1.4
    06-Jan-2015 2.5
    07-Jan-2015 -0.57143
    08-Jan-2015 -1.3333
    09-Jan-2015 1
    10-Jan-2015 -1.5
```


## See Also

totalreturnprice

## Topics

"Portfolio Construction Examples" on page 3-7
"Portfolio Optimization Functions" on page 3-4

## Introduced before R2006a

## plot

Plot data series

Note plot has been partially removed and will no longer accept a fints object (tsobj). Use timetable or table instead for financial time series.

Use fts2timetable to convert a fints object to a timetable object.

## Syntax

```
plot(tsobj)
hp = plot(tsobj)
plot(tsobj,linefmt)
hp = plot(tsobj,linefmt)
plot(..., volumename,bar)
hp = plot(..., volumename,bar)
```


## Arguments

| tsobj | Financial time series object. |
| :--- | :--- |
| linefmt | (Optional) Line format. |
| volumename | (Optional) Specifies which data series is the volume series. <br> volumename must be the exact data series name for the volume <br> column (case sensitive). |
| bar | (Optional) <br> - bar $=0-$ (Default) Plot volume as a line. <br> - bar $=1-$ Plot volume as a bar chart. The width of each bar is <br> the same as the default in bar, barh. |

## Description

plot (tsobj) plots the data series contained in the object tsobj. Each data series is a line. plot automatically generates a legend and dates on the $x$-axis. Grid is turned on by default. plot uses the default color order as if plotting a matrix.

The plot command automatically creates subplots when multiple time series are encountered, and they differ greatly on their decimal scales. For example, subplots are generated if one time series data set is in the 10 s and another is in the $10,000 \mathrm{~s}$.
$\mathrm{hp}=\mathrm{plot}(\mathrm{tsobj})$ also returns the handle(s) to the object(s) inside the plot figure. If there are multiple lines in the plot, hp is a vector of multiple handles.
plot(tsobj, linefmt) plots the data series in tsobj using the line format specified. For a list of possible line formats, see plot. The plot legend is not generated, but the dates on the $x$-axis and the plot grid are. The specified line format is applied to all data series; that is, all data series have the same line type.
$h p=p l o t(t s o b j, l i n e f m t)$ plots the data series in tsobj using the format specified. The plot legend is not generated, but the dates on the $x$-axis and the plot grid are. The specified line format is applied to all data series, that is, all data series can have the same line type. If there are multiple lines in the plot, hp is a vector of multiple handles.
plot(..., volumename,bar) also specifies which data series is the volume. The volume is plotted in a subplot below the other data series. If bar $=1$, the volume is plotted as a bar chart. Otherwise, a line plot is used.
$\mathrm{hp}=\mathrm{plot}(\ldots$, volumename, bar) returns handles for each line. If bar $=1$, the handle to the patch for the bars is also returned.

Note To turn off the legend, enter legend off at the MATLAB command line. Once you turn it off, the legend is deleted. To turn it back on, recreate it using the legend command as if you are creating it for the first time. To turn off the grid, enter grid off. To turn it back on, enter grid on.

## See Also

candle|chartfts|grid|highlow|legend|plot

## Introduced before R2006a

## plotFrontier

Plot efficient frontier

## Syntax

```
[prsk,pret] = plotFrontier(obj)
[prsk,pret] = plotFrontier(obj,NumPorts)
[prsk,pret] = plotFrontier(obj,PortWeights)
[prsk,pret] = plotFrontier(obj,PortRisk,PortReturn)
```


## Description

[prsk, pret] = plotFrontier(obj) estimates the efficient frontier with default number of 10 portfolios on the frontier, and plots the corresponding efficient frontier for Portfolio, PortfolioCVaR, or PortfolioMAD objects. For details on the respective workflows when using these different objects, see "Portfolio Object Workflow" on page 421, "PortfolioCVaR Object Workflow" on page 5-20, and "PortfolioMAD Object Workflow" on page 6-19.
[prsk, pret] = plotFrontier(obj,NumPorts) estimates the efficient frontier with a specified number portfolios on the frontier, and plots the corresponding efficient frontier. The number of portfolios is defined by NumPorts.
[prsk,pret] = plotFrontier(obj,PortWeights) estimates efficient portfolio risks and returns with PortWeights, and plots the efficient frontier with those portfolios. This syntax assumes that you provide valid efficient portfolio weights as input. PortWeights is a NumAsset-by-NumPorts matrix.
[prsk,pret] = plotFrontier(obj,PortRisk,PortReturn) plots the efficient frontier with the given risks and returns. This syntax assumes that you provide valid inputs for efficient portfolio risks and returns. PortRisk and PortReturn are vectors with the same size.

Note plotFrontier handles multiple input formats as described above. Given an asset universe with NumAssets assets and an efficient frontier with NumPorts portfolios,
remember that portfolio weights are NumAsset-by-NumPorts matrices and that portfolio risks and returns are NumPorts column vectors.

## Examples

## Plot the Efficient Frontier for the Portfolio Object

Given a portfolio p, plot the efficient frontier.
load CAPMuniverse
p = Portfolio('AssetList',Assets(1:12));
p = estimateAssetMoments(p, Data(:,1:12),'missingdata',true);
p = setDefaultConstraints(p);
plotFrontier(p);


## Plot the Efficient Frontier for the Portfolio Object with BoundType and MaxNumAssets Constraints

Create a Portfolio object for 12 stocks based on CAPMuniverse.mat.
load CAPMuniverse
p0 = Portfolio('AssetList',Assets(1:12));
p0 = estimateAssetMoments(p0, Data(:, 1:12),'missingdata',true);
$\mathrm{p} 0=$ setDefaultConstraints(p0);
Use setMinMaxNumAssets to define a maximum number of 3 assets.
pWithMaxNumAssets = setMinMaxNumAssets(p0, [], 3);
Use setBounds to define a lower and upper bound and a BoundType of 'Conditional'.
pWithConditionalBound $=$ setBounds(p0, 0.1, 0.5,'BoundType', 'Conditional');
Use plotFrontier to compare the different portfolio objects.
figure;
plotFrontier(p0); hold on;
plotFrontier(pWithMaxNumAssets); hold on;
plotFrontier(pWithConditionalBound); hold off;
legend('p0', 'with Max 3 assets invested', ' with each asset weight 0 or [0.1, 0.5]',


Define a target return and use estimateFrontierByReturn to compare the three portfolio objects.

```
targetRetn = 2.0e-3;
pwgt0 = estimateFrontierByReturn(p0, targetRetn);
pwgtWithMaxNumAssets = estimateFrontierByReturn(pWithMaxNumAssets, targetRetn);
pwgtConditionalBound = estimateFrontierByReturn(pWithConditionalBound, targetRetn);
```

The following table shows the final allocation for specified target return among the three portfolio objects. You can see that the small positions in 'AAPL' and 'HPQ' are avoided in pwgtConditionalBound, and only three assets are invested in pwgtWithMaxNumAssets.

```
result = table(p0.AssetList',pwgt0,pwgtWithMaxNumAssets,pwgtConditionalBound)
result=12\times4 table
    Var1 pwgt0 pwgtWithMaxNumAssets pwgtConditionalBound
    'AAPL' 0.076791
    'AMZN' 0
    'CSCO'
    'DEL'' 
    'DELL' (1)
    'GOOG' 0.44841 0.47297
    'HPQ' 0.022406 0
        0.34763
    lrrr' 
    lMS'' 
    'YHOO' 0
                                    0.10002
    0
                                0
0 0.44254
    'IBM' 0.31139
                0
                                    - 
    ~
    MAPL
\begin{tabular}{rr}
0 & 0.10002 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
7 & 0.44254 \\
0 & 0 \\
63 & 0.31591 \\
0 & 0 \\
4 & 0.14153 \\
0 & 0 \\
0 & 0
\end{tabular}
```


## Plot the Efficient Frontier for the PortfolioCVaR Object

Given a PortfolioCVaR p, plot the efficient frontier.

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
```

$\mathrm{m}=\mathrm{m} / 12$;
$C=C / 12$;
AssetScenarios = mvnrnd(m, C, 20000);
p = PortfolioCVaR;
p = setScenarios(p, AssetScenarios);
p = setDefaultConstraints(p);
$p=\operatorname{set}$ ProbabilityLevel(p, 0.95);
plotFrontier(p);


## Plot Efficient Frontier for PortfolioMAD Object

Given a PortfolioMAD p, plot the efficient frontier.

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;
AssetScenarios = mvnrnd(m, C, 20000);
p = PortfolioMAD;
p = setScenarios(p, AssetScenarios);
p = setDefaultConstraints(p);
plotFrontier(p);
```



## Input Arguments

## obj - Object for portfolio

object
Object for portfolio, specified using Portfolio, PortfolioCVaR, or PortfolioMAD object. For more information on creating a portfolio object, see

- Portfolio
- PortfolioCVaR
- PortfolioMAD

Data Types: object

## NumPorts - Number of points to obtain on efficient frontier

value from hidden property defaultNumPorts (default value is 10) (default) | scalar integer

Number of points to obtain on the efficient frontier, specified as a scalar integer.

Note If no value is specified for NumPorts, the default value is obtained from the hidden property defaultNumPorts (default value is 10). If NumPorts $=1$, this function returns the portfolio specified by the hidden property defaultFrontierLimit (current default value is 'min').

## Data Types: double

## PortRisk - Standard deviations of portfolio returns for each portfolio

[] (default) | vector
Standard deviations of portfolio returns for each portfolio, specified as a vector.

Note PortRisk and PortReturn must be vectors with the same size.

## Data Types: double

PortReturn - Means of portfolio returns for each portfolio [] (default) | vector

Means of portfolio returns for each portfolio, specified as a vector.

Note PortRisk and PortReturn must be vectors with the same size.

## Data Types: double

## PortWeights - Optimal portfolios on the efficient frontier

[^10]Optimal portfolios on the efficient frontier, specified as a NumAsset-by-NumPorts matrix.

## Data Types: double

## Output Arguments

## prsk - Estimated efficient portfolio risks (standard deviation of returns

vector
Estimated efficient portfolio risks (standard deviation of returns, returned as a vector for a Portfolio, PortfolioCVaR, or PortfolioMAD input object (obj).

## Note

- If the portfolio object has a name in the Name property, the name is displayed as the title of the plot. Otherwise, the plot is labeled "Efficient Frontier."
- If the portfolio object has an initial portfolio in the InitPort property, the initial portfolio is plotted and labeled.
- If portfolio risks and returns are inputs, make sure that risks come first in the calling sequence. In addition, if portfolio risks and returns are not sorted in ascending order, this method performs the sort. On output, the sorted moments are returned.


## pret - Estimated efficient portfolio returns

vector
Estimated efficient portfolio returns, returned as a vector for a Portfolio, PortfolioCVaR, or PortfolioMAD input object (obj).

## Note

- If the portfolio object has a name in the Name property, the name is displayed as the title of the plot. Otherwise, the plot is labeled "Efficient Frontier."
- If the portfolio object has an initial portfolio in the InitPort property, the initial portfolio is plotted and labeled.
- If portfolio risks and returns are inputs, make sure that risks come first in the calling sequence. In addition, if portfolio risks and returns are not sorted in ascending order, this method performs the sort. On output, the sorted moments are returned.


## Tips

You can also use dot notation to plot the efficient frontier.
[prsk, pret] = obj.plotFrontier;

## See Also

estimateFrontier|estimateFrontierByReturn|estimateFrontierByRisk| estimateFrontierLimits

## Topics

"Plotting the Efficient Frontier for a Portfolio Object" on page 4-143
"Plotting the Efficient Frontier for a PortfolioCVaR Object" on page 5-122
"Plotting the Efficient Frontier for a PortfolioMAD Object" on page 6-115
"Portfolio Optimization Examples" on page 4-167

## Introduced in R2011a

## plus

Financial time series addition

Note plus will be removed in a future release. Use timetable instead. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

```
newfts = tsobj_1 + tsobj_2
newfts = tsobj + array
newfts = array + tsobj
```


## Arguments

| tsobj_1, tsobj_2 | A pair of financial time series objects. |
| :--- | :--- |
| array | A scalar value or array with the number of rows equal to the <br> number of dates in tsobj and the number of columns equal <br> to the number of data series in tsobj. |

## Description

plus is an element-by-element addition of the components.
newfts $=$ tsobj_1 + tsobj_2 adds financial time series objects. If an object is to be added to another object, both objects must have the same dates and data series names, although the order need not be the same. The order of the data series, when one financial time series object is added to another, follows the order of the first object.
newfts = tsobj + array adds an array element by element to a financial time series object.
newfts $=$ array + tsobj adds a financial time series object element by element to an array.

See Also<br>minus|rdivide|times<br>\section*{Topics}<br>"Financial Time Series Operations" on page 13-7<br>Introduced before R2006a

## pointfig

## Point and figure chart

Note pointfig is updated to accept data input as a matrix, timetable, or table.

## Syntax

pointfig(Data)
h = pointfig(ax,Data)

## Description

pointfig(Data) plots a point and figure chart from a series of prices of a security. Upward price movements are plotted as X's and downward price movements are plotted as 0 's.
$\mathrm{h}=$ pointfig(ax,Data) adds an optional argument for ax .

## Examples

## Generate a Point and Figure Chart for a Data Series for a Stock

Load the file SimulatedStock.mat, which provides a timetable (TMW) for financial data for TMW stock. This Point and Figure chart is for closing prices of the stock TMW for the most recent 21 days. Note that the variable name of asset price is be renamed to 'Price' (case insensitive).
load SimulatedStock.mat
TMW.Properties.VariableNames\{'Close'\} = 'Price'; pointfig(TMW(1:200,:))
title('Point and figure chart for TMW')


## Input Arguments

## Data - Data for a series of prices

matrix | table | timetable
Data for a series of prices, specified as a matrix, table, or timetable. Timetables and tables with M rows must contain a variable named 'Price' (case insensitive).

Data Types: double| table|timetable

## ax - Valid axis object

current axes (ax = gca) (default) | axes object
(Optional) Valid axis object, specified as an axes object. The point and figure plot is created in the axes specified by ax instead of in the current axes ( $a x=g c a$ ). The option ax can precede any of the input argument combinations.

## Data Types: object

## Output Arguments

## h - Graphic handle of the figure <br> handle object

Graphic handle of the figure, returned as a handle object.

## See Also <br> candle| highlow|kagi|linebreak|movavg|priceandvol|renko|table| timetable|volarea

## Topics

"Using Timetables in Finance"
"Convert Financial Time Series Objects fints to Timetables" on page 12-2

## Introduced in R2006a

## portalloc

Optimal capital allocation to efficient frontier portfolios

## Syntax

[RiskyRisk,RiskyReturn,RiskyWts,RiskyFraction,OverallRisk,OverallReturn] = portalloc(P)

## Arguments

| PortRisk | Standard deviation of each risky asset efficient frontier <br> portfolio. A number of portfolios (NPORTS-by-1 vector). |
| :--- | :--- |
| PortReturn | Expected return of each risky asset efficient frontier <br> portfolio. An NPORTS-by-1 vector. |
| PortWts | Weights allocated to each asset. An NPORTS by number of <br> assets (NASSETS) matrix of weights allocated to each asset. <br> Each row represents an efficient frontier portfolio of risky <br> assets. Total of all weights in a portfolio is 1. |
| RisklessRate | Risk-free lending rate. A decimal number. |
| BorrowRate | (Optional) Borrowing rate. A decimal number. If borrowing is <br> not desired, or not an option, set to NaN (default). |
| RiskAversion | (Optional) Coefficient of investor's degree of risk aversion. <br> Higher numbers indicate greater risk aversion. Typical <br> coefficients range from 2.0 through 4.0 (Default = 3). |
|  | Note Consider that a less risk-averse investor would be <br> expected to accept much greater risk and, consequently, a <br> more risk-averse investor would accept less risk for a given <br> level of return. Therefore, making the <br> RiskAversionargument higher reflects the risk-return <br> tradeoff in the data. |

## Description

[RiskyRisk,RiskyReturn,RiskyWts,RiskyFraction,OverallRisk,OverallRet urn] = portalloc(PortRisk,PortReturn,PortWts, RisklessRate,BorrowRate,RiskAversion) computes the optimal risky portfolio, and the optimal allocation of funds between the risky portfolio and the risk-free asset.

RiskyRisk is the standard deviation of the optimal risky portfolio.
RiskyReturn is the expected return of the optimal risky portfolio.
RiskyWts is a 1-by-NASSETS vector of weights allocated to the optimal risky portfolio. The total of all weights in the portfolio is 1 .

RiskyFraction is the fraction of the complete portfolio allocated to the risky portfolio.
OverallRisk is the standard deviation of the optimal overall portfolio.
OverallReturn is the expected rate of return of the optimal overall portfolio.
portalloc generates a plot of the optimal capital allocation if you invoke it without output arguments.

## Examples

## Compute the Optimal Risky Portfolio

This example shows how to compute the optimal risky portfolio by generating the efficient frontier from the asset data and then finding the optimal risky portfolio and allocate capital. The risk-free investment return is $8 \%$, and the borrowing rate is $12 \%$.

ExpReturn = [0.1 0.2 0.15];
ExpCovariance $\left.=\left[\begin{array}{rrr}0.005 & -0.010 & 0.004 \\ & -0.010 & 0.040 \\ & 0.004 & -0.002\end{array}\right) 0.023\right] ;$
[PortRisk, PortReturn, PortWts] = portopt(ExpReturn,... ExpCovariance);

RisklessRate = 0.08;

```
BorrowRate = 0.12;
RiskAversion = 3;
[RiskyRisk, RiskyReturn, RiskyWts, RiskyFraction, ...
OverallRisk, OverallReturn] = portalloc(PortRisk, PortReturn,...
PortWts, RisklessRate, BorrowRate, RiskAversion)
RiskyRisk = 0.1283
RiskyReturn = 0.1788
RiskyWts = 1×3
    0.0265 0.6023 0.3712
RiskyFraction = 1.1898
OverallRisk = 0.1527
OverallReturn = 0.1899
```


## References

Bodie, Kane, and Marcus. Investments. Second Edition. Chapters 6 and 7.

## See Also

portrand|portstats

## Topics

"Portfolio Construction Examples" on page 3-7
"Portfolio Optimization Functions" on page 3-4

Introduced before R2006a

## portalpha

Compute risk-adjusted alphas and returns for one or more assets

## Syntax

```
portalpha(Asset,Benchmark)
portalpha(Asset,Benchmark,Cash)
portalpha(Asset,Benchmark,Cash,Choice)
Alpha = portalpha(Asset,Benchmark,Cash,Choice)
[Alpha,RAReturn] = portalpha(Asset,Benchmark,Cash,Choice)
```


## Arguments

| Asset | NUMSAMPLES x NUMSERIES matrix with NUMSAMPLES <br> observations of asset returns for NUMSERIES asset return <br> series. |
| :--- | :--- |
| Benchmark | NUMSAMPLES vector of returns for a benchmark asset. The <br> periodicity must be the same as the periodicity of Asset. For <br> example, if Asset is monthly data, then Benchmark should <br> be monthly returns. |
| Cash | (Optional) Either a scalar return for a riskless asset or a <br> vector of asset returns to be a proxy for a "riskless" asset. In <br> either case, the periodicity must be the same as the <br> periodicity of Asset. For example, if Asset is monthly data, <br> then Cash must be monthly returns. If no value is supplied, <br> the default value for Cash returns is 0. |


| Choice | (Optional) A character vector, or cell array of character vectors to indicate one or more measures to be computed from among various risk-adjusted alphas and return measures. The number of choices selected in Choice is NUMCHOICES. The list of choices is given in the following table: |  |
| :---: | :---: | :---: |
|  | Code | Description |
|  | 'xs ' | Excess Return (no risk adjustment) |
|  | 'sml' | Security Market Line: The security market line shows that the relationship between risk and return is linear for individual securities (that is, increased risk $=$ increased return). |
|  | ' capm' | Jensen's Alpha: Risk-adjusted performance measure that represents the average return on a portfolio or investment, above or below that predicted by the capital asset pricing model (CAPM), given the portfolio's or investment's beta and the average market return. |
|  | 'mm' | Modigliani \& Modigliani: Measures the returns of an investment portfolio for the amount of risk taken relative to some benchmark portfolio. |
|  | 'gh1' | Graham-Harvey 1: Performance measure developed by John Graham and Campbell Harvey. The idea is to lever a fund's portfolio to exactly match the volatility of the $S \& P 500$. The difference between the fund's levered return and the S\&P 500 return is the performance measure. |
|  | 'gh2' | Graham-Harvey 2: In this measure, the idea is to lever up or down the fund's recommended investment strategy (using |

19-1322


## Description

Given NUMSERIES assets with NUMSAMPLES returns in a NUMSAMPLES-by-NUMSERIES matrix Asset, a NUMSAMPLES vector of Benchmark returns, and either a scalar Cash return or a NUMSAMPLES vector of Cash returns, compute risk-adjusted alphas and returns for one or more methods specified by Choice.

To summarize the outputs of portalpha:

- Alpha is a NUMCHOICES-by-NUMSERIES matrix of risk-adjusted alphas for each series in Asset with each row corresponding to a specified measure in Choice.
- RAReturn is a NUMCHOICES-by-NUMSERIES matrix of risk-adjusted returns for each series in Asset with each row corresponding to a specified measure in Choice.

Note NaN values in the data are ignored and, if NaNs are present, some results could be unpredictable. Although the alphas are comparable across measures, risk-adjusted returns depend on whether the Asset or Benchmark is levered or unlevered to match its risk with the alternative. If Choice = 'all', the order of rows in Alpha and RAReturn follows the order in the table. In addition, Choice = 'all' overrides all other choices.

## Examples

See "Risk-Adjusted Return" on page 7-13.

## References

John Lintner. "The Valuation of Risk Assets and the Selection of Risky Investments in Stocks Portfolios and Capital Budgets." Review of Economics and Statistics. Vol. 47, No. 1, February 1965, pp. 13-37.

John R. Graham and Campbell R. Harvey. "Market Timing Ability and Volatility Implied in Investment Newsletters' Asset Allocation Recommendations." Journal of Financial Economics. Vol. 42, 1996, pp. 397-421.

Franco Modigliani and Leah Modigliani. "Risk-Adjusted Performance: How to Measure It and Why." Journal of Portfolio Management. Vol. 23, No. 2, Winter 1997, pp. 45-54.

Jan Mossin. "Equilibrium in a Capital Asset Market." Econometrica. Vol. 34, No. 4, October 1966, pp. 768-783.

William F. Sharpe. "Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk." Journal of Finance. Vol. 19, No. 3, September 1964, pp. 425-442.

## See Also

inforatio|sharpe

## Topics

"Performance Metrics Illustration" on page 7-4
"Performance Metrics Overview" on page 7-2
Introduced in R2006b

## portcons

Portfolio constraints
As an alternative to portcons, use the Portfolio object (Portfolio) for mean-variance portfolio optimization. This object supports gross or net portfolio returns as the return proxy, the variance of portfolio returns as the risk proxy, and a portfolio set that is any combination of the specified constraints to form a portfolio set. For information on the workflow when using Portfolio objects, see "Portfolio Object Workflow" on page 4-21.

## Syntax

ConSet $=$ portcons(varargin)

## Description

Using linear inequalities, portcons generates a matrix of constraints for a portfolio of asset investments. The matrix ConSet is defined as ConSet $=$ [A b]. A is a matrix and b a vector such that A*PortWts ' <= b sets the value, where PortWts is a 1-by-number of assets (NASSETS) vector of asset allocations.

ConSet = portcons('ConstType',Datal, ..., DataN) creates a matrix ConSet, based on the constraint type ConstType, and the constraint parameters Datal, ..., DataN.

ConSet = portcons('ConstType1',Data11, ..., Data21, ...,
Data2N, ...) creates a matrix ConSet, based on the constraint types ConstTypeN, and the corresponding constraint parameters DataN1, ..., DataNN.

| Constraint Type | Description | Values |
| :--- | :--- | :--- |
| Default | All allocations are <br> $>=0 ;$ no short selling <br> allowed. Combined <br> value of portfolio <br> allocations <br> normalized to 1. | NumAssets (required). Scalar representing number <br> of assets in portfolio. |

\(\left.$$
\begin{array}{|l|l|l|}\hline \text { Constraint Type } & \text { Description } & \text { Values } \\
\hline \text { PortValue } & \begin{array}{l}\text { Fix total value of } \\
\text { portfolio to PVal. }\end{array} & \begin{array}{l}\text { PVal (required). Scalar representing total value of } \\
\text { portfolio. } \\
\text { NumAssets (required). Scalar representing } \\
\text { number of assets in portfolio. See pcpval. }\end{array} \\
\hline \text { AssetLims } & \begin{array}{l}\text { Minimum and } \\
\text { maximum allocation } \\
\text { per asset. }\end{array} & \begin{array}{l}\text { AssetMin (required). Scalar or vector of length } \\
\text { NASSETS, specifying minimum allocation per asset. }\end{array} \\
\hline \text { GroupLims } & \begin{array}{l}\text { MinsetMax (required). Scalar or vector of length } \\
\text { maximum allocations } \\
\text { to asset group. }\end{array} & \begin{array}{l}\text { NASSETS, specifying maximum allocation per asset. } \\
\text { NumAssets (optional). See pcalims. }\end{array}
$$ <br>
specifying which assets belong to each group. <br>
GroupMin (required). Scalar or a vector of length <br>
NGROUPS, specifying minimum combined allocations <br>
in each group. <br>

GroupMax (required). Scalar or a vector of length\end{array}\right\}\)| NGROUPS, specifying maximum combined |
| :--- |
| allocations in each group. |
| See pcglims. |

\(\left.$$
\begin{array}{|l|l|l|}\hline \text { Constraint Type } & \text { Description } & \text { Values } \\
\hline \text { GroupComparison } & \begin{array}{l}\text { Group-to-group } \\
\text { comparison } \\
\text { constraints. }\end{array} & \begin{array}{l}\text { GroupA (required). NGROUPS-by-NASSETS matrix } \\
\text { specifying first group in the comparison. }\end{array} \\
& \begin{array}{ll}\text { AtoBmin (required). Scalar or vector of length } \\
\text { NGROUPS specifying minimum ratios of allocations } \\
\text { in GroupA to allocations in GroupB. } \\
\text { AtoBmax (required). Scalar or vector of length } \\
\text { NGROUPS specifying maximum ratios of allocations } \\
\text { in GroupA to allocations in GroupB. } \\
\text { GroupB (required). NGROUPS-by-NASSETS matrix }\end{array}
$$ <br>

specifying second group in the comparison.\end{array}\right\}\)| See pcgcomp. |
| :--- |

## Examples

Constrain a portfolio of three assets:

| Asset | IBM | HPQ | XOM |
| :--- | :--- | :--- | :--- |
| Group | A | A | B |
| Minimum Weight | 0 | 0 | 0 |
| Maximum Weight | 0.5 | 0.9 | 0.8 |

NumAssets = 3;
PVal = 1; \% Scale portfolio value to 1.
AssetMin $=0$;

```
AssetMax = [0.5 0.9 0.8];
GroupA = [ll 1 0];
GroupB = [0 0 1];
AtoBmax = 1.5 % Value of assets in Group A at most 1.5 times value
    % in group B.
ConSet = portcons('PortValue', PVal, NumAssets,'AssetLims',...
AssetMin, AssetMax, NumAssets, 'GroupComparison',GroupA, NaN,...
AtoBmax, GroupB)
ConSet =
\begin{tabular}{rrrr}
1.0000 & 1.0000 & 1.0000 & 1.0000 \\
-1.0000 & -1.0000 & -1.0000 & -1.0000 \\
1.0000 & 0 & 0 & 0.5000 \\
0 & 1.0000 & 0 & 0.9000 \\
0 & 0 & 1.0000 & 0.8000 \\
-1.0000 & 0 & 0 & 0 \\
0 & -1.0000 & 0 & 0 \\
0 & 0 & -1.0000 & 0 \\
1.0000 & 1.0000 & -1.5000 & 0
\end{tabular}
```

For instance, one possible solution for portfolio weights that satisfy the constraints is $30 \%$ in IBM, $30 \%$ in HPQ, and $40 \%$ in XOM.

## See Also

Portfolio|pcalims|pcgcomp|pcglims | pcpval|portopt

## Topics

"Portfolio Construction Examples" on page 3-7
"Portfolio Selection and Risk Aversion" on page 3-9
"Active Returns and Tracking Error Efficient Frontier" on page 3-43
"Analyzing Portfolios" on page 3-2
"Portfolio Optimization Functions" on page 3-4

## Introduced before R2006a

## Portfolio

Create Portfolio object for mean-variance portfolio optimization and analysis

## Description

Use the Portfolio function to create a Portfolio object for mean-variance portfolio optimization.

The main workflow for portfolio optimization is to create an instance of a Portfolio object that completely specifies a portfolio optimization problem and to operate on the Portfolio object using supported functions to obtain and analyze efficient portfolios. For details on this workflow, see "Portfolio Object Workflow" on page 4-21.

You can use the Portfolio object in several ways. To set up a portfolio optimization problem in a Portfolio object, the simplest syntax is:

```
p = Portfolio;
```

This syntax creates a Portfolio object, p , such that all object properties are empty.
The Portfolio object also accepts collections of name-value pair arguments for properties and their values. The Portfolio object accepts inputs for properties with the general syntax:

```
p = Portfolio('property1',value1,'property2',value2, ... );
```

If a Portfolio object exists, the syntax permits the first (and only the first argument) of the Portfolio object to be an existing object with subsequent name-value pair arguments for properties to be added or modified. For example, given an existing Portfolio object in $p$, the general syntax is:
p = PortfolioCVaR(p,'property1',value1,'property2', value2, ... );
Input argument names are not case-sensitive, but must be completely specified. In addition, several properties can be specified with alternative argument names (see "Shortcuts for Property Names" on page 19-1349). The Portfolio object tries to detect problem dimensions from the inputs and, once set, subsequent inputs can undergo various scalar or matrix expansion operations that simplify the overall process to
formulate a problem. In addition, a Portfolio object is a value object so that, given portfolio $p$, the following code creates two objects, $p$ and $q$, that are distinct:
$\mathrm{q}=$ Portfolio(p, ...)
After creating a Portfolio object, you can use the associated object functions to set portfolio constraints, analyze the efficient frontier, and validate the portfolio model.

For more detailed information on the theoretical basis for mean-variance optimization, see "Portfolio Optimization Theory" on page 4-3.

## Creation

## Syntax

p = Portfolio
p = Portfolio(Name, Value)
p = Portfolio(p,Name, Value)

## Description

p = Portfolio creates an empty Portfolio object for mean-variance portfolio optimization and analysis. You can then add elements to the Portfolio object using the supported "add" and "set" functions. For more information, see "Creating the Portfolio Object" on page 4-28.
p = Portfolio(Name, Value) creates a Portfolio object (p) and sets Properties on page 19-1331 using name-value pairs. For example, $p=$ Portfolio('AssetList', Assets(1:12)). You can specify multiple name-value pairs.
p = Portfolio ( $p$, Name, Value) creates a Portfolio object ( $p$ ) using a previously created Portfolio object $p$ and sets Properties on page 19-1331 using name-value pairs. You can specify multiple name-value pairs.

## Input Arguments

p - Previously constructed Portfolio object
object
Previously constructed Portfolio object, specified using Portfolio.

## Properties

## Setting Up the Object

## AssetList - Names or symbols of assets in universe

[ ] (default) | cell array of character vectors | string array
Names or symbols of assets in the universe, specified as a cell array of character vectors or a string array.

Data Types: cell|string

## InitPort - Initial portfolio

[] (default) | vector
Initial portfolio, specified as a vector.

## Data Types: double

## Name - Name for instance of Portfolio object

[ ] (default) | character vector | string
Name for instance of the Portfolio object, specified as a character vector or a string.
Data Types: char|string

## NumAssets - Number of assets in the universe

[] (default) | integer scalar
Number of assets in the universe, specified as an integer scalar.
Data Types: double

## Portfolio Object Constraints

## AEquality - Linear equality constraint matrix [] (default) | matrix

Linear equality constraint matrix, specified as a matrix.
Data Types: double

## AInequality - Linear inequality constraint matrix

[ ] (default) | matrix

Linear inequality constraint matrix, specified as a matrix.
Data Types: double

## bEquality - Linear equality constraint vector

[] (default) | vector
Linear equality constraint vector, specified as a vector.

## Data Types: double

## bInequality - Linear inequality constraint [] (default) | vector

Linear inequality constraint vector, specified as a vector.

## Data Types: double

## GroupA - Group A weights to be bounded by weights in group B [ ] (default) | matrix

Group A weights to be bounded by weights in group B, specified as a matrix.
Data Types: double

## GroupB - Group B weights

## [] (default) | matrix

Group B weights, specified as a matrix.
Data Types: double

## GroupMatrix - Group membership matrix

[^11]Group membership matrix, specified as a matrix.

## Data Types: double

## LowerBound - Lower-bound constraint

[] (default) | vector
Lower-bound constraint, specified as a vector.

## Data Types: double

## LowerBudget - Lower-bound budget constraint

[] (default) | scalar
Lower-bound budget constraint, specified as a scalar.
Data Types: double
LowerGroup - Lower-bound group constraint
[] (default) | vector
Lower-bound group constraint, specified as a vector.
Data Types: double

## LowerRatio - Minimum ratio of allocations between Groups A and B <br> [] (default) | vector

Minimum ratio of allocations between GroupA and GroupB, specified as a vector.
Data Types: double
TrackingError - Upper bound for tracking error constraint
[] (default) | scalar
Upper bound for tracking error constraint, specified as a scalar.

## Data Types: double

TrackingPort - Tracking portfolio for tracking error constraint [] (default) | vector

Tracking portfolio for tracking error constraint, specified as a vector.
Data Types: double

## UpperBound - Upper-bound constraint [] (default) | vector

Upper-bound constraint, specified as a vector.

## Data Types: double

## UpperBudget - Upper-bound budget constraint

 [] (default) | scalarUpper-bound budget constraint, specified as a scalar.

## Data Types: double

## UpperGroup - Upper-bound group constraint

 [] (default) | vectorUpper-bound group constraint, specified as a vector.

## Data Types: double

## UpperRatio - Maximum ratio of allocations between Groups A and B [] (default) | vector

Maximum ratio of allocations between GroupA and GroupB, specified as a vector.

## Data Types: double

## BoundType - Type of bounds for each asset weight

'Simple' (default)| character vector with value 'Simple' or 'Conditional' | string with value "Simple" or "Conditional" | cell array of character vectors with values 'Simple' or 'Conditional' | string array with values "Simple" or "Conditional"

Type of bounds for each asset weight, specified as a scalar character vector or string, or a cell array of character vectors or a string array. For more information, see setBounds.

Data Types: char|cell|string

## MinNumAssets - Minimum number of assets allocated in portfolio

[] (default) | numeric
Minimum number of assets allocated in portfolio, specified as a scalar numeric value. For more information, see setMinMaxNumAssets.

Data Types: double

## MaxNumAssets - Maximum number of assets allocated in portfolio [] (default)| numeric

Maximum number of assets allocated in portfolio, specified as a scalar numeric value. For more information, see setMinMaxNumAssets.

Data Types: double

## SellTurnover - Turnover constraint on sales

[] (default) | scalar
Turnover constraint on sales, specified as a scalar.
Data Types: double

## BuyTurnover - Turnover constraint on purchases

[] (default) | scalar
Turnover constraint on purchases, specified as a scalar.
Data Types: double

## Turnover - Turnover constraint

[] (default) | scalar
Turnover constraint, specified as a scalar.
Data Types: double

## Portfolio Object Modeling

## AssetCovar - Covariance of asset returns

[] (default) | square matrix
Covariance of asset returns, specified as a square matrix.
Data Types: double
AssetMean - Mean of asset returns
[] (default) | vector
Mean of asset returns, specified as a vector.
Data Types: double

## BuyCost - Proportional purchase costs

[] (default) | vector
Proportional purchase costs, specified as a vector.
Data Types: double
RiskFreeRate - Risk-free rate
[] (default) | scalar
Risk-free rate, specified as a scalar.
Data Types: double
SellCost - Proportional sales costs
[] (default) | vector
Proportional sales costs, specified as a vector.
Data Types: double

## Object Functions

setAssetList
setInitPort
setDefaultConstraints
getAssetMoments
setAssetMoments
estimateAssetMoments
setCosts
addEquality
addGroupRatio
addGroups
addInequality
getBounds

Set up list of identifiers for assets
Set up initial or current portfolio
Set up portfolio constraints with nonnegative weights that sum to 1
Obtain mean and covariance of asset returns from Portfolio object
Set moments (mean and covariance) of asset returns for Portfolio object
Estimate mean and covariance of asset returns from data Set up proportional transaction costs Add linear equality constraints for portfolio weights to existing constraints Add group ratio constraints for portfolio weights to existing group ratio constraints
Add group constraints for portfolio weights to existing group constraints
Add linear inequality constraints for portfolio weights to existing constraints
Obtain bounds for portfolio weights from portfolio object

| getBudget | Obtain budget constraint bounds from portfolio object |
| :--- | :--- |
| getCosts | Obtain buy and sell transaction costs from portfolio object |
| getEquality | Obtain equality constraint arrays from portfolio object |
| getGroupRatio | Obtain group ratio constraint arrays from portfolio object |
| getGroups | Obtain group constraint arrays from portfolio object |
| getInequality | Obtain inequality constraint arrays from portfolio object |
| getOneWayTurnover | Obtain one-way turnover constraints from portfolio object |
| setGroups | Set up group constraints for portfolio weights |
| setInequality | Set up linear inequality constraints for portfolio weights |
| setBounds | Set up bounds for portfolio weights for a Portfolio object |
| setBudget | Set up budget constraints |
| setCosts | Set up proportional transaction costs |
| setDefaultConstraints | Set up portfolio constraints with nonnegative weights that |
|  | sum to 1 |
| setEquality | Set up linear equality constraints for portfolio weights |
| setGroupRatio | Set up group ratio constraints for portfolio weights |
| setInitPort | Set up initial or current portfolio |
| setOneWayTurnover | Set up one-way portfolio turnover constraints |
| setTurnover | Set up maximum portfolio turnover constraint |
| setTrackingPort | Set up benchmark portfolio for tracking error constraint |
| setTrackingError | Set up maximum portfolio tracking error constraint |
| setMinMaxNumAssets | Set cardinality constraints on the number of assets invested |
|  | in a Portfolio object |
| checkFeasibility | Check feasibility of input portfolios against portfolio object |
| estimateBounds | Estimate global lower and upper bounds for set of portfolios |
| estimateFrontier | Estimate specified number of optimal portfolios on the |
|  | efficient frontier |
| estimateFrontierByReturn | Estimate optimal portfolios with targeted portfolio returns |
| estimateFrontierByRisk | Estimate optimal portfolios with targeted portfolio risks |
| estimateFrontierLimits | Estimate optimal portfolios at endpoints of efficient frontier |
| plotFrontier | Plot efficient frontier |
| estimateMaxSharpeRatio | Estimate efficient portfolio to maximize Sharpe ratio for |
| estimatePortMoments | Portfolio object |
| estimatePortReturn | Estimate moments of portfolio returns for Portfolio object |
| estimatePortRisk | Estimate mean of portfolio returns |
| setSolver | with corresponding object |
|  | Choose main solver and specify associated solver options for |
| portfolio optimization |  |

getBudget
getCosts
getEquality
getGroupRatio
getGroups
getInequality
getOneWayTurnover
setGroups
setInequality
setBounds
setBudget
setCosts
setDefaultConstraints
setEquality
setGroupRatio
setInitPort
setOneWayTurnover
setTurnover
setTrackingPort
setTrackingError
setMinMaxNumAssets
checkFeasibility
estimateBounds
estimateFrontier
estimateFrontierByReturn
estimateFrontierByRisk
estimateFrontierLimits
plotFrontier
estimateMaxSharpeRatio
estimatePortMoments
estimatePortReturn
estimatePortRisk
setSolver

Obtain budget constraint bounds from portfolio object Obtain buy and sell transaction costs from portfolio object Obtain equality constraint arrays from portfolio object Obtain group ratio constraint arrays from portfolio object Obtain group constraint arrays from portfolio object Obtain inequality constraint arrays from portfolio object Obtain one-way turnover constraints from portfolio object Set up group constraints for portfolio weights
Set up linear inequality constraints for portfolio weights
Set up bounds for portfolio weights for a Portfolio object
Set up budget constraints
Set up proportional transaction costs
sum to 1
Set up linear equality constraints for portfolio weights
Set up group ratio constraints for portfolio weights
Set up initial or current portfolio
Set up one-way portfolio turnover constraints
Set up maximum portfolio turnover constraint
Set up benchmark portfolio for tracking error constraint
Set up maximum portfolio tracking error constraint in a Portfolio object
Check feasibility of input portfolios against portfolio object
Estimate global lower and upper bounds for set of portfolios
Estimate specified number of optimal portfolios on the efficient frontier
Estimate optimal portfolios with targeted portfolio returns Estimate optimal portfolios with targeted portfolio risks Estimate optimal portfolios at endpoints of efficient frontier Plot efficient frontier
Estimate efficient portfolio to maximize Sharpe ratio for Portfolio object
Estimate moments of portfolio returns for Portfolio object Estimate mean of portfolio returns
Estimate portfolio risk according to risk proxy associated with corresponding object
Choose main solver and specify associated solver options for portfolio optimization

setSolverMINLP

Choose mixed integer nonlinear programming (MINLP) solver for portfolio optimization

## Examples

## Create an Empty Portfolio Object

You can create a Portfolio object, p, with no input arguments and display it using disp.

```
p = Portfolio;
disp(p);
    Portfolio with properties:
```

            BuyCost: []
            SellCost: []
        RiskFreeRate: []
            AssetMean: []
            AssetCovar: []
    TrackingError: []
        TrackingPort: []
            Turnover: []
        BuyTurnover: []
        SellTurnover: []
            Name: []
            NumAssets: []
            AssetList: []
            InitPort: []
            AInequality: []
            bInequality: []
            AEquality: []
            bEquality: []
            LowerBound: []
            UpperBound: []
            LowerBudget: []
            UpperBudget: []
            GroupMatrix: []
            LowerGroup: []
            UpperGroup: []
            GroupA: []
            GroupB: []
            LowerRatio: []
    UpperRatio: []
MinNumAssets: []
MaxNumAssets: []
BoundType: []
This approach provides a way to set up a portfolio optimization problem with the Portfolio function. You can then use the associated set functions to set and modify collections of properties in the Portfolio object.

## Create a Portfolio Object Using a Single-Step Setup

You can use the Portfolio object directly to set up a "standard" portfolio optimization problem, given a mean and covariance of asset returns in the variables $m$ and $C$.

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
p = Portfolio('assetmean', m, 'assetcovar', C, ...
'lowerbudget', 1, 'upperbudget', 1, 'lowerbound', 0)
p =
    Portfolio with properties:
            BuyCost: []
            SellCost: []
        RiskFreeRate: []
            AssetMean: [4x1 double]
            AssetCovar: [4\times4 double]
        TrackingError: []
            TrackingPort: []
            Turnover: []
            BuyTurnover: []
            SellTurnover: []
                Name: []
            NumAssets: 4
            AssetList: []
            InitPort: []
            AInequality: []
            bInequality: []
```

```
        AEquality: []
        bEquality: []
        LowerBound: [4x1 double]
        UpperBound: []
    LowerBudget: 1
    UpperBudget: 1
    GroupMatrix: []
    LowerGroup: []
    UpperGroup: []
        GroupA: []
        GroupB: []
    LowerRatio: []
    UpperRatio: []
MinNumAssets: []
MaxNumAssets: []
    BoundType: []
```

Note that the LowerBound property value undergoes scalar expansion since AssetMean and AssetCovar provide the dimensions of the problem.

## Create a Portfolio Object Using a Sequence of Steps

Using a sequence of steps is an alternative way to accomplish the same task of setting up a "standard" portfolio optimization problem, given a mean and covariance of asset returns in the variables m and C (which also illustrates that argument names are not case sensitive).

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
p = Portfolio;
p = Portfolio(p, 'assetmean', m, 'assetcovar', C);
p = Portfolio(p, 'lowerbudget', 1, 'upperbudget', 1);
p = Portfolio(p, 'lowerbound', 0);
plotFrontier(p);
```

Efficient Frontier


This way works because the calls to Portfolio are in this particular order. In this case, the call to initialize AssetMean and AssetCovar provides the dimensions for the problem. If you were to do this step last, you would have to explicitly dimension the LowerBound property as follows:

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
p = Portfolio;
p = Portfolio(p, 'LowerBound', zeros(size(m)));
p = Portfolio(p, 'LowerBudget', 1, 'UpperBudget', 1);
```

```
p = Portfolio(p, 'AssetMean', m, 'AssetCovar', C);
plotFrontier(p);
```

Efficient Frontier


If you did not specify the size of LowerBound but, instead, input a scalar argument, the Portfolio object assumes that you are defining a single-asset problem and produces an error at the call to set asset moments with four assets.

## Create a Portfolio Object Using Shortcuts for Property Names

You can create a Portfolio object, p with Portfolio using shortcuts for property names.

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
p = Portfolio('mean', m, 'covar', C, 'budget', 1, 'lb', 0)
p =
    Portfolio with properties:
            BuyCost: []
            SellCost: []
        RiskFreeRate: []
            AssetMean: [4x1 double]
            AssetCovar: [4\times4 double]
        TrackingError: []
            TrackingPort: []
            Turnover: []
        BuyTurnover: []
        SellTurnover: []
            Name: []
            NumAssets: 4
            AssetList: []
            InitPort: []
            AInequality: []
            bInequality: []
            AEquality: []
            bEquality: []
            LowerBound: [4x1 double]
            UpperBound: []
            LowerBudget: 1
            UpperBudget: 1
            GroupMatrix: []
            LowerGroup: []
            UpperGroup: []
            GroupA: []
            GroupB: []
            LowerRatio: []
            UpperRatio: []
            MinNumAssets: []
            MaxNumAssets: []
            BoundType: []
```


## Direct Setting of Portfolio Object Properties

Although not recommended, you can set properties directly, however no error-checking is done on your inputs.

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
p = Portfolio;
p.NumAssets = numel(m);
p.AssetMean = m;
p.AssetCovar = C;
p.LowerBudget = 1;
p.UpperBudget = 1;
p.LowerBound = zeros(size(m));
disp(p)
```

    Portfolio with properties:
            BuyCost: []
            SellCost: []
    RiskFreeRate: []
        AssetMean: [4x1 double]
        AssetCovar: [4×4 double]
        TrackingError: []
        TrackingPort: []
            Turnover: []
        BuyTurnover: []
        SellTurnover: []
            Name: []
        NumAssets: 4
        AssetList: []
            InitPort: []
        AInequality: []
        bInequality: []
            AEquality: []
            bEquality: []
            LowerBound: [4x1 double]
            UpperBound: []
            LowerBudget: 1
    UpperBudget: 1
GroupMatrix: []
LowerGroup: []
UpperGroup: []
GroupA: []
GroupB: []
LowerRatio: []
UpperRatio: []
MinNumAssets: []
MaxNumAssets: []BoundType: []

## Create a Portfolio Object and Determine Efficient Portfolios

Create efficient portfolios:
load CAPMuniverse
p = Portfolio('AssetList',Assets(1:12));
p = estimateAssetMoments(p, Data(:,1:12),'missingdata',true);
$\mathrm{p}=$ setDefaultConstraints(p);
plotFrontier(p);


```
pwgt = estimateFrontier(p, 5);
pnames = cell(1,5);
for i = 1:5
    pnames{i} = sprintf('Port%d',i);
end
Blotter = dataset([{pwgt},pnames],'obsnames',p.AssetList);
disp(Blotter);
\begin{tabular}{llllll} 
& Port1 & Port2 & Port3 & Port4 & Port5 \\
AAPL & 0.017926 & 0.058247 & 0.097816 & 0.12955 & 0
\end{tabular}
```

| AMZN | 0 | 0 | 0 | 0 | 0 |
| :--- | ---: | ---: | ---: | ---: | ---: |
| CSCO | 0 | 0 | 0 | 0 | 0 |
| DELL | 0.0041906 | 0 | 0 | 0 | 0 |
| EBAY | 0 | 0 | 0 | 0 | 0 |
| GOOG | 0.16144 | 0.35678 | 0.55228 | 0.75116 | 1 |
| HPQ | 0.052566 | 0.032302 | 0.011186 | 0 | 0 |
| IBM | 0.46422 | 0.36045 | 0.25577 | 0.11928 | 0 |
| INTC | 0 | 0 | 0 | 0 | 0 |
| MSFT | 0.29966 | 0.19222 | 0.082949 | 0 | 0 |
| ORCL | 0 | 0 | 0 | 0 | 0 |
| YHOO | 0 | 0 | 0 | 0 | 0 |

## Definitions

## Mean-Variance Portfolio Optimization

A mean-variance optimization problem is completely specified with three elements.
The three elements for a mean-variance optimization problem are:

- A universe of assets with estimates for the prospective mean and covariance of asset total returns for a period of interest.
- A portfolio set that specifies the set of portfolio choices in terms of a collection of constraints.
- A model for portfolio return and risk, which, for mean-variance optimization, is either the gross or net mean of portfolio returns and the standard deviation of portfolio returns.

After you specify these three elements in an unambiguous way, you can solve and analyze portfolio optimization problems. The simplest mean-variance portfolio optimization problem has:

- A mean and covariance of asset total returns
- Nonnegative weights for all portfolios that sum to 1 (the summation constraint is known as a budget constraint)
- Built-in models for portfolio return and risk that use the mean and covariance of asset total returns

For more information on the theory and definition of mean-variance optimization supported by portfolio optimization tools in Financial Toolbox, see "Portfolio Optimization Theory" on page 4-3.

## Portfolio Problem Sufficiency

A mean-variance portfolio optimization is completely specified with the Portfolio object if two condition are met.

The following are the two conditions that must be met:

- The moments of asset returns must be specified such that the property AssetMean contains a valid finite mean vector of asset returns and the property AssetMean contains a valid symmetric positive-semidefinite matrix for the covariance of asset returns.

The first condition is satisfied by setting the properties associated with the moments of asset returns.

- The set of feasible portfolios must be a nonempty compact set, where a compact set is closed and bounded.

The second condition is satisfied by an extensive collection of properties that define different types of constraints to form a set of feasible portfolios. Since such sets must be bounded, either explicit or implicit constraints can be imposed, and several functions, such as estimateBounds, provide ways to ensure that your problem is properly formulated.

Given mean and covariance of asset returns in the variables AssetMean and AssetCovar, this problem is completely specified by:

```
p = Portfolio('AssetMean', AssetMean, 'AssetCovar', AssetCovar,...
'LowerBound', 0, 'UpperBudget',1, 'LowerBudget',1)
```

or equivalently by:
p = Portfolio;
p = setAssetMoments(p, AssetMean, AssetCovar);
p = setDefaultConstraints(p);
Although the general sufficiency conditions for mean-variance portfolio optimization go beyond these two conditions, the Portfolio object implemented in Financial Toolbox implicitly handles all these additional conditions. For more information on the Markowitz
model for mean-variance portfolio optimization, see "Portfolio Optimization" on page A7.

## Shortcuts for Property Names

The Portfolio object has shorter argument names that replace longer argument names associated with specific properties of the Portfolio object.

For example, rather than enter 'assetcovar', the Portfolio object accepts the caseinsensitive name ' covar' to set the AssetCovar property in a Portfolio object. Every shorter argument name corresponds with a single property in the Portfolio object. The one exception is the alternative argument name 'budget', which signifies both the LowerBudget and UpperBudget properties. When 'budget ' is used, then the LowerBudget and UpperBudget properties are set to the same value to form an equality budget constraint.

## Shortcuts for Property Names

| Shortcut Argument Name | Equivalent Argument / Property Name |
| :--- | :--- |
| ae | AEquality |
| ai | AInequality |
| covar | AssetCovar |
| assetnames or assets | AssetList |
| mean | AssetMean |
| be | bEquality |
| bi | bInequality |
| group | GroupMatrix |
| lb | LowerBound |
| n or num | NumAssets |
| rfr | RiskFreeRate |
| ub | UpperBound |
| budget | UpperBudget and LowerBudget |

## References

[1] For a complete list of references for the Portfolio object, see "Portfolio Optimization" on page A-7.

## See Also

PortfolioCVaR|PortfolioMAD | estimateFrontier | plotFrontier

## Topics

"Creating the Portfolio Object" on page 4-28
"Working with Portfolio Constraints Using Defaults" on page 4-68
"Estimate Efficient Portfolios for Entire Efficient Frontier for Portfolio Object" on page 4115
"Estimate Efficient Frontiers for Portfolio Object" on page 4-140
"Asset Allocation Case Study" on page 4-196
"Portfolio Optimization Examples" on page 4-167
"Portfolio Optimization Theory" on page 4-3
"Portfolio Object Workflow" on page 4-21
"Portfolio Object Properties and Functions" on page 4-23
"Working with Portfolio Objects" on page 4-23
"Setting and Getting Properties" on page 4-24
"Displaying Portfolio Objects" on page 4-25
"Saving and Loading Portfolio Objects" on page 4-25
"Estimating Efficient Portfolios and Frontiers" on page 4-25
"Arrays of Portfolio Objects" on page 4-25
"Subclassing Portfolio Objects" on page 4-26
"Conventions for Representation of Data" on page 4-26

## External Websites

Getting Started with Portfolio Optimization (4 min 12 sec )

## Introduced in R2011a

## PortfolioCVaR

Creates PortfolioCVaR object for conditional value-at-risk portfolio optimization and analysis

## Description

Use PortfolioCVaR to create a PortfolioCVaR object for conditional value-at-risk portfolio optimization.

The main workflow for CVaR portfolio optimization is to create an instance of a PortfolioCVaR object that completely specifies a portfolio optimization problem and to operate on the PortfolioCVaR object using supported functions to obtain and analyze efficient portfolios. For details on this workflow, see "PortfolioCVaR Object Workflow" on page 5-20.

You can use the PortfolioCVaR object in several ways. To set up a portfolio optimization problem in a PortfolioCVaR object, the simplest syntax is:
p = PortfolioCVaR;
This syntax creates a PortfolioCVaR object, p, such that all object properties are empty.
The PortfolioCVaR object also accepts collections of name-value pair arguments for properties and their values. The PortfolioCVaR function accepts inputs for properties with the general syntax:

```
p = PortfolioCVaR('property1',value1,'property2',value2, ... );
```

If a PortfolioCVaR object already exists, the syntax permits the first (and only the first argument) of the PortfolioCVaR object to be an existing object with subsequent namevalue pair arguments for properties to be added or modified. For example, given an existing PortfolioCVaR object in $p$, the general syntax is:

```
p = PortfolioCVaR(p,'property1',value1,'property2',value2, ... );
```

Input argument names are not case sensitive, but must be completely specified. In addition, several properties can be specified with alternative argument names (see "Shortcuts for Property Names" on page 19-1371). The PortfolioCVaR object tries to
detect problem dimensions from the inputs and, once set, subsequent inputs can undergo various scalar or matrix expansion operations that simplify the overall process to formulate a problem. In addition, a PortfolioCVaR object is a value object so that, given portfolio $p$, the following code creates two objects, $p$ and $q$, that are distinct:
$\mathrm{q}=\operatorname{PortfolioCVaR(p,} . .$.
After creating a PortfolioCVaR object, you can use the associated object functions to set portfolio constraints, analyze the efficient frontier, and validate the portfolio model.

For more detailed information on the theoretical basis for conditional value-at-risk portfolio optimization, see "Portfolio Optimization Theory" on page 5-3.

## Creation

## Syntax

p = PortfolioCVaR
p = PortfolioCVaR(Name, Value)
p = PortfolioCVaR(p,Name, Value)

## Description

p = PortfolioCVaR creates an empty PortfolioCVaR object for conditional value-at-risk portfolio optimization and analysis. You can then add elements to the PortfolioCVaR object using the supported "add" and "set" functions. For more information, see "Creating the PortfolioCVaR Object" on page 5-27.
$p=$ PortfolioCVaR(Name, Value) creates a PortfolioCVaR object (p) and sets Properties on page 19-1353 using name-value pairs. For example, p = PortfolioCVaR('AssetList',Assets(1:12)). You can specify multiple name-value pairs.
p = PortfolioCVaR( $p$, Name, Value) creates a PortfolioCVaR object (p) using a previously created PortfolioCVaR object p and sets Properties on page 19-1353 using name-value pairs. You can specify multiple name-value pairs.

## Input Arguments

p - Previously constructed PortfolioCVaR object
object
Previously constructed PortfolioCVaR object, specified using PortfolioCVaR.

## Properties

## Setting Up the Object

## AssetList - Names or symbols of assets in universe

[ ] (default) | cell array of character vectors | string array
Names or symbols of assets in the universe, specified as a cell array of character vectors or a string array.

Data Types: cell|string

## InitPort - Initial portfolio

[] (default) | vector
Initial portfolio, specified as a vector.

## Data Types: double

## Name - Name for instance of Portfolio object

[ ] (default) | character vector | string
Name for instance of the Portfolio object, specified as a character vector or string.
Data Types: char|string

## NumAssets - Number of assets in the universe

[] (default) | integer scalar
Number of assets in the universe, specified as an integer scalar.
Data Types: double

## Portfolio Object Constraints

## AEquality - Linear equality constraint matrix [] (default) | matrix

Linear equality constraint matrix, specified as a matrix.
Data Types: double

## AInequality - Linear inequality constraint matrix

[ ] (default) | matrix

Linear inequality constraint matrix, specified as a matrix.
Data Types: double

## bEquality - Linear equality constraint vector

[] (default) | vector
Linear equality constraint vector, specified as a vector.

## Data Types: double

## bInequality - Linear inequality constraint [] (default) | vector

Linear inequality constraint vector, specified as a vector.

## Data Types: double

## GroupA - Group A weights to be bounded by weights in group B [] (default) | matrix

Group A weights to be bounded by weights in group B, specified as a matrix.
Data Types: double

## GroupB - Group B weights

## [] (default) | matrix

Group B weights, specified as a matrix.
Data Types: double

## GroupMatrix - Group membership matrix

[^12]Group membership matrix, specified as a matrix.

## Data Types: double

## LowerBound - Lower-bound constraint

[] (default) | vector
Lower-bound constraint, specified as a vector.
Data Types: double

## LowerBudget - Lower-bound budget constraint

[] (default) | scalar
Lower-bound budget constraint, specified as a scalar.
Data Types: double
LowerGroup - Lower-bound group constraint
[] (default) | vector
Lower-bound group constraint, specified as a vector.
Data Types: double

## LowerRatio - Minimum ratio of allocations between Groups A and B <br> [] (default) | vector

Minimum ratio of allocations between GroupA and GroupB, specified as a vector.

## Data Types: double

## UpperBound - Upper-bound constraint

[] (default) | vector
Upper-bound constraint, specified as a vector.

## Data Types: double

## UpperBudget - Upper-bound budget constraint

[] (default) | scalar
Upper-bound budget constraint, specified as a scalar.
Data Types: double

## UpperGroup - Upper-bound group constraint [] (default) | vector

Upper-bound group constraint, specified as a vector.

## Data Types: double

## UpperRatio - Maximum ratio of allocations between Groups A and B <br> [] (default) | vector

Maximum ratio of allocations between GroupA and GroupB, specified as a vector.
Data Types: double

## Portfolio Object Modeling

BuyCost - Proportional purchase costs
[] (default) | vector
Proportional purchase costs, specified as a vector.
Data Types: double

## BuyTurnover - Turnover constraint on purchases

[] (default) | scalar
Turnover constraint on purchases, specified as a scalar.
Data Types: double

## RiskFreeRate - Risk-free rate

[] (default) | scalar
Risk-free rate, specified as a scalar.
Data Types: double

## ProbabilityLevel - Value-at-risk probability level which is 1 - (loss probability)

[] (default) | scalar
Value-at-risk probability level which is 1 - (loss probability), specified as a scalar.
Data Types: double

## NumScenarios - Number of scenarios <br> [] (default) | integer scalar

Number of scenarios, specified as an integer scalar.
Data Types: double
SellCost - Proportional sales costs
[] (default) | vector
Proportional sales costs, specified as a vector.

## Data Types: double

## SellTurnover - Turnover constraint on sales

[] (default) | scalar
Turnover constraint on sales, specified as a scalar.
Data Types: double

## Turnover - Turnover constraint

[] (default) | scalar
Turnover constraint, specified as a scalar.
Data Types: double

## Object Functions

setAssetList
setInitPort
setDefaultConstraints
estimateAssetMoments
setCosts
addEquality
addGroupRatio

Set up list of identifiers for assets
Set up initial or current portfolio
Set up portfolio constraints with nonnegative weights that sum to 1
Estimate mean and covariance of asset returns from data
Set up proportional transaction costs
Add linear equality constraints for portfolio weights to existing constraints Add group ratio constraints for portfolio weights to existing group ratio constraints

| addGroups | Add group constraints for portfolio weights to <br> existing group constraints |
| :--- | :--- |
| addInequality | Add linear inequality constraints for portfolio <br> weights to existing constraints |
| getBounds | Obtain bounds for portfolio weights from portfolio <br> object |
| getBudget | Obtain budget constraint bounds from portfolio |
| object |  |
| getCosts | Obtain buy and sell transaction costs from |
| getEquality | portfolio object |
| getGroupRatio | Obtain equality constraint arrays from portfolio |
|  | object |
| getGroups | Obtain group ratio constraint arrays from |
| getInequality | portfolio object |
| Obtain group constraint arrays from portfolio |  |
| getOneWayTurnover | object |
| setGroups | Obtain inequality constraint arrays from portfolio |
| setInequality | object |
| setBounds | Obtain one-way turnover constraints from |
|  | portfolio object |
| setBudget | Set up group constraints for portfolio weights |
| setCosts | Set up linear inequality constraints for portfolio |
| setDefaultConstraints | weights |
| setEquality | Set up bounds for portfolio weights for |
| setGroupRatio | PortfolioCVaR or PortfolioMAD objects |
| setInitPort | Set up budget constraints |
| setOneWayTurnover | Set up proportional transaction costs |
| setTurnover | Set up portfolio constraints with nonnegative |
| checkFeasibility | weights that sum to 1 |
| estimateBounds | Set up linear equality constraints for portfolio |
|  | weights |
| Set up group ratio constraints for portfolio |  |
| weights |  |


| estimateFrontier | Estimate specified number of optimal portfolios <br> on the efficient frontier |
| :--- | :--- |
| estimateFrontierByReturn | Estimate optimal portfolios with targeted <br> portfolio returns |
| estimateFrontierByRisk | Estimate optimal portfolios with targeted <br> portfolio risks <br> Estimate optimal portfolios at endpoints of <br> efficient frontier |
| estimateFrontierLimits | Plot efficient frontier |
| plotFrontier | Estimate mean of portfolio returns <br> estimatePortReturn <br> estimatePortRisk |
| associated wortfolio risk according to risk proxy |  |
| setSolver | Choose main solver and specify associated solver <br> options for portfolio optimization |
| setProbabilityLevel | Set probability level for VaR and CVaR <br> calculations |
| setScenarios | Set asset returns scenarios by direct matrix <br> getScenarios <br> simulateNormalScenariosByData |
| Obtain scenarios from portfolio object <br> Simulate multivariate normal asset return |  |
| simulateNormalScenariosByMoments | scenarios from data <br> Simulate multivariate normal asset return <br> scenarios from mean and covariance of asset |
| estimateScenarioMoments | returns |
| estimatePortVaR | Estimate mean and covariance of asset return |
| scenarios |  |

## Examples

## Create an Empty PortfolioCVaR Object

You can create a PortfolioCVaR object, $p$, with no input arguments and display it using disp.
p = PortfolioCVaR;
disp(p);

```
PortfolioCVaR with properties:
            BuyCost: []
            SellCost: []
        RiskFreeRate: []
    ProbabilityLevel: []
            Turnover: []
        BuyTurnover: []
        SellTurnover: []
        NumScenarios: []
            Name: []
        NumAssets: []
        AssetList: []
        InitPort: []
        AInequality: []
        bInequality: []
            AEquality: []
            bEquality: []
            LowerBound: []
            UpperBound: []
            LowerBudget: []
            UpperBudget: []
            GroupMatrix: []
            LowerGroup: []
            UpperGroup: []
            GroupA: []
            GroupB: []
            LowerRatio: []
            UpperRatio: []
```

This approach provides a way to set up a portfolio optimization problem with the PortfolioCVaR function. You can then use the associated set functions to set and modify collections of properties in the PortfolioCVaR object.

## Create a PortfolioCVaR Object Using a Single-Step Setup

You can use the PortfolioCVaR object directly to set up a "standard" portfolio optimization problem. Given scenarios of asset returns in the variable AssetScenarios, this problem is completely specified as follows:

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
```

```
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;
AssetScenarios = mvnrnd(m, C, 20000);
p = PortfolioCVaR('Scenarios', AssetScenarios,
'LowerBound', 0, 'LowerBudget', 1, 'UpperBudget', 1, ...
'ProbabilityLevel', 0.95)
p =
    PortfolioCVaR with properties:
            BuyCost: []
            SellCost: []
        RiskFreeRate: []
        ProbabilityLevel: 0.9500
            Turnover: []
        BuyTurnover: []
        SellTurnover: []
        NumScenarios: 20000
            Name: []
        NumAssets: 4
        AssetList: []
            InitPort: []
            AInequality: []
            bInequality: []
            AEquality: []
            bEquality: []
            LowerBound: [4x1 double]
            UpperBound: []
            LowerBudget: 1
            UpperBudget: 1
            GroupMatrix: []
            LowerGroup: []
            UpperGroup: []
            GroupA: []
            GroupB: []
            LowerRatio: []
            UpperRatio: []
```

Note that the LowerBound property value undergoes scalar expansion since AssetScenarios provides the dimensions of the problem.

## Create a PortfolioCVaR Object Using a Sequence of Steps

Using a sequence of steps is an alternative way to accomplish the same task of setting up a "standard" CVaR portfolio optimization problem, given AssetScenarios variable is:

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
0.00408 0.0289 0.0204 0.0119;
0.00192 0.0204 0.0576 0.0336;
0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;
AssetScenarios = mvnrnd(m, C, 20000);
p = PortfolioCVaR;
p = setScenarios(p, AssetScenarios);
p = PortfolioCVaR(p, 'LowerBound', 0);
p = PortfolioCVaR(p, 'LowerBudget', 1, 'UpperBudget', 1);
p = setProbabilityLevel(p, 0.95);
plotFrontier(p);
```



This way works because the calls to PortfolioCVaR are in this particular order. In this case, the call to initialize AssetScenarios provides the dimensions for the problem. If you were to do this step last, you would have to explicitly dimension the LowerBound property as follows:
m = [ 0.05; 0.1; 0.12; 0.18 ];
$C=[0.00640 .00408$ 0.00192 0;
0.004080 .02890 .02040 .0119 ;
0.001920 .02040 .05760 .0336 ;
00.01190 .03360 .1225 ];
$\mathrm{m}=\mathrm{m} / 12$;
C $=\mathrm{C} / 12$;

```
AssetScenarios = mvnrnd(m, C, 20000);
p = PortfolioCVaR;
p = PortfolioCVaR(p, 'LowerBound', zeros(size(m)));
p = PortfolioCVaR(p, 'LowerBudget', 1, 'UpperBudget', 1);
p = setProbabilityLevel(p, 0.95);
p = setScenarios(p, AssetScenarios);
plotFrontier(p);
```



If you did not specify the size of LowerBound but, instead, input a scalar argument, the PortfolioCVaR object assumes that you are defining a single-asset problem and produces an error at the call to set asset scenarios with four assets.

## Create a PortfolioCVaR Object Using Shortcuts for Property Names

You can create a PortfolioCVaR object, $p$ with the PortfolioCVaR object using shortcuts for property names.

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
0.00408 0.0289 0.0204 0.0119;
0.00192 0.0204 0.0576 0.0336;
0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;
```

AssetScenarios $=$ mvnrnd(m, C, 20000);
p = PortfolioCVaR('scenario', AssetScenarios, 'lb', 0, 'budget', 1, 'plevel', 0.95)
$p=$
PortfolioCVaR with properties:
BuyCost: []
SellCost: []
RiskFreeRate: []
ProbabilityLevel: 0.9500
Turnover: []
BuyTurnover: []
SellTurnover: []
NumScenarios: 20000
Name: []
NumAssets: 4
AssetList: []
InitPort: []
AInequality: []
bInequality: []
AEquality: []
bEquality: []
LowerBound: [4x1 double]
UpperBound: []
LowerBudget: 1
UpperBudget: 1
GroupMatrix: []
LowerGroup: []

UpperGroup: []
GroupA: []
GroupB: []
LowerRatio: []
UpperRatio: []

## Direct Setting of PortfolioCVaR Object Properties

Although not recommended, you can set properties directly, however no error-checking is done on your inputs.

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;
```

AssetScenarios $=$ mvnrnd(m, C, 20000);
p = PortfolioCVaR;
p = setScenarios(p, AssetScenarios);
p.ProbabilityLevel = 0.95;
p.LowerBudget = 1;
p.UpperBudget = 1;
p.LowerBound = zeros(size(m));
disp(p)
PortfolioCVaR with properties:
BuyCost: []
SellCost: []
RiskFreeRate: []
ProbabilityLevel: 0.9500
Turnover: []
BuyTurnover: []
SellTurnover: []
NumScenarios: 20000

```
            Name: []
    NumAssets: 4
    AssetList: []
        InitPort: []
    AInequality: []
bInequality: []
    AEquality: []
    bEquality: []
    LowerBound: [4x1 double]
    UpperBound: []
LowerBudget: 1
UpperBudget: 1
GroupMatrix: []
    LowerGroup: []
    UpperGroup: []
        GroupA: []
        GroupB: []
    LowerRatio: []
    UpperRatio: []
```

Scenarios cannot be assigned directly to a PortfolioCVaR object. Scenarios must always be set through either the PortfolioCVaR function, the setScenarios function, or any of the scenario simulation functions.

## Construct a PortfolioCVaR Object and Determine Efficient Portfolios

Create efficient portfolios:
load CAPMuniverse
p = PortfolioCVaR('AssetList',Assets(1:12));
p = simulateNormalScenariosByData(p, Data(:,1:12), 20000, 'missingdata',true);
$\mathrm{p}=$ setDefaultConstraints(p);
$\mathrm{p}=$ setProbabilityLevel(p, 0.95);
plotFrontier(p);


```
pwgt = estimateFrontier(p, 5);
pnames = cell(1,5);
for i = 1:5
    pnames{i} = sprintf('Port%d',i);
end
Blotter = dataset([{pwgt},pnames],'obsnames',p.AssetList);
disp(Blotter);
```

```
AAPL
                    Port1 
```

Port3
0.11933

Port4
0.13068

Port5
1.5092e-14

| AMZN | 0 | 0 | $1.1755 \mathrm{e}-38$ | 0 | $2.8997 \mathrm{e}-14$ |
| :--- | ---: | ---: | ---: | ---: | ---: |
| CSCO | $5.8775 \mathrm{e}-39$ | 0 | $5.8775 \mathrm{e}-39$ | $1.4106 \mathrm{e}-37$ | $4.1869 \mathrm{e}-14$ |
| DELL | 0.022454 | $9.484 \mathrm{e}-33$ | 0 | $3.7616 \mathrm{e}-37$ | $3.9048 \mathrm{e}-14$ |
| EBAY | 0 | 0 | $1.782 \mathrm{e}-38$ | $2.351 \mathrm{e}-37$ | $1.3394 \mathrm{e}-15$ |
| GOOG | 0.20335 | 0.38055 | 0.56242 | 0.75932 | 1 |
| HPQ | 0.041724 | 0.0099223 | 0 | $1.822 \mathrm{e}-37$ | $3.8894 \mathrm{e}-14$ |
| IBM | 0.44482 | 0.36453 | 0.26282 | 0.11 | $3.7902 \mathrm{e}-14$ |
| INTC | 0 | 0 | $4.1142 \mathrm{e}-38$ | 0 | $3.8264 \mathrm{e}-14$ |
| MSFT | 0.27667 | 0.17175 | 0.055435 | $2.9387 \mathrm{e}-37$ | $4.0873 \mathrm{e}-14$ |
| ORCL | 0 | 0 | $7.053 \mathrm{e}-38$ | 0 | $3.7811 \mathrm{e}-14$ |
| YHOO | 0 | 0 | 0 | 0 | $3.535 \mathrm{e}-14$ |

## Definitions

## Conditional Value-at-Risk Portfolio Optimization

A CVaR optimization problem is completely specified with four elements.
The four elements for a CVaR optimization problem are:

- A universe of assets with scenarios of asset total returns for a period of interest, where scenarios comprise a collection of samples from the underlying probability distribution for asset total returns. This collection must be sufficiently large for asymptotic convergence of sample statistics. Asset return moments and related statistics are derived exclusively from the scenarios.
- A portfolio set that specifies the set of portfolio choices in terms of a collection of constraints.
- A model for portfolio return and risk proxies, which, for CVaR optimization, is either the gross or net mean of portfolio returns and the conditional value-at-risk of portfolio returns.
- A probability level that specifies the probability that a loss is less than or equal to the value-at-risk. Typical values are 0.9 and 0.95 , which indicate $10 \%$ and $5 \%$ loss probabilities.

After these four elements have been specified in an unambiguous way, it is possible to solve and analyze CVaR portfolio optimization problems.

The simplest CVaR portfolio optimization problem has:

- Scenarios of asset total returns
- A requirement that all portfolios have nonnegative weights that sum to 1 (the summation constraint is known as a budget constraint)
- Built-in models for portfolio return and risk proxies that use scenarios of asset total returns
- A probability level of 0.95

Given scenarios of asset returns in the variable AssetScenarios, this problem is completely specified by:

```
p = PortfolioCVaR('Scenarios', AssetScenarios, 'LowerBound', 0, 'Budget', 1, ...
'ProbabilityLevel', 0.95);
or equivalently by:
p = PortfolioCVaR;
p = setScenarios(p, AssetScenarios);
p = setDefaultConstraints(p);
p = setProbabilityLevel(p, 0.95);
```

To confirm that this is a valid portfolio optimization problem, the following function determines whether the set of PortfolioCVaR choices is bounded (a necessary condition for portfolio optimization).

```
[lb, ub, isbounded] = estimateBounds(p);
```

Given the problem specified in the PortfolioCVaR object $p$, the efficient frontier for this problem can be displayed with:

```
plotFrontier(p);
```

and efficient portfolios can be obtained with:

```
pwgt = estimateFrontier(p);
```

For more information on the theory and definition of conditional value-at-risk optimization supported by portfolio optimization tools in Financial Toolbox, see "Portfolio Optimization Theory" on page 5-3.

## PortfolioCVaR Problem Sufficiency

A CVaR portfolio optimization problem is completely specified with the PortfolioCVaR object if three conditions are met.

The following are the three conditions that must be met:

- You must specify a collection of asset returns or prices known as scenarios such that all scenarios are finite asset returns or prices. These scenarios are meant to be samples from the underlying probability distribution of asset returns. This condition can be satisfied by the setScenarios function or with several canned scenario simulation functions.
- The set of feasible portfolios must be a nonempty compact set, where a compact set is closed and bounded. You can satisfy this condition using an extensive collection of properties that define different types of constraints to form a set of feasible portfolios. Since such sets must be bounded, either explicit or implicit constraints can be imposed and several tools, such as the estimateBounds function, provide ways to ensure that your problem is properly formulated.
- You must specify a probability level to locate the level of tail loss above which the conditional value-at-risk is to be minimized. This condition can be satisfied by the setProbabilityLevel function.

Although the general sufficient conditions for CVaR portfolio optimization go beyond the first three conditions, the PortfolioCVaR object handles all these additional conditions.

## Shortcuts for Property Names

The PortfolioCVaR object has shorter argument names that replace longer argument names associated with specific properties of the PortfolioCVaR object.

For example, rather than enter 'ProbabilityLevel', the PortfolioCVaR object accepts the case-insensitive name 'plevel' to set the ProbabilityLevel property in a PortfolioCVaR object. Every shorter argument name corresponds with a single property in the PortfolioCVaR object. The one exception is the alternative argument name 'budget ', which signifies both the LowerBudget and UpperBudget properties. When 'budget ' is used, then the LowerBudget and UpperBudget properties are set to the same value to form an equality budget constraint.

## Shortcuts for Property Names

| Shortcut Argument Name | Equivalent Argument / Property Name |
| :--- | :--- |
| ae | AEquality |
| ai | AInequality |
| assetnames or assets | AssetList |
| be | bEquality |
| bi | bInequality |
| budget | UpperBudget and LowerBudget |
| group | GroupMatrix |
| lb | LowerBound |
| n or num | NumAssets |
| level, problevel, or plevel | ProbabilityLevel |
| rfr | RiskFreeRate |
| scenario or <br> assetscenarios | Scenarios |
| ub | UpperBound |

## References

[1] For a complete list of references for the PortfolioCVaR object, see "Portfolio
Optimization" on page A-7.

## See Also

Portfolio|PortfolioMAD|estimateFrontier|plotFrontier|setScenarios

## Topics

"Creating the PortfolioCVaR Object" on page 5-27
"Common Operations on the PortfolioCVaR Object" on page 5-36
"Working with CVaR Portfolio Constraints Using Defaults" on page 5-63
"Asset Returns and Scenarios Using PortfolioCVaR Object" on page 5-44
"Estimate Efficient Portfolios for Entire Frontier for PortfolioCVaR Object" on page 5-101
"Estimate Efficient Frontiers for PortfolioCVaR Object" on page 5-118
"Postprocessing Results to Set Up Tradable Portfolios" on page 5-129
"Portfolio Optimization Theory" on page 5-3
"PortfolioCVaR Object Workflow" on page 5-20
"PortfolioCVaR Object Properties and Functions" on page 5-22
"Working with PortfolioCVaR Objects" on page 5-22
"Setting and Getting Properties" on page 5-23
"Displaying PortfolioCVaR Objects" on page 5-24
"Saving and Loading PortfolioCVaR Objects" on page 5-24
"Estimating Efficient Portfolios and Frontiers" on page 5-24
"Arrays of PortfolioCVaR Objects" on page 5-24
"Subclassing PortfolioCVaR Objects" on page 5-25
"Conventions for Representation of Data" on page 5-25

## External Websites

Analyzing Investment Strategies with CVaR Portfolio Optimization in MATLAB (50 min 42 sec )

## Introduced in R2012b

## PortfolioMAD

Create PortfolioMAD object for mean-absolute deviation portfolio optimization and analysis

## Description

The PortfolioMAD object implements mean-absolute deviation portfolio optimization, where MAD stands for "mean-absolute deviation." PortfolioMAD objects support functions that are specific to MAD portfolio optimization.

The main workflow for MAD portfolio optimization is to create an instance of a PortfolioMAD object that completely specifies a portfolio optimization problem and to operate on the PortfolioMAD object to obtain and analyze efficient portfolios. For more information on the workflow when using PortfolioMAD objects, see "PortfolioMAD Object Workflow" on page 6-19.

You can use the PortfolioMAD object in several ways. To set up a portfolio optimization problem in a PortfolioMAD object, the simplest syntax is:
p = PortfolioMAD;
This syntax creates a PortfolioMAD object, p, such that all object properties are empty.
The PortfolioMAD object also accepts collections of name-value pair arguments for properties and their values. The PortfolioMAD object accepts inputs for properties with the general syntax:
p = PortfolioMAD('property1', value1,'property2',value2, ... );

If a PortfolioMAD object exists, the syntax permits the first (and only the first argument) of the PortfolioMAD object to be an existing object with subsequent name-value pair arguments for properties to be added or modified. For example, given an existing PortfolioMAD object in $p$, the general syntax is:
p = PortfolioMAD(p,'property1',value1,'property2',value2, ... );
Input argument names are not case-sensitive, but must be completely specified. In addition, several properties can be specified with alternative argument names (see
"Shortcuts for Property Names" on page 19-1394). The PortfolioMAD object tries to detect problem dimensions from the inputs and, once set, subsequent inputs can undergo various scalar or matrix expansion operations that simplify the overall process to formulate a problem. In addition, a PortfolioMAD object is a value object so that, given portfolio $p$, the following code creates two objects, $p$ and $q$, that are distinct:

```
q = PortfolioMAD(p, ...)
```

After creating a PortfolioMAD object, you can use the associated object functions to set portfolio constraints, analyze the efficient frontier, and validate the portfolio model.

For more detailed information on the theoretical basis for conditional value-at-risk portfolio optimization, see "Portfolio Optimization Theory" on page 6-3.

## Creation

## Syntax

p = PortfolioMAD
p = PortfolioMAD(Name,Value)
p = PortfolioMAD(p,Name,Value)

## Description

p = PortfolioMAD creates an empty PortfolioMAD object for mean-absolute deviation portfolio optimization and analysis. You can then add elements to the PortfolioMAD object using the supported "add" and "set" functions. For more information, see "Creating the PortfolioMAD Object" on page 6-26.
p = PortfolioMAD(Name,Value) creates a PortfolioMAD object (p) and sets Properties on page 19-1376 using name-value pairs. For example, p = PortfolioMAD('AssetList', Assets(1:12)). You can specify multiple name-value pairs.
$p=\operatorname{PortfolioMAD(p,Name,Value)~creates~a~PortfolioMAD~object~(p)~using~a~}$ previously created PortfolioMAD object p and sets Properties on page 19-1376 using name-value pairs. You can specify multiple name-value pairs.

## Input Arguments

p - Previously constructed PortfolioMAD object
object
Previously constructed PortfolioMAD object, specified using PortfolioMAD.

## Properties

## Setting Up the Object

## AssetList - Names or symbols of assets in universe

[ ] (default) | cell array of character vectors | string array
Names or symbols of assets in the universe, specified as a cell array of character vectors or a string array.

Data Types: cell|string

## InitPort - Initial portfolio

[] (default) | vector
Initial portfolio, specified as a vector.

## Data Types: double

## Name - Name for instance of Portfolio object

[ ] (default) | character vector | string
Name for instance of the Portfolio object, specified as a character vector.
Data Types: char|string

## NumAssets - Number of assets in the universe

[] (default) | integer scalar
Number of assets in the universe, specified as an integer scalar.
Data Types: double

## Portfolio Object Constraints

## AEquality - Linear equality constraint matrix [] (default) | matrix

Linear equality constraint matrix, specified as a matrix.
Data Types: double

## AInequality - Linear inequality constraint matrix

## [] (default) | matrix

Linear inequality constraint matrix, specified as a matrix.
Data Types: double

## bEquality - Linear equality constraint vector [] (default) | vector

Linear equality constraint vector, specified as a vector.

## Data Types: double

## bInequality - Linear inequality constraint [] (default) | vector

Linear inequality constraint vector, specified as a vector.

## Data Types: double

## GroupA - Group A weights to be bounded by weights in group B [] (default) | matrix

Group A weights to be bounded by weights in group B, specified as a matrix.
Data Types: double

## GroupB - Group B weights

[ ] (default) | matrix
Group B weights, specified as a matrix.
Data Types: double

## GroupMatrix - Group membership matrix

[^13]Group membership matrix, specified as a matrix.

## Data Types: double

## LowerBound - Lower-bound constraint

[] (default) | vector
Lower-bound constraint, specified as a vector.

## Data Types: double

## LowerBudget - Lower-bound budget constraint

[] (default) | scalar
Lower-bound budget constraint, specified as a scalar.
Data Types: double
LowerGroup - Lower-bound group constraint
[] (default) | vector
Lower-bound group constraint, specified as a vector.
Data Types: double

## LowerRatio - Minimum ratio of allocations between Groups A and B <br> [] (default) | vector

Minimum ratio of allocations between GroupA and GroupB, specified as a vector.

## Data Types: double

UpperBound - Upper-bound constraint
[] (default) | vector
Upper-bound constraint, specified as a vector.

## Data Types: double

## UpperBudget - Upper-bound budget constraint

[] (default) | scalar
Upper-bound budget constraint, specified as a scalar.
Data Types: double

## UpperGroup - Upper-bound group constraint

 [] (default) | vectorUpper-bound group constraint, specified as a vector.

## Data Types: double

## UpperRatio - Maximum ratio of allocations between Groups A and B <br> [] (default) | vector

Maximum ratio of allocations between GroupA and GroupB, specified as a vector.
Data Types: double

## Portfolio Object Modeling

BuyCost - Proportional purchase costs
[] (default) | vector
Proportional purchase costs, specified as a vector.
Data Types: double

## BuyTurnover - Turnover constraint on purchases

[] (default) | scalar
Turnover constraint on purchases, specified as a scalar.
Data Types: double

## RiskFreeRate - Risk-free rate

[] (default) | scalar
Risk-free rate, specified as a scalar.
Data Types: double

## ProbabilityLevel - Value-at-risk probability level which is $\mathbf{1}$ - (loss probability)

[] (default) | scalar
Value-at-risk probability level which is 1 - (loss probability), specified as a scalar.
Data Types: double

## NumScenarios - Number of scenarios <br> [ ] (default) | integer scalar

Number of scenarios, specified as an integer scalar.
Data Types: double
SellCost - Proportional sales costs
[] (default) | vector
Proportional sales costs, specified as a vector.

## Data Types: double

## SellTurnover - Turnover constraint on sales

[] (default) | scalar
Turnover constraint on sales, specified as a scalar.
Data Types: double

## Turnover - Turnover constraint

[] (default) | scalar
Turnover constraint, specified as a scalar.
Data Types: double

## Object Functions

setAssetList
setInitPort
setDefaultConstraints
estimateAssetMoments
setCosts
addEquality
addGroupRatio

Set up list of identifiers for assets
Set up initial or current portfolio
Set up portfolio constraints with nonnegative weights that sum to 1
Estimate mean and covariance of asset returns from data
Set up proportional transaction costs
Add linear equality constraints for portfolio weights to existing constraints Add group ratio constraints for portfolio weights to existing group ratio constraints

| addGroups | Add group constraints for portfolio weights to <br> existing group constraints |
| :--- | :--- |
| addInequality | Add linear inequality constraints for portfolio <br> weights to existing constraints |
| getBounds | Obtain bounds for portfolio weights from portfolio <br> object |
| getBudget | Obtain budget constraint bounds from portfolio |
| object |  |
| getCosts | Obtain buy and sell transaction costs from |
| getEquality | portfolio object |
| getGroupRatio | Obtain equality constraint arrays from portfolio |
|  | object |
| getGroups | Obtain group ratio constraint arrays from |
| getInequality | portfolio object |
| Obtain group constraint arrays from portfolio |  |
| getOneWayTurnover | object |
| setGroups | Obtain inequality constraint arrays from portfolio |
| setInequality | object |
| setBounds | Obtain one-way turnover constraints from |
|  | portfolio object |
| setBudget | Set up group constraints for portfolio weights |
| setCosts | Set up linear inequality constraints for portfolio |
| setDefaultConstraints | weights |
| setEquality | Set up bounds for portfolio weights for |
| setGroupRatio | PortfolioCVaR or PortfolioMAD objects |
| setInitPort | Set up budget constraints |
| setOneWayTurnover | Set up proportional transaction costs |
| setTurnover | Set up portfolio constraints with nonnegative |
| checkFeasibility | weights that sum to 1 |
| estimateBounds | Set up linear equality constraints for portfolio |
|  | weights |
| Set up group ratio constraints for portfolio |  |
| weights |  |

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| estimateFrontier | Estimate specified number of optimal portfolios <br> on the efficient frontier |
| :--- | :--- |
| estimateFrontierByReturn | Estimate optimal portfolios with targeted <br> portfolio returns |
| estimateFrontierByRisk | Estimate optimal portfolios with targeted <br> portfolio risks |
| estimateFrontierLimits | Estimate optimal portfolios at endpoints of <br> efficient frontier |
| plotFrontier | Plot efficient frontier |
| estimatePortReturn | Estimate mean of portfolio returns <br> estimatePortRisk |
| Estimate portfolio risk according to risk proxy |  |
| setSolver | associated with corresponding object |
| Choose main solver and specify associated solver |  |
| setProbabilityLevel | options for portfolio optimization |
| Set probability level for VaR and CVaR |  |
| setScenarios | calculations |
| getScenarios | Set asset returns scenarios by direct matrix |
| simulateNormalScenariosByData | Obtain scenarios from portfolio object |
|  | Simulate multivariate normal asset return <br> scenarios from data |
| simulateNormalScenariosByMoments | Simulate multivariate normal asset return <br> scenarios from mean and covariance of asset |
| estimateScenarioMoments | returns |
| estimatePortStd | Estimate mean and covariance of asset return |
| scenarios |  |

## Examples

## Create an Empty PortfolioMAD Object

You can create a PortfolioMAD object, p , with no input arguments and display it using disp.
p = PortfolioMAD;
disp(p);
PortfolioMAD with properties:
BuyCost: []
SellCost: []
RiskFreeRate: []
Turnover: []
BuyTurnover: []
SellTurnover: []
NumScenarios: []
Name: []
NumAssets: []
AssetList: []
InitPort: []
AInequality: []
bInequality: []
AEquality: []
bEquality: []
LowerBound: []
UpperBound: []
LowerBudget: []
UpperBudget: []
GroupMatrix: []
LowerGroup: []
UpperGroup: []
GroupA: []
GroupB: []
LowerRatio: []
UpperRatio: []

This approach provides a way to set up a portfolio optimization problem with the PortfolioMAD function. You can then use the associated set functions to set and modify collections of properties in the PortfolioMAD object.

## Create a PortfolioMAD Object Using a Single-Step Setup

You can use the PortfolioMAD object directly to set up a "standard" portfolio optimization problem. Given scenarios of asset returns in the variable AssetScenarios, this problem is completely specified as follows:

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
```

```
m = m/12;
C = C/12;
AssetScenarios = mvnrnd(m, C, 20000);
p = PortfolioMAD('Scenarios', AssetScenarios, ...
'LowerBound', 0, 'LowerBudget', 1, 'UpperBudget', 1)
p =
    PortfolioMAD with properties:
            BuyCost: []
            SellCost: []
    RiskFreeRate: []
            Turnover: []
            BuyTurnover: []
        SellTurnover: []
        NumScenarios: 20000
            Name: []
        NumAssets: 4
        AssetList: []
            InitPort: []
        AInequality: []
        bInequality: []
            AEquality: []
            bEquality: []
            LowerBound: [4x1 double]
            UpperBound: []
            LowerBudget: 1
            UpperBudget: 1
            GroupMatrix: []
            LowerGroup: []
            UpperGroup: []
            GroupA: []
            GroupB: []
            LowerRatio: []
            UpperRatio: []
```

Note that the LowerBound property value undergoes scalar expansion since AssetScenarios provides the dimensions of the problem.

## Create a PortfolioMAD Object Using a Sequence of Steps

Using a sequence of steps is an alternative way to accomplish the same task of setting up a "standard" MAD portfolio optimization problem, given AssetScenarios variable is:
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
0.004080 .02890 .0204 0.0119;
0.001920 .02040 .0576 0.0336;
00.01190 .03360 .1225 ];
$\mathrm{m}=\mathrm{m} / 12$;
$C=C / 12$;
AssetScenarios = mvnrnd(m, C, 20000);
p = PortfolioMAD;
p = setScenarios(p, AssetScenarios);
p = PortfolioMAD(p, 'LowerBound', 0);
p = PortfolioMAD(p, 'LowerBudget', 1, 'UpperBudget', 1);
plotFrontier(p);


This way works because the calls to PortfolioMAD are in this particular order. In this case, the call to initialize AssetScenarios provides the dimensions for the problem. If you were to do this step last, you would have to explicitly dimension the LowerBound property as follows:
m = [ 0.05; 0.1; 0.12; 0.18 ];
$C=[0.00640 .00408$ 0.00192 0;
0.00408 0.0289 0.0204 0.0119;
0.001920 .02040 .0576 0.0336;
00.01190 .03360 .1225 ];
$\mathrm{m}=\mathrm{m} / 12$;
C $=\mathrm{C} / 12$;

```
AssetScenarios = mvnrnd(m, C, 20000);
p = PortfolioMAD;
p = PortfolioMAD(p, 'LowerBound', zeros(size(m)));
p = PortfolioMAD(p, 'LowerBudget', 1, 'UpperBudget', 1);
p = setScenarios(p, AssetScenarios);
plotFrontier(p);
```



If you did not specify the size of LowerBound but, instead, input a scalar argument, the PortfolioMAD object assumes that you are defining a single-asset problem and produces an error at the call to set asset scenarios with four assets.

## Create a PortfolioMAD Object Using Shortcuts for Property Names

You can create a PortfolioMAD object, p with the PortfolioMAD object using shortcuts for property names.

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
0.00408 0.0289 0.0204 0.0119;
0.00192 0.0204 0.0576 0.0336;
0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;
```

AssetScenarios = mvnrnd(m, C, 20000);
p = PortfolioMAD('scenario', AssetScenarios, 'lb', 0, 'budget', 1)
p =
PortfolioMAD with properties:
BuyCost: []
SellCost: []
RiskFreeRate: []
Turnover: []
BuyTurnover: []
SellTurnover: []
NumScenarios: 20000
Name: []
NumAssets: 4
AssetList: []
InitPort: []
AInequality: []
bInequality: []
AEquality: []
bEquality: []
LowerBound: [4x1 double]
UpperBound: []
LowerBudget: 1
UpperBudget: 1
GroupMatrix: []
LowerGroup: []
UpperGroup: []

GroupA: []
GroupB: []
LowerRatio: []
UpperRatio: []

## Direct Setting of PortfolioMAD Object Properties

Although not recommended, you can set properties directly, however no error-checking is done on your inputs.

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;
AssetScenarios = mvnrnd(m, C, 20000);
p = PortfolioMAD;
p = setScenarios(p, AssetScenarios);
p.LowerBudget = 1;
p.UpperBudget = 1;
p.LowerBound = zeros(size(m));
disp(p);
PortfolioMAD with properties:
    BuyCost: []
        SellCost: []
    RiskFreeRate: []
        Turnover: []
        BuyTurnover: []
    SellTurnover: []
    NumScenarios: 20000
            Name: []
        NumAssets: 4
        AssetList: []
```

```
        InitPort: []
AInequality: []
bInequality: []
    AEquality: []
    bEquality: []
    LowerBound: [4x1 double]
    UpperBound: []
LowerBudget: 1
UpperBudget: 1
GroupMatrix: []
    LowerGroup: []
    UpperGroup: []
        GroupA: []
        GroupB: []
    LowerRatio: []
    UpperRatio: []
```

Scenarios cannot be assigned directly to a PortfolioMAD object. Scenarios must always be set through either the PortfolioMAD function, the setScenarios function, or any of the scenario simulation functions.

## Create a PortfolioMAD Object and Determine Efficient Portfolios

Create efficient portfolios:
load CAPMuniverse
p = PortfolioMAD('AssetList',Assets(1:12));
p = simulateNormalScenariosByData(p, Data(:,1:12), 20000 ,'missingdata',true);
p = setDefaultConstraints(p);
plotFrontier(p);


```
pwgt = estimateFrontier(p, 5);
pnames = cell(1,5);
for i = 1:5
    pnames{i} = sprintf('Port%d',i);
end
Blotter = dataset([{pwgt},pnames],'obsnames',p.AssetList);
disp(Blotter);
```

```
AAPL
                    Port1 }\quad\mathrm{ Port2 
```

Port3
0.11278

Port4
0.13456

Port5
1.5092e-14

| AMZN | $2.0475 \mathrm{e}-21$ | 0 | $7.7965 \mathrm{e}-23$ | 0 | $2.8997 \mathrm{e}-14$ |
| :--- | ---: | ---: | ---: | ---: | ---: |
| CSCO | $9.6659 \mathrm{e}-22$ | $3.4382 \mathrm{e}-23$ | $4.0358 \mathrm{e}-23$ | $3.7616 \mathrm{e}-37$ | $4.1869 \mathrm{e}-14$ |
| DELL | 0.0089659 | 0 | 0 | 0 | $3.9048 \mathrm{e}-14$ |
| EBAY | $1.3835 \mathrm{e}-21$ | $2.9706 \mathrm{e}-23$ | $2.9385 \mathrm{e}-23$ | $4.9619 \mathrm{e}-37$ | $1.3394 \mathrm{e}-15$ |
| GOOG | 0.16117 | 0.35201 | 0.54486 | 0.74888 | 1 |
| HPQ | 0.056551 | 0.024037 | 0 | $3.7616 \mathrm{e}-36$ | $3.8894 \mathrm{e}-14$ |
| IBM | 0.45905 | 0.37891 | 0.29383 | 0.11656 | $3.7902 \mathrm{e}-14$ |
| INTC | 0.28403 | $7.0923 \mathrm{e}-22$ | $3.7644 \mathrm{e}-23$ | $3.0093 \mathrm{e}-36$ | $3.8264 \mathrm{e}-14$ |
| MSFT | 0.28406 | 0.048527 | $2.8212 \mathrm{e}-36$ | $4.0873 \mathrm{e}-14$ |  |
| ORCL | $3.9607 \mathrm{e}-22$ | 0.169 | 0 | $5.6691 \mathrm{e}-23$ | $2.6449 \mathrm{e}-37$ |
| YHOO | $8.27 \mathrm{e}-22$ | $3.6285 \mathrm{e}-23$ | $7.3462 \mathrm{e}-24$ | 0 | $0.7811 \mathrm{e}-14$ |
|  |  |  |  | $3.535 \mathrm{e}-14$ |  |

## Definitions

## Mean-Absolute Deviation Portfolio Optimization

A MAD optimization problem is completely specified with three elements.
The three elements for a A MAD optimization problem are:

- A universe of assets with scenarios of asset total returns for a period of interest, where scenarios comprise a collection of samples from the underlying probability distribution for asset total returns. This collection must be sufficiently large for asymptotic convergence of sample statistics. Asset return moments and related statistics are derived exclusively from the scenarios.
- A portfolio set that specifies the set of portfolio choices in terms of a collection of constraints.
- A model for portfolio return and risk proxies, which, for MAD optimization, is either the gross or net mean of portfolio returns and the mean-absolute deviation of portfolio returns.

After these three elements have been specified unambiguously, it is possible to solve and analyze MAD portfolio optimization problems.

The simplest MAD portfolio optimization problem has:

- Scenarios of asset total returns
- A requirement that all portfolios have nonnegative weights that sum to 1 (the summation constraint is known as a budget constraint)
- Built-in models for portfolio return and risk proxies that use scenarios of asset total returns

Given scenarios of asset returns in the variable AssetScenarios, this problem is completely specified by:

```
p = PortfolioMAD('Scenarios', AssetScenarios, 'LowerBound', 0, 'Budget', 1);
```

or equivalently by:

```
p = PortfolioMAD;
p = setScenarios(p, AssetScenarios);
p = setDefaultConstraints(p);
```

To confirm that this is a valid portfolio optimization problem, the following function determines whether the set of PortfolioMAD choices is bounded (a necessary condition for portfolio optimization).

```
[lb, ub, isbounded] = estimateBounds(p);
```

Given the problem specified in the PortfolioMAD object $p$, the efficient frontier for this problem can be displayed with:

```
plotFrontier(p);
```

and efficient portfolios can be obtained with:

```
pwgt = estimateFrontier(p);
```

For more detailed information on the theoretical basis for mean-absolute deviation optimization, see "Portfolio Optimization Theory" on page 6-3.

## PortfolioMAD Problem Sufficiency

A MAD portfolio optimization problem is completely specified with the PortfolioMAD object if three conditions are met.

The following are the three conditions that must be met:

- You must specify a collection of asset returns or prices known as scenarios such that all scenarios are finite asset returns or prices. These scenarios are meant to be samples from the underlying probability distribution of asset returns. This condition can be satisfied by the setScenarios function or with several canned scenario simulation functions.
- The set of feasible portfolios must be a nonempty compact set, where a compact set is closed and bounded. You can satisfy this condition using an extensive collection of properties that define different types of constraints to form a set of feasible portfolios. Since such sets must be bounded, either explicit or implicit constraints can be imposed and several tools, such as the estimateBounds function, provide ways to ensure that your problem is properly formulated.

Although the general sufficient conditions for MAD portfolio optimization go beyond these conditions, the PortfolioMAD object handles all these additional conditions.

## Shortcuts for Property Names

The PortfolioMAD object has shorter argument names that replace longer argument names associated with specific properties of the PortfolioMAD object.

For example, rather than enter 'AInequality ', PortfolioMAD accepts the caseinsensitive name 'ai' to set the AInequality property in a PortfolioMAD object. Every shorter argument name corresponds with a single property in the PortfolioMAD function. The one exception is the alternative argument name 'budget', which signifies both the LowerBudget and UpperBudget properties. When 'budget ' is used, then the LowerBudget and UpperBudget properties are set to the same value to form an equality budget constraint.

## Shortcuts for Property Names

| Shortcut Argument Name | Equivalent Argument / Property Name |
| :--- | :--- |
| ae | AEquality |
| ai | AInequality |
| assetnames or assets | AssetList |
| be | bEquality |
| bi | bInequality |
| budget | UpperBudget and LowerBudget |
| group | GroupMatrix |
| lb | LowerBound |
| n or num | NumAssets |
| rfr | RiskFreeRate |
| scenario or <br> assetscenarios | Scenarios |
| ub | UpperBound |

## References

[1] For a complete list of references for the PortfolioMAD object, see "Portfolio Optimization" on page A-7.

## See Also

Portfolio| PortfolioCVaR|estimateFrontier|plotFrontier| setScenarios

## Topics

"Creating the PortfolioMAD Object" on page 6-26
"Common Operations on the PortfolioMAD Object" on page 6-34
"Working with MAD Portfolio Constraints Using Defaults" on page 6-61
"Asset Returns and Scenarios Using PortfolioMAD Object" on page 6-42
"Validate the MAD Portfolio Problem" on page 6-91
"Estimate Efficient Portfolios Along the Entire Frontier for PortfolioMAD Object" on page 6-96
"Estimate Efficient Frontiers for PortfolioMAD Object" on page 6-111
"Postprocessing Results to Set Up Tradable Portfolios" on page 6-122
"Portfolio Optimization Theory" on page 6-3
"PortfolioMAD Object Workflow" on page 6-19
"PortfolioMAD Object Properties and Functions" on page 6-21
"Working with PortfolioMAD Objects" on page 6-21
"Setting and Getting Properties" on page 6-22
"Displaying PortfolioMAD Objects" on page 6-23
"Saving and Loading PortfolioMAD Objects" on page 6-23
"Estimating Efficient Portfolios and Frontiers" on page 6-23
"Arrays of PortfolioMAD Objects" on page 6-23
"Subclassing PortfolioMAD Objects" on page 6-24
"Conventions for Representation of Data" on page 6-24

## Introduced in R2013b

## portopt

Portfolios on constrained efficient frontier

Note portopt has been partially removed and will no longer accept ConSet or varargin arguments. Use Portfolio instead to solve portfolio problems that are more than a long-only fully-invested portfolio. For information on the workflow when using Portfolio objects, see "Portfolio Object Workflow". For more information on migrating portopt code to Portfolio, see "portopt Migration to Portfolio Object".

## Syntax

```
[PortRisk,PortReturn,PortWts] = portopt(ExpReturn,ExpCovariance)
[PortRisk,PortReturn,PortWts] = portopt(ExpReturn,ExpCovariance,NumPorts)
[PortRisk,PortReturn,PortWts] = portopt(ExpReturn,ExpCovariance,NumPorts,PortReturn)
```


## Arguments

| ExpReturn | 1 by number of assets (NASSETS) vector specifying the <br> expected (mean) return of each asset. |
| :--- | :--- |
| ExpCovariance | NASSETS-by-NASSETS matrix specifying the covariance of <br> the asset returns. |
| NumPorts | (Optional) Number of portfolios generated along the efficient <br> frontier. Returns are equally spaced between the maximum <br> possible return and the minimum risk point. If NumPorts is <br> empty (entered as [ ]), computes 10 equally spaced points. |
| PortReturn | (Optional) Expected return of each portfolio. A number of <br> portfolios (NPORTS-by-1 vector). If not entered or empty, <br> NumPorts equally spaced returns between the minimum and <br> maximum possible values are used. |

## Description

[PortRisk,PortReturn,PortWts] = portopt(ExpReturn,ExpCovariance) sets up the most basic portfolio problem with weights greater than or equal to 0 that must sum to 1 . All that is necessary to solve this problem is the mean and covariance of asset returns. The problem returns 10 equally-spaced points on the efficient frontier by return.
[PortRisk,PortReturn,PortWts] =
portopt (ExpReturn,ExpCovariance,NumPorts) sets up the basic portfolio problem but lets you specify how many equally-spaced points on the efficient frontier that you want in NumPorts. If you specify 1 , it returns the minimum-risk portfolio.
[PortRisk,PortReturn,PortWts] = portopt (ExpReturn, ExpCovariance,NumPorts,PortReturn) sets up the basic portfolio problem but lets you specify target returns on the efficient frontier in the vector PortReturn. This functionality requires that if you set PortReturn, NumPorts should be empty.

Note portopt generates a warning if have returns outside the range and returns the portfolios at the endpoints of the efficient frontier.

The outputs for portopt are:
PortRisk is an NPORTS-by-1 vector of the standard deviation of each portfolio.
PortReturn is an NPORTS-by-1 vector of the expected return of each portfolio.
PortWts is an NPORTS-by-NASSETS matrix of weights allocated to each asset. Each row represents a portfolio. The total of all weights in a portfolio is 1.

If portopt is invoked without output arguments, it writes to the current figure window.

## Examples

## Plot the Risk-Return Efficient Frontier

Use portopt to connect 20 portfolios along the efficient frontier having evenly spaced returns. By default, choose among portfolios without short-selling and scale the value of the portfolio to 1 .

ExpReturn = [0.1 0.2 0.15];

| ExpCovariance $=$ | $[0.005$ | -0.010 |
| ---: | ---: | ---: |
|  | -0.010 | 0.040 |
|  | 0.004 | -0.002 |
|  | 0.002 |  |
|  | $0.023] ;$ |  |

NumPorts = 20; portopt(ExpReturn, ExpCovariance, NumPorts)

See AlsoPortfolio|ewstats|frontier|portcons|portstats
Topics
"Portfolio Construction Examples" on page 3-7
"Plotting an Efficient Frontier Using portopt" on page 10-26
"Portfolio Selection and Risk Aversion" on page 3-9
"Active Returns and Tracking Error Efficient Frontier" on page 3-43
"portopt Migration to Portfolio Object" on page 3-14
"Analyzing Portfolios" on page 3-2
"Portfolio Optimization Functions" on page 3-4
Introduced before R2006a

## portrand

Randomized portfolio risks, returns, and weights

## Syntax

[PortRisk,PortReturn, PortWts] = portrand(Asset,Return, Points, Method)
portrand(Asset, Return, Points,Method)

## Arguments

| Asset | Matrix of time series data. Each row is an observation and each <br> column represents a single security. |
| :--- | :--- |
| Return | (Optional) Row vector where each column represents the rate of <br> return for the corresponding security in Asset. By default, Return <br> is computed by taking the average value of each column of Asset. |
| Points | (Optional) Scalar that specifies how many random points should be <br> generated. Default = 1000. |
| Method | (Optional) A character vector that specifies how to generate random <br> portfolios from the set of portfolios with two possible methods: <br> - uniform ' - Uniformly distributed portfolio weights (default <br> method). The ' uniform ' method generates portfolio weights <br> that are uniformly distributed on the set of portfolio weights. <br> ' geometric ' - Concentrated portfolio weights around the <br> geometric center of the set of portfolios. The 'geometric' <br> method generates portfolio weights that are concentrated <br> around the geometric center of the set of portfolio weights. |

## Description

[PortRisk,PortReturn,PortWts] = portrand(Asset,Return,Points,Method) returns the risks, rates of return, and weights of random portfolio configurations.

| PortRisk | Points-by-1 vector of standard deviations. |
| :--- | :--- |
| PortReturn | Points-by-1 vector of expected rates of return. |
| PortWts | Points by number of securities matrix of asset weights. Each row <br> of PortWts is a different portfolio configuration. |

portrand(Asset, Return, Points, Method) plots the points representing each portfolio configuration. It does not return any data to the MATLAB workspace.

Note Portfolios are selected at random from a set of portfolios such that portfolio weights are nonnegative and sum to 1 . The sample mean and covariance of asset returns are used to compute portfolio returns for each random portfolio.

## References

Bodie, Kane, and Marcus. Investments. Chapter 7.

## See Also

portror|portvar

## Topics

"Portfolio Construction Examples" on page 3-7
"Portfolio Optimization Functions" on page 3-4
Introduced before R2006a

## portror

Portfolio expected rate of return

## Syntax

```
R = portror(Return,Weight)
```


## Arguments

| Return | 1 -by-N matrix of rates of return. Each column of Return represents <br> the rate of return for a single security |
| :--- | :--- |
| Weight | M-by-N matrix of weights. Each row of Weight represents a different <br> weighting combination of the assets in the portfolio. |

## Description

$R=$ portror (Return, Weight) returns a 1-by-M vector for the expected rate of return.

## Examples

## Portfolio Expected Rate of Return

This example shows a portfolio that is made up of two assets ABC and XYZ having expected rates of return of $10 \%$ and $14 \%$, respectively. If $40 \%$ percent of the portfolio's funds are allocated to asset ABC and the remaining funds are allocated to asset XYZ, the portfolio's expected rate of return is:

```
r = portror([.1 .14],[.4 .6])
r = 0.1240
```


## References

Bodie, Kane, and Marcus. Investments. Chapter 7.

## See Also

portrand|portvar

Topics<br>"Portfolio Construction Examples" on page 3-7<br>"Portfolio Optimization Functions" on page 3-4<br>Introduced before R2006a

## portsim

Monte Carlo simulation of correlated asset returns

## Syntax

RetSeries = portsim(ExpReturn,ExpCovariance,NumObs,RetIntervals,NumSim,Method)

## Arguments

| ExpReturn | 1 by number of assets (NASSETS) vector specifying the <br> expected (mean) return of each asset. |
| :--- | :--- |
| ExpCovariance | NASSETS-by-NASSETS matrix of asset return covariances. <br> ExpCovariance must be symmetric and positive <br> semidefinite (no negative eigenvalues). The standard <br> deviations of the returns are <br> ExpSigma = sqrt (diag (ExpCovariance) ). |
| NumObs | Positive scalar integer indicating the number of consecutive <br> observations in the return time series. If NumObs is entered <br> as the empty matrix [ ], the length of Ret Intervals is <br> used. |
| RetIntervals | (Optional) Positive scalar or number of observations NUMOBS- <br> by-1 vector of interval times between observations. If <br> Ret Intervals is not specified, all intervals are assumed to <br> have length 1. |
| NumSim | (Optional) Positive scalar integer indicating the number of <br> simulated sample paths (realizations) of NUMOBS <br> observations. Default = 1 (single realization of NUMOBS <br> correlated asset returns). |


| Method | Optional) Character vector indicating the type of Monte <br> Carlo simulation: <br> ' Exact ' (default) generates correlated asset returns in <br> which the sample mean and covariance match the input <br> mean (ExpReturn) and covariance (ExpCovariance) <br> specifications. <br> 'Expected ' generates correlated asset returns in which the |
| :--- | :--- |
| sample mean and covariance are statistically equal to the |  |
| input mean and covariance specifications. (The expected |  |
| value of the sample mean and covariance are equal to the |  |
| input mean (ExpReturn) and covariance (ExpCovariance) |  |
| specifications.) |  |
| For either method, the sample mean and covariance |  |
| returned are appropriately scaled by RetIntervals. |  |

## Description

portsim simulates correlated returns of NASSETS assets over NUMOBS consecutive observation intervals. Asset returns are simulated as the proportional increments of constant drift, constant volatility stochastic processes, thereby approximating continuoustime geometric Brownian motion.

RetSeries is a NUMOBS-by-NASSETS-by-NUMSIM three-dimensional array of correlated, normally distributed, proportional asset returns. Asset returns over an interval of length $d t$ are given by

$$
\frac{d S}{S}=\mu d t+\sigma d z=\mu d t+\sigma \varepsilon \sqrt{d t}
$$

where $S$ is the asset price, $\mu$ is the expected rate of return, $\sigma$ is the volatility of the asset price, and $\varepsilon$ represents a random drawing from a standardized normal distribution.

## Notes

- When Method is 'Exact ', the sample mean and covariance of all realizations (scaled by RetIntervals) match the input mean and covariance. When the returns are then
converted to asset prices, all terminal prices for a given asset are in close agreement. Although all realizations are drawn independently, they produce similar terminal asset prices. Set Method to 'Expected ' to avoid this behavior.
- The returns from the portfolios in PortWts are given by PortReturn = PortWts * RetSeries(:,:,1)', where PortWts is a matrix in which each row contains the asset allocations of a portfolio. Each row of PortReturn corresponds to one of the portfolios identified in PortWts, and each column corresponds to one of the observations taken from the first realization (the first plane) in RetSeries. See portopt and portstats for portfolio specification and optimization.


## Examples

## Distinction Between Simulation Methods

This example shows the distinction between the Exact and Expected methods of simulation.

Consider a portfolio of five assets with the following expected returns, standard deviations, and correlation matrix based on daily asset returns (where ExpReturn and Sigmas are divided by 100 to convert percentages to returns).

| ExpReturn | $=[0.0246$ | 0.0189 | 0.0273 | 0.0141 | 0.0311]/100; |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Sigmas | = [0.9509 | 1.4259 | 1.5227 | 1.1062 | 1.0877]/100; |
| Correlations | = [1.0000 | 0.4403 | 0.4735 | 0.4334 | 0.6855 |
|  | 0.4403 | 1.0000 | 0.7597 | 0.7809 | 0.4343 |
|  | 0.4735 | 0.7597 | 1.0000 | 0.6978 | 0.4926 |
|  | 0.4334 | 0.7809 | 0.6978 | 1.0000 | 0.4289 |
|  | 0.6855 | 0.4343 | 0.4926 | 0.4289 | 1.0000]; |

Convert the correlations and standard deviations to a covariance matrix.
ExpCovariance = corr2cov(Sigmas, Correlations)
ExpCovariance $=5 \times 5$
$10^{-3} \times$

| 0.0904 | 0.0597 | 0.0686 | 0.0456 | 0.0709 |
| :--- | :--- | :--- | :--- | :--- |
| 0.0597 | 0.2033 | 0.1649 | 0.1232 | 0.0674 |


| 0.0686 | 0.1649 | 0.2319 | 0.1175 | 0.0816 |
| :--- | :--- | :--- | :--- | :--- |
| 0.0456 | 0.1232 | 0.1175 | 0.1224 | 0.0516 |
| 0.0709 | 0.0674 | 0.0816 | 0.0516 | 0.1183 |

Assume that there are 252 trading days in a calendar year, and simulate two sample paths (realizations) of daily returns over a two-year period. Since ExpReturn and ExpCovariance are expressed daily, set RetIntervals = 1.

```
StartPrice = 100;
NumObs = 504; % two calendar years of daily returns
NumSim = 2;
RetIntervals = 1; % one trading day
NumAssets = 5;
```

To illustrate the distinction between methods, simulate two paths by each method, starting with the same random number state.

```
rng('default');
RetExact = portsim(ExpReturn, ExpCovariance, NumObs, ...
RetIntervals, NumSim, 'Exact');
rng(0);
RetExpected = portsim(ExpReturn, ExpCovariance, NumObs, ...
RetIntervals, NumSim, 'Expected');
```

Compare the mean and covariance of RetExact with the inputs (ExpReturn and ExpCovariance), you will observe that they are almost identical.

At this point, RetExact and RetExpected are both 504-by-5-by-2 arrays. Now assume an equally weighted portfolio formed from the five assets and create arrays of portfolio returns in which each column represents the portfolio return of the corresponding sample path of the simulated returns of the five assets. The portfolio arrays PortRetExact and PortRetExpected are 504-by-2 matrices.

```
Weights = ones(NumAssets, 1)/NumAssets;
PortRetExact = zeros(NumObs, NumSim);
PortRetExpected = zeros(NumObs, NumSim);
for i = 1:NumSim
    PortRetExact(:,i) = RetExact(:,:,i) * Weights;
    PortRetExpected(:,i) = RetExpected(:,:,i) * Weights;
end
```

Finally, convert the simulated portfolio returns to prices and plot the data. In particular, note that since the Exact method matches expected return and covariance, the terminal
portfolio prices are virtually identical for each sample path. This is not true for the Expected simulation method. Although this example examines portfolios, the same methods apply to individual assets as well. Thus, Exact simulation is most appropriate when unique paths are required to reach the same terminal prices.

```
PortExact = ret2tick(PortRetExact, ...
repmat(StartPrice,1,NumSim));
```

Warning: This syntax will be removed in a future release. See the documentation for red
PortExpected $=$ ret2tick(PortRetExpected, ... repmat(StartPrice,1,NumSim));

Warning: This syntax will be removed in a future release. See the documentation for red
subplot(2,1,1), plot(PortExact, '-r')
ylabel('Portfolio Prices')
title('Exact Method')
subplot(2,1,2), plot(PortExpected, '-b')
ylabel('Portfolio Prices')
title('Expected Method')


## Interaction Between ExpReturn, ExpCovariance, and RetIntervals

This example shows the interplay among ExpReturn, ExpCovariance, and RetIntervals. Recall that portsim simulates correlated asset returns over an interval of length $d t$, given by the equation

$$
\frac{d S}{S}=\mu d t+\sigma d z=\mu d t+\sigma \varepsilon \sqrt{d t}
$$

where $S$ is the asset price, $\mu$ is the expected rate of return, $\sigma$ is the volatility of the asset price, and $\varepsilon$ represents a random drawing from a standardized normal distribution.

The time increment $d t$ is determined by the optional input RetIntervals, either as an explicit input argument or as a unit time increment by default. Regardless, the periodicity of ExpReturn, ExpCovariance, and RetIntervals must be consistent. For example, if ExpReturn and ExpCovariance are annualized, then RetIntervals must be in years. This point is often misunderstood.

To illustrate the interplay among ExpReturn, ExpCovariance, and RetIntervals, consider a portfolio of five assets with the following expected returns, standard deviations, and correlation matrix based on daily asset returns.

| ExpReturn | $=[0.0246$ | 0.0189 | 0.0273 | 0.0141 | 0.0311]/100; |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Sigmas | $=[0.9509$ | 1.4259 | 1.5227 | 1.1062 | 1.0877]/100; |
| Correlations | $=[1.0000$ | 0.4403 | 0.4735 | 0.4334 | 0.6855 |
|  | 0.4403 | 1.0000 | 0.7597 | 0.7809 | 0.4343 |
|  | 0.4735 | 0.7597 | 1.0000 | 0.6978 | 0.4926 |
|  | 0.4334 | 0.7809 | 0.6978 | 1.0000 | 0.4289 |
|  | 0.6855 | 0.4343 | 0.4926 | 0.4289 | 1.0000]; |

Convert the correlations and standard deviations to a covariance matrix of daily returns.

```
ExpCovariance = corr2cov(Sigmas, Correlations);
```

Assume 252 trading days per calendar year, and simulate a single sample path of daily returns over a four-year period. Since the ExpReturn and ExpCovariance inputs are expressed daily, set RetIntervals $=1$.

```
StartPrice = 100;
NumObs = 1008; % four calendar years of daily returns
RetIntervals = 1; % one trading day
NumAssets = length(ExpReturn);
randn('state',0);
RetSeries1 = portsim(ExpReturn, ExpCovariance, Num0bs, ...
RetIntervals, 1, 'Expected');
```

Now annualize the daily data, thereby changing the periodicity of the data, by multiplying ExpReturn and ExpCovariance by 252 and dividing RetIntervals by 252
(RetIntervals $=1 / 252$ of a year). Resetting the random number generator to its initial state, you can reproduce the results.
rng('default');
RetSeries 2 = portsim(ExpReturn*252, ExpCovariance*252, ...
Num0bs, RetIntervals/252, 1, 'Expected');

Assume an equally weighted portfolio and compute portfolio returns associated with each simulated return series.

```
Weights = ones(NumAssets, 1)/NumAssets;
PortRet1 = RetSeries2 * Weights;
PortRet2 = RetSeries2 * Weights;
```

Comparison of the data reveals that PortRet1 and PortRet2 are identical.

## Univariate Geometric Brownian Motion

This example shows how to simulates a univariate geometric Brownian motion process. It is based on an example found in Hull, Options, Futures, and Other Derivatives, 5th Edition (see example 12.2 on page 236). In addition to verifying Hull's example, it also graphically illustrates the lognormal property of terminal stock prices by a rather large Monte Carlo simulation.

Assume that you own a stock with an initial price of $\$ 20$, an annualized expected return of $20 \%$ and volatility of $40 \%$. Simulate the daily price process for this stock over the course of one full calendar year ( 252 trading days).

```
StartPrice = 20;
ExpReturn = 0.2;
ExpCovariance = 0.4^2;
NumObs = 252;
NumSim = 10000;
RetIntervals = 1/252;
```

Note that RetIntervals is expressed in years, consistent with the fact that ExpReturn and ExpCovariance are annualized. Also, ExpCovariance is entered as a variance rather than the more familiar standard deviation (volatility).

Set the random number generator state, and simulate 10,000 trials (realizations) of stock returns over a full calendar year of 252 trading days.

```
rng('default');
RetSeries = squeeze(portsim(ExpReturn, ExpCovariance, NumObs, ...
RetIntervals, NumSim, 'Expected'));
```

The squeeze function reformats the output array of simulated returns from a 252-by-1-by- 10000 array to more convenient 252 -by- 10000 array. (Recall that portsim is fundamentally a multivariate simulation engine).

In accordance with Hull's equations 12.4 and 12.5 on page 236

$$
\begin{aligned}
E\left(S_{T}\right) & =S_{0} e^{\mu T} \\
\operatorname{var}\left(S_{T}\right) & =S_{0}^{2} e^{2 \mu T}\left(e^{\sigma^{2} T}-1\right)
\end{aligned}
$$

convert the simulated return series to a price series and compute the sample mean and the variance of the terminal stock prices.

```
StockPrices = ret2tick(RetSeries, repmat(StartPrice, 1, NumSim));
SampMean = mean(StockPrices(end,:))
SampVar = var(StockPrices(end,:))
SampMean =
    24.4489
```


## SampVar =

101.4243

Compare these values with the values you obtain by using Hull's equations.

```
ExpValue = StartPrice*exp(ExpReturn)
ExpVar = ...
StartPrice*StartPrice*exp(2*ExpReturn)*(exp((ExpCovariance)) - 1)
```

ExpValue =
24.4281
ExpVar =
103.5391

These results are very close to the results shown in Hull's example 12.2.
Display the sample density function of the terminal stock price after one calendar year. From the sample density function, the lognormal distribution of terminal stock prices is apparent.

```
[count, BinCenter] = hist(StockPrices(end,:), 30);
figure
bar(BinCenter, count/sum(count), 1, 'r')
xlabel('Terminal Stock Price')
ylabel('Probability')
title('Lognormal Terminal Stock Prices')
```



## References

Hull, John, C. Options, Futures, and Other Derivatives. 5th Edition. Upper Saddle River, New Jersey: Prentice-Hall, 2003, ISBN 0-13-009056-5.

## See Also

ewstats| portopt|portstats| randn| ret2tick| squeeze

## Topics

"Portfolio Construction Examples" on page 3-7
"Portfolio Optimization Functions" on page 3-4

## Introduced before R2006a

## portstats

Portfolio expected return and risk

## Syntax

[PortRisk,PortReturn] = portstats(ExpReturn,ExpCovariance,PortWts)

## Arguments

| ExpReturn | 1-by-number of assets (NASSETS) vector specifying the <br> expected (mean) return of each asset. |
| :--- | :--- |
| ExpCovariance | NASSETS-by-NASSETS matrix specifying the covariance of <br> the asset returns. |
| PortWts | (Optional) Number of portfolios (NPORTS) by NASSETS <br> matrix of weights allocated to each asset. Each row <br> represents a different weighting combination. Default $=1 /$ <br> NASSETS (equally weighted). |

## Description

[PortRisk,PortReturn] = portstats(ExpReturn,ExpCovariance,PortWts) computes the expected rate of return and risk for a portfolio of assets.

PortRisk is an NPORTS-by-1 vector of the standard deviation of each portfolio.
PortReturn is an NPORTS-by-1 vector of the expected return of each portfolio.

## Examples

## Computes the Expected Rate of Return and Risk for a Portfolio of Assets

This example shows how to calculate the expected rate of return and risk for a portfolio of assets.

```
ExpReturn = [0.1 0.2 0.15];
```

$\left.\begin{array}{rrrr}\text { ExpCovariance }= & {[0.0100} & -0.0061 & 0.0042 \\ & -0.0061 & 0.0400 & -0.0252 \\ & 0.0042 & -0.0252 & 0.0225\end{array}\right] ;$

```
PortWts=[0.4 0.2 0.4; 0.2 0.4 0.2];
```

```
[PortRisk, PortReturn] = portstats(ExpReturn, ExpCovariance,...
```

PortWts)
PortRisk $=2 \times 1$
0.0560
0.0550

```
PortReturn = 2×1
```

0.1400
0.1300

## See Also

ewstats|portalloc

## Topics

"Portfolio Construction Examples" on page 3-7
"Portfolio Selection and Risk Aversion" on page 3-9
"Active Returns and Tracking Error Efficient Frontier" on page 3-43
"Analyzing Portfolios" on page 3-2
"Portfolio Optimization Functions" on page 3-4

Introduced before R2006a

## portvar

Variance for portfolio of assets

## Syntax

V = portvar(Asset, Weight)

## Arguments

| Asset | M-by-N matrix of M asset returns for N securities. |
| :--- | :--- |
| Weight | R-by-N matrix of R portfolio weights for $N$ securities. Each row of <br> Weight constitutes a portfolio of securities in Asset. |

## Description

$\mathrm{V}=$ portvar(Asset, Weight) returns the portfolio variance as an R-by-1vector (assuming Weight is a matrix of size R -by- N ) with each row representing a variance calculation for each row of Weight.
$\mathrm{V}=$ portvar(Asset) assigns each security an equal weight when calculating the portfolio variance.

## References

Bodie, Kane, and Marcus. Investments. Chapter 7.

## See Also

portrand|portror

## Topics

"Portfolio Construction Examples" on page 3-7
"Portfolio Optimization Functions" on page 3-4
Introduced before R2006a

19-1418

## portvrisk

Portfolio value at risk (VaR)

## Syntax

```
ValueAtRisk = portvrisk(PortReturn,PortRisk)
ValueAtRisk = portvrisk(
```

$\qquad$

``` ,RiskThreshold,PortValue)
```


## Description

ValueAtRisk = portvrisk(PortReturn,PortRisk) returns the maximum potential loss in the value of a portfolio over one period of time (that is, monthly, quarterly, yearly, and so on) given the loss probability level. portvrisk calculates ValueAtRisk using a normal distribution.

ValueAtRisk = portvrisk( $\qquad$ ,RiskThreshold,PortValue) adds optional arguments for RiskThreshold and PortValue.

## Examples

## Compute the Maximum Potential Loss in the Value of a Portfolio Over One Period of Time

This example shows how to return the maximum potential loss in the value of a portfolio over one period of time, where ValueAtRisk is computed on a per-unit basis.

```
PortReturn = 0.29/100;
PortRisk = 3.08/100;
RiskThreshold = [0.01;0.05;0.10];
PortValue = 1;
ValueAtRisk = portvrisk(PortReturn,PortRisk,...
RiskThreshold,PortValue)
ValueAtRisk = 3×1
```

0.0688
0.0478
0.0366

## Compute the Maximum Potential Loss in the Value of a Portfolio Over One Period of Time Using Actual Values

This example shows how to return the maximum potential loss in the value of a portfolio over one period of time, where ValueAtRisk is computed with actual values.

```
PortReturn = [0.29/100;0.30/100];
PortRisk = [3.08/100;3.15/100];
RiskThreshold = 0.10;
PortValue = [1000000000;500000000];
ValueAtRisk = portvrisk(PortReturn,PortRisk,...
RiskThreshold,PortValue)
ValueAtRisk = 2×1
107 x
```

    3.6572
    1.8684
    
## Input Arguments

## PortReturn - Expected return of each portfolio over period <br> numeric

Expected return of each portfolio over the period, specified as a NPORTS-by-1 vector or scalar.

## Data Types: double

## PortRisk - Standard deviation of each portfolio over period

numeric
Standard deviation of each portfolio over period, specified as a NPORTS-by-1 vector or scalar.

## Data Types: double

## RiskThreshold - Loss probability

0.05 (5\%) (default) | decimal
(Optional) Loss probability, specified as a NPORTS-by-1 vector or scalar.

## Data Types: double

## PortValue - Total value of asset portfolio <br> 1 (default) | numeric

(Optional) Total value of asset portfolio, specified as a NPORTS-by-1 vector or scalar.

Note If PortReturn and PortRisk are in dollar units, then PortValue should be 1. If PortReturn and PortRisk are on a percentage basis, then PortValue should be the total value of the portfolio.

## Data Types: double

## Output Arguments

## ValueAtRisk - Estimated maximum loss in portfolio

vector
Estimated maximum loss in the portfolio, returned as an NPORTS-by-1 vector.
ValueAtRisk is predicted with a confidence probability of 1 - RiskThreshold.

Note If PortValue is not given, ValueAtRisk is presented on a per-unit basis. A value of 0 indicates no losses.

## See Also

Portfolio|portopt

## Topics

"Portfolio Construction Examples" on page 3-7
"Portfolio Optimization Functions" on page 3-4
Introduced before R2006a

19-1422

## posvolidx

Positive volume index

Note posvolidx has been partially removed and will no longer accept a fints object (tsobj) argument. Use a matrix, timetable, or table instead for financial time series.

Use fts2timetable to convert a fints object to a timetable object.

## Syntax

```
volume = posvolidx(Data)
volume = posvolidx(
```

$\qquad$

``` ,Name, Value)
```


## Description

volume = posvolidx(Data) calculates the positive volume index from the series of closing stock prices and trade volume.
volume = posvolidx( $\qquad$ ,Name, Value) adds optional name-value pair arguments.

## Examples

## Calculate the Positive Volume Index for a Data Series for a Stock

Load the file SimulatedStock. mat, which provides a timetable (TMW) for financial data for TMW stock.

Positive Volume Index for TMW


## Input Arguments

Data - Data with closing prices and trade volume
matrix | table | timetable
Data with closing prices and trade volume, specified as a matrix, table, or timetable. For matrix input, Data is an M-by-2 with closing prices and trade volume stored in the first and second columns. Timetables and tables with M rows must contain variables named 'Close' and 'Volume' (case insensitive).

Data Types: double | table | timetable

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1,Value1, ... , NameN, ValueN.

Example: volume = posvolidx(TMW,'InitialValue',500)

## InitialValue - Initial value for positive volume index <br> 100 (default) | positive integer

Initial value for positive volume index, specified as the comma-separated pair consisting of 'InitialValue' and a scalar positive integer.

Data Types: double

## Output Arguments

## volume - Positive volume index

matrix | table | timetable
Positive volume index, returned with the same number of rows $(\mathrm{M})$ and the same type (matrix, table, or timetable) as the input Data.

## Definitions

## Positive Volume Index

Positive volume index shows the days when the trading volume of a particular security is substantially higher than other days.

## References

[1] Achelis, S. B. Technical Analysis from A to Z. Second Edition. McGraw-Hill, 1995, pp. 236-238.

See Also<br>negvolidx| onbalvol|table|timetable<br>\section*{Topics}<br>"Using Timetables in Finance"<br>"Convert Financial Time Series Objects fints to Timetables" on page 12-2<br>Introduced before R2006a

## power

Financial time series power

Note power will be removed in a future release. Use timetable instead. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

```
newfts = tsobj .^ array
newfts = array .^tsobj
newfts = tsobj_1 .^ tsobj_2
```


## Arguments

| tsobj | Financial time series object. |
| :--- | :--- |
| array | Scalar value or array with the number of rows equal to the <br> number of dates in tsobj and the number of columns equal <br> to the number of data series in tsobj. |
| tsobj_1, tsobj_2 | Pair of financial time series objects. |

## Description

newfts $=$ tsobj.$^{\wedge}$ array raises all values in the data series of the financial time series object tsobj element by element to the power indicated by the array value. The results are stored in another financial time series object newfts. The newfts object contains the same data series names as tsobj.
newfts $=$ array.$^{\wedge}$ tsobj raises the array values element by element to the values contained in the data series of the financial time series object tsobj. The results are stored in another financial time series object newfts. The newfts object contains the same data series names as tsobj.
newfts = tsobj_1 .^ tsobj_2 raises the values in the object tsobj_1 element by element to the values in the object tsobj_2. The data series names, the dates, and the number of data points in both series must be identical. newfts contains the same data series names as the original time series objects.

See Also<br>minus | plus | rdivide | times

## Topics

"Financial Time Series Operations" on page 13-7

## Introduced before R2006a

## prbyzero

Price bonds in portfolio by set of zero curves

## Syntax

```
BondPrices = prbyzero(Bonds,Settle,ZeroRates,ZeroDates)
BondPrices = prbyzero(
```

$\qquad$

``` , Compounding)
```


## Description

BondPrices = prbyzero(Bonds,Settle,ZeroRates,ZeroDates) computes the bond prices in a portfolio using a set of zero curves.

BondPrices = prbyzero( $\qquad$ , Compounding) adds an optional argument for Compounding.

## Examples

## Compute the Bond Prices in a Portfolio Using a Set of Zero Curves

This example uses the function zbtprice to compute a zero curve given a portfolio of coupon bonds and their prices. It then reverses the process, using the zero curve as input to the function prbyzero to compute the prices.

```
Bonds = [datenum('6/1/1998') 0.0475 100 2 0 0;
    datenum('7/1/2000') 0.06 100 2 0 0;
    datenum('7/1/2000') 0.09375 100 6 1 0;
    datenum('6/30/2001') 0.05125 100 1 3 1;
    datenum('4/15/2002') 0.07125 100 4 1 0;
    datenum('1/15/2000') 0.065 100 2 0 0;
    datenum('9/1/1999') 0.08 100 3 3 0;
    datenum('4/30/2001') 0.05875 100 2 0 0;
    datenum('11/15/1999') 0.07125 100 2 0 0;
    datenum('6/30/2000') 0.07 100 2 3 1;
```

```
    datenum('7/1/2001') 0.0525 100 2 3 0;
    datenum('4/30/2002') 0.07 100 2 0 0];
Prices = [ 99.375;
    99.875;
    105.75 ;
    96.875;
    103.625;
    101.125;
    103.125;
        99.375;
        101.0 ;
        101.25 ;
        96.375;
        102.75 ];
Settle = datenum('12/18/1997');
```

Set semiannual compounding for the zero curve, on an actual/365 basis.
OutputCompounding $=2$;
Execute the function zbtprice which returns the zero curve at the maturity dates.
[ZeroRates, ZeroDates] = zbtprice(Bonds, Prices, Settle,... OutputCompounding)

```
ZeroRates = 11×1
```

    0.0616
    0.0609
    0.0658
    0.0590
    0.0647
    0.0655
    0.0606
    0.0601
    0.0642
    0.0621
    ZeroDates = $11 \times 1$

729907

730364
730439
730500
730667
730668
730971
731032
731033
731321

Execute the function prbyzero.
BondPrices $=$ prbyzero(Bonds, Settle, ZeroRates, ZeroDates)
BondPrices $=12 \times 1$
99.3750
98.7980
106.8270
96.8750
103.6249
101. 1250
103.1250
99.3637
101.0000
101.2500

In this example zbtprice and prbyzero do not exactly reverse each other. Many of the bonds have the end-of-month rule off (EndMonthRule $=0$ ). The rule subtly affects the time factor computation. If you set the rule on (EndMonthRule $=1$ ) everywhere in the Bonds matrix, then prbyzero returns the original prices, except when the two incompatible prices fall on the same maturity date.

## Compute the Bond Prices in a Portfolio Using a Set of Zero Curves and datetime Inputs

This example uses the function zbtprice to compute a zero curve given a portfolio of coupon bonds and their prices. It then reverses the process, using the zero curve as input to the function prbyzero with datetime inputs to compute the prices.

```
Bonds = [datenum('6/1/1998') 0.0475 100 2 0 0;
    datenum('7/1/2000') 0.06 100 2 0 0;
    datenum('7/1/2000') 0.09375 100 6 1 0;
    datenum('6/30/2001') 0.05125 100 1 3 1;
    datenum('4/15/2002') 0.07125 100 4 1 0;
    datenum('1/15/2000') 0.065 100 2 0 0;
    datenum('9/1/1999') 0.08 100 3 3 0;
    datenum('4/30/2001') 0.05875 100 2 0 0;
    datenum('11/15/1999') 0.07125 100 2 0 0;
    datenum('6/30/2000') 0.07 100 2 3 1;
    datenum('7/1/2001') 0.0525 100 2 3 0;
    datenum('4/30/2002') 0.07 100 2 0 0];
Prices = [ 99.375;
        99.875;
        105.75 ;
        96.875;
        103.625;
        101.125;
        103.125;
        99.375;
        101.0 ;
        101.25;
        96.375;
        102.75 ];
Settle = datenum('12/18/1997');
OutputCompounding = 2;
```

[ZeroRates, ZeroDates] = zbtprice(Bonds, Prices, Settle, OutputCompounding);
dates = datetime(Bonds(:,1),'ConvertFrom','datenum','Locale','en_US');
data = Bonds(:,2:end);
$\mathrm{t}=[$ table(dates) array2table(data)];
BondPrices = prbyzero(t, datetime(Settle,'ConvertFrom','datenum','Locale','en_US'),...
ZeroRates, datetime(ZeroDates,'ConvertFrom','datenum','Locale','en_US'))

```
BondPrices = 12\times1
```

99.3750
98.7980
106.8270
96.8750
103.6249
101.1250
103.1250
99.3637
101.0000
101.2500

## Input Arguments

## Bonds - Coupon bond information to compute prices

## table | matrix

Coupon bond information to compute prices, specified as a 6-column table or a NumBonds-by- 6 matrix of bond information where the table columns or matrix columns contains:

- Maturity (Required) Maturity date of the bond, as a serial date number. Use datenum to convert date character vectors to serial date numbers. If the input Bonds is a table, the Maturity dates can be serial date numbers, date character vectors, or datetime arrays.
- CouponRate (Required) Decimal number indicating the annual percentage rate used to determine the coupons payable on a bond.
- Face (Optional) Face or par value of the bond. Default $=100$.
- Period (Optional) Coupons per year of the bond. Allowed values are 0, 1, 2 (default), 3, 4, 6, and 12.
- Basis (Optional) Day-count basis of the bond. A vector of integers.
- $0=$ actual/actual (default)
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- $4=30 / 360$ (BMA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- $9=\mathrm{actual} / 360$ (ICMA)
- $10=$ actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252
- For more information, see basis on page Glossary-0
- EndMonthRule (Optional) End-of-month rule. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days. $0=$ ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. $1=$ set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month


## Note

- If Bonds is a table, the table columns have the same meaning as when a matrix is used, but the Maturity dates can be serial date numbers, date character vectors, or datetime arrays.
- If Bonds is a matrix, it is a NUMBONDS-by-6 matrix of bonds where each row describes one bond. The first two columns are required; the remaining columns are optional but must be added in order. All rows in Bonds must have the same number of columns. The columns are Maturity, CouponRate, Face, Period, Basis, and EndMonthRule.


## Data Types: double | table

## Settle - Settlement date <br> serial date number | date character vector | datetime

Settlement date, specified as serial date numbers, date character vectors, or datetime arrays.

## Data Types: double | datetime | char

## ZeroRates - Observed zero rates

decimal fractions
Observed zero rates, specified as NUMDATES-by-NUMCURVES matrix of decimal fractions. Each column represents a rate curve. Each row represents an observation date.
Data Types: double \| datetime \| char

## ZeroDates - Observed dates for ZeroRates

serial date number | date character vector | datetime
Observed dates for ZeroRates, specified as a NUMDATES-by-1 column using serial date numbers, date character vectors, or datetime arrays.
Data Types: double | datetime | char

## Compounding - Compounding frequency of input ZeroRates when annualized 2 (default) | numeric values: $1,2,3,4,6,12$,

(Optional) Compounding frequency of input ZeroRates when annualized, specified using the allowed values:

- 1 - Annual compounding
- 2 - Semiannual compounding (default)
- 3-Compounding three times per year
- 4-Quarterly compounding
- 6 - Bimonthly compounding
- 12 - Monthly compounding

Data Types: double

## Output Arguments

## BondPrices - Clean bond prices

numeric
Clean bond prices, returned as a NUMBONDS-by-NUMCURVES matrix. Each column is derived from the corresponding zero curve in ZeroRates.

In addition, you can use the Financial Instruments Toolbox method getZeroRates for an IRDataCurve object with a Dates property to create a vector of dates and data acceptable for prbyzero. For more information, see "Converting an IRDataCurve or IRFunctionCurve Object" (Financial Instruments Toolbox).

## See Also

datetime|tr2bonds|zbtprice

## Topics

"Term Structure of Interest Rates" on page 2-42
"Fixed-Income Terminology" on page 2-22

## Introduced before R2006a

## prcroc

Price rate of change

Note prcroc has been partially removed and will no longer accept a fints object (tsobj) argument. Use a matrix, timetable, or table instead for financial time series.

Use fts2timetable to convert a fints object to a timetable object.

## Syntax

PriceChangeRate $=$ prcroc $($ Data $)$
PriceChangeRate = prcroc( __, ,Name, Value)

## Description

PriceChangeRate $=$ prcroc (Data) calculates the price rate-of-change, PriceChangeRate, from the series of closing stock prices. By default, the volume rate-of-change is calculated between the current closing price and the closing price 12 periods ago.

PriceChangeRate = prcroc( $\qquad$ ,Name, Value) adds optional name-value pair arguments.

## Examples

## Calculate the Price Rate-of-Change for a Data Series for a Stock

Load the file SimulatedStock. mat, which provides a timetable (TMW) for financial data for TMW stock.
load SimulatedStock.mat
PriceChangeRate $=$ prcroc(TMW);

```
plot(PriceChangeRate.Time,PriceChangeRate.PriceRoc)
title('Price Rate of Change for TMW')
```

Price Rate of Change for TMW


## Input Arguments

## Data - Data for closing prices

matrix | table | timetable
Data for closing prices, specified as a matrix, table, or timetable. For matrix input, Data is an M-by-1 matrix of closing prices. Timetables and tables with M rows must contain a variable named 'Close' (case insensitive).

Data Types: double | table | timetable

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN , ValueN.

Example: PriceChangeRate $=$ prcroc(TMW,'NumPeriods',18)

## NumPeriods - Period difference

12 (default) | positive integer
Period difference, specified as the comma-separated pair consisting of 'NumPeriods ' and a scalar positive integer.
Data Types: double

## Output Arguments

PriceChangeRate - Closing price rate-of-change
matrix | table | timetable
Closing price rate-of-change, returned with the same number of rows ( $M$ ) and the same type (matrix, table, or timetable) as the input Data.

## References

[1] Achelis, S. B. Technical Analysis from A to Z. Second Edition. McGraw-Hill, 1995, pp. 243-245.

## See Also

table|timetable|volroc

## Topics

"Using Timetables in Finance"

[^14]
## prdisc

Price of discounted security

## Syntax

```
Price = prdisc(Settle,Maturity,FaceDiscount)
```

Price = prdisc(
$\qquad$ ,Basis)

## Description

Price $=$ prdisc(Settle, Maturity, FaceDiscount) returns the price of a security whose yield is quoted as a bank discount rate (for example, U. S. Treasury bills).

Price = prdisc( __ , Basis) adds an optional argument for Basis.

## Examples

## Calculate the Price of a Security Whose Yield is Quoted as a Bank Discount Rate

This example shows how to return the price of a security whose yield is quoted as a bank discount rate (for example, U. S. Treasury bills).

```
Settle = '10/14/2000';
Maturity = '03/17/2001';
Face = 100;
Discount = 0.087;
Basis = 2;
Price = prdisc(Settle, Maturity, Face, Discount, Basis)
Price = 96.2783
```


## Calculate the Price of a Security Whose Yield is Quoted as a Bank Discount Rate Using datetime Inputs

This example shows how to use datetime inputs to return the price of a security whose yield is quoted as a bank discount rate (for example, U. S. Treasury bills).

```
Settle = '10/14/2000';
Maturity = '03/17/2001';
Face = 100;
Discount = 0.087;
Basis = 2;
Price = prdisc(datetime(Settle,'Locale','en_US'),datetime(Maturity,'Locale','en_US'),
Price = 96.2783
```


## Input Arguments

## Settle - Settlement date

serial date number | date character vector | datetime
Settlement date, specified as serial date numbers, date character vectors, or datetime arrays.

Settle must be earlier than Maturity.
Data Types: double | datetime | char

## Maturity - Maturity date

serial date number | date character vector | datetime
Maturity date, specified as serial date numbers, date character vectors, or datetime arrays.

Data Types: double | datetime | char

## Face - Redemption (par, face) value

numeric
Redemption (par, face) value, specified as a numeric value.
Data Types: double

## Discount - Bank discount rate of the security <br> decimal fraction

Bank discount rate of the security, specified as a decimal fraction value.

## Data Types: double

## Basis - Day-count basis of instrument

0 (default) | numeric values: $0,1,2,3,4,6,7,8,9,10,11,12,13$
(Optional) Day-count basis of the instrument, specified as a numeric value. Allowed values are:

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- 2 = actual/360
- 3 = actual/365
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- $7=$ actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- $9=$ actual/360 (ICMA)
- $10=$ actual/365 (ICMA)
- $11=30 / 360 \mathrm{E}$ (ICMA)
- $12=$ actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-0
Data Types: double

## Output Arguments

## Price - Price of discounted security

numeric

Price of discounted security, returned as a numeric value.

## References

[1] Mayle. "Standard Securities Calculation Methods." Volumes I-II, 3rd edition. Formula 2.

## See Also

acrudisc| bndprice|datetime|discrate|prmat|ylddisc

## Topics

"Pricing Functions" on page 2-32
"Fixed-Income Terminology" on page 2-22

Introduced before R2006a

## priceandvol

Price and Volume chart

Note priceandvol is updated to accept data input as a matrix, timetable, or table.
The syntax for priceandvol has changed. Previously, when using table input, the first column of dates could be serial date numbers, date character vectors, or datetime arrays, and you were required to have specific number of columns.

When using table input, the new syntax for priceandvol supports:

- No need for time information. If you want to pass in date information, use timetable input.
- No requirement of specific number of columns. However, you must provide valid column names. linebreak must contain columns named 'open', 'high', 'low', 'close', and 'volume' (case insensitive).


## Syntax

priceandvol(Data)
h = priceandvol(Data)

## Description

priceandvol(Data) plots two charts from a series of opening, high, low, closing prices, and traded volume. Opening, high, low, and closing prices are on one axis and the volume series are on a second axis.
$\mathrm{h}=$ priceandvol(Data) adds a Graphic handle for the figure.

## Examples

## Generate a Price and Volume Chart for a Data Series for a Stock

Load the file SimulatedStock.mat, which provides a timetable (TMW) for financial data for TMW stock. This price and volume chart for the TMW stock contains the open, high, low, close, and volume for the most recent 21 days.
load SimulatedStock.mat priceandvol(TMW(end-20:end,:));



## Input Arguments

## Data - Data for opening, high, low, closing prices, and volume traded matrix | table | timetable

Data for opening, high, low, closing prices, and volume traded, specified as a matrix, table, or timetable. For matrix input, Data is an M-by-5 matrix of opening, high, low, closing prices and traded volume. Timetables and tables with M rows must contain variables named 'Open', 'High', 'Low', 'Close', and 'Volume' (case insensitive).

Data Types: double| table| timetable

## Output Arguments

## h - Graphic handle of the figure <br> handle object

Graphic handle of the figure, returned as a handle object.

## See Also

candle|highlow|kagi|linebreak|movavg|pointfig|table|timetable| volarea

## Topics

"Using Timetables in Finance"
"Convert Financial Time Series Objects fints to Timetables" on page 12-2
Introduced in R2008a

## prmat

Price with interest at maturity

## Syntax

```
[Price,AccruInterest] = prmat(Settle,Maturity,Issue,Face,CouponRate
Yield)
[Price,AccruInterest] = prmat(
```

$\qquad$

``` ,Basis)
```


## Description

[Price,AccruInterest] = prmat(Settle,Maturity, Issue,Face, CouponRate Yield) returns the price and accrued interest of a security that pays interest at maturity. This function also applies to zero coupon bonds or pure discount securities by setting CouponRate $=0$.
[Price,AccruInterest] = prmat( $\qquad$ , Basis) adds an optional argument for Basis.

## Examples

## Find the Yield of a Security Paying Interest at Maturity

This example shows how to find the yield of a security paying interest at maturity for the following.

```
Settle = '02/07/2000';
Maturity = '04/13/2000';
Issue = '10/11/1999';
Face = 100;
Price = 99.98;
CouponRate = 0.0608;
Basis = 1;
```

```
Yield = yldmat(Settle, Maturity, Issue, Face, Price,...
CouponRate, Basis)
```

Yield = 0.0607

## Find the Yield of a Security Paying Interest at Maturity Using datetime Inputs

This example shows how to use datetime inputs find the yield of a security paying interest at maturity for the following:

```
Settle = '7-Feb-2000';
Maturity = '13-Apr-2000';
Issue = '11-0ct-1999';
Face = 100;
Price = 99.98;
CouponRate = 0.0608;
Basis = 1;
Settle = datetime(Settle,'Locale','en_US');
Maturity = datetime(Maturity,'Locale','en_US');
Issue = datetime(Issue,'Locale','en_US');
Yield = yldmat(Settle, Maturity, Issue, Face, Price,...
CouponRate, Basis)
Yield = 0.0607
```


## Input Arguments

## Settle - Settlement date of security

serial date number | date character vector | datetime
Settlement date of the security, specified as serial date numbers, date character vectors, or datetime arrays. The Settle date must be before the Maturity date.

Data Types: double | char | datetime

## Maturity - Maturity date of security

serial date number | date character vector | datetime

Maturity date of the security, specified as serial date numbers, date character vectors, or datetime arrays.

## Data Types: double | char | datetime

## Issue - Issue date

serial date number | date character vector | datetime
Issue date of the security, specified as serial date numbers, date character vectors, or datetime arrays.
Data Types: double | char | datetime

## Face - Redemption value

numeric
Redemption value (par value), specified as a numeric value.
Data Types: double

## CouponRate - Coupon rate <br> decimal fraction

Coupon rate, specified as a decimal fraction value.

## Data Types: double

## Yield - Annual yield <br> decimal fraction

Annual yield, specified as a decimal fraction value.
Data Types: double

## Basis - Day-count basis

0 (actual/actual) (default) | integers of the set [0...13] | vector of integers of the set [0...13]
(Optional) Day-count basis of the security, specified using the following values:

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- 2 = actual/360
- 3 = actual/365
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- $7=$ actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- $9=$ actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- $12=$ actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-0
Data Types: double

## Output Arguments

## Price - Security price

numeric
Security price, returned as a numeric value.

## AccruInterest - Accrued interest

numeric
Accrued interest for security, returned as a numeric value.

## References

[1] Mayle, J. Standard Securities Calculation Methods. Volumes I-II, 3rd edition. Formula 3.

## See Also

acrudisc| bndprice| bndyield|datetime|prmat|ylddisc|yldtbill

## Topics

"Yield Functions" on page 2-32
"Yield Conventions" on page 2-31

## Introduced before R2006a

## prtbill

Price of Treasury bill

## Syntax

Price = prtbill(Settle,Maturity,Face,Discount)

## Description

Price = prtbill(Settle,Maturity,Face,Discount) returns the price for a Treasury bill.

## Examples

## Calculate the Price for a Treasury Bill

This example shows how to return the price for a Treasury bill, where the settlement date of a Treasury bill is February 10, 2002, the maturity date is August 6, 2002, the discount rate is $3.77 \%$, and the par value is $\$ 1000$.

```
Price = prtbill('2/10/2002', '8/6/2002', 1000, 0.0377)
```

Price $=981.4642$

## Calculate the Price for a Treasury Bill Using datetime Inputs

This example shows how to use datetime inputs to return the price for a Treasury bill, where the settlement date of a Treasury bill is February 10, 2002, the maturity date is August 6,2002 , the discount rate is $3.77 \%$, and the par value is $\$ 1000$.

```
Price = prtbill(datetime('10-Feb-2002','Locale','en_US'), datetime('6-Aug-2002','Locale
```


## Input Arguments

## Settle - Settlement date for Treasury bill

serial date number | date character vector | datetime
Settlement date for the Treasury bill, specified as serial date numbers, date character vectors, or datetime arrays. The Settle date must be before the Maturity date.
Data Types: double | char | datetime

## Maturity - Maturity date for Treasury bill

serial date number | date character vector | datetime
Maturity date for the Treasury bill, specified as serial date numbers, date character vectors, or datetime arrays.
Data Types: double | char | datetime

## Face - Redemption value of Treasury bill

numeric
Redemption value (par value) of the Treasury bill, specified as a numeric value.
Data Types: double
Discount - Discount rate of Treasury bill
decimal fraction
Discount rate of the Treasury bill, specified as a decimal fraction value.
Data Types: double

## Output Arguments

## Price - Treasury bill price

numeric
Treasury bill price, returned as a numeric value.

## References

[1] Bodie, Kane, and Marcus. Investments. McGraw-Hill Education, 2013.

## See Also <br> beytbill|datetime|yldtbill

## Topics

"Computing Treasury Bill Price and Yield" on page 2-38
"Treasury Bills Defined" on page 2-37

Introduced before R2006a

## pvfix

Present value with fixed periodic payments

## Syntax

PresentVal = pvfix(Rate,NumPeriods, Payment, ExtraPayment,Due)

## Arguments

| rate | Periodic interest rate, as a decimal fraction. |
| :--- | :--- |
| NumPeriods | Number of periods. |
| Payment | Periodic payment. |
| ExtraPayment | (Optional) Payment received other than Payment in the last <br> period. Default $=0$. |
| Due | (Optional) When payments are due or made: $0=$ end of <br> period (default), or $1=$ beginning of period. |

## Description

PresentVal = pvfix(Rate,NumPeriods, Payment, ExtraPayment, Due) returns the present value of a series of equal payments.

## Examples

## Calculate the Present Value of a Series of Equal Payments

This example shows how to return the present value of a series of equal payments, where $\$ 200$ is paid monthly into a savings account earning $6 \%$. The payments are made at the end of the month for five years.

PresentVal $=$ pvfix(0.06/12, 5*12, 200, 0, 0)

```
PresentVal = 1.0345e+04
```


## See Also

fvfix|fvvar| payper | pvvar

## Topics

"Analyzing and Computing Cash Flows" on page 2-18

Introduced before R2006a

## pvtrend

Price and Volume Trend (PVT)

Note pvtrend has been partially removed and will no longer accept a fints object (tsobj) argument. Use a matrix, timetable, or table instead for financial time series.

Use fts2timetable to convert a fints object to a timetable object.

## Syntax

trend $=$ pvtrend (Data)

## Description

trend $=$ pvtrend (Data) calculates the Price and Volume Trend (PVT) from the series of closing stock prices and trade volume.

## Examples

## Calculate the Price and Volume Trend for a Data Series for a Stock

Load the file SimulatedStock. mat, which provides a timetable (TMW) for financial data for TMW stock.

```
load SimulatedStock.mat
trend = pvtrend(TMW);
plot(trend.Time,trend.PriceVolumeTrend)
title('Price and Volume Trend for TMW')
```



## Input Arguments

## Data - Data for closing prices and trade volume

matrix | table | timetable
Data for closing prices and trade volume, specified as a matrix, table, or timetable. For matrix input, Data is an M-by-2 matrix of closing prices and trade volume. Timetables and tables with $M$ rows must contain variables named 'Close' and 'Volume' (case insensitive).

Data Types: double| table | timetable

## Output Arguments

## trend - Price and volume trend <br> matrix | table | timetable

Price and volume trend (PVT), returned with the same number of rows (M) and the same type (matrix, table, or timetable) as the input Data.

## References

[1] Achelis, S. B. Technical Analysis from A to Z. Second Edition. McGraw-Hill, 1995, pp. 239-240.

## See Also

onbalvol|table| timetable|tsaccel|volroc

## Topics

"Using Timetables in Finance"
"Convert Financial Time Series Objects fints to Timetables" on page 12-2
Introduced before R2006a

## pvvar

Present value of varying cash flow

## Syntax

PresentVal = pvvar(CashFlow,Rate,CFDates)

## Arguments

| CashFlow | A vector of varying cash flows. Include the initial investment as the <br> initial cash flow value (a negative number). If CashFlow is a matrix, <br> each column is treated as a separate cash-flow stream. |
| :--- | :--- |
| Rate | Periodic interest rate. Enter as a decimal fraction. If CashFlow is a <br> matrix, a scalar Rate is allowed when the same rate applies to all <br> cash-flow streams in CashFlow. When multiple cash-flow streams <br> require different discount rates, Rate must be a vector whose <br> length equals the number of columns in CashFlow. |
| CFDates | (Optional) A vector of serial date numbers, date character vectors, <br> or datetime arrays on which the cash flows occur. Specify CFDates <br> when there are irregular (nonperiodic) cash flows. The default <br> assumes that CashFlow contains regular (periodic) cash flows. If <br> CashFlow is a matrix, and all cash-flow streams share the same <br> dates, CFDates can be a vector whose length matches the number <br> of rows in CashFlow. When different cash-flow streams have <br> different payment dates, specify CFDates as a matrix the same size <br> as CashFlow. |

## Description

PresentVal = pvvar(CashFlow, Rate, CFDates) returns the net present value of a varying cash flow. Present value is calculated at the time the first cash flow occurs.

## Examples

This cash flow represents the yearly income from an initial investment of $\$ 10,000$. The annual interest rate is $8 \%$.

| Year 1 | $\$ 2000$ |
| :--- | :--- |
| Year 2 | $\$ 1500$ |
| Year 3 | $\$ 3000$ |
| Year 4 | $\$ 3800$ |
| Year 5 | $\$ 5000$ |

To calculate the net present value of this regular cash flow

```
PresentVal = pvvar([-10000 2000 1500 3000 3800 5000], 0.08)
```

returns
PresentVal =
1715.39

An investment of $\$ 10,000$ returns this irregular cash flow. The original investment and its date are included. The periodic interest rate is $9 \%$.

| Cash Flow | Dates |
| :--- | :--- |
| $(\$ 10000)$ | January 12, 1987 |
| $\$ 2500$ | February 14,1988 |
| $\$ 2000$ | March 3, 1988 |
| $\$ 3000$ | June 14, 1988 |
| $\$ 4000$ | December 1, 1988 |

To calculate the net present value of this irregular cash flow
CashFlow = [-10000, 2500, 2000, 3000, 4000];
CFDates $=[$ '01/12/1987'
'02/14/1988'
'03/03/1988'
'06/14/1988'

```
    '12/01/1988'];
PresentVal = pvvar(CashFlow, 0.09, CFDates)
returns
PresentVal =
    142.16
```

The net present value of the same investment under different discount rates of $7 \%, 9 \%$, and $11 \%$ is obtained in a single call:

```
PresentVal = pvvar(repmat(CashFlow,3,1)', [.07 . 09 .11], CFDates)
pv =
    419.0136 142.1648-122.1275
```


## See Also

datetime|fvfix|fvvar|irr|payuni|pvfix

## Topics

"Analyzing and Computing Cash Flows" on page 2-18

## Introduced before R2006a

## pyld2zero

Zero curve given par yield curve

Note In R2017b, the specification of optional input arguments has changed. While the previous ordered inputs syntax is still supported, it may no longer be supported in a future release. Use the new optional name-value pair inputs: InputCompounding, InputBasis, OutputCompounding, and OutputBasis.

## Syntax

[ZeroRates,CurveDates] = pyld2zero(ParRates,CurveDates,Settle)
[ZeroRates,CurveDates] = pyld2zero( $\qquad$ , Name, Value)

## Description

[ZeroRates,CurveDates] = pyld2zero(ParRates,CurveDates,Settle) returns a zero curve given a par yield curve and its maturity dates. If either input for CurveDates or Settle is a datetime array, CurveDates is returned as a datetime array. Otherwise, CurveDates is returned as a serial date number.
[ZeroRates,CurveDates] = pyld2zero( __ , Name, Value) adds optional namevalue pair arguments

## Examples

## Compute Zero Curve Given Par Yield Curve

Define the settlement date, maturity, and zero rates.

```
Settle = datenum('01-Feb-2013');
CurveDates = datemnth(Settle,12*[1 2 3 5 7 10 20 30]');
ZeroRates = [.11 0.30 0.64 1.44 2.07 2.61 3.29 3.55]'/100;
InputCompounding = 2;
```

InputBasis = 1;
OutputCompounding = 2;
OutputBasis = 1;
Compute par yield curve from zero rates.
ParRates = zero2pyld(ZeroRates, CurveDates, Settle,'InputCompounding',2,... 'InputBasis',1,'OutputCompounding',2,'OutputBasis',1)

ParRates $=8 \times 1$
0.0011
0.0030
0.0064
0.0142
0.0201
0.0251
0.0309
0.0330

Compute zero curve from the par yield curve.
ZeroRates = pyld2zero(ParRates, CurveDates, Settle,'InputCompounding',2,... 'InputBasis',1,'OutputCompounding',2,'OutputBasis',1)

ZeroRates = 8×1
0.0011
0.0030
0.0064
0.0144
0.0207
0.0261
0.0329
0.0355

## Compute Zero Curve Given Par Yield Curve Using datetime Inputs

Use datetime inputs to compute the zero curve given the par yield curve.
Settle = datenum('01-Feb-2013');

```
CurveDates = [datenum('01-Feb-2014')
    datenum('01-Feb-2015')
    datenum('01-Feb-2016')
    datenum('01-Feb-2018')
    datenum('01-Feb-2020')
    datenum('01-Feb-2023')
    datenum('01-Feb-2033')
    datenum('01-Feb-2043')];
OriginalParRates = [0.11 0.30 0.64 1.42 2.02 2.51 3.10 3.31]'/100;
InputCompounding = 1;
InputBasis = 0;
OutputCompounding = 1;
OutputBasis = 0;
Settle = datetime(Settle, 'ConvertFrom', 'datenum','Locale','en_US');
CurveDates = datetime(CurveDates, 'ConvertFrom', 'datenum','Locale','en_US');
[ZeroRates Dates] = pyld2zero(OriginalParRates, CurveDates, Settle, ...
'OutputCompounding', OutputCompounding, 'OutputBasis', OutputBasis, ...
'InputCompounding', InputCompounding, 'InputBasis', InputBasis)
ZeroRates = 8×1
    0.0011
    0.0030
    0.0064
    0.0144
    0.0207
    0.0261
    0.0329
    0.0356
Dates = 8x1 datetime array
    01-Feb-2014 00:00:00
    01-Feb-2015 00:00:00
    01-Feb-2016 00:00:00
    01-Feb-2018 00:00:00
    01-Feb-2020 00:00:00
    01-Feb-2023 00:00:00
    01-Feb-2033 00:00:00
    01-Feb-2043 00:00:00
```


## Demonstrate a Roundtrip From pyld2zero to zero2pyld

Given the following a par yield curve and its maturity dates, return the ZeroRates.

```
Settle = datenum('01-Feb-2013');
CurveDates = [datenum('01-Feb-2014')
    datenum('01-Feb-2015')
    datenum('01-Feb-2016')
    datenum('01-Feb-2018')
    datenum('01-Feb-2020')
    datenum('01-Feb-2023')
    datenum('01-Feb-2033')
    datenum('01-Feb-2043')];
OriginalParRates = [0.11 0.30 0.64 1.42 2.02 2.51 3.10 3.31]'/100;
InputCompounding = 1;
InputBasis = 0;
OutputCompounding = 1;
OutputBasis = 0;
ZeroRates = pyld2zero(OriginalParRates, CurveDates, Settle, ...
'OutputCompounding', OutputCompounding, 'OutputBasis', OutputBasis, ...
'InputCompounding', InputCompounding, 'InputBasis', InputBasis)
ZeroRates = 8×1
    0.0011
    0.0030
    0.0064
    0.0144
    0.0207
    0.0261
    0.0329
    0.0356
```

With the ZeroRates, use the zero2pyld function to return the ParRates0ut and determine the roundtrip error.

```
ParRatesOut = zero2pyld(ZeroRates, CurveDates, Settle, ...
'OutputCompounding', OutputCompounding, 'OutputBasis', OutputBasis, ...
'InputCompounding', InputCompounding, 'InputBasis', InputBasis)
ParRatesOut = 8×1
    0.0011
    0.0030
    0.0064
    0.0142
    0.0202
    0.0251
    0.0310
    0.0331
max(abs(OriginalParRates - ParRatesOut)) % Roundtrip error
ans = 1.2750e-16
```


## Input Arguments

## ParRates - Annualized par yields <br> decimal fraction

Annualized par yields (coupon rates), specified as a NUMBONDS-by-1 vector using decimal fractions. In aggregate, the rates constitute an implied zero curve for the investment horizon represented by CurveDates.

## Data Types: double

## CurveDates - Maturity dates

serial date number | date character vector | datetime
Maturity dates which correspond to the input ParRates, specified as a NUMBONDS-by-1 vector using serial date numbers, date character vectors, or datetime arrays.

Data Types: double | datetime | char

## Settle - Common settlement date for ZeroRates

serial date number | date character vector | datetime

Common settlement date for input ParRates, specified as serial date numbers, date character vectors, or datetime arrays.

Data Types: double | datetime | char

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, ... , NameN, ValueN.

Example: [ZeroRates,CurveDates] = pyld2zero(ParRates,CurveDates,Settle, 'OutputCompounding', 3,'OutputBasis',5,'InputCompounding',4,'InputBasis',5)

## OutputCompounding - Compounding frequency of output ZeroRates

2 (default) | numeric values: $0,1,2,3,4,6,12,365,-1$
Compounding frequency of output ZeroRates, specified using the allowed values:

- 0 - Simple interest (no compounding)
- 1 - Annual compounding
- 2 - Semiannual compounding (default)
- 3-Compounding three times per year
- 4-Quarterly compounding
- 6-Bimonthly compounding
- 12 - Monthly compounding
- 365 - Daily compounding
-     - 1 - Continuous compounding


## Note

- If OutputCompounding is set to 0 (simple), - 1 (continuous), or 365 (daily), the InputCompounding must also be specified using a valid value.
- If OutputCompounding is not specified, then OutputCompounding is assigned the value specified for InputCompounding.
- If either OutputCompounding or InputCompounding are not specified, the default is 2 (semiannual) for both.


## Data Types: double

## OutputBasis - Day-count basis of output ZeroRates

0 (default) | numeric values: $0,1,2,3,4,6,7,8,9,10,11,12,13$
Day count basis of output ZeroRates, specified using allowed values:

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- 2 = actual/360
- 3 = actual/365
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- $7=$ actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- $9=$ actual/360 (ICMA)
- $10=$ actual/365 (ICMA)
- $11=30 / 360 \mathrm{E}$ (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-0

Note If OutputBasis is not specified, then OutputBasis is assigned the value specified for InputBasis. If either InputBasis or OutputBasis are not specified, the default is 0 (actual/actual) for both.

## Data Types: double

## InputCompounding - Compounding frequency of input ParRates

## 2 (default) | numeric values: $0,1,2,3,4,6,12,365,-1$

Compounding frequency of input ParRates, specified using allowed values:

- 1 - Annual compounding
- 2 - Semiannual compounding (default)
- 3-Compounding three times per year
- 4 - Quarterly compounding
- 6 - Bimonthly compounding
- 12 - Monthly compounding


## Note

- If OutputCompounding is $1,2,3,4,6$, or 12 and InputCompounding is not specified, the value of OutputCompounding is used.
- If OutputCompounding is 0 (simple), - 1 (continuous), or 365 (daily), a valid InputCompounding value must also be specified.
- If either InputCompounding or OutputCompounding are not specified, the default is 2 (semiannual) for both.


## Data Types: double

## InputBasis - Day-count basis of input ParRates

0 (default) | numeric values: $0,1,2,3,4,6,7,8,9,10,11,12,13$
Day count basis of the input ParRates, specified using allowed values:

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- 2 = actual/360
- 3 = actual/365
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- $9=\mathrm{actual} / 360$ (ICMA)
- $10=$ actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-0

Note If InputBasis is not specified, then InputBasis is assigned the value specified for OutputBasis. If either InputBasis or Outputbasis are not specified, the default is 0 (actual/actual) for both.

Data Types: double

## Output Arguments

## ZeroRates - Zero rates

numeric
Zero rates, returned as a NUMBONDS-by-1 numeric vector. In aggregate, the rates in ZeroRates constitute a zero curve for the investment horizon represented by CurveDates. ZeroRates are ordered by ascending maturity.

CurveDates - Maturity dates that correspond to ZeroRates
serial date number | date character vector | datetime
Maturity dates that correspond to the ZeroRates, returned as a NUMBONDS-by-1 vector of maturity dates that correspond to each par rate contained in ZeroRates. CurveDates are ordered by ascending maturity.

If either input for CurveDates or Settle is a datetime array, CurveDates is returned as a datetime array. Otherwise, CurveDates are returned as a serial date numbers.
See Alsodatetime | datetime | disc2zero| fwd2zero|getForwardRates|zbtprice|
zbtyield|zero2disc|zero2fwd|zero2pyld

## Topics

"Term Structure of Interest Rates" on page 2-42
"Sensitivity of Bond Prices to Interest Rates" on page 10-2
"Bond Prices and Yield Curve Parallel Shifts" on page 10-10
"Bond Prices and Yield Curve Nonparallel Shifts" on page 10-15
"Term Structure Analysis and Interest-Rate Swaps" on page 10-22
"Fixed-Income Terminology" on page 2-22
Introduced before R2006a

## quarter

Returns the quarter of given date

## Syntax

```
q = quarter(date)
q = quarter(
,month1)
q = quarter(\overline{date,month1,dateformat)}
```


## Description

$\mathrm{q}=$ quarter(date) returns the quarter of the given date, assuming the standard calendar (starting on January 1st).
$\mathrm{q}=$ quarter( $\qquad$ , month1) returns the quarter of the date for a calendar which starts on the month specified by month1. month1 must be an integer between 1-12 representing Jan-Dec respectively.
$\mathrm{q}=$ quarter(date, month1, dateformat) returns the quarter of the date for a calendar which starts on the month specified by month1. month1 must be an integer between 1-12 representing January to December respectively. The dateformat input is a character vector to specify the format of your date character vector in case it is not normally recognized by the datenum function.

## Examples

## Determine the Quarter for a Given Date

quarter returns the quarter of the given date, assuming the standard calendar that starts on Jan 1st.

```
quarter('7-Apr-2013')
ans = 2
```


## Determine the Quarter for a Given Date for an Off-Cycle Calendar

quarter returns the quarter of the date for a calendar which starts on the month specified by month1.

```
quarter('7-Apr-2013',5)
ans = 4
```

If the financial calendar starts in May (where month1 = 5), April would be the last month and fall in the last quarter.

## Determine the Quarter for a Given Date When the Date is Not Recognized by datenum

When using quarter, the optional input argument for dateformat is a character vector which lets you specify the format of your date character vector in case it isn't normally recognized by the datenum function.

```
quarter('07-04-2013',1)
ans = 3
```

This gives the quarter 3 because by default datenum interprets the date as July 7th, 2013.

If you really meant April 7th, 2013, you can use dateformat to specify the intended format.
quarter('07-04-2013',1,'dd-mm-yyyy')
ans $=2$

## Input Arguments

## date - Date in a quarter

serial date number | date character vector

Date in a quarter, specified as serial date numbers or date character vectors.

## Data Types: double | char

## month1 - First month in a calendar

integer with value between 1-12
First month in a calendar, specified as an integer with a value between 1-12, representing January to December respectively. Use month1 when the standard calendar that starts on January 1st does not apply.
Data Types: double
dateformat - Format of date
character vector
Format of date, specified as a character vector. dateformat input is a character vector to specify the format of your date character vector in case it is not normally recognized by the datenum function.

Data Types: char

## Output Arguments

$q$ - Quarter for given date
integer
Quarter for given date, returned as an integer between 1-4.

## See Also

datenum

Introduced in R2016a

## rdivide

Financial time series division

Note rdivide will be removed in a future release. Use timetable instead. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

```
newfts = tsobj_1 ./ tsobj_2
newfts = tsobj ./ array
newfts = array ./ tsobj
```


## Arguments

| tsobj_1, tsobj_2 | Pair of financial time series objects. |
| :--- | :--- |
| array | Scalar value or array with the number of rows equal to the <br> number of dates in tsobj and the number of columns equal <br> to the number of data series in tsobj. |

## Description

The rdivide method divides, element by element, the components of one financial time series object by the components of the other. You can also divide the whole object by an array or divide a financial time series object into an array.

If an object is to be divided by another object, both objects must have the same dates and data series names, although the order need not be the same. The order of the data series, when an object is divided by another object, follows the order of the first object.
newfts = tsobj_1 ./ tsobj_2 divides financial time series objects element by element.
> newfts = tsobj ./ array divides a financial time series object element by element by an array.
> newfts = array ./ tsobj divides an array element by element by a financial time series object.

> For financial time series objects, the rdivide operation is identical to the mrdivide operation.

See Also<br>minus|mrdivide|plus|times<br>\section*{Topics}<br>"Financial Time Series Operations" on page 13-7<br>Introduced before R2006a

## renko

Renko

Note renko is updated to accept data input as a matrix, timetable, or table.
The syntax for renko has changed. Previously, when using table input, the first column of dates could be serial date numbers, date character vectors, or datetime arrays, and you were required to have specific number of columns.

When using table input, the new syntax for renko supports:

- No need for time information. If you want to pass in date information, use timetable input.
- No requirement of specific number of columns. However, you must provide valid column names. renko must contain a column named 'price' (case insensitive).


## Syntax

```
renko(Data)
renko(Data,Threshold,)
h = renko(ax,
```

$\qquad$

``` )
```


## Description

renko (Data) plots a Renko chart from a series of prices of a security.
renko (Data, Threshold, ) adds an optional argument for Threshold.
h = renko(ax, $\qquad$ ) adds an optional argument for ax.

## Examples

## Generate a Renko Chart for a Data Series for a Stock

Load the file SimulatedStock.mat, which provides a timetable (TMW) for financial data for TMW stock. This example shows how to plot a Renko chart for the most recent 21 days. Note that the variable name of asset price is be renamed to 'Price ' (case insensitive).
load SimulatedStock.mat
TMW.Properties.VariableNames\{'Close'\} = 'Price'; renko(TMW(end-20:end,:))
title('Renko Chart for TMW')
Renko Chart for TMW



## Input Arguments

## Data - Data for a series of prices

matrix | table | timetable
Data for a series of prices, specified as a matrix, table, or timetable. Timetables and tables with M rows must contain a variable named 'Price' (case insensitive).

## Data Types: double | table | timetable

## Threshold - (Optional) Least price change value when adding a new box 1 (default) | positive numeric

Least price change value when adding a new box, specified as a scalar positive numeric value.

Data Types: double

## ax - Valid axis object

current axes (ax = gca) (default) | axes object
(Optional) Valid axis object, specified as an axes object. The renko plot is created in the axes specified by ax instead of in the current axes ( $\mathrm{ax}=\mathrm{gca}$ ). The option ax can precede any of the input argument combinations.

Data Types: object

## Output Arguments

## h - Graphic handle of the figure

handle object
Graphic handle of the figure, returned as a handle object.

## See Also

candle|highlow|kagi|linebreak|movavg|pointfig|priceandvol|table| timetable|volarea

## Topics

"Using Timetables in Finance"
"Convert Financial Time Series Objects fints to Timetables" on page 12-2

Introduced in R2008a

19-1482

## resamplets

Downsample data

Note resamplets will be removed in a future release. Use timetable instead. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

newfts $=$ resamplets(oldfts,samplestep)

## Description

newfts = resamplets(oldfts, samplestep) downsamples the data contained in the financial time series object oldfts every samplestep periods. For example, to have the new financial time series object contain every other data element from oldfts, set samplestep to 2.
newfts is a financial time series object containing the same data series (names) as the input oldfts.

## See Also

filter

## Topics

"Data Transformation and Frequency Conversion" on page 13-13

Introduced before R2006a

## ret2tick

Convert return series to price series

Note ret2tick accepts Data as a matrix, timetable, or table.

## Syntax

[TickSeries,TickTimes] = ret2tick(Data) [TickSeries,TickTimes] = ret2tick( $\qquad$ ,Name, Value)

## Description

[TickSeries,TickTimes] = ret2tick(Data) computes prices from the start prices of NASSET assets and NUMOBS return observations.
[TickSeries,TickTimes] = ret2tick( $\qquad$ , Name, Value) adds optional namevalue pair arguments.

## Examples

## Convert a Return Series to a Price Series

Compute the price increase of two stocks over a year's time based on three incremental return observations.

```
RetSeries = [0.10 0.12
    0.050 .04
    -0.05 0.05];
RetIntervals = [182
    91
    92];
```

```
StartTime = datetime('18-Dec-2000');
[TickSeries,TickTimes] = ret2tick(RetSeries,'ReturnIntervals',RetIntervals,...
'StartTime',StartTime)
TickSeries = 4×2
    1.0000 1.0000
    1.1000 1.1200
    1.1550 1.1648
    1.0973 1.2230
TickTimes = 4x1 datetime array
    18-Dec-2000
    18-Jun-2001
    17-Sep-2001
    18-Dec-2001
```


## Convert a Price Series to a Return Series Using timetable Input

Use timetable input to convert a price series to a return series, given periodic returns of two stocks observed in the first, second, third, and fourth quarters.

```
RetSeries = [0.10 0.12
    0.05 0.04
    -0.05 0.05];
```

RetTimes = datetime(\{'6/18/2001','9/17/2001','12/18/2001'\},'InputFormat','MM/dd/uuuu',
RetSeries = array2timetable(RetSeries,'RowTimes',RetTimes);
StartTime = datetime('18-Dec-2000');
[TickSeries,TickTimes] = ret2tick(RetSeries,'StartTime',StartTime)
TickSeries=4×2 timetable
Time RetSeries1 RetSeries2
18-Dec-2000 1
18-Jun-2001
1.1
1.12

19-1485

```
17-Sep-2001 1.155 1.1648
18-Dec-2001 1.0973
1.223
TickTimes = 4x1 datetime array
    18-Dec-2000
    18-Jun-2001
    17-Sep-2001
    18-Dec-2001
```


## Input Arguments

## Data - Data for asset returns

matrix | table | timetable
Data for asset returns, specified as a matrix, table, or timetable of NUMOBS-by-NASSETS asset returns. The returns are not normalized by the time increments between successive price observations.

Data Types: double | table | timetable

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Example: [TickSeries,TickTimes] = ret2tick(RetSeries,'StartTime',StartTime)

## StartPrice - Initial prices for each asset

1 for all assets (default) | numeric
Initial prices for each asset, specified as the comma-separated pair consisting of 'StartPrice' and a NASSETS-by-1 vector indicating initial prices for each asset, or a single scalar initial price applied to all assets.

Data Types: double

## ReturnIntervals - Return interval between prices

## 1 for all assets (default) | numeric | duration | calendar duration

Return interval between prices, specified as the comma-separated pair consisting of 'ReturnIntervals' and a scalar return interval applied to all returns, or a vector of length NUMOBS return intervals between successive returns. ReturnIntervals is defined as:

ReturnIntervals(t) = TickTimes(t) - TickTimes(t-1).

Note If the type of Data is a timetable, ReturnIntervals is ignored.

## Data Types: double

## StartTime - Starting time for first observation applied to the prices of all assets

## 0 if ReturnIntervals is numeric (default) | numeric | duration | calendar duration

Starting time for first observation applied to the price series of all assets, specified as the comma-separated pair consisting of 'StartTime' and a scalar string, character vector, double, or datetime.

Note If ReturnIntervals is a duration or calendar duration value, the default for StartTime is datetime('today').

If Data is a timetable and StartTime is not specified, the resulting asset prices in the first period are not reported.

## Data Types: double | string | char|datetime

## Method - Method to convert asset returns to prices

'Simple' (default)| character vector with value of 'Simple' or 'Continuous' | string with value of "Simple" or "Continuous"

Method to convert asset returns to prices, specified as the comma-separated pair consisting of 'Method ' and a string or character vector indicating the method to convert asset prices to returns.

If the method is 'Simple', then simple periodic returns are used:

TTickSeries(t) $=$ TickSeries(t-1)*(1 + ReturnSeries(t)).
If the method is 'Continuous' , then continuous returns are used:
TickSeries(t) $=$ TickSeries(t-1)*exp(ReturnSeries(t)).
Data Types: char|string

## Output Arguments

## TickSeries - Time series array of asset prices matrix | table | timetable

Time series array of asset prices, returned as NUMBOBS+1-by-NASSETS time series of asset prices of the same type (matrix, table, or timetable) as the input Data. The first row contains the oldest prices and the last row contains the most recent. Prices across a given row are assumed to occur at the same time for all columns, and each column is a price series of an individual asset.

## TickTimes - Observation times associated with prices in TickSeries vector

Observation times associated with the prices in TickSeries, returned as a NUMB0BS+1 length column vector of monotonically increasing observation times associated with the prices in TickSeries. The initial time is StartTime. For matrix and table Data, sequential observations occur at increments specified in ReturnIntervals and for Data timetables, sequential observations are derived from times and dates in Data.

## Extended Capabilities

## Tall Arrays

Calculate with arrays that have more rows than fit in memory.
This function supports input Data that is specified as a tall column vector, a tall table, or a tall timetable. For more information, see tall and "Tall Arrays" (MATLAB).
See Alsodatetime|tick2ret
Topics"Data Transformation and Frequency Conversion" on page 13-13
Introduced before R2006a

## ret2tick (fts)

Convert return series to price series for time series object

Note ret2tick (fts) will be removed in a future release. Use ret2tick instead.

## Syntax

```
priceFts = ret2tick(returnFts)
priceFts = ret2tick(returnFts,'PARAM1',VALUE1,'PARAM2',VALUE2', ...)
```


## Arguments

| returnFts | Financial time series object of returns. |
| :--- | :--- |
| 'PARAM1' | (Optional) StartPrice is a Numeric value and is a scalar <br> or 1-by-N vector of initial prices for each asset. If <br> StartPrice is unspecified or empty, the initial price of all <br> assets is 1. |
| 'PARAM2' | (Optional) StartTime is Date value for a scalar date <br> number or a single date character vector specifying the <br> starting time for the first observation. This date is applied to <br> the price series of all assets. |
| Note The first period price value of the resulting price <br> series will not be reported if StartTime is not specified. The <br> resulting price series are scaled based on the StartPrice, <br> even if StartTime is not supplied. |  |


| 'PARAM3' | (Optional) Method is a character vector indicating the <br> method to convert asset returns to prices. The value must be <br> defined as 'Simple' (default) or 'Continuous '. If Method <br> is 'Simple ' , ret2tick uses simple periodic returns. If <br> Method is 'Continuous ' , the function uses continuously <br> compounded returns. Case is ignored for Method. |
| :--- | :--- |

## Description

priceFts = ret2tick(returnFts,'PARAM1',VALUE1,'PARAM2',VALUE2', ...) generates a financial time series object of prices.

If Method is unspecified or 'Simple', the prices are
PriceSeries(i+1) = PriceSeries(i)*[1 + ReturnSeries(i)]
If Method is 'Continuous', the prices are

```
PriceSeries(i+1) = PriceSeries(i)*exp[ReturnSeries(i)]
```


## Examples

Compute the price series from the following return series:

```
RetSeries = [0.10 0.12
    0.05 0.04
    -0.05 0.05]
```

Use the following dates:

```
Dates = {'18-Jun-2001'; '17-Sep-2001'; '18-Dec-2001'}
```

where

```
ret = fints(Dates, RetSeries)
ret =
desc: (none)
freq: Unknown (0)
```

'dates: (3)' 'series1: (3)' 'series2: (3)'

| '18-Jun-2001' | [ | $0.1000]$ | $[$ | $0.1200]$ |
| :--- | :--- | ---: | :--- | :--- |
| '17-Sep-2001' | [ | $0.0500]$ | $[$ | $0.0400]$ |
| '18-Dec-2001' | $[$ | $-0.0500]$ | $[$ | $0.0500]$ |

PriceFtS is computed as:

```
PriceFts = ret2tick(ret, 'StartPrice', 100, 'StartTime', '18-Dec-2000')
```

PriceFts =
desc: (none)
freq: Unknown (0)

| dates: (4)' | 'series1: (4)' | 'series2: (4)' |
| :---: | :---: | :---: |
| '18-Dec-2000' | [ 100] | [ 100] |
| '18-Jun-2001' | [ 110.0000] | [ 112.0000] |
| '17-Sep-2001' | [ 115.5000] | $116.4800]$ |
| 18-Dec-2001' | 109.7250] | 122.3040] |

## See Also

portsim|tick2ret

## Introduced before R2006a

## rmfield

Remove data series

Note rmfield will be removed in a future release. Use timetable instead. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

fts = rmfield(tsobj,fieldname)

## Arguments

| tsobj | Financial time series object. |
| :--- | :--- |
| fieldname | Character vector containing the data series name to remove a single <br> series from the object. Cell array of character vectors for the data <br> series names to remove multiple data series from the object at the <br> same time. |

## Description

fts $=$ rmfield(tsobj,fieldname) removes the data series fieldname and its contents from the financial time series object tsobj.

## See Also

chfield|extfield|fieldnames | getfield|isfield

## Topics

"What Is the Financial Time Series App?" on page 14-2

Introduced before R2006a

## rsindex

Relative Strength Index (RSI)

Note rsindex has been partially removed and will no longer accept a fints object (tsobj) argument. Use a matrix, timetable, or table instead for financial time series.

Use fts2timetable to convert a fints object to a timetable object.

## Syntax

```
index = rsindex(Data)
index = rsindex(
,Name,Value)
```


## Description

index = rsindex(Data) calculates the Relative Strength Index (RSI) from the series of closing stock prices.
index $=$ rsindex (__ , Name, Value $)$ adds optional name-value pair arguments.

## Examples

## Calculate the Relative Strength Index for a Data Series for a Stock

Load the file SimulatedStock. mat, which provides a timetable (TMW) for financial data for TMW stock.

```
load SimulatedStock.mat
index = rsindex(TMW);
plot(index.Time,index.RelativeStrengthIndex)
title('Relative Strength Index for TMW')
```



## Input Arguments

## Data - Data with closing prices

matrix | table | timetable
Data with closing prices, specified as a matrix, table, or timetable. For matrix input, Data is M-by-1 with closing prices. Timetables and tables with M rows must contain a variable named 'Close' (case insensitive).

Data Types: double| table| timetable

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Example: index = rsindex(TMW, 'WindowSize',10)

## WindowSize - Moving window size for relative strength index

## 14 (default) | positive integer

Moving window size for relative strength index, specified as the comma-separated pair consisting of 'WindowSize' and a scalar positive integer.

Data Types: double

## Output Arguments

## index - Relative strength index <br> matrix | table | timetable

Relative strength index, returned with the same number of rows $(M)$ and the same type (matrix, table, or timetable) as the input Data.

## Definitions

## Relative Strength Index

Relative strength index is calculated by dividing the average of the gains by the average of the losses within a specified period.

RS = (average gains) / (average losses)

## References

[1] Murphy, John J. Technical Analysis of the Futures Market. New York Institute of Finance, 1986, pp. 295-302.

See Also<br>negvolidx| posvolidx| table|timetable<br>\section*{Topics}<br>"Using Timetables in Finance"<br>"Convert Financial Time Series Objects fints to Timetables" on page 12-2<br>Introduced before R2006a

## second

Seconds of date or time

## Syntax

```
Seconds = second(Date)
Seconds = second(
F)
```


## Description

Seconds $=$ second(Date) returns the seconds given a serial date number or a date character vector.

Seconds $=$ second (__, F) returns the second of one or more date character vectors, Date, using format defined by the optional input F. Date can be a character array where each row corresponds to one date character vector, or a one-dimensional cell array of character vectors. All the character vectors in Date must have the same format F. F must designate a supported date format symbol. For more information on supported date formats, see datestr.

## Examples

## Determine the Seconds of the Date for Various Dates

Find the seconds of the day (Date) using a serial date number.
Seconds $=$ second(738647.558427893)
Seconds = 8.1700
Find the seconds of the day (Date) using a date character vector format.

```
Seconds = second('06-May-2022, 13:24:08.17')
Seconds = 8.1700
```


## Input Arguments

## Date - Date to determine second

serial date number | date character vector | cell array of date character vectors
Date to determine second, specified as a serial date number or date character vector.
Date can be an array of date character vectors, where each row corresponds to one date character vector, or a one-dimensional cell array of character vectors. All the character vectors in Date must have the same format F. F must designate a supported date format symbol. For more information on supported date formats, see datestr
Data Types: single \| double \| char \| cell

## F - Date format symbol

character vector designating date format
Date format symbol, specified as a character vector to designate the date format symbol for input argument Date. For more information on supported date character vector formats, see datestr. Note, formats with ' $Q$ ' are not accepted.
Data Types: char

## Output Arguments

## Seconds - Seconds of date or time

serial date number | datetime array
Seconds of date or time, returned as a serial date number or date character vector.

## See Also

datevec|minute | second

## Topics

"Handle and Convert Dates" on page 2-2

Introduced before R2006a

## selectreturn

Portfolio configurations from 3-D efficient frontier

## Syntax

```
PortConfigs = selectreturn(AllMean,AllCovariance,Target)
```


## Arguments

| AllMean | Number of curves (NCURVES-by-1 cell array), where each <br> element is a 1-by-NASSETS (number of assets) vector of the <br> expected asset returns used to generate each curve on the <br> surface. |
| :--- | :--- |
| AllCovariance | NCURVES-by-1 cell array where each element is an NASSETS- <br> by-NASSETS vector of the covariance matrix used to <br> generate each curve on the surface. |
| Target | Target return value for each curve in the frontier. |

## Description

PortConfigs $=$ selectreturn(AllMean,AllCovariance, Target) returns the portfolio configurations for a target return given the average return and covariance for a rolling efficient frontier.

PortConfigs is a NASSETS-by-NCURVES matrix of asset allocation weights needed to obtain the target rate of return.

## See Also

frontier

## Topics

"Portfolio Construction Examples" on page 3-7
"Portfolio Optimization Functions" on page 3-4

## Introduced before R2006a

## setAssetList

Set up list of identifiers for assets

## Syntax

obj = setAssetList(obj,AssetList)
obj = setAssetList(obj,'asset1','asset2',asset3',...)

## Description

obj = setAssetList(obj,AssetList) sets up the list of identifiers for assets for Portfolio, PortfolioCVaR, or PorfolioMAD objects. For details on the respective workflows when using these different objects, see "Portfolio Object Workflow" on page 421, "PortfolioCVaR Object Workflow" on page 5-20, and "PortfolioMAD Object Workflow" on page 6-19.
obj $=$ setAssetList(obj,'asset1','asset2', asset3',...) sets up a list of asset identifiers, specified as a comma-separated list of character vectors, a cell array of character vectors, or string array where each character vector or string is an asset identifier.

## Note

- If an asset list is entered as an input, this function overwrites an existing asset list in the object if one exists.
- If no asset list is entered as an input, three actions can occur:
- If NumAssets is nonempty and AssetList is empty, AssetList becomes a numbered list of assets with default names according to the hidden property in defaultforAssetList ('Asset').
- If NumAssets is nonempty and AssetList is nonempty, nothing happens.
- If NumAssets is empty and AssetList is empty, the default NumAssets $=1$ is set and a default asset list is created ('Asset1').


## Examples

## Create a Default List of Asset Names with Three Assets for a Portfolio Object

Create a default list of asset names with three assets.

```
p = Portfolio('NumAssets',3);
p = setAssetList(p);
disp(p.AssetList);
    'Asset1' 'Asset2' 'Asset3'
```


## Create an Explicitly Named List of Asset Names with Three Assets for a Portfolio Object

Create a list of asset names for three equities AGG, EEM, and VEU.

```
p = Portfolio;
p = setAssetList(p, 'AGG', 'EEM', 'VEU');
disp(p.AssetList);
    'AGG' 'EEM' 'VEU'
```


## Create a Default List of Asset Names with Three Assets for a PortfolioCVaR Object

Create a default list of asset names with three assets.

```
p = PortfolioCVaR('NumAssets',3);
p = setAssetList(p);
disp(p.AssetList);
```

```
    'Asset1' 'Asset2' 'Asset3'
```


## Create an Explicitly Named List of Asset Names with Three Assets for a PortfolioCVaR Object

Create a list of asset names for three equities AGG, EEM, and VEU.

```
p = PortfolioCVaR;
p = setAssetList(p, 'AGG', 'EEM', 'VEU');
disp(p.AssetList);
    'AGG' 'EEM' 'VEU'
```


## Create a Default List of Asset Names with Three Assets for a PortfolioMAD Object

Create a default list of asset names with three assets.

```
p = PortfolioMAD('NumAssets',3);
p = setAssetList(p);
disp(p.AssetList);
```

'Asset1' 'Asset2' 'Asset3'

## Create an Explicitly Named List of Asset Names with Three Assets for a PortfolioMAD Object

Create a list of asset names for three equities AGG, EEM, and VEU.
p = PortfolioMAD;
p = setAssetList(p, 'AGG', 'EEM', 'VEU');
disp(p.AssetList);
'AGG' 'EEM' 'VEU'

## Input Arguments

obj - Object for portfolio
object

Object for portfolio, specified using Portfolio, PortfolioCVaR, or PortfolioMAD object. For more information on creating a portfolio object, see

- Portfolio
- PortfolioCVaR
- PortfolioMAD

Data Types: object

## AssetList - List of assets

character vector | cell array of character vectors | string array
List of assets, specified using a character vector, cell array of character vectors, or string array where each character vector or string is an asset identifier.

Data Types: char|cell|string

## Output Arguments

## obj - Updated portfolio object <br> object for portfolio

Updated portfolio object, returned as a Portfolio, PortfolioCVaR, or PortfolioMAD object. For more information on creating a portfolio object, see

- Portfolio
- PortfolioCVaR
- PortfolioMAD

The underlying object (obj) has a number of public hidden properties to format the asset list:

- defaultforAssetList - Default name for assets ('Asset'). Change this name to create default asset names such as 'ETF', 'Bond'.
- sortAssetList - Reserved for future implementation.
- uppercaseAssetList - If true, make all asset identifiers uppercase character vectors. Otherwise do nothing. Default is false.


## Tips

- You can also use dot notation to set up list of identifiers for assets.
obj = obj.setAssetList(AssetList);
- To clear an AssetList, call this function with [] or \{[]\}.


## See Also <br> estimateFrontier|estimateFrontierByReturn|estimateFrontierByRisk| estimateFrontierLimits

## Topics

"Common Operations on the Portfolio Object" on page 4-37
"Common Operations on the PortfolioCVaR Object" on page 5-36
"Common Operations on the PortfolioMAD Object" on page 6-34
"Portfolio Optimization Examples" on page 4-167

Introduced in R2011a

## setAssetMoments

Set moments (mean and covariance) of asset returns for Portfolio object

## Syntax

```
obj = setAssetMoments(obj,AssetMean)
obj = setAssetMoments(obj,AssetMean,AssetCovar,NumAssets)
```


## Description

obj $=$ setAssetMoments(obj,AssetMean) obtains mean and covariance of asset returns for a Portfolio object. For details on the workflow, see "Portfolio Object Workflow" on page 4-21.
obj = setAssetMoments(obj,AssetMean,AssetCovar,NumAssets) obtains mean and covariance of asset returns for a Portfolio object with additional options for AssetCovar and NumAssets.

## Examples

## Set Asset Moments for a Portfolio Object

Set the asset moment properties, given the mean and covariance of asset returns in the variables $m$ and $C$.

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;
p = Portfolio;
```

```
p = setAssetMoments(p, m, C);
[assetmean, assetcovar] = getAssetMoments(p)
assetmean = 4×1
    0.0042
    0.0083
    0.0100
    0.0150
assetcovar = 4×4
    0.0005 0.0003 0.0002 0
    0.0003 0.0024 0.0017 0.0010
    0.0002 0.0017 0.0048 0.0028
        0 0.0010 0.0028 0.0102
```


## Input Arguments

## obj - Object for portfolio

object
Object for portfolio, specified using a Portfolio object. For more information on creating a portfolio object, see

- Portfolio

Data Types: object

## AssetMean - Mean of asset returns

vector
Mean of asset returns, specified as a vector.

Note If AssetMean is a scalar and the number of assets is known, scalar expansion occurs. If the number of assets cannot be determined, this method assumes that NumAssets = 1 .

## Data Types: double

## AssetCovar - Covariance of asset returns

symmetric positive-semidefinite matrix
Covariance of asset returns, specified as a symmetric positive-semidefinite matrix.

Note If AssetCovar is a scalar and the number of assets is known, a diagonal matrix is formed with the scalar value along the diagonals. If it is not possible to determine the number of assets, this method assumes that NumAssets $=1$.

If AssetCovar is a vector, a diagonal matrix is formed with the vector along the diagonal.

## Data Types: double

## NumAssets - Number of assets

integer
Number of assets, specified as an integer.

Note If NumAssets is not already set in the object, NumAssets can be entered to resolve array expansions with AssetMean or AssetCovar.

## Data Types: double

## Output Arguments

## obj — Updated portfolio object

object for portfolio
Updated portfolio object, returned as a Portfolio object. For more information on creating a portfolio object, see

- Portfolio


## Tips

- You can also use dot notation to set moments (mean and covariance) of the asset returns.
obj = obj.setAssetMoments(obj, AssetMean, AssetCovar, NumAssets);
- To clear NumAssets and AssetCovar, use this function to set these respective inputs to [].


## See Also

estimateAssetMoments|estimateFrontierByRisk

## Topics

"Asset Returns and Moments of Asset Returns Using Portfolio Object" on page 4-48
"Portfolio Optimization Examples" on page 4-167
"Portfolio Optimization Theory" on page 4-3

## Introduced in R2011a

## setBounds

Set up bounds for portfolio weights for PortfolioCVaR or PortfolioMAD objects

## Syntax

```
obj = setBounds(obj,LowerBound)
obj = setBounds(obj,LowerBound,UpperBound,NumAssets)
```


## Description

obj = setBounds(obj,LowerBound) sets up bounds for portfolio weights for PortfolioCVaR, or PortfolioMAD objects. For details on the respective workflows when using these different objects, see "PortfolioCVaR Object Workflow" on page 5-20 and "PortfolioMAD Object Workflow" on page 6-19.
obj = setBounds(obj,LowerBound,UpperBound,NumAssets) sets up bounds for portfolio weights for portfolio objects with additional options for UpperBound and NumAssets.

Given bound constraints LowerBound and UpperBound, every weight in a portfolio Port must satisfy the following:

LowerBound <= Port <= UpperBound

## Examples

## Set Bound Constraints for a PortfolioCVaR Object

Suppose you have a balanced fund with stocks that can range from $50 \%$ to $75 \%$ of your portfolio and bonds that can range from $25 \%$ to $50 \%$ of your portfolio. To set the bound constraints for a balanced fund.

```
lb = [ 0.5; 0.25 ];
ub = [ 0.75; 0.5 ];
```

```
p = PortfolioCVaR;
p = setBounds(p, lb, ub);
disp(p.NumAssets);
    2
disp(p.LowerBound);
    0.5000
    0.2500
disp(p.UpperBound);
    0.7500
    0.5000
```


## Set Bound Constraints for a PortfolioMAD Object

Suppose you have a balanced fund with stocks that can range from $50 \%$ to $75 \%$ of your portfolio and bonds that can range from $25 \%$ to $50 \%$ of your portfolio. To set the bound constraints for a balanced fund.

```
lb = [ 0.5; 0.25 ];
ub = [ 0.75; 0.5 ];
p = PortfolioMAD;
p = setBounds(p, lb, ub);
disp(p.NumAssets);
    2
disp(p.LowerBound);
    0.5000
    0.2500
disp(p.UpperBound);
    0.7500
    0.5000
```


## Input Arguments

## obj - Object for portfolio

object
Object for portfolio, specified using PortfolioCVaR or PortfolioMAD object. For more information on creating a portfolio object, see

- PortfolioCVaR
- PortfolioMAD


## Data Types: object

## LowerBound - Lower-bound weight for each asset vector

Lower-bound weight for each asset, specified as a vector for a PortfolioCVaR or PortfolioMAD input object (obj).

## Note

- If either LowerBound or UpperBound are input as empties with [ ], the corresponding attributes in the portfolio object are cleared and set to [].
- If LowerBound or UpperBound are specified as scalars and NumAssets exists or can be imputed, then they undergo scalar expansion. The default value for NumAssets is 1.
- If both LowerBound and UpperBound exist and they are not ordered correctly, the setBounds function switches bounds if necessary.


## Data Types: double

## UpperBound - Upper-bound weight for each asset vector

Upper-bound weight for each asset, specified as a vector for a PortfolioCVaR or PortfolioMAD input object (obj).

## Note

- If either LowerBound or UpperBound are input as empties with [ ], the corresponding attributes in the portfolio object are cleared and set to [].
- If LowerBound or UpperBound are specified as scalars and NumAssets exists or can be imputed, then they undergo scalar expansion. The default value for NumAssets is 1.
- If both LowerBound and UpperBound exist and they are not ordered correctly, the setBounds function switches bounds if necessary.


## Data Types: double

## NumAssets - Number of assets in portfolio scalar

Number of assets in portfolio, specified as a scalar for a PortfolioCVaR or PortfolioMAD input object (obj).

Note NumAssets cannot be used to change the dimension of a portfolio object.

- If either LowerBound or UpperBound are input as empties with [], the corresponding attributes in the portfolio object are cleared and set to [].
- If LowerBound or UpperBound are specified as scalars and NumAssets exists or can be imputed, then they undergo scalar expansion. The default value for NumAssets is 1.
- If both LowerBound and UpperBound exist and they are not ordered correctly, the setBounds function switches bounds if necessary.


## Data Types: double

## Output Arguments

## obj - Updated portfolio object <br> object for portfolio

Updated portfolio object, returned as a PortfolioCVaR or PortfolioMAD object. For more information on creating a portfolio object, see

- PortfolioCVaR
- PortfolioMAD


## Tips

> You can also use dot notation to set up the bounds for portfolio weights. $$
\text { obj }=\text { obj.setBounds(LowerBound, UpperBound, NumAssets) ; }
$$

## See Also

getBounds

## Topics

"Working with Bound Constraints Using PortfolioCVaR Object" on page 5-68
"Working with Bound Constraints Using PortfolioMAD Object" on page 6-66
"Portfolio Optimization Examples" on page 4-167
"Portfolio Set for Optimization Using PortfolioCVaR Object" on page 5-10
"Portfolio Set for Optimization Using PortfolioMAD Object" on page 6-10

## Introduced in R2011a

## setBounds

Set up bounds for portfolio weights for a Portfolio object

## Syntax

```
obj = setBounds(obj,LowerBound)
obj = setBounds(___,Name,Value)
obj = setBounds(obj,LowerBound,UpperBound)
obj = setBounds(___ ,Name,Value)
```


## Description

obj = setBounds(obj, LowerBound) sets up bounds for portfolio weights for a Portfolio object. For details on the workflow when using a Portfolio object, see "Portfolio Object Workflow" on page 4-21.
obj = setBounds (__ ,Name, Value) specifies options using one or more name-value pair arguments in addition to the input arguments in the previous syntax, including BoundType as 'Simple' or 'Conditional'.
obj = setBounds(obj,LowerBound, UpperBound) sets up bounds for portfolio weights for portfolio objects with an additional option for UpperBound.

Given bound constraints LowerBound and UpperBound and 'Simple' BoundType, every weight in a portfolio Port must satisfy the following:

LowerBound <= Port <= UpperBound
Given bound constraints LowerBound and UpperBound, and 'Conditional'
BoundType, every weight in a portfolio Port must satisfy the following:
Port = 0 or LowerBound <= Port <= UpperBound
obj = setBounds (__ , Name, Value) specifies options using one or more name-value pair arguments in addition to the input arguments in the previous syntax, including BoundType as 'Simple' or 'Conditional'.

## Examples

## Set Bound Constraints for a Portfolio Object

Suppose you have a balanced fund with stocks that can range from $50 \%$ to $75 \%$ of your portfolio and bonds that can range from $25 \%$ to $50 \%$ of your portfolio. Use setBounds to set the bound constraints for a balanced fund. Note that this sets default 'Simple' BoundType, which enforces $0.5<=\mathrm{x} 1<=0.75,0.25<=\mathrm{x} 2<=0.5$.

```
lb = [ 0.5; 0.25 ];
ub = [ 0.75; 0.5 ];
p = Portfolio;
p = setBounds(p, lb, ub);
disp(p.NumAssets);
    2
disp(p.LowerBound);
    0.5000
    0.2500
disp(p.UpperBound);
    0.7500
    0.5000
```


## Set Bound Constraints to Define 'Conditional' BoundType Constraints for a Portfolio Object

Suppose you have the mean and covariance of the asset returns for a three asset portfolio:

```
AssetMean = [ 0.0101110; 0.0043532; 0.0137058 ];
AssetCovar = [ 0.00324625 0.00022983 0.00420395;
    0.00022983 0.00049937 0.00019247;
    0.00420395 0.00019247 0.00764097 ];
```

The following uses setBounds with 'Conditional' BoundType (semicontinuous) constraints to set $x i=0$ or $0.02<=x i<=0.5$ for all $i=1, \ldots$ NumAssets.

```
p = Portfolio('AssetMean', AssetMean, 'AssetCovar', AssetCovar);
p = setBounds(p, 0.02, 0.5,'BoundType', 'Conditional', 'NumAssets', 3)
p =
    Portfolio with properties:
            BuyCost: []
            SellCost: []
        RiskFreeRate: []
            AssetMean: [3x1 double]
            AssetCovar: [3x3 double]
        TrackingError: []
            TrackingPort: []
            Turnover: []
        BuyTurnover: []
        SellTurnover: []
            Name: []
            NumAssets: 3
            AssetList: []
            InitPort: []
        AInequality: []
        bInequality: []
            AEquality: []
            bEquality: []
            LowerBound: [3x1 double]
            UpperBound: [3x1 double]
            LowerBudget: []
            UpperBudget: []
            GroupMatrix: []
            LowerGroup: []
            UpperGroup: []
                GroupA: []
            GroupB: []
            LowerRatio: []
            UpperRatio: []
            MinNumAssets: []
            MaxNumAssets: []
            BoundType: [3x1 categorical]
disp(p.LowerBound);
    0.0200
    0.0200
    0.0200
```

```
disp(p.UpperBound);
    0.5000
    0.5000
    0.5000
disp(p.BoundType);
    conditional
    conditional
    conditional
```


## Set Bound Constraints to Define Mixed BoundTypes for a Portfolio Object

Suppose you have the mean and covariance of the asset returns for a three asset portfolio:

```
AssetMean = [ 0.0101110; 0.0043532; 0.0137058 ];
AssetCovar = [ 0.00324625 0.00022983 0.00420395;
    0.00022983 0.00049937 0.00019247;
    0.00420395 0.00019247 0.00764097 ];
```

The following uses setBounds with both 'Simple' and 'Conditional' BoundType constraints for all $i=1, \ldots$ NumAssets .

```
p = Portfolio('AssetMean', AssetMean, 'AssetCovar', AssetCovar);
p = setBounds(p, 0.1, 0.5, 'BoundType',["simple"; "conditional"; "conditional"])
p =
    Portfolio with properties:
```

            BuyCost: []
            SellCost: []
        RiskFreeRate: []
        AssetMean: [3x1 double]
        AssetCovar: [3x3 double]
    TrackingError: []
        TrackingPort: []
            Turnover: []
        BuyTurnover: []
        SellTurnover: []
            Name: []
            NumAssets: 3
    ```
        AssetList: []
            InitPort: []
        AInequality: []
        bInequality: []
        AEquality: []
        bEquality: []
        LowerBound: [3x1 double]
        UpperBound: [3x1 double]
    LowerBudget: []
    UpperBudget: []
    GroupMatrix: []
        LowerGroup: []
        UpperGroup: []
        GroupA: []
        GroupB: []
    LowerRatio: []
    UpperRatio: []
    MinNumAssets: []
    MaxNumAssets: []
    BoundType: [3x1 categorical]
disp(p.LowerBound);
    0.1000
    0.1000
    0.1000
disp(p.UpperBound);
    0.5000
    0.5000
    0.5000
disp(p.BoundType);
    simple
    conditional
    conditional
```

You can use supply lower and upper bounds as vectors, which defines different values for each asset. The following have $-0.8<=x 1<=0.2 ; x 2=0$ or $0.1<=x 2<=0.5 ; x 3=0$ or $0.1<=x 3<=0.5$. Note that as 'Simple' BoundType, the assets can be held as short or long positions. However, when using 'Conditional' BoundType, the assets can only be long positions.

```
p = setBounds(p, [-0.8, 0.1, 0.1], [-0.2,0.5,0.5], 'BoundType',["simple"; "conditional
disp(p.LowerBound);
    -0.8000
    0.1000
    0.1000
disp(p.UpperBound);
    -0.2000
    0.5000
    0.5000
disp(p.BoundType);
    simple
    conditional
    conditional
```


## Set MinNumAssets Constraint for a Portfolio Object

Set the minimum cardinality constraint for a three-asset portfolio for which you have the mean and covariance values of the asset returns.

```
AssetMean = [ 0.0101110; 0.0043532; 0.0137058 ];
AssetCovar = [ 0.00324625 0.00022983 0.00420395;
    0.00022983 0.00049937 0.00019247;
    0.00420395 0.00019247 0.00764097 ];
p = Portfolio('AssetMean', AssetMean, 'AssetCovar', AssetCovar);
p = setDefaultConstraints(p);
```

When working with a Portfolio object, the setMinMaxNumAssets function enables you to set up limits on the number of assets invested. These limits are also known as cardinality constraints. When managing a portfolio, it is common that you want to invest in at least a certain number of assets. In addition, you should also clearly define the weight requirement for each invested asset. You can do this using setBounds with a 'Conditional ' BoundType. If you do not specify a 'Conditional ' BoundType, the optimizer cannot understand which assets are invested assets and cannot formulate the MinNumAssets constraint.

The following example specifies that at least two assets should be invested and the investments should be greater than $16 \%$.

```
p = setMinMaxNumAssets(p, 2, []);
p = setBounds(p, 0.16, 'BoundType', 'conditional');
```

Use estimateFrontierByReturn to estimate optimal portfolios with targeted portfolio returns.

```
pwgt = estimateFrontierByReturn(p,[ 0.008, 0.01 ])
pwgt = 3\times2
    0.2861 0.3967
    0.5001 0.2438
    0.2138 0.3595
```


## Input Arguments

## obj - Object for portfolio

object
Object for portfolio, specified using Portfolio object. For more information on creating a portfolio object, see Portfolio.

Data Types: object

## LowerBound - Lower-bound weight for each asset

scalar | vector
Lower-bound weight for each asset, specified as a scalar or vector for a Portfolio input object (obj).

## Note

- If either LowerBound or UpperBound are input as empties with [ ], the corresponding attributes in the Portfolio object are cleared and set to [].
- If LowerBound or UpperBound are specified as scalars and NumAssets exists or can be computed, then they undergo scalar expansion. The default value for NumAssets is 1.
- If both LowerBound and UpperBound exist and they are not ordered correctly, the setBounds function switches bounds if necessary.
- If 'Conditional 'BoundType is specified, the LowerBound cannot be a negative value.


## Data Types: double

## UpperBound - Upper-bound weight for each asset

scalar | vector
(Optional) Upper-bound weight for each asset, specified as a scalar or vector for a Portfolio input object (obj).

## Note

- If either LowerBound or UpperBound are input as empties with [ ], the corresponding attributes in the Portfolio object are cleared and set to [].
- If LowerBound or UpperBound are specified as scalars and NumAssets exists or can be computed, then they undergo scalar expansion. The default value for NumAssets is 1.
- If both LowerBound and UpperBound exist and they are not ordered correctly, the setBounds function switches bounds if necessary.
- If 'Conditional'BoundType is specified, the UpperBound cannot be a negative value.


## Data Types: double

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, ... , NameN, ValueN.

Example: obj = setBounds(p,0.02,'BoundType','Conditional');

## BoundType - Type of bounds for each asset weight

'Simple' (default)| character vector with value 'Simple' or 'Conditional' | string with value "Simple" or "Conditional" | cell array of character vectors with values 'Simple' or 'Conditional' | string array with values "Simple" or "Conditional"

Type of bounds for each asset weight, specified as the comma-separated pair consisting of
'BoundType' and a scalar character vector or string with a value of 'Simple' or
'Conditional' or a cell array of character vectors with values of 'Simple' or
'Conditional'.

- 'Simple' is LowerBound <=AssetWeight <= UpperBound.
- 'Conditional ' is LowerBound <= AssetWeight <= UpperBound or AssetWeight = 0.

Warning If you specify the Bound range to be inclusive of zero (using either a 'Simple' or 'Conditional' BoundType), when you use setMinMaxNumAssets to specify the MinNumAssets constraint, and then use one of the estimate functions, it is ambiguous for the optimizer to define the minimum requirement for an allocated asset. In this case, the optimizer considers that an asset with zero weight is a valid allocated asset and the optimization proceeds, but with the warning that the allocation has less than the MinNumAssets required. For more information, see "Troubleshooting for Setting 'Conditional' BoundType, MinNumAssets, and MaxNumAssets Constraints" on page 4-156.

Data Types: char|cell | string

## NumAssets - Number of assets in portfolio

1 (default) | scalar numeric
Number of assets in portfolio, specified as the comma-separated pair consisting of 'NumAssets' and a scalar numeric value.

Note NumAssets cannot be used to change the dimension of a Portfolio object.

- If either LowerBound or UpperBound are input as empties with [], the corresponding attributes in the Portfolio object are cleared and set to [].
- If LowerBound or UpperBound are specified as scalars and NumAssets exists or can be imputed, then they undergo scalar expansion. The default value for NumAssets is 1.
- If both LowerBound and UpperBound exist and they are not ordered correctly, the setBounds function switches bounds if necessary.


## Data Types: double

## Output Arguments

## obj — Updated Portfolio object <br> object

Updated Portfolio object, returned as a Portfolio object. For more information on creating a Portfolio object, see Portfolio.

## Tips

You can also use dot notation to set up the bounds for portfolio weights.

```
obj = obj.setBounds(LowerBound, UpperBound, Name,Value);
```

If any of LowerBound, UpperBound, or BoundTypeare input as empties with [ ], the corresponding attributes in the Portfolio object are cleared and set to [ ]. If BoundType is cleared as [ ], the bound type defaults to 'Simple'.

```
p = setBounds(p, LowerBound, [ ], 'BoundType',[ ]);
```

To reset a Portfolio object to be a continuous problem, run the following:
$\mathrm{p}=$ setMinMaxNumAssets(p, [],[]);
$\mathrm{p}=$ setBounds(p, p. LowerBound, p. UpperBound, 'BoundType', 'Simple');

## See Also

estimateAssetMoments|estimateFrontier|estimateFrontierByReturn| estimateFrontierByRisk|estimateFrontierLimits|
estimateMaxSharpeRatio|estimatePortMoments|estimatePortReturn| estimatePortRisk|estimatePortSharpeRatio|getBounds|
setMinMaxNumAssets| setSolverMINLP

## Topics

"Working with 'Simple' Bound Constraints Using Portfolio Object" on page 4-73
"Working with 'Conditional' BoundType, MinNumAssets, and MaxNumAssets Constraints
Using Portfolio Object" on page 4-94
"Troubleshooting for Setting 'Conditional' BoundType, MinNumAssets, and MaxNumAssets Constraints" on page 4-156
"Portfolio Optimization Examples" on page 4-167
"Portfolio Optimization with Semicontinuous and Cardinality Constraints"
"Portfolio Set for Optimization Using Portfolio Object" on page 4-10

## Introduced in R2011a

## setBudget

Set up budget constraints

## Syntax

```
obj = setBudget(obj,LowerBudget)
obj = setBudget(obj,LowerBudget,UpperBudget)
```


## Description

obj = setBudget(obj,LowerBudget) sets up budget constraints for Portfolio, PortfolioCVaR, or PortfolioMAD objects. For details on the respective workflows when using these different objects, see "Portfolio Object Workflow" on page 4-21, "PortfolioCVaR Object Workflow" on page 5-20, and "PortfolioMAD Object Workflow" on page 6-19.
obj = setBudget(obj,LowerBudget,UpperBudget) sets up budget constraints for portfolio objects with an additional option for UpperBudget.

## Examples

## Set Budget Constraint for a Portfolio Object

Assume you have a fund that permits up to $10 \%$ leverage, which means that your portfolio can be from $100 \%$ to $110 \%$ invested in risky assets. Given a Portfolio object p, set the budget constraint.

```
p = Portfolio;
p = setBudget(p, 1, 1.1);
disp(p.LowerBudget);
    1
disp(p.UpperBudget);
```


## Set Budget Constraint for a PortfolioCVaR Object

Assume you have a fund that permits up to $10 \%$ leverage, which means that your portfolio can be from $100 \%$ to $110 \%$ invested in risky assets. Given a CVaR portfolio object $p$, set the budget constraint.

```
p = PortfolioCVaR;
p = setBudget(p, 1, 1.1);
disp(p.LowerBudget);
```

1
disp(p.UpperBudget);
1.1000

## Set Budget Constraint for a PortfolioMAD Object

Assume you have a fund that permits up to $10 \%$ leverage, which means that your portfolio can be from $100 \%$ to $110 \%$ invested in risky assets. Given PortfolioMAD object $p$, set the budget constraint.
p = PortfolioMAD;
$\mathrm{p}=\operatorname{setBudget}(\mathrm{p}, 1,1.1)$;
disp(p.LowerBudget);
1
disp(p.UpperBudget);
1.1000

## Control the Allocation of a Risk-Free Asset Using setBudget

Define mean and covariance of risk asset returns.

```
m = [ 0.05; 0.1; 0.12; 0.18; ];
C = [ 0.0064 0.00408 0.00192 0,;
    0.00408 0.0289 0.0204 0.0119,;
    0.00192 0.0204 0.0576 0.0336,;
    0 0.0119 0.0336 0.1225];
```

Create a Portfolio object defining the risk-free rate.

```
p = Portfolio('RiskFreeRate',0.03, 'assetmean', m, 'assetcovar', C, ...
'lowerbudget', 1, 'upperbudget', 1, 'lowerbound', 0);
```

Create multiple Portfolio objects with different budgets on risky assets. By defining the risky assets, you can control how much is invested in a risk-free asset.

```
p = setBudget(p, 1, 1); % allow 0% risk-free asset allocation, meaning fully investe
p1 = setBudget(p, 0, 1); % allow 0 to 100% risk-free asset allocation
p2 = setBudget(p, 0.7, 1); % allow 0 to 30% risk-free asset allocation
plotFrontier(p); hold on;
plotFrontier(p1);hold on;
plotFrontier(p2);
legend('Without risk-free asset', 'With risk-free asset in range [0,1]', 'With risk-fr
```


setBudget defines the bound for total weights for the allocated risky assets, and the remaining is automatically the bound for a risk-free asset. Use setBudget to control the level of allowed allocation to a risk-free asset. For additional information on using setBudget with a risk-free asset, see "Leverage in Portfolio Optimization with a RiskFree Asset".

## Input Arguments

## obj - Object for portfolio

object

Object for portfolio, specified using Portfolio, PortfolioCVaR, or PortfolioMAD object. For more information on creating a portfolio object, see

- Portfolio
- PortfolioCVaR
- PortfolioMAD


## Data Types: object

LowerBudget - Lower-bound for budget constraint
scalar
Lower-bound for budget constraint, specified as a scalar for a Portfolio, PortfolioCVaR, or PortfolioMAD input object (obj).

Note Given bounds for a budget constraint in either LowerBudget or UpperBudget, budget constraints require any portfolio in Port to satisfy:

LowerBudget <= sum(Port) <= UpperBudget
One or both constraints may be specified. The usual budget constraint for a fully invested portfolio is to have LowerBudget = UpperBudget $=1$. However, if the portfolio has allocations in cash, the budget constraints can be used to specify the cash constraints. For example, if the portfolio can hold between $0 \%$ and $10 \%$ in cash, the budget constraint would be set up with

```
obj = setBudget(obj, 0.9, 1)
```


## Data Types: double

## UpperBudget - Upper-bound for budget constraint scalar

Upper-bound for budget constraint, specified as a scalar for a Portfolio, PortfolioCVaR, or PortfolioMAD input object (obj).

Note Given bounds for a budget constraint in either LowerBudget or UpperBudget, budget constraints require any portfolio in Port to satisfy:

```
LowerBudget <= sum(Port) <= UpperBudget
```

One or both constraints may be specified. The usual budget constraint for a fully invested portfolio is to have LowerBudget $=$ UpperBudget $=1$. However, if the portfolio has allocations in cash, the budget constraints can be used to specify the cash constraints.
For example, if the portfolio can hold between $0 \%$ and $10 \%$ in cash, the budget constraint would be set up with

```
obj = setBudget(obj, 0.9, 1)
```


## Data Types: double

## Output Arguments

## obj — Updated portfolio object

object for portfolio
Updated portfolio object, returned as a Portfolio, PortfolioCVaR, or PortfolioMAD object. For more information on creating a portfolio object, see

- Portfolio
- PortfolioCVaR
- PortfolioMAD


## Tips

You can also use dot notation to set up the budget constraints.

```
obj = obj.setBudget(LowerBudget, UpperBudget);
```


## See Also

getBudget

## Topics

"Working with Budget Constraints Using Portfolio Object" on page 4-77
"Working with Budget Constraints Using PortfolioCVaR Object" on page 5-71
"Working with Budget Constraints Using PortfolioMAD Object" on page 6-69
"Portfolio Optimization Examples" on page 4-167
"Leverage in Portfolio Optimization with a Risk-Free Asset"
"Portfolio Set for Optimization Using Portfolio Object" on page 4-10
"Portfolio Set for Optimization Using PortfolioCVaR Object" on page 5-10
"Portfolio Set for Optimization Using PortfolioMAD Object" on page 6-10
Introduced in R2011a

## setCosts

Set up proportional transaction costs

## Syntax

```
obj = setCosts(obj,BuyCost)
obj = setCosts(obj,BuyCost,SellCost,InitPort,NumAssets)
```


## Description

obj = setCosts(obj, BuyCost) sets up proportional transaction costs for Portfolio, PortfolioCVaR, or PortfolioMAD objects. For details on the respective workflows when using these different objects, see "Portfolio Object Workflow" on page 421, "PortfolioCVaR Object Workflow" on page 5-20, and "PortfolioMAD Object Workflow" on page 6-19.
obj = setCosts(obj,BuyCost,SellCost,InitPort,NumAssets) sets up proportional transaction costs for portfolio objects with additional options specified for SellCost, InitPort, and NumAssets.

Given proportional transaction costs and an initial portfolio in the variables BuyCost, SellCost, and InitPort, the transaction costs for any portfolio Port reduce expected portfolio return by:

```
BuyCost' * max{0, Port - InitPort} + SellCost' * max{0, InitPort - Port}
```


## Examples

## Set Up Transaction Costs for a Portfolio Object

Given a Portfolio object $p$ with an initial portfolio already set, use the setCosts function to set up transaction costs.

```
bc = [ 0.00125; 0.00125; 0.00125; 0.00125; 0.00125 ];
sc = [ 0.00125; 0.007; 0.00125; 0.00125; 0.0024 ];
x0 = [ 0.4; 0.2; 0.2; 0.1; 0.1 ];
p = Portfolio('InitPort', x0);
p = setCosts(p, bc, sc);
disp(p.NumAssets);
    5
disp(p.BuyCost);
    0.0013
    0.0013
    0.0013
    0.0013
    0.0013
disp(p.SellCost);
    0.0013
    0.0070
    0.0013
    0.0013
    0.0024
disp(p.InitPort);
    0.4000
    0.2000
    0.2000
    0.1000
    0.1000
```


## Set Up Transaction Costs for a PortfolioCVaR Object

Given a CVaR portfolio object $p$ with an initial portfolio already set, use the setCosts function to set up transaction costs.

```
bc = [ 0.00125; 0.00125; 0.00125; 0.00125; 0.00125 ];
sc = [ 0.00125; 0.007; 0.00125; 0.00125; 0.0024 ];
x0 = [ 0.4; 0.2; 0.2; 0.1; 0.1 ];
```

```
p = PortfolioCVaR('InitPort', x0);
p = setCosts(p, bc, sc);
disp(p.NumAssets);
    5
disp(p.BuyCost);
    0.0013
    0.0013
    0.0013
    0.0013
    0.0013
disp(p.SellCost);
    0.0013
    0.0070
    0.0013
    0.0013
    0.0024
disp(p.InitPort);
    0.4000
    0.2000
    0.2000
    0.1000
    0.1000
```


## Set Up Transaction Costs for a PortfolioMAD Object

Given PortfolioMAD object p with an initial portfolio already set, use the setCosts function to set up transaction costs.

```
bc = [ 0.00125; 0.00125; 0.00125; 0.00125; 0.00125 ];
sc = [ 0.00125; 0.007; 0.00125; 0.00125; 0.0024 ];
x0 = [ 0.4; 0.2; 0.2; 0.1; 0.1 ];
p = PortfolioMAD('InitPort', x0);
p = setCosts(p, bc, sc);
```

```
disp(p.NumAssets);
    5
disp(p.BuyCost);
    0.0013
    0.0013
    0.0013
    0.0013
    0.0013
disp(p.SellCost);
    0.0013
    0.0070
    0.0013
    0.0013
    0.0024
disp(p.InitPort);
    0.4000
    0.2000
    0.2000
    0.1000
    0.1000
```


## Input Arguments

## obj - Object for portfolio

object
Object for portfolio, specified using Portfolio, PortfolioCVaR, or PortfolioMAD object. For more information on creating a portfolio object, see

- Portfolio
- PortfolioCVaR
- PortfolioMAD

Data Types: object

## BuyCost - Proportional transaction cost to purchase each asset vector

Proportional transaction cost to purchase each asset, specified as a vector for a Portfolio, PortfolioCVaR, or PortfolioMAD input object (obj).

## Note

- If BuyCost, SellCost, or InitPort are specified as scalars and NumAssets exists or can be imputed, then these values undergo scalar expansion. The default value for NumAssets is 1.
- Transaction costs in BuyCost and SellCost are positive valued if they introduce a cost to trade. In some cases, they can be negative valued, which implies trade credits.


## Data Types: double

## SellCost - Proportional transaction cost to sell each asset

 vectorProportional transaction cost to sell each asset, specified as a vector for a Portfolio, PortfolioCVaR, or PortfolioMAD input object (obj).

## Note

- If BuyCost, SellCost, or InitPort are specified as scalars and NumAssets exists or can be imputed, then these values undergo scalar expansion. The default value for NumAssets is 1.
- Transaction costs in BuyCost and SellCost are positive valued if they introduce a cost to trade. In some cases, they can be negative valued, which implies trade credits.


## Data Types: double

## InitPort - Initial or current portfolio weights

vector
Initial or current portfolio weights, specified as a vector for a Portfolio, PortfolioCVaR, or PortfolioMAD input object (obj).

Note If no InitPort is specified, that value is assumed to be 0 .

- If BuyCost, SellCost, or InitPort are specified as scalars and NumAssets exists or can be imputed, then these values undergo scalar expansion. The default value for NumAssets is 1.
- Transaction costs in BuyCost and SellCost are positive valued if they introduce a cost to trade. In some cases, they can be negative valued, which implies trade credits.


## Data Types: double

## NumAssets - Number of assets in portfolio <br> scalar

Number of assets in portfolio, specified as a scalar for a Portfolio, PortfolioCVaR, or PortfolioMAD input object (obj).

Note NumAssets cannot be used to change the dimension of a portfolio object.

- If BuyCost, SellCost, or InitPort are specified as scalars and NumAssets exists or can be imputed, then these values undergo scalar expansion. The default value for NumAssets is 1.
- Transaction costs in BuyCost and SellCost are positive valued if they introduce a cost to trade. In some cases, they can be negative valued, which implies trade credits.


## Data Types: double

## Output Arguments

## obj - Updated portfolio object <br> object for portfolio

Updated portfolio object, returned as a Portfolio, PortfolioCVaR, or PortfolioMAD object. For more information on creating a portfolio object, see

- Portfolio
- PortfolioCVaR
- PortfolioMAD


## Tips

- You can also use dot notation to set up proportional transaction costs.

```
obj = obj.setCosts(BuyCost, SellCost, InitPort, NumAssets);
```

- If BuyCost or SellCost are input as empties with [], the corresponding attributes in the portfolio object are cleared and set to [ ]. If InitPort is set to empty with [], it will only be cleared and set to [] if BuyCost, SellCost, and Turnover are also empty. Otherwise, it is an error.


## See Also

getCosts| setInitPort

## Topics

"Working with Transaction Costs" on page 4-63
"Working with Transaction Costs" on page 5-58
"Working with Transaction Costs" on page 6-56
"Portfolio Analysis with Turnover Constraints"
"Portfolio Optimization Examples" on page 4-167
"Portfolio Set for Optimization Using Portfolio Object" on page 4-10
"Portfolio Set for Optimization Using PortfolioCVaR Object" on page 5-10
"Portfolio Set for Optimization Using PortfolioMAD Object" on page 6-10

## Introduced in R2011a

## setDefaultConstraints

Set up portfolio constraints with nonnegative weights that sum to 1

## Syntax

```
obj = setDefaultConstraints(obj)
```

obj = setDefaultConstraints(obj,NumAssets)

## Description

obj = setDefaultConstraints(obj) sets up portfolio constraints with nonnegative weights that sum to 1 for Portfolio, PortfolioCVaR, or PortfolioMAD objects. For details on the respective workflows when using these different objects, see "Portfolio Object Workflow" on page 4-21, "PortfolioCVaR Object Workflow" on page 5-20, and "PortfolioMAD Object Workflow" on page 6-19.
obj = setDefaultConstraints(obj,NumAssets) sets up portfolio constraints with nonnegative weights that sum to 1 with an additional option for NumAssets.

A "default" portfolio set has LowerBound = 0 and LowerBudget = UpperBudget = 1 such that a portfolio Port must satisfy sum(Port) $=1$ with Port >= 0 .

## Examples

## Define Default Constraints for the Portfolio Object

Assuming you have 20 assets, you can define the "default" portfolio set.

```
p = Portfolio('NumAssets', 20);
p = setDefaultConstraints(p);
disp(p);
```

Portfolio with properties:

```
BuyCost: []
SellCost: []
RiskFreeRate: []
AssetMean: []
AssetCovar: []
TrackingError: []
TrackingPort: []
Turnover: []
BuyTurnover: []
SellTurnover: []
Name: []
NumAssets: 20
AssetList: []
InitPort: []
AInequality: []
bInequality: []
AEquality: []
bEquality: []
LowerBound: [20x1 double]
UpperBound: []
LowerBudget: 1
UpperBudget: 1
GroupMatrix: []
LowerGroup: []
UpperGroup: []
GroupA: []
GroupB: []
LowerRatio: []
UpperRatio: []
MinNumAssets: []
MaxNumAssets: []
BoundType: [20x1 categorical]
```


## Define Default Constraints for the PortfolioCVaR Object

Assuming you have 20 assets, you can define the "default" portfolio set.

```
p = PortfolioCVaR('NumAssets', 20);
p = setDefaultConstraints(p);
disp(p);
```

PortfolioCVaR with properties:

```
BuyCost: []
SellCost: []
RiskFreeRate: []
ProbabilityLevel: []
Turnover: []
BuyTurnover: []
SellTurnover: []
NumScenarios: []
            Name: []
    NumAssets: 20
    AssetList: []
    InitPort: []
AInequality: []
bInequality: []
    AEquality: []
    bEquality: []
    LowerBound: [20x1 double]
    UpperBound: []
LowerBudget: 1
UpperBudget: 1
GroupMatrix: []
    LowerGroup: []
    UpperGroup: []
        GroupA: []
        GroupB: []
    LowerRatio: []
    UpperRatio: []
```


## Define Default Constraints for the PortfolioMAD Object

Assuming you have 20 assets, you can define the "default" portfolio set.

```
p = PortfolioMAD('NumAssets', 20);
p = setDefaultConstraints(p);
disp(p);
    PortfolioMAD with properties:
        BuyCost: []
        SellCost: []
    RiskFreeRate: []
        Turnover: []
```

BuyTurnover: ..... []
SellTurnover: ..... []
NumScenarios: []Name: []
NumAssets: 20
AssetList:
InitPort: [
AInequality: [
bInequality: [
AEquality: []
bEquality: []
UpperBound: [
LowerBudget: ..... 1
UpperBudget: ..... 1
GroupMatrix: ..... []
LowerGroup: ..... []
UpperGroup: [
GroupA: []
GroupB: []
LowerRatio: []
UpperRatio: []
LowerBound: [20x1 double]
Input Arguments
obj - Object for portfolio
object
Object for portfolio, specified using Portfolio, PortfolioCVaR, or PortfolioMADobject. For more information on creating a portfolio object, see

- Portfolio
- PortfolioCVaR
- PortfolioMAD
Data Types: object
NumAssets - Number of assets in portfolio
scalar

Number of assets in portfolio, specified as a scalar for a Portfolio, PortfolioCVaR, or PortfolioMAD input object (obj).

Note NumAssets cannot be used to change the dimension of a portfolio object. The default for NumAssets is 1.

## Data Types: double

## Output Arguments

## obj — Updated portfolio object <br> object for portfolio

Updated portfolio object, returned as a Portfolio, PortfolioCVaR, or PortfolioMAD object. For more information on creating a portfolio object, see

- Portfolio
- PortfolioCVaR
- PortfolioMAD


## Tips

- You can also use dot notation to set up the default portfolio set.
obj = obj.setDefaultConstraints(NumAssets);
- This function does not modify any existing constraints in a portfolio object other than the bound and budget constraints. If an UpperBound constraint exists, it is cleared and set to [].


## See Also

getBounds | setBounds | setBudget

## Topics

"Setting Default Constraints for Portfolio Weights Using Portfolio Object" on page 4-68
"Setting Default Constraints for Portfolio Weights Using PortfolioCVaR Object" on page 563
"Setting Default Constraints for Portfolio Weights Using PortfolioMAD Object" on page 661
"Portfolio Optimization Examples" on page 4-167
"Portfolio Set for Optimization Using Portfolio Object" on page 4-10
"Portfolio Set for Optimization Using PortfolioCVaR Object" on page 5-10
"Portfolio Set for Optimization Using PortfolioMAD Object" on page 6-10

## Introduced in R2011a

## setEquality

Set up linear equality constraints for portfolio weights

## Syntax

```
obj= setEquality(obj,AEquality,bEquality)
```


## Description

obj= setEquality(obj,AEquality,bEquality) sets up linear equality constraints for portfolio weights for Portfolio, PortfolioCVaR, or PortfolioMAD objects. For details on the respective workflows when using these different objects, see "Portfolio Object Workflow" on page 4-21, "PortfolioCVaR Object Workflow" on page 5-20, and "PortfolioMAD Object Workflow" on page 6-19.

Given linear equality constraint matrix AEquality and vector bEquality, every weight in a portfolio Port must satisfy the following:

```
AEquality * Port = bEquality
```


## Examples

## Set Linear Equality Constraints for a Portfolio Object

Suppose you have a portfolio of five assets, and you want to ensure that the first three assets are $50 \%$ of your portfolio. Given a Portfolio object p, set the linear equality constraints with the following.

```
A = [ 1 1 1 1 0 0 ];
b = 0.5;
p = Portfolio;
p = setEquality(p, A, b);
disp(p.NumAssets);
```

5

```
disp(p.AEquality);
```

$\begin{array}{lllll}1 & 1 & 1 & 0 & 0\end{array}$
disp(p.bEquality);
0.5000

## Set Linear Equality Constraints for a PortfolioCVaR Object

Suppose you have a portfolio of five assets and you want to ensure that the first three assets are $50 \%$ of your portfolio. Given a PortfolioCVaR object p, set the linear equality constraints and obtain the values for AEquality and bEquality:
$A=\left[\begin{array}{lllll}1 & 1 & 1 & 0 & 0\end{array}\right] ;$
b $=0.5$;
p = PortfolioCVaR;
$p=\operatorname{setEquality}(p, A, b) ;$
disp(p.NumAssets);
5
disp(p.AEquality);
$\begin{array}{lllll}1 & 1 & 1 & 0 & 0\end{array}$
disp(p.bEquality);
0.5000

## Set Linear Equality Constraints for a PortfolioMAD Object

Suppose you have a portfolio of five assets and you want to ensure that the first three assets are $50 \%$ of your portfolio. Given a PortfolioMAD object p, set the linear equality constraints and obtain the values for AEquality and bEquality:
$A=\left[\begin{array}{llllll}1 & 1 & 1 & 0 & 0\end{array}\right] ;$
b $=0.5$;

```
p = PortfolioMAD;
p = setEquality(p, A, b);
[AEquality, bEquality] = getEquality(p)
AEquality = 1×5
    1 1 1 1 0 0
bEquality = 0.5000
```


## Input Arguments

## obj - Object for portfolio

## object

Object for portfolio, specified using Portfolio, PortfolioCVaR, or PortfolioMAD object. For more information on creating a portfolio object, see

- Portfolio
- PortfolioCVaR
- PortfolioMAD

Data Types: object
AEquality - Matrix to form linear equality constraints matrix

Matrix to form linear equality constraints, returned as a matrix for a Portfolio, PortfolioCVaR, or PortfolioMAD input object (obj).

Note An error results if AEquality is empty and bEquality is nonempty.

## Data Types: double

## bEquality - Vector to form linear equality constraints

vector

Vector to form linear equality constraints, returned as a vector for a Portfolio, PortfolioCVaR, or PortfolioMAD input object (obj).

## Note An error results if AEquality is nonempty and bEquality is empty.

## Data Types: double

## Output Arguments

## obj - Updated portfolio object <br> object for portfolio

Updated portfolio object, returned as a Portfolio, PortfolioCVaR, or PortfolioMAD object. For more information on creating a portfolio object, see

- Portfolio
- PortfolioCVaR
- PortfolioMAD


## Tips

- You can also use dot notation to set up linear equality constraints for portfolio weights. obj = obj.setEquality(AEquality, bEquality);
- Linear equality constraints can be removed from a portfolio object by entering [] for each property you want to remove.


## See Also

addEquality|getEquality

## Topics

"Working with Linear Equality Constraints Using Portfolio Object" on page 4-88
"Working with Linear Equality Constraints Using PortfolioCVaR Object" on page 5-82
"Working with Linear Equality Constraints Using PortfolioMAD Object" on page 6-79
"Portfolio Optimization Examples" on page 4-167
"Portfolio Set for Optimization Using Portfolio Object" on page 4-10
"Portfolio Set for Optimization Using PortfolioCVaR Object" on page 5-10
"Portfolio Set for Optimization Using PortfolioMAD Object" on page 6-10

Introduced in R2011a

## setGroupRatio

Set up group ratio constraints for portfolio weights

## Syntax

```
obj = setGroupRatio(obj,GroupA,GroupB,LowerRatio)
obj = setGroupRatio(
,UpperRatio)
```


## Description

obj $=$ setGroupRatio(obj,GroupA, GroupB, LowerRatio) sets up group ratio constraints for portfolio weights for Portfolio, PortfolioCVaR, or PortfolioMAD objects. For details on the respective workflows when using these different objects, see "Portfolio Object Workflow" on page 4-21, "PortfolioCVaR Object Workflow" on page 5-20, and "PortfolioMAD Object Workflow" on page 6-19.
obj = setGroupRatio( __ ,UpperRatio) sets up group ratio constraints for portfolio weights for portfolio objects with an additional optional argument for UpperRatio.

Given base and comparison group matrices GroupA and GroupB and LowerRatio or UpperRatio bounds, group ratio constraints require any portfolio in Port to satisfy the following:

```
(GroupB * Port) .* LowerRatio <= GroupA * Port <= (GroupB * Port) .* UpperRatio
```

Caution This collection of constraints usually requires that portfolio weights be nonnegative and that the products GroupA * Port and GroupB * Port are always nonnegative. Although negative portfolio weights and non-Boolean group ratio matrices are supported, use with caution.

## Examples

## Set Group Ratio Constraints for a Portfolio Object

Suppose you want to ensure that the ratio of financial to nonfinancial companies in your portfolio never exceeds 50\%. Assume you have six assets with three financial companies (assets 1-3) and three nonfinancial companies (assets 4-6). Group ratio constraints can be set with:

```
GA = [ true true true false false false ]; % financial companies
GB = [ false false false true true true ]; % nonfinancial companies
p = Portfolio;
p = setGroupRatio(p, GA, GB, [], 0.5);
disp(p.NumAssets);
    6
disp(p.GroupA);
    1 1 1 1 1 0 0 0
disp(p.GroupB);
    0
disp(p.UpperRatio);
0.5000
```


## Set Group Ratio Constraints for a PortfolioCVaR Object

Suppose you want to ensure that the ratio of financial to nonfinancial companies in your portfolio never exceeds 50\%. Assume you have six assets with three financial companies (assets 1-3) and three nonfinancial companies (assets 4-6). Group ratio constraints can be set with:

```
GA = [ true true true false false false ]; % financial companies
GB = [ false false false true true true ]; % nonfinancial companies
p = PortfolioCVaR;
p = setGroupRatio(p, GA, GB, [], 0.5);
disp(p.NumAssets);
```

    6
    ```
disp(p.GroupA);
    1
disp(p.GroupB);
    0
disp(p.UpperRatio);
    0.5000
```


## Set Group Ratio Constraints for a PortfolioMAD Object

Suppose you want to ensure that the ratio of financial to nonfinancial companies in your portfolio never exceeds 50\%. Assume you have six assets with three financial companies (assets 1-3) and three nonfinancial companies (assets 4-6). Group ratio constraints can be set with:

```
GA = [ true true true false false false ]; % financial companies
GB = [ false false false true true true ]; % nonfinancial companies
p = PortfolioMAD;
p = setGroupRatio(p, GA, GB, [], 0.5);
disp(p.NumAssets);
    6
disp(p.GroupA);
    1
disp(p.GroupB);
    0
disp(p.UpperRatio);
    0.5000
```


## Input Arguments

## obj - Object for portfolio

object
Object for portfolio, specified using Portfolio, PortfolioCVaR, or PortfolioMAD object. For more information on creating a portfolio object, see

- Portfolio
- PortfolioCVaR
- PortfolioMAD

Data Types: object

## GroupA - Matrix that forms base groups for comparison matrix

Matrix that forms base groups for comparison, specified as a matrix for a Portfolio, PortfolioCVaR, or PortfolioMAD input object (obj).

Note The group matrices GroupA and GroupB are usually indicators of membership in groups, which means that their elements are usually either 0 or 1 . Because of this interpretation, GroupA and GroupB matrices can be either logical or numerical arrays.

## Data Types: double | logical

## GroupB - Matrix that forms comparison groups

matrix
Matrix that forms comparison groups, specified as a matrix Portfolio, PortfolioCVaR, or PortfolioMAD input object (obj).

Note The group matrices GroupA and GroupB are usually indicators of membership in groups, which means that their elements are usually either 0 or 1 . Because of this interpretation, GroupA and GroupB matrices can be either logical or numerical arrays.

## LowerRatio - Lower bound for ratio of GroupB groups to GroupA groups vector

Lower bound for ratio of GroupB groups to GroupA groups, specified as a vector for a Portfolio, PortfolioCVaR, or PortfolioMAD input object (obj).

Note If input is scalar, LowerRatio undergoes scalar expansion to be conformable with the group matrices.

## Data Types: double

## UpperRatio - Upper bound for ratio of GroupB groups to GroupA groups vector

Upper bound for ratio of GroupB groups to GroupA groups, specified as a vector for a Portfolio, PortfolioCVaR, or PortfolioMAD input object (obj).

Note If input is scalar, UpperRatio undergoes scalar expansion to be conformable with the group matrices.

## Data Types: double

## Output Arguments

## obj - Updated portfolio object

object for portfolio
Updated portfolio object, returned as a Portfolio, PortfolioCVaR, or PortfolioMAD object. For more information on creating a portfolio object, see

- Portfolio
- PortfolioCVaR
- PortfolioMAD


## Tips

- You can also use dot notation to set up group ratio constraints for portfolio weight. obj = obj.setGroupRatio(GroupA, GroupB, LowerRatio, UpperRatio);
- To remove group ratio constraints, enter empty arrays for the corresponding arrays. To add to existing group ratio constraints, use addGroupRatio.


## See Also

addGroupRatio|getGroupRatio

## Topics

"Working with Group Ratio Constraints Using Portfolio Object" on page 4-84
"Working with Group Constraints Using PortfolioCVaR Object" on page 5-74
"Working with Group Constraints Using PortfolioMAD Object" on page 6-71
"Portfolio Optimization Examples" on page 4-167
"Portfolio Set for Optimization Using Portfolio Object" on page 4-10
"Portfolio Set for Optimization Using PortfolioCVaR Object" on page 5-10
"Portfolio Set for Optimization Using PortfolioMAD Object" on page 6-10

## Introduced in R2011a

## setGroups

Set up group constraints for portfolio weights

## Syntax

```
obj = setGroups(obj,GroupMatrix,LowerGroup)
obj = setGroups(obj,GroupMatrix,LowerGroup,UpperGroup)
```


## Description

obj = setGroups(obj,GroupMatrix,LowerGroup) sets up group constraints for portfolio weights for Portfolio, PortfolioCVaR, or PortfolioMAD objects. For details on the respective workflows when using these different objects, see "Portfolio Object Workflow" on page 4-21, "PortfolioCVaR Object Workflow" on page 5-20, and "PortfolioMAD Object Workflow" on page 6-19.
obj = setGroups(obj,GroupMatrix,LowerGroup,UpperGroup) sets up group constraints for portfolio weights for portfolio objects with an additional option specified for UpperGroup.

Given GroupMatrix and either LowerGroup or UpperGroup, a portfolio Port must satisfy the following:

```
LowerGroup <= GroupMatrix * Port <= UpperGroup
```


## Examples

## Set Group Constraints for a Portfolio Object

Suppose you have a portfolio of five assets and you want to ensure that the first three assets constitute at most $30 \%$ of your portfolio. Given a Portfolio object p, set the group constraints with the following.

```
G = [ true true true false false ];
p = Portfolio;
```

```
p = setGroups(p, G, [], 0.3);
disp(p.NumAssets);
    5
disp(p.GroupMatrix);
    1 1 1 1 1 0
disp(p.UpperGroup);
    0.3000
```


## Set Group Constraints for a PortfolioCVaR Object

Suppose you have a portfolio of five assets and you want to ensure that the first three assets constitute at most $30 \%$ of your portfolio. Given a CVaR portfolio object $p$, set the group constraints with the following.

```
G = [ true true true false false ];
p = PortfolioCVaR;
p = setGroups(p, G, [], 0.3);
disp(p.NumAssets);
    5
disp(p.GroupMatrix);
    1
disp(p.UpperGroup);
    0.3000
```


## Set Group Constraints for a PortfolioMAD Object

Suppose you have a portfolio of five assets and you want to ensure that the first three assets constitute at most $30 \%$ of your portfolio. Given PortfolioMAD object p, set the group constraints with the following.

```
G = [ true true true false false ];
p = PortfolioMAD;
p = setGroups(p, G, [], 0.3);
disp(p.NumAssets);
    5
disp(p.GroupMatrix);
    1 1 1 1 1 0 0
disp(p.UpperGroup);
    0.3000
```


## Input Arguments

## obj - Object for portfolio

object
Object for portfolio, specified using Portfolio, PortfolioCVaR, or PortfolioMAD object. For more information on creating a portfolio object, see

- Portfolio
- PortfolioCVaR
- PortfolioMAD


## Data Types: object

## GroupMatrix - Group constraint matrix

logical or numeric matrix
Group constraint matrix, specified as a matrix for a Portfolio, PortfolioCVaR, or PortfolioMAD input object (obj).

Note The group matrix GroupMatrix is usually an indicator of membership in groups, which means that its elements are usually either 0 or 1 . Because of this interpretation, GroupMatrix can be either a logical or numerical matrix.

## Data Types: double

## LowerGroup - Lower bound for group constraints

vector
Lower bound for group constraints, specified as a vector for a Portfolio, PortfolioCVaR, or PortfolioMAD input object (obj).

Note If input is scalar, LowerGroup undergoes scalar expansion to be conformable with GroupMatrix.

## Data Types: double

## UpperGroup - Upper bound for group constraints

vector
Upper bound for group constraints, returned as a vector for a Portfolio, PortfolioCVaR, or PortfolioMAD input object (obj).

Note If input is scalar, UpperGroup undergoes scalar expansion to be conformable with GroupMatrix.

## Data Types: double

## Output Arguments

## obj - Updated portfolio object

object for portfolio
Updated portfolio object, returned as a Portfolio, PortfolioCVaR, or PortfolioMAD object. For more information on creating a portfolio object, see

- Portfolio
- PortfolioCVaR
- PortfolioMAD


## Tips

- You can also use dot notation to set up group constraints for portfolio weights. obj = obj.setGroups(GroupMatrix, LowerGroup, UpperGroup);
- To remove group constraints, enter empty arrays for the corresponding arrays. To add to existing group constraints, use addGroups.


## See Also

addGroups | getGroups

## Topics

"Working with Group Constraints Using Portfolio Object" on page 4-80
"Working with Group Constraints Using PortfolioCVaR Object" on page 5-74
"Working with Group Constraints Using PortfolioMAD Object" on page 6-71
"Constraint Specification Using a Portfolio Object" on page 3-34
"Portfolio Optimization Examples" on page 4-167
"Portfolio Set for Optimization Using Portfolio Object" on page 4-10
"Portfolio Set for Optimization Using PortfolioCVaR Object" on page 5-10
"Portfolio Set for Optimization Using PortfolioMAD Object" on page 6-10

## Introduced in R2011a

## setInequality

Set up linear inequality constraints for portfolio weights

## Syntax

```
obj = setInequality(obj,AInequality,bInequality)
```


## Description

obj = setInequality(obj,AInequality,bInequality) sets up linear inequality constraints for portfolio weights for Portfolio, PortfolioCVaR, or PortfolioMAD objects. For details on the respective workflows when using these different objects, see "Portfolio Object Workflow" on page 4-21, "PortfolioCVaR Object Workflow" on page 5-20, and "PortfolioMAD Object Workflow" on page 6-19.

Given a linear inequality constraint matrix AInequality and vector bInequality, every weight in a portfolio Port must satisfy the following:

AInequality * Port <= bInequality

## Examples

## Set Linear Inequality Constraints for a Portfolio Object

Suppose you have a portfolio of five assets and you want to ensure that the first three assets are no more than $50 \%$ of your portfolio. Given a Portfolio object p, set the linear inequality constraints with the following.

```
A = [ 1 1 1 1 0 0 ];
b = 0.5;
p = Portfolio;
p = setInequality(p, A, b);
disp(p.NumAssets);
```

5

```
disp(p.AInequality);
```

$\begin{array}{lllll}1 & 1 & 1 & 0 & 0\end{array}$

```
disp(p.bInequality);
```

0.5000

## Set Linear Inequality Constraints for a PortfolioCVaR Object

Suppose you have a portfolio of five assets and you want to ensure that the first three assets are no more than $50 \%$ of your portfolio. Given a CVaR portfolio object p, set the linear inequality constraints with the following.
$\mathrm{A}=\left[\begin{array}{lllll}1 & 1 & 1 & 0\end{array}\right] ;$
b $=0.5$;
p = PortfolioCVaR;
$\mathrm{p}=\operatorname{setInequality(p,~A,~b);~}$
disp(p.NumAssets);
5
disp(p.AInequality);
$\begin{array}{lllll}1 & 1 & 1 & 0 & 0\end{array}$
disp(p.bInequality);
0.5000

## Set Linear Inequality Constraints for a PortfolioMAD Object

Suppose you have a portfolio of five assets and you want to ensure that the first three assets are no more than $50 \%$ of your portfolio. Given PortfolioMAD object p, set the linear inequality constraints with the following.
$A=\left[\begin{array}{llllll}1 & 1 & 1 & 0 & 0\end{array}\right] ;$
b $=0.5$;

```
p = PortfolioMAD;
p = setInequality(p, A, b);
disp(p.NumAssets);
    5
disp(p.AInequality);
    1 1 1 
disp(p.bInequality);
    0.5000
```


## Input Arguments

## obj - Object for portfolio

object
Object for portfolio, specified using Portfolio, PortfolioCVaR, or PortfolioMAD object. For more information on creating a portfolio object, see

- Portfolio
- PortfolioCVaR
- PortfolioMAD

Data Types: object

## AInequality - Matrix to form linear inequality constraints matrix

Matrix to form linear inequality constraints, specified as a matrix for a Portfolio, PortfolioCVaR, or PortfolioMAD input object (obj).

Note An error results if AInequality is empty and bInequality is nonempty.

## bInequality - Vector to form linear inequality constraints <br> vector

Vector to form linear inequality constraints, specified as a vector for a Portfolio, PortfolioCVaR, or PortfolioMAD input object (obj).

Note An error results if AInequality is nonempty and bInequality is empty.

## Data Types: double

## Output Arguments

## obj - Updated portfolio object

object for portfolio
Updated portfolio object, returned as a Portfolio, PortfolioCVaR, or PortfolioMAD object. For more information on creating a portfolio object, see

- Portfolio
- PortfolioCVaR
- PortfolioMAD


## Tips

- You can also use dot notation to set up linear inequality constraints for portfolio weights.

```
obj = obj.setInequality(AInequality, bInequality);
```

- To remove inequality constraints, enter empty arguments. To add to existing inequality constraints, use addInequality.


## See Also

addInequality|getInequality

## Topics

"Working with Linear Inequality Constraints Using Portfolio Object" on page 4-91
"Working with Linear Inequality Constraints Using PortfolioCVaR Object" on page 5-85
"Working with Linear Inequality Constraints Using PortfolioMAD Object" on page 6-82
"Portfolio Optimization Examples" on page 4-167
"Portfolio Set for Optimization Using Portfolio Object" on page 4-10
"Portfolio Set for Optimization Using PortfolioCVaR Object" on page 5-10
"Portfolio Set for Optimization Using PortfolioMAD Object" on page 6-10

## Introduced in R2011a

## setlnitPort

Set up initial or current portfolio

## Syntax

```
obj = setInitPort(obj,InitPort)
obj = setInitPort(obj,InitPort,NumAssets)
```


## Description

obj = setInitPort(obj,InitPort) sets up initial or current portfolio for Portfolio, PortfolioCVaR, or PortfolioMAD objects. For details on the respective workflows when using these different objects, see "Portfolio Object Workflow" on page 421, "PortfolioCVaR Object Workflow" on page 5-20, and "PortfolioMAD Object Workflow" on page 6-19.
obj = setInitPort(obj,InitPort,NumAssets) sets up initial or current portfolio for portfolio objects with an additional options specified for NumAssets.

## Examples

## Set the InitPort Property for a Portfolio Object

Given an initial portfolio in x0, use the setInitPort function to set the InitPort property.

```
p = Portfolio('NumAssets', 4);
x0 = [ 0.3; 0.2; 0.2; 0.0 ];
p = setInitPort(p, x0);
disp(p.InitPort);
    0.3000
    0.2000
    0.2000
        0
```


## Set InitPort to Create an Equally-Weighted Portfolio of Four Assets for a Portfolio Object

Create an equally weighted portfolio of four assets using the setInitPort function.

```
p = Portfolio('NumAssets', 4);
p = setInitPort(p, 1/4, 4);
disp(p.InitPort);
0.2500
0.2500
0.2500
0.2500
```


## Set the InitPort Property for a PortfolioCVaR Object

Given an initial portfolio in $\times 0$, use the setInitPort function to set the InitPort property.

```
p = PortfolioCVaR('NumAssets', 4);
x0 = [ 0.3; 0.2; 0.2; 0.0 ];
p = setInitPort(p, x0);
disp(p.InitPort);
0.3000
0.2000
0.2000
0
```


## Set InitPort to Create an Equally-Weighted Portfolio of Four Assets for a PortfolioCVaR Object

Create an equally weighted portfolio of four assets using the setInitPort function.

```
p = PortfolioCVaR('NumAssets', 4);
p = setInitPort(p, 1/4, 4);
disp(p.InitPort);
```

0.2500
0.2500
0.2500
0.2500

## Set the InitPort Property for a PortfolioMAD Object

Given an initial portfolio in $\times 0$, use the setInitPort function to set the InitPort property.

```
p = PortfolioMAD('NumAssets', 4);
x0 = [ 0.3; 0.2; 0.2; 0.0 ];
p = setInitPort(p, x0);
disp(p.InitPort);
    0.3000
    0.2000
    0.2000
        0
```


## Set InitPort to Create an Equally-Weighted Portfolio of Four Assets for a PortfolioMAD Object

Create an equally weighted portfolio of four assets using the setInitPort function.

```
p = PortfolioMAD('NumAssets', 4);
p = setInitPort(p, 1/4, 4);
disp(p.InitPort);
```

0.2500
0.2500
0.2500
0.2500

## Input Arguments

## obj - Object for portfolio

## object

Object for portfolio, specified using Portfolio, PortfolioCVaR, or PortfolioMAD object. For more information on creating a portfolio object, see

- Portfolio
- PortfolioCVaR
- PortfolioMAD


## Data Types: object

## InitPort - Initial or current portfolio weights

vector

Initial or current portfolio weights, specified as a vector for a Portfolio, PortfolioCVaR, or PortfolioMAD input object (obj).

Note If InitPort is specified as a scalar and NumAssets exists, then InitPort undergoes scalar expansion.

## Data Types: double

## NumAssets - Number of assets in portfolio 1 (default) | scalar

Number of assets in portfolio, specified as a scalar for a Portfolio, PortfolioCVaR, or PortfolioMAD input object (obj).

Note If it is not possible to obtain a value for NumAssets, it is assumed that NumAssets is 1 .

## Output Arguments

obj - Updated portfolio object<br>object for portfolio

Updated portfolio object, returned as a Portfolio, PortfolioCVaR, or PortfolioMAD object. For more information on creating a portfolio object, see

- Portfolio
- PortfolioCVaR
- PortfolioMAD


## Tips

- You can also use dot notation to set up an initial or current portfolio.
obj = obj.setInitPort(InitPort, NumAssets);
- To remove an initial portfolio, call this function with an empty argument [] for InitPort.


## See Also

setCosts | setTurnover

## Topics

"Common Operations on the Portfolio Object" on page 4-37
"Common Operations on the PortfolioCVaR Object" on page 5-36
"Common Operations on the PortfolioMAD Object" on page 6-34
"Portfolio Optimization Examples" on page 4-167
"Portfolio Set for Optimization Using Portfolio Object" on page 4-10
"Portfolio Set for Optimization Using PortfolioCVaR Object" on page 5-10
"Portfolio Set for Optimization Using PortfolioMAD Object" on page 6-10

## Introduced in R2011a

## setMinMaxNumAssets

Set cardinality constraints on the number of assets invested in a Portfolio object

## Syntax

obj $=$ setMinMaxNumAssets(obj,MinNumAssets,MaxNumAssets)

## Description

obj = setMinMaxNumAssets(obj,MinNumAssets,MaxNumAssets) sets cardinality constraints for a Portfolio object. MinNumAssets and MaxNumAssets are the minimum and maximum number of assets invested in the portfolio, respectively. The total number of allocated assets satisfying the Bound constraints is between [MinNumAssets, MaxNumAssets]. For details on the workflow when using a Portfolio object, see "Portfolio Object Workflow" on page 4-21.

## Examples

## Set MaxNumAssets Constraint for a Portfolio Object

Set the maximum cardinality constraint for a three-asset portfolio for which you have the mean and covariance values of the asset returns.

```
AssetMean = [ 0.0101110; 0.0043532; 0.0137058 ];
AssetCovar = [ 0.00324625 0.00022983 0.00420395;
    0.00022983 0.00049937 0.00019247;
    0.00420395 0.00019247 0.00764097 ];
p = Portfolio('AssetMean', AssetMean, 'AssetCovar', AssetCovar);
p = setDefaultConstraints(p);
```

When working with a Portfolio object, the setMinMaxNumAssets function enables you to set up the limits on the number of assets invested. Use setMinMaxNumAssets to limit the total number of allocated assets to no more than two.

```
p = setMinMaxNumAssets(p, [], 2);
```

Use estimateFrontierByReturn to estimate optimal portfolios with targeted portfolio returns.

```
pwgt = estimateFrontierByReturn(p,[ 0.008, 0.01 ])
pwgt = 3\times2
```

| 0 | 0 |
| ---: | ---: |
| 0.6101 | 0.3962 |
| 0.3899 | 0.6038 |

## Set MinNumAssets Constraint for a Portfolio Object

Set the minimum cardinality constraint for a three-asset portfolio for which you have the mean and covariance values of the asset returns.

```
AssetMean = [ 0.0101110; 0.0043532; 0.0137058 ];
AssetCovar = [ 0.00324625 0.00022983 0.00420395;
    0.00022983 0.00049937 0.00019247;
    0.00420395 0.00019247 0.00764097 ];
p = Portfolio('AssetMean', AssetMean, 'AssetCovar', AssetCovar);
p = setDefaultConstraints(p);
```

When working with a Portfolio object, the setMinMaxNumAssets function enables you to set up limits on the number of assets invested. These limits are also known as cardinality constraints. When managing a portfolio, it is common that you want to invest in at least a certain number of assets. In addition, you should also clearly define the weight requirement for each invested asset. You can do this using setBounds with a 'Conditional' BoundType. If you do not specify a 'Conditional' BoundType, the optimizer cannot understand which assets are invested assets and cannot formulate the MinNumAssets constraint.

The following example specifies that at least two assets should be invested and the investments should be greater than $16 \%$.
p = setMinMaxNumAssets(p, 2, []);
p = setBounds(p, 0.16, 'BoundType', 'conditional');

Use estimateFrontierByReturn to estimate optimal portfolios with targeted portfolio returns.

```
pwgt = estimateFrontierByReturn(p,[ 0.008, 0.01 ])
pwgt = 3×2
\begin{tabular}{ll}
0.2861 & 0.3967 \\
0.5001 & 0.2438 \\
0.2138 & 0.3595
\end{tabular}
```


## Set 'Conditional' BoundType, MinNumAssets, and MaxNumAssets Constraints for a Portfolio Object

Set the minimum and maximum cardinality constraints and a 'Conditional'
BoundType for a three-asset portfolio for which you have the mean and covariance values of the asset returns.

```
AssetMean = [ 0.0101110; 0.0043532; 0.0137058 ];
AssetCovar = [ 0.00324625 0.00022983 0.00420395;
    0.00022983 0.00049937 0.00019247;
    0.00420395 0.00019247 0.00764097 ];
```

p = Portfolio('AssetMean', AssetMean, 'AssetCovar', AssetCovar);
p = setDefaultConstraints(p);

When working with a Portfolio object, the setMinMaxNumAssets function enables you to set up the limits on the number of assets invested. The following example specifies that exactly two assets should be invested using setMinMaxNumAssets and the investment should be equally allocated among the two assets using setBounds.

```
p = setMinMaxNumAssets(p, 2, 2);
p = setBounds(p, 0.5, 0.5, 'BoundType', 'conditional');
```

Use estimateFrontierByReturn to estimate optimal portfolios with targeted portfolio returns.

```
pwgt = estimateFrontierByReturn(p,[ 0.008, 0.01 ])
pwgt = 3\times2
```

| 0 | 0.5000 |
| ---: | ---: |
| 0.5000 | 0 |
| 0.5000 | 0.5000 |

## Input Arguments

## obj - Object for portfolio

## object

Object for portfolio, specified using a Portfolio object. For more information on creating a portfolio object, see Portfolio.
Data Types: object

## MinNumAssets - Minimum number of assets allocated in portfolio scalar numeric

Minimum number of assets allocated in the portfolio, specified using a scalar numeric.
Data Types: double

## MaxNumAssets - Maximum number of assets allocated in portfolio scalar numeric

Maximum number of assets allocated in the portfolio, specified using a scalar numeric.
Data Types: double

## Output Arguments

## obj - Updated portfolio object

object for portfolio
Updated portfolio object, returned as a Portfolio object. For more information on creating a portfolio object, see Portfolio.

## Tips

- You can also use dot notation to set up a list of identifiers for assets.
obj = obj.setMinMaxNumAssets(MinNumAssets,MaxNumAssets);
- Specifying empty values ([[]) for MinNumAssets and MaxNumAsssets removes limit constraints from the Portfolio object.


## See Also

estimateAssetMoments|estimateFrontier|estimateFrontierByReturn| estimateFrontierByRisk|estimateFrontierLimits | estimateMaxSharpeRatio|estimatePortMoments|estimatePortReturn| estimatePortRisk|estimatePortSharpeRatio| setBounds|setSolverMINLP

## Topics

"Working with 'Conditional' BoundType, MinNumAssets, and MaxNumAssets Constraints Using Portfolio Object" on page 4-94
"Troubleshooting for Setting 'Conditional' BoundType, MinNumAssets, and MaxNumAssets Constraints" on page 4-156
"Common Operations on the Portfolio Object" on page 4-37
"Portfolio Optimization Examples" on page 4-167
"Portfolio Optimization with Semicontinuous and Cardinality Constraints"
"Mixed-Integer Quadratic Programming Portfolio Optimization: Problem-Based" (Optimization Toolbox)

## Introduced in R2018b

## setOneWayTurnover

Set up one-way portfolio turnover constraints

## Syntax

obj = setOneWayTurnover(obj,BuyTurnover)
obj = setOneWayTurnover(obj,BuyTurnover,SellTurnover,InitPort, NumAssets)

## Description

obj = setOneWayTurnover(obj,BuyTurnover) sets up one-way portfolio turnover constraints for Portfolio, PortfolioCVaR, or PortfolioMAD objects. For details on the respective workflows when using these different objects, see "Portfolio Object Workflow" on page 4-21, "PortfolioCVaR Object Workflow" on page 5-20, and "PortfolioMAD Object Workflow" on page 6-19.
obj = setOneWayTurnover(obj,BuyTurnover,SellTurnover,InitPort, NumAssets) sets up one-way portfolio turnover constraints for portfolio objects with additional options specified for SellTurnover, InitPort, and NumAssets.

Given an initial portfolio in InitPort and an upper bound for portfolio turnover on purchases in BuyTurnover or sales in SellTurnover, the one-way turnover constraints require any portfolio Port to satisfy the following:

```
1' * max{0, Port - InitPort} <= BuyTurnover
1' * max{0, InitPort - Port} <= SellTurnover
```

Note If Turnover = BuyTurnover = SellTurnover, the constraint is not equivalent to:

```
1' * | Port - InitPort | <= Turnover
```

To set this constraint, use setTurnover.

## Examples

## Set One-Way Turnover Constraints for a Portfolio Object

Set one-way turnover constraints.

```
p = Portfolio('AssetMean',[0.1, 0.2, 0.15], 'AssetCovar',...
[ 0.005, -0.010, 0.004; -0.010, 0.040, -0.002; 0.004, -0.002, 0.023]);
p = setBudget(p, 1, 1);
p = setOneWayTurnover(p, 1.3, 0.3, 0); %130-30 portfolio
plotFrontier(p);
```



## Set One-Way Turnover Constraints for a PortfolioCVaR Object

Set one-way turnover constraints.

```
x0 = [ 0.12; 0.09; 0.08; 0.07; 0.1; 0.1; 0.15; 0.11; 0.08; 0.1 ];
p = PortfolioCVaR('InitPort', x0);
p = setOneWayTurnover(p, 0.3, 0.2);
disp(p.NumAssets);
    1 0
disp(p.BuyTurnover)
    0.3000
disp(p.SellTurnover)
    0.2000
disp(p.InitPort);
    0.1200
    0.0900
    0.0800
    0.0700
    0.1000
    0.1000
    0.1500
    0.1100
    0.0800
    0.1000
```


## Set One-Way Turnover Constraints for a PortfolioMAD Object

Set one-way turnover constraints.

```
x0 = [ 0.12; 0.09; 0.08; 0.07; 0.1; 0.1; 0.15; 0.11; 0.08; 0.1 ];
p = PortfolioMAD('InitPort', x0);
p = setOneWayTurnover(p, 0.3, 0.2);
disp(p.NumAssets);
```

```
disp(p.BuyTurnover)
    0.3000
disp(p.SellTurnover)
    0.2000
disp(p.InitPort);
    0.1200
    0.0900
    0.0800
    0.0700
    0.1000
    0.1000
    0.1500
    0.1100
    0.0800
    0.1000
```


## Input Arguments

## obj - Object for portfolio

object
Object for portfolio, specified using Portfolio, PortfolioCVaR, or PortfolioMAD object. For more information on creating a portfolio object, see

- Portfolio
- PortfolioCVaR
- PortfolioMAD


## Data Types: object

## BuyTurnover - Turnover constraint on purchases

## nonnegative and finite scalar

Turnover constraint on purchases, specified as a nonnegative and finite scalar for a Portfolio, PortfolioCVaR, or PortfolioMAD input object (obj).

## SellTurnover - Turnover constraint on sales nonnegative and finite scalar

Turnover constraint on sales, specified as a nonnegative and finite scalar for a Portfolio, PortfolioCVaR, or PortfolioMAD input object (obj).

## InitPort - Initial or current portfolio weights <br> 0 (default) | finite vector with NumAssets > 0 elements

Initial or current portfolio weights, specified as a finite vector with NumAssets $>0$ elements for a Portfolio, PortfolioCVaR, PortfolioMAD input object (obj).

Note If no InitPort is specified, that value is assumed to be 0 .
If InitPort is specified as a scalar and NumAssets exists, then InitPort undergoes scalar expansion.

## Data Types: double

## NumAssets - Number of assets in portfolio <br> 1 (default) | scalar

Number of assets in portfolio, specified as a scalar for a Portfolio, PortfolioCVaR, or PortfolioMAD input object (obj).

Note If it is not possible to obtain a value for NumAssets, it is assumed that NumAssets is 1 .

## Data Types: double

## Output Arguments

## obj - Updated portfolio object

object for portfolio
Updated portfolio object, returned as a Portfolio, PortfolioCVaR, or PortfolioMAD object. For more information on creating a portfolio object, see

- Portfolio
- PortfolioCVaR
- PortfolioMAD


## Definitions

## One-way Turnover Constraint

One-way turnover constraints ensure that estimated optimal portfolios differ from an initial portfolio by no more than specified amounts according to whether the differences are purchases or sales.

The constraints take the form

$$
\begin{aligned}
& 1^{T} \max \left\{0, x-x_{0}\right\} \leq \tau_{B} \\
& 1^{T} \max \left\{0, x_{0}-x\right\} \leq \tau_{S}
\end{aligned}
$$

with

- x - The portfolio (NumAssets vector)
- $\mathrm{x}_{0}$ - Initial portfolio (NumAssets vector)
- $\tau_{B}$ - Upper-bound for turnover constraint on purchases (scalar)
- $\tau_{S}$ - Upper-bound for turnover constraint on sales (scalar)

Specify one-way turnover constraints using the following properties in a supported portfolio object: BuyTurnover for $\tau_{B}$, SellTurnover for $\tau_{S}$, and InitPort for $\mathrm{x}_{0}$.

Note The average turnover constraint (which is set using setTurnover) is not just the combination of the one-way turnover constraints with the same value for the constraint.

## Tips

You can also use dot notation to set up one-way portfolio turnover constraints.

## See Also

getOneWayTurnover| setCosts | setInitPort| setTurnover

## Topics

"Working with One-Way Turnover Constraints Using Portfolio Object" on page 4-102
"Working with One-Way Turnover Constraints Using PortfolioCVaR Object" on page 5-92
"Working with One-Way Turnover Constraints Using PortfolioMAD Object" on page 6-88
"Portfolio Optimization Examples" on page 4-167
"Portfolio Set for Optimization Using Portfolio Object" on page 4-10
"Portfolio Set for Optimization Using PortfolioCVaR Object" on page 5-10
"Portfolio Set for Optimization Using PortfolioMAD Object" on page 6-10

## Introduced in R2011a

## setProbabilityLevel

Set probability level for VaR and CVaR calculations

## Syntax

```
obj = setProbabilityLevel(obj,ProbabilityLevel)
```


## Description

obj = setProbabilityLevel(obj,ProbabilityLevel) sets probability level for VaR and CVaR calculations for a PortfolioCVaR object. For details on the workflow, see "PortfolioCVaR Object Workflow" on page 5-20.

## Examples

## Set Probability Level for a PortfolioCVaR Object

Set the ProbabilityLevel for a CVaR portfolio object.

```
p = PortfolioCVaR;
p = setProbabilityLevel(p, 0.95);
disp(p.ProbabilityLevel)
```

0.9500

## Input Arguments

## obj - Object for portfolio

object
Object for portfolio, specified using a PortfolioCVaR object.

For more information on creating a PortfolioCVaR object, see

- PortfolioCVaR

Data Types: object

## ProbabilityLevel - Probability level which is $\mathbf{1}$ minus the probability of losses greater than the value-at-risk

scalar with value from 0 to 1
Probability level which is 1 minus the probability of losses greater than the value-at-risk, specified as a scalar with value from 0 to 1 .

Note ProbabilityLevel must be a value from 0 to 1 and, in most cases, should be a value from 0.9 to 0.99 .

## Data Types: double

## Output Arguments

obj - Updated portfolio object
object for portfolio
Updated portfolio object, returned as a PortfolioCVaR object. For more information on creating a portfolio object, see

- PortfolioCVaR


## Tips

You can also use dot notation to set the probability level for VaR and CVaR calculations:
obj = obj.setProbabilityLevel(ProbabilityLevel)

## See Also

setScenarios

## Topics

"What Are Scenarios?" on page 5-45
"Conditional Value-at-Risk" on page 5-6

## External Websites

Analyzing Investment Strategies with CVaR Portfolio Optimization in MATLAB (50 min 42 sec )

Introduced in R2012b

## setSolver

Choose main solver and specify associated solver options for portfolio optimization

## Syntax

```
obj = setSolver(obj,solverType)
obj = setSolver(obj,solverType,Name,Value)
obj = setSolver(obj,solverType,optimoptions)
```


## Description

obj = setSolver(obj, solverType) selects the main solver and enables you to specify associated solver options for portfolio optimization for Portfolio, PortfolioCVaR, or PortfolioMAD objects. For details on the respective workflows when using these different objects, see "Portfolio Object Workflow" on page 4-21, "PortfolioCVaR Object Workflow" on page 5-20, and "PortfolioMAD Object Workflow" on page 6-19.
obj = setSolver(obj, solverType,Name,Value) selects the main solver and enables you to specify associated solver options for portfolio optimization for portfolio objects with additional options specified by using one or more Name, Value pair arguments.
obj = setSolver(obj, solverType,optimoptions) selects the main solver and enables you to specify associated solver options for portfolio optimization for portfolio objects with an optimoptions object.

## Examples

## Set Solver Type for a Portfolio Object

If you use the quadprog function as the solverType, the default is the interior-point-convex version of quadprog.
load CAPMuniverse
p = Portfolio('AssetList',Assets(1:12));
$p=$ setDefaultConstraints(p);
p = setSolver(p, 'quadprog');
display(p.solverType);
quadprog
You can switch back to lcprog with:
p = setSolver(p, 'lcprog');
display(p.solverType);
lcprog

## Set the Solver Type as 'fmincon' for a PortfolioCVaR Object

Use 'fmincon' as the solverType.
p = PortfolioCVaR;
p = setSolver(p, 'fmincon');
display(p.solverType);
fmincon

## Set the Solver Type as 'fmincon' and Use Name-Value Pair Arguments to Set the Algorithm for a PortfolioCVaR Object

Use 'fmincon' as the solverType and use name-value pair arguments to set the algorithm to 'trust-region-reflective' and to turn off the display.

```
p = PortfolioCVaR;
p = setSolver(p, 'fmincon', 'Algorithm', 'trust-region-reflective', 'Display', 'off');
display(p.solverOptions.Algorithm);
```

```
trust-region-reflective
display(p.solverOptions.Display);
off
```


## Set the Solver Type as 'fmincon' and Use an optimoptions Object to Set the Algorithm for a PortfolioCVaR Object

Use 'fmincon' as the solverType and use an optimoptions object to set the algorithm to 'trust-region-reflective' and to turn off the display.

```
p = PortfolioCVaR;
options = optimoptions('fmincon', 'Algorithm', 'trust-region-reflective', 'Display',
p = setSolver(p, 'fmincon', options);
display(p.solverOptions.Algorithm);
trust-region-reflective
```

display(p.solverOptions.Display);
off

## Set 'cuttingplane' as the Solver Type with Default Options for a PortfolioCVaR Object

Use 'cuttingplane' as the solverType with default options.
p = PortfolioCVaR;
p = setSolver(p,'cuttingplane');
display(p.solverType);
cuttingplane

## Set 'cuttingplane' as the Solver Type with Maximum Iterations for a PortfolioCVaR Object

Use the Name-Value pair 'MaxIter' to set the maximum number of iterations to 1500.

```
p = PortfolioCVaR;
p = setSolver(p,'cuttingplane','MaxIter',1500);
display(p.solverType);
cuttingplane
display(p.solverOptions);
    MaxIter: 1500
    AbsTol: 1.0000e-06
    RelTol: 1.0000e-05
    MasterSolverOptions: [1x1 optim.options.Linprog]
```


## Set 'cuttingplane' as the Solver Type and Change the Master Solver Option for a PortfolioCVaR Object

For the master solver, continue using the dual-simplex algorithm with no display, but tighten its termination tolerance to le-8.
p = PortfolioCVaR;
options = optimoptions('linprog','Algorithm','Dual-Simplex','Display','off','Optimalit p = setSolver(p,'cuttingplane','MasterSolverOptions',options); display(p.solverType)

## cuttingplane

display(p.solverOptions)
MaxIter: 1000
AbsTol: 1.0000e-06
RelTol: 1.0000e-05
MasterSolverOptions: [1x1 optim.options.Linprog]
display(p.solverOptions.MasterSolverOptions.Algorithm)
dual-simplex
display(p.solverOptions.MasterSolverOptions.Display)
off
display(p.solverOptions.MasterSolverOptions.TolFun)
1.0000e-08

For the master solver, use the interior-point algorithm with no display, and with a termination tolerance of 1e-7.

```
p = PortfolioCVaR;
options = optimoptions('linprog','Algorithm','interior-point','Display','off','Optimal
p = setSolver(p,'cuttingplane','MasterSolverOptions',options);
display(p.solverType)
```


## cuttingplane

```
display(p.solverOptions)
```

MaxIter: 1000
AbsTol: 1.0000e-06
RelTol: 1.0000e-05
MasterSolverOptions: [1x1 optim.options.Linprog]
display(p.solverOptions.MasterSolverOptions.Algorithm)
interior-point
display(p.solverOptions.MasterSolverOptions.Display)

off
display(p.solverOptions.MasterSolverOptions.TolFun)
1.0000e-07

## Set Solver Type as 'fmincon' for a PortfolioMAD Object

Use 'fmincon' as the solverType.
p = PortfolioMAD;
p = setSolver(p, 'fmincon');
display(p.solverType);
fmincon

## Set the Solver Type as 'fmincon' and Use Name-Value Pair Arguments to Set the Algorithm for a PortfolioMAD Object

Use 'fmincon' as the solverType and use name-value pair arguments to set the algorithm to 'sqp' and to turn on the display.

```
p = PortfolioMAD;
p = setSolver(p, 'fmincon', 'Algorithm', 'sqp', 'Display', 'final');
display(p.solverOptions.Algorithm);
```

sqp
display(p.solverOptions.Display);
final

## Set Solver Type as 'fmincon' and Use an optimoptions Structure to Set the Algorithm for a PortfolioMAD Object

Use 'fmincon' as the solverType and use an optimoptions object to set the algorithm to 'trust-region-reflective' and to turn off the display.

```
p = PortfolioMAD;
options = optimoptions('fmincon', 'Algorithm', 'trust-region-reflective', 'Display',
p = setSolver(p, 'fmincon', options);
display(p.solverOptions.Algorithm);
trust-region-reflective
display(p.solverOptions.Display);
off
```


## Set the Solver Type as 'fmincon' and Use an optimoptions Structure to Set the Algorithm and Use of Gradients for a PortfolioMAD Object

Use 'fmincon' as the solverType and use an optimoptions object to set the algorithm to 'active-set' and to set the gradients flag 'on' for 'GradObj' and turn off the display.

```
p = PortfolioMAD;
options = optimoptions('fmincon','algorithm','active-set','display','off','gradobj','oו
p = setSolver(p, 'fmincon', options);
display(p.solverOptions.Algorithm);
active-set
display(p.solverOptions.Display);
off
```


## Input Arguments

## obj - Object for portfolio <br> object

Object for portfolio, specified using Portfolio, PortfolioCVaR, or PortfolioMAD object. For more information on creating a portfolio object, see

- Portfolio
- PortfolioCVaR
- PortfolioMAD

Data Types: object

## solverType - Solver to use for portfolio optimization

character vector | string | optimoptions object
Solver to use for portfolio optimization, specified using a character vector, string, or optimoptions object.

The SolverType input argument depends on which type of object (obj) is being used for a portfolio optimization.

For a Portfolio object:

- The default solverType for the Portfolio object is 'lcprog' with the control variables 'maxiter','tiebreak', 'tolpiv'
- The Portfolio object can also use 'quadprog', which has several different options that can be set with optimoptions. Like Optimization Toolbox software which uses the
interior-point-convex algorithm as the default algorithm for quadprog, the portfolio optimization tools also uses the interior-point-convex algorithm. For more information about quadprog and quadratic programming algorithms and options, see "Quadratic Programming Algorithms" (Optimization Toolbox).


## For a PortfolioCVaR object:

- The supported solverType are:
- 'fmincon'
- 'cuttingplane'
- The default is 'fmincon' using the 'sqp' algorithm.

For a PortfolioMAD object:

- The supported solverType is:
- 'fmincon'
- The default is 'fmincon' using the 'sqp' algorithm and 'GradObj' set to 'on'


## Data Types: char|string|object

## optimoptions - optimoptions object

## object

(Optional) optimoptions object, specified as an optimoptions object that is created using optimoptions from Optimization Toolbox. For example:

```
p = setSolver(p,'fmincon',optimoptions('fmincon','Display','iter'));
```

Note optimoptions is the default and recommended method to set solver options, however optimset is also supported.

## Data Types: object

## Name-Value Pair Arguments or optimoptions Object

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes.

You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Example: p = setSolver(p,'cuttingplane','MasterSolverOptions',options)

## MaxIter - Maximum number of iterations

1000 (default) | positive integer
Maximum number of iterations, specified as the comma-separated pair consisting of 'MaxIter' and a positive integer when using a PortfolioCVaR object.
Data Types: double

## AbsTol - Absolute stopping tolerance

1e-6 (default) | positive scalar
Absolute stopping tolerance, specified as the comma-separated pair consisting of 'AbsTol' and a positive scalar when using a PortfolioCVaR object.

Data Types: double

## RelTol - Relative stopping tolerance

1e-5 (default) | positive scalar
Relative stopping tolerance, specified as the comma-separated pair consisting of 'RelTol' and a positive scalar when using a PortfolioCVaR object.

Data Types: double

## MasterSolverOptions - Options for the master solver

optimoptions('linprog','Algorithm','Dual-Simplex','Display','off') (default)|optimoptions object

Options for the master solver linprog, specified as the comma-separated pair consisting of 'MasterSolverOptions' and an optimoptions object when using a PortfolioCVaR object. For more information about linprog and linear programming algorithms and options, see "Linear Programming Algorithms" (Optimization Toolbox).
Data Types: double

## Output Arguments

## obj - Updated portfolio object

object for portfolio
Updated portfolio object, returned as a Portfolio, PortfolioCVaR, or PortfolioMAD object. For more information on creating a portfolio object, see

- Portfolio
- PortfolioCVaR
- PortfolioMAD


## Definitions

## Name-Value Options to Control the Solver Specified in solverType

The name-value pair argument options depend on which type of object (obj) is being used for a portfolio optimization.

For a Portfolio object:

- The default solverType for the Portfolio object is 'lcprog' with the control variables 'maxiter','tiebreak','tolpiv'
- The Portfolio object can also use a solverType of 'quadprog' , which has several different options that can be set with optimoptions. Like Optimization Toolbox software which uses the interior-point-convex algorithm as the default algorithm for quadprog, the portfolio optimization tools also uses the interior-point-convex algorithm. For more information about quadprog and quadratic programming algorithms and options, see "Quadratic Programming Algorithms" (Optimization Toolbox).

For a PortfolioCVaR object:

- For the default solverType, ' fmincon', PortfolioCVaR by default sets the algorithm to 'sqp', uses objective function gradients, and turns off the display. All fmincon options are supported, either as name-value pair arguments or using an optimoptions object. For more information about fmincon and quadratic
programming algorithms and options, see "Quadratic Programming Algorithms" (Optimization Toolbox).
- For the cuttingplane solver, the following solver options are available as namevalue pair arguments:
- MaxIter
- AbsTol
- RelTol
- MasterSolverOptions


## For a PortfolioMAD object:

- For the default solverType ' fmincon', PortfolioMAD, by default, sets the algorithm to 'sqp' and turns off the display. While all variations of fmincon from Optimization Toolbox are accepted, use of 'sqp' and 'active-set' algorithms for fmincon are recommended and the use of 'interior-point' and 'trust-region-reflective' algorithms are not recommended for MAD portfolio optimization.
- All fmincon options are supported either as name-value pair arguments or using an optimoptions object.
- optimoptions is the default and recommended method to set solver options, however optimset is also supported. For details about fmincon and constrained nonlinear optimization algorithms and options, see "Constrained Nonlinear Optimization Algorithms" (Optimization Toolbox).


## Tips

You can also use dot notation to choose the solver and specify associated solver options.

```
obj = obj.setSolver(solverType,Name,Value);
```


## Algorithms

One version of the CVaR portfolio optimization problem minimizes risk, measured as the CVaR of the portfolio, subject to a target return and other linear constraints on the portfolio. For the definition of the CVaR function, see "Risk Proxy" on page 5-6.

```
\(\operatorname{minimize}_{x} C V a R_{\alpha}(x)\)
subject to \(\bar{y}^{T} x \geq\) TargetReturn
\(A x \leq b\)
\(A_{e q} x=b_{e q}\)
\(l b \leq x \leq u b\)
```

Vector $\bar{y}$ is the mean return vector (the column-wise mean of the scenario matrix $Y$ ), so that $\bar{y}^{T} x$ is the expected return of portfolio $x$. The first constraint says that the expected return must be at least as good as a target return.

An alternative version of the CVaR portfolio optimization problem maximizes the expected return of the portfolio, subject to a target risk and other linear constraints on the portfolio.

```
maximize \(\bar{y}^{T} x\)
subject to \(C V a R_{\alpha}(x) \leq C V a R L i m i t\)
\(A x \leq b\)
\(A_{e q} x=b_{e q}\)
\(l b \leq x \leq u b\)
```

The first constraint in this case says that the portfolio CVaR cannot exceed a given CVaR limit.

By default, the CVaR portfolio object uses fmincon to solve the CVaR portfolio optimization problems. For information about fmincon and quadratic programming algorithms and options, see "Quadratic Programming Algorithms" (Optimization Toolbox).

Alternatively, the CVaR portfolio optimization problems can be solved with ' cuttingplane' , an implementation of Kelley's cutting-plane method. For more information, see Kelley [45] at "Portfolio Optimization" on page A-7.

## References

[1] Kelley, J. E. "The Cutting-Plane Method for Solving Convex Programs." Journal of the Society for Industrial and Applied Mathematics. Vol. 8, No. 4, December 1960, pp. 703-712.
[2] Rockafellar, R. T. and S. Uryasev "Optimization of Conditional Value-at-Risk." Journal of Risk. Vol. 2, No. 3, Spring 2000, pp. 21-41.
[3] Rockafellar, R. T. and S. Uryasev "Conditional Value-at-Risk for General Loss Distributions." Journal of Banking and Finance. Vol. 26, 2002, pp. 1443-1471.

## See Also

getOneWayTurnover | setCosts | setInitPort| setSolverMINLP| setTurnover

## Topics

"Working with One-Way Turnover Constraints Using Portfolio Object" on page 4-102
"Working with One-Way Turnover Constraints Using PortfolioCVaR Object" on page 5-92
"Working with One-Way Turnover Constraints Using PortfolioMAD Object" on page 6-88
"Portfolio Optimization Examples" on page 4-167
"Portfolio Set for Optimization Using Portfolio Object" on page 4-10
"Portfolio Set for Optimization Using PortfolioCVaR Object" on page 5-10
"Portfolio Set for Optimization Using PortfolioMAD Object" on page 6-10
"Choosing and Controlling the Solver for Mean-Variance Portfolio Optimization" on page 4-131

## Introduced in R2011a

## setSolverMINLP

Choose mixed integer nonlinear programming (MINLP) solver for portfolio optimization

## Syntax

```
obj = setSolverMINLP(obj,solverTypeMINLP)
obj = setSolverMINLP(___ ,Name,Value)
```


## Description

obj = setSolverMINLP(obj, solverTypeMINLP) selects the mixed integer nonlinear programming (MINLP) solver and enables you to specify associated solver options for portfolio optimization for a Portfolio object.

When any one or any combination of 'Conditional' BoundType, MinNumAssets, or MaxNumAssets constraints are active, the portfolio problem is formulated by adding NumAssets binary variables. The binary variable 0 indicates that an asset is not invested and the binary variable 1 indicates that an asset is invested. For more information on using 'Conditional' BoundType, see setBounds. For more information on specifying MinNumAssets and MaxNumAssets, see setMinMaxNumAssets.

If you use the estimate functions with a Portfolio object for which any of the 'Conditional' BoundType, MinNumAssets, or MaxNumAssets constraints are active, MINLP solver is automatically used. For details on MINLP, see "Algorithms" on page 191612.
obj = setSolverMINLP( $\qquad$ ,Name, Value) specifies options using one or more name-value pair arguments in addition to the input arguments in the previous syntax.

## Examples

## Configure MINLP Solver and Options

Configure the MINLP solver for a three-asset portfolio for which you have the mean and covariance values of the asset returns.

```
AssetMean = [ 0.0101110; 0.0043532; 0.0137058 ];
AssetCovar = [ 0.00324625 0.00022983 0.00420395;
    0.00022983 0.00049937 0.00019247;
    0.00420395 0.00019247 0.00764097 ];
p = Portfolio('AssetMean', AssetMean, 'AssetCovar', AssetCovar);
```

When working with a Portfolio object, use setBounds with a
'Conditional' BoundType constraint to set $x i=0$ or $0.02<=x i<=0.5$ for all $i=$ 1,...NumAssets.
$p=$ setBounds(p, 0.02, 0.5,'BoundType', 'Conditional', 'NumAssets', 3);
When working with a Portfolio object, use setMinMaxNumAssets function to set up MinNumAssets and MaxNumAssets constraints for a portfolio. This sets limit constraints for the Portfolio object, where the total number of allocated assets satisfying the constraints is between MinNumAssets and MaxNumAssets. Setting MinNumAssets = MaxNumAssets $=2$, specifies that only two of the three assets are invested in the portfolio.

```
p = setMinMaxNumAssets(p, 2, 2);
```

Three different MINLP solvers (OuterApproximation, ExtendedCP, TrustRegionCP) use the cutting plane method. Use the setSolverMINLP function to configure the OuterApproximation solver and options.

```
pint = setSolverMINLP(p,'OuterApproximation', 'NonlinearScalingFactor', 1e4, 'Display'
pint.solverTypeMINLP
ans =
'OuterApproximation'
pint.solverOptionsMINLP
ans = struct with fields:
            MaxIterations: 1000
        AbsoluteGapTolerance: 1.0000e-07
        RelativeGapTolerance: 1.0000e-05
        NonlinearScalingFactor: 10000
        ObjectiveScalingFactor: 1000
```

```
    IntMasterSolverOptions: [1×1 optim.options.Intlinprog]
            Display: 'iter'
            CutGeneration: 'basic'
        MaxIterationsInactiveCut: 30
        ActiveCutTolerance: 1.0000e-07
NumIterationsEarlyIntegerConvergence: 30
```

You can also configure the options for intlinprog, which is the Master solver for mixed integer linear programming problems in the MINLP solver.

```
pint = setSolverMINLP(p,'OuterApproximation', 'IntMasterSolverOptions', optimoptions('
pint.solverOptionsMINLP.IntMasterSolverOptions
ans =
    intlinprog options:
```

        Set properties:
            Display: 'off'
        Default properties:
            AbsoluteGapTolerance: 0
                            BranchRule: 'maxpscost'
            ConstraintTolerance: 1.0000e-04
                    CutGeneration: 'basic'
                    CutMaxIterations: 10
                    Heuristics: 'basic'
            HeuristicsMaxNodes: 50
                            IntegerPreprocess: 'basic'
                            IntegerTolerance: 1.0000e-05
                            LPMaxIterations: 'max(30000,10*(number0fEqualities+number0fInequalit
            LPOptimalityTolerance: 1.0000e-07
                MaxFeasiblePoints: Inf
                    MaxNodes: 10000000
                            MaxTime: 7200
                            NodeSelection: 'simplebestproj'
                            ObjectiveCutOff: Inf
        ObjectiveImprovementThreshold: 0
                    OutputFcn: []
                            PlotFcn: []
                            RelativeGapTolerance: 1.0000e-04
                            RootLPAlgorithm: 'dual-simplex'
                            RootLPMaxIterations: 'max(30000,10*(numberOfEqualities+numberOfInequalit:
    
## Input Arguments

## obj - Object for portfolio <br> object

Object for portfolio, specified using Portfolio.
Data Types: object
solverTypeMINLP - MINLP solver for portfolio optimization

## character vector with a value of 'OuterApproximation', 'ExtendedCP' , or <br> 'TrustRegionCP' | string with a value of "OuterApproximation", "ExtendedCP", or <br> "TrustRegionCP"

MINLP solver for portfolio optimization when any one or any combination of 'Conditional' BoundType, MinNumAssets, or MaxNumAssets constraints are active. Specify solverTypeMINLP using a character vector or string with a value of 'OuterApproximation', 'ExtendedCP', or 'TrustRegionCP'.

For a Portfolio object, the default value of the solverTypeMINLP is 'OuterApproximation' with the following default settings:

- MaxIterations-1000
- AbsoluteGapTolerance-1.0000e-07
- RelativeGapTolerance-1.0000e-05
- Display - 'off'
- NonlinearScalingFactor-1000
- ObjectiveScalingFactor-1000
- CutGeneration - 'basic'
- MaxIterationsInactiveCut - 30
- NumIterationsEarlyIntegerConvergence-30
- ActiveCutTolerance-1.0000e-07
- TrustRegionStartIteration-2
- ShrinkRatio-0.75
- DeltaLimit-1
- IntMasterSolverOptions optimoptions('intlinprog','Algorithm', 'DualSimplex','Display','off')

Data Types: char|string

## Name-Value Pair Arguments or optimoptions Object

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1,Value1, . . . , NameN, ValueN.

Example: $p$ = setSolverMINLP(p,'ExtendedCP','MaxIterations', 10000,'NonlinearScalingFactor',1000)

## MaxIterations - Maximum number of iterations <br> 1000 (default) | nonnegative integer

Maximum number of iterations, specified as the comma-separated pair consisting of 'MaxIterations ' and a nonnegative integer value when using a Portfolio object.

Data Types: double

## NonlinearScalingFactor - Scaling factor for nonlinear function and gradient 1000 (default) | nonnegative real

Scaling factor for nonlinear function and gradient, specified as the comma-separated pair consisting of 'NonlinearScalingFactor' and a nonnegative real value when using a Portfolio object.

Data Types: double

## ObjectiveScalingFactor - Scales the objective function used by MasterSolver by a factor

```
1000 (default) | nonnegative real
```

Scales the objective function used by MasterSolver by a factor, specified as the commaseparated pair consisting of 'ObjectiveScalingFactor' and a nonnegative real value when using a Portfolio object.

Data Types: double

## AbsoluteGapTolerance - Solver stops if absolute difference between approximated nonlinear function value and its true value is less than or equal to AbsoluteGapTolerance <br> 0.0000001 (default) | nonnegative real

The solver stops if the absolute difference between the approximated nonlinear function value and its true value is less than or equal to AbsoluteGapTolerance.
AbsoluteGapTolerance is specified as the comma-separated pair consisting of 'AbsoluteGapTolerance' and a nonnegative real value when using a Portfolio object.

Data Types: double

## RelativeGapTolerance - Solver stops if the relative difference between approximated nonlinear function value and its true value is less than or equal to RelativeGapTolerance

0.00001 (default) | nonnegative real

The solver stops if the relative difference between the approximated nonlinear function value and its true value is less than or equal to RelativeGapTolerance. RelativeGapTolerance is specified as the comma-separated pair consisting of 'AbsoluteGapTolerance' and a nonnegative real value when using a Portfolio object.

## Data Types: double

## Display - Display output format

'off' (default)| character vector with value of 'iter', 'final', or 'off'
Display output format, specified as the comma-separated pair consisting of 'Display' and a character vector with a value of:

- 'off' - Display no output
- 'iter' - Display output at each iteration and the technical exit message
- 'final' - Display only the final output and the final technical exit message


## Data Types: char

## CutGeneration - Cut specification

'basic' (default)| character vector with value of 'midway ' or 'basic'
Cut specification, specified as the comma-separated pair consisting of 'CutGeneration' and a character vector with one of these values:

- 'midway' - Add the new cut at the midpoint between the latest and previous solutions found.
- 'basic' - Add the new cut at the latest solution found.

Data Types: char

## MaxIterationsInactiveCut - Removes constraints that are not active for last MaxIterationsInactiveCut iterations

30 (default) | nonnegative integer
Removes constraints that are not active for the last MaxIterationsInactiveCut iterations, specified as the comma-separated pair consisting of 'MaxIterationsInactiveCut' and a nonnegative integer value when using a Portfolio object. Generally, the MaxIterationsInactiveCut value is larger than 10.
Data Types: double
NumIterationsEarlyIntegerConvergence - When integer variable solution is stable for the last NumIterationsEarlyIntegerConvergence iterations, the solver computes a final NLP by using latest integer variable solution in the MILP 30 (default) | nonnegative integer

When the integer variable solution is stable for the last NumIterationsEarlyIntegerConvergence iterations, the solver computes a final NLP by using the latest integer variable solution in the MILP.
NumIterationsEarlyIntegerConvergence is specified as the comma-separated pair consisting of 'NumIterationsEarlyIntegerConvergence' and a nonnegative integer value.

Data Types: double

## ActiveCutTolerance - Determines if the cuts are active 0.0000001 (default) | nonnegative real

Determines if the cuts are active, specified as the comma-separated pair consisting of 'ActiveCutTolerance' and a nonnegative real value. ActiveCutTolerance is used together with MaxIterationsInactiveCut to decide which cuts to remove from the MILP subproblem.

Data Types: double

## TrustRegionStartIteration - Solver starts to apply trust region heuristic at TrustRegionStartIteration <br> 2 (default) | nonnegative integer

Solver starts to apply the trust region heuristic at TrustRegionStartIteration, specified as the comma-separated pair consisting of 'TrustRegionStartIteration' and a nonnegative integer.

Note The TrustRegionStartIteration name-value pair argument can only be used with a solverTypeMINLP of 'TrustRegionCP'.

## Data Types: double

## ShrinkRatio - Ratio to shrink size of trust region

0.75 (default) | nonnegative real between 0 and 1

Ratio to shrink size of trust region, specified as the comma-separated pair consisting of 'ShrinkRatio' and a nonnegative real value between 0 and 1. If the approximated functions do not have good agreement in the previous iterations, the algorithm uses this ratio to shrink the trust-region size.

Note The ShrinkRatio name-value pair argument can only be used with a solverTypeMINLP of 'TrustRegionCP'.

## Data Types: double

## DeltaLimit - Trust region of the approximated functions is bounded by DeltaLimit during iterations

1 (default) | nonnegative real
Trust region of the approximated functions is bounded by DeltaLimit during the iterations, specified as the comma-separated pair consisting of 'DeltaLimit' and a nonnegative real value. For portfolio problems, 1 is generally a good value for DeltaLimit, since portfolio weights are between 0 and 1 .

Note The DeltaLimit name-value pair argument can only be used with a solverTypeMINLP of 'TrustRegionCP'.

Data Types: double

## IntMasterSolverOptions - Options for master solver optimoptions('intlinprog','Algorithm','DualSimplex','Display', 'off') (default)|optimoptions object

Options for the master solver intlinprog, specified as the comma-separated pair consisting of 'IntMasterSolverOptions' and an optimoptions object.

Example: 'IntMasterSolverOptions',
optimoptions('intlinprog','Display','off')
Data Types: object

## Output Arguments

## obj — Updated portfolio object

object
Updated portfolio object, returned as a Portfolio object.

## Definitions

## MINLP Solvers

All three MINLP solvers ('OuterApproximation', 'ExtendedCP', and 'TrustRegionCP') defined by solverTypeMINLP rely on the cutting plane concept.

These MINLP solvers approximate the nonlinear convex function $f(x)$ by a piecewise linear approximation, which is a sequence of linear cuts around the original function. In this way, the original MINLP is reduced to a sequence of MILP subproblems, each one with a more refined approximation to $f(x)$ than previous MILPs, and yields a more optimal solution. The process continues until the solution found from MILP converges to the true function value within a certain tolerance.

- The 'ExtendedCP' solver iteratively adds a linear cut at the latest solution found to approximate $f(x)$.
- The 'OuterApproximation' solver is similar to 'ExtendedCP', but they differ in where to add the cut. Instead of using the solution from the latest MILP,

OuterApproximation uses the values of integer variables from the latest MILP solution and fixes them to reduce the MINLP to a nonlinear programming (NLP) problem. The cut is added at the solution from this NLP problem.

- The 'TrustRegionCP' solver is a version of 'ExtendedCP' that is modified to speed up the optimization process. In general, the trust region method uses a model to approximate the true function within a region at each iteration. In the context of the MINLP solver, the model is the maximum of all the added cuts. The true function is the nonlinear function $f(x)$ in the optimization problem. The region of the model is updated based on how well the model approximates the true function for the iteration. This approximation is the comparison of the predicted reduction of the objective function using the model vs. the true reduction.


## Tips

You can also use dot notation to specify associated name-value pair options.

```
obj = obj.setSolverMINLP(Name,Value);
```

Note The solverTypeMINLP and solverOptionsMINLP properties cannot be set using dot notation because they are hidden properties. To set the solverTypeMINLP and solverOptionsMINLP properties, use the setSolverMINLP function directly.

## Algorithms

When any one, or any combination of 'Conditional' BoundType, MinNumAssets, or MaxNumAssets constraints is active, the portfolio problem is formulated by adding NumAssets binary variables. The binary variable 0 indicates that an asset is not invested and the binary variable 1 indicates that an asset is invested.

The MinNumAssets and MaxNumAssets constraints narrow down the number of active positions in a portfolio to the range of $[\min N, \max N]$. In addition, the 'Conditional' BoundType constraint is to set a lower and upper bound so that the position is either 0 or lies in the range [minWgt, maxWgt]. These two types of constraints are incorporated into the mean-variance optimization model by introducing $n$ variables, $\nu_{i}$, which only take binary values 0 and 1 to indicate whether the corresponding asset is invested (1) or not invested (0). Here $n$ is the total number of assets and the constraints can be formulated as the following linear inequality constraints:

$$
\begin{aligned}
& \min N \leq \sum_{i=1}^{n} v_{i} \leq \max N \\
& \operatorname{minWg} t * v_{i} \leq x_{i} \leq \max W g t * v_{i} \\
& 0 \leq v \leq 1 \\
& v_{i} \text { are integers }
\end{aligned}
$$

In this equation, $\min N$ and $\max N$ are representations for MinNumAsset and MaxNumAsset that are set using setMinMaxNumAssets. Also, minWgt and maxWgt are representations for LowerBound and UpperBound that are set using setBounds.

The portfolio optimization problem to minimize the variance of the portfolio, subject to achieving a target expected return and some additional linear constraints on the portfolio weights, is formulated as

```
minimize \(x_{x}^{T} H x\)
s.t. \(m^{T} x \geq\) TargetReturn
\(A x \leq b\)
\(A_{e q} x=b_{e q}\)
\(l b \leq x \leq u b\)
```

In this equation, $H$ represents the covariance and $m$ represents the asset returns.
The portfolio optimization problem to maximize the return, subject to an upper limit on the variance of the portfolio return and some additional linear constraints on the portfolio weights, is formulated as

$$
\begin{aligned}
& \text { maximize }_{x} \quad m^{T} x \\
& \text { s.t. } x^{T} H x \leq \text { TargetRisk } \\
& A x \leq b \\
& A_{e q} x=b_{e q} \\
& l b \leq x \leq u b
\end{aligned}
$$

When the 'Conditional' BoundType, MinNumAssets, and MaxNumAssets constraints are added to the two optimization problems, the problems become:
minimize $_{x v} x^{T} H x$
s.t. $m^{T} x \geq$ TargetReturn
$A^{\prime}[x ; v] \leq b$,
$A_{e q} x=b_{e q}$
$\min N \leq \sum_{i=1}^{n} v_{i} \leq \max N$
$\operatorname{minWgt}\left(v_{i}\right) \leq x_{i} \leq \operatorname{maxWgt}\left(v_{i}\right)$
$l b \leq x \leq u b$
$0 \leq v \leq 1$
$v_{i}$ are integers
maximize $_{x v} \quad m^{T} x$
s.t. $x^{T} H x \geq$ TargetRisk
$A^{\prime}[x ; v] \leq b$,
$A_{e q} x=b_{e q}$
$\min N \leq \sum_{i=1}^{n} v_{i} \leq \max N$
$\operatorname{minWgt} * v_{i} \leq x_{i} \leq \operatorname{maxWgt}\left(v_{i}\right)$
$0 \leq v \leq 1$
$v_{i}$ are integers

## References

[1] Bonami, P., Kilinc, M., and J. Linderoth. "Algorithms and Software for Convex Mixed Integer Nonlinear Programs." Technical Report \#1664. Computer Sciences Department, University of Wisconsin-Madison, 2009.
[2] Kelley, J. E. "The Cutting-Plane Method for Solving Convex Programs." Journal of the Society for Industrial and Applied Mathematics. Vol. 8, Number 4, 1960, pp. 703712.
[3] Linderoth, J. and S. Wright. "Decomposition Algorithms for Stochastic Programming on a Computational Grid." Computational Optimization and Applications. Vol. 24, Issue 2-3, 2003, pp. 207-250.
[4] Nocedal, J., and S. Wright. Numerical Optimization. New York: Springer-Verlag, 1999.

## See Also

estimateFrontier|estimateFrontierByReturn|estimateFrontierByRisk| estimateFrontierLimits |estimateMaxSharpeRatio| setBounds | setMinMaxNumAssets | setSolver

## Topics

"Portfolio Optimization with Semicontinuous and Cardinality Constraints"
"Mixed-Integer Quadratic Programming Portfolio Optimization: Problem-Based" (Optimization Toolbox)
"Portfolio Set for Optimization Using Portfolio Object" on page 4-10
"Choosing and Controlling the Solver for Mean-Variance Portfolio Optimization" on page 4-131

Introduced in R2018b

## setScenarios

Set asset returns scenarios by direct matrix

Note setScenarios has been partially removed and will no longer accept a fints object for the GetAssetList argument. Use timetable instead for financial time series.

Use fts2timetable to convert a fints object to a timetable object.

## Syntax

```
obj = setScenarios(obj,AssetScenarios)
obj = setScenarios(obj,AssetScenarios,Name,Value)
```


## Description

obj = setScenarios(obj,AssetScenarios) sets asset returns scenarios by direct matrix for PortfolioCVaR or PortfolioMAD objects. For details on the workflows, see "PortfolioCVaR Object Workflow" on page 5-20, and "PortfolioMAD Object Workflow" on page 6-19.
obj = setScenarios(obj,AssetScenarios,Name,Value) set asset returns scenarios by direct matrix for PortfolioCVaR or PortfolioMAD objects using additional options specified by one or more Name, Value pair arguments.

## Examples

## Set Asset Returns Scenarios for a PortfolioCVaR Object

Given a PortfolioCVaR object p, use the setScenarios function to set asset return scenarios.
m = [ 0.05; 0.1; 0.12; 0.18 ];
$C=[0.00640 .004080 .001920$;

```
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;
AssetScenarios = mvnrnd(m, C, 20000);
p = PortfolioCVaR;
p = setScenarios(p, AssetScenarios);
p = setDefaultConstraints(p);
p = setProbabilityLevel(p, 0.95);
disp(p)
    PortfolioCVaR with properties:
    BuyCost: []
        SellCost: []
        RiskFreeRate: []
        ProbabilityLevel: 0.9500
            Turnover: []
        BuyTurnover: []
        SellTurnover: []
        NumScenarios: 20000
        Name: []
        NumAssets: 4
        AssetList: []
        InitPort: []
        AInequality: []
        bInequality: []
        AEquality: []
        bEquality: []
        LowerBound: [4x1 double]
        UpperBound: []
        LowerBudget: 1
        UpperBudget: 1
        GroupMatrix: []
            LowerGroup: []
            UpperGroup: []
        GroupA: []
        GroupB: []
    LowerRatio: []
    UpperRatio: []
```


## Set Asset Returns Scenarios for a PortfolioMAD Object

Given PortfolioMAD object $p$, use the setScenarios function to set asset return scenarios.

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;
```

AssetScenarios = mvnrnd(m, C, 20000);
$\mathrm{p}=$ PortfolioMAD;
p = setScenarios(p, AssetScenarios);
$p=$ setDefaultConstraints(p);
$\operatorname{disp}(p)$
PortfolioMAD with properties:
BuyCost: []
SellCost: []
RiskFreeRate: []
Turnover: []
BuyTurnover: []
SellTurnover: []
NumScenarios: 20000
Name: []
NumAssets: 4
AssetList: []
InitPort: []
AInequality: []
bInequality: []
AEquality: []
bEquality: []
LowerBound: [4x1 double]
UpperBound: []
LowerBudget: 1
UpperBudget: 1
GroupMatrix: []
LowerGroup: []

UpperGroup: []
GroupA: []
GroupB: []
LowerRatio: []
UpperRatio: []

## Set Asset Returns Scenarios for a PortfolioCVaR Object Using Timetable Data

To illustrate using the setScenarios function with AssetScenarios data continued in a timetable object, use the CAPMuniverse. mat which contains a timetable object (AssetTimeTable) for returns data.

```
load CAPMuniverse;
AssetsTimeTable.Properties;
head(AssetsTimeTable,5)
```

| Time |  | AAPL | AMZN | CSCO | DELL | EBAY |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 03-Jan-2000 | 00:00:00 | 0.088805 | 0.1742 | 0.008775 | -0.002353 | 0.12 |
| 04-Jan-2000 | 00:00:00 | -0.084331 | -0.08324 | -0.05608 | -0.08353 | -0.0938 |
| 05-Jan-2000 | 00:00:00 | 0.014634 | -0.14877 | -0.003039 | 0.070984 | 0.0668 |
| 06-Jan-2000 | 00:00:00 | -0.086538 | -0.060072 | -0.016619 | -0.038847 | -0.012 |
| 07-Jan-2000 | 00:00:00 | 0.047368 | 0.061013 | 0.0587 | -0.037708 | - 0.000 |

setScenarios accepts a name-value pair argument name 'DataFormat' with a corresponding value set to 'prices ' to indicate that the input to the function is in the form of asset prices and not returns (the default value for the 'DataFormat' argument is 'returns').
r = PortfolioCVaR;
r = setScenarios(r,AssetsTimeTable,'dataformat','returns');
In addition, the setScenarios function also extracts asset names or identifiers from a timetable object when the name-value argument 'GetAssetList' set to true (its default value is false). If the 'GetAssetList' value is true, the timetable column identifiers are used to set the AssetList property of the PortfolioCVaR object. To show this, the formation of the PortfolioCVaR object $r$ is repeated with the 'GetAssetList' flag set to true.

```
r = setScenarios(r,AssetsTimeTable,'GetAssetList',true);
disp(r.AssetList')
    'AAPL'
    'AMZN'
    'CSCO'
    'DELL'
    'EBAY'
    'GOOG '
    'HPQ'
    'IBM'
    'INTC'
    'MSFT'
    'ORCL
    'YHOO'
    'MARKET'
    'CASH'
```


## Input Arguments

## obj - Object for portfolio

## object

Object for portfolio, specified using a PortfolioCVaR or PortfolioMAD object.
For more information on creating a PortfolioCVaR or PortfolioMAD object, see

- PortfolioCVaR
- PortfolioMAD


## Data Types: object

## AssetScenarios - Scenarios for asset returns or prices matrix | table | timetable

Scenarios for asset returns or prices, specified as a matrix, table, or timetable that contains asset data that can be converted into asset returns ([NumSamples-byNumAssets] matrix).

AssetReturns data can be:

- NumSamples-by-NumAssets matrix.
- Table of NumSamples prices or returns at a given periodicity for a collection of NumAssets assets
- Timetable object with NumSamples observations and NumAssets time series

If the input data are prices, they can be converted into returns with the DataFormat name-value pair argument, where the default format is assumed to be 'Returns'. Be careful using price data because portfolio optimization usually requires total returns and not simply price returns.

This function sets up a function handle to indirectly access input AssetScenarios without needing to make a copy of the data.

Data Types: double|table | timetable

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Example: $p$ = setScenarios(p,
AssetScenarios,'DataFormat', 'Returns', 'GetAssetList',false);

## DataFormat - Flag to convert input data as prices into returns

'Returns' (default) | character vector with values 'Returns' or 'Prices'
Flag to convert input data as prices into returns, specified as the comma-separated pair consisting of 'DataFormat ' and a character vector with the values:

- 'Returns ' - Data in AssetReturns contains asset total returns.
- 'Prices' - Data in AssetReturns contains asset total return prices.

Data Types: char

## GetAssetList - Flag indicating which asset names to use for the asset list false (default) | logical with value true or false

Flag indicating which asset names to use for the asset list, specified as the commaseparated pair consisting of 'GetAssetList' and a logical with a value of true or false. Acceptable values for GetAssetList are:

- false - Do not extract or create asset names.
- true - Extract or create asset names from table or timetable.

If a table or timetable is passed into this function as AssetScenarios and the GetAssetList flag is true, the column names from the table or timetable are used as asset names in obj. AssetList.

If a matrix is passed and the GetAssetList flag is true, default asset names are created based on the AbstractPortfolio property defaultforAssetList, which is currently 'Asset'.

If the GetAssetList flag is false, no action occurs, which is the default behavior.
Data Types: logical

## Output Arguments

## obj - Updated portfolio object

object for portfolio
Updated portfolio object, returned as a PortfolioCVaR or PortfolioMAD object. For more information on creating a portfolio object, see

- PortfolioCVaR
- PortfolioMAD


## Tips

You can also use dot notation to set asset return scenarios.

```
obj = obj.setScenarios(AssetScenarios);
```


## See Also

getScenarios

## Topics

"Asset Returns and Scenarios Using PortfolioCVaR Object" on page 5-44
"Asset Returns and Scenarios Using PortfolioMAD Object" on page 6-42

## External Websites

Analyzing Investment Strategies with CVaR Portfolio Optimization in MATLAB (50 min 42 sec)

Introduced in R2012b

## setTrackingError

Set up maximum portfolio tracking error constraint

## Syntax

```
obj = setTrackingError(obj,TrackingError)
obj = setTrackingError(
```

$\qquad$

``` ,TrackingPort,NumAssets)
```


## Description

obj = setTrackingError(obj,TrackingError) sets up a maximum portfolio tracking error constraint for a Portfolio object. For details on the workflow when using a Portfolio object, see "Portfolio Object Workflow" on page 4-21.
obj = setTrackingError (__ , TrackingPort, NumAssets) sets up a maximum portfolio tracking error constraint using optional arguments for TrackingPort and NumAssets.

## Examples

## Set up a Tracking Error Constraint

Create a Portfolio object.
AssetMean = [ 0.05; 0.1; 0.12; 0.18 ];
AssetCovar = [ 0.0064 0.00408 0.00192 0;
0.004080 .02890 .0204 0.0119;
0.001920 .02040 .0576 0.0336;
00.01190 .03360 .1225 ];
p = Portfolio('mean', AssetMean, 'covar', AssetCovar, 'lb', 0, 'budget', 1)
$p=$
Portfolio with properties:

```
            BuyCost: []
            SellCost: []
    RiskFreeRate: []
        AssetMean: [4x1 double]
        AssetCovar: [4x4 double]
TrackingError: []
    TrackingPort: []
            Turnover: []
        BuyTurnover: []
        SellTurnover: []
            Name: []
        NumAssets: 4
            AssetList: []
            InitPort: []
        AInequality: []
        bInequality: []
            AEquality: []
            bEquality: []
            LowerBound: [4x1 double]
            UpperBound: []
            LowerBudget: 1
            UpperBudget: 1
            GroupMatrix: []
            LowerGroup: []
            UpperGroup: []
            GroupA: []
            GroupB: []
            LowerRatio: []
            UpperRatio: []
MinNumAssets: []
MaxNumAssets: []
            BoundType: []
```

Estimate the Sharpe ratio for the Portfolio object p and define the tracking error.

```
x0 = estimateMaxSharpeRatio(p);
```

te $=0.08$;
$p=\operatorname{set} T r a c k i n g E r r o r(p, t e, x 0) ;$
display(p.NumAssets);
4
display(p.TrackingError);
0.0800
display(p.TrackingPort);
0.6608
0.1622
0.0626
0.1143

## Input Arguments

## obj - Object for portfolio

object
Object for portfolio, specified using a Portfolio object. For more information on creating a portfolio object, see Portfolio.

Data Types: object

## TrackingError - Upper bound for portfolio tracking error nonnegative and finite scalar

Upper bound for portfolio tracking error, specified using a nonnegative and finite scalar.
Given an upper bound for portfolio tracking error in TrackingError and a tracking portfolio in TrackingPort, the tracking error constraint requires any portfolio in Port to satisfy

```
(Port - TrackingPort)'*AssetCovar*(Port - TrackingPort) <= TrackingError^2 .
```

For more information, see "Tracking Error Constraints" on page 4-17.
Data Types: double

## TrackingPort - Tracking portfolio weights <br> finite vector

Tracking portfolio weights, specified using a vector. TrackingPort must be a finite vector with NumAssets > 0 elements.

If no TrackingPort is specified, it is assumed to be 0. If TrackingPort is specified as a scalar and NumAssets exists, then TrackingPort undergoes scalar expansion.

## Data Types: double

## NumAssets - Number of assets in portfolio scalar

Number of assets in portfolio, specified using a scalar. If it is not possible to obtain a value for NumAssets, it is assumed that NumAssets is 1 .

## Data Types: double

## Output Arguments

obj — Updated portfolio object<br>object for portfolio

Updated portfolio object, returned as a Portfolio object. For more information on creating a portfolio object, see Portfolio.

Note The tracking error constraints can be used with any of the other supported constraints in the Portfolio object without restrictions. However, since the portfolio set necessarily and sufficiently must be a non-empty compact set, the application of a tracking error constraint can result in an empty portfolio set. Use estimateBounds to confirm that the portfolio set is non-empty and compact.

## Tips

You can also use dot notation to set up a maximum portfolio tracking error constraint.

```
obj = obj.setTrackingError(TrackingError, NumAssets);
```

To remove a tracking portfolio, call this function with an empty argument ([ ]) for TrackingError.
obj = setTrackingError(obj, [ ]);

## See Also

Portfolio|setTrackingPort

## Topics

"Working with Tracking Error Constraints Using Portfolio Object" on page 4-106
"Portfolio Optimization Examples" on page 4-167
"Tracking Error Constraints" on page 4-17
"Setting Up a Tracking Portfolio" on page 4-45

## Introduced in R2015b

## setTrackingPort

Set up benchmark portfolio for tracking error constraint

## Syntax

```
obj = setTrackingPort(obj,TrackingPort)
obj = setTrackingPort(
```

$\qquad$

``` ,NumAssets)
```


## Description

obj = setTrackingPort(obj,TrackingPort) sets up a benchmark portfolio for a tracking error constraint for a Portfolio object. For details on the workflow when using a Portfolio object, see "Portfolio Object Workflow" on page 4-21.
obj = setTrackingPort( $\qquad$ ,NumAssets) sets up a benchmark portfolio for a tracking error constraint using an optional input argument for NumAssets.

## Examples

## Set up a Tracking Port

Create a Portfolio object.

```
AssetMean = [ 0.05; 0.1; 0.12; 0.18 ];
AssetCovar = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
p = Portfolio('mean', AssetMean, 'covar', AssetCovar, 'lb', 0, 'budget', 1)
p =
    Portfolio with properties:
```

BuyCost: []

```
            SellCost: []
    RiskFreeRate: []
        AssetMean: [4x1 double]
        AssetCovar: [4x4 double]
TrackingError: []
    TrackingPort: []
            Turnover: []
        BuyTurnover: []
    SellTurnover: []
            Name: []
        NumAssets: 4
        AssetList: []
            InitPort: []
        AInequality: []
        bInequality: []
            AEquality: []
            bEquality: []
            LowerBound: [4x1 double]
            UpperBound: []
    LowerBudget: 1
    UpperBudget: 1
    GroupMatrix: []
            LowerGroup: []
            UpperGroup: []
            GroupA: []
            GroupB: []
            LowerRatio: []
            UpperRatio: []
MinNumAssets: []
MaxNumAssets: []
    BoundType: []
```

Estimate the Sharpe ratio for the Portfolio object p and define the tracking port.

```
x0 = estimateMaxSharpeRatio(p);
```

p = setTrackingPort(p, x0);
display(p.NumAssets);
4
display(p.TrackingPort);
0.6608
0.1622

## Input Arguments

## obj - Object for portfolio

## object

Object for portfolio, specified using a Portfolio object. For more information on creating a portfolio object, see Portfolio.

Data Types: object

## TrackingPort - Tracking portfolio weights <br> vector

Tracking portfolio weights, specified using a vector. If TrackingPort is specified as a scalar and NumAssets exists, then TrackingPort undergoes scalar expansion.

Data Types: double

## NumAssets - Number of assets in portfolio

scalar
Number of assets in portfolio, specified using a scalar. If it is not possible to obtain a value for NumAssets, it is assumed that NumAssets is 1.

Data Types: double

## Output Arguments

obj - Updated portfolio object
object for portfolio
Updated portfolio object, returned as a Portfolio object. For more information on creating a portfolio object, see Portfolio.

Note The tracking error constraints can be used with any of the other supported constraints in the Portfolio object without restrictions. However, since the portfolio set
necessarily and sufficiently must be a non-empty compact set, the application of a tracking error constraint can result in an empty portfolio set. Use estimateBounds to confirm that the portfolio set is non-empty and compact.

## Tips

You can also use dot notation to set up a benchmark portfolio for tracking error constraint.
obj = obj.setTrackingPort(TrackingPort, NumAssets);
To remove a tracking portfolio, call this function with an empty argument ([]) for TrackingPort.

```
obj = setTrackingPort(obj, [ ]);
```


## See Also

Portfolio|setTrackingError

## Topics

"Working with Tracking Error Constraints Using Portfolio Object" on page 4-106
"Portfolio Optimization Examples" on page 4-167
"Tracking Error Constraints" on page 4-17
"Setting Up a Tracking Portfolio" on page 4-45
Introduced in R2015b

19-1632

## setTurnover

Set up maximum portfolio turnover constraint

## Syntax

```
obj = setTurnover(obj,Turnover)
obj = setTurnover(obj,Turnover,InitPort,NumAssets)
```


## Description

obj = setTurnover(obj,Turnover) sets up maximum portfolio turnover constraint for Portfolio, PortfolioCVaR, or PortfolioMAD objects. For details on the respective workflows when using these different objects, see "Portfolio Object Workflow" on page 4-21, "PortfolioCVaR Object Workflow" on page 5-20, and "PortfolioMAD Object Workflow" on page 6-19.
obj = setTurnover(obj,Turnover,InitPort,NumAssets) sets up maximum portfolio turnover constraint for portfolio objects with additional options specified for Turnover, InitPort, and NumAssets.

Given an upper bound for portfolio turnover in Turnover and an initial portfolio in InitPort, the turnover constraint requires any portfolio in Port to satisfy the following:

```
1' *1/2* | Port - InitPort | <= Turnover
```


## Examples

## Set Turnover Constraint for a Portfolio Object

Given a Portfolio object p, to ensure that average turnover is no more than $30 \%$ with an initial portfolio of 10 assets in a variable $\times 0$, use the setTurnover method to set the turnover constraint.

```
x0 = [ 0.12; 0.09; 0.08; 0.07; 0.1; 0.1; 0.15; 0.11; 0.08; 0.1 ];
p = Portfolio('InitPort', x0);
```

```
p = setTurnover(p, 0.3);
disp(p.NumAssets);
    1 0
disp(p.Turnover);
    0.3000
disp(p.InitPort);
    0.1200
    0.0900
    0.0800
    0.0700
    0.1000
    0.1000
    0.1500
    0.1100
    0.0800
    0.1000
```


## Set Turnover Constraint for a CVaR Portfolio Object

Given a CVaR portfolio object $p$, to ensure that average turnover is no more than $30 \%$ with an initial portfolio of 10 assets in a variable $\times 0$, use the setTurnover method to set the turnover constraint.

```
x0 = [ 0.12; 0.09; 0.08; 0.07; 0.1; 0.1; 0.15; 0.11; 0.08; 0.1 ];
p = PortfolioCVaR('InitPort', x0);
p = setTurnover(p, 0.3);
disp(p.NumAssets);
    1 0
disp(p.Turnover);
    0.3000
disp(p.InitPort);
```

0.1200
0.0900
0.0800
0.0700
0.1000
0.1000
0.1500
0.1100
0.0800
0.1000

## Set Turnover Constraint for a MAD Portfolio Object

Given PortfolioMAD object $p$, to ensure that average turnover is no more than $30 \%$ with an initial portfolio of 10 assets in a variable $\times 0$, use the setTurnover method to set the turnover constraint.

```
x0 = [ 0.12; 0.09; 0.08; 0.07; 0.1; 0.1; 0.15; 0.11; 0.08; 0.1 ];
p = PortfolioMAD('InitPort', x0);
p = setTurnover(p, 0.3);
disp(p.NumAssets);
    1 0
disp(p.Turnover);
    0.3000
disp(p.InitPort);
    0.1200
    0.0900
    0.0800
    0.0700
    0.1000
    0.1000
    0.1500
    0.1100
    0.0800
    0.1000
```


## Input Arguments

## obj - Object for portfolio

object
Object for portfolio, specified using Portfolio, PortfolioCVaR, or PortfolioMAD object. For more information on creating a portfolio object, see

- Portfolio
- PortfolioCVaR
- PortfolioMAD


## Data Types: object

## Turnover - Portfolio turnover constraint

nonnegative and finite scalar
Portfolio turnover constraint, specified as a nonnegative and finite scalar for a Portfolio, PortfolioCVaR, or PortfolioMADinput object (obj).

## InitPort - Initial or current portfolio weights

0 (default) | finite vector with NumAssets > 0 elements.
Initial or current portfolio weights, specified as a finite vector with NumAssets >0 elements for a Portfolio, PortfolioCVaR, or PortfolioMAD input object (obj).

Note If no InitPort is specified, that value is assumed to be 0 .
If InitPort is specified as a scalar and NumAssets exists, then InitPort undergoes scalar expansion.

## Data Types: double

## NumAssets - Number of assets in portfolio

1 (default) | scalar
Number of assets in portfolio, specified as a scalar for a Portfolio, PortfolioCVaR, or PortfolioMAD input object (obj).

Note If it is not possible to obtain a value for NumAssets, it is assumed that NumAssets is 1 .

## Data Types: double

## Output Arguments

## obj — Updated portfolio object <br> object for portfolio

Updated portfolio object, returned as a Portfolio, PortfolioCVaR, or PortfolioMAD object. For more information on creating a portfolio object, see

- Portfolio
- PortfolioCVaR
- PortfolioMAD


## Tips

You can also use dot notation to set up the maximum portfolio turnover constraint.

```
obj = obj.setTurnover(Turnover, InitPort, NumAssets);
```


## See Also

getOneWayTurnover| setInitPort|setOneWayTurnover

## Topics

"Working with Average Turnover Constraints Using Portfolio Object" on page 4-98
"Working with Average Turnover Constraints Using PortfolioCVaR Object" on page 5-88
"Portfolio Optimization Examples" on page 4-167
"Portfolio Analysis with Turnover Constraints"
"Portfolio Set for Optimization Using Portfolio Object" on page 4-10
"Portfolio Set for Optimization Using PortfolioCVaR Object" on page 5-10
"Portfolio Set for Optimization Using PortfolioMAD Object" on page 6-10

Introduced in R2011a

19-1638

## simulateNormalScenariosByData

Simulate multivariate normal asset return scenarios from data

Note simulateNormalScenariosByData has been partially removed and will no longer accept a fints object for the AssetReturns argument. Use timetable instead for financial time series.

Use fts2timetable to convert a fints object to a timetable object.

## Syntax

obj = simulateNormalScenariosByData(obj,AssetReturns)
obj = simulateNormalScenariosByData(obj,AssetReturns,NumScenarios, Name, Value)

## Description

obj = simulateNormalScenariosByData(obj,AssetReturns) simulates multivariate normal asset return scenarios from data for portfolio object for PortfolioCVaR or PortfolioMAD objects. For details on the workflows, see "PortfolioCVaR Object Workflow" on page 5-20, and "PortfolioMAD Object Workflow" on page 6-19
obj = simulateNormalScenariosByData(obj,AssetReturns,NumScenarios, Name, Value) simulates multivariate normal asset return scenarios from data for portfolio object for PortfolioCVaR or PortfolioMAD objects using additional options specified by one or more Name, Value pair arguments.

This function estimates the mean and covariance of asset returns from either price or return data and then uses these estimates to generate the specified number of scenarios with the function mvnrnd.

Data can be in a NumSamples-by-NumAssets matrix of NumSamples prices or returns at a given periodicity for a collection of NumAssets assets, a table or a timetable.

Note If you want to use the method multiple times and you want to simulate identical scenarios each time the function is called, precede each function call with rng(seed) using a specified integer seed.

## Examples

## Simulate Multivariate Normal Asset Return Scenarios from Data for a PortfolioCVaR Object

Given a PortfolioCVaR object p, use the simulateNormalScenariosByData function to simulate multivariate normal asset return scenarios from data.

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;
RawData = mvnrnd(m, C, 240);
NumScenarios = 2000;
p = PortfolioCVaR;
p = simulateNormalScenariosByData(p, RawData, NumScenarios)
p =
    PortfolioCVaR with properties:
            BuyCost: []
            SellCost: []
        RiskFreeRate: []
        ProbabilityLevel: []
            Turnover: []
            BuyTurnover: []
            SellTurnover: []
            NumScenarios: 2000
                Name: []
            NumAssets: 4
            AssetList: []
            InitPort: []
```

AInequality: []
bInequality: []
AEquality: []bEquality: []
LowerBound: []
UpperBound: []
LowerBudget: []
UpperBudget: []
GroupMatrix: []
LowerGroup: []
UpperGroup: []
GroupA: []
GroupB: []
LowerRatio: []
UpperRatio: []
p = setDefaultConstraints(p);
$p=$ setProbabilityLevel(p, 0.9);
disp(p)
PortfolioCVaR with properties:
BuyCost: []
SellCost: []
RiskFreeRate: []
ProbabilityLevel: 0.9000
Turnover: [
BuyTurnover: [
SellTurnover: []
NumScenarios: 2000
Name: []
NumAssets: 4
AssetList: []
InitPort: []
AInequality: []
bInequality: []
AEquality: ..... []
bEquality: []
LowerBound: [4x1 double]
UpperBound: []
LowerBudget: 1
UpperBudget: 1
GroupMatrix: []

LowerGroup: []
UpperGroup: []
GroupA: []
GroupB: []
LowerRatio: []
UpperRatio: []

## Estimate Mean and Covariance of Asset Returns from Market Data for a PortfolioCVaR Object

Create a PortfolioCVaR object $p$ and use the simulateNormalScenariosByData function with market data loaded from CAPMuniverse. mat to simulate multivariate normal asset return scenarios. The market data, AssetsTimeTable, is a timetable of asset returns.
load CAPMuniverse

```
p = PortfolioCVaR('AssetList',Assets);
```

disp(p);

PortfolioCVaR with properties:
BuyCost: []
SellCost: []
RiskFreeRate: []
ProbabilityLevel: []
Turnover: []
BuyTurnover: []
SellTurnover: []
NumScenarios: []
Name: []
NumAssets: 14
AssetList: \{1x14 cell\}
InitPort: []
AInequality: []
bInequality: []
AEquality: []
bEquality: []
LowerBound: []
UpperBound: []
LowerBudget: []
UpperBudget: []

```
GroupMatrix: []
    LowerGroup: []
    UpperGroup: []
        GroupA: []
        GroupB: []
    LowerRatio: []
    UpperRatio: []
```

Simulate the scenarios from the timetable data for each of the assets from CAPMuniverse.mat and plot the efficient frontier.
p = simulateNormalScenariosByData(p,AssetsTimeTable,10000,'missingdata',true);
p = setDefaultConstraints(p);
p = setProbabilityLevel(p, 0.9);
plotFrontier(p);


## Estimate Mean and Covariance of Asset Returns from Data for a PortfolioMAD Object

Given a PortfolioMAD object p, use the simulateNormalScenariosByData function to simulate multivariate normal asset return scenarios from data.

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
```

```
m = m/12
C = C/12;
```

RawData $=$ mvnrnd(m, C, 240);
NumScenarios = 2000;
p = PortfolioMAD;
p = simulateNormalScenariosByData(p, RawData, NumScenarios);
p = setDefaultConstraints(p);
disp(p);
PortfolioMAD with properties:

```
    BuyCost: []
        SellCost: []
    RiskFreeRate: []
            Turnover: []
        BuyTurnover: []
        SellTurnover: []
        NumScenarios: 2000
            Name: []
        NumAssets: 4
        AssetList: []
            InitPort: []
        AInequality: []
        bInequality: []
            AEquality: []
            bEquality: []
            LowerBound: [4x1 double]
            UpperBound: []
            LowerBudget: 1
            UpperBudget: 1
            GroupMatrix: []
            LowerGroup: []
            UpperGroup: []
            GroupA: []
            GroupB: []
            LowerRatio: []
            UpperRatio: []
```


## Estimate Mean and Covariance of Asset Returns from Market Data for a PortfolioMAD Object

Create a PortfolioMAD object p and use the simulateNormalScenariosByData function with market data loaded from CAPMuniverse. mat to simulate multivariate normal asset return scenarios. The market data, AssetsTimeTable, is a timetable of asset returns.
load CAPMuniverse
p = PortfolioMAD('AssetList',Assets);
disp(p.AssetList');
'AAPL'
'AMZN'
'CSCO'
'DELL'
'EBAY'
'G00G '
'HPQ'
'IBM'
'INTC'
'MSFT'
' ORCL'
'YHOO '
'MARKET'
'CASH'
Simulate the scenarios from the timetable data for each of the assets from CAPMuniverse.mat and plot the efficient frontier.
p = simulateNormalScenariosByData(p,AssetsTimeTable,10000,'missingdata',true);
$p=$ setDefaultConstraints(p);
plotFrontier(p);


## Input Arguments

obj - Object for portfolio
object
Object for portfolio, specified using a PortfolioCVaR or PortfolioMAD object.
For more information on creating a PortfolioCVaR or PortfolioMAD object, see

- PortfolioCVaR
- PortfolioMAD

Data Types: object

## AssetReturns - Asset data that can be converted into asset returns matrix | table | timetable

Asset data that can be converted into asset returns ([NumSamples-by-NumAssets] matrix), specified as a matrix, table, or timetable.

AssetReturns data can be:

- NumSamples-by-NumAssets matrix.
- Table of NumSamples prices or returns at a given periodicity for a collection of NumAssets assets
- Timetable object with NumSamples observations and NumAssets time series

Data Types: double|table | timetable

## NumScenarios - Number of scenarios to simulate <br> positive integer

Number of scenarios to simulate, specified as a positive integer.
Data Types: double

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Example: $\mathrm{p}=$
simulateNormalScenariosByData(p,RawData,NumScenarios,'DataFormat', 'R eturns','MissingData',true,' GetAssetList',true)

## DataFormat - Flag to convert input data as prices into returns

'Returns' (default)| character vector with values 'Returns' or 'Prices'
Flag to convert input data as prices into returns, specified as the comma-separated pair consisting of 'DataFormat' and a character vector with the values:

- 'Returns' - Data in AssetReturns contains asset total returns.
- 'Prices ' - Data in AssetReturns contains asset total return prices.

Data Types: char

## MissingData - Flag to use ECM algorithm to handle NaN values

false (default) | logical with values true or false
Flag to use ECM algorithm to handle NaN values, specified as the comma-separated pair consisting of 'MissingData' and a logical with a value of true or false.

- false - Do not use ECM algorithm to handle NaN values (exclude NaN values).
- true - Use ECM algorithm to handle NaN values.

Data Types: logical

## GetAssetList - Flag indicating which asset names to use for the asset list

 false (default) | logical with values true or falseFlag indicating which asset names to use for the asset list, specified as the commaseparated pair consisting of 'GetAssetList' and a logical with a value of true or false.

- false - Do not extract or create asset names.
- true - Extract or create asset names from the table or timetable.

If a table or timetable is passed into this function using the AssetReturns argument and the GetAssetList flag is true, the column names from the table or timetable are used as asset names in obj. AssetList.

If a matrix is passed and the GetAssetList flag is true, default asset names are created based on the AbstractPortfolio property defaultforAssetList, which is 'Asset'.

If the GetAssetList flag is false, no action occurs, which is the default behavior.
Data Types: logical

## Output Arguments

obj - Updated portfolio object
object for portfolio
Updated portfolio object, returned as a PortfolioCVaR or PortfolioMAD object. For more information on creating a portfolio object, see

- PortfolioCVaR
- PortfolioMAD


## Tips

You can also use dot notation to simulate multivariate normal asset return scenarios from data for a PortfolioCVaR or PortfolioMAD object.
obj = obj.simulateNormalScenariosByData(AssetReturns,NumScenarios,Name, Value);

## See Also

fints | rng| simulateNormalScenariosByMoments

## Topics

"Asset Returns and Scenarios Using PortfolioCVaR Object" on page 5-44
"Asset Returns and Scenarios Using PortfolioMAD Object" on page 6-42

## External Websites

Analyzing Investment Strategies with CVaR Portfolio Optimization in MATLAB (50 min 42 sec )

## Introduced in R2012b

## simulateNormalScenariosByMoments

Simulate multivariate normal asset return scenarios from mean and covariance of asset returns

## Syntax

```
obj = simulateNormalScenariosByMoments(obj,AssetMean,AssetCovar,
NumScenarios)
obj = simulateNormalScenariosByMoments(obj,AssetMean,AssetCovar
NumScenarios,NumAssets)
```


## Description

obj = simulateNormalScenariosByMoments(obj,AssetMean,AssetCovar, NumScenarios) simulates multivariate normal asset return scenarios from mean and covariance of asset returns for PortfolioCVaR or PortfolioMAD objects. For details on the workflows, see "PortfolioCVaR Object Workflow" on page 5-20, and "PortfolioMAD Object Workflow" on page 6-19.
obj = simulateNormalScenariosByMoments(obj,AssetMean,AssetCovar NumScenarios, NumAssets) simulates multivariate normal asset return scenarios from mean and covariance of asset returns for PortfolioCVaR or PortfolioMAD objects using the optional input NumScenarios.

Note This function overwrites existing scenarios associated with PortfolioCVaR or PortfolioMAD objects, and also, possibly, NumScenarios.

If you want to use the function multiple times and you want to simulate identical scenarios each time the function is called, precede each function call with rng(seed) using a specified integer seed.

## Examples

## Simulate Multivariate Normal Asset Return Scenarios from Moments for a PortfolioCVaR Object

Given PortfolioCVaR object p, use the simulateNormalScenariosByMoments function to simulate multivariate normal asset return scenarios from moments.

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;
AssetScenarios = mvnrnd(m, C, 20000);
p = PortfolioCVaR;
p = setScenarios(p, AssetScenarios);
p = setDefaultConstraints(p);
p = setProbabilityLevel(p, 0.95);
AssetMean = [.5]
AssetMean = 0.5000
AssetCovar = [.5]
AssetCovar = 0.5000
NumScenarios = 100
NumScenarios = 100
p = simulateNormalScenariosByMoments(p, AssetMean, AssetCovar, NumScenarios)
p =
    PortfolioCVaR with properties:
                    BuyCost: []
            SellCost: []
            RiskFreeRate: []
        ProbabilityLevel: 0.9500
            Turnover: []
            BuyTurnover: []
            SellTurnover: []
```

```
NumScenarios: 100
    Name: []
    NumAssets: 4
    AssetList: []
        InitPort: []
AInequality: []
bInequality: []
    AEquality: []
        bEquality: []
    LowerBound: [4x1 double]
    UpperBound: []
LowerBudget: 1
UpperBudget: 1
GroupMatrix: []
    LowerGroup: []
    UpperGroup: []
        GroupA: []
        GroupB: []
    LowerRatio: []
    UpperRatio: []
```


## Simulate Multivariate Normal Asset Return Scenarios from Moments for a PortfolioMAD Object

Given PortfolioMAD object $p$, use the simulateNormalScenariosByMoments function to simulate multivariate normal asset return scenarios from moments.

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;
AssetScenarios = mvnrnd(m, C, 20000);
p = PortfolioMAD;
p = setScenarios(p, AssetScenarios);
p = setDefaultConstraints(p);
```

```
AssetMean = [.5]
AssetMean = 0.5000
AssetCovar = [.5]
AssetCovar = 0.5000
NumScenarios = 100
NumScenarios = 100
p = simulateNormalScenariosByMoments(p, AssetMean, AssetCovar, NumScenarios)
p =
    PortfolioMAD with properties:
    BuyCost: []
        SellCost: []
        RiskFreeRate: []
            Turnover: []
        BuyTurnover: []
        SellTurnover: []
        NumScenarios: 100
            Name: []
        NumAssets: 4
        AssetList: []
            InitPort: []
        AInequality: []
        bInequality: []
            AEquality: []
            bEquality: []
            LowerBound: [4x1 double]
            UpperBound: []
            LowerBudget: 1
            UpperBudget: 1
            GroupMatrix: []
            LowerGroup: []
            UpperGroup: []
            GroupA: []
            GroupB: []
            LowerRatio: []
            UpperRatio: []
```


## Input Arguments

## obj - Object for portfolio <br> object

Object for portfolio, specified using a PortfolioCVaR or PortfolioMAD object.
For more information on creating a PortfolioCVaR or PortfolioMAD object, see

- PortfolioCVaR
- PortfolioMAD


## Data Types: object

## AssetMean - Mean of asset returns

vector
Mean of asset returns, specified as a vector.

Note If AssetMean is a scalar and the number of assets is known, scalar expansion occurs. If the number of assets cannot be determined, this function assumes that NumAssets = 1 .

## Data Types: double

## AssetCovar - Covariance of asset returns

symmetric positive-semidefinite matrix
Covariance of asset returns, specified as a symmetric positive-semidefinite matrix.

## Note

- If AssetCovar is a scalar and the number of assets is known, a diagonal matrix is formed with the scalar value along the diagonals. If it is not possible to determine the number of assets, this method assumes that NumAssets $=1$.
- If AssetCovar is a vector, a diagonal matrix is formed with the vector along the diagonal.


## Data Types: double

## NumScenarios - Number of scenarios to simulate positive integer

Number of scenarios to simulate, specified as a positive integer.
Data Types: double

## NumAssets - Number of assets

scalar
Number of assets, specified as a scalar.
Data Types: double

## Output Arguments

## obj - Updated portfolio object

object for portfolio
Updated portfolio object, returned as a PortfolioCVaR or PortfolioMAD object. For more information on creating a portfolio object, see

- PortfolioCVaR
- PortfolioMAD


## Tips

You can also use dot notation to simulate multivariate normal asset return scenarios from a mean and covariance of asset returns for a PortfolioCVaR or PortfolioMAD object.
obj = obj.simulateNormalScenariosByMoments(AssetMean, AssetCovar, NumScenarios, NumAssets);

## See Also

rng|simulateNormalScenariosByData

## Topics

"Asset Returns and Scenarios Using PortfolioCVaR Object" on page 5-44
"Asset Returns and Scenarios Using PortfolioMAD Object" on page 6-42

## External Websites

Analyzing Investment Strategies with CVaR Portfolio Optimization in MATLAB (50 min 42 sec)

Introduced in R2012b

## setfield

Set content of specific field

Note setfield will be removed in a future release. Use timetable instead. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

```
newfts = setfield(tsobj,field,V)
newfts = setfield(tsobj,field,{dates},V)
```


## Description

setfield treats the contents of fields in a time series object (tsobj) as fields in a structure.
newfts $=$ setfield(tsobj, field, $V$ ) sets the contents of the specified field to the value V . This is equivalent to the syntax S .field $=\mathrm{V}$.
newfts $=$ setfield(tsobj,field,\{dates\}, V ) sets the contents of the specified field for the specified dates. dates can be individual cells of date character vectors or a cell of a date character vector's range using the : : operator, for example, '03/01/99: :03/31/99'. Dates can contain time-of-day information.

## Examples

Example 1. Set the closing value for all days to 3890.

```
load dji30short
format bank
myfts1 = setfield(myfts1, 'Close', 3890)
```

Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/setfield (line 84)

Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/display (line 66)
myfts1 =
desc: DJI30MAR94.dat
freq: Daily (1)

| 'dates: (20)' | 'Open: (20)' | 'High: (20)' | 'Low: (20)' | 'Close: (20)' |
| :---: | :---: | :---: | :---: | :---: |
| '04-Mar-1994' | [ 3830.90] | [ 3868.04] | [ 3800.50] | [ 3890.00] |
| '07-Mar-1994' | [ 3851.72] | [ 3882.40] | $3824.71]$ | [ 3890.00] |
| '08-Mar-1994' | [ 3858.48] | [ 3881.55] | 3822.45 ] | [ 3890.00] |
| '09-Mar-1994' | [ 3853.97] | [ 3874.52] | [ 3817.95] | [ 3890.00] |
| '10-Mar-1994' | [ 3852.57] | [ 3865.51] | 3801.63] | [ 3890.00] |
| '11-Mar-1994' | [ 3832.58] | [ 3872.83] | 3806.69] | [ 3890.00] |
| '14-Mar-1994' | [ 3870.29] | [ 3894.21] | [ 3835.96] | [ 3890.00] |
| '15-Mar-1994' | [ 3863.41] | [ 3888.46] | $3826.85]$ | [ 3890.00] |
| '16-Mar-1994' | [ 3851.03] | [ 3879.53] | $3819.94]$ | [ 3890.00] |
| '17-Mar-1994' | [ 3853.62] | [ 3891.34] | [ 3821.66] | [ 3890.00] |
| '18-Mar-1994' | [ 3865.42] | [ 3911.78] | [ 3838.65] | [ 3890.00] |
| '21-Mar-1994' | [ 3878.38] | [ 3898.25] | $3838.65]$ | [ 3890.00] |
| '22-Mar-1994' | [ 3865.71] | [ 3896.23] | [ 3840.66] | [ 3890.00] |
| ' 23-Mar-1994' | [ 3868.88] | [ 3901.41] | [ 3839.80] | [ 3890.00] |
| '24-Mar-1994' | [ 3849.88] | [ 3865.42] | $3792.58]$ | [ 3890.00] |
| '25-Mar-1994' | [ 3827.13] | [ 3826.85] | [ 3774.73] | [ 3890.00] |
| ' 28-Mar-1994' | [ 3776.46] | [ 3793.45] | [ 3719.74] | [ 3890.00] |
| '29-Mar-1994' | [ 3757.17] | [ 3771.86] | $3689.23]$ | [ 3890.00] |
| '30-Mar-1994' | [ 3688.36] | [ 3718.88] | [ 3612.36] | [ 3890.00] |
| '31-Mar-1994' | [ 3639.71] | [ 3673.10] | [ 3544.12] | [ 3890.00] |

Example 2. Set values for specific times on specific days.
First create a financial time series containing time-of-day data.

```
dates = ['01-Jan-2001';'01-Jan-2001'; '02-Jan-2001'; ...
    '02-Jan-2001'; '03-Jan-2001';'03-Jan-2001'];
times = ['11:00';'12:00';'11:00';'12:00';'11:00';'12:00'];
dates times = cellstr([dates, repmat(' ',size(dates,1),1),...
    times]);
myfts = fints(dates_times,[(1:4)'; nan; 6],{'Datal'},1,...
    'My FİNTS')
```

Warning: FINTS will be removed in a future release. Use TIMETABLE instead. > In fints (line 165)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead. > In fints/display (line 66)
myfts =
desc: My FINTS
freq: Daily (1)

| 'dates: (6)' | 'times: (6)' | 'Datal: (6)' |
| :---: | :---: | :---: |
| 01-Jan-2001' | '11:00' | $1]$ |
| " ' | '12:00' | $2]$ |

19-1659


Now use setfield to replace the data in myfts with new data starting at 12:00 on January 1, 2001 and ending at 11:00 on January 3, 2001.

```
S = setfield(myfts,'Data1',...
    {'01-Jan-2001 12:00::03-Jan-2001 11:00'},(102:105)')
```

Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/setfield (line 84)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/display (line 66)
S =
desc: My FINTS
freq: Daily (1)

| 'dates: (6)' | 'times: | (6)' | 'Datal: (6)' |
| :---: | :---: | :---: | :---: |
| '01-Jan-2001 | '11:00' |  | 1.00] |
| ' " | '12:00' |  | $102.00]$ |
| '02-Jan-2001 | '11:00' |  | $103.00]$ |
| ' " | '12:00' |  | 104.00] |
| '03-Jan-2001 | '11:00' |  | 105.00] |
|  | '12:00' |  | 6.00] |

## See Also

chfield|fieldnames | getfield|isfield|rmfield

## Topics

"What Is the Financial Time Series App?" on page 14-2

Introduced before R2006a

## sharpe

Compute Sharpe ratio for one or more assets

## Syntax

```
sharpe(Asset)
sharpe(Asset,Cash)
Ratio = sharpe(Asset,Cash)
```


## Arguments

| Asset | NUMSAMPLES-by-NUMSERIES matrix with NUMSAMPLES <br> observations of asset returns for NUMSERIES asset return <br> series. |
| :--- | :--- |
| Cash | (Optional) Either a scalar return for a riskless asset or a <br> vector of asset returns to be a proxy for a riskless asset. In <br> either case, the return periodicity must be the same as the <br> periodicity of Asset. For example, if Asset is monthly data, <br> then Cash must be monthly returns. If no value is supplied, <br> the default value for Cash returns is 0. |

## Description

Given NUMSERIES assets with NUMSAMPLES returns for each asset in a NUMSAMPLES-byNUMSERIES matrix Asset and given either a scalar Cash asset return or a vector of Cash asset returns, the Sharpe ratio is computed for each asset.

The output is Ratio, a 1-by-NUMSERIES row vector of Sharpe ratios for each series in Asset. Any series in Asset with standard deviation of returns equal to 0 has a $N a N$ value for its Sharpe ratio.

Note If Cash is a vector, Asset and Cash need not have the same number of returns but must have the same periodicity of returns. The classic Sharpe ratio assumes that Cash is
riskless. In reality, a short-term cash rate is not necessarily riskless. NaN values in the data are ignored.

## Examples

See "Sharpe Ratio" on page 7-7.

## References

William F. Sharpe. "Mutual Fund Performance." Journal of Business. Vol. 39, No. 1, Part 2, January 1966, pp. 119-138.

## See Also

inforatio|portalpha

## Topics

"Sharpe Ratio" on page 7-7
"Performance Metrics Illustration" on page 7-4
"Performance Metrics Overview" on page 7-2

## Introduced in R2006b

## size

Number of dates and data series

Note size will be removed in a future release. Use timetable instead. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

```
szfts = size(tsobj,dim)
[numRows,numCols] = size(tsobj)
```


## Arguments

| tsobj | Financial time series object. |
| :--- | :--- |
| dim | (Optional) A scalar that specifies the following dimension: |
|  | dim $=1$ returns number of dates (rows). |
|  | dim $=2$ returns number of data series (columns). |

## Description

szfts = size(tsobj) returns the number of dates (rows) and the number of data series (columns) in the financial time series object tsobj. The result is returned in the vector szfts, whose first element is the number of dates and second is the number of data series.
szfts $=$ size(tsobj, dim) specifies the dimension you want to extract.
numRows returns a scalar representing the number of dates (rows).
numCols returns a scalar representing the number of data series (columns).
See Alsolength|size
Topics"Financial Time Series Operations" on page 13-7
Introduced before R2006a

## smoothts

## Smooth data

Note smoothts will be removed in a future release. Use smoothdata instead.

## Syntax

```
output = smoothts(input)
output = smoothts(input,'b',wsize)
output = smoothts(input,'g',wsize,stdev)
output = smoothts(input,'e',n)
```


## Arguments

| input | Financial time series object or a row-oriented matrix. In a row- <br> oriented matrix, each row represents an individual set of <br> observations. |
| :--- | :--- |
| 'b', 'g', or 'e' | Smoothing method (essentially the type of filter used). Can be <br> Exponential (e), Gaussian (g), or Box (b). Default = b. |
| wsize | Window size (scalar). Default = 5. |
| stdev | Scalar that represents the standard deviation of the Gaussian <br> window. Default $=0.65$. |


| n | For Exponential method, specifies window size or exponential factor, <br> depending upon value. <br> - $n>1$ (window size) or period length <br> $-n<1$ and $>0$ (exponential factor: alpha) <br> - $n=1$ (either window size or alpha) <br> If $n$ is not supplied, the defaults are wsize $=5$ and alpha $=$ <br> 0.3333. |
| :--- | :--- |

## Description

smoothts smooths the input data using the specified method.
output = smoothts(input) smooths the input data using the default Box method with window size, wsize, of 5 .
output = smoothts(input,'b', wsize) smooths the input data using the Box (simple, linear) method. wsize specifies the width of the box to be used.
output = smoothts(input,'g',wsize,stdev) smooths the input data using the Gaussian window method.
output $=$ smoothts(input, 'e', n ) smooths the input data using the Exponential method. n can represent the window size (period length) or alpha. If $\mathrm{n}>1, \mathrm{n}$ represents the window size. If $0<n<1$, $n$ represents alpha, where

$$
\alpha=\frac{2}{w \operatorname{size}+1} .
$$

If input is a financial time series object, output is a financial time series object identical to input except for contents. If input is a row-oriented matrix, output is a row-oriented matrix of the same length.

## See Also

tsmovavg

[^15]
## sortfts

Sort financial time series

Note sortfts will be removed in a future release. Use timetable instead. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

```
sfts = sortfts(tsobj)
sfts = sortfts(tsobj,flag)
sfts = sortfts(tsobj,seriesnames,flag)
[sfts,sidx] = sortfts(...)
```


## Arguments

| tsobj | Financial time series object. |
| :--- | :--- |
| flag | (Optional) Sort order: |
|  | flag $=1$; increasing order (default) |
| flag $=-1$; decreasing order |  |

## Description

sfts $=$ sortfts(tsobj) sorts the financial time series object tsobj in increasing order based only upon the 'dates ' vector if tsobj does not contain time-of-day information. If the object includes time-of-day information, the sort is based upon a combination of the 'dates' and 'times' vectors. The 'times' vector cannot be sorted individually.
sfts $=$ sortfts (tsobj, flag) sets the order of the sort. $\mathrm{flag}=1$ : increasing date and time order. $\mathrm{flag}=-1$ : decreasing date and time order.
sfts = sortfts(tsobj, seriesnames, flag) sorts the financial time series object tsobj based upon the data series name(s) seriesnames. The seriesnames argument can be a single character vector containing a data series name or a cell array of character vectors containing a list of data series names. If the optional flag is set to -1, the sort is in decreasing order.
[sfts, sidx] = sortfts(...) also returns the index of the original object tsobj sorted based on 'dates ' or specified data series name(s).

## See Also

issorted| sort|sortrows

## Topics

"What Is the Financial Time Series App?" on page 14-2

## Introduced before R2006a

## spctkd

Slow stochastics

Note spctkd will be removed in a future release. Use stochosc instead.

## Syntax

```
[spctk,spctd] = spctkd(fastpctk,fastpctd)
[spctk,spctd] = spctkd([fastpctk fastpctd])
[spctk,spctd] = spctkd(fastpctk,fastpctd,dperiods,dmamethod)
[spctk,spctd] = spctkd([fastpctk fastpctd], dperiods, dmamethod)
skdts = spctkd(tsobj)
skdts = spctkd(tsobj,dperiods,dmamethod)
skdts = spctkd(tsobj,dperiods,dmamethod,'ParameterName',ParameterValue, ...)
```


## Arguments

| fastpctk | Fast stochastic F\%K (vector). |
| :--- | :--- |
| fastpctd | Fast stochastic F\%D (vector). |
| dperiods | (Optional) \%D periods. Default = 3. |
| dmamethod | (Optional) \%D moving average method. Default = 'e' (exponential). |
| tsobj | Financial time series object. |

## Description

[spctk,spctd] = spctkd(fastpctk,fastpctd) calculates the slow stochastics S \%K and S\%D. spctk and spctd are column vectors representing the respective slow
stochastics. The inputs must be single column-oriented vectors containing the fast stochastics F\%K and F\%D.
[spctk,spctd] = spctkd([fastpctk fastpctd]) accepts a two-column matrix as input. The first column contains the fast stochastic F\%K values, and the second contains the fast stochastic F\%D values.
[spctk,spctd] = spctkd(fastpctk,fastpctd,dperiods,dmamethod) calculates the slow stochastics, $\mathrm{S} \% \mathrm{~K}$ and $\mathrm{S} \% \mathrm{D}$, using the value of dperiods to set the number of periods and dmamethod to indicate the moving average method. The inputs fastpctk and fastpctk must contain the fast stochastics, F\%K and F\%D, in column orientation. spctk and spctd are column vectors representing the respective slow stochastics.

Valid moving average methods for \%D are exponential ('e'), triangular ('t'), and modified ('m'). See tsmovavg for explanations of these methods.
[spctk,spctd] = spctkd([fastpctk fastpctd],dperiods,dmamethod) accepts a two-column matrix rather than two separate vectors. The first column contains the F\%K values, and the second contains the F\%D values.
skdts = spctkd(tsobj) calculates the slow stochastics, S\%K and S\%D. tsobj must contain the fast stochastics, F\%K and F\%D, in data series named PercentK and PercentD. The skdts output is a financial time series object with the same dates as tsobj. Within tsobj the two series SlowPctK and SlowPctD represent the respective slow stochastics.
skdts = spctkd(tsobj,dperiods,dmamethod) lets you specify the length and the method of the moving average used to calculate S\%D values.
skdts =
spctkd(tsobj,dperiods,dmamethod,'ParameterName',ParameterValue, ...) accepts parameter name/parameter value pairs as input. These pairs specify the name(s) for the required data series if it is different from the expected default name(s). Valid parameter names are

- KName: F\%K series name
- DName: F\%D series name

Parameter values are the character vectors that represent the valid parameter names.

## Examples

## Calculate the Slow Stochastics

This example shows how to calculate the slow stochastics for Disney stock and plot the results.
load disney.mat dis_FastStoch = fpctkd(dis);
Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in dis_SlowStoch = spctkd(dis_FastStoch);
Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in plot(dis_SlowStoch)

Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in title('Slow Stochastics for Disney')


## References

Achelis, Steven B. Technical Analysis from A to Z. Second Edition. McGraw-Hill, 1995, pp. 268-271.

See Also<br>fpctkd|stochosc|tsmovavg

Introduced before R2006a

## std

Standard deviation

Note std will be removed in a future release. Use timetable instead. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

```
tsstd = std(tsobj)
tsstd = std(tsobj,flag)
```


## Arguments

| tsobj | Financial time series object. |
| :--- | :--- |
| flag | (Optional) Normalization factor: |
|  | flag $=1$ normalizes by n (number of observations). |
|  | flag $=0$ normalizes by n-1. |

## Description

tsstd $=$ std(tsobj) computes the standard deviation of each data series in the financial time series object tsobj and returns the results in tsstd. The tsstd output is a structure with field name(s) identical to the data series name(s).
tsstd $=$ std(tsobj, flag) normalizes the data as indicated by flag.

## See Also

hist|mean

## Topics

"Financial Time Series Operations" on page 13-7

Introduced before R2006a

## stochosc

Stochastic oscillator

Note stochosc has been partially removed and will no longer accept a fints object (tsobj) argument. Use a matrix, timetable, or table instead for financial time series.

Use fts2timetable to convert a fints object to a timetable object.

## Syntax

```
percentKnD = stochosc(Data)
percentKnD = stochosc(
```

$\qquad$

``` ,Name, Value)
```


## Description

percentKnD = stochosc(Data) calculates the stochastic oscillator.
percentKnD = stochosc( $\qquad$ ,Name, Value) adds optional name-value pair arguments.

## Input Arguments

## Data - Data with high, low, open, close information matrix | table | timetable

Data with high, low, open, close information, specified as a matrix, table, or timetable. For matrix input, Data is an M-by-3 matrix of high, low, and closing prices stored in the corresponding columns, respectively. Timetables and tables with M rows must contain variables named 'High', 'Low', and 'Close' (case insensitive).

Data Types: double|table|timetable

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.
Example: percentKnD = stochosc(TMW,'NumPeriodsD',10,'NumPeriodsK', 3,'Type','exponential')

## NumPeriodsK - Period difference for PercentK

10 (default) | positive integer
Period difference for PercentK, specified as the comma-separated pair consisting of 'NumPeriodsK' and a scalar positive integer.

## Data Types: double

## NumPeriodsD - Length of moving average in periods for PercentD 3 (default) | positive integer

Length of moving average in periods for PercentD, specified as the comma-separated pair consisting of 'NumPeriodsD' and a scalar positive integer.
Data Types: double

## Type - Moving average method for PercentD calculation

'e' (exponential) (default) | character vector with values 'exponential' or 'triangular'

Moving average method for PercentD calculation, specified as the comma-separated pair consisting of 'Type' and a character vector with a value of:

- 'exponential' - Exponential moving average is a weighted moving average. Exponential moving averages reduce the lag by applying more weight to recent prices. For example, a 10 period exponential moving average weights the most recent price by $18.18 \%$.
- 'triangular' - Triangular moving average is a double-smoothing of the data. The first simple moving average is calculated and then a second simple moving average is calculated on the first moving average with the same window size.

[^16]
## Output Arguments

## percentKnD - PercentK and PercentD

## matrix | table | timetable

PercentK and PercentD, returned with the same number of rows (M) and type (matrix, table, or timetable) as the input Data.

## Examples

## Calculate the Stochastic Oscillator for a Stock

Load the file SimulatedStock.mat, which provides a timetable (TMW) for financial data for TMW stock.
load SimulatedStock.mat
oscillator = stochosc(TMW,'NumPeriodsD',7,'NumPeriodsK',10,'Type','exponential'); plot(oscillator.Time, oscillator. FastPercentK, oscillator.Time,oscillator. FastPercentD) title('Stochastic Oscillator for TMW')


## Definitions

## Stochastic Oscillator

The stochastic oscillator calculates the Fast PercentK (F\%K), Fast PercentD (F\%D), Slow PercentK (S\%K), and Slow PercentD (S\%D) from the series of high, low, and closing stock prices.

By default, the stochastic oscillator is based on 10-period difference for PercentK and a 3period exponential moving average for PercentD.

## References

[1] Achelis, S. B. Technical Analysis from A to Z. Second Edition. McGraw-Hill, 1995, pp. 268-271.

## See Also

chaikosc|table|timetable

## Topics

"Using Timetables in Finance"
"Convert Financial Time Series Objects fints to Timetables" on page 12-2

Introduced before R2006a

## subsasgn

Content assignment

Note subsasgn will be removed in a future release. Use timetable instead. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

subasgn

## Description

subasgn assigns content to a component within a financial time series object. subasgn supports integer indexing or date character vector indexing into the time series object with values assigned to the designated components. Serial date numbers cannot be used as indices. To use date character vector indexing, enclose the date character vector(s) in a pair of single quotation marks ' '.

You can use integer indexing on the object as in any other MATLAB matrix. It will return the appropriate entry(ies) from the object.

You must specify the component to which you want to assign values. An assigned value must be either a scalar or a column vector.

## Examples

Given a time series myfts with a default data series name of series1, myfts.series1('07/01/98::07/03/98') = [1 2 3]';
assigns the values 1, 2, and 3 corresponding to the first three days of July, 1998.

```
myfts('07/01/98::07/05/98')
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/subsref (line 106)
```

```
desc: Data Assignment
freq: Daily (1)
    'dates: (5)' 'series1: (5)
    '01-Jul-1998' [ 1]
    '02-Jul-1998' [ 2]
    '03-Jul-1998' [ 3]
    '04-Jul-1998' [ 4561.2]
    '05-Jul-1998' [ 5612.3]
```

When the financial time series object contains a time-of-day specification, you can assign data to a specific time on a specific day. For example, create a financial time series object called timeday containing both dates and times:

```
dates = ['01-Jan-2001';'01-Jan-2001'; '02-Jan-2001'; ...
'02-Jan-2001'; '03-Jan-2001';'03-Jan-2001'];
times = ['11:00';'12:00';'11:00';'12:00';'11:00';'12:00'];
dates times = cellstr([dates, repmat(' ',size(dates,1),1),...
times]})
timeday = fints(dates_times,(1:6)',{'Datal'},1,'My first FINTS')
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints (line 165)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/display (line 66)
timeday =
desc: My first FINTS
freq: Daily (1)
\begin{tabular}{|c|c|c|c|}
\hline ates: (6)' & 'times: & (6) \({ }^{\prime}\) & 'Datal: (6)' \\
\hline '01-Jan-2001' & '11:00' & & \(1]\) \\
\hline " ' & ' 12:00' & & \(2]\) \\
\hline '02-Jan-2001' & ' 11:00' & & \(3]\) \\
\hline " ' & ' 12:00' & & 4] \\
\hline '03-Jan-2001' & '11:00' & & 5] \\
\hline " ' & ' 12:00' & & \(6]\) \\
\hline
\end{tabular}
```

Use integer indexing to assign the value 999 to the first item in the object.
timeday(1) = 999

```
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/subsasgn (line 118)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/display (line 66)
timeday =
    desc: My first FINTS
    freq: Daily (1)
\begin{tabular}{|c|c|c|c|c|}
\hline 'dates: (6)' & 'times: & (6) \({ }^{\prime}\) & 'Datal: & (6) ' \\
\hline '01-Jan-2001' & '11:00' & & & 999] \\
\hline , " , & '12:00' & & [ & 2] \\
\hline '02-Jan-2001' & '11:00' & & [ & \(3]\) \\
\hline " ' & '12:00' & & [ & \(4]\) \\
\hline '03-Jan-2001' & '11:00' & & & 5] \\
\hline & '12:00' & & & 6] \\
\hline
\end{tabular}
```

For value assignment using date character vectors, enclose the character vector in single quotation marks. If a date has multiple times, designating only the date and assigning a value results in every element of that date taking on the assigned value. For example, to assign the value 0.5 to all times-of-day on January 1, 2001, enter

```
timedata('01-Jan-2001') = 0.5
```

The result is

```
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/size (line 28)
    In help (line 53)
timedata =
desc: My first FINTS
freq: Daily (1)
\begin{tabular}{|c|c|c|c|}
\hline 'dates: (6)' & 'times: & (6) ' & 'Datal: (6)' \\
\hline '01-Jan-2001' & '11:00' & & \(0.5000]\) \\
\hline & '12:00' & & \(0.5000]\) \\
\hline '02-Jan-2001' & '11:00' & & \(3]\) \\
\hline & '12:00' & & 4] \\
\hline '03-Jan-2001' & '11:00' & & 5] \\
\hline & '12:00' & & 1 \\
\hline
\end{tabular}
```

To access the individual components of the financial time series object, use the structure syntax. For example, to assign a range of data to all the data items in the series Datal, you can use

```
timedata.Datal = (0: .1 : .5)'
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/size (line 28)
    In help (line 53)
```

timedata =
desc: My first FINTS
freq: Daily (1)

| 'dates: (6)' | 'times: | (6) ' | 'Datal: (6)' |
| :---: | :---: | :---: | :---: |
| '01-Jan-2001' | '11:00' |  | [ 0] |
| ' " ' | '12:00' |  | [ 0.1000] |
| '02-Jan-2001' | '11:00' |  | [ 0.2000] |
| ' " ' | '12:00' |  | [ 0.3000] |
| '03-Jan-2001' | '11:00' |  | [ 0.4000] |
|  | '12:00' |  | $0.5000]$ |

## See Also

datestr| subsref

## Topics

"Financial Time Series Operations" on page 13-7

## Introduced before R2006a

## subsref

Subscripted reference

Note subsref will be removed in a future release. Use timetable instead. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

```
subref
```


## Description

subsref implements indexing for a financial time series object. Integer indexing or date (and time) character vector indexing is allowed. Serial date numbers cannot be used as indices.

To use date character vector indexing, enclose the date character vector(s) in a pair of single quotation marks ' ' .

You can use integer indexing on the object as in any other MATLAB matrix. It returns the appropriate entry(ies) from the object.

Additionally, subsref lets you access the individual components of the object using the structure syntax.

## Examples

Create a time series named myfts:

```
myfts = fints((datenum('07/01/98'):datenum('07/01/98')+4)',...
[1234.56; 2345.61; 3456.12; 4561.23; 5612.34], [], 'Daily',...
'Data Reference')
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints (line 165)
```

```
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/display (line 66)
myfts =
    desc: Data Reference
    freq: Daily (1)
    'dates: (5)' 'series1: (5)'
    '01-Jul-1998' [ 1.2346e+03]
    '02-Jul-1998' [ 2.3456e+03]
    '03-Jul-1998' [ 3.4561e+03]
    '04-Jul-1998' [ 4.5612e+03]
    '05-Jul-1998' [ 5.6123e+03]
```

Extract the data for the single day July 1, 1998:

```
myfts('07/01/98')
```

```
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/subsref (line 106)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/display (line 66)
ans =
    desc: Data Reference
    freq: Daily (1)
    'dates: (1)' 'series1: (1)'
    '01-Jul-1998' [ 1.2346e+03]
```

Now, extract the data for the range of dates July 1, 1998, through July 5, 1998:

```
myfts('07/01/98::07/03/98')
```

```
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/subsref (line 106)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/display (line 66)
ans =
    desc: Data Reference
    freq: Daily (1)
```

```
'dates: (3)' 'series1: (3)'
'01-Jul-1998' [ 1.2346e+03]
'02-Jul-1998' [ 2.3456e+03]
'03-Jul-1998' [ 3.4561e+03]
```

You can use the MATLAB structure syntax to access the individual components of a financial time series object. To get the description field of myfts, enter
myfts.desc
at the command line, which returns

```
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/subsref (line 106)
ans =
    'Data Reference'
```

Similarly
myfts.series1
returns

```
    Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/subsref (line 106)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/display (line 66)
ans =
    desc: Data Reference
    freq: Daily (1)
    'dates: (5)' 'seriesl: (5)'
    '01-Jul-1998' [ 1.2346e+03]
    '02-Jul-1998' [ 2.3456e+03]
    '03-Jul-1998' [ 3.4561e+03]
    '04-Jul-1998' [ 4.5612e+03]
    '05-Jul-1998' [ 5.6123e+03]
```

The syntax for integer indexing is the same as for any other MATLAB matrix. Create a new financial time series object containing both dates and times:

```
dates = ['01-Jan-2001';'01-Jan-2001'; '02-Jan-2001'; ...
    '02-Jan-2001'; '03-Jan-2001';'03-Jan-2001'];
times = ['11:00';'12:00';'11:00';'12:00';'11:00';'12:00'];
dates_times = cellstr([dates, repmat(' ',size(dates,1),1),...
    times]);
anewfts = fints(dates_times,(1:6)',{'Datal'},1,'Another FinTs')
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints (line 165)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/display (line 66)
anewfts =
    desc: Another FinTs
    freq: Daily (1)
\begin{tabular}{|c|c|c|c|}
\hline dates: (6)' & 'times: & (6) \({ }^{\prime}\) & 'Datal: (6)' \\
\hline 01-Jan-2001' & '11:00' & & \(1]\) \\
\hline " ' & '12:00' & & 2] \\
\hline '02-Jan-2001' & '11:00' & & \(3]\) \\
\hline " ' & '12:00' & & \(4]\) \\
\hline '03-Jan-2001' & '11:00' & & 5] \\
\hline & '12:00' & & 61 \\
\hline
\end{tabular}
```

Use integer indexing to extract the second and third data items from the object.
anewfts(2:3)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead. > In fints/subsref (line 106)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead. > In fints/display (line 66)
ans =
desc: Another FinTs
freq: Daily (1)

| 'dates: (2)' | 'times: | (2)' | 'Datal: | (2)' |
| :--- | :--- | :--- | :--- | :--- |
| '01-Jan-2001' | '12:00' | $[$ | $2]$ |  |
| '02-Jan-2001' | '11:00' | $[$ | $3]$ |  |

For date character vector, enclose the indexing character vector in a pair of single quotation marks.

If there is one date with multiple times, indexing with only the date returns all the times for that specific date:

```
anewfts('01-Jan-2001')
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/subsref (line 106)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/display (line 66)
ans =
    desc: Another FinTs
    freq: Daily (1)
    'dates: (2)' 'times: (2)' 'Datal: (2)
```



To specify one specific date and time, index with that date and time:

```
anewfts('01-Jan-2001 12:00')
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/subsref (line 106)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/display (line 66)
ans =
    desc: Another FinTs
    freq: Daily (1)
    'dates: (1)' 'times: (1)' 'Datal: (1)
    '01-Jan-2001' '12:00' [ 2]
```

To specify a range of dates and times, use the double colon (: :) operator:

```
anewfts('01-Jan-2001 12:00::03-Jan-2001 11:00')
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/subsref (line 106)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/display (line 66)
ans =
```

```
desc: Another FinTs
freq: Daily (1)
\begin{tabular}{|c|c|c|c|}
\hline 'dates: (4)' & 'times: & \((4)^{\prime}\) & 'Datal: (4)' \\
\hline '01-Jan-2001' & '12:00' & & \(2]\) \\
\hline '02-Jan-2001' & '11:00' & & [ 3] \\
\hline ' " & '12:00' & & [ 4] \\
\hline '03-Jan-2001' & '11:00' & & \(5]\) \\
\hline
\end{tabular}
```

To request all the dates, times, and data, use the : : operator without specifying any specific date or time:

```
anewfts('::')
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/subsref (line 106)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/display (line 66)
ans =
    desc: Another FinTs
    freq: Daily (1)
    'dates: (6)' 'times: (6)' 'Datal: (6)
    '01-Jan-2001' '11:00' [ 1]
    ' " ' '12:00' [ 2]
    '02-Jan-2001' '11:00' [ 3]
    ' " ' '12:00' [ 4]
    '03-Jan-2001' '11:00' [ 5]
    '" ' '12:00' [ 6]
```


## See Also

datestr|fts2mat|subsasgn

## Topics

"Financial Time Series Operations" on page 13-7

## Introduced before R2006a

## targetreturn

Portfolio weight accuracy

## Syntax

return = targetreturn(Universe, Window,Offset, Weights)

## Arguments

| Universe | Number of observations (NUMOBS) by number of assets plus one <br> (NASSETS + 1) array containing total return data for a group of <br> securities. Each row represents an observation. Column 1 contains <br> MATLAB serial date numbers. The remaining columns contain the <br> total return data for each security. |
| :--- | :--- |
| Window | Number of data periods used to calculate frontier. |
| Offset | Increment in number of periods at which each frontier is generated. |
| Weights | Number of assets (NASSETS) by number of curves (NCURVES) matrix <br> of asset allocation weights needed to obtain the target rate of <br> return. |

## Description

return = targetreturn(Universe,Window, Offset, Weights) computes target return values for each window of data and given portfolio weights. These values should match the input target return used with selectreturn.

## See Also

frontier|portopt

## Topics

"Portfolio Construction Examples" on page 3-7
"Portfolio Optimization Functions" on page 3-4
Introduced before R2006a

## taxedrr

After-tax rate of return

## Syntax

Return = taxedrr(PreTaxReturn,TaxRate)

## Arguments

| PreTaxReturn | Nominal rate of return. Enter as a decimal fraction. |
| :--- | :--- |
| TaxRate | Tax rate. Enter as a decimal fraction. |

## Description

Return = taxedrr(PreTaxReturn,TaxRate) calculates the after-tax rate of return.

## Examples

## Calculate the After-Tax Rate of Return

This example shows how to calculate the after-tax rate of return, given an investment that has a $12 \%$ nominal rate of return and is taxed at a $30 \%$ rate.

```
Return \(=\) taxedrr(0.12, 0.30)
```

Return = 0.0840

## See Also

effrr|irr|mirr|nomrr|xirr

## Topics

"Analyzing and Computing Cash Flows" on page 2-18

Introduced before R2006a

## tbilldisc2yield

Convert Treasury bill discount to equivalent yield

## Syntax

```
[BEYield,MMYield] = tbilldisc2yield(Discount,Settle,Maturity)
```


## Description

[BEYield,MMYield] = tbilldisc2yield(Discount,Settle,Maturity) converts the discount rate on Treasury bills into their respective money-market or bond-equivalent yields.

## Examples

## Convert the Discount Rate on Treasury Bills

This example shows how to convert the discount rate on Treasury bills into their respective money-market or bond-equivalent yields, given a Treasury bill with the following characteristics.

```
Discount = 0.0497;
Settle = '01-0ct-02';
Maturity = '31-Mar-03';
[BEYield MMYield] = tbilldisc2yield(Discount, Settle, Maturity)
BEYield = 0.0517
MMYield = 0.0510
```


## Convert the Discount Rate on Treasury Bills Using datetime Inputs

This example shows how to use datetime inputs to convert the discount rate on Treasury bills into their respective money-market or bond-equivalent yields, given a Treasury bill with the following characteristics.

```
Discount = 0.0497;
Settle = datetime('01-0ct-02','Locale','en US');
Maturity = datetime('31-Mar-03','Locale','en US');
[BEYield MMYield] = tbilldisc2yield(Discount, Settle, Maturity)
BEYield = 0.0517
MMYield = 0.0510
```


## Input Arguments

## Discount - Discount rate of Treasury bills

decimal
Discount rate of the Treasury bills, specified as a scalar of a NTBILLS-by-1 vector of decimal values. The discount rate basis is actual/360.

Data Types: double

## Settle - Settlement date of Treasury bill <br> serial date number | date character vector | datetime

Settlement date of the Treasury bill, specified as a scalar or a NTBILLS-by-1 vector of serial date numbers, date character vectors, or datetime arrays. Settle must be earlier than Maturity.
Data Types: double | char | datetime

## Maturity - Maturity date of Treasury bill <br> serial date number | date character vector | datetime

Maturity date of the Treasury bill, specified as a scalar or a NTBILLS-by-1 vector of serial date numbers, date character vectors, or datetime arrays.

Data Types: double|char|datetime

## Output Arguments

## BEYield - Bond equivalent yields of Treasury bills

numeric
Bond equivalent yields of the Treasury bills, returned as a NTBILLS-by-1 vector. The bond-equivalent yield basis is actual/365.

## MMYield - Money-market yields of Treasury bills

## numeric

Money-market yields of the Treasury bills, returned as a NTBILLS-by-1 vector. The money-market yield basis is actual/360.

## References

[1] SIA Fixed Income Securities Formulas for Price, Yield, and Accrued Interest. Volume 1, 3rd edition, pp. 44-45.
[2] Krgin, D. Handbook of Global Fixed Income Calculations. Wiley, 2002.
[3] Stigum, M., Robinson, F. Money Market and Bond Calculation. McGraw-Hill, 1996.

## See Also

datetime |tbillyield2disc|zeroyield

## Topics

"Computing Treasury Bill Price and Yield" on page 2-38
"Treasury Bills Defined" on page 2-37

## Introduced before R2006a

## tbillprice

Price Treasury bill

## Syntax

```
Price = tbillprice(Rate,Settle,Maturity)
Price = tbillprice(
```

$\qquad$

``` ,Type)
```


## Description

Price = tbillprice(Rate, Settle, Maturity) computes the price of a Treasury bill given a yield or discount rate.

Price = tbillprice( __ , Type) adds an optional argument for Type.

## Examples

## Compute the Price of a Treasury Bill Using the Bond-Equivalent Yield

Given a Treasury bill with the following characteristics, compute the price of the Treasury bill using the bond-equivalent yield (Type $=2$ ) as input.

```
Rate = 0.045;
Settle = '01-0ct-02';
Maturity = '31-Mar-03';
Type = 2;
Price = tbillprice(Rate, Settle, Maturity, Type)
Price = 97.8172
```


## Price a Portfolio of Treasury Bills

Use tbillprice to price a portfolio of Treasury bills.
Rate $=[0.045 ; 0.046] ;$

```
Settle = {'02-Jan-02'; '01-Mar-02'};
```

Maturity = \{'30-June-02'; '30-June-02'\};
Type = [2 3];
Price = tbillprice(Rate, Settle, Maturity, Type)
Price $=2 \times 1$
97.8408
98.4539

## Price a Portfolio of Treasury Bills Using datetime Input

Use tbillprice to price a portfolio of Treasury bills using datetime input.

```
Rate = [0.045; 0.046];
Type = [2 3];
```

Settle = datetime(\{'02-Jan-2002'; '01-Mar-2002'\},'Locale','en_US');
Maturity = datetime(\{'30-June-2002'; '30-June-2002'\},'Locale','en_US');
Price = tbillprice(Rate, Settle, Maturity, Type)
Price $=2 \times 1$
97.8408
98.4539

## Input Arguments

## Rate - Bond-equivalent yield, money-market yield, or discount rate decimal

Bond-equivalent yield, money-market yield, or discount rate (defined by the input Type), specified as a scalar of a NTBILLS-by-1 vector of decimal values.

## Data Types: double

## Settle - Settlement date of Treasury bill

serial date number | date character vector | datetime
Settlement date of the Treasury bill, specified as a scalar or a NTBILLS-by-1 vector of serial date numbers, date character vectors, or datetime arrays.

Data Types: double|char|datetime

## Maturity - Maturity date of Treasury bill <br> serial date number | date character vector | datetime

Maturity date of the Treasury bill, specified as a scalar or a NTBILLS-by-1 vector of serial date numbers, date character vectors, or datetime arrays.

Data Types: double | char|datetime

## Type - (Optional) Rate type

2 (default) | numeric with values 1 = money market, 2 = bond-equivalent, 3 = discount rate

Rate type (determines how to interpret values entered in Rate), specified as a numeric value of 1,2, or 3 using a scalar or a NTBILLS-by-1 vector.

Note The bond-equivalent yield basis is actual/365. The money-market yield basis is actual/360. The discount rate basis is actual/360.

## Data Types: double

## Output Arguments

Price - Treasury bill price for every $\$ 100$ face
numeric
Treasury bill prices for every $\$ 100$ face, returned as a NTBILLS-by-1 vector.

## References

[1] SIA Fixed Income Securities Formulas for Price, Yield, and Accrued Interest. Volume 1, 3rd edition, pp. 44-45.
[2] Krgin, D. Handbook of Global Fixed Income Calculations. Wiley, 2002.
[3] Stigum, M., Robinson, F. Money Market and Bond Calculation. McGraw-Hill, 1996.

## See Also

datetime|tbillyield|zeroprice

## Topics

"Computing Treasury Bill Price and Yield" on page 2-38
"Treasury Bills Defined" on page 2-37

Introduced before R2006a

## tbillrepo

Break-even discount of repurchase agreement

## Syntax

TBEDiscount = tbillrepo(RepoRate,InitialDiscount,PurchaseDate, SaleDate, Maturity)

## Description

TBEDiscount = tbillrepo(RepoRate,InitialDiscount,PurchaseDate, SaleDate, Maturity) computes the true break-even discount of a repurchase agreement.

## Examples

## Compute the True Break-Even Discount

This example shows how to compute the true break-even discount of a Treasury bill repurchase agreement.

```
RepoRate = [0.045; 0.0475];
InitialDiscount = 0.0475;
PurchaseDate = '3-Jan-2002';
SaleDate = '3-Feb-2002';
Maturity = '3-Apr-2002';
TBEDiscount = tbillrepo(RepoRate, InitialDiscount,...
PurchaseDate, SaleDate, Maturity)
```

TBEDiscount $=2 \times 1$
0.0491
0.0478

## Compute the True Break-Even Discount Using datetime Inputs

This example shows how to use datetime inputs to compute the true break-even discount of a Treasury bill repurchase agreement.

```
RepoRate = [0.045; 0.0475];
InitialDiscount = 0.0475;
PurchaseDate = datetime('3-Jan-2002','Locale','en_US');
SaleDate = datetime('3-Feb-2002','Locale','en_US');
Maturity = datetime('3-Apr-2002','Locale','en_US');
TBEDiscount = tbillrepo(RepoRate, InitialDiscount,...
PurchaseDate, SaleDate, Maturity)
TBEDiscount = 2×1
0.0491
0.0478
```


## Input Arguments

## RepoRate - Annualized, 360-day based repurchase rate

decimal
Annualized, 360-day based repurchase rate, specified as a scalar of a NTBILLS-by-1 vector of decimal values.
Data Types: double

## InitialDiscount - Discount on Treasury bill on day of purchase decimal

Discount on the Treasury bill on the day of purchase, specified as a scalar of a NTBILLS-by-1 vector of decimal values.

## Data Types: double

## PurchaseDate - Date Treasury bill is purchased <br> serial date number | date character vector | datetime

Date the Treasury bill is purchased, specified as a scalar or a NTBILLS-by-1 vector of serial date numbers, date character vectors, or datetime arrays.

Data Types: double | char | datetime

## SaleDate - Date Treasury bill repurchase term is due serial date number | date character vector | datetime

Date the Treasury bill repurchase term is due, specified as a scalar or a NTBILLS-by-1 vector of serial date numbers, date character vectors, or datetime arrays.

## Data Types: double | char |datetime

## Maturity - Maturity date of Treasury bill <br> serial date number | date character vector | datetime

Maturity date of the Treasury bill, specified as a scalar or a NTBILLS-by-1 vector of serial date numbers, date character vectors, or datetime arrays.
Data Types: double | char | datetime

## Output Arguments

## TBEDiscount - True break-even discount of repurchase agreement numeric

True break-even discount of a repurchase agreement, returned as a scalar or NTBILLS-by-1 vector.

## References

[1] SIA Fixed Income Securities Formulas for Price, Yield, and Accrued Interest. Volume 1, 3rd edition, pp. 44-45.
[2] Krgin, D. Handbook of Global Fixed Income Calculations. Wiley, 2002.
[3] Stigum, M., Robinson, F. Money Market and Bond Calculation. McGraw-Hill, 1996.

See Also<br>datetime|tbillprice|tbillval01|tbillyield

## Topics

"Computing Treasury Bill Price and Yield" on page 2-38
"Treasury Bills Defined" on page 2-37

## Introduced before R2006a

## tbillval01

Value of one basis point

## Syntax

[Val01Disc, Val01MMY,Val01BEY] = tbillval01(Settle,Maturity)

## Description

[Val01Disc,Val01MMY,Val01BEY] = tbillval01(Settle,Maturity) calculates the value of one basis point of $\$ 100$ Treasury bill face value on the discount rate, moneymarket yield, or bond-equivalent yield.

## Examples

## Compute the Value of One Basis Point

This example shows how to compute the value of one basis point, given a Treasury bill with the following settle and maturity dates.

```
Settle = '01-Mar-03';
Maturity = '30-June-03';
[Val01Disc, Val01MMY, Val01BEY] = tbillval01(Settle, Maturity)
Val01Disc = 0.0034
Val01MMY = 0.0034
Val01BEY = 0.0033
```


## Compute the Value of One Basis Point Using datetime Inputs

This example shows how to use datetime inputs to compute the value of one basis point, given a Treasury bill with the following settle and maturity dates.

```
Settle = datetime('01-Mar-03','Locale','en_US');
Maturity = datetime('30-June-03','Locale','en_US');
[Val01Disc, Val01MMY, Val01BEY] = tbillval01(Settle, Maturity)
Val01Disc = 0.0034
Val01MMY = 0.0034
Val01BEY = 0.0033
```


## Input Arguments

## Settle - Settlement date of Treasury bill

serial date number | date character vector | datetime
Settlement date of the Treasury bill, specified as a scalar or a NTBILLS-by-1 vector of serial date numbers, date character vectors, or datetime arrays. Settle must be earlier than Maturity.

Data Types: double | char|datetime

## Maturity - Maturity date of Treasury bill

serial date number | date character vector | datetime
Maturity date of the Treasury bill, specified as a scalar or a NTBILLS-by-1 vector of serial date numbers, date character vectors, or datetime arrays.

Data Types: double | char|datetime

## Output Arguments

Val01Disc - Value of one basis point of discount rate for every $\mathbf{\$ 1 0 0}$ face numeric

Value of one basis point of discount rate for every $\$ 100$ face, returned as a NTBILLS-by-1 vector.

## Val01MMY - Value of one basis point of money-market yield for every \$100 face numeric

Value of one basis point of money-market yield for every $\$ 100$ face, returned as a NTBILLS-by-1 vector.

## Val01BEY - Value of one basis point of bond-equivalent yield for every \$100 face

numeric
Value of one basis point of bond-equivalent yield for every $\$ 100$ face, returned as a NTBILLS-by-1 vector.

## References

[1] SIA Fixed Income Securities Formulas for Price, Yield, and Accrued Interest. Volume 1, 3rd edition, pp. 44-45.
[2] Krgin, D. Handbook of Global Fixed Income Calculations. Wiley, 2002.
[3] Stigum, M., Robinson, F. Money Market and Bond Calculation. McGraw-Hill, 1996.

## See Also

datetime|tbillyield2disc|zeroyield

## Topics

"Computing Treasury Bill Price and Yield" on page 2-38
"Treasury Bills Defined" on page 2-37

## Introduced before R2006a

## tbillyield

Yield on Treasury bill

## Syntax

[MMYield,BEYield,Discount] = tbillyield(Price,Settle,Maturity)

## Description

[MMYield, BEYield,Discount] = tbillyield(Price,Settle,Maturity) computes the yield of US Treasury bills given Price, Settle, and Maturity.

## Examples

## Compute the Yield of U.S. Treasury Bills

This example shows how to compute the yield of U.S. Treasury bills, given a Treasury bill with the following characteristics.

```
Price = 98.75;
Settle = '01-0ct-02';
Maturity = '31-Mar-03';
[MMYield, BEYield, Discount] = tbillyield(Price, Settle,...
Maturity)
MMYield = 0.0252
BEYield = 0.0255
Discount = 0.0249
```


## Compute the Yield of U.S. Treasury Bills Using datetime Inputs

This example shows how to use datetime inputs to compute the yield of U.S. Treasury bills, given a Treasury bill with the following characteristics.

```
Price = 98.75;
Settle = datetime('01-0ct-2002','Locale','en US');
Maturity = datetime('31-Mar-2003','Locale','en US');
[MMYield, BEYield, Discount] = tbillyield(Price, Settle,Maturity)
MMYield = 0.0252
BEYield = 0.0255
Discount = 0.0249
```


## Input Arguments

## Price - Price of Treasury bills for every $\$ 100$ face value

numeric
Price of Treasury bills for every $\$ 100$ face value, specified as a scalar of a NTBILLS-by-1 vector of decimal values.

## Data Types: double

## Settle - Settlement date of Treasury bill <br> serial date number | date character vector | datetime

Settlement date of the Treasury bill, specified as a scalar or a NTBILLS-by-1 vector of serial date numbers, date character vectors, or datetime arrays. Settle must be earlier than Maturity.

Data Types: double| char|datetime

## Maturity - Maturity date of Treasury bill <br> serial date number | date character vector | datetime

Maturity date of the Treasury bill, specified as a scalar or a NTBILLS-by-1 vector of serial date numbers, date character vectors, or datetime arrays.

Data Types: double| char|datetime

## Output Arguments

## MMYield - Money-market yields of Treasury bills numeric

Money-market yields of the Treasury bills, returned as a NTBILLS-by-1 vector.

## BEYield - Bond equivalent yields of Treasury bills numeric

Bond equivalent yields of the Treasury bills, returned as a NTBILLS-by-1 vector.

## Discount - Discount rates of Treasury bills

numeric
Discount rates of the Treasury bills, returned as a NTBILLS-by-1 vector.

## References

[1] SIA Fixed Income Securities Formulas for Price, Yield, and Accrued Interest. Volume 1, 3rd edition, pp. 44-45.
[2] Krgin, D. Handbook of Global Fixed Income Calculations. Wiley, 2002.
[3] Stigum, M., Robinson, F. Money Market and Bond Calculation. McGraw-Hill, 1996.

See Also<br>datetime|tbilldisc2yield|tbillprice|tbillyield2disc|zeroyield<br>Topics<br>"Computing Treasury Bill Price and Yield" on page 2-38<br>"Treasury Bills Defined" on page 2-37<br>Introduced before R2006a

## tbillyield2disc

Convert Treasury bill yield to equivalent discount

## Syntax

Discount = tbillyield2disc(Yield,Settle, Maturity) Discount = tbillyield2disc( $\qquad$ ,Type)

## Description

Discount = tbillyield2disc(Yield, Settle, Maturity) converts the yield on some Treasury bills into their respective discount rates.

Discount = tbillyield2disc( $\qquad$ ,Type) adds an optional argument for Type.

## Examples

## Compute the Discount Rate on a Money-Market Basis

Given a Treasury bill with these characteristics, compute the discount rate.

```
Yield = 0.0497;
Settle = '01-0ct-02';
Maturity = '31-Mar-03';
Discount = tbillyield2disc(Yield,Settle,Maturity)
Discount = 0.0485
```


## Compute the Discount Rate on a Money-Market Basis Using datetime Inputs

Given a Treasury bill with these characteristics, compute the discount rate using datetime inputs.

```
Yield = 0.0497;
Settle = datetime('01-0ct-2002','Locale','en_US');
Maturity = datetime('31-Mar-2003','Locale','en_US');
Discount = tbillyield2disc(Yield,Settle,Maturity)
Discount = 0.0485
```


## Input Arguments

## Yield - Yield of Treasury bills <br> decimal

Yield of Treasury bills, specified as a scalar of a NTBILLS-by-1 vector of decimal values.
Data Types: double

## Settle - Settlement date of Treasury bill <br> serial date number | date character vector | datetime

Settlement date of the Treasury bill, specified as a scalar or a NTBILLS-by-1 vector of serial date numbers, date character vectors, or datetime arrays.
Data Types: double | char | datetime

## Maturity - Maturity date of Treasury bill <br> serial date number | date character vector | datetime

Maturity date of the Treasury bill, specified as a scalar or a NTBILLS-by-1 vector of serial date numbers, date character vectors, or datetime arrays.
Data Types: double | char \| datetime

## Type - Yield type

1 (default) | numeric with values 1 = money market, 2 = bond-equivalent
(Optional) Yield type (determines how to interpret values entered in Yield), specified as a numeric value of 1 or 2 using a scalar or a NTBILLS-by-1 vector.

Note The bond-equivalent yield basis is actual/365. The money-market yield basis is actual/360.

## Data Types: double

## Output Arguments

## Discount - Discount rates of Treasury bills

numeric
Discount rates of the Treasury bills, returned as a NTBILLS-by-1 vector.

## References

[1] SIA Fixed Income Securities Formulas for Price, Yield, and Accrued Interest. Volume 1, 3rd edition, pp. 44-45.
[2] Krgin, D. Handbook of Global Fixed Income Calculations. Wiley, 2002.
[3] Stigum, M., Robinson, F. Money Market and Bond Calculation. McGraw-Hill, 1996.

## See Also

datetime|tbilldisc2yield

## Topics

"Computing Treasury Bill Price and Yield" on page 2-38
"Treasury Bills Defined" on page 2-37

## Introduced before R2006a

## tbl2bond

Treasury bond parameters given Treasury bill parameters

## Syntax

[TBondMatrix,Settle] = tbl2bond(TBillMatrix)

## Description

[TBondMatrix,Settle] = tbl2bond(TBillMatrix)restates US Treasury bill market parameters in US Treasury bond form as zero-coupon bonds. This function makes Treasury bills directly comparable to Treasury bonds and notes.

## Examples

## Restate U.S. Treasury Bill in U.S. Treasury Bond Form

This example shows how to restate U.S. Treasury bill market parameters in U.S. Treasury bond form, given published Treasury bill market parameters for December 22, 1997.

```
TBill = [datenum('jan 02 1998') 10 0.0526 0.0522 0.053
    datenum('feb 05 1998') 44 0.0537 0.0533 0.0544
    datenum('mar 05 1998') 72 0.0529 0.0527 0.0540];
TBond = tbl2bond(TBill)
TBond = 3\times5
105 x
\begin{tabular}{lllll}
0 & 7.2976 & 0.0010 & 0.0010 & 0.0000 \\
0 & 7.2979 & 0.0010 & 0.0010 & 0.0000 \\
0 & 7.2982 & 0.0010 & 0.0010 & 0.0000
\end{tabular}
```


## Restate U.S. Treasury Bill in U.S. Treasury Bond Form Using datetime Input

This example shows how to use datetime input to restate U.S. Treasury bill market parameters in U.S. Treasury bond form, given published Treasury bill market parameters for December 22, 1997.

```
TBill = [datenum('jan 02 1998') 10 0.0526 0.0522 0.0530
    datenum('feb 05 1998') 44 0.0537 0.0533 0.0544
    datenum('mar 05 1998') 72 0.0529 0.0527 0.0540];
dates = datetime(TBill(:,1), 'ConvertFrom', 'datenum','Locale','en_US');
data = TBill(:,2:end);
t=[table(dates) array2table(data)];
[TBond, Settle] = tbl2bond(t)
TBond=3\times5 table
    CouponRate Maturity Bid Asked AskYield
    _ ____
\begin{tabular}{lrrrrr}
0 & \(02-\) Jan-1998 & \(00: 00: 00\) & 99.854 & 99.855 & 0.053 \\
0 & \(05-\) Feb-1998 & \(00: 00: 00\) & 99.344 & 99.349 & 0.0544 \\
0 & \(05-\) Mar-1998 & \(00: 00: 00\) & 98.942 & 98.946 & 0.054
\end{tabular}
```

Settle = 3x1 datetime array
22-Dec-1997 00:00:00
22-Dec-1997 00:00:00
22-Dec-1997 00:00:00

## Input Arguments

## TBillMatrix - Treasury bill parameters

table | matrix
Treasury bill parameters, specified as a 5-column table or a N -by- 5 matrix of bond information where the table columns or matrix columns contains:

- Maturity (Required) Maturity date of Treasury bills, specified as a serial date number when using a matrix. Use datenum to convert date character vectors to serial
date numbers. If the input TBillMatrix is a table, the Maturity dates can be serial date numbers, date character vectors, or datetime arrays.
- DaysMaturity (Required) Days to maturity, specified as an integer. Days to maturity are quoted on a skip-day basis; the actual number of days from settlement to maturity is DaysMaturity + 1 .
- Bid (Required) Bid bank-discount rate (the percentage discount from face value at which the bill could be bought, annualized on a simple-interest basis), specified as a decimal fraction.
- Asked (Required) Asked bank-discount rate, specified as a decimal fraction.
- AskYield (Required) Asked yield (the bond-equivalent yield from holding the bill to maturity, annualized on a simple-interest basis and assuming a 365-day year), specified as a decimal fraction.


## Data Types: double | table

## Output Arguments

## TBondMatrix - Treasury bond parameters <br> table | matrix

Treasury bond parameters, returned as a table or matrix depending on the TBillMatrix input.

When TBillMatrix is a table, TBondMatrix is also a table, and the variable type for the Maturity dates in TBondMatrix (column 1) matches the variable type for Maturity in TBillMatrix. For example, if Maturity dates are datetime arrays in TBillMatrix, they will also be datetime arrays in TBondMatrix.

When TBillMatrix input is a N-by-5 matrix, then each row describes a Treasury bond.
The parameters or columns returned for TBondMatrix are:

- CouponRate (Column 1) Coupon rate, which is always 0 since the Treasury bills are, by definition, a zero coupon instrument.
- Maturity (Column 2) Maturity date for each bond in the portfolio as a serial date number. The format of the dates matches the format used for Maturity in TBillMatrix (serial date number, date character vector, or datetime array).
- Bid (Column 3) Bid price based on $\$ 100$ face value.
- Asked (Column 4) Asked price based on $\$ 100$ face value.
- AskYield (Column 5) Asked yield to maturity: the effective return from holding the bond to maturity, annualized on a compound-interest basis.


## Settle - Settlement dates implied by maturity dates and number of days to maturity quote

## serial date number | datetime

Settlement dates implied by the maturity dates and the number of days to maturity quote, returned as a $N$-by- 5 vector containing serial date numbers, by default. Settle is returned as a datetime array only if the input TBillMatrix is a table containing datetime arrays for Maturity in the first column.

## See Also

datetime|tr2bonds

## Topics

"Term Structure of Interest Rates" on page 2-42
"Computing Treasury Bill Price and Yield" on page 2-38
"Treasury Bills Defined" on page 2-37

## Introduced before R2006a

## thirdwednesday

Find third Wednesday of month

## Syntax

```
[BeginDates,EndDates] = thirdwednesday(Month,Year)
[BeginDates,EndDates] = thirdwednesday( __ ,outputType)
```


## Description

[BeginDates,EndDates] = thirdwednesday(Month,Year) computes the beginning and end period date for a LIBOR contract (third Wednesdays of delivery months).
[BeginDates,EndDates] = thirdwednesday( $\qquad$ , outputType), using optional input arguments, computes the beginning and end period date for a LIBOR contract (third Wednesdays of delivery months).

The type of the outputs depends on the input outputType. If this variable is 'datenum', BeginDates and EndDates are serial date numbers. If outputType is 'datetime', then BeginDates and EndDates are datetime arrays. By default, outputType is set to 'datenum'.

## Examples

## Determine the Third Wednesday for Given Months and Years

Find the third Wednesday dates for swaps commencing in the month of October in the years 2002, 2003, and 2004.

```
Months = [10; 10; 10];
Year = [2002; 2003; 2004];
[BeginDates, EndDates] = thirdwednesday(Months, Year);
datestr(BeginDates)
```

```
ans = 3x11 char array
    '16-0ct-2002'
    '15-0ct-2003'
    '20-0ct-2004'
datestr(EndDates)
ans = 3x11 char array
    '16-Jan-2003'
    '15-Jan-2004
    '20-Jan-2005'
```

Find the third Wednesday dates for swaps commencing in the month of October in the years 2002, 2003, and 2004 using an outputType of 'datetime'.

Months = [10; 10; 10];
Year = [2002; 2003; 2004];
[BeginDates, EndDates] = thirdwednesday(Months, Year,'datetime')
BeginDates = 3x1 datetime array
16-0ct-2002
15-0ct-2003
20-0ct-2004

EndDates = 3x1 datetime array
16-Jan-2003
15-Jan-2004
20-Jan-2005

## Input Arguments

## Month - Month of delivery for Eurodollar futures

integer from 1 through 12 | vector of integers from 1 through 12
Month of delivery for Eurodollar futures, specified as an N-by-1 vector of integers from 1 through 12.

Duplicate dates are returned when identical months and years are supplied.

Data Types: single | double

## Year - Delivery year for Eurodollar futures/Libor contracts corresponding to Month <br> four-digit nonnegative integer | vector of four-digit nonnegative integers

Delivery year for Eurodollar futures/Libor contracts corresponding to Month, specified as an N -by-1 vector of our-digit nonnegative integers.

Duplicate dates are returned when identical months and years are supplied.

## Data Types: single | double

## outputType - Output date format

'datenum' (default)| character vector with values 'datenum' or 'datetime'
Output date format, specified as a character vector with values 'datenum' or 'datetime'. If outputType is 'datenum', then BeginDates and EndDates are serial date numbers. However, if outputType is 'datetime', then BeginDates and EndDates are datetime arrays.
Data Types: char

## Output Arguments

## BeginDates - Third Wednesday of given month and year

serial date number | date character vector
Third Wednesday of given month and year, returned as serial date numbers or date character vectors, or datetime arrays. This is also the beginning of the 3-month period contract.

The type of the outputs depends on the input outputType. If this variable is 'datenum', BeginDates and EndDates are serial date numbers. If outputType is 'datetime', thenBeginDates and EndDates are datetime arrays. By default, outputType is set to 'datenum'.

## EndDates - End of three-month period contract for given month and year serial date number | date character vector

End of three-month period contract for given month and year, returned as serial date numbers or date character vectors, or datetime arrays.

The type of the outputs depends on the input outputType. If this variable is 'datenum ', BeginDates and EndDates are serial date numbers. If outputType is 'datetime', thenBeginDates and EndDates are datetime arrays. By default, outputType is set to 'datenum'.

## See Also

datetime|tr2bonds

## Topics

"Handle and Convert Dates" on page 2-2
"Trading Calendars User Interface" on page 16-2
"UICalendar User Interface" on page 16-4

Introduced before R2006a

## thirtytwo2dec

Thirty-second quotation to decimal

## Syntax

OutNumber = thirtytwo2dec(InNumber,InFraction)

## Description

OutNumber $=$ thirtytwo2dec(InNumber, InFraction) changes the price quotation for a bond or bond future from a fraction with a denominator of 32 to a decimal.

## Examples

## Change the Price Quotation for a Bond or Bond Future From a Fraction

This example shows how to change the price quotation for a bond or bond future from a fraction with a denominator of 32 to a decimal, given two bonds that are quoted as 101-25 and 102-31.

```
InNumber = [101; 102];
InFraction = [25; 31];
OutNumber = thirtytwo2dec(InNumber, InFraction)
OutNumber = 2\times1
```

    101.7813
    102.9688
    
## Input Arguments

## InNumber - Input number <br> integer

Input number, specified as a scalar or an N -by-1 vector of integers representing price without the fractional components.

## Data Types: double

## InFraction - Fractional portions of each element in InNumber numeric decimal fraction

Fractional portions of each element in InNumber, specified as a scalar or an N -by- 1 vector of numeric decimal fractions.

Data Types: double

## Output Arguments

## OutNumber - Output number that represents sum of InNumber and InFraction decimal

Output number that represents sum of InNumber and InFraction, returned as a decimal.

See Also<br>dec2thirtytwo<br>Introduced before R2006a

## tick2ret

Convert price series to return series

> Note tick2ret accepts Data as a matrix, timetable, or table.

## Syntax

```
[Returns,Intervals] = tick2ret(Data)
[Returns,Intervals] = tick2ret(
,Name,Value)
```


## Description

[Returns,Intervals] = tick2ret(Data) computes asset returns for NUMOBS price observations of NASSETS assets.
[Returns,Intervals] = tick2ret( $\qquad$ ,Name, Value) adds optional name-value pair arguments.

## Examples

## Convert Price Series to Return Series

Load the file SimulatedStock. mat, which provides a timetable (TMW) for financial data for TMW stock. Then convert a price series to a return series, given the first 10 periodic returns of TMW.

```
load SimulatedStock.mat
TMW_Close = TMW(1:10,'Close');
[Returns,Intervals] = tick2ret(TMW_Close)
Returns=9\times1 timetable
    Time Close
```

```
    05-Sep-2012
    0.0017955
    06-Sep-2012
    0.013741
    07-Sep-2012
    10-Sep-2012
    11-Sep-2012
    -0.011557
    -0.014843
12-Sep-2012
    -0.0012384
13-Sep-2012
    0.0081628
    14-Sep-2012 -0.00051245
    17-Sep-2012 -0.02902
Intervals = 9x1 duration array
    24:00:00
    24:00:00
    24:00:00
    72:00:00
    24:00:00
    24:00:00
    24:00:00
    24:00:00
    72:00:00
```


## Convert Price Series to Return Series Using datetime Input

Use datetime input to convert a price series to a return series, given periodic returns of two stocks observed in the first, second, third, and fourth quarters.

TickSeries $=[10080$
11090
11588
110 91];
TickTimes = datetime(\{'1/1/2015','1/7/2015','1/16/2015','1/28/2015'\},'InputFormat', 'MM [Returns,Intervals] = tick2ret(TickSeries,'TickTimes',TickTimes)

Returns $=3 \times 2$
$\begin{array}{lr}0.1000 & 0.1250 \\ 0.0455 & -0.0222\end{array}$

19-1727

## Input Arguments

## Data - Data for asset prices

matrix | table | timetable
Data for asset prices, specified as a matrix, table, or timetable, specified as NUMOBS-byNASSETS. Prices across a given row are assumed to occur at the same time for all columns, and each column is a price series of an individual asset.

Data Types: double|table | timetable

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, ... , NameN, ValueN.
Example: [Returns,Intervals] = tick2ret(TickSeries,'TickTimes',TickTimes)

## TickTimes - Observation times associated with prices

sequential observation times from 1,2,...NUMOBS assumed for all assets (default) | vector
Observation times associated with prices, specified as the comma-separated pair consisting of 'TickTimes ' and a NUMOBS element column vector of monotonically increasing observation times associated with the prices in Data. Times are taken either as serial date numbers (day units), date strings, datetime arrays, or as decimal numbers in arbitrary units (for example, yearly).

Note If the input Data type is a timetable, the row times information in the timetable overwrites the TickTimes input.

## Data Types: double | datetime | string

## Method - Method to convert asset prices to returns

## 'Simple' (default)| character vector with value of 'Simple' or 'Continuous' | string with value of "Simple" or "Continuous"

Method to convert asset prices to returns, specified as the comma-separated pair consisting of 'Method ' and a string or character vector indicating the method to convert asset prices to returns.

If the method is 'Simple', then simple periodic returns at time $t$ are computed as:
Returns(t) = Data(t)/Data(t-1) - 1 .
If the method is 'Continuous ', the continuous returns are computed as:
Returns(t) $=\log ($ Data(t) $/ \operatorname{Data}(t-1))$.
Data Types: char|string

## Output Arguments

## Returns - Time series array of asset returns

matrix | table | timetable
Time series array of asset returns, returned as a NUMOBS-1-by-NASSETS array of asset returns with the same type (matrix, table, or timetable) as the input Data. The first row contains the oldest returns and the last row contains the most recent. Returns across a given row are assumed to occur at the same time for all columns, and each column is a return series of an individual asset.

## Intervals - Interval times between successive prices <br> vector

Interval times between successive prices, returned as a NUMOBS-1 length column vector where $\operatorname{Intervals}(t)=\operatorname{TickTimes}(t)$ - TickTimes $(t-1)$.

## Extended Capabilities

## Tall Arrays

Calculate with arrays that have more rows than fit in memory.
This function supports input Data that is specified as a tall column vector, a tall table, or a tall timetable. For more information, see tall and "Tall Arrays" (MATLAB).

## See Also

datetime| ret2tick

## Topics

"Data Transformation and Frequency Conversion" on page 13-13

## Introduced before R2006a

## tick2ret (fts)

Convert price series to return series for time series object

Note tick2ret (fts) will be removed in a future release. Use tick2ret instead.

## Syntax

```
returnFts = tick2ret(priceFts)
returnFts = tick2ret(priceFts,'PARAM1',VALUE1,'PARAM2',VALUE2', ....)
```


## Arguments

| priceFts | Financial time series object of prices. |
| :--- | :--- |
| 'PARAM1' | (Optional) Method is a character vector indicating the <br> method to convert asset returns to prices. The value must be <br> defined as 'Simple' (default) or 'Continuous '. If Method |
| is 'Simple', tick2 ret uses simple periodic returns. If |  |
| Method is 'Continuous ' , the function uses continuously |  |
| compounded returns. Case is ignored for Method. |  |

## Description

returnFts = tick2ret(priceFts,'PARAM1',VALUE1,'PARAM2',VALUE2', ...) generates a financial time series object of returns.

Note The $i$ 'th return is quoted for the period PriceSeries $(i)$ to PriceSeries( $i+1$ ) and is not normalized by the time increment between successive price observations.

If Method is unspecified or 'Simple', the prices are

```
ReturnSeries(i) = PriceSeries(i+1)/PriceSeries(i)-1
```

If Method is 'Continuous', the prices are
ReturnSeries(i) $=$ log[PriceSeries(i+1)/PriceSeries(i)]

## Examples

## Convert Price Series to Return Series for a fints Object

Compute the return series from the following price series:

```
PriceSeries = [100.0000 100.0000
110.0000 112.0000
115.5000 116.4800
109.7250 122.3040]
PriceSeries = 4×2
    100.0000 100.0000
    110.0000 112.0000
    115.5000 116.4800
    109.7250 122.3040
```

Use the following dates:

```
Dates = {'18-Dec-2000'
'18-Jun-2001'
'17-Sep-2001'
'18-Dec-2001'}
Dates = 4x1 cell array
    {'18-Dec-2000'}
    {'18-Jun-2001'}
    {'17-Sep-2001'}
    {'18-Dec-2001'}
```

The fints object is:
p = fints(Dates, PriceSeries)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in

## $\mathrm{p}=$

desc: $\quad$ (none)
freq: Unknown (0)

| 4) | (4) | (4) ' |
| :---: | :---: | :---: |
| 18-Dec-2000' | 100] | [ 100] |
| 18-Jun-2001' | 110] | [ 112] |
| 17-Sep-2001' | $115.5000]$ | $116.4800]$ |
| 18-Dec-2001' | 109.7250] | 122.3040] |

returnFts is computed as:

## tick2ret(p)

Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in
ans $=$
desc: (none)
freq: Unknown (0)

| 'dates: (3)' | 'series1: (3)' | 'series2: (3)' |  |
| :--- | :--- | :--- | :--- |
| '18-Jun-2001' | [ | $0.1000]$ | $[$ |

Note that for $n$ dates in the original time series, there are ( $n-1$ ) dates returned for returnFts from tick2ret. The formula for the date output dates is described as: RetDate(i) = PriceDate (i+1).

## See Also

portsim|ret2tick

## Introduced before R2006a

## time2date

Dates from time and frequency

## Syntax

```
Dates = time2date(Settle,TFactors)
Dates = time2date( ___ ,Compounding,Basis,EndMonthRule)
```


## Description

Dates = time2date(Settle,TFactors) computes Dates corresponding to compounded rate quotes between Settle and TFactors. time2date is the inverse of date2time.

Dates = time2date( __ , Compounding,Basis,EndMonthRule) computes Dates corresponding to compounded rate quotes between Settle and TFactors using optional input arguments for Compounding, Basis, and EndMonthRule. time2date is the inverse of date2time.

## Examples

## Calculate Dates Using time2date

Show that date2time and time2date are the inverse of each other. First compute the time factors using date2time.

```
Settle = '1-Sep-2002';
Dates = datenum(['31-Aug-2005'; '28-Feb-2006'; '15-Jun-2006';
    '31-Dec-2006']);
Compounding = 2;
Basis = 0;
EndMonthRule = 1;
TFactors = date2time(Settle, Dates, Compounding, Basis,...
EndMonthRule)
```

```
TFactors = 4×1
```

5.9945
6.9945
7.5738
8.6576

Now use the calculated TFactors in time2date and compare the calculated dates with the original set.

```
Dates calc = time2date(Settle, TFactors, Compounding, Basis,...
EndMonthRule)
Dates_calc = 4\times1
    732555
    732736
    732843
    733042
datestr(Dates_calc)
ans = 4x11 char array
    '31-Aug-2005'
    ' 28-Feb-2006'
    ' 15-Jun-2006'
    '31-Dec-2006'
```

Show time2date support for datetime input for Settle.

```
Settle = '1-Sep-2002';
Dates = datenum(['31-Aug-2005'; '28-Feb-2006'; '15-Jun-2006';
    '31-Dec-2006']);
Compounding = 2;
Basis = 0;
EndMonthRule = 1;
TFactors = date2time(Settle, Dates, Compounding, Basis,...
EndMonthRule);
Dates calc = time2date(datetime(Settle,'Locale','en US'), TFactors,...
Compounding, Basis, EndMonthRule)
Dates calc = 4x1 datetime array
    31-Aug-2005
```


## Input Arguments

## Settle - Settlement date

serial date number | date character vector | datetime object
Settlement date, specified as a serial date number, date character vector, or datetime array.

Data Types: double | char | datetime

## TFactors - Time factors

vector
Time factors, corresponding to the compounding value, specified as a vector. TFactors must be equal to or greater than zero.

## Data Types: double

## Compounding - Rate at which input zero rates are compounded when annualized

2 (Semiannual compounding) (default) | scalar with numeric values of $0,1,2,3,4,5,6$, 12, 365, -1

Rate at which input zero rates are compounded when annualized, specified as a scalar with numeric values of: $0,1,2,3,4,5,6,12,365$, or -1 . Allowed values are defined as:

- 0 - Simple interest (no compounding)
- 1 - Annual compounding
- 2 - Semiannual compounding (default)
- 3-Compounding three times per year
- 4-Quarterly compounding
- 6 - Bimonthly compounding
- 12 - Monthly compounding
- 365 - Daily compounding
-     - 1 - Continuous compounding

The optional Compounding argument determines the formula for the discount factors (Disc):

- Compounding $=1,2,3,4,6,12$
- Disc $=(1+Z / F)^{\wedge}(-T)$, where $F$ is the compounding frequency, $Z$ is the zero rate, and $T$ is the time in periodic units, for example, $T=F$ is one year.
- Compounding $=365$
- Disc $=(1+Z / F)^{\wedge}(-T)$, where $F$ is the number of days in the basis year and $T$ is a number of days elapsed computed by basis.
- Compounding $=-1$
- Disc $=\exp \left(-\mathrm{T}^{*} \mathrm{Z}\right)$, where T is time in years.


## Basis - Day-count basis

## 0 (actual/actual) (default) | numeric with value 0 through $13 \mid$ vector of numerics with values 0 through 13

Day-count basis, specified as an integer with a value of 0 through 13 or a N -by- 1 vector of integers with values 0 through 13.

- $0=$ actual/actual (default)
- $1=30 / 360$ (SIA)
- 2 = actual/360
- 3 = actual/365
- $4=30 / 360$ (BMA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- $7=$ actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- $9=$ actual/360 (ICMA)
- $10=$ actual/365 (ICMA)
- $11=30 / 360 \mathrm{E}$ (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-0
Data Types: single | double

## EndMonthRule - End-of-month rule flag for month having 30 or fewer days

 1 (in effect) (default) | nonnegative integer [0,1]End-of-month rule flag for month having 30 or fewer days, specified as scalar nonnegative integer $[0,1]$ or a using a N -by-1 vector of values. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days.

- $0=$ Ignore rule, meaning that a payment date is always the same numerical day of the month.
- $1=$ Set rule on, meaning that a payment date is always the last actual day of the month.


## Data Types: logical

## Output Arguments

## Dates - Dates corresponding to compounded rate quotes between Settle and TFactors <br> serial date number | datetime array

Dates corresponding to compounded rate quotes between Settle and TFactors, returned as a scalar or a N -by-1 vector using serial date numbers or datetime arrays.
Data Types: double |datetime

## See Also

cfamounts|cftimes|date2time|datetime

## Topics

"Handle and Convert Dates" on page 2-2

## Introduced before R2006a

## times

Financial time series multiplication

Note times will be removed in a future release. Use timetable instead. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

```
newfts = tsobj_1 .* tsobj_2
newfts = tsobj .* array
newfts = array .* tsobj
```


## Arguments

| tsobj_1, tsobj_2 | Pair of financial time series objects. |
| :--- | :--- |
| array | A scalar value or array with the number of rows equal to the <br> number of dates in tsobj and the number of columns equal <br> to the number of data series in tsobj. |

## Description

The times method multiplies element by element the components of one financial time series object by the components of the other. You can also multiply the entire object by an array.

If an object is to be multiplied by another object, both objects must have the same dates and data series names, although the order need not be the same. The order of the data series, when an object is multiplied by another object, follows the order of the first object.
newfts = tsobj_1 .* tsobj_2 multiplies financial time series objects element by element.

```
newfts = tsobj .* array multiplies a financial time series object element by element
by an array.
newfts = array .* tsobj and newfts = array / tsobj multiplies an array
element by element by a financial time series object.
For financial time series objects, the times operation is identical to the mtimes operation.
```

See Also<br>minus |mtimes |plus| rdivide<br>\section*{Topics}<br>"Financial Time Series Operations" on page 13-7<br>Introduced before R2006a

## tmfactor

Time factors of arbitrary dates

## Syntax

TFactors = tmfactor(Settle,Maturity)

## Arguments

| Settle | Settlement date. A vector of serial date numbers, date character <br> vectors, or datetime arrays. Settle must be earlier than <br> Maturity. |
| :--- | :--- |
| Maturity | Maturity date. A vector of serial date numbers, date character <br> vectors, or datetime arrays. |

## Description

TFactors = tmfactor(Settle, Maturity) determines the time factors from a vector of Settlement dates to a vector of Maturity dates.

## Examples

Find the TFactors for Settle and Maturity dates.

```
TFactors = tmfactor('1-Jan-2015','1-Jan-2016')
TFactors =
```

2

Find the TFactors for Settle and Maturity dates using a datetime array.

```
TFactors = tmfactor(datetime('1-Jan-2015','Locale','en US'),'1-Jan-2016')
```

TFactors =2
See Alsocfamounts|cftimes|datetime
Topics
"Analyzing and Computing Cash Flows" on page 2-18
Introduced before R2006a

## toannual

Convert to annual

Note toannual will be removed in a future release. Use timetable instead. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

```
newfts = toannual(oldfts)
newfts = toannual(oldfts,'ParameterName',ParameterValue, ...)
```


## Arguments

| oldfts | Financial time series object. |
| :--- | :--- |

## Description

newfts $=$ toannual (oldfts) converts a financial time series of any frequency to one of an annual frequency. The default end-of-year is the last business day of the December. toannual uses holidays.m to determine valid trading days.

Note If oldfts contains time-of-day information, newfts displays the time-of-day as '00:00' for those days that did not previously exist in oldfts.

Empty ([ ]) passed as inputs for parameter pair values for toannual triggers the use of the defaults.
newfts = toannual(oldfts,'ParameterName',ParameterValue, ...) accepts parameter name/parameter value pairs as input, as specified in the following table.

| Parameter <br> Name | Parameter <br> Value | Description |
| :--- | :---: | :--- |
| CalcMethod | CumSum | Returns the cumulative sum of the values within each <br> year. Data for missing dates are given the value 0. |
| CalcMethod | Exact | Returns the exact value at the end-of-year date. No <br> data manipulation occurs. <br> (Default) Returns the values located at the end-of- <br> year dates. If there is missing data, Nearest returns <br> the nearest data point preceding the end-of-year date. <br> Returns an averaged annual value that only takes into <br> account dates with data (non-NaN) within each year. <br> This mode is compatible with previous versions of this <br> function (Version 2.1.x and earlier). It returns an |
| CalcMethod | SimpAvg | v21x |
| CalcMethod |  | toannual algorithm. This algorithm takes into <br> account all dates and data. For dates that do not <br> contain any data, the data is assumed to be 0. |
| Note If you set CalcMethod to v2lx, settings for all the following parameter name/ |  |  |
| parameter value pairs are not supported. |  |  |

\(\left.$$
\begin{array}{lll}\begin{array}{l}\text { Parameter } \\
\text { Name }\end{array} & \begin{array}{l}\text { Parameter } \\
\text { Value } \\
\text { DateFilter } \\
\text { Absolute }\end{array} & \begin{array}{l}\text { Description } \\
\text { (Default) Returns all annual dates between the start } \\
\text { and end dates of oldfts. Some dates may be } \\
\text { disregarded if BusDays = } 1 .\end{array}
$$ <br>
\hline Note The default is to create a time series with every <br>
date at the specified periodicity, which is with <br>
DateFilter = Absolute. If you use DateFilter = <br>
Relative, the endpoint effects do not apply since <br>

only your data defines which dates appear in the\end{array}\right]\)| output time series object. |
| :--- |


| Parameter <br> Name | Parameter <br> Value | Description |
| :--- | :--- | :--- |
| [Begin, End] | Denotes the minimum number of days that constitute <br> an odd annual period at the endpoints of the time <br> series (before the first-time series date and after the <br> last end-of-year date). <br> Begin and End must be -1 or any positive integer <br> greater than or equal to 0. |  |
|  | A single value input for ' EndPtTol ' is the same as <br> specifying that single value for Begin and End. |  |
|  | -1 Exclude odd annual period dates and data from <br> calculations. |  |
|  | $0 \quad$ (Default) Include odd annual period dates and <br> data in calculations. |  |
|  | n Number of days (any positive integer) that <br> constitute an odd annual period. If there are <br> insufficient days for a complete year, the endpoint <br> data is ignored. |  |

The following diagram is a general depiction of the factors involved in the determination of endpoints for this function.


TimeSpec First Returns only the observation that occurs at the first (earliest) time for a specific date.

| Parameter <br> Name | Parameter <br> Value | Description |
| :--- | :--- | :--- |
| TimeSpec | Last | (Default) Returns only the observation that occurs at <br> the last (latest) time for a specific date. |
| AltHolidays |  | Vector of dates specifying an alternate set of market <br> closure dates. |
| AltHolidays | -1 | Excludes all holidays. <br> Veekend |
|  |  | Vector of length 7 containing 0's and 1's. The value 1 <br> indicates a weekend day. The first element of this <br> vector corresponds to Sunday. For example, when <br> Saturday and Sunday are weekend days (default) then <br> Weekend $=\left[\begin{array}{lllllll}1 & 0 & 0 & 0 & 0 & 1\end{array}\right]$. |

## Examples

## Transform Time Series Object from Weekly to Annual Values

This example shows how to transform a time series object from weekly to annual values.
Load the data from the file predict_ret_data.mat and use the fints function to create a time series object with a weekly frequency.
load predict ret data.mat x0 = fints(expdā̄es, expdata, \{'Metric'\}, 'w', 'Index')

Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in
$x 0=$

```
    desc: Index
    freq: Weekly (2)
    'dates: (53)' 'Metric
    '01-Jan-1999' [ 97.8872]
    '08-Jan-1999' [ 97.0847]
    '15-Jan-1999' [ 109.6312]
    '22-Jan-1999' [ 105.5743]
    '29-Jan-1999' [ 108.4028]
    'Metric: (53)
```



| '10-Dec-1999' | [ | $33.8457]$ |
| :---: | :---: | :--- |
| '17-Dec-1999' | [ | $33.3868]$ |
| '24-Dec-1999' | [ | $32.7737]$ |
| '31-Dec-1999' | [ | $28.5665]$ |

Use toannual to obtain the annual aggregate for the x 0 times series.
x1 = toannual(x0)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in
x1 =
desc: TOANNUAL: Index
freq: Annual (6)
'dates: (1)' 'Metric: (1)
'31-Dec-1999' [ 28.5665]

## See Also

convertto|fints|todaily|tomonthly|toquarterly|tosemi|toweekly

## Topics

"Data Transformation and Frequency Conversion" on page 13-13

## Introduced before R2006a

## todaily

Convert to daily

Note todaily will be removed in a future release. Use timetable instead. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

```
newfts = todaily(oldfts)
newfts = todaily(oldfts,'ParameterName',ParameterValue, ...)
```


## Arguments

| oldfts | Financial time series object |
| :--- | :--- |

## Description

newfts $=$ todaily(oldfts) converts a financial time series of any frequency to a daily frequency. todaily uses holidays.m to determine valid trading days.

Note If oldfts contains time-of-day information, newfts displays the time-of-day as '00:00' for those days that did not previously exist in oldfts.

Empty ([ ]) passed as inputs for parameter pair values for todaily trigger the use of the defaults.
newfts = todaily(oldfts,'ParameterName',ParameterValue, ...) accepts parameter name/parameter value pairs as input, as specified in the following table.

| Parameter Name | Parameter Value | Description |
| :---: | :---: | :---: |
| CalcMethod | Exact | Returns the value at specific dates/times. No data manipulation occurs. |
| CalcMethod | v21x | This mode is compatible with previous versions of this function (Version 2.1.x and earlier). It returns a fiveday business week that starts on Monday and ends on Friday. |
| Note If you set CalcMethod to v21x, settings for all the following parameter name/ parameter value pairs are not supported. |  |  |
| BusDays | 0 | Generates a financial time series that ranges from (or between) the first date to the last date in oldfts (includes NYSE nonbusiness days and holidays). |
| BusDays | 1 | (Default) Generates a daily financial time series that ranges from the first date to the last date in oldfts (excludes NYSE nonbusiness days and holidays and weekends based on AltHolidays and Weekend). |
|  |  | NYSE market closures, holidays, and weekends are observed if AltHolidays and Weekend are not supplied or empty ([]). |
| DateFilter | Absolute | (Default) Displays all daily dates between the start and end dates of oldfts. Some dates may be disregarded if BusDays $=1$. |
|  |  | Note The default is to create a time series with every date at the specified periodicity, which is with DateFilter = Absolute. If you use DateFilter = Relative, the endpoint effects do not apply since only your data defines which dates appear in the output time series object. |
| DateFilter | Relative | Displays only dates that exist in oldfts. Some dates may be disregarded if BusDays $=1$. |
| TimeSpec | First | Returns only the observation that occurs at the first (earliest) time for a specific date. |


| Parameter <br> Name | Parameter <br> Value | Description |
| :--- | :--- | :--- |
| TimeSpec | Last | (Default) Returns only the observation that occurs at <br> the last (latest) time for a specific date. |
| AltHolidays |  | Vector of dates specifying an alternate set of market <br> closure dates. |
| AltHolidays | -1 | Excludes all holidays. <br> Weekend |
|  | Vector of length 7 containing 0's and 1's. The value 1 <br> indicates a weekend day. The first element of this <br> vector corresponds to Sunday. For example, when <br> Saturday and Sunday are weekend days (default) then <br> Weekend $=\left[\begin{array}{llllll}1 & 0 & 0 & 0 & 0 & 1\end{array}\right]$. |  |

## Examples

## Transform Time Series Object from Weekly to Daily Values

Load the data from the file predict_ret data.mat and use the fints function to create a time series object with a weekly frequency.

```
load predict_ret_data.mat
x0 = fints(expdates, expdata, {'Metric'}, 'w', 'Index')
```

Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in
$x 0=$
desc: Index
freq: Weekly (2)
'dates: (53)' 'Metric: (53)'
'01-Jan-1999' [ 97.8872]
'08-Jan-1999' [ 97.0847]
'15-Jan-1999' [ 109.6312]
'22-Jan-1999' [ 105.5743]
'29-Jan-1999' [ 108.4028]
'05-Feb-1999' [ 134.4882]
'12-Feb-1999' [ 117.5581]

| ' 19-Feb-1999 ' | 106.6683] |
| :---: | :---: |
| ' 26-Feb-1999 ' | 118.2912] |
| '05-Mar-1999' | 105.6835] |
| '12-Mar-1999' | 128.5836] |
| '19-Mar-1999' | $115.1746]$ |
| '26-Mar-1999' | 131.2854] |
| '02-Apr-1999' | 130.7116] |
| '09-Apr-1999' | 123.1684] |
| '16-Apr-1999' | 107.2975] |
| '23-Apr-1999' | 91.5625] |
| '30-Apr-1999' | 78.5738] |
| '07-May-1999' | $65.2904]$ |
| ' 14-May-1999' | 70.8581] |
| '21-May-1999' | $72.4807]$ |
| '28-May-1999' | $72.9190]$ |
| '04-Jun-1999' | $64.3460]$ |
| '11-Jun-1999' | $59.8743]$ |
| '18-Jun-1999' | $55.0026]$ |
| ' 25-Jun-1999' | $49.4032]$ |
| '02-Jul-1999' | $49.9485]$ |
| '09-Jul-1999' | $47.8061]$ |
| '16-Jul-1999' | $61.0517]$ |
| '23-Jul-1999' | $58.9313]$ |
| '30-Jul-1999' | $53.9584]$ |
| '06-Aug-1999' | 44.8472 ] |
| '13-Aug-1999' | 45.0463 ] |
| '20-Aug-1999' | 45.1088 ] |
| '27-Aug-1999' | $56.4897]$ |
| '03-Sep-1999' | $61.2449]$ |
| '10-Sep-1999' | $58.1012]$ |
| '17-Sep-1999' | $50.8974]$ |
| '24-Sep-1999' | $46.5143]$ |
| '01-0ct-1999' | $38.0806]$ |
| '08-0ct-1999' | $33.6664]$ |
| '15-0ct-1999' | $34.2992]$ |
| '22-0ct-1999' | $33.4202]$ |
| '29-0ct-1999' | $36.9287]$ |
| ' 05-Nov-1999' | 35.1278 ] |
| ' 12-Nov-1999' | 41.8128 ] |
| '19-Nov-1999' | $35.8199]$ |
| '26-Nov-1999' | $36.9495]$ |
| '03-Dec-1999' | $36.2880]$ |
| '10-Dec-1999' | $33.8457]$ |
| '17-Dec-1999' | 33.3868 ] |

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| '24-Dec-1999' | $[$ | $32.7737]$ |
| :---: | :---: | :---: |
| '31-Dec-1999' | $[\quad 28.5665]$ |  |

Use todaily to obtain the daily aggregate for the $x 0$ times series.

```
xl = todaily(x0)
```

Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in

```
x1 =
    desc: TODAILY: Index
    freq: Daily (1)
    'dates: (252)' 'Metric: (252)
    '04-Jan-1999' [ NaN]
    '05-Jan-1999' [ NaN]
    '06-Jan-1999' [ NaN]
    '07-Jan-1999' [ NaN]
    '08-Jan-1999' [ 97.0847]
    '11-Jan-1999' [ NaN]
    '12-Jan-1999' [ NaN]
    '13-Jan-1999' [ NaN]
    '14-Jan-1999' [ NaN]
    '15-Jan-1999' [ 109.6312]
    '19-Jan-1999' [ NaN]
    '20-Jan-1999'
    '21-Jan-1999'
    '22-Jan-1999'
    '25-Jan-1999'
    '26-Jan-1999'
    '27-Jan-1999'
    '28-Jan-1999'
    '29-Jan-1999'
    '01-Feb-1999'
    ' 02-Feb-1999'
    '03-Feb-1999'
    '04-Feb-1999'
    '05-Feb-1999'
    ' 08-Feb-1999'
    ' 09-Feb-1999'
    ' 10-Feb-1999'
    '11-Feb-1999'
    ' 12-Feb-1999'
\begin{tabular}{|c|c|}
\hline 'Metric: & c: (252)' \\
\hline [ & NaN ] \\
\hline [ & NaN ] \\
\hline [ & NaN ] \\
\hline [ & \(\mathrm{NaN}]\) \\
\hline [ & 97.0847 ] \\
\hline [ & \(\mathrm{NaN}]\) \\
\hline [ & \(\mathrm{NaN}]\) \\
\hline [ & NaN ] \\
\hline [ & \(\mathrm{NaN}]\) \\
\hline 1 & 109.6312] \\
\hline [ & \(\mathrm{NaN}]\) \\
\hline [ & \(\mathrm{NaN}]\) \\
\hline [ & \(\mathrm{NaN}]\) \\
\hline 1 & 105.5743] \\
\hline [ & NaN ] \\
\hline [ & NaN ] \\
\hline [ & NaN ] \\
\hline [ & \(\mathrm{NaN}]\) \\
\hline 1 & 108.4028] \\
\hline [ & NaN ] \\
\hline [ & NaN ] \\
\hline [ & NaN ] \\
\hline [ & NaN ] \\
\hline 1 & 134.4882] \\
\hline [ & NaN] \\
\hline [ & NaN ] \\
\hline [ & \(\mathrm{NaN}]\) \\
\hline [ & \(\mathrm{NaN}]\) \\
\hline 1 & 117.5581] \\
\hline
\end{tabular}
```

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| ' 20-Apr-1999' | NaN ] |
| :---: | :---: |
| '21-Apr-1999' | NaN ] |
| '22-Apr-1999' | NaN ] |
| '23-Apr-1999' | $91.5625]$ |
| '26-Apr-1999' | NaN ] |
| '27-Apr-1999' | NaN ] |
| '28-Apr-1999' | NaN ] |
| '29-Apr-1999' | $\mathrm{NaN}]$ |
| '30-Apr-1999' | 78.5738] |
| '03-May-1999' | NaN ] |
| '04-May-1999' | NaN ] |
| ' 05-May-1999' | NaN ] |
| '06-May-1999' | $\mathrm{NaN}]$ |
| ' 07-May-1999' | $65.2904]$ |
| ' 10-May-1999' | NaN ] |
| '11-May-1999' | NaN ] |
| ' 12-May-1999' | NaN ] |
| '13-May-1999' | NaN ] |
| '14-May-1999' | 70.8581] |
| '17-May-1999' | NaN ] |
| '18-May-1999' | NaN ] |
| ' 19-May-1999' | NaN ] |
| '20-May-1999' | $\mathrm{NaN}]$ |
| '21-May-1999' | 72.4807] |
| '24-May-1999' | $\mathrm{NaN}]$ |
| '25-May-1999' | $\mathrm{NaN}]$ |
| '26-May-1999' | $\mathrm{NaN}]$ |
| '27-May-1999' | $\mathrm{NaN}]$ |
| '28-May-1999' | 72.9190] |
| '01-Jun-1999' | NaN ] |
| ' 02-Jun-1999' | NaN ] |
| '03-Jun-1999' | $\mathrm{NaN}]$ |
| '04-Jun-1999' | $64.3460]$ |
| '07-Jun-1999' | NaN ] |
| '08-Jun-1999' | NaN ] |
| '09-Jun-1999' | NaN ] |
| '10-Jun-1999' | NaN ] |
| '11-Jun-1999' | $59.8743]$ |
| '14-Jun-1999' | NaN ] |
| '15-Jun-1999' | NaN ] |
| '16-Jun-1999' | NaN ] |
| '17-Jun-1999' | NaN ] |
| '18-Jun-1999' | $55.0026]$ |
| '21-Jun-1999' | NaN ] |

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| ' 22-Jun-1999' | NaN] |
| :---: | :---: |
| ' 23-Jun-1999' | NaN ] |
| '24-Jun-1999' | NaN ] |
| '25-Jun-1999' | 49.4032] |
| '28-Jun-1999' | NaN ] |
| ' 29-Jun-1999' | NaN ] |
| '30-Jun-1999' | NaN ] |
| '01-Jul-1999' | NaN ] |
| '02-Jul-1999' | $49.9485]$ |
| '06-Jul-1999' | NaN ] |
| ' 07-Jul-1999' | NaN ] |
| ' 08-Jul-1999' | NaN ] |
| '09-Jul-1999' | $47.8061]$ |
| '12-Jul-1999' | NaN ] |
| '13-Jul-1999' | NaN ] |
| '14-Jul-1999' | NaN ] |
| '15-Jul-1999' | NaN ] |
| '16-Jul-1999' | $61.0517]$ |
| '19-Jul-1999' | NaN ] |
| '20-Jul-1999' | NaN ] |
| '21-Jul-1999' | NaN ] |
| '22-Jul-1999' | NaN ] |
| '23-Jul-1999' | $58.9313]$ |
| '26-Jul-1999' | NaN ] |
| '27-Jul-1999' | NaN ] |
| ' 28-Jul-1999' | NaN ] |
| ' 29-Jul-1999' | $\mathrm{NaN}]$ |
| '30-Jul-1999' | $53.9584]$ |
| ' 02-Aug-1999' | $\mathrm{NaN}]$ |
| ' 03-Aug-1999' | NaN ] |
| ' 04-Aug-1999' | $\mathrm{NaN}]$ |
| ' 05-Aug-1999' | NaN ] |
| ' 06-Aug-1999' | 44.8472 ] |
| ' 09-Aug-1999' | $\mathrm{NaN}]$ |
| '10-Aug-1999' | NaN ] |
| '11-Aug-1999' | NaN ] |
| '12-Aug-1999' | NaN ] |
| '13-Aug-1999' | 45.0463 ] |
| '16-Aug-1999' | NaN ] |
| '17-Aug-1999' | NaN ] |
| '18-Aug-1999' | NaN ] |
| '19-Aug-1999' | $\mathrm{NaN}]$ |
| ' 20-Aug-1999' | 45.1088] |
| '23-Aug-1999' | NaN ] |

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| ' 24-Aug-1999' | NaN ] |
| :---: | :---: |
| ' 25-Aug-1999' | NaN] |
| ' 26-Aug-1999' | NaN ] |
| ' 27-Aug-1999' | $56.4897]$ |
| '30-Aug-1999' | NaN] |
| '31-Aug-1999' | NaN] |
| '01-Sep-1999' | NaN] |
| '02-Sep-1999' | NaN ] |
| '03-Sep-1999' | $61.2449]$ |
| '07-Sep-1999' | NaN ] |
| ' 08-Sep-1999' | NaN ] |
| ' 09-Sep-1999' | $\mathrm{NaN}]$ |
| ' 10-Sep-1999' | $58.1012]$ |
| '13-Sep-1999' | NaN ] |
| '14-Sep-1999' | NaN ] |
| '15-Sep-1999' | NaN ] |
| '16-Sep-1999' | NaN ] |
| '17-Sep-1999' | $50.8974]$ |
| '20-Sep-1999' | NaN ] |
| '21-Sep-1999' | NaN ] |
| '22-Sep-1999' | NaN ] |
| '23-Sep-1999' | NaN ] |
| '24-Sep-1999' | 46.5143 ] |
| '27-Sep-1999' | NaN ] |
| '28-Sep-1999' | NaN ] |
| '29-Sep-1999' | NaN ] |
| ' 30-Sep-1999' | $\mathrm{NaN}]$ |
| '01-0ct-1999' | $38.0806]$ |
| ' 04-0ct-1999' | NaN ] |
| ' 05-0ct-1999' | NaN ] |
| ' 06-0ct-1999' | NaN ] |
| '07-0ct-1999' | NaN ] |
| ' 08-0ct-1999' | $33.6664]$ |
| '11-0ct-1999' | NaN ] |
| '12-0ct-1999' | NaN ] |
| '13-0ct-1999' | NaN ] |
| '14-0ct-1999' | NaN ] |
| '15-0ct-1999' | $34.2992]$ |
| '18-0ct-1999' | NaN ] |
| '19-0ct-1999' | NaN ] |
| '20-0ct-1999' | NaN ] |
| '21-0ct-1999' | NaN ] |
| '22-0ct-1999' | $33.4202]$ |
| '25-0ct-1999' | $\mathrm{NaN}]$ |


| '26-0ct-1999' | NaN ] |
| :---: | :---: |
| '27-0ct-1999' | NaN ] |
| '28-0ct-1999' | NaN ] |
| '29-0ct-1999' | 36.9287 ] |
| '01-Nov-1999' | $\mathrm{NaN}]$ |
| '02-Nov-1999' | NaN ] |
| '03-Nov-1999' | NaN ] |
| '04-Nov-1999' | NaN ] |
| ' 05-Nov-1999' | 35.1278 ] |
| '08-Nov-1999' | NaN ] |
| '09-Nov-1999' | NaN ] |
| '10-Nov-1999' | NaN ] |
| '11-Nov-1999' | NaN ] |
| ' 12-Nov-1999' | $41.8128]$ |
| '15-Nov-1999' | NaN ] |
| '16-Nov-1999' | NaN ] |
| '17-Nov-1999' | NaN ] |
| '18-Nov-1999' | NaN ] |
| ' 19-Nov-1999' | $35.8199]$ |
| '22-Nov-1999' | NaN ] |
| ' 23 -Nov-1999' | NaN ] |
| ' 24 -Nov-1999' | NaN ] |
| ' 26 -Nov-1999' | $36.9495]$ |
| '29-Nov-1999' | NaN ] |
| '30-Nov-1999' | NaN ] |
| '01-Dec-1999' | NaN ] |
| '02-Dec-1999' | $\mathrm{NaN}]$ |
| '03-Dec-1999' | $36.2880]$ |
| '06-Dec-1999' | NaN ] |
| '07-Dec-1999' | NaN ] |
| '08-Dec-1999' | NaN ] |
| '09-Dec-1999' | NaN ] |
| '10-Dec-1999' | $33.8457]$ |
| '13-Dec-1999' | NaN ] |
| '14-Dec-1999' | NaN ] |
| '15-Dec-1999' | NaN ] |
| '16-Dec-1999' | $\mathrm{NaN}]$ |
| '17-Dec-1999' | 33.3868 ] |
| '20-Dec-1999' | NaN ] |
| '21-Dec-1999' | NaN ] |
| '22-Dec-1999' | NaN ] |
| '23-Dec-1999' | NaN ] |
| '27-Dec-1999' | NaN ] |
| '28-Dec-1999' | NaN ] |

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```
'29-Dec-1999'
'30-Dec-1999'
'31-Dec-1999'
```

NaN]
NaN]
$28.5665]$

## See Also

convertto|fints|toannual|tomonthly| toquarterly| tosemi | toweekly

## Topics

"Data Transformation and Frequency Conversion" on page 13-13

## Introduced before R2006a

## today

Current date

## Syntax

```
Date = today
Date = today(outputType)
```


## Description

Date $=$ today returns the current date as a serial date number.
Date $=$ today(outputType) returns the current date using an optional outputType. The type of output is determined by an optional outputType variable input.

## Examples

## Return the Current Date

Use today to return the current date with the default serial date number.

```
Date = today
```

Date $=737299$

Use the optional argument outputType to return a datetime array.

```
Date = today('datetime')
Date = datetime
    27-Aug-2018
```


## Input Arguments

## outputType - Type of output

## 'datenum' (default)| character vector with values 'datetime' or 'datenum'

Type of output, specified as a character vector with values 'datetime' or 'datenum'.
If outputType is 'datenum', then Date is a serial date number. If outputType is 'datetime', then Date is a datetime array. By default, outputType is 'datenum'.

## Data Types: char

## Output Arguments

## Date - Current date

serial date number or datetime array
Current date, returned as a serial date number or datetime array, depending on the optional input argument outputType.

## See Also

datenum|datestr|datetime|now

## Topics

"Data Transformation and Frequency Conversion" on page 13-13

## Introduced before R2006a

## todecimal

Fractional to decimal conversion

## Syntax

```
usddec = todecimal(quote,fracpart)
```


## Description

usddec = todecimal(quote,fracpart) returns the decimal equivalent, usddec, of a security whose price is normally quoted as a whole number and a fraction (quote).
fracpart indicates the fractional base (denominator) with which the security is normally quoted (default = 32).

## Examples

In the Wall Street Journal, bond prices are quoted in fractional form based on a denominator of 32 . For example, if you see the quoted price is $100: 05$ it means $1005 / 32$. To find the equivalent decimal value, enter

```
usddec = todecimal(100.05)
usddec =
    100.1563
usddec = todecimal(97.04, 16)
usddec =
    97.2500
```

Note The convention of using . (period) as a substitute for : (colon) in the input is adopted from Excel software.
See Alsotoquoted
Topics"Data Transformation and Frequency Conversion" on page 13-13
Introduced before R2006a

## tomonthly

Convert to monthly

Note tomonthly will be removed in a future release. Use timetable instead. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

```
newfts = tomonthly(oldfts)
newfts = tomonthly(oldfts,'ParameterName',ParameterValue, ...)
```


## Arguments

| oldfts | Financial time series object. |
| :--- | :--- |

## Description

newfts = tomonthly(oldfts) converts a financial time series of any frequency to a monthly frequency. The default end-of-month day is the last business day of the month. tomonthly uses holidays.m to determine valid trading days.

Note If oldfts contains time-of-day information, newfts displays the time-of-day as 00:00 for those days that did not previously exist in oldfts.

Empty ([ ]) passed as inputs for parameter pair values for tomonthly triggers the use of the defaults.
newfts = tomonthly(oldfts,'ParameterName',ParameterValue, ...) accepts parameter name/parameter value pairs as input, as specified in the following table.

| Parameter Name | Parameter Value | Description |
| :---: | :---: | :---: |
| CalcMethod | CumSum | Returns the cumulative sum of the values within each month. Data for missing dates are given the value 0 . |
|  | Exact | Returns the exact value at the end-of-month date. No data manipulation occurs. |
|  | Nearest | (Default) Returns the values located at the end-ofmonth date. If there is missing data, 'Nearest' returns the nearest data point preceding the end-of-month date. |
|  | SimpAvg | Returns an averaged monthly value that only takes into account dates with data (non-NaN) within each month. |
|  | v21x | This mode is compatible with previous versions of this function (Version 2.1. $x$ and earlier). It returns an averaged end-of-month value using a previous tomonthly algorithm. This algorithm takes into account all dates and data. For dates that do not contain any data, the data is assumed to be 0 . |
|  | Note If you set CalcMethod to v21x, settings for all the following parameter name/parameter value pairs are not supported. |  |
| BusDays | 0 | Generates a monthly financial time series that ranges from the first date to the last date in oldfts (includes NYSE nonbusiness days and holidays). |
|  | 1 | (Default) Generates a monthly financial time series that ranges from the first date to the last date in oldfts (excludes NYSE nonbusiness days and holidays and weekends based on AltHolidays and Weekend). If an end-of-month date falls on a nonbusiness day or NYSE holiday, returns the last business day of the month. |
|  |  | NYSE market closures, holidays, and weekends are observed if AltHolidays and Weekend are not supplied or empty ([ ]). |


| Parameter Name | Parameter Value | Description |
| :---: | :---: | :---: |
| DateFilter | Absolute | (Default) Returns all monthly dates between the start and end dates of oldfts. Some dates may be disregarded if BusDays $=1$. |
|  |  | Note The default is to create a time series with every date at the specified periodicity, which is with DateFilter = Absolute. If you use DateFilter = Relative, the endpoint effects do not apply since only your data defines which dates appear in the output time series object. |
|  | Relative | Returns only monthly dates that exist in oldfts. Some dates may be disregarded if BusDays $=1$. |
| ED | 0 | (Default) The end-of-month date is the last day (or last business day) of the month. |
|  | 1-31 | Returns values on the specified end-of-month day. Months that do not contain the specified end-ofmonth day return the last day of the month instead (for example, ED = 31 does not exist for February). |
|  |  | If end-of-month falls on a NYSE non-business day or holiday, the previous business day is returned if BusDays = 1 . |


| Parameter Name | Parameter Value | Description |
| :---: | :---: | :---: |
| EndPtTol | [Begin, End] | Denotes the minimum number of days that constitute an odd month at the end points of the time series (before the first whole period and after the last whole period). |
|  |  | Begin and End must be -1 or any positive integer greater than or equal to 0 . |
|  |  | A single value input for EndPtTol is the same as specifying that single value for Begin and End. |
|  |  | - 1 Do not include odd month dates and data in calculations. |
|  |  | 0 (Default) Include all odd month dates and data in calculations. |
|  |  | n Number of days that constitute an odd month. If the minimum number of days is not met, the odd month dates and data are ignored. |
| The followin of end points | ram is a gen his function. | depiction of the factors involved in the determination |


$\begin{array}{ll}\text { TimeSpec First } & \begin{array}{l}\text { Returns only the observation that occurs at the first } \\ \text { (earliest) time for a specific date. } \\ \text { (Default) Returns only the observation that occurs at } \\ \text { the last (latest) time for a specific date. }\end{array}\end{array}$

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| Parameter Name | Parameter Value | Description |
| :---: | :---: | :---: |
| AltHolidays |  | Vector of dates specifying an alternate set of market closure dates. |
|  | -1 | Excludes all holidays. |
| Weekend |  | Vector of length 7 containing 0 's and 1 's. The value 1 indicates a weekend day. The first element of this vector corresponds to Sunday. For example, when Saturday and Sunday are weekend days (default) then Weekend $=\left[\begin{array}{lllllll}1 & 0 & 0 & 0 & 0 & 0 & 1\end{array}\right]$. |

## Examples

## Transform Time Series Object from Weekly to Monthly Values

This example shows how to transform a time series object from weekly to monthly values.
Load the data from the file predict_ret_data.mat and use the fints function to create a time series object with a weekly frequency.
load predict_ret data.mat
x0 = fints(expdā̄es, expdata, \{'Metric'\}, 'w', 'Index')
Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in
$x 0=$
desc: Index
freq: Weekly (2)
'dates: (53)' 'Metric: (53)'
'01-Jan-1999' [ 97.8872]
'08-Jan-1999' [ 97.0847]
'15-Jan-1999'
109.6312]
'22-Jan-1999'
'29-Jan-1999'
'05-Feb-1999'
105.5743]
108.4028]
'12-Feb-1999'
134.4882]
117.5581]

| ' 19-Feb-1999 ' | 106.6683] |
| :---: | :---: |
| '26-Feb-1999' | 118.2912] |
| '05-Mar-1999' | 105.6835] |
| '12-Mar-1999' | 128.5836] |
| '19-Mar-1999' | $115.1746]$ |
| '26-Mar-1999' | [ 131.2854] |
| '02-Apr-1999' | 130.7116] |
| '09-Apr-1999' | [ 123.1684] |
| '16-Apr-1999' | 107.2975] |
| '23-Apr-1999' | $91.5625]$ |
| '30-Apr-1999' | $78.5738]$ |
| '07-May-1999' | $65.2904]$ |
| '14-May-1999' | 70.8581] |
| '21-May-1999' | $72.4807]$ |
| ' 28-May-1999' | $72.9190]$ |
| '04-Jun-1999' | [ 64.3460] |
| '11-Jun-1999' | [ 59.8743] |
| ' 18-Jun-1999' | [ 55.0026] |
| ' 25-Jun-1999' | [ 49.4032] |
| '02-Jul-1999' | [ 49.9485] |
| '09-Jul-1999' | $47.8061]$ |
| '16-Jul-1999' | [ 61.0517] |
| ' 23-Jul-1999' | [ 58.9313] |
| '30-Jul-1999' | [ 53.9584] |
| '06-Aug-1999' | [ 44.8472] |
| ' 13-Aug-1999' | $45.0463]$ |
| ' 20-Aug-1999' | 45.1088 ] |
| '27-Aug-1999' | [ 56.4897] |
| '03-Sep-1999' | $61.2449]$ |
| ' 10-Sep-1999' | [ 58.1012] |
| '17-Sep-1999' | $50.8974]$ |
| '24-Sep-1999' | [ 46.5143] |
| '01-0ct-1999' | [ 38.0806] |
| '08-0ct-1999' | [ 33.6664] |
| '15-0ct-1999' | [ 34.2992] |
| '22-0ct-1999' | $33.4202]$ |
| '29-0ct-1999' | $36.9287]$ |
| '05-Nov-1999' | [ 35.1278] |
| ' 12-Nov-1999' | 41.8128 ] |
| '19-Nov-1999' | $35.8199]$ |
| '26-Nov-1999' | $36.9495]$ |
| '03-Dec-1999' | $36.2880]$ |
| ' 10-Dec-1999' | $33.8457]$ |
| '17-Dec-1999' | 33.3868 ] |

```
'24-Dec-1999' [ 32.7737]
'31-Dec-1999'
28.5665]
```

Use tomonthly to obtain the monthly aggregate for the $x 0$ times series.

```
x1 = tomonthly(x0)
```

Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in
x1 =
desc: TOMONTHLY: Index
freq: Monthly (3)
'dates: (12)' 'Metric: (12)'
'29-Jan-1999' [ 108.4028]
'26-Feb-1999' [ 118.2912]
'31-Mar-1999' [ 131.2854]
'30-Apr-1999' [ 78.5738]
'28-May-1999' [ 72.9190]
'30-Jun-1999' [ 49.4032]
'30-Jul-1999' [ 53.9584]
'31-Aug-1999' [ 56.4897]
'30-Sep-1999' [ 46.5143]
'29-0ct-1999' [ 36.9287]
'30-Nov-1999' [ 36.9495]
'31-Dec-1999' [ 28.5665]

## See Also

convertto|fints|toannual|todaily|toquarterly|tosemi|toweekly

## Topics

"Data Transformation and Frequency Conversion" on page 13-13

## Introduced before R2006a

## toquarterly

Convert to quarterly

Note toquarterly will be removed in a future release. Use timetable instead. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

```
newfts = toquarterly(oldfts)
newfts = toquarterly(oldfts,'ParameterName',ParameterValue, ...)
```


## Arguments

oldfts $\quad$ Financial time series object

## Description

newfts = toquarterly(oldfts) converts a financial time series of any frequency to a quarterly frequency. The default quarterly days are the last business day of March, June, September, and December. toquarterly uses holidays.m to determine valid trading days.

Note If oldfts contains time-of-day information, newfts displays the time-of-day as 00:00 for those days that did not previously exist in oldfts.

Empty ([ ]) passed as inputs for parameter pair values for toquarterly triggers the use of the defaults.
newfts = toquarterly(oldfts,'ParameterName',ParameterValue, ...) accepts parameter name/parameter value pairs as input, as specified in the following table.

| Parameter Name | Parameter Value | Description |
| :---: | :---: | :---: |
| CalcMethod | CumSum | Returns the cumulative sum of the values between each quarter. Data for missing dates are given the value 0. |
|  | Exact | Returns the exact value at the end-of-quarter date. No data manipulation occurs. |
|  | Nearest | (Default) Returns the values located at the end-ofquarter date. If there is missing data, Nearest returns the nearest data point preceding the end-ofquarter date. |
|  | SimpAvg | Returns an averaged quarterly value that only takes into account dates with data (non-NaN) within each quarter. |
|  | v21x | This mode is compatible with previous versions of this function (Version 2.1.x and earlier). It returns an averaged end-of-quarter value using a previous toquarterly algorithm. This algorithm takes into account all dates and data. For dates that do not contain any data, the data is assumed to be 0 . |
|  | Note If you set CalcMethod to v21x, settings for all the following parameter name/parameter value pairs are not supported. |  |
| BusDays | 0 | Generates a financial time series that ranges from (or between) the first date to the last date in oldfts (includes NYSE nonbusiness days and holidays). |
|  | 1 | (Default) Generates a financial time series that ranges from the first date to the last date in oldfts (excludes NYSE nonbusiness days and holidays and weekends based on AltHolidays and Weekend). If an end-of-quarter date falls on a nonbusiness day or NYSE holiday, returns the last business day of the quarter. |
|  |  | NYSE market closures, holidays, and weekends are observed if AltHolidays and Weekend are not supplied or empty ([]). |


| Parameter Name | Parameter Value | Description |
| :---: | :---: | :---: |
| DateFilter | Absolute | (Default) Returns all quarterly dates between the start and end dates of oldfts. Some dates may be disregarded if BusDays $=1$. |
|  |  | Note The default is to create a time series with every date at the specified periodicity, which is with DateFilter = Absolute. If you use DateFilter = Relative, the endpoint effects do not apply since only your data defines which dates appear in the output time series object. |
|  | Relative | Returns only quarterly dates that exist in oldfts. Some dates may be disregarded if BusDays $=1$. |
| ED | 0 | (Default) The end-of-quarter date is the last day (or last business day) of the quarter. |
|  | 1-31 | Specifies a particular end-of-quarter day. Months that do not contain the specified end-of-quarter day return the last day of the quarter instead (for example, ED $=31$ does not exist for February). |
| EM | 1-12 | Last month of the first quarter. All subsequent quarterly dates are based on this month. The default end-of-first-quarter month is March (3). |


| Parameter Name | Parameter Value | Description |
| :---: | :---: | :---: |
| EndPtTol | [Begin, End] | Denotes the minimum number of days that constitute an odd quarter at the endpoints of the time series (before the first whole period and after the last whole period). |
|  |  | Begin and End must be -1 or any positive integer greater than or equal to 0 . |
|  |  | A single value input for EndPtTol is the same as specifying that single value for Begin and End. |
|  |  | -1 Do not include odd quarter dates and data in calculations. |
|  |  | 0 (Default) Include all odd quarter dates and data in calculations. |
|  |  | n Number of days (any positive integer) that constitute an odd quarter. If there are insufficient days for a complete quarter, the odd quarter dates and data are ignored. |
| The followin | gram is a gen | depiction of the factors involved in the determination |



TimeSpec First Returns only the observation that occurs at the first (earliest) time for a specific date.

| Parameter <br> Name | Parameter <br> Value | Description |
| :--- | :--- | :--- |
|  | Last | (Default) Returns only the observation that occurs at <br> the last (latest) time for a specific date. |
| AltHolidays |  | Vector of dates specifying an alternate set of market <br> closure dates. |
| Weekend | -1 | Excludes all holidays. <br> Vector of length 7 containing 0's and 1's. The value 1 <br> indicates a weekend day. The first element of this <br> vector corresponds to Sunday. For example, when <br> Saturday and Sunday are weekend days (default) then <br> Weekend $=\left[\begin{array}{llllll}1 & 0 & 0 & 0 & 0 & 1\end{array}\right]$. |

## Examples

## Transform Time Series Object from Weekly to Quarterly Values

This example shows how to transform a time series object from weekly to quarterly values.

Load the data from the file predict_ret_data.mat and use the fints function to create a time series object with a weekly frequency.
load predict_ret_data.mat

```
x0 = fints(expda\overline{tes, expdata, {'Metric'}, 'w', 'Index')}
```

Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in

```
x0 =
```

    desc: Index
    freq: Weekly (2)
    'dates: (53)' 'Metric: (53)'
    '01-Jan-1999' [ 97.8872]
    '08-Jan-1999' [ 97.0847]
    '15-Jan-1999' [ 109.6312]
    '22-Jan-1999' [ 105.5743]
    | ' 29-Jan-1999 ' | 108.4028] |
| :---: | :---: |
| '05-Feb-1999' | $134.4882]$ |
| ' 12-Feb-1999' | 117.5581] |
| ' 19-Feb-1999' | 106.6683] |
| ' 26-Feb-1999 ' | 118.2912] |
| '05-Mar-1999' | 105.6835] |
| '12-Mar-1999' | 128.5836] |
| ' 19-Mar-1999' | $115.1746]$ |
| ' 26-Mar-1999' | 131.2854] |
| ' 02-Apr-1999' | 130.7116] |
| ' 09-Apr-1999' | 123.1684] |
| ' 16-Apr-1999' | 107.2975] |
| '23-Apr-1999' | $91.5625]$ |
| '30-Apr-1999' | $78.5738]$ |
| '07-May-1999' | $65.2904]$ |
| '14-May-1999' | $70.8581]$ |
| '21-May-1999' | $72.4807]$ |
| ' 28-May-1999' | $72.9190]$ |
| '04-Jun-1999' | $64.3460]$ |
| '11-Jun-1999' | $59.8743]$ |
| '18-Jun-1999' | $55.0026]$ |
| ' 25-Jun-1999' | $49.4032]$ |
| '02-Jul-1999' | $49.9485]$ |
| '09-Jul-1999' | $47.8061]$ |
| '16-Jul-1999' | $61.0517]$ |
| '23-Jul-1999' | $58.9313]$ |
| ' 30-Jul-1999' | $53.9584]$ |
| ' 06-Aug-1999' | 44.8472 ] |
| ' 13-Aug-1999' | $45.0463]$ |
| ' 20-Aug-1999' | 45.1088 ] |
| '27-Aug-1999' | $56.4897]$ |
| '03-Sep-1999' | $61.2449]$ |
| '10-Sep-1999' | $58.1012]$ |
| '17-Sep-1999' | $50.8974]$ |
| '24-Sep-1999' | $46.5143]$ |
| '01-0ct-1999' | $38.0806]$ |
| '08-0ct-1999' | $33.6664]$ |
| '15-0ct-1999' | $34.2992]$ |
| '22-0ct-1999' | $33.4202]$ |
| ' 29-0ct-1999' | $36.9287]$ |
| ' $05-$ Nov-1999' | 35.1278 ] |
| ' 12-Nov-1999' | 41.8128 ] |
| ' 19-Nov-1999' | $35.8199]$ |
| ' 26 -Nov-1999 ' | $36.9495]$ |

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| '03-Dec-1999' | [ | $36.2880]$ |
| :--- | :--- | :--- |
| '10-Dec-1999' | [ | $33.8457]$ |
| '17-Dec-1999' | [ | $33.3868]$ |
| '24-Dec-1999' | [ | $32.7737]$ |
| '31-Dec-1999' | [ | $28.5665]$ |

Use toquarterly to obtain the quarterly aggregate for the $x 0$ times series.

```
x1 = toquarterly(x0)
```

Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in
x1 =
desc: TOQUARTERLY: Index
freq: Quarterly (4)
'dates: (4)' 'Metric: (4)'
'31-Mar-1999' [ 131.2854]
'30-Jun-1999' [ 49.4032]
'30-Sep-1999' [ 46.5143]
'31-Dec-1999' [ 28.5665]

## Transform Time Series Object from Weekly to Quarterly Values Including NYSE Nonbusiness Days and Holidays

Load the data from the file predict_ret_data.mat and use the fints function to create a time series object with a weekly frequency.
load predict ret data.mat

```
x0 = fints(expdates, expdata, {'Metric'}, 'w', 'Index')
```

Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in
x0 =
desc: Index
freq: Weekly (2)

```
'dates: (53)' 'Metric: (53)'
'01-Jan-1999' [ 97.8872]
```

| ' 08-Jan-1999' | $97.0847]$ |
| :---: | :---: |
| '15-Jan-1999' | $109.6312]$ |
| '22-Jan-1999' | $105.5743]$ |
| '29-Jan-1999' | 108.4028] |
| '05-Feb-1999' | 134.4882] |
| '12-Feb-1999' | $117.5581]$ |
| '19-Feb-1999' | 106.6683] |
| ' 26-Feb-1999' | 118.2912] |
| '05-Mar-1999' | 105.6835] |
| '12-Mar-1999' | $128.5836]$ |
| '19-Mar-1999' | 115.1746] |
| '26-Mar-1999' | [ 131.2854] |
| '02-Apr-1999' | 130.7116] |
| '09-Apr-1999' | [ 123.1684] |
| '16-Apr-1999' | [ 107.2975] |
| '23-Apr-1999' | 91.5625] |
| '30-Apr-1999' | 78.5738 ] |
| '07-May-1999' | $65.2904]$ |
| '14-May-1999' | $70.8581]$ |
| '21-May-1999' | $72.4807]$ |
| '28-May-1999' | $72.9190]$ |
| '04-Jun-1999' | $64.3460]$ |
| '11-Jun-1999' | 59.8743 ] |
| ' 18-Jun-1999' | $55.0026]$ |
| '25-Jun-1999' | $49.4032]$ |
| '02-Jul-1999' | $49.9485]$ |
| '09-Jul-1999' | $47.8061]$ |
| '16-Jul-1999' | $61.0517]$ |
| '23-Jul-1999' | $58.9313]$ |
| '30-Jul-1999' | $53.9584]$ |
| '06-Aug-1999' | 44.8472 ] |
| '13-Aug-1999' | [ 45.0463] |
| '20-Aug-1999' | [ 45.1088] |
| '27-Aug-1999' | 56.4897 ] |
| '03-Sep-1999' | $61.2449]$ |
| '10-Sep-1999' | 58.1012 ] |
| '17-Sep-1999' | $50.8974]$ |
| '24-Sep-1999' | $46.5143]$ |
| '01-0ct-1999' | $38.0806]$ |
| '08-0ct-1999' | $33.6664]$ |
| '15-0ct-1999' | $34.2992]$ |
| '22-0ct-1999' | $33.4202]$ |
| '29-0ct-1999' | $36.9287]$ |
| '05-Nov-1999' | 35.1278 ] |

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Use toquarterly with the optional BusDays argument set to 0 to obtain the quarterly cumulative sums for the x 0 times series that includes NYSE nonbusiness days and holidays.

```
x1 = toquarterly(x0,'CalcMethod','CumSum','Busdays',0)
```

Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in

```
x1 =
    desc: TOQUARTERLY: Index
    freq: Quarterly (4)
    'dates: (4)' 'Metric: (4)'
    '31-Mar-1999' [ 1.4763e+03]
    '30-Jun-1999' [ 1.0415e+03]
    '30-Sep-1999' [ 679.9459]
    '31-Dec-1999' [ 490.9659]
```


## See Also

convertto|fints|toannual|todaily|tomonthly|tosemi|toweekly

## Topics

"Data Transformation and Frequency Conversion" on page 13-13

## Introduced before R2006a

## toquoted

Decimal to fractional conversion

## Syntax

```
quote = toquoted(usddec,fracpart)
```


## Description

quote $=$ toquoted(usddec, fracpart) returns the fractional equivalent, quote, of the decimal figure, usddec, based on the fractional base (denominator), fracpart. The fractional bases are the ones used for quoting equity prices in the United States (denominator $2,4,8,16$, or 32 ). If fracpart is not entered, the denominator 32 is assumed.

## Examples

A United States equity price in decimal form is 101.625. To convert this to fractional form in eighths of a dollar:

```
quote = toquoted(101.625, 8)
quote =
    101.05
```

The answer is interpreted as $1015 / 8$.

Note The convention of using . (period) as a substitute for : (colon) in the output is adopted from Excel software.

## See Also

todecimal

[^17]
## tosemi

Convert to semiannual

Note tosemi will be removed in a future release. Use timetable instead. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

```
newfts = tosemi(oldfts)
newfts = tosemi(oldfts,'ParameterName',ParameterValue, ...)
```


## Arguments

| oldfts | Financial time series object. |
| :--- | :--- |

## Description

newfts $=$ tosemi (oldfts) converts a financial time series of any frequency to a semiannual frequency. The default semiannual days are the last business day of June and December. tosemi uses holidays.m to determine valid trading days.

Note If oldfts contains time-of-day information, newfts displays the time-of-day as 00:00 for those days that did not previously exist in oldfts.

Empty ([ ]) passed as inputs for parameter pair values for tosemi triggers the use of the defaults.
newfts = tosemi(oldfts,'ParameterName', ParameterValue, ...) accepts parameter name/parameter value pairs as input, as specified in the following table.

| Parameter Name | Parameter Value | Description |
| :---: | :---: | :---: |
| CalcMethod | CumSum | Returns the cumulative sum of the values within each semiannual period. Data for missing dates are given the value 0 . |
|  | Exact | Returns the exact value at the end-of-period date. No data manipulation occurs. |
|  | Nearest | (Default) Returns the values located at the end-ofperiod date. If there is missing data, Nearest returns the nearest data point preceding the end-of-period date. |
|  | SimpAvg | Returns an averaged semiannual value that only takes into account dates with data (non-NaN) within each semiannual period. |
|  | v21x | This mode is compatible with previous versions of this function (Version 2.1.x and earlier). It returns an averaged end-of-period value using a previous tosemi algorithm. This algorithm takes into account all dates and data. For dates that do not contain any data, the data is assumed to be 0 . |
|  | Note If you set CalcMethod to v21x, settings for all the following parameter name/parameter value pairs are not supported. |  |
| BusDays | 0 | Generates a financial time series that ranges from (or between) the first date to the last date in oldfts (includes NYSE nonbusiness days and holidays). |
|  | 1 | (Default) Generates a financial time series that ranges from the first date to the last date in oldfts (excludes NYSE nonbusiness days and holidays and weekends based on AltHolidays and Weekend). If an end-of-quarter date falls on a nonbusiness day or NYSE holiday, returns the last business day of the quarter. |
|  |  | NYSE market closures, holidays, and weekends are observed if AltHolidays and Weekend are not supplied or empty ([ ]). |


| Parameter Name | Parameter Value | Description |
| :---: | :---: | :---: |
| DateFilter | Absolute | (Default) Returns all semiannual dates between the start and end dates of oldfts. Some dates may be disregarded if BusDays $=1$. |
|  |  | Note The default is to create a time series with every date at the specified periodicity, which is with DateFilter = Absolute. If you use DateFilter = Relative, the endpoint effects do not apply since only your data defines which dates appear in the output time series object. |
|  | Relative | Returns only semiannual dates that exist in oldfts. Some dates may be disregarded if BusDays $=1$. |
| ED | 0 | (Default) The end-of-period date is the last day (or last business day) of the semiannual period. |
|  | 1-31 | Specifies a particular end-of-period day. Months that do not contain the specified end-of-period day return the last day of the semiannual period instead (for example, ED = 31 does not exist for February). |
| EM | 1-12 | End month of the first semiannual period. All subsequent period dates are based on this month. The default end-of-period months are June (6) and December (12). |


| Parameter Name | Parameter Value | Description |
| :---: | :---: | :---: |
| EndPtTol | [Begin, End] | Denotes the minimum number of days that constitute an odd semiannual period at the endpoints of the time series (before the first whole period and after the last whole period). |
|  |  | Begin and End must be -1 or any positive integer greater than or equal to 0 . |
|  |  | A single value input for EndPtTol is the same as specifying that single value for Begin and End. |
|  |  | -1 Do not include odd period dates and data in calculations. |
|  |  | 0 (Default) Include all odd period dates and data in calculations. |
|  |  | n Number of days (any positive integer) that constitute an odd period. If there are insufficient days for a complete semiannual period, the odd period dates and data are ignored. |
| The followin of endpoints | ram is a gene is function. | depiction of the factors involved in the determination |



TimeSpec First Returns only the observation that occurs at the first (earliest) time for a specific date.

| Parameter <br> Name | Parameter <br> Value | Description |
| :--- | :--- | :--- |
|  | Last | (Default) Returns only the observation that occurs at <br> the last (latest) time for a specific date. |
| AltHolidays |  | Vector of dates specifying an alternate set of market <br> closure dates. |
| Weekend | -1 | Excludes all holidays. <br> Vector of length 7 containing 0's and 1's. The value 1 <br> indicates a weekend day. The first element of this <br> vector corresponds to Sunday. For example, when <br> Saturday and Sunday are weekend days (default) then <br> Weekend $=\left[\begin{array}{llllll}1 & 0 & 0 & 0 & 0 & 1\end{array}\right]$. |

## Examples

## Transform Time Series Object from Weekly to Semiannual Values

This example shows how to transform a time series object from weekly to semiannual values.

Load the data from the file predict_ret_data.mat and use the fints function to create a time series object with a weekly frequency.
load predict_ret_data.mat
x 0 = fints(expdā̄es, expdata, \{'Metric'\}, 'w', 'Index')
Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in
$x 0=$

```
desc: Index
    freq: Weekly (2)
    'dates: (53)' 'Metric: (53)'
    '01-Jan-1999' [ 97.8872]
    '08-Jan-1999' [ 97.0847]
    '15-Jan-1999' [ 109.6312]
    '22-Jan-1999' [ 105.5743]
```

| ' 29-Jan-1999 ' | 108.4028] |
| :---: | :---: |
| ' $05-\mathrm{Feb}-1999$ ' | 134.4882] |
| ' 12-Feb-1999' | 117.5581] |
| ' 19-Feb-1999' | 106.6683] |
| '26-Feb-1999' | 118.2912] |
| '05-Mar-1999' | [ 105.6835] |
| '12-Mar-1999' | 128.5836] |
| ' 19-Mar-1999' | $115.1746]$ |
| ' 26 -Mar-1999' | [ 131.2854] |
| '02-Apr-1999' | 130.7116] |
| ' 09-Apr-1999' | 123.1684] |
| ' 16-Apr-1999' | 107.2975] |
| '23-Apr-1999' | $91.5625]$ |
| '30-Apr-1999' | $78.5738]$ |
| '07-May-1999' | $65.2904]$ |
| '14-May-1999' | $70.8581]$ |
| '21-May-1999' | $72.4807]$ |
| '28-May-1999' | [ 72.9190] |
| '04-Jun-1999' | [ 64.3460] |
| '11-Jun-1999' | [ 59.8743] |
| ' 18-Jun-1999' | $55.0026]$ |
| ' 25-Jun-1999' | $49.4032]$ |
| '02-Jul-1999' | $49.9485]$ |
| '09-Jul-1999' | [ 47.8061] |
| ' 16-Jul-1999' | [ 61.0517] |
| '23-Jul-1999' | $58.9313]$ |
| ' 30-Jul-1999' | $53.9584]$ |
| ' 06-Aug-1999' | [ 44.8472] |
| ' 13-Aug-1999' | $45.0463]$ |
| ' 20-Aug-1999' | [ 45.1088] |
| '27-Aug-1999' | $56.4897]$ |
| '03-Sep-1999' | [ 61.2449] |
| '10-Sep-1999' | [ 58.1012] |
| '17-Sep-1999' | [ 50.8974] |
| '24-Sep-1999' | $46.5143]$ |
| '01-0ct-1999' | [ 38.0806] |
| '08-0ct-1999' | [ 33.6664] |
| '15-0ct-1999' | [ 34.2992] |
| '22-0ct-1999' | $33.4202]$ |
| '29-0ct-1999' | $36.9287]$ |
| '05-Nov-1999' | 35.1278 ] |
| '12-Nov-1999' | 41.8128 ] |
| '19-Nov-1999' | $35.8199]$ |
| '26-Nov-1999' | $36.9495]$ |


| '03-Dec-1999' | $[$ | $36.2880]$ |
| :--- | :--- | :--- |
| '10-Dec-1999' | $[$ | $33.8457]$ |
| '17-Dec-1999' | $[$ | $33.3868]$ |
| '24-Dec-1999' | $[$ | $32.7737]$ |
| '31-Dec-1999' $^{\prime}$ | $[$ | $28.5665]$ |

Use tosemi to obtain the semiannual aggregate for the x 0 times series.

```
x1 = tosemi(x0)
```

Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in
$x 1=$
desc: TOSEMI: Index
freq: Semiannual (5)
'dates: (2)' 'Metric: (2)
'30-Jun-1999' [ 49.4032]
'31-Dec-1999' [ 28.5665]

## See Also

convertto|fints|toannual|todaily|tomonthly|toquarterly|toweekly

## Topics

"Data Transformation and Frequency Conversion" on page 13-13

Introduced before R2006a

## totalreturnprice

Total return price time series

## Syntax

Return = totalreturnprice(Price, Action, Dividend)

## Arguments

| Price | Price can be a table or a NUMOBS-by-2 matrix. If Price is a table, <br> the dates can either be serial date numbers, date character vectors. <br> If Price is a number of observations NUMOBS-by-2 matrix of price <br> data, column 1 contains MATLAB serial date numbers and column 2 <br> contains price values. |
| :--- | :--- |
| Action | Action can be a table or a NUMOBS-by-2 matrix. If Action is a <br> table, the dates can either be serial date numbers, date character <br> vectors. If Action is a NUMOBS-by-2 matrix of price data, column 1 <br> contains MATLAB serial date numbers and column 2 contains split <br> ratios. |
| Dividend | Dividend can be a table or a NUMOBS-by-2 matrix. If Dividend is a <br> table, the dates can either be serial date numbers, date character <br> vectors. If Dividend is a NUMOBS-by-2 matrix of price data, column <br> 1 contains MATLAB serial date numbers and column 2 contains <br> dividend payouts. |

The number of observations (NUMOBS) for the three input arguments differ from each other.

## Description

Return = totalreturnprice(Price,Action, Dividend) generates a total return price time series given price data, action or split data, and dividend data.

If all three inputs are matrices, then Return is a NUMOBS-by-2 array of price data, where NUMOBS reflects the number of observations of price data. Column 1 contains MATLAB serial date numbers. Column 2 contains total return price values.

However, if any inputs are tables, then Return will also be a table. The class of the first column depends on the classes used in the input tables. If any tables used datetimes for dates, then Returns will have datetimes in the first column. If there were no datetimes in the inputs, but if any inputs used date character vectors, then Returns will use date character vectors in the first column. For any other case, serial date numbers are used.

## Examples

## Compute Return Using datetime Input for Price and Action

Compute Return returned as a table using datetime input in tables for Price and Action.

```
act = [732313, 2; 732314 ,2];
div = [732313, 0.0800; 732314, 0.0800];
prc = [732313, 12; 732314, 13];
prcTableDateTime=table(datetime(prc(:,1),'ConvertFrom','datenum'),prc(:,2));
acttableString=table(datestr(act(:,1)),act(:,2));
divTableNum = array2table(div);
Return = totalreturnprice(prcTableDateTime,acttableString,divTableNum)
Return=2\times2 table
    Date Return
    01-Jan-2005 00:00:00 1
02-Jan-2005 00:00:00 1.0833
```


## See Also

datetime|periodicreturns

## Topics

"Portfolio Construction Examples" on page 3-7
"Portfolio Optimization Functions" on page 3-4
Introduced before R2006a

## toweekly

Convert to weekly

Note toweekly will be removed in a future release. Use timetable instead. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

```
newfts = toweekly(oldfts)
newfts = toweekly(oldfts,'ParameterName',ParameterValue, ...)
```


## Arguments

| oldfts | Financial time series object. |
| :--- | :--- |

## Description

newfts = toweekly(oldfts) converts a financial time series of any frequency to a weekly frequency. The default weekly days are Fridays or the last business day of the week. toweekly uses holidays.m to determine valid trading days.

Note If oldfts contains time-of-day information, newfts displays the time-of-day as 00:00 for those days that did not previously exist in oldfts.

Empty ([ ]) passed as inputs for parameter pair values for toweekly triggers the use of the defaults.
newfts = toweekly(oldfts,'ParameterName',ParameterValue, ...) accepts parameter name/parameter value pairs as input, as specified in the following table.

| Parameter Name | Parameter Value | Description |
| :---: | :---: | :---: |
| CalcMethod | CumSum | Returns the cumulative sum of the values within each week. Data for missing dates are given the value 0 . |
|  | Exact | Returns the exact value at the end-of-week dates. No data manipulation occurs. |
|  | Nearest | (Default) Returns the values located at the end-ofweek dates. If there is missing data, Nearest returns the nearest data point preceding the end-of-week date. |
|  | SimpAvg | Returns an averaged weekly value that only takes into account dates with data (non-NaN) within each week. |
|  | v21x | This mode is compatible with previous versions of this function (Version 2.1.x and earlier). It returns an averaged end-of-weekly value using a previous toquarterly algorithm. This algorithm takes into account all dates and data. For dates that do not contain any data, the data is assumed to be 0 . |
|  | Note If you set CalcMethod to v21x, settings for all the following parameter name/parameter value pairs are not supported. |  |
| BusDays | 0 | Generates a financial time series that ranges from (or between) the first date to the last date in oldfts (includes NYSE nonbusiness days and holidays). |
|  | 1 | (Default) Generates a financial time series that ranges from the first date to the last date in oldfts (excludes NYSE nonbusiness days and holidays and weekends based on AltHolidays and Weekend). If an end-of-quarter date falls on a nonbusiness day or NYSE holiday, returns the last business day of the quarter. |
|  |  | NYSE market closures, holidays, and weekends are observed if AltHolidays and Weekend are not supplied or empty ([]). |


| Parameter Name | Parameter Value | Description |
| :---: | :---: | :---: |
| DateFilter | Absolute | (Default) Returns all weekly dates between the start and end dates of oldfts. Some dates may be disregarded if BusDays $=1$. |
|  |  | Note The default is to create a time series with every date at the specified periodicity, which is with DateFilter = Absolute. If you use DateFilter = Relative, the endpoint effects do not apply since only your data defines which dates appear in the output time series object. |
|  | Relative | Returns only end-of-week dates that exist in oldfts. Some dates may be disregarded if BusDays = 1 . |
| EndPtTol | [Begin, End] | Denotes the minimum number of days that constitute an odd week at the endpoints of the time series (before the first whole period and after the last whole period). |
|  |  | Begin and End must be -1 or any positive integer greater than or equal to 0 . |
|  |  | A single value input for EndPtTol is the same as specifying that single value for Begin and End. |
|  |  | -1 Do not include odd week dates and data in calculations. |
|  |  | 0 (Default) Include all odd week dates and data in calculations. |
|  |  | n Number of days (any positive integer) that constitute an odd week. If there are insufficient days for a complete week, the odd week dates and data are ignored. |


| Parameter |  |
| :--- | :--- |
| Name | Parameter <br> Value |$\quad$ Description

The following diagram is a general depiction of the factors involved in the determination of endpoints for this function.

$\left.\begin{array}{lll}\begin{array}{ll}\text { Parameter } \\ \text { Name }\end{array} & \begin{array}{l}\text { Parameter } \\ \text { Value }\end{array} & \text { Description } \\ \text { Weekend } & & \begin{array}{l}\text { Vector of length } 7 \text { containing 0's and 1's. The value } 1 \\ \text { indicates a weekend day. The first element of this } \\ \\ \\ \end{array} \\ & & \text { Sector corresponds to Sunday. For example, when }\end{array}\right\}$

## Examples

## Transform Time Series Object from Quarterly to Weekly Values

This example shows how to transform a time series object from quarterly to weekly values.

Load the data from the file predict_ret_data.mat and use the fints function to create a time series object with a quarterly frequency.

```
load predict_ret_data.mat
x0 = fints(expdates, expdata, {'Metric'}, 'q', 'Index')
```

Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in
$x 0=$
desc: Index
freq: Quarterly (4)

| 'dates: (53)' | 'Metric: (53)' |  |
| :--- | :--- | ---: |
| '01-Jan-1999' | [ | $97.8872]$ |
| '08-Jan-1999' | [ | $97.0847]$ |
| '15-Jan-1999' | [ | $109.6312]$ |
| '22-Jan-1999' | [ | $105.5743]$ |
| '29-Jan-1999' | [ | $108.4028]$ |
| '12-Feb-1999' | [ | $134.4882]$ |
| '19-Feb-1999' | [ | $117.5581]$ |
| '26-Feb-1999' | [ | $106.6683]$ |
| '05-Mar-1999' | [ | $118.2912]$ |


| '12-Mar-1999' | 128.5836] |
| :---: | :---: |
| '19-Mar-1999' | $115.1746]$ |
| '26-Mar-1999' | 131.2854] |
| '02-Apr-1999' | 130.7116] |
| '09-Apr-1999' | 123.1684] |
| '16-Apr-1999' | 107.2975] |
| '23-Apr-1999' | $91.5625]$ |
| '30-Apr-1999' | $78.5738]$ |
| '07-May-1999' | $65.2904]$ |
| '14-May-1999' | 70.8581] |
| '21-May-1999' | $72.4807]$ |
| '28-May-1999' | $72.9190]$ |
| '04-Jun-1999' | $64.3460]$ |
| ' 11-Jun-1999' | 59.8743 ] |
| ' 18-Jun-1999' | $55.0026]$ |
| '25-Jun-1999' | $49.4032]$ |
| '02-Jul-1999' | 49.9485 ] |
| '09-Jul-1999' | $47.8061]$ |
| '16-Jul-1999' | $61.0517]$ |
| '23-Jul-1999' | 58.9313] |
| '30-Jul-1999' | $53.9584]$ |
| '06-Aug-1999' | 44.8472 ] |
| ' 13-Aug-1999' | 45.0463 ] |
| '20-Aug-1999' | 45.1088 ] |
| '27-Aug-1999' | $56.4897]$ |
| '03-Sep-1999' | $61.2449]$ |
| ' 10-Sep-1999' | $58.1012]$ |
| '17-Sep-1999 ' | $50.8974]$ |
| '24-Sep-1999' | $46.5143]$ |
| '01-0ct-1999' | $38.0806]$ |
| '08-0ct-1999' | $33.6664]$ |
| '15-0ct-1999' | $34.2992]$ |
| '22-0ct-1999' | $33.4202]$ |
| '29-0ct-1999' | $36.9287]$ |
| '05-Nov-1999' | 35.1278 ] |
| ' 12-Nov-1999' | 41.8128 ] |
| ' 19-Nov-1999' | $35.8199]$ |
| '26-Nov-1999' | $36.9495]$ |
| '03-Dec-1999' | $36.2880]$ |
| '10-Dec-1999' | $33.8457]$ |
| '17-Dec-1999' | 33.3868 ] |
| '24-Dec-1999' | $32.7737]$ |
| '31-Dec-1999' | $28.5665]$ |

Use toweekly to obtain the weekly aggregate for the x 0 times series.

```
x1 = toweekly(x0)
```

Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in

```
x1 =
    desc: TOWEEKLY: Index
    freq: Weekly (2)
    'dates: (53)' 'Metric: (53)'
    '31-Dec-1998' [ NaN]
    '08-Jan-1999' [ 97.0847]
    '15-Jan-1999' [ 109.6312]
    '22-Jan-1999' [ 105.5743]
    '29-Jan-1999' [ 108.4028]
    '05-Feb-1999' [ 134.4882]
    '12-Feb-1999' [ 117.5581]
    '19-Feb-1999' [ 106.6683]
    '26-Feb-1999' [ 118.2912]
    '05-Mar-1999' [ 105.6835]
    '12-Mar-1999' [ 128.5836]
    '19-Mar-1999' [ 115.1746]
    '26-Mar-1999' [ 131.2854]
    '01-Apr-1999' [ NaN]
    '09-Apr-1999' [ 123.1684]
    '16-Apr-1999' [ 107.2975]
    '23-Apr-1999' [ 91.5625]
    '30-Apr-1999' [ 78.5738]
    '07-May-1999' [ 65.2904]
    '14-May-1999' [ 70.8581]
    '21-May-1999' [ 72.4807]
    '28-May-1999' [ 72.9190]
    '04-Jun-1999' [ 64.3460]
    '11-Jun-1999' [ 59.8743]
    '18-Jun-1999' [ 55.0026]
    '25-Jun-1999' [ 49.4032]
    '02-Jul-1999' [ 49.9485]
    '09-Jul-1999' [ 47.8061]
    '16-Jul-1999' [ 61.0517]
    '23-Jul-1999' [ 58.9313]
    '30-Jul-1999' [ 53.9584]
    '06-Aug-1999' [ 44.8472]
```

| '13-Aug-1999' | [ | $45.0463]$ |
| :--- | :--- | :--- |
| '20-Aug-1999' | [ | $45.1088]$ |
| '27-Aug-1999' | [ | $56.4897]$ |
| '03-Sep-1999' | [ | $61.2449]$ |
| '10-Sep-1999' | [ | $58.1012]$ |
| '17-Sep-1999' | [ | $50.8974]$ |
| '24-Sep-1999' | [ | $46.5143]$ |
| '01-0ct-1999' | [ | $38.0806]$ |
| '15-0ct-1999' | [ | $33.6664]$ |
| '22-0ct-1999' | [ | $34.2992]$ |
| '29-0ct-1999' | [ | $33.4202]$ |
| '05-Nov-1999' | [ | $36.9287]$ |
| '12-Nov-1999' | [ | $45.1278]$ |
| '19-Nov-1999' | [ | $31.8128]$ |
| '26-Nov-1999' | [ | $36.8199]$ |
| '03-Dec-1999' | [ | $36.2896]$ |
| '10-Dec-1999' | [ | $33.8457]$ |
| '17-Dec-1999' | [ | $33.3868]$ |
| '23-Dec-1999' | [ | NaN] |
| '31-Dec-1999' | [ | $28.5665]$ |

## See Also

convertto|fints|toannual|todaily|tomonthly|toquarterly|tosemi

## Topics

"Data Transformation and Frequency Conversion" on page 13-13

## Introduced before R2006a

## tr2bonds

Term-structure parameters given Treasury bond parameters

## Syntax

```
[Bonds,Prices,Yields] = tr2bonds(TreasuryMatrix,Settle)
[Bonds,Prices,Yields] = tr2bonds(
```

$\qquad$

``` ,Settle)
```


## Description

[Bonds,Prices,Yields] = tr2bonds(TreasuryMatrix,Settle) returns termstructure parameters (Bonds, Prices, and Yields) sorted by ascending maturity date, given Treasury bond parameters. The formats of the output matrix and vectors meet requirements for input to the zbtprice and zbtyield zero-curve bootstrapping functions.
[Bonds,Prices,Yields] = tr2bonds( $\qquad$ ,Settle) adds an optional argument for Settle.

## Examples

## Return Term-Structure Parameters Given Treasury Bond Parameters

This example shows how to return term-structure parameters (bond information, prices, and yields) sorted by ascending maturity date, given Treasury bond market parameters for December 22, 1997.

```
Matrix =[0.0650 datenum('15-apr-1999') 101.03125 101.09375 0.0564
    0.05125 datenum('17-dec-1998') 99.4375 99.5 0.0563
    0.0625 datenum('30-jul-1998') 100.3125 100.375 0.0560
    0.06125 datenum('26-mar-1998') 100.09375 100.15625 0.0546];
[Bonds, Prices, Yields] = tr2bonds(Matrix)
```

```
Bonds = 4×6
105 x
\begin{tabular}{llllll}
7.2984 & 0.0000 & 0.0010 & 0.0000 & 0 & 0.0000 \\
7.2997 & 0.0000 & 0.0010 & 0.0000 & 0 & 0.0000 \\
7.3011 & 0.0000 & 0.0010 & 0.0000 & 0 & 0.0000 \\
7.3022 & 0.0000 & 0.0010 & 0.0000 & 0 & 0.0000
\end{tabular}
Prices = 4×1
    100.1563
    100.3750
    99.5000
    101.0938
Yields = 4×1
    0.0546
    0.0560
    0.0563
    0.0564
```


## Return Term-Structure Parameters Given Treasury Bond Parameters Using datetime Input

This example shows how to use datetime input to return term-structure parameters (bond information, prices, and yields) sorted by ascending maturity date, given Treasury bond market parameters for December 22, 1997.

```
Matrix =[0.0650 datenum('15-apr-1999') 101.03125 101.09375 0.0564
    0.05125 datenum('17-dec-1998') 99.4375 99.5 0.0563
    0.0625 datenum('30-jul-1998') 100.3125 100.375 0.0560
    0.06125 datenum('26-mar-1998') 100.09375 100.15625 0.0546];
t=array2table(Matrix);
t.Matrix2=datetime(t{:,2},'ConvertFrom','datenum','Locale','en_US');
[Bonds, Prices, Yields] = tr2bonds(t,datetime('1-Jan-1997','Locale','en_US'))
Bonds=4\times6 table
    Maturity CouponRate Face Period Basis EndMonthRule
```

```
26-Mar-1998 00:00:00
    0.06125 100
    2 0
-1
30-Jul-1998 00:00:00 0.0625 100 1 0 1
17-Dec-1998 00:00:00 0.05125 100 1
15-Apr-1999 00:00:00 0.065 100 1
Prices = 4×1
    100.1563
    100.3750
    99.5000
    101.0938
Yields = 4×1
    0.0598
    0.0599
    0.0540
    0.0598
```


## Input Arguments

## TreasuryMatrix - Treasury bond parameters

table | matrix
Treasury bond parameters, specified as a 5-column table or a NumBonds-by-5 matrix of bond information where the table columns or matrix columns contains:

- CouponRate (Required) Coupon rate of the Treasury bond, specified as a decimal indicating the coupon rates for each bond in the portfolio.
- Maturity (Required) Maturity date of the Treasury bond, specified as a serial date number when using a matrix. Use datenum to convert date character vectors to serial date numbers. If the input TreasuryMatrix is a table, the Maturity dates can be serial date numbers, date character vectors, or datetime arrays.
- Bid (Required) Bid prices, specified using an integer-decimal form for each bond in the portfolio.
- Asked (Required) Asked prices, specified using an integer-decimal form for each bond in the portfolio.
- AskYield (Required) Quoted ask yield, specified using a decimal form for each bond in the portfolio.


## Data Types: double | table

## Settle - Settlement date of Treasury bond <br> serial date number | date character vector | datetime

(Optional) Settlement date of the Treasury bond, specified as a scalar or a NINST-by-1 vector of serial date numbers, date character vectors, or datetime arrays. The Settle date must be before the Maturity date.
Data Types: double | char \| datetime

## Output Arguments

## Bonds - Coupon bond information

numeric
Coupon bond information, returned as a table or matrix depending on the TreasuryMatrix input.

When TreasuryMatrix is a table, Bonds is also a table, and the variable type for the Maturity dates in Bonds (column 1) matches the variable type for Maturity in TreasuryMatrix.

When TreasuryMatrix input is a n-by-5 matrix, then each row describes a bond.
The parameters or columns returned for Bonds are:

- Maturity (Column 1) Maturity date for each bond in the portfolio as a serial date number. The format of the dates matches the format used for Maturity in TreasuryMatrix (serial date number, date character vector, or datetime array).
- CouponRate (Column 2) Coupon rate for each bond in the portfolio in decimal form.
- Face (Column 3, Optional) Face or par value for each bond in the portfolio. The default is 100.
- Period (Column 4, Optional) Number of coupon payments per year for each bond in the portfolio with allowed values: $1,2,3,4,6$, and 12 . The default is 2 , unless you are dealing with zero coupons, then Period is 0 instead of 2.
- Basis (Column 5, Optional) Day-count basis for each bond in the portfolio with possible values:
- $0=$ actual/actual (default)
- $1=30 / 360$ (SIA)
- 2 = actual/360
- 3 = actual/365
- $4=30 / 360$ (BMA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- $9=\mathrm{actual} / 360$ (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252
- For more information, see basis on page Glossary-0
- EndMonthRule (Column 6, Optional) End-of-month rule flag for each bond in the portfolio. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days. $0=$ ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. 1 = set rule on, meaning that a bond's coupon payment date is always the last actual day of the month. The default is 1 .


## Prices - Bond prices

numeric
Bond prices, returned as a column vector containing the price of each bond in Bonds, respectively. The number of rows ( $n$ ) matches the number of rows in Bonds.

## Yields - Bond yields

numeric

Bond yields, returned as a column vector containing the yield to maturity of each bond in Bonds, respectively. The number of rows ( $n$ ) matches the number of rows in Bonds.

If the optional input argument Settle is used, Yields is computed as a semiannual yield to maturity. If the input Settle is not used, the quoted input yields are used.

## See Also

datetime|tbl2bond|zbtprice|zbtyield

## Topics

"Term Structure of Interest Rates" on page 2-42
"Treasury Bills Defined" on page 2-37

## Introduced before R2006a

## transprob

Estimate transition probabilities from credit ratings data

## Syntax

```
[transMat,sampleTotals,idTotals] = transprob(data)
[transMat,sampleTotals,idTotals] = transprob( __ ,Name, Value)
```


## Description

[transMat,sampleTotals,idTotals] = transprob(data) constructs a transition matrix from historical data of credit ratings.
[transMat,sampleTotals,idTotals] = transprob( $\qquad$ ,Name, Value) adds optional name-value pair arguments.

## Examples

## Construct a Transition Matrix From a Table of Historical Data of Credit Ratings

Using the historical credit rating table as input data from Data_TransProb.mat display the first ten rows and compute the transition matrix:

```
load Data_TransProb
data(1:10,:)
ans=10\times3 table
```

'00010283'
'00010283'
'00010283'
'00010283'
'00013326'

Date
' 10-Nov-1984
' 12-May-1986
'29- Jun-1988'
'12-Dec-1991'
'09-Feb-1985'

Rating

' CCC'
'B'
'CCC'
'D'
'A'

| '00013326' | ' 24 -Feb-1994' | ' $\mathrm{AA}^{\prime}$ |
| :---: | :---: | :---: |
| '00013326' | ' 10-Nov-2000' | ' BBB |
| '00014413' | ' 23-Dec-1982' | 'B' |
| '00014413' | ' 20-Apr-1988' | 'BB' |
| '00014413' | '16-Jan-1998' | B' |

```
% Estimate transition probabilities with default settings
transMat = transprob(data)
transMat = 8\times8
```

| 93.1170 | 5.8428 | 0.8232 | 0.1763 | 0.0376 | 0.0012 | 0.0001 | 0.0017 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 1.6166 | 93.1518 | 4.3632 | 0.6602 | 0.1626 | 0.0055 | 0.0004 | 0.0396 |
| 0.1237 | 2.9003 | 92.2197 | 4.0756 | 0.5365 | 0.0661 | 0.0028 | 0.0753 |
| 0.0236 | 0.2312 | 5.0059 | 90.1846 | 3.7979 | 0.4733 | 0.0642 | 0.2193 |
| 0.0216 | 0.1134 | 0.6357 | 5.7960 | 88.9866 | 3.4497 | 0.2919 | 0.7050 |
| 0.0010 | 0.0062 | 0.1081 | 0.8697 | 7.3366 | 86.7215 | 2.5169 | 2.4399 |
| 0.0002 | 0.0011 | 0.0120 | 0.2582 | 1.4294 | 4.2898 | 81.2927 | 12.7167 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 100.0000 |

Using the historical credit rating table input data from Data_TransProb.mat, compute the transition matrix using the cohort algorithm:
\%Estimate transition probabilities with 'cohort' algorithm transMatCoh $=$ transprob(data,'algorithm','cohort')
transMatCoh $=8 \times 8$

| 93.1345 | 5.9335 | 0.7456 | 0.1553 | 0.0311 | 0 | 0 | 0 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 1.7359 | 92.9198 | 4.5446 | 0.6046 | 0.1560 | 0 | 0 | 0.0390 |
| 0.1268 | 2.9716 | 91.9913 | 4.3124 | 0.4711 | 0.0544 | 0 | 0.0725 |
| 0.0210 | 0.3785 | 5.0683 | 89.7792 | 4.0379 | 0.4627 | 0.0421 | 0.2103 |
| 0.0221 | 0.1105 | 0.6851 | 6.2320 | 88.3757 | 3.6464 | 0.2873 | 0.6409 |
| 0 | 0 | 0.0761 | 0.7230 | 7.9909 | 86.1872 | 2.7397 | 2.2831 |
| 0 | 0 | 0 | 0.3094 | 1.8561 | 4.5630 | 80.8971 | 12.3743 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 100.0000 |

Using the historical credit rating data with ratings investment grade ('IG '), speculative grade ('SG'), and default ('D'), from Data_TransProb.mat display the first ten rows and compute the transition matrix:

```
dataIGSG(1:10,:)
```

```
ans=10\times3 table
    ID
    '00011253'
    '00012751' '17-Feb-1985'
    'SG'
    '00012751' '19-May-1986' 'D'
    '00014690' '17-Jan-1983' 'IG'
    '00012144' '21-Nov-1984' 'IG'
    '00012144' '25-Mar-1992' 'SG'
    '00012144' '07-May-1994' 'IG'
    '00012144' '23-Jan-2000' 'SG'
    '00012144' '20-Aug-2001' 'IG'
    '00012937' '07-Feb-1984' 'IG'
transMatIGSG = transprob(dataIGSG,'labels',{'IG','SG','D'})
transMatIGSG = 3\times3
\begin{tabular}{rrr}
98.6719 & 1.2020 & 0.1261 \\
3.5781 & 93.3318 & 3.0901 \\
0 & 0 & 100.0000
\end{tabular}
```

Using the historical credit rating data with numeric ratings for investment grade (1), speculative grade (2), and default (3), from Data_TransProb. mat display the first ten rows and compute the transition matrix:
dataIGSGnum(1:10,:)

| ID | Date | Rating |
| :---: | :---: | :---: |
| '00011253' | '04-Apr-1983' | 1 |
| '00012751' | '17-Feb-1985' | 2 |
| '00012751' | '19-May-1986' | 3 |
| '00014690' | '17-Jan-1983' | 1 |
| '00012144' | '21-Nov-1984' | 1 |
| '00012144' | '25-Mar-1992' | 2 |
| '00012144' | '07-May-1994' | 1 |
| '00012144' | '23-Jan-2000' | 2 |
| '00012144' | '20-Aug-2001' | 1 |

'00012937' '07-Feb-1984' 1

```
transMatIGSGnum = transprob(dataIGSGnum,'labels',{1,2,3})
transMatIGSGnum = 3 3 
```

| 98.6719 | 1.2020 | 0.1261 |
| ---: | ---: | ---: |
| 3.5781 | 93.3318 | 3.0901 |
| 0 | 0 | 100.0000 |

## Create a Transition Matrix Using a Cell Array for Historical Data of Credit Ratings

Using a MATLAB® table containing the historical credit rating cell array input data (dataCellFormat) from Data_TransProb.mat, estimate the transition probabilities with default settings.

```
load Data TransProb
transMat = transprob(dataCellFormat)
transMat = 8\times8
```

| 93.1170 | 5.8428 | 0.8232 | 0.1763 | 0.0376 | 0.0012 | 0.0001 | 0.0017 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 1.6166 | 93.1518 | 4.3632 | 0.6602 | 0.1626 | 0.0055 | 0.0004 | 0.0396 |
| 0.1237 | 2.9003 | 92.2197 | 4.0756 | 0.5365 | 0.0661 | 0.0028 | 0.0753 |
| 0.0236 | 0.2312 | 5.0059 | 90.1846 | 3.7979 | 0.4733 | 0.0642 | 0.2193 |
| 0.0216 | 0.1134 | 0.6357 | 5.7960 | 88.9866 | 3.4497 | 0.2919 | 0.7050 |
| 0.0010 | 0.0062 | 0.1081 | 0.8697 | 7.3366 | 86.7215 | 2.5169 | 2.4399 |
| 0.0002 | 0.0011 | 0.0120 | 0.2582 | 1.4294 | 4.2898 | 81.2927 | 12.7167 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 100.0000 |

Using the historical credit rating cell array input data (dataCellFormat), compute the transition matrix using the cohort algorithm:

```
%Estimate transition probabilities with 'cohort' algorithm
transMatCoh = transprob(dataCellFormat,'algorithm','cohort')
transMatCoh = 8 8 8
```

93.1345
5.9335
0.7456
0.1553
0.0311
0
0

| 1.7359 | 92.9198 | 4.5446 | 0.6046 | 0.1560 | 0 | 0 | 0.0390 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 0.1268 | 2.9716 | 91.9913 | 4.3124 | 0.4711 | 0.0544 | 0 | 0.0725 |
| 0.0210 | 0.3785 | 5.0683 | 89.7792 | 4.0379 | 0.4627 | 0.0421 | 0.2103 |
| 0.0221 | 0.1105 | 0.6851 | 6.2320 | 88.3757 | 3.6464 | 0.2873 | 0.6409 |
| 0 | 0 | 0.0761 | 0.7230 | 7.9909 | 86.1872 | 2.7397 | 2.2831 |
| 0 | 0 | 0 | 0.3094 | 1.8561 | 4.5630 | 80.8971 | 12.3743 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 100.0000 |

## Input Arguments

## data - Credit migration data

table | cell array of character vectors | preprocessed data structure
Using transprob to estimate transition probabilities given credit ratings historical data (that is, credit migration data), the data input can be one of the following:

- An nRecords-by-3 MATLAB table containing the historical credit ratings data of the form:

| ID | Date | Rating |
| :--- | :--- | :--- |
| '00010283' | '10-Nov-1984' |  |
| 'CCC' |  |  |
| '00010283' | '12-May-1986' | 'B' |
| '00010283' | '29-Jun-1988' | 'CCC' |
| '00010283' | '12-Dec-1991' | 'D' |
| '00013326' | '09-Feb-1985' | 'A' |
| '00013326' | '24-Feb-1994' | 'AA' |
| $00014413 '$ | '10-Nov-2000' | 'BBB' |
| $23-$ Dec-1982' | 'B' |  |

where each row contains an ID (column 1), a date (column 2), and a credit rating (column 3). Column 3 is the rating assigned to the corresponding ID on the corresponding date. All information corresponding to the same ID must be stored in contiguous rows. Sorting this information by date is not required, but recommended for efficiency. When using a MATLAB table input, the names of the columns are irrelevant, but the ID, date and rating information are assumed to be in the first, second, and third columns, respectively. Also, when using a table input, the first and third columns can be categorical arrays, and the second can be a datetime array. The following summarizes the supported data types for table input:

| Data Input Type | ID (1st Column) | Date (2nd Column) | Rating (3rd Column) |
| :---: | :---: | :---: | :---: |
| Table | - Numeric array <br> - Cell array of character vectors <br> - Categorical array | - Numeric array <br> - Cell array of character vectors <br> - Datetime array | - Numeric array <br> - Cell array of character vectors <br> - Categorical array |

- An nRecords-by-3 cell array of character vectors containing the historical credit ratings data of the form:

```
'00010283' '10-Nov-1984' 'CCC'
'00010283' '12-May-1986' 'B'
'00010283' '29-Jun-1988' 'CCC'
'00010283' '12-Dec-1991' 'D'
'00013326' '09-Feb-1985' 'A'
'00013326' '24-Feb-1994' 'AA'
'00013326' '10-Nov-2000' 'BBB'
'00014413' '23-Dec-1982' 'B'
```

where each row contains an ID (column 1), a date (column 2), and a credit rating (column 3). Column 3 is the rating assigned to the corresponding ID on the corresponding date. All information corresponding to the same ID must be stored in contiguous rows. Sorting this information by date is not required, but recommended for efficiency. IDs, dates, and ratings are stored in character vector format, but they can also be entered in numeric format. The following summarizes the supported data types for cell array input:

| Data Input Type | ID (1st Column) | Date (2nd Column) | Rating (3rd Column) |
| :---: | :---: | :---: | :---: |
| Cell | - Numeric elements <br> - Character vector elements | - Numeric elements <br> - Character vector elements | - Numeric elements <br> - Character vector elements |

- A preprocessed data structure obtained using transprobprep. This data structure contains the fields'idStart', 'numericDates', 'numericRatings', and 'ratingsLabels'.

Data Types: table|cell|struct

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1,Value1, ... , NameN, ValueN.

Example: transMat = transprob(data,'algorithm','cohort')

## algorithm - Estimation algorithm

'duration' (default)| character vector with values are 'duration' or 'cohort'
Estimation algorithm, specified as the comma-separated pair consisting of 'algorithm' and a character vector with a value of 'duration' or 'cohort'.

Data Types: char

## endDate - End date of the estimation time window

latest date in data (default) | character vector | serial date number | datetime
End date of the estimation time window, specified as the comma-separated pair consisting of 'endDate' and a date character vector, serial date number, or datetime object. The endDate cannot be a date before the startDate.

Data Types: char|double |datetime

## labels - Credit-rating scale

\{'AAA', 'AA', 'A','BBB', 'BB', 'B', 'CCC', 'D'\} (default)| cell array of character vectors

Credit-rating scale, specified as the comma-separated pair consisting of 'labels' and a nRatings-by-1, or 1-by-nRatings cell array of character vectors.
labels must be consistent with the ratings labels used in the third column of data. Use a cell array of numbers for numeric ratings, and a cell array for character vectors for categorical ratings.

Note When the input argument data is a preprocessed data structure obtained from a previous call to transprobprep, this optional input for ' labels is unused because the labels in the 'ratingsLabels' field of transprobprep take priority.

## snapsPerYear - Number of credit-rating snapshots per year 1 (default) | numeric values are $1,2,3,4,6$, or 12

Number of credit-rating snapshots per year to be considered for the estimation, specified as the comma-separated pair consisting of 'snapsPerYear' and a numeric value of 1, 2, $3,4,6$, or 12 .

Note This parameter is only used with the 'cohort' algorithm.

## Data Types: double

## startDate - Start date of the estimation time window

earliest date in data (default) | character vector | serial date number | datetime
Start date of the estimation time window, specified as the comma-separated pair consisting of 'startDate' and a date character vector, serial date number, or datetime object.

Data Types: char|double | datetime

## transInterval - Length of the transition interval in years

1 (one year transition probability) (default) | numeric
Length of the transition interval, in years, specified as the comma-separated pair consisting of 'transInterval' and a numeric value.

Data Types: double

## Output Arguments

transMat - Matrix of transition probabilities in percent matrix

Matrix of transition probabilities in percent, returned as a nRatings-by-nRatings transition matrix.

## sampleTotals - Structure with sample totals

structure
Structure with sample totals, returned with fields:

- totalsVec - A vector of size 1-by-nRatings.
- totalsMat - A matrix of size nRatings-by-nRatings.
- algorithm - A character vector with values 'duration' or 'cohort'.

For the 'duration' algorithm, totalsMat( $i, j)$ contains the total transitions observed out of rating $i$ into rating (all the diagonal elements are zero). The total time spent on rating $i$ is stored in totalsVec $(i)$. For example, if there are three rating categories, Investment Grade (IG), Speculative Grade (SG), and Default (D), and the following information:

| Total time spent | $\begin{gathered} \text { IG } \\ 4859.09 \end{gathered}$ |  | $\begin{gathered} \text { SG } \\ 1503.36 \end{gathered}$ | $\begin{gathered} \text { D } \\ 1162.05 \end{gathered}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| in rating: |  |  |  |  |  |
| Transitions |  | IG | SG | D |  |
| out of (row) | IG | 0 | 89 | 7 |  |
| into (column): | SG | 202 | 0 | 32 |  |
|  | D | 0 | 0 | 0 |  |

Then

```
totals.totalsVec = [4859.09 1503.36 1162.05]
totals.totalsMat = [ 0 89 7
    202 0 32
    0 0 0]
totals.algorithm = 'duration'
```

For the ' cohort ' algorithm, totalsMat $(i, j)$ contains the total transitions observed from rating $i$ to rating $j$, and totalsVec(i) is the initial count in rating $i$. For example, given the following information:

| Initial count | IG | SG | D |
| :--- | ---: | ---: | ---: |
| in rating: | 4808 | 1572 | 1145 |
|  |  |  |  |
| Transitions |  | IG | SG |
| from (row) | IG | 4721 | 80 |
| to (column): | SG | 193 | 1347 |
|  | D | 0 | 0 |
|  |  | 1145 |  |

Then

```
totals.totalsVec = [4808 1572
    1145]
totals.totalsMat = [4721 80 7
    193 1347 32
```

```
    0 0 1145
totals.algorithm = 'cohort'
```


## idTotals - IDs totals

```
struct array
```

IDs totals, returned as a struct array of size nIDs-by-1, where nIDs is the number of distinct IDs in column 1 of data when this is a table or cell array or, equivalently, equal to the length of the idStart field minus 1 when data is a preprocessed data structure from transprobprep. For each ID in the sample, idTotals contains one structure with the following fields:

- totalsVec - A sparse vector of size 1-by-nRatings.
- totalsMat - A sparse matrix of size nRatings-by-nRatings.
- algorithm - A character vector with values 'duration' or 'cohort '.

These fields contain the same information described for the output sampleTotals, but at an ID level. For example, for 'duration', idTotals(k).totalsVec contains the total time that the $k$-th company spent on each rating.

## Definitions

## Cohort Estimation

The cohort algorithm estimates the transition probabilities based on a sequence of snapshots of credit ratings at regularly spaced points in time.

If the credit rating of a company changes twice between two snapshot dates, the intermediate rating is overlooked and only the initial and final ratings influence the estimates.

## Duration Estimation

Unlike the cohort method, the duration algorithm estimates the transition probabilities based on the full credit ratings history, looking at the exact dates on which the credit rating migrations occur.

There is no concept of snapshots in this method, and all credit rating migrations influence the estimates, even when a company's rating changes twice within a short time.

## Algorithms

## Cohort Estimation

The algorithm first determines a sequence $t_{0, \ldots,} t_{K}$ of snapshot dates. The elapsed time, in years, between two consecutive snapshot dates $t_{k-1}$ and $t_{k}$ is equal to $1 / n s$, where $n s$ is the number of snapshots per year. These $K+1$ dates determine $K$ transition periods.

The algorithm computes $N_{i}^{n}$, the number of transition periods in which obligor $n$ starts at rating $i$. These are added up over all obligors to get $N_{i}$, the number of obligors in the sample that start a period at rating $i$. The number periods in which obligor $n$ starts at rating $i$ and ends at rating $j$, or migrates from $i$ to $j$, denoted by $N_{i j}^{n}$, is also computed.

These are also added up to get $N_{i j}$, the total number of migrations from $i$ to $j$ in the sample.

The estimate of the transition probability from $i$ to $j$ in one period, denoted by $P_{i j}$, is given by:

$$
P_{i j}=\frac{N_{i j}}{N_{i}}
$$

These probabilities are arranged in a one-period transition matrix $P_{0}$, where the $i, j$ entry in $P_{0}$ is $P_{i j}$.

If the number of snapshots per year $n s$ is 4 (quarterly snapshots), the probabilities in $P_{0}$ are 3-month (or 0.25 -year) transition probabilities. You may, however, be interested in 1year or 2-year transition probabilities. The latter time interval is called the transition interval, $\Delta t$, and it is used to convert $P_{0}$ into the final transition matrix, $P$, according to the formula:

$$
P=P_{0}^{n s \Delta t}
$$

For example, if $n s=4$ and $\Delta t=2, P$ contains the 2-year transition probabilities estimated from quarterly snapshots.

```
Note For the cohort algorithm, optional output arguments idTotals and
sampleTotals from transprob contain the following information:
```

```
    idTotals(n).totalsVec = (NN
```

- idTotals( n ). .totalsMat $=\left(N_{i, j}^{n}\right)$ vij
- idTotals(n).algorithm = 'cohort'
- sampleTotals.totalsVec $=\left(N_{i}\right)_{\forall i}$
- sampleTotals.totalsMat $=\left(N_{i, j}\right)$ vij
- sampleTotals.algorithm = 'cohort'

For efficiency, the vectors and matrices in idTotals are stored as sparse arrays.

## Duration Estimation

The algorithm computes $T_{i}^{n}$, the total time that obligor $n$ spends in rating $i$ within the estimation time window. These quantities are added up over all obligors to get $T_{i}$, the total time spent in rating $i$, collectively, by all obligors in the sample. The algorithm also computes $T_{i j}^{n}$, the number times that obligor $n$ migrates from rating $i$ to rating $j$, with $i$ not equal to $j$, within the estimation time window. And it also adds them up to get $T_{i j}$, the total number of migrations, by all obligors in the sample, from the rating $i$ to $j$, with $i$ not equal to $j$.

To estimate the transition probabilities, the duration algorithm first computes a generator matrix $\Lambda$. Each off-diagonal entry of this matrix is an estimate of the transition rate out of rating $i$ into rating $j$, and is given by:

$$
\lambda_{i j}=\frac{T_{i j}}{T_{i}}, i \neq j
$$

The diagonal entries are computed as:

$$
\lambda_{i i}=-\sum_{j \neq i} \lambda_{i j}
$$

With the generator matrix and the transition interval $\Delta t$ (e.g., $\Delta t=2$ corresponds to 2year transition probabilities), the transition matrix is obtained as $P=\exp (\Delta t \Lambda)$, where exp denotes matrix exponentiation (expm in MATLAB).

Note For the duration algorithm, optional output arguments idTotals and sampleTotals from transprob contain the following information:

```
    idTotals(n).totalsVec = (Ti
```

- idTotals(n).totalsMat $=\left(T_{i, j}^{n}\right) \forall i j$
- idTotals(n).algorithm = 'duration'
- sampleTotals.totalsVec $=\left(T_{i}\right) \forall i$
- sampleTotals.totalsMat $=\left(T_{i, j}\right) \forall i j$
- sampleTotals.algorithm = 'duration'

For efficiency, the vectors and matrices in idTotals are stored as sparse arrays.

## References

[1] Hanson, S., T. Schuermann. "Confidence Intervals for Probabilities of Default." Journal of Banking \& Finance. Vol. 30(8), Elsevier, August 2006, pp. 2281-2301.
[2] Löffler, G., P. N. Posch. Credit Risk Modeling Using Excel and VBA. West Sussex, England: Wiley Finance, 2007.
[3] Schuermann, T. "Credit Migration Matrices." in E. Melnick, B. Everitt (eds.), Encyclopedia of Quantitative Risk Analysis and Assessment. Wiley, 2008.
See Alsotable|transprobbytotals | transprobprep
Topics"Estimation of Transition Probabilities" on page 8-2"Estimate Transition Probabilities for Different Rating Scales" on page 8-5
External WebsitesCredit Risk Modeling with MATLAB (53 min 09 sec )Forecasting Corporate Default Rates with MATLAB ( 54 min 36 sec )
Introduced in R2010b

## transprobbytotals

Estimate transition probabilities using totals structure input

## Syntax

```
[transMat,sampleTotals] = transprobbytotals(totals)
[transMat,sampleTotals] = transprobbytotals(
```

$\qquad$

``` ,Name, Value)
```


## Description

[transMat,sampleTotals] = transprobbytotals(totals) estimates transition probabilities using a totals structure input. transprobbytotals is useful for removing outlier information, obtaining bootstrapped confidence intervals, or computing transition probability estimates for different periodicity parameters (1-year transitions, 2year transitions, and so on) efficiently.
[transMat,sampleTotals] = transprobbytotals( $\qquad$ ,Name, Value) adds optional name-value pair arguments.

## Examples

## Estimate Transition Probabilities Using a totals Structure Input

Use historical credit rating input data from Data_TransProb.mat and transprob to generate input for transprobbytotals:

```
load Data_TransProb
```

```
% Call TRANSPROB with three output arguments
[transMat, sampleTotals, idTotals] = transprob(data);
transMat
transMat = 8\times8
```

| 93.1170 | 5.8428 | 0.8232 | 0.1763 | 0.0376 | 0.0012 | 0.0001 | 0.0017 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 1.6166 | 93.1518 | 4.3632 | 0.6602 | 0.1626 | 0.0055 | 0.0004 | 0.0396 |
| 0.1237 | 2.9003 | 92.2197 | 4.0756 | 0.5365 | 0.0661 | 0.0028 | 0.0753 |
| 0.0236 | 0.2312 | 5.0059 | 90.1846 | 3.7979 | 0.4733 | 0.0642 | 0.2193 |
| 0.0216 | 0.1134 | 0.6357 | 5.7960 | 88.9866 | 3.4497 | 0.2919 | 0.7050 |
| 0.0010 | 0.0062 | 0.1081 | 0.8697 | 7.3366 | 86.7215 | 2.5169 | 2.4399 |
| 0.0002 | 0.0011 | 0.0120 | 0.2582 | 1.4294 | 4.2898 | 81.2927 | 12.7167 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 100.0000 |

Suppose companies 4 and 27 are outliers and you want to remove them from the preprocessed idTotals struct array and estimate the new transition probabilities.

```
idTotals([4 27]) = [];
[transMat1, sampleTotals1] = transprobbytotals(idTotals);
transMat1
transMat1 = 8\times8
\begin{tabular}{rrrrrrrr}
93.1172 & 5.8427 & 0.8231 & 0.1763 & 0.0377 & 0.0012 & 0.0001 & 0.0017 \\
1.6213 & 93.1501 & 4.3584 & 0.6614 & 0.1631 & 0.0055 & 0.0004 & 0.0397 \\
0.1239 & 2.9027 & 92.2297 & 4.0628 & 0.5367 & 0.0661 & 0.0028 & 0.0753 \\
0.0236 & 0.2313 & 5.0070 & 90.1825 & 3.7986 & 0.4734 & 0.0642 & 0.2193 \\
0.0216 & 0.1134 & 0.6357 & 5.7959 & 88.9866 & 3.4497 & 0.2920 & 0.7050 \\
0.0010 & 0.0062 & 0.1081 & 0.8697 & 7.3367 & 86.7217 & 2.5171 & 2.4395 \\
0.0002 & 0.0011 & 0.0120 & 0.2591 & 1.4340 & 4.3034 & 81.3027 & 12.6875 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 100.0000
\end{tabular}
```

Obtain the 1 -year, 2 -year, 3 -year, 4 -year, and 5 -year default probabilities, without the outlier information (i.e., using sampleTotals1).

```
DefProb = zeros(7,5);
for t = 1:5
    transMatTemp = transprobbytotals(sampleTotals1,'transInterval',t);
    DefProb(:,t) = transMatTemp(1:7,8);
end
DefProb
DefProb = 7\times5
\begin{tabular}{lllll}
0.0017 & 0.0070 & 0.0159 & 0.0285 & 0.0450 \\
0.0397 & 0.0828 & 0.1299 & 0.1813 & 0.2377 \\
0.0753 & 0.1606 & 0.2567 & 0.3640 & 0.4831 \\
0.2193 & 0.4675 & 0.7430 & 1.0445 & 1.3700
\end{tabular}
```

| 0.7050 | 1.4668 | 2.2759 | 3.1232 | 4.0000 |
| ---: | ---: | ---: | ---: | ---: |
| 2.4395 | 4.9282 | 7.4071 | 9.8351 | 12.1847 |
| 12.6875 | 23.1184 | 31.7177 | 38.8282 | 44.7266 |

## Input Arguments

## totals - Total transitions observed

structure | struct array
Total transitions observed, specified as a structure, or a struct array of length nTotals, with fields:

- totalsVec - A sparse vector of size 1-by-nRatings1.
- totalsMat - A sparse matrix of size nRatings1-by-nRatings2 with nRatings1 $\leq$ nRatings2.
- algorithm - A character vector with values 'duration' or 'cohort'.

For the 'duration' algorithm, totalsMat $(i, j)$ contains the total transitions observed out of rating $i$ into rating $j$ (all the diagonal elements are 0 ). The total time spent on rating $i$ is stored in totalsVec(i). For example, you have three rating categories, Investment Grade (IG), Speculative Grade (SG), and Default (D), and the following information:

```
Total time spent IG SG D
in rating: 4859.09 1503.36 1162.05
\begin{tabular}{lrrrr} 
Transitions & & IG & SG & D \\
out of (row) & IG & 0 & 89 & 7 \\
into (column): & SG & 202 & 0 & 32 \\
& D & 0 & 0 & 0
\end{tabular}
```

Then:

```
totals.totalsVec = [4859.09 1503.36 1162.05]
totals.totalsMat = [ 0 89 7
    202 0 32
    0 0 0]
totals.algorithm = 'duration'
```

For the ' cohort ' algorithm, totalsMat $(i, j)$ contains the total transitions observed from rating $i$ to rating $j$, and totalsVec(i) is the initial count in rating $i$. For example, given the following information:

| Initial count | IG | SG | D |
| :--- | ---: | ---: | :---: |
| in rating: |  | 4808 | 1572 |
|  |  |  |  |
|  |  | IG | SG |

Then:

```
totals.totalsVec = [\begin{array}{lll}{4808}&{1572 1145]}\end{array}]
totals.totalsMat = [4721 80 7
        193 1347 32
    0 0 1145
totals.algorithm = 'cohort'
```

Common totals structures are the optional output arguments from transprob:

- sampleTotals - A single structure summarizing the totals information for the whole dataset.
- idTotals - A struct array with the totals information at the ID level.


## Data Types: struct| structure

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Example: transMat = transprobbytotals(Totalsl,'transInterval',5)

## snapsPerYear - Number of credit-rating snapshots per year

1 (default) | numeric values are $1,2,3,4,6$, or 12
Number of credit-rating snapshots per year to be considered for the estimation, specified as the comma-separated pair consisting of 'snapsPerYear' and a numeric value of 1,2 , $3,4,6$, or 12 .

Note This parameter is only used with the 'cohort ' algorithm.

## Data Types: double

## transInterval - Length of the transition interval in years

1 (one year transition probability) (default) | numeric
Length of the transition interval, in years, specified as the comma-separated pair consisting of 'transInterval' and a numeric value.

Data Types: double

## Output Arguments

## transMat - Matrix of transition probabilities in percent

matrix
Matrix of transition probabilities in percent, returned as a nRatings1-by-nRatings2 transition matrix.

## sampleTotals - Structure with sample totals

structure
Structure with sample totals, returned with fields:

- totalsVec - A vector of size 1-by-nRatings1.
- totalsMat - A matrix of size nRatings1-by-nRatings2 with nRatings1 $\leq$ nRatings2.
- algorithm - A character vector with values 'duration' or 'cohort'.

If totals is a struct array, sampleTotals contains the aggregated information. That is, sampleTotals.totalsVec is the sum of totals(k).totalsVec over all $k$, and similarly for totalsMat. When totals is itself a single structure, sampleTotals and totals are the same.

## Definitions

## Cohort Estimation

The cohort algorithm estimates the transition probabilities based on a sequence of snapshots of credit ratings at regularly spaced points in time.

If the credit rating of a company changes twice between two snapshot dates, the intermediate rating is overlooked and only the initial and final ratings influence the estimates.

## Duration Estimation

Unlike the cohort method, the duration algorithm estimates the transition probabilities based on the full credit ratings history, looking at the exact dates on which the credit rating migrations occur.

There is no concept of snapshots in this method, and all credit rating migrations influence the estimates, even when a company's rating changes twice within a short time.

## References

[1] Hanson, S., T. Schuermann. "Confidence Intervals for Probabilities of Default." Journal of Banking \& Finance. Vol. 30(8), Elsevier, August 2006, pp. 2281-2301.
[2] Löffler, G., P. N. Posch. Credit Risk Modeling Using Excel and VBA. West Sussex, England: Wiley Finance, 2007.
[3] Schuermann, T. "Credit Migration Matrices." in E. Melnick, B. Everitt (eds.), Encyclopedia of Quantitative Risk Analysis and Assessment. Wiley, 2008.

## See Also

transprob|transprobgrouptotals

## Topics

"Estimation of Transition Probabilities" on page 8-2
"Estimate Transition Probabilities for Different Rating Scales" on page 8-5

## External Websites

Credit Risk Modeling with MATLAB ( 53 min 09 sec )
Forecasting Corporate Default Rates with MATLAB (54 min 36 sec )

Introduced in R2010b

## transprobfromthresholds

Convert from credit quality thresholds to transition probabilities

## Syntax

```
trans = transprobfromthresholds(thresh)
```


## Description

trans $=$ transprobfromthresholds (thresh) transforms credit quality thresholds into transition probabilities

## Examples

Transform Credit Quality Thresholds Into Transition Probabilities
Use historical credit rating input data from Data_TransProb.mat, estimate transition probabilities with default settings.

```
load Data_TransProb
% Estimate transition probabilities with default settings
transMat = transprob(data)
transMat = 8\times8
```

| 93.1170 | 5.8428 | 0.8232 | 0.1763 | 0.0376 | 0.0012 | 0.0001 | 0.0017 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 1.6166 | 93.1518 | 4.3632 | 0.6602 | 0.1626 | 0.0055 | 0.0004 | 0.0396 |
| 0.1237 | 2.9003 | 92.2197 | 4.0756 | 0.5365 | 0.0661 | 0.0028 | 0.0753 |
| 0.0236 | 0.2312 | 5.0059 | 90.1846 | 3.7979 | 0.4733 | 0.0642 | 0.2193 |
| 0.0216 | 0.1134 | 0.6357 | 5.7960 | 88.9866 | 3.4497 | 0.2919 | 0.7050 |
| 0.0010 | 0.0062 | 0.1081 | 0.8697 | 7.3366 | 86.7215 | 2.5169 | 2.4399 |
| 0.0002 | 0.0011 | 0.0120 | 0.2582 | 1.4294 | 4.2898 | 81.2927 | 12.7167 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 100.0000 |

Obtain the credit quality thresholds.

```
thresh = transprobtothresholds(transMat)
thresh = 8×8
\begin{tabular}{rrrrrrrr} 
Inf & -1.4846 & -2.3115 & -2.8523 & -3.3480 & -4.0083 & -4.1276 & -4.1413 \\
Inf & 2.1403 & -1.6228 & -2.3788 & -2.8655 & -3.3166 & -3.3523 & -3.3554 \\
Inf & 3.0264 & 1.8773 & -1.6690 & -2.4673 & -2.9800 & -3.1631 & -3.1736 \\
Inf & 3.4963 & 2.8009 & 1.6201 & -1.6897 & -2.4291 & -2.7663 & -2.8490 \\
Inf & 3.5195 & 2.9999 & 2.4225 & 1.5089 & -1.7010 & -2.3275 & -2.4547 \\
Inf & 4.2696 & 3.8015 & 3.0477 & 2.3320 & 1.3838 & -1.6491 & -1.9703 \\
Inf & 4.6241 & 4.2097 & 3.6472 & 2.7803 & 2.1199 & 1.5556 & -1.1399 \\
Inf & Inf & Inf & Inf & Inf & Inf & Inf & Inf
\end{tabular}
```

Recover the transition probabilities.
trans = transprobfromthresholds(thresh)
trans $=8 \times 8$

| 93.1170 | 5.8428 | 0.8232 | 0.1763 | 0.0376 | 0.0012 | 0.0001 | 0.0017 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 1.6166 | 93.1518 | 4.3632 | 0.6602 | 0.1626 | 0.0055 | 0.0004 | 0.0396 |
| 0.1237 | 2.9003 | 92.2197 | 4.0756 | 0.5365 | 0.0661 | 0.0028 | 0.0753 |
| 0.0236 | 0.2312 | 5.0059 | 90.1846 | 3.7979 | 0.4733 | 0.0642 | 0.2193 |
| 0.0216 | 0.1134 | 0.6357 | 5.7960 | 88.9866 | 3.4497 | 0.2919 | 0.7050 |
| 0.0010 | 0.0062 | 0.1081 | 0.8697 | 7.3366 | 86.7215 | 2.5169 | 2.4399 |
| 0.0002 | 0.0011 | 0.0120 | 0.2582 | 1.4294 | 4.2898 | 81.2927 | 12.7167 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 100.0000 |

## Input Arguments

## thresh - Credit quality thresholds

matrix
Credit quality thresholds, specified as a M-by-N matrix of credit quality thresholds.
In each row, the first element must be Inf and the entries must satisfy the following monotonicity condition:

```
thresh(i,j) >= thresh(i,j+1), for 1<=j<N
```

The M-by-N input thresh and the M-by-N output trans are related as follows. The thresholds thresh( $i, j$ ) are critical values of a standard normal distribution $z$, such that:

```
trans(i,N) = P[z < thresh(i,N)],
trans(i,j) = P[z < thresh(i,j)] - P[z < thresh(i,j+1)], for 1<=j<N
```

Any given row in the output matrix trans determines a probability distribution over a discrete set of $N$ ratings 'R1' , ..., 'RN', so that for any row $i \operatorname{trans}(i, j)$ is the probability of migrating into ' Rj '. trans can be a standard transition matrix, with $\mathrm{M} \leq \mathrm{N}$, in which case row $i$ contains the transition probabilities for issuers with rating 'Ri'. But trans does not have to be a standard transition matrix. trans can contain individual transition probabilities for a set of M-specific issuers, with $M>N$.

For example, suppose that there are only N=3 ratings, 'High', 'Low', and 'Default', with these credit quality thresholds:

|  | High | Low | Default |
| :--- | :--- | ---: | ---: |
| High | Inf | -2.0814 | -3.1214 |
| Low | Inf | 2.4044 | -1.7530 |

The matrix of transition probabilities is then:

|  | High | Low | Default |
| :--- | ---: | :---: | :---: |
|  | 98.13 | 1.78 | 0.09 |
| High | 0.81 | 95.21 | 3.98 |

This means the probability of default for 'High' is equivalent to drawing a standard normal random number smaller than -3.1214 , or $0.09 \%$. The probability that a 'High' ends up the period with a rating of 'Low' or lower is equivalent to drawing a standard normal random number smaller than -2.0814 , or $1.87 \%$. From here, the probability of ending with a 'Low' rating is:
$\mathrm{P}[z<-2.0814]-\mathrm{P}[z<-3.1214]=1.87 \%-0.09 \%=1.78 \%$
And the probability of ending with a 'High ' rating is:
$100 \%-1.87 \%=98.13 \%$
where $100 \%$ is the same as:
$\mathrm{P}[z<\operatorname{Inf}]$
Data Types: double

## Output Arguments

## trans - Matrix of transition probabilities in percent

matrix
Matrix of transition probabilities in percent, returned as a M-by-N matrix.

## References

[1] Gupton, G. M., C. C. Finger, and M. Bhatia. "CreditMetrics." Technical Document, RiskMetrics Group, Inc., 2007.

## See Also

transprob|transprobbytotals |transprobtothresholds

## Topics

"Estimation of Transition Probabilities" on page 8-2
"Estimate Transition Probabilities for Different Rating Scales" on page 8-5
"Estimate Probabilities for Different Segments" on page 8-21

## External Websites

Credit Risk Modeling with MATLAB (53 min 09 sec )
Forecasting Corporate Default Rates with MATLAB (54 min 36 sec )

## Introduced in R2011b

## transprobgrouptotals

Aggregate credit ratings information into fewer rating categories

## Syntax

```
totalsGrouped = transprobgrouptotals(totals,groupingEdges)
```


## Description

totalsGrouped = transprobgrouptotals(totals,groupingEdges) aggregates the credit ratings information stored in the totals input into fewer ratings categories, which are defined by the groupingEdges argument.

## Examples

## Aggregate the Credit Ratings Information Stored in the totals Input

Use historical credit rating input data from Data_TransProb.mat. Load input data from file Data_TransProb.mat.

```
load Data_TransProb
% Call TRANSPROB with two output arguments
[transMat, sampleTotals] = transprob(data);
transMat
transMat = 8×8
```

| 93.1170 | 5.8428 | 0.8232 | 0.1763 | 0.0376 | 0.0012 | 0.0001 | 0.0017 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 1.6166 | 93.1518 | 4.3632 | 0.6602 | 0.1626 | 0.0055 | 0.0004 | 0.0396 |
| 0.1237 | 2.9003 | 92.2197 | 4.0756 | 0.5365 | 0.0661 | 0.0028 | 0.0753 |
| 0.0236 | 0.2312 | 5.0059 | 90.1846 | 3.7979 | 0.4733 | 0.0642 | 0.2193 |
| 0.0216 | 0.1134 | 0.6357 | 5.7960 | 88.9866 | 3.4497 | 0.2919 | 0.7050 |
| 0.0010 | 0.0062 | 0.1081 | 0.8697 | 7.3366 | 86.7215 | 2.5169 | 2.4399 |
| 0.0002 | 0.0011 | 0.0120 | 0.2582 | 1.4294 | 4.2898 | 81.2927 | 12.7167 |

Group into investment grade (ratings 1-4) and speculative grade (ratings 5-7); note, the default is the last rating (number 8).

```
edges = [4 7 8];
sampleTotalsGrp = transprobgrouptotals(sampleTotals,edges);
% Transition matrix at investment grade / speculative grade level
transMatIGSG = transprobbytotals(sampleTotalsGrp)
transMatIGSG = 3×3
\begin{tabular}{rrr}
98.5336 & 1.3608 & 0.1056 \\
3.9155 & 92.9692 & 3.1153 \\
0 & 0 & 100.0000
\end{tabular}
```

Obtain the 1-year, 2-year, 3-year, 4-year, and 5-year default probabilities at investment grade and speculative grade level.

```
DefProb = zeros(2,5);
for t = 1:5
transMatTemp = transprobbytotals(sampleTotalsGrp,'transInterval',t);
DefProb(:,t) = transMatTemp(1:2,3);
end
DefProb
DefProb = 2×5
\begin{tabular}{rrrrr}
0.1056 & 0.2521 & 0.4359 & 0.6537 & 0.9027 \\
3.1153 & 6.0157 & 8.7179 & 11.2373 & 13.5881
\end{tabular}
```


## Input Arguments

## totals - Total transitions observed

structure | struct array
Total transitions observed, specified as a structure, or a struct array of length nTotals, with fields:

- totalsVec - A sparse vector of size 1-by-nRatings1.
- totalsMat - A sparse matrix of size nRatings1-by-nRatings2 with nRatings1 $\leq$ nRatings2.
- algorithm - A character vector with values 'duration' or 'cohort'.

For the 'duration' algorithm, totalsMat $(i, j)$ contains the total transitions observed out of rating $i$ into rating $j$ (all the diagonal elements are 0 ). The total time spent on rating $i$ is stored in totalsVec(i). For example, you have three rating categories, Investment Grade (IG), Speculative Grade (SG), and Default (D), and the following information:

\left.| Total time spent | IG | SG |  | D |  |  |  |
| :--- | ---: | ---: | ---: | ---: | :---: | :---: | :---: |
| in rating: | 4859.09 | 1503.36 | 1162.05 |  |  |  |  |
| Transitions | IG |  |  |  |  | SG | D |
| out of (row) | IG | 0 | 89 | 7 |  |  |  |
| into (column): | SG | 202 | 0 | 32 |  |  |  |
|  |  | D | 0 | 0 |  |  |  |$\right)$

Then:

```
totals.totalsVec = [4859.09 1503.36 1162.05]
totals.totalsMat = [ 0 89 7
    202 0 32
    0 0 0]
totals.algorithm = 'duration'
```

For the ' cohort' algorithm, totalsMat $(i, j)$ contains the total transitions observed from rating $i$ to rating $j$, and totalsVec(i) is the initial count in rating $i$. For example, given the following information:

| Initial count | IG | SG | D |
| :--- | ---: | ---: | ---: |
| in rating: | 4808 | 1572 | 1145 |
|  |  |  |  |
| Transitions |  | IG | SG |
| from (row) | IG | 4721 | 80 |
| to (column): | SG | 193 | 1347 |
|  | D | 0 | 0 |
|  |  | 1145 |  |

Then:

```
totals.totalsVec = [4808
    1572 1145]
totals.totalsMat = [4721 80 7
    193 1347 32
```

$$
\begin{array}{ccc} 
\\
\text { totals.algorithm }=\text { 'cohort' } & 0 & 1145
\end{array}
$$

Common totals structures are the optional output arguments from transprob:

- sampleTotals - A single structure summarizing the totals information for the whole dataset.
- idTotals - A struct array with the totals information at the ID level.


## Data Types: struct|structure

## groupingEdges - Indicator for grouping credit ratings into categories numeric array

Indicator for grouping credit ratings into categories, specified as a numeric array.
This table illustrates how to group a list of whole ratings into investment grade (IG) and speculative grade (SG) categories. Eight ratings are in the original list. Ratings 1 to 4 are IG, ratings 5 to 7 are SG, and rating 8 is a category of its own. In this example, the array of grouping edges is [4 7 8].

| Original ratings: 'AAA | AA ${ }^{\prime}$ | A' | 'BBB' | 'BB' | 'B' | ' CCC' | D' |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Relative ordering: (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
| Grouped ratings: |  |  |  |  | 'SG' |  | D' |
| Grouping edges: |  |  | (4) |  |  | (7) | (8) |

In general, if groupingEdges has $K$ elements edge1 < edge $2<\ldots$. $<e d g e K$, ratings 1 to edge1 (inclusive) are grouped in the first category, ratings edge1+1 to edge2 in the second category, and so forth.

Regarding the last element, edgeK:

- If $n$ Ratings 1 equals $n$ Ratings2, then edge $K$ must equal $n$ Ratings1. This leads to $K$ groups, and $n$ RatingsGrouped $1=n$ RatingsGrouped $2=K$.
- If nRatings1 < nRatings2, then either:
- edge $K$ equals $n$ Ratings1, in which case ratings edge $K+1, \ldots, n$ Ratings2 are treated as categories of their own. This results in $K+$ (nRatings2-edgeK) groups, with nRatingsGrouped1 $=K$ and nRatingsGrouped2 $=K+$ (nRatings2 edgeK); or
- edge $K$ equals nRatings2, in which case there must be a jth edge element, edgej, such that edgej equals nRatings1. This leads to $K$ groups, and nRatingsGrouped1 $=j$ and nRatingsGrouped2 $=K$.


## Data Types: double

## Output Arguments

## totalsGrouped - Aggregated information by categories

structure | struct array
Aggregated information by categories, returned as a structure, or a struct array of length nTotals, with fields:

- totalsVec - A vector of size 1-by-nRatingsGrouped1.
- totalsMat - A matrix of size nRatingsGrouped1-by-nRatingsGrouped2.
- algorithm - A character vector, 'duration' or 'cohort'.
nRatingsGrouped1 and nRatingsGrouped2 are defined in the description of groupingEdges. Each structure contains aggregated information by categories, based on the information provided in the corresponding structure in totals, according to the grouping of ratings defined by groupingEdges and consistent with the algorithm choice.

Following the examples in the description of the totals input, suppose IG and SG are grouped into a single ND (Not-Defaulted) category, using the edges[2 3]. For the ' cohort ' algorithm, the output is:

```
totalsGrouped.totalsVec = [6380 1145]
totalsGrouped.totalsMat = [6341 39
0, 1145]
```

totalsGrouped.algorithm = 'cohort'
and for the 'duration ' algorithm:
totalsGrouped.totalsVec $=\left[\begin{array}{ll}6362.45 & 1162.05\end{array}\right]$
totalsGrouped.totalsMat $=\left[\begin{array}{ll}0 & 39\end{array}\right.$
0 0]
totalsGrouped.algorithm = 'duration'

## Definitions

## Cohort Estimation

The cohort algorithm estimates the transition probabilities based on a sequence of snapshots of credit ratings at regularly spaced points in time.

If the credit rating of a company changes twice between two snapshot dates, the intermediate rating is overlooked and only the initial and final ratings influence the estimates. For more information, see the Algorithms section of transprob.

## Duration Estimation

Unlike the cohort algorithm, the duration algorithm estimates the transition probabilities based on the full credit ratings history, looking at the exact dates on which the credit rating migrations occur.

There is no concept of snapshots in this method, and all credit rating migrations influence the estimates, even when a company's rating changes twice within a short time. For more information, see the Algorithms section of transprob.

## References

[1] Hanson, S., T. Schuermann. "Confidence Intervals for Probabilities of Default." Journal of Banking \& Finance. Vol. 30(8), Elsevier, August 2006, pp. 2281-2301.
[2] Löffler, G., P. N. Posch. Credit Risk Modeling Using Excel and VBA. West Sussex, England: Wiley Finance, 2007.
[3] Schuermann, T. "Credit Migration Matrices." in E. Melnick, B. Everitt (eds.), Encyclopedia of Quantitative Risk Analysis and Assessment. Wiley, 2008.

## See Also

transprob|transprobbytotals

## Topics

"Group Credit Ratings" on page 8-16
"Estimation of Transition Probabilities" on page 8-2
"Estimate Transition Probabilities for Different Rating Scales" on page 8-5
"Estimate Probabilities for Different Segments" on page 8-21

## External Websites

Credit Risk Modeling with MATLAB ( 53 min 09 sec )
Forecasting Corporate Default Rates with MATLAB (54 min 36 sec )

## Introduced in R2011b

## transprobprep

Preprocess credit ratings data to estimate transition probabilities

## Syntax

[prepData] = transprobprep(data)
[prepData] = transprobprep( $\qquad$ ,Name, Value)

## Description

[prepData] = transprobprep(data) preprocesses credit ratings historical data (that is, credit migration data) for the subsequent estimation of transition probabilities.
[prepData] = transprobprep( $\qquad$ ,Name, Value) adds optional name-value pair arguments.

## Examples

## Aggregate the Credit Ratings Information Stored in the totals Input

Load input data from the file Data_TransProb.mat and display the first ten rows. In this example, the inputs are provided in character vector format.

```
load Data_TransProb
% Preprocess credit ratings data.
prepData = transprobprep(data)
prepData = struct with fields:
            idStart: [1506x1 double]
    numericDates: [4315x1 double]
    numericRatings: [4315\times1 double]
        ratingsLabels: {'AAA' 'AA' 'A' 'BBB' 'BB' 'B' 'CCC' 'D'}
```

Estimate transition probabilities with the default settings.

```
transMat = transprob(prepData)
transMat = 8\times8
```

| 93.1170 | 5.8428 | 0.8232 | 0.1763 | 0.0376 | 0.0012 | 0.0001 | 0.0017 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 1.6166 | 93.1518 | 4.3632 | 0.6602 | 0.1626 | 0.0055 | 0.0004 | 0.0396 |
| 0.1237 | 2.9003 | 92.2197 | 4.0756 | 0.5365 | 0.0661 | 0.0028 | 0.0753 |
| 0.0236 | 0.2312 | 5.0059 | 90.1846 | 3.7979 | 0.4733 | 0.0642 | 0.2193 |
| 0.0216 | 0.1134 | 0.6357 | 5.7960 | 88.9866 | 3.4497 | 0.2919 | 0.7050 |
| 0.0010 | 0.0062 | 0.1081 | 0.8697 | 7.3366 | 86.7215 | 2.5169 | 2.4399 |
| 0.0002 | 0.0011 | 0.0120 | 0.2582 | 1.4294 | 4.2898 | 81.2927 | 12.7167 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 100.0000 |

Estimate transition probabilities with the 'cohort' algorithm.
transMatCoh = transprob(prepData,'algorithm','cohort')
transMatCoh $=8 \times 8$

| 93.1345 | 5.9335 | 0.7456 | 0.1553 | 0.0311 | 0 | 0 | 0 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 1.7359 | 92.9198 | 4.5446 | 0.6046 | 0.1560 | 0 | 0 | 0.0390 |
| 0.1268 | 2.9716 | 91.9913 | 4.3124 | 0.4711 | 0.0544 | 0 | 0.0725 |
| 0.0210 | 0.3785 | 5.0683 | 89.7792 | 4.0379 | 0.4627 | 0.0421 | 0.2103 |
| 0.0221 | 0.1105 | 0.6851 | 6.2320 | 88.3757 | 3.6464 | 0.2873 | 0.6409 |
| 0 | 0 | 0.0761 | 0.7230 | 7.9909 | 86.1872 | 2.7397 | 2.2831 |
| 0 | 0 | 0 | 0.3094 | 1.8561 | 4.5630 | 80.8971 | 12.3743 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 100.0000 |

## Input Arguments

data - Historical data for credit ratings
table | cell array of character vectors
Historical input data for credit ratings, specified as one of the following:

- A MATLAB table of size nRecords-by-3 containing the credit ratings. Each row contains an ID (column 1), a date (column 2), and a credit rating (column 3). The assigned credit rating corresponds to the associated ID on the associated date. All
information corresponding to the same ID must be stored in contiguous rows. Sorting this information by date is not required, but recommended for efficiency. When using a MATLAB table input, the names of the columns are irrelevant, but the ID, date and rating information are assumed to be in the first, second, and third columns, respectively. Also, when using a table input, the first and third columns can be categorical arrays, and the second can be a datetime array. Here is an example with all the information in table format:

| ID | Date | Rating |
| :---: | :---: | :---: |
| '00010283' | '10-Nov-1984' | 'CCC' |
| '00010283' | '12-May-1986' | 'B' |
| '00010283' | '29-Jun-1988' | 'CCC' |
| '00010283' | '12-Dec-1991' | 'D' |
| '00013326' | '09-Feb-1985' | 'A' |
| '00013326' | '24-Feb-1994' | 'AA' |

The following summarizes the supported data types for table input:

| Data Input Type | ID (1st Column) | Date (2nd Column) | Rating (3rd Column) |
| :---: | :---: | :---: | :---: |
| Table | - Numeric array <br> - Cell array of character vectors <br> - Categorical array | - Numeric array <br> - Cell array of character vectors <br> - Datetime array | - Numeric array <br> - Cell array of character vectors <br> - Categorical array |

- A cell array of size nRecords-by-3 containing the credit ratings. Each row contains an ID (column 1), a date (column 2), and a credit rating (column 3). The assigned credit rating corresponds to the associated ID on the associated date. All information corresponding to the same ID must be stored in contiguous rows. Sorting this information by date is not required but is recommended. IDs, dates, and ratings are stored in character vector format, but they can also be entered in numeric format. Here is an example with all the information in character vector format:

```
'00010283' '10-Nov-1984' 'CCC'
'00010283' '12-May-1986' 'B'
'00010283' '29-Jun-1988' 'CCC'
'00010283' '12-Dec-1991' 'D'
'00013326' '09-Feb-1985' 'A'
'00013326' '24-Feb-1994' 'AA'
```

The following summarizes the supported data types for cell array input:

| Data Input Type | ID (1st Column) | Date (2nd Column) | Rating (3rd Column) |
| :---: | :---: | :---: | :---: |
| Cell | - Numeric elements <br> - Character vector elements | - Numeric elements <br> - Character vector elements | - Numeric elements <br> - Character vector elements |

## Data Types: table |cell

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Example: prepData = transprobprep(data,'labels', \{'AAA','AA','A','BBB','BB','B','CCC','F'\})

## labels - Credit-rating scale

\{'AAA','AA','A','BBB','BB','B','CCC','D'\} (default)| cell array of character vectors

Credit-rating scale, specified as the comma-separated pair consisting of 'labels' and a nRatings-by-1, or 1-by-nRatings cell array of character vectors.
labels must be consistent with the ratings labels used in the third column of data. Use a cell array of numbers for numeric ratings, and a cell array for character vectors for categorical ratings.
Data Types: cell

## Output Arguments

## prepData - Summary where credit ratings information corresponding to each company starts and ends

structure

Summary where the credit ratings information corresponding to each company starts and ends, returned as a structure with the following fields:

- idStart - Array of size (nIDs+1)-by-1, where nIDs is the number of distinct IDs in column 1 of data. This array summarizes where the credit ratings information corresponding to each company starts and ends. The dates and ratings corresponding to company $j$ in data are stored from row idStart $(j)$ to row idStart $(j+1)-1$ of numericDates and numericRatings.
- numericDates - Array of size $n$ Records-by-1, containing the dates in column 2 of data, in numeric format.
- numericRatings - Array of size $n$ Records-by-1, containing the ratings in column 3 of data, mapped into numeric format.
- ratingsLabels - Cell array of sizel-by-nRatings, containing the credit rating scale.


## See Also

table|transprob|transprobbytotals

## Topics

"Group Credit Ratings" on page 8-16
"Estimation of Transition Probabilities" on page 8-2
"Estimate Transition Probabilities for Different Rating Scales" on page 8-5
"Estimate Probabilities for Different Segments" on page 8-21

## External Websites

Credit Risk Modeling with MATLAB ( 53 min 09 sec )
Forecasting Corporate Default Rates with MATLAB ( 54 min 36 sec )

## Introduced in R2011b

## transprobtothresholds

Convert from transition probabilities to credit quality thresholds

## Syntax

```
thresh = transprobtothresholds(trans)
```


## Description

thresh = transprobtothresholds(trans) transforms transition probabilities into credit quality thresholds.

## Examples

## Transform Transition Probabilities Into Credit Quality Thresholds

Use historical credit rating input data from Data_TransProb.mat. Load input data from file Data_TransProb.mat.

```
load Data_TransProb
```

\% Estimate transition probabilities with default settings
transMat = transprob(data)
transMat $=8 \times 8$

| 93.1170 | 5.8428 | 0.8232 | 0.1763 | 0.0376 | 0.0012 | 0.0001 | 0.0017 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 1.6166 | 93.1518 | 4.3632 | 0.6602 | 0.1626 | 0.0055 | 0.0004 | 0.0396 |
| 0.1237 | 2.9003 | 92.2197 | 4.0756 | 0.5365 | 0.0661 | 0.0028 | 0.0753 |
| 0.0236 | 0.2312 | 5.0059 | 90.1846 | 3.7979 | 0.4733 | 0.0642 | 0.2193 |
| 0.0216 | 0.1134 | 0.6357 | 5.7960 | 88.9866 | 3.4497 | 0.2919 | 0.7050 |
| 0.0010 | 0.0062 | 0.1081 | 0.8697 | 7.3366 | 86.7215 | 2.5169 | 2.4399 |
| 0.0002 | 0.0011 | 0.0120 | 0.2582 | 1.4294 | 4.2898 | 81.2927 | 12.7167 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 100.0000 |

Obtain the credit quality thresholds.

```
thresh = transprobtothresholds(transMat)
thresh = 8×8
\begin{tabular}{rrrrrrrr} 
Inf & -1.4846 & -2.3115 & -2.8523 & -3.3480 & -4.0083 & -4.1276 & -4.1413 \\
Inf & 2.1403 & -1.6228 & -2.3788 & -2.8655 & -3.3166 & -3.3523 & -3.3554 \\
Inf & 3.0264 & 1.8773 & -1.6690 & -2.4673 & -2.9800 & -3.1631 & -3.1736 \\
Inf & 3.4963 & 2.8009 & 1.6201 & -1.6897 & -2.4291 & -2.7663 & -2.8490 \\
Inf & 3.5195 & 2.9999 & 2.4225 & 1.5089 & -1.7010 & -2.3275 & -2.4547 \\
Inf & 4.2696 & 3.8015 & 3.0477 & 2.3320 & 1.3838 & -1.6491 & -1.9703 \\
Inf & 4.6241 & 4.2097 & 3.6472 & 2.7803 & 2.1199 & 1.5556 & -1.1399 \\
Inf & Inf & Inf & Inf & Inf & Inf & Inf & Inf
\end{tabular}
```


## Input Arguments

## trans - Transition probabilities in percent <br> matrix

Transition probabilities in percent, specified as a M-by-N matrix. Entries cannot be negative and cannot exceed 100, and all rows must add up to 100 .

Any given row in the M-by-N input matrix trans determines a probability distribution over a discrete set of $N$ ratings. If the ratings are 'R1',...,'RN', then for any row $i$ trans $(i, j)$ is the probability of migrating into ' $R j$ '. If trans is a standard transition matrix, then $\mathrm{M} \leqq \mathrm{N}$ and row $i$ contains the transition probabilities for issuers with rating 'Ri'. But trans does not have to be a standard transition matrix. trans can contain individual transition probabilities for a set of M -specific issuers, with $\mathrm{M}>\mathrm{N}$.

The credit quality thresholds thresh(i,j) are critical values of a standard normal distribution $z$, such that:

```
trans(i,N) = P[z < thresh(i,N)],
trans(i,j) = P[z < thresh(i,j)] - P[z < thresh(i,j+1)], for 1<=j<N
```

This implies that thresh $(i, 1)=$ Inf, for all $i$. For example, suppose that there are only $\mathrm{N}=3$ ratings, 'High', 'Low', and 'Default', with the following transition probabilities:

|  | High | Low | Default |
| :--- | :---: | :---: | :---: |
| High | 98.13 | 1.78 | 0.09 |
| Low | 0.81 | 95.21 | 3.98 |

The matrix of credit quality thresholds is:

|  | High | Low | Default |
| :--- | :--- | ---: | :---: |
| High | Inf | -2.0814 | -3.1214 |
| Low | Inf | 2.4044 | -1.7530 |

This means the probability of default for 'High ' is equivalent to drawing a standard normal random number smaller than -3.1214 , or $0.09 \%$. The probability that a 'High' ends up the period with a rating of 'Low' or lower is equivalent to drawing a standard normal random number smaller than -2.0814 , or $1.87 \%$. From here, the probability of ending with a 'Low' rating is:
$P[z<-2.0814]-P[z<-3.1214]=1.87 \%-0.09 \%=1.78 \%$
And the probability of ending with a 'High ' rating is:
$100 \%-1.87 \%=98.13 \%$
where $100 \%$ is the same as:
$\mathrm{P}[z<$ Inf]
Data Types: double

## Output Arguments

## thresh - Credit quality thresholds

matrix
Credit quality thresholds, returned as a M-by-N matrix.

## References

[1] Gupton, G. M., C. C. Finger, and M. Bhatia. "CreditMetrics." Technical Document, RiskMetrics Group, Inc., 2007.
See Alsotransprob|transprobbytotals|transprobfromthresholds
Topics
"Credit Quality Thresholds" on page 8-57
"Group Credit Ratings" on page 8-16
"Estimation of Transition Probabilities" on page 8-2
"Estimate Transition Probabilities for Different Rating Scales" on page 8-5
"Estimate Probabilities for Different Segments" on page 8-21
External Websites
Credit Risk Modeling with MATLAB ( 53 min 09 sec )
Forecasting Corporate Default Rates with MATLAB (54 min 36 sec )
Introduced in R2011b

## tsaccel

Acceleration between times

Note tsaccel has been partially removed and will no longer accept a fints object (tsobj) argument. Use a vector, matrix, timetable, or table instead for financial time series.

Use fts2timetable to convert a fints object to a timetable object.

## Syntax

```
acceleration = tsaccel(Data)
acceleration = tsaccel(___,Name,Value)
```


## Description

acceleration = tsaccel (Data) calculates the acceleration of a data series with time distance of $n$ periods.
$n$ periods.
acceleration $=$ tsaccel( $\qquad$ , Name, Value) adds optional name-value pair arguments.

## Examples

## Calculate the Acceleration of a Data Series for a Stock

Load the file SimulatedStock. mat, which provides a timetable (TMW) for financial data for TMW stock.
load SimulatedStock.mat
TMW.Volume $=$ []; \% remove VOLUME field

```
acceleration = tsaccel(TMW);
plot(acceleration.Time,acceleration.Variables)
legend('OPEN','HIGH','LOW','CLOSE')
title('Acceleration for TMW')
```

Acceleration for TMW


## Input Arguments

## Data - Data with high, low, open, close information

vector | matrix | table | timetable
Data with high, low, open, close information, specified as a vector, matrix, table, or timetable. For vector input, Data is a column vector of high, low, open, and closing prices
stored in the corresponding columns. For matrix input, Data is an M-by-N column-oriented matrix of high, low, open, and closing prices stored in the corresponding columns.
Timetables and tables with M rows must contain variables named 'High', 'Low', 'Open ', and 'Close' (case insensitive).

Data Types: double | table|timetable

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, ... , NameN, ValueN.

Example: accelearation = tsaccel(TMW,'NumPeriods',10,'Datatype',1)

## NumPeriods - Period difference for acceleration

12 (default) | positive integer
Period difference for acceleration, specified as the comma-separated pair consisting of ' NumPeriods ' and a scalar positive integer.

## Data Types: double

## Datatype - Indicates if Data contains the data itself or the momentum of the data

0 (Data contains the data itself) (default) | integer with values 0 or 1
Indicates if Data contains the data itself or the momentum of the data, specified as the comma-separated pair consisting of 'Datatype' and a scalar integer with a value of:

- 0 - Data contains the data itself.
- 1 - Data contains the momentum of the data.

Data Types: double

## Output Arguments

acceleration - Acceleration series<br>vector | matrix | table | timetable

Acceleration series, returned with the same number of rows $(M)$ and columns $(N)$ and the same type (vector, matrix, table, or timetable) as the input Data.

## Definitions

## Acceleration

Acceleration is defined as the difference of two momentum series separated by $n$ periods.
Acceleration is the difference of the current momentum with the momentum $n$ periods ago. By default, acceleration is based on 12-period difference.

## References

[1] Kaufman, P. J. The New Commodity Trading Systems and Methods. John Wiley and Sons, New York, 1987.

## See Also

table|timetable

## Topics

"Using Timetables in Finance"
"Convert Financial Time Series Objects fints to Timetables" on page 12-2

## Introduced before R2006a

## tsmom

Momentum between times

Note tsmom has been partially removed and will no longer accept a fints object (tsobj) argument. Use a vector, matrix, timetable, or table instead for financial time series.

Use fts2timetable to convert a fints object to a timetable object.

## Syntax

```
momentum = tsmom(Data)
momentum = tsmom(
```

$\qquad$

``` ,Name, Value)
```


## Description

momentum $=$ tsmom(Data) calculates the momentum of a data series with time distance of $n$ periods.

```
momentum = tsmom(
```

$\qquad$

``` , Name, Value) adds optional name-value pair arguments.
```


## Examples

## Calculate the Momentum for a Data Series for a Stock

Load the file SimulatedStock. mat, which provides a timetable (TMW) for financial data for TMW stock.

```
load SimulatedStock.mat
TMW.Volume = []; % remove VOLUME field
momentum = tsmom(TMW);
plot(momentum.Time,momentum.Variables)
legend('OPEN','HIGH','LOW','CLOSE')
title('Acceleration for TMW')
```


## Acceleration for TMW



## Input Arguments

## Data - Data with high, low, open, close information

vector | matrix | table | timetable
Data with high, low, open, close information, specified as a vector, matrix, table, or timetable. For vector input, Data is a column vector. For matrix input, Data is an M-by-N column oriented matrix. Timetables and tables with M rows can contain variables named 'High', 'Low', 'Open', and 'Close' (case insensitive).

Data Types: double| table|timetable

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Example: momentum = tsmom(TMW,'NumPeriods',15)

## NumPeriods - Period difference for momentum

12 (default) | positive integer
Period difference for momentum, specified as the comma-separated pair consisting of ' NumPeriods ' and a scalar positive integer.

## Data Types: double

## Output Arguments

## momentum - Momentum series

matrix | table | timetable
Momentum series, returned with the same number of rows $(M)$ and the same type (matrix, table, or timetable) as the input Data.

## Definitions

## Momentum Series

Momentum series is the difference of the current data with the data $n$ periods ago. By default, momentum is based on 12-period difference.

## References

[1] Kaufman, P. J. The New Commodity Trading Systems and Methods. John Wiley and Sons, New York, 1987.
See Alsotable|timetable|tsaccel
Topics
"Using Timetables in Finance"
"Convert Financial Time Series Objects fints to Timetables" on page 12-2
Introduced before R2006a

## tsmovavg

Moving average

Note tsmovavg will be removed in a future release. Use timetable instead. For more information, see "Convert Financial Time Series Objects fints to Timetables".
tsmovavg calculates the simple, exponential, triangular, weighted, and modified moving average of a vector or fints object of data. For information on working with financial time series (fints objects) data, see "Working with Financial Time Series Objects" on page 13-2.

## Syntax

```
output = tsmovavg(tsobj,'s',lag)
output = tsmovavg(vector,'s',lag,dim)
output = tsmovavg(tsobj,'e',timeperiod)
output = tsmovavg(vector,'e',timeperiod,dim)
output = tsmovavg(tsobj,'t',numperiod)
output = tsmovavg(vector,'t',numperiod,dim)
output = tsmovavg(tsobj,'w',weights)
output = tsmovavg(vector,'w',weights,dim)
output = tsmovavg(tsobj,'m',numperiod)
output = tsmovavg(vector,'m',numperiod,dim)
```


## Description

output = tsmovavg(tsobj,'s',lag) returns the simple moving average by for financial time series object, tsobj. lag indicates the number of previous data points used with the current data point when calculating the moving average.
output = tsmovavg(vector,'s',lag, dim) returns the simple moving average for a vector. lag indicates the number of previous data points used with the current data point when calculating the moving average.
output = tsmovavg(tsobj,'e',timeperiod) returns the exponential weighted moving average for financial time series object, tsobj. The exponential moving average is a weighted moving average, where timeperiod specifies the time period. Exponential moving averages reduce the lag by applying more weight to recent prices. For example, a 10 -period exponential moving average weights the most recent price by 18.18\%. Exponential Percentage $=2 /($ TIMEPER +1$)$ or $2 /($ WINDOW_SIZE +1$)$.
output = tsmovavg(vector,'e',timeperiod,dim) returns the exponential weighted moving average for a vector. The exponential moving average is a weighted moving average, where timeperiod specifies the time period. Exponential moving averages reduce the lag by applying more weight to recent prices. For example, a 10period exponential moving average weights the most recent price by 18.18\%. (2) (timeperiod + 1)).
output = tsmovavg(tsobj, 't', numperiod) returns the triangular moving average for financial time series object, tsobj. The triangular moving average double-smooths the data. tsmovavg calculates the first simple moving average with window width of ceil (numperiod +1 )/2. Then it calculates a second simple moving average on the first moving average with the same window size.
output = tsmovavg(vector,'t', numperiod, dim) returns the triangular moving average for a vector. The triangular moving average double-smooths the data. tsmovavg calculates the first simple moving average with window width of ceil (numperiod $+1) / 2$. Then it calculates a second simple moving average on the first moving average with the same window size.
output = tsmovavg(tsobj, 'w', weights) returns the weighted moving average for the financial time series object, tsobj, by supplying weights for each element in the moving window. The length of the weight vector determines the size of the window. If larger weight factors are used for more recent prices and smaller factors for previous prices, the trend is more responsive to recent changes.
output = tsmovavg(vector, 'w' , weights, dim) returns the weighted moving average for the vector by supplying weights for each element in the moving window. The length of the weight vector determines the size of the window. If larger weight factors are used for more recent prices and smaller factors for previous prices, the trend is more responsive to recent changes.
output = tsmovavg(tsobj,'m', numperiod) returns the modified moving average for the financial time series object, tsobj. The modified moving average is similar to the simple moving average. Consider the argument numperiod to be the lag of the simple moving average. The first modified moving average is calculated like a simple moving average. Subsequent values are calculated by adding the new price and subtracting the last average from the resulting sum.
output = tsmovavg(vector,'m', numperiod, dim) returns the modified moving average for the vector. The modified moving average is similar to the simple moving average. Consider the argument numperiod to be the lag of the simple moving average. The first modified moving average is calculated like a simple moving average. Subsequent values are calculated by adding the new price and subtracting the last average from the resulting sum.

## Examples

## Compute Five Forms of Moving Averages Using a Financial Time Series Object

Load the financial time series object, dis for Disney stock and look at the weekly data for this time series.

```
load disney.mat
weekly = toweekly(dis);
```

Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in
dates $=$ (weekly.dates);
price $=$ fts2mat(weekly.CLOSE);

Warning: FINTS will be removed in a future release. Use TIMETABLE instead. For more in
Set the|lag| input argument for the window size for the moving average.
window_size = 12;
Calculate the simple moving average.

```
simple = tsmovavg(price,'s',window_size,1);
Warning: TSMOVAVG will be removed in a future release. Use MOVAVG instead.
```

Calculate the exponential weighted moving average moving average.

```
exp = tsmovavg(price,'e',window_size,1);
```

Warning: TSMOVAVG will be removed in a future release. Use MOVAVG instead.
Calculate the triangular moving average moving average.
tri = tsmovavg(price,'t',window_size,1);
Warning: TSMOVAVG will be removed in a future release. Use MOVAVG instead.
Calculate the weighted moving average moving average.

```
semi_gaussian = [0.026 0.045 0.071 0.1 0.12 0.138];
semi_gaussian = [semi_gaussian fliplr(semi_gaussian)];
weighted = tsmovavg(price,'w',semi_gaussian,1);
Warning: TSMOVAVG will be removed in a future release. Use MOVAVG instead.
```

Calculate the modified moving average moving average.

```
modif = tsmovavg(price,'m',window_size,1);
```

Warning: TSMOVAVG will be removed in a future release. Use MOVAVG instead.

Plot the results for the five moving average calculations for Disney stock.

```
plot(dates,price,...
    dates,simple,...
    dates,exp,...
    dates,tri,...
    dates,weighted,...
    dates,modif)
datetick
legend('Stock Price','Simple','Exponential','Triangular','Weighted',...
    'Modified','Location','NorthWest')
title('Disney Weekly Price & Moving Averages')
```



## Input Arguments

## tsobj - Financial time series object

object
Financial time series object specified using a time series object created using fints.
's' - Indicator for simple moving average

## character vector

lag is the parameter indicating the number of previous data points to be used in conjunction with the current data point when calculating the simple moving average.

## lag - Number of previous data points

## nonnegative integer

Number of previous data points specified as a nonnegative integer. Lag indicates the window size or number of periods of the moving average.

## vector - Set of observations

vector or matrix
Set of observations specified as a vector or matrix.

## dim - dimension to operate along

positive integer with value 1 or 2
Dimension to operate along, specified as a positive integer with a value of 1 or 2 . dim is an optional input argument, and if it is not included as an input, the default value 2 is assumed. The default of dim = 2 indicates a row-oriented matrix, where each row is a variable and each column is an observation.

If $\operatorname{dim}=1$, the input is assumed to be a column vector or column-oriented matrix, where each column is a variable and each row an observation.

## 'e' - Indicator for exponential moving average

character vector
Exponential moving average is a weighted moving average, where timeperiod is the time period of the exponential moving average. Exponential moving averages reduce the lag by applying more weight to recent prices. For example, a 10 period exponential moving average weights the most recent price by $18.18 \%$.

```
Exponential Percentage = 2/(TIMEPER + 1) or 2/(WINDOW_SIZE + 1)
```

timeperiod - Length of time period
nonnegative integer
Length of time period specified as a nonnegative integer.

## 't' - Indicator for triangular moving average <br> character vector

Triangular moving average is a double-smoothing of the data. The first simple moving average is calculated with a window width of ceil (numperiod +1 )/2. Then a second
simple moving average is calculated on the first moving average with the same window size.
' $m$ ' - Indicator for modified moving average
character vector
The modified moving average is similar to the simple moving average. Consider the argument numperiod to be the lag of the simple moving average. The first modified moving average is calculated like a simple moving average. Subsequent values are calculated by adding the new price and subtracting the last average from the resulting sum.

## numperiod - Number of periods considered

nonnegative integer
Number of periods considered specified as a nonnegative integer.

## 'w' - Indicator for weighted moving average

character vector
A weighted moving average is calculated with a weight vector, weights. The length of the weight vector determines the size of the window. If larger weight factors are used for more recent prices and smaller factors for previous prices, the trend is more responsive to recent changes.

## weights - Weights for each element in the moving window <br> vector of weights

Weights for each element in the window specified as a vector of weights.

## Output Arguments

## output - Moving average calculation

vector or matrix
Moving average calculation returned as a vector or matrix. The output returned from tsmovavg is identical in format to the input.

## References

[1] Achelis, Steven B. Technical Analysis from A to Z. Second Edition. McGraw-Hill, 1995, pp. 184-192.

See Also<br>boxcox|convert2sur|convertto|diff|fillts|filter|lagts|leadts|mean | peravg| resamplets | smoothts

## Topics

"Data Transformation and Frequency Conversion" on page 13-13
"Working with Financial Time Series Objects" on page 13-2
"Creating a Financial Time Series Object" on page 14-11
"Indexing a Financial Time Series Object" on page 13-22
"Financial Time Series Operations" on page 13-7

## Introduced before R2006a

## typprice

Typical price

Note typprice has been partially removed and will no longer accept a fints object (tsobj) argument. Use a matrix, timetable, or table instead for financial time series.

Use fts2timetable to convert a fints object to a timetable object.

## Syntax

TypicalPrice = typprice(Data)

## Description

TypicalPrice = typprice(Data) calculates the typical prices from the series of high, low, and closing prices. The typical price is the average of the high, low, and closing prices for each period.

## Examples

## Calculate the Typical Price for a Data Series for a Stock

Load the file SimulatedStock.mat, which provides a timetable (TMW) for financial data for TMW stock.

```
load SimulatedStock.mat
TypicalPrice = typprice(TMW);
plot(TypicalPrice.Time,TypicalPrice.TypicalPrice)
title('Typical Price for TMW')
```

Typical Price for TMW


## Input Arguments

## Data - Data for high, low, and closing prices

matrix | table | timetable
Data for high, low, and closing prices, specified as a matrix, table, or timetable. For matrix input, Data is an M-by-3 matrix of high, low, and closing prices stored in the corresponding columns. Timetables and tables with M rows must contain variables named 'High', 'Low', and 'Close' (case insensitive).
Data Types: double| table | timetable

## Output Arguments

## TypicalPrice - Typical price series <br> matrix | table | timetable

Typical price series, returned with the same number of rows $(M)$ and the same type (matrix, table, or timetable) as the input Data.

## References

[1] Achelis, S. B. Technical Analysis from A to Z. Second Edition. McGraw-Hill, 1995, pp. 291-292.

## See Also

medprice|table|timetable|wclose

## Topics

"Using Timetables in Finance"
"Convert Financial Time Series Objects fints to Timetables" on page 12-2
Introduced before R2006a

## uicalendar

Graphical calendar

## Syntax

```
uicalendar(Name,Value)
```


## Description

uicalendar(Name, Value) supports a customizable graphical calendar that interfaces with one or more uicontrol. uicalendar populates one or more uicontrol with userselected dates.

## Examples

## Use uicalendar with an uicontrol

Create an uicontrol:

```
textH1 = uicontrol('style', 'edit', 'position', [10 10 100 20])
```

textH1 =
UIControl with properties:
Style: 'edit'
String:
BackgroundColor: [0.9400 0.9400 0.9400]
Callback:
Value: 0
Position: [10 10100 20]
Units: 'pixels'
Call UICalendar:


Select a date and click OK.
Alternatively, you can use datetime arrays for InitDate and Holiday.
uicalendar('InitDate',datetime('15-Mar-2015','Locale','en_US'),'Holiday',datetime('16-Mar-2015','Locale','en_US'))

| A UICalendar |  |  |  | $\square$ | 回 | - |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 4 | March |  |  | 2015 | $\checkmark$ | - |
| Su | Mo | Tu | We | Th | Fr | Sa |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 8 | 9 | 10 | 11 | 12 | 13 | 14 |
| 15 | 16 | 17 | 18 | 19 | 20 | 21 |
| 22 | 23 | 24 | 25 | 26 | 27 | 28 |
| 29 | 30 | 31 | 1 | 2 | 3 | 4 |
| 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| Clear | all |  |  | OK |  | cel |

Select a date and click OK. For more information on using uicalendar with an application, see "Example of Using UICalendar with an Application" on page 16-5.

## Input Arguments

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1,Value1, ... , NameN, ValueN.

Example: uicalendar('InitDate',datetime('15-
Mar-2015','Locale','en_US'),'Holiday', datetime('16-
Mar-2015','Locale','en_US'))

## BusDays - Flag to indicate nonbusiness days

0 (Standard calendar without nonbusiness day indicators) (default) | numeric values of 0 or 1

Flag to indicate nonbusiness days, specified using numeric values of 0 or 1 . The values are:

- 0 - (Default) Standard calendar without nonbusiness day indicators.
- 1 - Marks NYSE nonbusiness days in red.


## Data Types: logical

## BusDaySelect - Flag to indicate whether business and nonbusiness days

1 (Allows selections of business and nonbusiness days) (default) | numeric values of 0 or 1
Flag to indicate whether business and nonbusiness days, specified using numeric values of 0 or 1 . The values are:

- 0 - Only allow selection of business days. Nonbusiness days are determined from the following parameters:
- 'BusDays'
- 'Holiday'
- 'Weekend'
- 1 - (Default) Allows selections of business and nonbusiness days.

Data Types: logical

## DateBoxColor - Color of date squares [date R G B]

Color of date squares, specified using [date $R \quad G B$ ], where $[R G B$ ] is the color.

## Data Types: double

## DateStrColor - Color of numeric date number in the date square

[date R G B]
Color of numeric date number in the date square, specified using [date R G B], where [ $R$ G B] is the color.

## Data Types: double

## DestinationUI - Destination object's handles

'string' (default UI property populated with dates) (default) | values are H or \{H, \{Prop\}\}

Destination object's handles, specified with values H or $\{\mathrm{H}$, $\{$ Prop $\}\}$. The values are:

- H - Scalar or vector of the destination object's handles. The default UI property that is populated with the dates is a character vector.
- $\{\mathrm{H}, \quad\{$ Prop $\}\}$ - Cell array of handles and the destination object's UI properties. H must be a scalar or vector and Prop must be a single property character vector or a cell array of property character vectors.

Data Types: char | cell

## Holiday - Holiday dates in calendar

serial date numbers | datetime arrays
Holiday dates in calendar, specified using a scalar or vector of serial date numbers or datetime arrays. The corresponding date character vector of the holiday appears Red.

Data Types: double | datetime

## InitDate - Initial start date when calendar is initialized <br> TODAY (default) | serial date number | datetime array | date character vector

Initial start date when calendar is initialized, specified with date values using a serial date number, datetime array, or date character vector. The values are:

- Datenum - Numeric or datetime array date value specifying the initial start date when the calendar is initialized. The default date is TODAY.
- Datestr - Date character vector value specifying the initial start date when the calendar is initialized. Datestr must include a Year, Month, and Day (for example, 01-Jan-2006).


## Data Types: double|char|datetime

## InputDateFormat - Format of initial start date

character vector
Format of initial start date (InitDate), specified using a character vector. Seedatestr for date format values.

Data Types: double | datetime

## OutputDateFormat - Format of output date

character vector
Format of output date, specified using a character vector. Seedatestr for date format values.

Data Types: double | datetime

## OutputDateStyle - Style for output date

0 (default) | numeric value of $0,1,2$, or 3
Style for output date, specified using a value of $0,1,2$, or 3 . The values are:

- 0 - (Default) Returns a single date character vector or a cell array (row) of date character vectors. For example, \{'01-Jan-2001, 02-Jan-2001, ...'\}.
- 1 - Returns a single date character vector or a cell (column) array of date character vectors. For example, \{'01-Jan-2001; 02-Jan-2001; ...'\}.
- 2 - Returns a character vector representation of a row vector of datenums. For example, '[732758, 732759, 732760, 732761]'.
- 3 - Returns a character vector representation of a column vector of datenums. For example, '[732758; 732759; 732760; 732761]'.


## Data Types: double

## SelectionType - Flag for date selection

1 (default) | numeric value of 0 or 1
Flag for date selection, specified with using a value of 0 or 1 . The values are:

- 0 - Allows multiple date selections.
- 1 - (Default) Allows only a single date selection.

Data Types: logical

## Weekend - Define weekend days

1 (default) | numeric values of 1 through 7
Define weekend days, specified using a value of 1 through 7 . Weekend days are marked in red. Day0fWeek can be a vector containing the following numeric values:

- 1 - Sunday
- 2 - Monday
- 3-Tuesday
- 4 - Wednesday
- 5 - Thursday
- 6 - Friday
- 7 - Saturday

Also this value can be a vector of length 7 containing 0 's and 1 's. The value 1 indicates a weekend day. The first element of this vector corresponds to Sunday. For example, when Saturday and Sunday are weekend days then WEEKEND = $\left[\begin{array}{llllll}1 & 0 & 0 & 0 & 0\end{array}\right]$.

Data Types: double

## WindowStyle - Window figure properties

Normal (default) | character vector with value of Normal or Model
Window figure properties, specified with using a character vector with a value of Normal or Model. The values are:

- Normal - (Default) Standard figure properties.
- Modal - Modal figures remain stacked above all normal figures and the MATLAB Command Window.


## Data Types: char

## See Also

datetime| holidays

## Topics

"Handle and Convert Dates" on page 2-2
"Trading Calendars User Interface" on page 16-2
"UICalendar User Interface" on page 16-4

Introduced before R2006a

## uminus

Unary minus of financial time series object

Note uminus will be removed in a future release. Use timetable instead. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

uminus

## Description

uminus implements unary minus for a financial time series object.

## See Also <br> uplus

## Topics

"Financial Time Series Operations" on page 13-7

Introduced before R2006a

## uplus

Unary plus of financial time series object

Note uplus will be removed in a future release. Use timetable instead. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

uplus

## Description

uplus implements unary plus for a financial time series object.

## See Also

uminus

## Topics

"Financial Time Series Operations" on page 13-7

Introduced before R2006a

## var

Variance

Note var will be removed in a future release. Use timetable instead. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

```
y = var(X)
y = var(X,1)
y = var(X,W)
y = var(X,W,DIM)
```


## Arguments

| $X$ | Financial times series object. |
| :--- | :--- |
| $W$ | Weight vector used in calculating variance. |
| DIM | Dimension of $X$ used in calculating variance. |

## Description

var supports financial time series objects based on the MATLAB var function. See var. $y=\operatorname{var}(X)$, if $X$ is a financial time series object and returns the variance of each series.
var normalizes y by $\mathrm{N}-1$ if $\mathrm{N}>1$, where N is the sample size. This is an unbiased estimator of the variance of the population from which $X$ is drawn, as long as $X$ consists of independent, identically distributed samples. For $N=1, y$ is normalized by $N$.
$y=\operatorname{var}(X, 1)$ normalizes by $N$ and produces the second moment of the sample about its mean. $\operatorname{var}(X, 0)$ is the same as $\operatorname{var}(X)$.
$y=\operatorname{var}(X, W)$ computes the variance using the weight vector $W$. The length of $W$ must equal the length of the dimension over which var operates, and its elements must be nonnegative. var normalizes $W$ to sum to 1 . Use a value of 0 for $W$ to use the default normalization by $N-1$, or use a value of 1 to use $N$.
$y=\operatorname{var}(X, W, D I M)$ takes the variance along the dimension DIM of $X$.

## Examples

The variance is the square of the standard deviation. Consider if

```
    f = fints((today:today+1)', [4 -2 1; 9 5 7])
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints (line 165)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/display (line 66)
f =
    desc: (none)
    freq: Unknown (0)
    'dates: (2)' 'series1: (2)' 'series2: (2)' 'series3: (2)
```



then

```
var(f, 0, 1)
```

is
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/var (line 49)
[12.5 24.5 18.0]
and
$\operatorname{var}(f, 0,2)$
is

```
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/var (line 49)
[9.0; 4.0]
```


## See Also

corrcoef | cov | mean | std

## Introduced before R2006a

## vertcat

Concatenate financial time series objects vertically

Note vertcat will be removed in a future release. Use timetable instead. For more information, see "Convert Financial Time Series Objects fints to Timetables".

## Syntax

vertcat

## Description

vertcat implements vertical concatenation of financial time series objects. vertcat essentially adds data points to a time series object. Objects to be vertically concatenated must not have any duplicate dates and/or times or any overlapping dates and/or times. The description fields are concatenated as well. They are separated by ||.

## Examples

Create two financial time series objects with daily frequencies:

```
myfts = fints((today:today+4)', (1:5)', 'DataSeries', 'd')
yourfts = fints((today+5:today+9)', (11:15)', 'DataSeries', 'd')
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints (line 165)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/display (line 66)
myfts =
    desc: (none)
    freq: Daily (1)
    'dates: (5)' 'DataSeries: (5)'
```

| '02-0ct-2017' | $[$ | $1]$ |
| :--- | :--- | :--- |
| '03-0ct-2017' | $[$ | $2]$ |
| '04-0ct-2017' | $[$ | $3]$ |
| '05-0ct-2017' | $[$ | $4]$ |
| '06-0ct-2017' | $[$ | $5]$ |

```
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints (line 165)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/display (line 66)
yourfts =
desc: (none)
freq: Daily (1)
'dates: (5)' 'DataSeries: (5)'
'07-0ct-2017' [ 11]
'08-0ct-2017' [ 12]
'09-0ct-2017' [ 13]
'10-0ct-2017' [ 14]
'11-0ct-2017' [ 15]
```

Use vertcat to concatenate them vertically:

```
newfts1 = [myfts; yourfts]
```

```
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/vertcat (line 35)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/display (line 66)
newfts1 =
desc: ||
freq: Daily (1)
'dates: (10)' 'DataSeries: (10)'
'11-Dec-2001' [ 1]
'12-Dec-2001' [ 2]
'13-Dec-2001' [ 3]
'14-Dec-2001' [ 4]
'15-Dec-2001' [ 5]
'16-Dec-2001' [ 11]
'17-Dec-2001' [ 12]
```

| '18-Dec-2001' | $[$ | $13]$ |
| :---: | :---: | :---: |
| '19-Dec-2001' | $[$ | $14]$ |
| '20-Dec-2001' | $[$ | $15]$ |

Create two financial time series objects with different frequencies:

```
myfts = fints((today:today+4)', (1:5)', 'DataSeries', 'd');
hisfts = fints((today+5:7:today+34)', (11:15)', 'DataSeries',..
    w')
    Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints (line 165)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints (line 165)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/display (line 66)
hisfts =
desc: (none)
freq: Weekly (2)
\begin{tabular}{lll} 
'dates: (5)' & 'DataSeries: & \((5)\) ' \\
'07-0ct-2017' & {\([\)} & \(11]\) \\
'14-0ct-2017' & {\([\)} & \(12]\) \\
'21-0ct-2017' & {\([\)} & \(13]\) \\
'28-0ct-2017' & {\([\)} & \(14]\) \\
'04-Nov-2017' & {\([\)} & \(15]\)
\end{tabular}
```

Concatenate these two objects vertically:

```
newfts2 = [myfts; hisfts]
```

    Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
    > In fints/vertcat (line 35)
Warning: FINTS will be removed in a future release. Use TIMETABLE instead.
> In fints/display (line 66)
newfts2 =
desc: ||
freq: Unknown (0)
'dates: (10)' 'DataSeries: (10)'
'11-Dec-2001' [ 1]
'12-Dec-2001' [ 2]

| '13-Dec-2001' | $[$ | $3]$ |
| ---: | :---: | ---: |
| '14-Dec-2001' | $[$ | $4]$ |
| '15-Dec-2001' | $[$ | $5]$ |
| '16-Dec-2001' | $[$ | $11]$ |
| '23-Dec-2001' $30-$ Dec-2001' | $[$ | $12]$ |
| '06-Jan-2002' | $[$ | $13]$ |
| '13-Jan-2002' | $[$ | $15]$ |

If all frequency indicators are the same, the new object has the same frequency indicator. However, if one of the concatenated objects has a different freq from the other(s), the frequency of the resulting object is set to Unknown (0). In these examples, newfts 1 has Daily frequency, while newfts2 has Unknown (0) frequency.

## See Also

horzcat

## Topics

"Financial Time Series Operations" on page 13-7

## Introduced before R2006a

## volarea

Price and volume chart

Note volarea is updated to accept data input as a matrix, timetable, or table.
The syntax for volarea has changed. Previously, when using table input, the first column of dates could be serial date numbers, date character vectors, or datetime arrays, and you were required to have specific number of columns.

When using table input, the new syntax for volarea supports:

- No need for time information. If you want to pass in date information, use timetable input.
- No requirement of specific number of columns. However, you must provide valid column names. volarea must contain a column named 'price' (case insensitive).


## Syntax

volarea(Data)
h = volarea(ax, Data)

## Description

volarea (Data) plots a chart from a series of price and traded volume of a security.
$\mathrm{h}=$ volarea(ax, Data) adds an optional argument for ax.

## Examples

## Plot Asset Date, Price, and Volume on a Single Axis for a Stock

Load the file SimulatedStock.mat, which provides a timetable (TMW) for financial data for TMW stock. This example shows how to plot asset date, price, and volume on a single
axis, given asset TMW containing asset price and volume in most recent 21 days. Note that the variable name of asset price is be renamed to 'Price' (case insensitive).
load SimulatedStock.mat;
TMW.Properties.VariableNames\{'Close'\} = 'Price'; volarea(TMW(end-20:end,:)) title('Price and Volume Chart for TMW')


Jul 27 Jul 30 Aug 02 Aug 05 Aug 08 Aug 11 Aug 14 Aug 17 Aug 20 Aug 23
2016

## Input Arguments

## Data - Data for prices and traded volume

matrix | table | timetable

Data for prices and traded volume, specified as a matrix, table, or timetable. For matrix input, Data is an M-by-2 matrix of prices and traded volume. Timetables and tables with M rows must contain variables named 'Price' and 'Volume' (case insensitive).

## Data Types: double|table|timetable

## ax - Valid axis object

current axes (ax = gca) (default) | axes object
(Optional) Valid axis object, specified as an axes object. The volarea plot is created in the axes specified by ax instead of in the current axes ( $a x=\mathrm{gca}$ ). The option ax can precede any of the input argument combinations.

## Data Types: object

## Output Arguments

## h - Graphic handle of the figure <br> handle object

Graphic handle of the figure, returned as a handle object.

## See Also

candle| highlow| kagi | linebreak|movavg|pointfig|table|timetable| volarea

## Topics

"Using Timetables in Finance"
"Convert Financial Time Series Objects fints to Timetables" on page 12-2
Introduced in R2008a

## volroc

Volume rate of change

Note volroc has been partially removed and will no longer accept a fints object (tsobj) argument. Use a vector, timetable, or table instead for financial time series.

Use fts2timetable to convert a fints object to a timetable object.

## Syntax

```
volumeChangeRate = volroc(Data)
volumeChangeRate = volroc(
```

$\qquad$

``` ,Name, Value)
```


## Description

volumeChangeRate $=$ volroc (Data) calculates the volume rate-of-change from a data series of volume traded. The volume rate-of-change is calculated between the current volume and the volume $n$ periods ago. By default, the Volume rate of change is based on a 12-period difference.
volumeChangeRate $=$ volroc $($ $\qquad$ ,Name, Value) adds optional name-value pair arguments.

## Examples

## Calculate the Volume Rate of Change for a Data Series for a Stock

Load the file SimulatedStock.mat, which provides a timetable (TMW) for financial data for TMW stock.
load SimulatedStock.mat
volumeChangeRate $=$ volroc(TMW);

```
plot(volumeChangeRate.Time,volumeChangeRate.VolumeChangeRate)
title('Volume Rate of Change for TMW')
```



## Input Arguments

## Data - Data for volume traded

vector | table | timetable
Data for volume traded, specified as a vector, table, or timetable. For vector input, Data is an M-by-1 column vector of volume traded. Timetables and tables with M rows must contain a variable named 'Volume' (case insensitive).

Data Types: double | table | timetable

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN , ValueN.

Example: volumeChangeRate $=$ volroc(TMW, 'NumPeriods',18)

## NumPeriods - Period difference for volumeChangeRate 12 (default) | positive integer

Period difference for volumeChangeRate, specified as the comma-separated pair consisting of 'NumPeriods' and a scalar positive integer.
Data Types: double

## Output Arguments

volumeChangeRate - Volume rate of change series
vector | table | timetable
Volume rate of change series, returned with the same number of rows ( M ) and the same type (vector, table, or timetable) as the input Data.

## References

[1] Achelis, S. B. Technical Analysis from A to Z. Second Edition. McGraw-Hill, 1995, pp. 310-311.

## See Also

prcroc|table|timetable

## Topics

"Using Timetables in Finance"

## "Convert Financial Time Series Objects fints to Timetables" on page 12-2

## Introduced before R2006a

## wclose

Weighted close

Note wclose has been partially removed and will no longer accept a fints object (tsobj) argument. Use a matrix, timetable, or table instead for financial time series.

Use fts2timetable to convert a fints object to a timetable object.

## Syntax

WeightedClose = wclose(Data)

## Description

WeightedClose $=$ wclose(Data) calculates the weighted closing prices from the series of high, low, and closing prices. The weighted closing price is the average of twice the closing price plus the high and low prices.

## Examples

## Calculate the Weighted Closing Prices for a Data Series for a Stock

Load the file SimulatedStock. mat, which provides a timetable (TMW) for financial data for TMW stock.

```
load SimulatedStock.mat
WeightedClose = wclose(TMW);
plot(WeightedClose.Time,WeightedClose.WeightedClose)
title('Weighted Closing Prices for TMW')
```


## Weighted Closing Prices for TMW



## Input Arguments

## Data - Data for high, low, and closing prices

matrix | table | timetable
Data for high, low, and closing prices, specified as a matrix, table, or timetable. For matrix input, Data is an M-by-3 matrix of high, low, and closing prices stored in the corresponding columns. Timetables and tables with M rows must contain variables named 'High', 'Low', and 'Close' (case insensitive).
Data Types: double | table | timetable

## Output Arguments

## WeightedClose - Weighted closing price series <br> matrix | table | timetable

Weighted closing price series, returned with the same number of rows ( $M$ ) and the same type (matrix, table, or timetable) as the input Data.

## References

[1] Achelis, S. B. Technical Analysis from A to Z. Second Edition. McGraw-Hill, 1995, pp. 312-313.

## See Also

medprice| table|timetable|typprice

## Topics

"Using Timetables in Finance"
"Convert Financial Time Series Objects fints to Timetables" on page 12-2
Introduced before R2006a

## weeknum

Week in year

## Syntax

[N] = weeknum(D)
[N] = weeknum( $\qquad$ , W, E)

## Description

[N] = weeknum(D) returns the week in year. The weeknum function considers the week containing January 1 to be the first week of the year.
[N] = weeknum (__ $\mathrm{W}, \mathrm{E}$ ) returns the week in year using the optional input arguments for $W$ and $E$. The weeknum function considers the week containing January 1 to be the first week of the year.

## Examples

## Determine the Week of the Year

Determine the week of the year using a serial date number.

```
N = weeknum(728647)
N = 52
```

Determine the week of the year using a character vector.

```
N = weeknum('19-Dec-1994')
\(\mathrm{N}=52\)
```

Determine the week of the year using a datetime array.

```
N = weeknum(datetime('19-Dec-1994','Locale','en_US'))
N = 52
```

The first week of the year must have at least four days in it. For example, January 8, 2004 was a Thursday. The European standard is used because the first week of the year is the first week longer than three days.

```
weeknum('08-jan-2004', 1, 1)
ans = 1
```

You can also use weeknum with datenum.

```
weeknum(datenum('01-Jan-2004'):datenum('08-Jan-2004'))
```

ans $=1 \times 8$
$\begin{array}{llllllll}1 & 1 & 1 & 2 & 2 & 2 & 2 & 2\end{array}$

The default start day of the week is Sunday. Every day after, and including the first Sunday of the year (04-Jan-2004), returns 2 denoting the second week. In this case, the first of week of the year started before January 1, 2004. You can also use weeknum with datenum and specify a $D$ value of 5 to indicate that the weeks start on Thursday.

```
weeknum(datenum('01-Jan-2004'):datenum('08-Jan-2004'),5)
ans = 1\times8
    1 1 1 1 1 
```

The first week of the year that has four or more days, based on the specified start day, is considered week one (even if this is not the first week in the calendar). Any day falling in (or before) this week is given a week number of 1 .

## Input Arguments

D - Date to determine week in year<br>serial date number | data character vector | datetime array

Date to determine week in year, specified as a serial date number, date character vector, or datetime array.

Serial date numbers can be a matrix. Date character vectors can be specified as a onedimensional cell array of character vectors. All the date character vectors must have the same format.

Use the function datestr to convert serial date numbers to formatted date character vectors.

## Data Types: single|double|char|datetime

## W - Day a week begins

1 (default) | integer with value 1 through $7 \mid$ vector of integers with values 1 through 7
Day a week begins, specified as an integer or a vector of integers from 1 through 7 .

- 1 - Sunday (default)
- 2 - Monday
- 3-Tuesday
- 4 - Wednesday
- 5 - Thursday
- 6 - Friday
- 7 - Saturday

The weeknum function considers the week containing January 1 to be the first week of the year.
Data Types: single | double

## E Flag indicates if week of year display is European standard <br> 0 (default) | numeric with values 1 or 0

Flag indicates if week of year display is European standard, specified as 1 (to use the European standard) or 0 (not to use the European standard).

The European standard considers first week of year to be first week longer than three days, offset by the given week's start day.

Data Types: logical

## Output Arguments

N - Week number of the year, given D
numeric | column vector
Week number of the year, given $D$, returned as a numeric value, given $D$, a serial date number, date character vector, or datetime array. If $D$ is a one-dimensional cell array of character vectors, then weeknum returns a column vector of $M$ week numbers, where $M$ is the number of character vectors in $D$.

If the optional input arguments $W$ and $E$ are defined, the week of the year is in the European standard.

## See Also

datenum|datestr|datetime|datevec|day

## Topics

"Handle and Convert Dates" on page 2-2
Introduced before R2006a

## weights2holdings

Portfolio values and weights into holdings

## Syntax

Holdings = weights2holdings(Values, Weights, Prices)

## Arguments

| Values | Scalar or number of portfolios (NPORTS) vector containing portfolio <br> values. |
| :--- | :--- |
| Weights | NPORTS by number of assets (NASSETS) matrix with portfolio <br> weights. The weights sum to the value of a Budget constraint, <br> which is usually 1. (See holdings2weights for information about <br> budget constraints.) |
| Prices | NASSETS vector of prices. |

## Description

Holdings $=$ weights2holdings(Values, Weights, Prices) converts portfolio values and weights into portfolio holdings.

Holdings is a NPORTS-by-NASSETS matrix containing the holdings of NPORTS portfolios that contain NASSETS assets.

Note This function does not create round-lot positions. Holdings are floating-point values.

## See Also

holdings2weights

[^18]
## willad

Williams Accumulation/Distribution line

Note willad has been partially removed and will no longer accept a fints object (tsobj) argument. Use a matrix, timetable, or table instead for financial time series.

Use fts2timetable to convert a fints object to a timetable object.

## Syntax

WADLine = willad(Data)

## Description

WADLine $=$ willad(Data) calculates the Williams Accumulation/Distribution line from the series of high, low, and closing prices.

## Examples

## Calculate the Williams Accumulation/Distribution Line for a Data Series for a Stock

Load the file SimulatedStock.mat, which provides a timetable (TMW) for financial data for TMW stock.
load SimulatedStock.mat
WADLine = willad(TMW);
plot(WADLine.Time,WADLine.Willad)
title('Williams A/D Line for TMW')


## Input Arguments

## Data - Data for high, low, and closing prices

matrix | table | timetable
Data for high, low, and closing prices, specified as a matrix, table, or timetable. For matrix input, Data is an M-by-3 matrix of high, low, and closing prices stored in the corresponding columns. Timetables and tables with M rows must contain variables named 'High', 'Low', and 'Close' (case insensitive).
Data Types: double| table | timetable

## Output Arguments

## WADLine - Williams Accumulation/Distribution line matrix | table | timetable

Williams Accumulation/Distribution line, returned with the same number of rows ( $M$ ) and the same type (matrix, table, or timetable) as the input Data.

## References

[1] Achelis, S. B. Technical Analysis from A to Z. Second Edition. McGraw-Hill, 1995, pp. 314-315.

## See Also

adline | adosc| table|timetable|willpctr

## Topics

"Using Timetables in Finance"
"Convert Financial Time Series Objects fints to Timetables" on page 12-2

Introduced before R2006a

## willpctr

Williams \%R

Note willpctr has been partially removed and will no longer accept a fints object (tsobj) argument. Use a matrix, timetable, or table instead for financial time series.

Use fts2timetable to convert a fints object to a timetable object.

## Syntax

PercentR = willpctr(Data)
PercentR = willpctr(__, , Name, Value)

## Description

PercentR = willpctr(Data) calculates the Williams PercentR (\%R) values for a data series of with high, low, and closing prices.

PercentR = willpctr(__ , Name, Value) adds optional name-value pair arguments.

## Examples

## Calculate the Williams \%R for a Data Series for a Stock

Load the file SimulatedStock. mat, which provides a timetable (TMW) for financial data for TMW stock.


## Input Arguments

## Data - Data with high, low, and close information

matrix | table | timetable
Data with high, low, open, close information, specified as a matrix, table, or timetable. For matrix input, Data is an M-by-3 with high, low, and closing prices stored in the corresponding columns. Timetables and tables with M rows must contain variables named 'High', 'Low', and 'Close' (case insensitive).

Data Types: double|table|timetable

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Example: PercentR = willpctr(TMW,'NumPeriods',15)

## NumPeriods - Moving window for Williams PercentR

14 (default) | positive integer
Moving window for Williams PercentR, specified as the comma-separated pair consisting of ' NumPeriods ' and a scalar positive integer.
Data Types: double

## Output Arguments

## PercentR - Williams PercentR series

## matrix | table | timetable

Williams PercentR series, returned with the same number of rows ( $M$ ) and the same type (matrix, table, or timetable) as the input Data.

## Definitions

## Williams \%R

Williams \%R shows the current closing price in relation to the high and low of the past $n$ days.

By default, Williams \%R values are based on 14 periods.

## References

[1] Achelis, S. B. Technical Analysis from A to Z. Second Edition. McGraw-Hill, 1995, pp. 316-317.
See Alsostochosc|table|timetable|willad
Topics"Using Timetables in Finance""Convert Financial Time Series Objects fints to Timetables" on page 12-2
Introduced before R2006a

## wrkdydif

Number of working days between dates

## Syntax

Days = wrkdydif(StartDate,EndDate,Holidays)

## Description

Days = wrkdydif(StartDate, EndDate,Holidays) returns the number of working days between dates StartDate and EndDate inclusive. Holidays is the number of holidays between the given dates, an integer.

## Examples

Determine the Number of Working Days Between a StartDate and EndDate
Determine Days using date character vectors for StartDate and EndDate.

```
Days = wrkdydif('9/1/2000', '9/11/2000', 1)
Days = 6
```

Determine Days using serial date numbers for StartDate and EndDate.
Days = wrkdydif(730730, 730740, 1)
Days $=6$
Determine Days using a datetime array for EndDate.
Days = wrkdydif('9/1/2000', datetime('11-Sep-2000','Locale','en_US'), 1)
Days $=6$

## Input Arguments

## StartDate - Start date <br> serial date number | date character vector | datetime object

Start date, specified as an N -by-1 or 1-by-N vector using serial date numbers, date character vectors, or datetime arrays.

Data Types: double | char | datetime

## EndDate - End date <br> serial date number | date character vector | datetime object

End date, specified as an N -by-1 or 1-by-N vector using serial date numbers, date character vectors, or datetime arrays.

Data Types: double|char|datetime
Holidays - Holidays between StartDate and EndDate vector of integers

Holidays between the StartDate and EndDate, specified as an N -by-1 or 1-by-N vector of integers.

Data Types: single | double

## Output Arguments

## Days - Number of working days between dates StartDate and EndDate inclusive

integer
Number of working days between dates StartDate and EndDate inclusive, returned an N -by-1 or 1 -by-N vector of integers.

## See Also

busdate | datetime | datewrkdy | days365|daysact | daysdif|holidays yearfrac

## Topics

"Handle and Convert Dates" on page 2-2
"Trading Calendars User Interface" on page 16-2
"UICalendar User Interface" on page 16-4

## Introduced before R2006a

## x2mdate

Excel serial date number to MATLAB serial date number or datetime format

## Syntax

MATLABDate = x2mdate(ExcelDateNumber,Convention) MATLABDate $=x 2 m d a t e($ $\qquad$ ,outputType)

## Description

MATLABDate $=$ x2mdate(ExcelDateNumber, Convention) converts Excel serial date numbers to MATLAB serial date numbers or datetime format.

MATLAB date numbers start with $1=$ January 1,0000 A.D., hence there is a difference of 693960 relative to the 1900 date system, or 695422 relative to the 1904 date system. This function is useful with Spreadsheet Link software.

MATLABDate $=x 2 m d a t e($ $\qquad$ , outputType) converts Excel serial date numbers to MATLAB serial date numbers or datetime format using an optional input argument for outputType.

The type of output is determined by an optional outputType input. If outputType is 'datenum ', then MATLABDate is a serial date number. If outputType is 'datetime', then MATLABDate is a datetime array. By default, outputType is 'datenum'.

## Examples

## Convert Excel Serial Date Numbers to MATLAB Dates

Given Excel ${ }^{\circledR}$ date numbers in the 1904 system, convert them to MATLAB ${ }^{\circledR}$ serial date numbers, and then to date character vectors.

ExDates $=\left[\begin{array}{lll}35423 & 35788 & 36153\end{array}\right] ;$
MATLABDate $=$ x2mdate $($ ExDates, 1)

```
MATLABDate = 1\times3
```

    730845731210
    731575
    ```
datestr(MATLABDate)
```

ans $=3 \times 11$ char array
'25-Dec-2000'
'25-Dec-2001'
'25-Dec-2002'

Alternatively, use the optional input outputType to specify 'datetime' to return datetime format.

```
ExDates = [[35423 35788 36153];
MATLABDate = x2mdate(ExDates, 1,'datetime')
MATLABDate = 1x3 datetime array
    25-Dec-2000 25-Dec-2001 25-Dec-2002
```


## Input Arguments

## ExcelDateNumber - Excel serial date number <br> serial date number

Excel serial date number, specified as a scalar or vector of Excel serial date numbers.
Data Types: double

## Convention - Flag for Excel date system

0 (Excel 1900 date system is in effect) (default) | numeric with value 0 or 1
Flag for Excel date system, specified as a scalar or vector as a numeric with a value 0 or 1. Convention must be either a scalar or else must be the same size as ExcelDateNumber.

When Convention $=0$ (default), the Excel 1900 date system is in effect. When Convention = 1, the Excel 1904 date system in used.

In the Excel 1900 date system, the Excel serial date number 1 corresponds to January 1, 1900 A.D. In the Excel 1904 date system, date number 0 is January 1, 1904 A.D.

Due to a software limitation in Excel software, the year 1900 is considered a leap year. As a result, all DATEVALUE's reported by Excel software between Jan. 1, 1900 and Feb. 28, 1900 (inclusive) differs from the values reported by 1. For example:

- In Excel software, Jan. 1, $1900=1$
- In MATLAB, Jan. 1, 1900-693960 (for 1900 date system) $=2$

```
datenum('Jan 1, 1900') - 693960
```

ans =

2

## Data Types: logical

## outputType - Output date format

'datenum' (default)| character vector with values 'datenum' or 'datetime'
Output date format, specified as a character vector with values 'datenum' or 'datetime'. The output MATLABDate is in serial date format if 'datenum' is specified or datetime format if 'datetime' is specified. By default the output is in serial date format.

## Data Types: char

## Output Arguments

## MATLABDate - MATLAB date

serial date numbers | datetime format
MATLAB date, returned as serial date numbers or datetime format.
The type of output is determined by an optional outputType input argument. If outputType is 'datenum', then MATLABDate is a serial date number. If outputType is 'datetime', then MATLABDate is a datetime array. By default, outputType is 'datenum'.

See Also<br>datenum | datestr|datetime |m2xdate<br>\section*{Topics}<br>"Handle and Convert Dates" on page 2-2<br>\section*{Introduced before R2006a}

## xirr

Internal rate of return for nonperiodic cash flow

## Syntax

Return = xirr(CashFlow, CashFlowDates)
Return = xirr(CashFlow, CashFlowDates, Guess, MaxIterations,Basis)

## Description

Return $=$ xirr(CashFlow, CashFlowDates) returns the internal rate of return for a schedule of nonperiodic cash flows.

Return = xirr(CashFlow, CashFlowDates, Guess, MaxIterations,Basis) returns the internal rate of return for a schedule of nonperiodic cash flows with optional inputs.

## Input Arguments

## CashFlow

A vector or matrix of cash flows. If CashFlow is a matrix, each column represents a separate stream of cash flows whose internal rate of return is calculated. The first cash flow of each stream is the initial investment, entered as a negative number.

## CashFlowDates

(Required) CashFlowDates is specified as serial date numbers, date character vectors, or datetime arrays. The size of the input date numbers for CashFlowDates must the same size as CashFlow, Each column of CashFlowDate represents the dates of the corresponding column of CashFlow.

## Guess

The initial estimate of the internal rate of return. Guess is a scalar applied to all streams, or a vector the same length as the number of streams.

Default: 0.1 (10\%)

## MaxIterations

The positive integer number of iterations used by Newton's method to solve the internal rate of return. MaxIterations is a scalar applied to all streams, or a vector the same length as the number of streams.

Default: 50

## Basis

Day-count basis of the instrument. A vector of integers.

- $0=$ actual/actual (default)
- $1=30 / 360$ (SIA)
- 2 = actual/360
- $3=$ actual $/ 365$
- $4=30 / 360$ (BMA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- $9=\mathrm{actual} / 360$ (ICMA)
- $10=$ actual/365 (ICMA)
- $11=30 / 360 \mathrm{E}$ (ICMA)
- $12=$ actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-0
Default: 0

## Output Arguments

## Return

Vector of the annualized internal rate of return of each cash flow stream. A NaN indicates that a solution is not found.

## Examples

Find the internal rate of return for an investment of \$10,000 that returns the following nonperiodic cash flow. The original investment is the first cash flow and is a negative number.

| Cash Flow | Dates |
| :--- | :--- |
| $(\$ 10000)$ | January 12,2007 |
| $\$ 2500$ | February 14,2008 |
| $\$ 2000$ | March 3,2008 |
| $\$ 3000$ | June 14, 2008 |
| $\$ 4000$ | December 1,2008 |

Calculate the internal rate of return for this nonperiodic cash flow:

```
CashFlow = [-10000, 2500, 2000, 3000, 4000];
CashFlowDates = ['01/12/2007'
    '02/14/2008'
    '03/03/2008'
    '06/14/2008'
    '12/01/2008'];
Return = xirr(CashFlow, CashFlowDates)
This returns:
Return =
```

```
    0.1006 (or 10.0644% per annum)
```

```
    0.1006 (or 10.0644% per annum)
```

Alternatively, you can use datetime input to calculate the internal rate of return for this nonperiodic cash flow:

CashFlow = [-10000, 2500, 2000, 3000, 4000];
CashFlowDates = ['01/12/2007'

```
    '02/14/2008'
    '03/03/2008'
    '06/14/2008'
    '12/01/2008'];
CashFlowDates = datetime(CashFlowDates,'Locale','en_US')';
Return = xirr(CashFlow, CashFlowDates)
This returns:
Return =
    0.1006 (or 10.0644% per annum)
```


## References

Brealey and Myers. Principles of Corporate Finance. McGraw-Hill Higher Education, Chapter 5, 2003.

Sharpe, William F., and Gordon J. Alexander. Investments. Englewood Cliffs, NJ: PrenticeHall. 4th ed., 1990.

## See Also

datetime|fvvar|irr|mirr|pvvar

## Topics

"Analyzing and Computing Cash Flows" on page 2-18

## Introduced before R2006a

## year

Year of date

## Syntax

```
Year = year(Date)
Year = year(__,F)
```


## Description

Year = year(Date) returns the year of date given a serial date number or a date character vector.

Year = year (__, F) returns the year of date given a serial date number or a date character vector, Date, using format defined by the optional input F. Date can be a character array where each row corresponds to one date character vector, or a onedimensional cell array of character vectors. All the character vectors in Date must have the same format F. F must designate a supported date format symbol. For more information on supported date formats, see datestr.

## Examples

## Determine the Year of the Date for Various Dates

Find the year for Date using a serial date number.

```
Year = year(731798.776)
Year = 2003
```

Find the year for Date using a date character vector format.

```
Year = year('05-Aug-2003')
Year = 2003
```

Use the optional F argument to designate a country-specific date format for a given Date.

```
Year = year('1999/05/09','yyyy/dd/mm')
Year = 1999
```


## Input Arguments

## Date - Date to determine year

serial date number | date character vector | cell array of date character vectors
Date to determine year, specified as a serial date number or date character vector.
Date can be an array of date character vectors, where each row corresponds to one date character vector, or a one-dimensional cell array of character vectors. All the character vectors in Date must have the same format F. F must designate a supported date format symbol. For more information on supported date formats, see datestr
Data Types: single | double | char | cell

## F - Date format symbol

character vector designating date format
Date format symbol, specified as a character vector to designate the date format symbol for input argument Date. For more information on supported date character vector formats, see datestr. Note, formats with ' Q ' are not accepted.
Data Types: char

## Output Arguments

Year - Numeric representation of the year
nonnegative integer
Numeric representation of the year, returned as a nonnegative integer.

## See Also

datevec | day | month | yeardays

## Topics

"Handle and Convert Dates" on page 2-2
Introduced before R2006a

## yeardays

## Number of days in year

## Syntax

```
Days = yeardays(Year)
Days = yeardays(___,Basis)
```


## Description

Days $=$ yeardays (Year) returns the number of days in the given Year.
Days = yeardays( $\qquad$ , Basis) returns the number of days in the given Year, based on the optional argument Basis for day-count.

## Examples

## Determine the Number of Days in a Given Year

Find the number of days in a given Year.
Days = yeardays(2000)
Days $=366$
Find the number of days in a given Year using the optional argument Basis.
Days = yeardays(2000, 1)
Days = 360

## Input Arguments

## Year - Year to determine days <br> 4 digit integer

## Data Types: single | double

## Basis - Day-count basis

0 (actual/actual) (default) | vector of integers with values $0,1,2,3,4,5,6,7,8,9,10,11,12,13$
Day-count basis, specified as a vector of integers with values
0,1,2,3,4,5,6,7,8,9,10,11,12,13.

- $0=$ actual/actual (default)
- $1=30 / 360$ (SIA)
- 2 = actual/360
- 3 = actual/365
- $4=30 / 360$ (BMA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- $9=$ actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- $11=30 / 360 \mathrm{E}$ (ICMA)
- $12=$ actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-0
Data Types: single| double

## Output Arguments

## Days - Number of days in given Year

nonnegative integer

Number of days in given Year, returned as a nonnegative integer.

## See Also

days360| days365 | daysact | year | yearfrac

## Topics

"Handle and Convert Dates" on page 2-2

## Introduced before R2006a

## yearfrac

Fraction of year between dates

## Syntax

```
YearFraction = yearfrac(StartDate,EndDate,Basis)
```


## Description

YearFraction = yearfrac(StartDate,EndDate,Basis) returns a fraction, in years, based on the number of days between dates StartDate and EndDate using the given day-count Basis.

The number of days in a year ( 365 or 366 ) is equal to the number of days in the calendar year after the StartDate. If EndDate is earlier than StartDate, YearFraction is negative.

All specified arguments must be number of instruments (NUMINST-by-1) or (1-byNUMINST) conforming vectors or scalar arguments.

## Examples

## Compute yearfrac When the Calendar Year After the StartDate is Not a Leap Year

Given a Basis of 0 and a Basis of 1, compute yearfrac.
Define the StartDate and EndDate using a Basis of 0.

```
YearFraction = yearfrac('14 mar 01', '14 sep 01', 0)
YearFraction = 0.5041
```

Define the StartDate and EndDate using a Basis of 1.

```
YearFraction = yearfrac('14 mar 01', '14 sep 01', 1)
YearFraction = 0.5000
```


## Compute yearfrac When the Calendar Year After the StartDate is a Leap Year

Given a Basis of 0, compute yearfrac when the calendar after StartDate is in a leap year.

Define the StartDate and EndDate using a Basis of 0.

```
yearFraction = yearfrac(' 14 mar 03', '14 sep 03', 0)
yearFraction = 0.5027
```

There are 184 days between March 14 and September 14, and the calendar year after the StartDate is a leap year, so yearfrac returns $184 / 366=0.5027$.

## Compute the Fraction of a Year Using an actual/actual Basis

To get the fraction of a year between '31-Jul-2015' and '30-Sep-2015' using the actual/ actual basis:

```
yearfrac('31-Jul-2015', '30-Sep-2015', 0)*2
ans = 0.3333
```

For the actual/actual basis, the fraction of a year is calculated as:
(Actual Days between Start Date and End Date)/(Actual Days between Start Date and exactly one year after Start Date)

There are 61 days between 31-Jul-2015 and 30-Sep-2015. Since the next year is a leap year, there are 366 days between 31-Jul-2015 and 31-Jul-2016. So, there is $61 / 366$ which is exactly $1 / 6$. So given this, exactly $2 / 6$ is the expected result for the fraction of the sixmonth period.

## Compute yearfrac When the Date Range Spans a Leap Year

Use a Basis of 12 to compute yearfrac when parts of the date range are in leap year and in a non-leap year. The output YearFraction is a fraction, in years, based on the number of days between the StartDate and EndDate.

```
YearFraction = yearfrac('1-Jan-2016','30-Jan-2017',12)
YearFraction = 1.0795
```


## Compute yearfrac When Specifying datetime Arrays

Given a Basis of 9, compute yearfrac when the StartDate and EndDate are specified using datetime arrays.

```
yearfrac(datetime('1-Jan-2000','Locale','en_US'), '1/1/2001', 9)
ans = 1.0167
```


## Input Arguments

## StartDate - Start date <br> serial date number | date character vector | datetime object

Start date, specified as an N-by-1 or 1-by-N vector using serial date numbers, date character vectors, or datetime arrays.

Data Types: double | char|datetime

## EndDate - End date

serial date number | date character vector | datetime object
End date, specified as an N-by-1 or 1-by-N vector using serial date numbers, date character vectors, or datetime arrays.

Data Types: double|char|datetime

## Basis - Day-count basis for each set of dates

0 (actual/actual) (default) | vector of numerics with values 0 through 13

Day-count basis for each set of dates, specified as an N -by-1 or 1-by-N vector of integers with values of 0 through 13.

- $0=$ actual/actual (default)
- $1=30 / 360$ (SIA)
- 2 = actual/360
- $3=$ actual $/ 365$
- $4=30 / 360$ (BMA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- $9=$ actual/360 (ICMA)
- $10=$ actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-0
Data Types: single | double

## Output Arguments

## YearFraction - Real numbers identifying interval, in years, between StartDate and EndDate <br> vector

Real numbers identifying the interval, in years, between StartDate and EndDate, returned an N -by- 1 or 1 -by- N vector.

## Definitions

## Difference Between yearfrac and date2time

The difference between yearfrac and date2time is that date2time counts full periods as a whole integer, even if the number of actual days in the periods are different. yearfrac does not count full periods.

For example,

```
yearfrac('1/1/2000', '1/1/2001', 9)
ans =
```

1.0167
yearfrac for Basis 9 (ACT/360 ICMA) calculates $366 / 360=1.0167$. So, even if the dates have the same month and date, with a difference of 1 in the year, the returned value may not be exactly 1 . On the other hand, date2time calculates one full year period:

```
date2time('1/1/2000', '1/1/2001', 1, 9)
```

ans =

## 1

## See Also

date2time | datetime | days360| days365|daysact|daysdif|months | wrkdydif | year | year | yeardays | yearfrac

## Topics

"Handle and Convert Dates" on page 2-2
"Trading Calendars User Interface" on page 16-2
"UICalendar User Interface" on page 16-4

## Introduced before R2006a

## ylddisc

Yield of discounted security

## Syntax

```
Yield = ylddisc(Settle,Maturity,Face,Price)
Yield = ylddisc(
```

$\qquad$

``` ,Basis)
```


## Description

Yield = ylddisc(Settle,Maturity,Face,Price) returns the yield of a discounted security.

Yield = ylddisc( __ , Basis) adds optional an argument for Basis.

## Examples

## Find the Yield of a Discounted Security

This example shows how to find the yield of the following discounted security.

```
Settle = '10/14/2000';
Maturity = '03/17/2001';
Face = 100;
Price = 96.28;
Basis = 2;
Yield = ylddisc(Settle, Maturity, Face, Price, Basis)
Yield = 0.0903
```


## Find the Yield of a Discounted Security Using datetime Inputs

This example shows how to use datetime inputs to find the yield of the following discounted security.

```
Settle = '10/14/2000';
Maturity = '03/17/2001';
Face = 100;
Price = 96.28;
Basis = 2;
Settle = datetime(Settle,'Locale','en_US');
Maturity = datetime(Maturity,'Locale','en_US');
Yield = ylddisc(Settle, Maturity, Face, Price, Basis)
Yield = 0.0903
```


## Input Arguments

## Settle - Settlement date of security <br> serial date number | date character vector | datetime

Settlement date of the security, specified as serial date numbers, date character vectors, or datetime arrays. The Settle date must be before the Maturity date.

## Data Types: double| char|datetime

## Maturity - Maturity date of security

serial date number | date character vector | datetime
Maturity date of the security, specified as serial date numbers, date character vectors, or datetime arrays.

Data Types: double| char|datetime

## Face - Redemption value of security

numeric
Redemption value (par value) of the security, specified as a numeric value.
Data Types: double

## Price - Discount price of security numeric

Discount price of the security, specified as a numeric value.

## Data Types: double

## Basis - (Optional) Day-count basis

0 (actual/actual) (default) | integers of the set [0...13] | vector of integers of the set [0...13]

Day-count basis for the security, specified using the following values:

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- 2 = actual/360
- 3 = actual/365
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- $7=$ actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- $9=\mathrm{actual} / 360$ (ICMA)
- $10=$ actual/365 (ICMA)
- $11=30 / 360 \mathrm{E}$ (ICMA)
- $12=$ actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-0
Data Types: double

## Output Arguments

## Yield - Yield of discounted security <br> numeric

Yield of discounted security, returned as a numeric value.

## References

[1] Mayle, J. Standard Securities Calculation Methods. Volumes I-II, 3rd edition. Formula 1.

## See Also

acrudisc| bndprice|bndyield|datetime|prdisc|yldmat|yldtbill

## Topics

"Yield Functions" on page 2-32
"Yield Conventions" on page 2-31

Introduced before R2006a

## yldmat

Yield with interest at maturity

## Syntax

```
Yield = yldmat(Settle,Maturity,Issue,Face,Price,CouponRate)
Yield = yldmat(
```

$\qquad$

``` ,Basis)
```


## Description

Yield = yldmat(Settle,Maturity,Issue,Face,Price,CouponRate) returns the yield of a security paying interest at maturity.

Yield = yldmat(__ , Basis) adds an optional argument for Basis.

## Examples

## Find the Yield of a Security Paying Interest at Maturity

This example shows how to find the yield of a security paying interest at maturity for the following.

```
Settle = '02/07/2000';
Maturity = '04/13/2000';
Issue = '10/11/1999';
Face = 100;
Price = 99.98;
CouponRate = 0.0608;
Basis = 1;
Yield = yldmat(Settle, Maturity, Issue, Face, Price,...
CouponRate, Basis)
Yield = 0.0607
```


## Find the Yield of a Security Paying Interest at Maturity Using datetime Inputs

This example shows how to use datetime inputs find the yield of a security paying interest at maturity for the following:

```
Settle = '7-Feb-2000';
Maturity = '13-Apr-2000';
Issue = '11-Oct-1999';
Face = 100;
Price = 99.98;
CouponRate = 0.0608;
Basis = 1;
Settle = datetime(Settle,'Locale','en_US');
Maturity = datetime(Maturity,'Locale','en_US');
Issue = datetime(Issue,'Locale','en US');
Yield = yldmat(Settle, Maturity, Issue, Face, Price,...
CouponRate, Basis)
Yield = 0.0607
```


## Input Arguments

## Settle - Settlement date of security <br> serial date number | date character vector | datetime

Settlement date of the security, specified as serial date numbers, date character vectors, or datetime arrays. The Settle date must be before the Maturity date.
Data Types: double | char \| datetime

## Maturity - Maturity date of security

serial date number | date character vector | datetime
Maturity date of the security, specified as serial date numbers, date character vectors, or datetime arrays.

Data Types: double | char|datetime

## Issue - Issue date of security

serial date number | date character vector | datetime
Issue date of the security, specified as serial date numbers, date character vectors, or datetime arrays.

Data Types: double|char|datetime

## Face - Redemption value of security

numeric
Redemption value (par value) of the security, specified as a numeric value.
Data Types: double

## Price - Price of security

numeric
Price of the security, specified as a numeric value.
Data Types: double

## CouponRate - Coupon rate of security

decimal fraction
Coupon rate of the security, specified as a decimal fraction.
Data Types: double

## Basis - Day-count basis

0 (actual/actual) (default) | integers of the set [0...13] | vector of integers of the set [0...13]
(Optional) Day-count basis for the security, specified using the following values:

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- 2 = actual/360
- 3 = actual/365
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-0
Data Types: double

## Output Arguments

## Yield - Yield of a security paying interest at maturity numeric

Yield of a security paying interest at maturity, returned as a numeric value.

## References

[1] Mayle, J. Standard Securities Calculation Methods. Volumes I-II, 3rd edition. Formula 3.

## See Also

acrudisc|bndprice|bndyield|datetime|prmat|ylddisc|yldtbill

## Topics

"Yield Functions" on page 2-32
"Yield Conventions" on page 2-31

Introduced before R2006a

## yldtbill

Yield of Treasury bill

## Syntax

```
Yield = yldtbill(Settle,Maturity,Face,Price)
```


## Description

Yield = yldtbill(Settle,Maturity,Face,Price)returns the yield for a Treasury bill.

## Examples

## Find the Yield for a Treasury Bill

This example shows how to return the yield for a Treasury bill, given the settlement date of a Treasury bill is February 10, 2000, the maturity date is August 6, 2000, the par value is $\$ 1000$, and the price is $\$ 981.36$.

Yield = yldtbill('2/10/2000', '8/6/2000', 1000, 981.36)
Yield $=0.0384$

## Find the Yield for a Treasury Bill Using datetime Inputs

This example shows how to use datetime inputs to return the yield for a Treasury bill, given the settlement date of a Treasury bill is February 10, 2000, the maturity date is August 6, 2000, the par value is $\$ 1000$, and the price is $\$ 981.36$.

```
Yield = yldtbill(datetime('10-Feb-2000','Locale','en_US'), datetime('6-Aug-2000','Loca'
```


## Input Arguments

## Settle - Settlement date of Treasury bill

serial date number | date character vector | datetime
Settlement date of the Treasury bill, specified as serial date numbers, date character vectors, or datetime arrays. The Settle date must be before the Maturity date.
Data Types: double | char | datetime

## Maturity - Maturity date of Treasury bill

serial date number | date character vector | datetime
Maturity date of the Treasury bill, specified as serial date numbers, date character vectors, or datetime arrays.

## Data Types: double|char|datetime

## Face - Redemption value of Treasury bill

numeric
Redemption value (par value) of the Treasury bill, specified as a numeric value.

## Data Types: double

## Price - Price of Treasury bill

numeric
Price of the Treasury bill, specified as a numeric value.
Data Types: double

## Output Arguments

## Yield - Yield for Treasury bill

numeric
Yield for Treasury bill, returned as a numeric value.

## References

[1] Bodie, Kane, and Marcus. Investments. McGraw-Hill Education, 2013.

See Also<br>beytbill|bndyield|datetime|prtbill|yldmat

## Topics

"Computing Treasury Bill Price and Yield" on page 2-38
"Yield Functions" on page 2-32
"Treasury Bills Defined" on page 2-37
"Yield Conventions" on page 2-31

Introduced before R2006a

## zbtprice

Zero curve bootstrapping from coupon bond data given price

## Syntax

[ZeroRates,CurveDates] = zbtprice(Bonds,Prices,Settle) ZeroRates,CurveDates = zbtprice( $\qquad$ ,OutputCompounding)

## Description

[ZeroRates,CurveDates] = zbtprice(Bonds,Prices,Settle) uses the bootstrap method to return a zero curve given a portfolio of coupon bonds and their prices.

A zero curve consists of the yields to maturity for a portfolio of theoretical zero-coupon bonds that are derived from the input Bonds portfolio. The bootstrap method that this function uses does not require alignment among the cash-flow dates of the bonds in the input portfolio. It uses theoretical par bond arbitrage and yield interpolation to derive all zero rates; specifically, the interest rates for cash flows are determined using linear interpolation. For best results, use a portfolio of at least 30 bonds evenly spaced across the investment horizon.

ZeroRates,CurveDates = zbtprice( $\qquad$ ,OutputCompounding) adds an optional argument for OutputCompounding.

## Examples

## Compute a Zero Curve Given a Portfolio of Coupon Bonds and Their Prices

Given data and prices for 12 coupon bonds, two with the same maturity date, and given the common settlement date.

```
Bonds = [datenum('6/1/1998') 0.0475 100 2 0 0;
    datenum('7/1/2000') 0.06 100 2 0 0;
    datenum('7/1/2000') 0.09375 100 6 1 0;
```

```
    datenum('6/30/2001') 0.05125 100 1 3 1;
datenum('4/15/2002') 0.07125 100 4 1 0;
datenum('1/15/2000') 0.065 100 2 0 0;
datenum('9/1/1999') 0.08 100 3 3 0;
datenum('4/30/2001') 0.05875 100 2 0 0;
datenum('11/15/1999') 0.07125 100 2 0 0;
datenum('6/30/2000') 0.07 100 2 3 1;
datenum('7/1/2001') 0.0525 100 2 3 0;
datenum('4/30/2002') 0.07 100 2 0 0];
Prices = [99.375;
    99.875;
    105.75 ;
    96.875;
    103.625;
    101.125;
    103.125;
        99.375;
    101.0 ;
    101.25;
        96.375;
    102.75 ];
Settle = datenum('12/18/1997');
```

Set semiannual compounding for the zero curve.
OutputCompounding $=2$;
Execute the function zbtprice which returns the zero curve at the maturity dates. Note the mean zero rate for the two bonds with the same maturity date.

```
[ZeroRates, CurveDates] = zbtprice(Bonds, Prices, Settle,...
OutputCompounding)
```

ZeroRates = $11 \times 1$
0.0616
0.0609
0.0658
0.0590
0.0647
0.0655
0.0606
0.0601

```
    0.0642
    0.0621
CurveDates = 11×1
    729907
    730364
    730439
    7 3 0 5 0 0
    730667
    7 3 0 6 6 8
    7 3 0 9 7 1
    7 3 1 0 3 2
    7 3 1 0 3 3
    7 3 1 3 2 1
```


## Compute a Zero Curve Given a Portfolio of Coupon Bonds and Their Prices Using datetime Inputs

Given data and prices for 12 coupon bonds, two with the same maturity date, and given the common settlement date, use datetime inputs to compute a zero curve.

```
Bonds = [datenum('6/1/1998') 0.0475 100 2 0 0;
    datenum('7/1/2000') 0.06 100 2 0 0;
    datenum('7/1/2000') 0.09375 100 6 1 0;
    datenum('6/30/2001') 0.05125 100 1 3 1;
    datenum('4/15/2002') 0.07125 100 4 1 0;
    datenum('1/15/2000') 0.065 100 2 0 0;
    datenum('9/1/1999') 0.08 100 3 3 0;
    datenum('4/30/2001') 0.05875 100 2 0 0;
    datenum('11/15/1999') 0.07125 100 2 0 0;
    datenum('6/30/2000') 0.07 100 2 3 1;
    datenum('7/1/2001') 0.0525 100 2 3 0;
    datenum('4/30/2002') 0.07 100 2 0 0];
Prices = [99.375;
    99.875;
    105.75 ;
```

96.875 ;
103.625;
101.125;
103.125;
99.375;
101.0 ;
101.25 ;
96.375 ;
102.75 ];

Settle = datenum('12/18/1997');
OutputCompounding = 2 ;
t=array2table(Bonds);
t.Bonds1=datetime(t.Bonds1,'ConvertFrom','datenum','Locale','en_US'); Settle = datetime(Settle,'ConvertFrom','datenum','Locale','en_US̄'); [ZeroRates, CurveDates] = zbtprice(t, Prices, Settle,... OutputCompounding)

ZeroRates = $11 \times 1$
0.0616
0.0609
0.0658
0.0590
0.0647
0.0655
0.0606
0.0601
0.0642
0.0621

CurveDates = 11x1 datetime array
01-Jun-1998 00:00:00
01-Sep-1999 00:00:00
15-Nov-1999 00:00:00
15-Jan-2000 00:00:00
30-Jun-2000 00:00:00
01-Jul-2000 00:00:00
30-Apr-2001 00:00:00
30-Jun-2001 00:00:00
01-Jul-2001 00:00:00
15-Apr-2002 00:00:00

## Input Arguments

## Bonds - Coupon bond information to generate zero curve

table | matrix
Coupon bond information to generate zero curve, specified as a 6-column table or a n-by- 2 to $n$-by- 6 matrix of bond information, where the table columns or matrix columns contains:

- Maturity (Column 1, Required) Maturity date of the bond, as a serial date number. Use datenum to convert date character vectors to serial date numbers. If the input Bonds is a table, the Maturity dates can be serial date numbers, date character vectors, or datetime arrays.
- CouponRate (Column 2, Required) Decimal fraction indicating the coupon rate of the bond.
- Face (Column 3, Optional) Redemption or face value of the bond. Default $=100$.
- Period (Column 4, Optional) Coupons per year of the bond. Allowed values are 0, 1, 2 (default), 3, 4, 6, and 12.
- Basis (Column 5, Optional) Day-count basis of the bond. A vector of integers.
- $0=$ actual/actual (default)
- $1=30 / 360$ (SIA)
- 2 = actual/360
- 3 = actual/365
- $4=30 / 360$ (BMA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- $9=$ actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- $11=30 / 360 \mathrm{E}$ (ICMA)
- $12=$ actual/365 (ISDA)
- 13 = BUS/252
- For more information, see basis on page Glossary-0
- EndMonthRule (Column 6, Optional) End-of-month rule. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days. $0=$ ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. 1 = set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month
:


## Note

- If Bonds is a table, the Maturity dates can be serial date numbers, date character vectors, or datetime arrays.
- If Bonds is a matrix, is an n-by-2 to $n$-by- 6 matrix where each row describes a bond, the first two columns (Maturity and CouponRate) are required. The remainder of the columns are optional but must be added in order. All rows in Bonds must have the same number of columns.


## Data Types: double | table

## Prices - Clean price (price without accrued interest) of each bond in Bonds numeric

Clean price (price without accrued interest) of each bond in Bonds, specified as a N -by-1 column vector. The number of rows ( $n$ ) must match the number of rows in Bonds.

## Data Types: double

## Settle - Settlement date representing time zero in derivation of zero curve serial date number | date character vector | datetime

Settlement date representing time zero in derivation of zero curve, specified as serial date number, date character vector, or datetime array. Settle represents time zero for deriving the zero curve, and it is normally the common settlement date for all the bonds.

## Data Types: double | char | datetime

## OutputCompounding - Compounding frequency of output ZeroRates <br> 2 (default) | numeric values: $0,1,2,3,4,6,12,365,-1$

(Optional) Compounding frequency of output ZeroRates, specified using the allowed values:

- 0 - Simple interest (no compounding)
- 1 - Annual compounding
- 2 - Semiannual compounding (default)
- 3-Compounding three times per year
- 4-Quarterly compounding
- 6 - Bimonthly compounding
- 12 - Monthly compounding
-     - 1 - Continuous compounding

Data Types: double

## Output Arguments

## ZeroRates - Implied zero rates for each point along the investment horizon defined by maturity date <br> decimal fractions

Implied zero rates for each point along the investment horizon defined by a maturity date, returned as a $m$-by- 1 vector of decimal fractions where $m$ is the number of bonds of unique maturity dates. In aggregate, the rates in ZeroRates constitute a zero curve.

If more than one bond has the same Maturity date, zbtprice returns the mean zero rate for that Maturity. Any rates before the first Maturity are assumed to be equal to the rate at the first Maturity, that is, the curve is assumed to be flat before the first Maturity.

## CurveDates - Maturity dates that correspond to ZeroRates <br> serial date number | date character vector | datetime

Maturity dates that correspond to the ZeroRates, returned as a m-by-1 vector of unique maturity dates, where m is the number of bonds of different maturity dates. These dates
begin with the earliest Maturity date and end with the latest Maturitydate in the Bonds table or matrix.

If either inputs for Bonds or Settle have datetime values, then CurveDates is datetimes. Otherwise CurveDates is serial date numbers.

## References

[1] Fabozzi, Frank J. "The Structure of Interest Rates." Ch. 6 in Fabozzi, Frank J. and T. Dessa Fabozzi, eds. The Handbook of Fixed Income Securities. 4th ed. New York, Irwin Professional Publishing, 1995.
[2] McEnally, Richard W. and James V. Jordan. "The Term Structure of Interest Rates." in Ch. 37 in Fabozzi and Fabozzi, ibid
[3] Das, Satyajit. "Calculating Zero Coupon Rates." in Swap and Derivative Financing. Appendix to Ch. 8, pp. 219-225. New York, Irwin Professional Publishing, 1994.

## See Also

datetime|zbtyield

## Topics

"Term Structure of Interest Rates" on page 2-42
"Fixed-Income Terminology" on page 2-22

## Introduced before R2006a

## zbtyield

Zero curve bootstrapping from coupon bond data given yield

## Syntax

[ZeroRates,CurveDates] = zbtyield(Bonds,YieldsSettle)
ZeroRates,CurveDates = zbtyield( $\qquad$ ,OutputCompounding)

## Description

[ZeroRates,CurveDates] = zbtyield(Bonds,YieldsSettle) uses the bootstrap method to return a zero curve given a portfolio of coupon bonds and their yields.

A zero curve consists of the yields to maturity for a portfolio of theoretical zero-coupon bonds that are derived from the input Bonds portfolio. The bootstrap method that this function uses does not require alignment among the cash-flow dates of the bonds in the input portfolio. It uses theoretical par bond arbitrage and yield interpolation to derive all zero rates; specifically, the interest rates for cash flows are determined using linear interpolation. For best results, use a portfolio of at least 30 bonds evenly spaced across the investment horizon.

ZeroRates,CurveDates = zbtyield( $\qquad$ ,OutputCompounding) adds an optional argument for OutputCompounding.

## Examples

## Compute a Zero Curve Given a Portfolio of Coupon Bonds and Their Yields

Given data and yields to maturity for 12 coupon bonds, two with the same maturity date; and given the common settlement date.

```
Bonds = [datenum('6/1/1998') 0.0475 100 2 0 0;
    datenum('7/1/2000') 0.06 100 2 0 0;
    datenum('7/1/2000') 0.09375 100 6 1 0;
```

```
datenum('6/30/2001') 0.05125 100 1 3 1;
datenum('4/15/2002') 0.07125 100 4 1 0;
datenum('1/15/2000') 0.065 100 2 0 0;
datenum('9/1/1999') 0.08 100 3 3 0;
datenum('4/30/2001') 0.05875 100 2 0 0;
datenum('11/15/1999') 0.07125 100 2 0 0;
datenum('6/30/2000') 0.07 100 2 3 1;
datenum('7/1/2001') 0.0525 100 2 3 0;
datenum('4/30/2002') 0.07 100 2 0 0];
```

```
Yields = [0.0616
    0.0605
    0.0687
    0.0612
    0.0615
    0.0591
    0.0603
    0.0608
    0.0655
    0.0646
    0.0641
    0.0627];
```

Settle = datenum('12/18/1997');

Set semiannual compounding for the zero curve.
OutputCompounding = 2;
Execute the function zbtyield which returns the zero curve at the maturity dates. Note the mean zero rate for the two bonds with the same maturity date.

```
[ZeroRates, CurveDates] = zbtyield(Bonds, Yields, Settle,...
OutputCompounding)
```

ZeroRates = $11 \times 1$
0.0616
0.0603
0.0657
0.0590
0.0649
0.0650
0.0606
0.0611

```
    0.0643
    0.0614
CurveDates = 11×1
    729907
    730364
    730439
    730500
    730667
    730668
    7 3 0 9 7 1
    731032
    731033
    7 3 1 3 2 1
```


## Compute a Zero Curve Given a Portfolio of Coupon Bonds and Their Yields Using datetime Inputs

Given data and yields to maturity for 12 coupon bonds, two with the same maturity date; and given the common settlement date, compute the zero curve using datetime inputs.

```
Bonds = [datenum('6/1/1998') 0.0475 100 2 0 0;
    datenum('7/1/2000') 0.06 100 2 0 0;
    datenum('7/1/2000') 0.09375 100 6 1 0;
    datenum('6/30/2001') 0.05125 100 1 3 1;
    datenum('4/15/2002') 0.07125 100 4 1 0;
    datenum('1/15/2000') 0.065 100 2 0 0;
    datenum('9/1/1999') 0.08 100 3 3 0;
    datenum('4/30/2001') 0.05875 100 2 0 0;
    datenum('11/15/1999') 0.07125 100 2 0 0;
    datenum('6/30/2000') 0.07 100 2 3 1;
    datenum('7/1/2001') 0.0525 100 2 3 0;
    datenum('4/30/2002') 0.07 100 2 0 0];
```

$\begin{aligned} \text { Yields }= & {[0.0616} \\ & 0.0605 \\ & 0.0687\end{aligned}$
0.0612
0.0615
0.0591
0.0603
0.0608
0.0655
0.0646
0.0641
0.0627];

Settle = datenum('12/18/1997');
OutputCompounding = 2;
t = array2table(Bonds,'VariableNames',\{'Maturity ','CouponRate ', 'Face' ,'Period', 'Ba disp(t)

| Maturity | CouponRate | Face | Period | Basis | EndMonthRule |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $7.2991 \mathrm{e}+05$ | 0.0475 | 100 | 2 | 0 | 0 |
| $7.3067 e+05$ | 0.06 | 100 | 2 | 0 | 0 |
| 7.3067e+05 | 0.09375 | 100 | 6 | 1 | 0 |
| 7.3103e+05 | 0.05125 | 100 | 1 | 3 | 1 |
| $7.3132 \mathrm{e}+05$ | 0.07125 | 100 | 4 | 1 | 0 |
| $7.305 \mathrm{e}+05$ | 0.065 | 100 | 2 | 0 | 0 |
| 7.3036e+05 | 0.08 | 100 | 3 | 3 | 0 |
| 7.3097e+05 | 0.05875 | 100 | 2 | 0 | 0 |
| $7.3044 \mathrm{e}+05$ | 0.07125 | 100 | 2 | 0 | 0 |
| $7.3067 \mathrm{e}+05$ | 0.07 | 100 | 2 | 3 | 1 |
| 7.3103e+05 | 0.0525 | 100 | 2 | 3 | 0 |
| 7.3134e+05 | 0.07 | 100 | 2 | 0 | 0 |

t.Maturity = datetime(t.Maturity,'ConvertFrom','datenum','Locale','en_US');

Settle = datetime(Settle,'ConvertFrom','datenum','Locale','en_US');
[ZeroRates, CurveDates] = zbtyield(t, Yields, Settle,...
OutputCompounding)
ZeroRates = 11×1
0.0616
0.0603
0.0657
0.0590
0.0649
0.0650
0.0606

```
    0.0611
    0.0643
    0.0614
```

CurveDates = 11x1 datetime array
01-Jun-1998 00:00:00
01-Sep-1999 00:00:00
15-Nov-1999 00:00:00
15-Jan-2000 00:00:00
30-Jun-2000 00:00:00
01-Jul-2000 00:00:00
30-Apr-2001 00:00:00
30-Jun-2001 00:00:00
01-Jul-2001 00:00:00
15-Apr-2002 00:00:00
30-Apr-2002 00:00:00

## Compute the Real Zero Rates From the Real Yields of Inflation-Linked Bonds

Use zbtyield to compute the real zero rates from the real yields of inflation-linked bonds.

```
% Load the data
load usbond 02Sep2008
Settle = datenum('02-Sep-2008');
```

Compute the real yields and then compute the real zero rates.

```
RealYields = bndyield(TIPSPrice,TIPSCoupon,Settle,TIPSMaturity);
TIPSBonds = [TIPSMaturity TIPSCoupon];
[RealZeroRates, CurveDates] = zbtyield(TIPSBonds, RealYields, Settle)
RealZeroRates = 26x1
    0.0069
    0.0094
    0.0092
    0.0111
    0.0110
```

```
    0.0119
    0.0116
    0.0128
    0.0126
    0.0136
CurveDates = 26x1
    7 3 4 1 5 3
    7 3 4 2 4 3
    7 3 4 5 1 8
    7 3 4 6 0 8
    734883
    7 3 4 9 7 4
    735065
    7 3 5 3 3 9
    7 3 5 4 3 0
    7 3 5 6 1 4
```


## Input Arguments

## Bonds - Coupon bond information to generate zero curve

## table | matrix

Coupon bond information to generate zero curve, specified as a 6-column table or a n -by- 2 to $n$-by- 6 matrix of bond information, where the table columns or matrix columns contains:

- Maturity (Column 1, Required) Maturity date of the bond, as a serial date number. Use datenum to convert date character vectors to serial date numbers. If the input Bonds is a table, the Maturity dates can be serial date numbers, date character vectors, or datetime arrays.
- CouponRate (Column 2, Required) Decimal fraction indicating the coupon rate of the bond.
- Face (Column 3, Optional) Redemption or face value of the bond. Default $=100$.
- Period (Column 4, Optional) Coupons per year of the bond. Allowed values are 0, 1, 2 (default), 3, 4, 6, and 12.
- Basis (Column 5, Optional) Day-count basis of the bond. A vector of integers.
- $0=$ actual/actual (default)
- $1=30 / 360$ (SIA)
- 2 = actual/360
- 3 = actual/365
- $4=30 / 360$ (BMA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- $9=$ actual/360 (ICMA)
- $10=$ actual/365 (ICMA)
- $11=30 / 360 E$ (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252
- For more information, see basis on page Glossary-0
- EndMonthRule (Column 6, Optional) End-of-month rule. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days. $0=$ ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. $1=$ set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month


## Note

- If Bonds is a table, the Maturity dates can be serial date numbers, date character vectors, or datetime arrays.
- If Bonds is a matrix, is an n-by-2 to $n$-by- 6 matrix where each row describes a bond, the first two columns (Maturity and CouponRate) are required. The remainder of the columns are optional but must be added in order. All rows in Bonds must have the same number of columns.


## Data Types: double | table

## Yields - Yield to maturity of each bond in Bonds <br> numeric

Yield to maturity of each bond in Bonds, specified as a N -by- 1 column vector. The number of rows ( $n$ ) must match the number of rows in Bonds.

Note Yield to maturity must be compounded semiannually.

## Data Types: double

## Settle - Settlement date representing time zero in derivation of zero curve serial date number | date character vector | datetime

Settlement date representing time zero in derivation of zero curve, specified as serial date number, date character vector, or datetime array. Settle represents time zero for deriving the zero curve, and it is normally the common settlement date for all the bonds.

## Data Types: double|char|datetime

## OutputCompounding - Compounding frequency of output ZeroRates

2 (default) | numeric values: $0,1,2,3,4,6,12,365,-1$
(Optional) Compounding frequency of output ZeroRates, specified using the allowed values:

- 0 - Simple interest (no compounding)
- 1 - Annual compounding
- 2 - Semiannual compounding (default)
- 3-Compounding three times per year
- 4-Quarterly compounding
- 6 - Bimonthly compounding
- 12 - Monthly compounding
-     - 1 - Continuous compounding

Data Types: double

## Output Arguments

## ZeroRates - Implied zero rates for each point along the investment horizon defined by maturity date

decimal fractions
Implied zero rates for each point along the investment horizon defined by a maturity date, returned as a m-by-1 vector of decimal fractions where $m$ is the number of bonds with unique maturity dates. In aggregate, the rates in ZeroRates constitute a zero curve.

If more than one bond has the same Maturity date, zbtyield returns the mean zero rate for that Maturity. Any rates before the first Maturity are assumed to be equal to the rate at the first Maturity, that is, the curve is assumed to be flat before the first Maturity.

## CurveDates - Maturity dates that correspond to ZeroRates

serial date number | date character vector | datetime
Maturity dates that correspond to the ZeroRates, returned as a m-by-1 vector of unique maturity dates, where m is the number of bonds of different maturity dates. These dates begin with the earliest Maturity date and end with the latest Maturitydate in the Bonds table or matrix.

If either inputs for Bonds or Settle have datetime values, then
CurveDatesCurveDates is datetimes. Otherwise CurveDates is serial date numbers.

## References

[1] Fabozzi, Frank J. "The Structure of Interest Rates." Ch. 6 in Fabozzi, Frank J. and T. Dessa Fabozzi, eds. The Handbook of Fixed Income Securities. 4th ed. New York, Irwin Professional Publishing, 1995.
[2] McEnally, Richard W. and James V. Jordan. "The Term Structure of Interest Rates." in Ch. 37 in Fabozzi and Fabozzi, ibid
[3] Das, Satyajit. "Calculating Zero Coupon Rates." in Swap and Derivative Financing. Appendix to Ch. 8, pp. 219-225. New York, Irwin Professional Publishing, 1994.

See Also<br>datetime| zbtprice

## Topics

"Term Structure of Interest Rates" on page 2-42
"Fixed-Income Terminology" on page 2-22

## Introduced before R2006a

## zero2disc

Discount curve given zero curve

Note In R2017b, the specification of optional input arguments has changed. While the previous ordered inputs syntax is still supported, it may no longer be supported in a future release. Use the optional name-value pair inputs: Compounding and Basis.

## Syntax

```
[DiscRates,CurveDates] = zero2disc(ZeroRates,CurveDates,Settle)
[DiscRates,CurveDates] = zero2disc( __ ,Name,Value)
```


## Description

[DiscRates,CurveDates] = zero2disc(ZeroRates,CurveDates,Settle) returns a discount curve given a zero curve and its maturity dates. If either inputs for or areCurveDatesSettle a datetime array, CurveDates is returned as a datetime array. Otherwise, CurveDates is returned as a serial date number. The DiscRates output is the same for any of these input data types.
[DiscRates,CurveDates] = zero2disc( ___ ,Name,Value) adds optional namevalue pair arguments

## Examples

## Compute a Discount Curve Given a Zero Curve and Maturity Dates

Given a zero curve over a set of maturity dates and a settlement date.

$$
\begin{aligned}
& \text { ZeroRates }= {[0.0464} \\
& 0.0509 \\
& 0.0524 \\
& 0.0525
\end{aligned}
$$

```
    0.0531
    0.0525
    0.0530
    0.0531
    0.0549
    0.0536];
CurveDates = [datenum('06-Nov-2000')
    datenum('11-Dec-2000')
    datenum('15-Jan-2001')
    datenum('05-Feb-2001')
    datenum('04-Mar-2001')
    datenum('02-Apr-2001')
    datenum('30-Apr-2001')
    datenum('25-Jun-2001')
    datenum('04-Sep-2001')
    datenum('12-Nov-2001')];
Settle = datenum('03-Nov-2000');
The zero curve is compounded daily on an actual/365 basis.
Compounding = 365;
Basis = 3;
```

Execute the function zero2disc which returns the discount curve DiscRates at the maturity dates CurveDates.
[DiscRates, CurveDates] = zero2disc(ZeroRates, CurveDates,... Settle, Compounding, Basis)

DiscRates $=10 \times 1$
0.9996
0.9947
0.9896
0.9866
0.9826
0.9787
0.9745
0.9665
0.9552
0.9466

## CurveDates $=10 \times 1$

730796
730831
730866
730887
730914
730943
730971
731027
731098
731167

For readability, ZeroRates and DiscRates are shown here only to the basis point. However, MATLAB® software computed them at full precision. If you enter ZeroRates as shown, DiscRates may differ due to rounding.

## Compute a Discount Curve Given a Zero Curve and Maturity Dates Using datetime Inputs

Given a zero curve over a set of maturity dates and a settlement date, compute a discount curve using datetime inputs.

```
ZeroRates = [0.0464
    0.0509
    0.0524
    0.0525
    0.0531
    0.0525
    0.0530
    0.0531
    0.0549
    0.0536];
CurveDates = [datenum('06-Nov-2000')
    datenum('11-Dec-2000')
    datenum('15-Jan-2001')
    datenum('05-Feb-2001')
    datenum('04-Mar-2001')
    datenum('02-Apr-2001')
    datenum('30-Apr-2001')
```

```
    datenum('25-Jun-2001')
datenum('04-Sep-2001')
datenum('12-Nov-2001')];
```

```
Settle = datenum('03-Nov-2000');
```

Settle = datenum('03-Nov-2000');
Compounding = 365;
Basis = 3;
CurveDates = datetime(CurveDates, 'ConvertFrom', 'datenum','Locale','en_US');
Settle = datetime(Settle,'ConvertFrom','datenum','Locale','en_US');
[DiscRates, CurveDates] = zero2disc(ZeroRates, CurveDates,...
Settle, Compounding, Basis)

```
```

DiscRates = 10\times1

```
DiscRates = 10\times1
    0.9996
    0.9947
    0.9896
    0.9866
    0.9826
    0.9787
    0.9745
    0.9665
    0.9552
    0.9466
CurveDates = 10x1 datetime array
    06-Nov-2000 00:00:00
    11-Dec-2000 00:00:00
    15-Jan-2001 00:00:00
    05-Feb-2001 00:00:00
    04-Mar-2001 00:00:00
    02-Apr-2001 00:00:00
    30-Apr-2001 00:00:00
    25-Jun-2001 00:00:00
    04-Sep-2001 00:00:00
    12-Nov-2001 00:00:00
```


## Input Arguments

## ZeroRates - Annualized zero rates

decimal fraction
Annualized zero rates, specified as a NUMBONDS-by-1 vector using decimal fractions. In aggregate, the zero rates constitute an implied zero curve for the investment horizon represented by CurveDates.

## Data Types: double

## CurveDates - Maturity dates <br> serial date number | date character vector | datetime

Maturity dates that correspond to the input ZeroRates, specified as NUMBONDS-by-1 vector using serial date numbers, date character vectors, or datetime arrays.

Data Types: double | datetime | char

## Settle - Common settlement date for ZeroRates <br> serial date number | date character vector | datetime

Common settlement date for ZeroRates, specified as serial date numbers, date character vectors, or datetime arrays. Settle is the settlement date for the bonds from which the zero curve was bootstrapped.

Data Types: double|datetime | char

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN , ValueN.

Example: [DiscRates,CurveDates] = zero2disc(ZeroRates,CurveDates,Settle,'Compounding',4,'Basis',6)

## Compounding - Rate at which input ZeroRateszero rates are compounded when annualized

2 (default) | numeric values: 0,1, 2, 3, 4, 6, 12, 365, - 1

Rate at which the input ZeroRates are compounded when annualized, specified as the comma-separated pair consisting of 'Compounding' and allowed numeric values:

- 0 - Simple interest (no compounding)
- 1 - Annual compounding
- 2 - Semiannual compounding (default)
- 3-Compounding three times per year
- 4-Quarterly compounding
- 6 - Bimonthly compounding
- 12 - Monthly compounding
- 365 - Daily compounding
-     - 1 - Continuous compounding


## Data Types: double

## Basis - Day-count basis used for annualizing input ZeroRates

0 (default) | numeric values: $0,1,2,3,4,6,7,8,9,10,11,12,13$
Day-count basis used for annualizing the input ZeroRates, specified as the commaseparated pair consisting of 'Basis' and allowed numeric values:

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- $2=$ actual $/ 360$
- $3=$ actual $/ 365$
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- $9=$ actual/360 (ICMA)
- $10=$ actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-0
Data Types: double

## Output Arguments

DiscRates - Discount factors
decimal
Discount factors, returned as a NUMBONDS-by-1 vector of decimal fractions. In aggregate, the discount factors constitute a discount curve for the investment horizon represented by CurveDates.

## CurveDates - Maturity dates that correspond to DiscRates <br> serial date number | date character vector | datetime

Maturity dates that correspond to the DiscRates, returned as a NUMBONDS-by-1 vector of maturity dates that correspond to the discount factors. This vector is the same as the input vector CurveDates, but is sorted by ascending maturity.

If either inputs for CurveDates or Settle are a datetime array, CurveDates is returned as a datetime array. Otherwise, CurveDates is returned as a serial date number.

## See Also

datetime|disc2zero|fwd2zero|prbyzero|pyld2zero|zbtprice|zbtyield| zero2fwd | zero2pyld

## Topics

"Term Structure of Interest Rates" on page 2-42
"Fixed-Income Terminology" on page 2-22

Introduced before R2006a

## zero2fwd

Forward curve given zero curve

Note In R2017b, the specification of optional input arguments has changed. While the previous ordered inputs syntax is still supported, it may no longer be supported in a future release. Use the new optional name-value pair inputs: InputCompounding, InputBasis, OutputCompounding, and OutputBasis.

## Syntax

[ForwardRates,CurveDates] = zero2fwd(ZeroRates,CurveDates,Settle) [ForwardRates,CurveDates] = zero2fwd( $\qquad$ , Name, Value)

## Description

[ForwardRates,CurveDates] = zero2fwd(ZeroRates,CurveDates,Settle) returns an implied forward rate curve given a zero curve and its maturity dates. If either input for CurveDates or Settle is a datetime array, CurveDates is returned as a datetime array. Otherwise, CurveDates is returned as a serial date number. ForwardRates is the same for any of these input data types.
[ForwardRates,CurveDates] = zero2fwd( $\qquad$ , Name, Value) adds optional namevalue pair arguments

## Examples

## Compute an Implied Forward Rate Curve Given a Zero Curve and Maturity Dates

Given a zero curve over a set of maturity dates, a settlement date, and a compounding rate, compute the forward rate curve.

```
ZeroRates = [0.0458
    0.0502
```

```
    0.0518
    0.0519
    0.0524
    0.0519
    0.0523
    0.0525
    0.0541
    0.0529];
CurveDates = [datenum('06-Nov-2000')
    datenum('11-Dec-2000')
    datenum('15-Jan-2001')
    datenum('05-Feb-2001')
    datenum('04-Mar-2001')
    datenum('02-Apr-2001')
    datenum('30-Apr-2001')
    datenum('25-Jun-2001')
    datenum('04-Sep-2001')
    datenum('12-Nov-2001')];
Settle = datenum('03-Nov-2000');
InputCompounding = 1;
InputBasis = 2;
OutputCompounding = 1;
OutputBasis = 2;
Execute the function zero2fwd to return the forward rate curve ForwardRates at the maturity dates CurveDates.
[ForwardRates, CurveDates] = zero2fwd(ZeroRates, CurveDates,...
Settle, 'InputCompounding',1,'InputBasis',2,'OutputCompounding',1,'OutputBasis',2)
ForwardRates = \(10 \times 1\)
0.0458
0.0506
0.0535
0.0522
0.0541
0.0498
0.0544
0.0531
0.0594
0.0476
```


## CurveDates $=10 \times 1$

730796
730831
730866
730887
730914
730943
730971
731027
731098
731167

## Compute an Implied Forward Rate Curve Given a Zero Curve and Maturity Dates Using datetime Inputs

Given a zero curve over a set of maturity dates, a settlement date, and a compounding rate, use datetime compute the forward rate curve.

```
ZeroRates = [0.0458
    0.0502
    0.0518
    0.0519
    0.0524
    0.0519
    0.0523
    0.0525
    0.0541
    0.0529];
CurveDates = [datenum('06-Nov-2000')
    datenum('11-Dec-2000')
    datenum('15-Jan-2001')
    datenum('05-Feb-2001')
    datenum('04-Mar-2001')
    datenum('02-Apr-2001')
    datenum('30-Apr-2001')
    datenum('25-Jun-2001')
    datenum('04-Sep-2001')
    datenum('12-Nov-2001')];
```

```
Settle = datenum('03-Nov-2000');
InputCompounding = 1;
InputBasis = 2;
OutputCompounding = 1;
OutputBasis = 2;
CurveDates = datetime(CurveDates, 'ConvertFrom', 'datenum','Locale','en_US');
Settle = datetime(Settle,'ConvertFrom','datenum','Locale','en_US');
[ForwardRates, CurveDates] = zero2fwd(ZeroRates, CurveDates,...
Settle,'InputCompounding',1,'InputBasis',2,'OutputCompounding',1,'OutputBasis',2)
ForwardRates = 10×1
    0.0458
    0.0506
    0.0535
    0.0522
    0.0541
    0.0498
    0.0544
    0.0531
    0.0594
    0.0476
CurveDates = 10x1 datetime array
    06-Nov-2000 00:00:00
    11-Dec-2000 00:00:00
    15-Jan-2001 00:00:00
    05-Feb-2001 00:00:00
    04-Mar-2001 00:00:00
    02-Apr-2001 00:00:00
    30-Apr-2001 00:00:00
    25-Jun-2001 00:00:00
    04-Sep-2001 00:00:00
    12-Nov-2001 00:00:00
```


## Input Arguments

## ZeroRates - Annualized zero rates

decimal fraction

Annualized zero rates, specified as a NUMBONDS-by-1 vector using decimal fractions. In aggregate, the rates constitute an implied zero curve for the investment horizon represented by CurveDates. The first element pertains to forward rates from the settlement date to the first curve date.

Data Types: double

## CurveDates - Maturity dates

serial date number | date character vector | datetime
Maturity dates, specified as a NUMBONDS-by-1 vector using serial date numbers, date character vectors, or datetime arrays, that correspond to the ZeroRates.

Data Types: double | datetime | char

## Settle - Common settlement date for ZeroRates <br> serial date number | date character vector | datetime

Common settlement date for input ZeroRates, specified as serial date numbers, date character vectors, or datetime arrays.
Data Types: double \| datetime | char

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, ... , NameN, ValueN.

Example: [ForwardRates,CurveDates] = zero2fwd(ZeroRates,CurveDates,Settle,' InputCompounding', 3,'InputBasis',5,'OutputCompounding',4,'OutputBasis',5)

## InputCompounding - Compounding frequency of input zero rates

2 (default) | numeric values: $0,1,2,3,4,6,12,365,-1$
Compounding frequency of input zero rates, specified as the comma-separated pair consisting of 'InputCompounding' and allowed values:

- 0 - Simple interest (no compounding)
- 1 - Annual compounding
- 2 - Semiannual compounding (default)
- 3-Compounding three times per year
- 4 - Quarterly compounding
- 6 - Bimonthly compounding
- 12 - Monthly compounding
- 365 - Daily compounding
-     - 1 - Continuous compounding

Note If InputCompounding is not specified, then InputCompounding is assigned the value specified for OutputCompounding. If either InputCompounding or OutputCompounding are not specified, the default is 2 (semiannual) for both.

## Data Types: double

## InputBasis - Day-count basis of input zero rates

0 (default) | numeric values: $0,1,2,3,4,6,7,8,9,10,11,12,13$
Day count basis of input zero rates, specified as the comma-separated pair consisting of ' InputBasis' and allowed values:

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- 2 = actual/360
- $3=$ actual/365
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- $9=$ actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-0

Note If InputBasis is not specified, then InputBasis is assigned the value specified for OutputBasis. If either InputBasis or Outputbasis are not specified, the default is 0 (actual/actual) for both.

## Data Types: double

## OutputCompounding - Compounding frequency of output forward rates

2 (default) | numeric values: $0,1,2,3,4,6,12,365,-1$
Compounding frequency of output forward rates, specified as the comma-separated pair consisting of 'OutputCompounding ' and allowed values:

- 0 - Simple interest (no compounding)
- 1 - Annual compounding
- 2 - Semiannual compounding (default)
- 3-Compounding three times per year
- 4-Quarterly compounding
- 6-Bimonthly compounding
- 12 - Monthly compounding
- 365 - Daily compounding
-     - 1 - Continuous compounding

Note If OutputCompounding is not specified, then OutputCompounding is assigned the value specified for InputCompounding. If either InputCompounding or OutputCompounding are not specified, the default is 2 (semiannual) for both.

## Data Types: double

OutputBasis - Day-count basis of output forward rates
0 (default) | numeric values: $0,1,2,3,4,6,7,8,9,10,11,12,13$

Day count basis of output forward rates, specified as the comma-separated pair consisting of 'OutputBasis' and allowed values:

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- 2 = actual/360
- 3 = actual/365
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- $9=$ actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- $12=$ actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-0

Note If OutputBasis is not specified, then OutputBasis is assigned the value specified for InputBasis. If either InputBasis or OutputBasis are not specified, the default is 0 (actual/actual) for both.

## Data Types: double

## Output Arguments

ForwardRates - Forward curve for investment horizon represented by CurveDates
numeric
Forward curve for the investment horizon represented by CurveDates, returned as a NUMBONDS-by-1 vector of decimal fractions. In aggregate, the rates in ForwardRates
constitute a forward curve over the dates in CurveDates. ForwardRates are ordered by ascending maturity.

## CurveDates - Maturity dates that correspond to ForwardRates serial date number | date character vector | datetime

Maturity dates that correspond to the ForwardRates, returned as a NUMBONDS-by-1 vector of maturity dates that correspond to the ForwardRates.

ForwardRates are expressed as serial date numbers (default) or datetimes (if CurveDates or Settle are datetime arrays), representing the maturity dates for each rate in ForwardRates. These dates are the same dates as those associated with the input ZeroRates, but are ordered by ascending maturity.

## See Also

datetime|disc2zero|fwd2zero|getForwardRates|pyld2zero|zbtprice| zbtyield|zero2disc|zero2pyld

## Topics

"Term Structure of Interest Rates" on page 2-42
"Fixed-Income Terminology" on page 2-22

## Introduced before R2006a

## zero2pyld

Par yield curve given zero curve

Note In R2017b, the specification of optional input arguments has changed. While the previous ordered inputs syntax is still supported, it may no longer be supported in a future release. Use the new optional name-value pair inputs: InputCompounding, InputBasis, OutputCompounding, and OutputBasis.

## Syntax

[ParRates,CurveDates] = zero2pyld(ZeroRates,CurveDates,Settle)
[ParRates,CurveDates] = zero2pyld( $\qquad$ ,Name, Value)

## Description

[ParRates,CurveDates] = zero2pyld(ZeroRates,CurveDates,Settle) returns a par yield curve given a zero curve and its maturity dates. If either input for CurveDates or Settle is a datetime array, CurveDates is returned as a datetime array. Otherwise, CurveDates is returned as a serial date number. ParRates is the same for any of these input data types.
[ParRates,CurveDates] = zero2pyld( $\qquad$ ,Name, Value) adds optional namevalue pair arguments

## Examples

## Compute Par Yield Curve Given a Zero Curve and Maturity Dates

Given a zero curve over a set of maturity dates, a settlement date, and annual compounding for the input zero curve and monthly compounding for the output par rates, compute a par yield curve.

ZeroRates $=$ [0.0457
0.0487

```
    0.0506
    0.0507
    0.0505
    0.0504
    0.0506
    0.0516
    0.0539
    0.0530];
CurveDates = [datenum('06-Nov-2000')
        datenum('11-Dec-2000')
        datenum('15-Jan-2001')
        datenum('05-Feb-2001')
        datenum('04-Mar-2001')
        datenum('02-Apr-2001')
        datenum('30-Apr-2001')
        datenum('25-Jun-2001')
        datenum('04-Sep-2001')
        datenum('12-Nov-2001')];
Settle = datenum('03-Nov-2000');
InputCompounding = 12;
InputBasis = 2;
OutputCompounding = 1;
OutputBasis = 2;
[ParRates, CurveDates] = zero2pyld(ZeroRates, CurveDates,...
Settle, 'InputCompounding',1,'InputBasis',1,'OutputCompounding',12,'OutputBasis',1)
ParRates = 10×1
    0.0448
    0.0477
    0.0495
    0.0496
    0.0494
    0.0493
    0.0495
    0.0504
    0.0526
    0.0517
CurveDates = 10×1
```

730796
730831
730866
730887
730914
730943
730971
731027
731098
731167

## Compute Par Yield Curve Given a Zero Curve and Maturity Dates Using datetime Inputs

Given a zero curve over a set of maturity dates, a settlement date, and annual compounding for the input zero curve and monthly compounding for the output par rates, use datetime inputs to compute a par yield curve.

```
ZeroRates = [0.0457
0.0487
0.0506
0.0507
0.0505
0.0504
0.0506
0.0516
0.0539
0.0530];
CurveDates = [datenum('06-Nov-2000')
datenum('11-Dec-2000')
datenum('15-Jan-2001')
datenum('05-Feb-2001')
datenum('04-Mar-2001')
datenum('02-Apr-2001')
datenum('30-Apr-2001')
datenum('25-Jun-2001')
datenum('04-Sep-2001')
datenum('12-Nov-2001')];
Settle = datenum('03-Nov-2000');
InputCompounding = 12;
InputBasis = 2;
```

```
OutputCompounding = 1;
OutputBasis = 2;
CurveDates = datetime(CurveDates, 'ConvertFrom', 'datenum','Locale','en_US');
Settle = datetime(Settle,'ConvertFrom','datenum','Locale','en_US');
[ParRates, CurveDates] = zero2pyld(ZeroRates, CurveDates,...
Settle, 'InputCompounding',12,'InputBasis',2,'OutputCompounding',1,'OutputBasis',2)
ParRates = 10\times1
    -0.0436
    0.0611
    0.0579
    0.0567
    0.0550
    0.0543
    0.0541
    0.0546
    0.0565
    0.0561
CurveDates = 10x1 datetime array
    06-Nov-2000 00:00:00
    11-Dec-2000 00:00:00
    15-Jan-2001 00:00:00
    05-Feb-2001 00:00:00
    04-Mar-2001 00:00:00
    02-Apr-2001 00:00:00
    30-Apr-2001 00:00:00
    25-Jun-2001 00:00:00
    04-Sep-2001 00:00:00
    12-Nov-2001 00:00:00
```


## Demonstrate a Roundtrip From zero2pyld to pyld2zero

Given the following zero curve and its maturity dates, return the ParRates.
Settle = datenum('01-Feb-2013');
CurveDates = [datenum('01-Feb-2014')

```
    datenum('01-Feb-2015')
    datenum('01-Feb-2016')
    datenum('01-Feb-2018')
    datenum('01-Feb-2020')
    datenum('01-Feb-2023')
    datenum('01-Feb-2033')
    datenum('01-Feb-2043')];
OriginalZeroRates = [.11 0.30 0.64 1.44 2.07 2.61 3.29 3.55]'/100;
OutputCompounding = 1;
OutputBasis = 0;
InputCompounding = 1;
InputBasis = 0;
ParRates = zero2pyld(OriginalZeroRates, CurveDates, Settle, ...
'OutputCompounding', OutputCompounding, 'OutputBasis', OutputBasis, ...
'InputCompounding', InputCompounding, 'InputBasis', InputBasis)
ParRates = 8×1
    0.0011
    0.0030
    0.0064
    0.0142
    0.0202
    0.0251
    0.0310
    0.0331
```

For the ParRates, use the pyld2zero function to return the ZeroRates0ut and determine the roundtrip error.

ZeroRatesOut = pyld2zero(ParRates, CurveDates, Settle, ...
'OutputCompounding', OutputCompounding, 'OutputBasis', OutputBasis, ... 'InputCompounding', InputCompounding, 'InputBasis', InputBasis)

ZeroRatesOut $=8 \times 1$
0.0011
0.0030
0.0064
0.0144
0.0207
0.0261
0.0329
0.0355

```
max(abs(OriginalZeroRates - ZeroRatesOut)) % Roundtrip error
ans = 1.4919e-16
```


## Input Arguments

## ZeroRates - Annualized zero rates

decimal fraction
Annualized zero rates, specified as a NUMBONDS-by-1 vector using decimal fractions. In aggregate, the rates constitute an implied zero curve for the investment horizon represented by CurveDates.
Data Types: double

## CurveDates - Maturity dates

serial date number | date character vector | datetime
Maturity dates which correspond to the input ZeroRates, specified as a NUMBONDS-by-1 vector using serial date numbers, date character vectors, or datetime arrays.

Data Types: double | datetime | char

## Settle - Common settlement date for ZeroRates

serial date number | date character vector | datetime
Common settlement date for input ZeroRates, specified as serial date numbers, date character vectors, or datetime arrays.
Data Types: double | datetime | char

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes.

You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Example: [ParRates,CurveDates] = zero2pyld(ZeroRates,CurveDates, Settle,'OutputCompounding',3,'OutputBasis',5,'InputCompounding', 4,'InputBasis',5)

## OutputCompounding - Compounding frequency of output ParRates

2 (default) | numeric values: $0,1,2,3,4,6,12,365,-1$
Compounding frequency of output ParRates, specified as the comma-separated pair consisting of 'OutputCompounding' and allowed values:

- 1 - Annual compounding
- 2 - Semiannual compounding (default)
- 3-Compounding three times per year
- 4 - Quarterly compounding
- 6-Bimonthly compounding
- 12 - Monthly compounding


## Note

- If InputCompounding is $1,2,3,4,6$, or 12 and OutputCompounding is not specified, the value of InputCompounding is used.
- If InputCompounding is 0 (simple), - 1 (continuous), or 365 (daily), a valid OutputCompounding value must also be specified.
- If either InputCompounding or OutputCompounding are not specified, the default is 2 (semiannual) for both.


## Data Types: double

## OutputBasis - Day-count basis of output ParRates

0 (default) | numeric values: 0,1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 13
Day count basis of output ParRates, specified as the comma-separated pair consisting of 'OutputBasis' and allowed values:

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- 2 = actual/360
- 3 = actual/365
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-0

Note If OutputBasis is not specified, then OutputBasis is assigned the value specified for InputBasis. If either InputBasis or OutputBasis are not specified, the default is 0 (actual/actual) for both.

## Data Types: double

## InputCompounding - Compounding frequency of input ZeroRates

2 (default) | numeric values: $0,1,2,3,4,6,12,365,-1$
Compounding frequency of input ZeroRates, specified as the comma-separated pair consisting of 'InputCompounding' and allowed values:

- 0 - Simple interest (no compounding)
- 1 - Annual compounding
- 2 - Semiannual compounding (default)
- 3 - Compounding three times per year
- 4-Quarterly compounding
- 6 - Bimonthly compounding
- 12 - Monthly compounding
- 365 - Daily compounding
-     - 1 - Continuous compounding


## Note

- If InputCompounding is set to 0 (simple), - 1 (continuous), or 365 (daily), the OutputCompounding must also be specified using a valid value.
- If InputCompounding is not specified, then InputCompounding is assigned the value specified for OutputCompounding.
- If either InputCompounding or OutputCompounding are not specified, the default is 2 (semiannual) for both.


## Data Types: double

## InputBasis - Day-count basis of input ZeroRates

0 (default) | numeric values: $0,1,2,3,4,6,7,8,9,10,11,12,13$
Day count basis of the input ZeroRates, specified as the comma-separated pair consisting of 'InputBasis' and allowed values:

- $0=$ actual/actual
- $1=30 / 360$ (SIA)
- 2 = actual/360
- $3=$ actual $/ 365$
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- $10=$ actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- $12=$ actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-0

Note If InputBasis is not specified, then InputBasis is assigned the value specified for OutputBasis. If either InputBasis or Outputbasis are not specified, the default is 0 (actual/actual) for both.

## Data Types: double

## Output Arguments

## ParRates - Par bond coupon rates

## numeric

Par bond coupon rates, returned as a NUMBONDS-by-1 numeric vector. ParRates are ordered by ascending maturity.

## CurveDates - Maturity dates that correspond to ParRates <br> serial date number | date character vector | datetime

Maturity dates that correspond to the ParRates, returned as a NUMBONDS-by-1 vector of maturity dates that correspond to each par rate contained in ParRates.

ParRates are expressed as serial date numbers (default) or datetimes (if CurveDates or Settle are datetime arrays). CurveDates are ordered by ascending maturity.

## See Also

datetime | datetime | disc2zero| fwd2zero| getForwardRates | pyld2zero| zbtprice|zbtyield|zero2disc|zero2fwd

## Topics

"Term Structure of Interest Rates" on page 2-42
"Fixed-Income Terminology" on page 2-22

## Introduced before R2006a

## score

Compute credit scores for given data

## Syntax

```
Scores = score(sc)
Scores = score(sc, data)
[Scores,Points] = score(sc)
[Scores,Points] = score(sc,data)
```


## Description

Scores $=$ score(sc) computes the credit scores for the creditscorecard object's training data. This data can be a "training" or a "live" dataset. If the data input argument is not explicitly provided, the score function determines scores for the existing creditscorecard object's data.
formatpoints supports multiple alternatives to modify the scaling of the scores and can also be used to control the rounding of points and scores, and whether the base points are reported separately or spread across predictors. Missing data translates into NaN values for the corresponding points, and therefore for the total score. Use formatpoints to modify the score behavior for rows with missing data.

Scores $=$ score(sc, data) computes the credit scores for the given input data. This data can be a "training" or a "live" dataset.
formatpoints supports multiple alternatives to modify the scaling of the scores and can also be used to control the rounding of points and scores, and whether the base points are reported separately or spread across predictors. Missing data translates into NaN values for the corresponding points, and therefore for the total score. Use formatpoints to modify the score behavior for rows with missing data.
[Scores,Points] = score(sc) computes the credit scores and points for the given data. If the data input argument is not explicitly provided, the score function determines scores for the existing creditscorecard object's data.
formatpoints supports multiple alternatives to modify the scaling of the scores and can also be used to control the rounding of points and scores, and whether the base points are reported separately or spread across predictors. Missing data translates into NaN values for the corresponding points, and therefore for the total score. Use formatpoints to modify the score behavior for rows with missing data.
[Scores,Points] = score(sc,data) computes the credit scores and points for the given input data. This data can be a "training" or a "live" dataset.
formatpoints supports multiple alternatives to modify the scaling of the scores and can also be used to control the rounding of points and scores, and whether the base points are reported separately or spread across predictors. Missing data translates into NaN values for the corresponding points, and therefore for the total score. Use formatpoints to modify the score behavior for rows with missing data.

## Examples

## Obtain Scores for Training Data

This example shows how to use score to obtain scores for the training data.
Create a creditscorecard object using the CreditCardData.mat file to load the data (using a dataset from Refaat 2011). Use the 'IDVar' argument in creditscorecard to indicate that 'CustID' contains ID information and should not be included as a predictor variable.

```
load CreditCardData
sc = creditscorecard(data,'IDVar','CustID');
```

Perform automatic binning to bin for all predictors.
sc = autobinning(sc);
Fit a linear regression model using default parameters.

```
sc = fitmodel(sc);
```

1. Adding CustIncome, Deviance $=1490.8527$, Chi2Stat $=32.588614$, PValue $=1.1387992 \mathrm{e}-($
2. Adding TmWBank, Deviance $=1467.1415$, Chi2Stat $=23.711203$, PValue $=1.1192909 \mathrm{e}-06$
3. Adding AMBalance, Deviance $=1455.5715$, Chi2Stat $=11.569967$, PValue $=0.0006702560$
4. Adding EmpStatus, Deviance $=1447.3451$, Chi2Stat $=8.2264038$, PValue $=0.0041285257$
5. Adding CustAge, Deviance $=1441.994$, Chi2Stat $=5.3511754$, PValue $=0.020708306$
6. Adding ResStatus, Deviance $=1437.8756$, Chi2Stat $=4.118404$, PValue $=0.042419078$
7. Adding OtherCC, Deviance $=1433.707$, Chi2Stat $=4.1686018$, PValue $=0.041179769$

Generalized linear regression model:
status ~ [Linear formula with 8 terms in 7 predictors]
Distribution = Binomial
Estimated Coefficients:

| Estimate | SE | tStat |  |  |
| ---: | ---: | ---: | ---: | ---: | pValue

1200 observations, 1192 error degrees of freedom
Dispersion: 1
Chi^2-statistic vs. constant model: 89.7, p-value $=1.4 \mathrm{e}-16$
Score training data using the score function without an optional input for data. By default, it returns unscaled scores. For brevity, only the first ten scores are displayed.

Scores = score(sc);
disp(Scores(1:10))

1. 0968
1.4646
0.7662
1.5779
1.4535
1.8944
-0.0872
0.9207
1.0399
0.8252

Scale scores and display both points and scores for each individual in the training data (for brevity, only the first ten rows are displayed). For other scaling methods, and other options for formatting points and scores, use the formatpoints function.

```
sc = formatpoints(sc,'WorstAndBestScores',[300 850]);
[Scores,Points] = score(sc);
disp(Scores(1:10))
    602.0394
    648.1988
    560.5569
    662.4189
    646.8109
    702.1398
    453.4572
    579.9475
    594.9064
    567.9533
disp(Points(1:10,:))
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline CustAge & ResStatus & EmpStatus & CustIncome & TmWBank & OtherCC & AMBalance \\
\hline 95.256 & 62.421 & 56.765 & 121.18 & 116.05 & 86.224 & 64.15 \\
\hline 126.46 & 82.276 & 105.81 & 121.18 & 62.107 & 86.224 & 64.15 \\
\hline 93.256 & 62.421 & 105.81 & 76.585 & 116.05 & 42.287 & 64.15 \\
\hline 95.256 & 82.276 & 105.81 & 121.18 & 60.719 & 86.224 & 110.96 \\
\hline 126.46 & 82.276 & 105.81 & 121.18 & 60.719 & 86.224 & 64.15 \\
\hline 126.46 & 82.276 & 105.81 & 121.18 & 116.05 & 86.224 & 64.15 \\
\hline 48.727 & 82.276 & 56.765 & 53.208 & 62.107 & 86.224 & 64.15 \\
\hline 95.256 & 113.58 & 105.81 & 121.18 & 62.107 & 42.287 & 39.729 \\
\hline 95.256 & 62.421 & 56.765 & 121.18 & 62.107 & 86.224 & 110.96 \\
\hline 95.256 & 82.276 & 56.765 & 121.18 & 62.107 & 86.224 & 64.15 \\
\hline
\end{tabular}
```


## Scores for Missing or Out-of-Range Data When Using the 'BinMissingData' Option

This example describes the assignment of points for missing data when the 'BinMissingData' option is set to true.

- Predictors that have missing data in the training set have an explicit bin for <missing> with corresponding points in the final scorecard. These points are computed from the Weight-of-Evidence (WOE) value for the <missing> bin and the logistic model coefficients. For scoring purposes, these points are assigned to missing values and to out-of-range values.
- Predictors with no missing data in the training set have no <missing> bin, therefore no WOE can be estimated from the training data. By default, the points for missing and out-of-range values are set to NaN , and this leads to a score of NaN when running score. For predictors that have no explicit <missing> bin, use the name-value argument 'Missing' in formatpoints to indicate how missing data should be treated for scoring purposes.

Create a creditscorecard object using the CreditCardData.mat file to load the dataMissing with missing values.

```
load CreditCardData.mat
head(dataMissing,5)
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline CustID & CustAge & TmAtAddress & ResStatus & EmpStatus & CustIncome & TmWB \\
\hline 1 & 53 & 62 & <undefined> & Unknown & 50000 & 55 \\
\hline 2 & 61 & 22 & Home Owner & Employed & 52000 & 25 \\
\hline 3 & 47 & 30 & Tenant & Employed & 37000 & 61 \\
\hline 4 & NaN & 75 & Home Owner & Employed & 53000 & 20 \\
\hline 5 & 68 & 56 & Home Owner & Employed & 53000 & 14 \\
\hline
\end{tabular}
fprintf('Number of rows: %d\n',height(dataMissing))
Number of rows: 1200
fprintf('Number of missing values CustAge: %d\n',sum(ismissing(dataMissing.CustAge)))
Number of missing values CustAge: 30
fprintf('Number of missing values ResStatus: %d\n',sum(ismissing(dataMissing.ResStatus
Number of missing values ResStatus: 40
```

Use creditscorecard with the name-value argument 'BinMissingData' set to true to bin the missing numeric or categorical data in a separate bin. Apply automatic binning.

```
sc = creditscorecard(dataMissing,'IDVar','CustID','BinMissingData',true);
sc = autobinning(sc);
disp(sc)
    creditscorecard with properties:
```

```
                        GoodLabel: 0
                ResponseVar: 'status'
                        WeightsVar:
                            VarNames: {1x11 cell}
        NumericPredictors: {1x6 cell}
        CategoricalPredictors: {'ResStatus' 'EmpStatus' 'OtherCC'}
        BinMissingData: 1
            IDVar: 'CustID'
        PredictorVars: {1x9 cell}
            Data: [1200x11 table]
```

Set a minimum value of zero for CustAge and CustIncome. With this, any negative age or income information becomes invalid or "out-of-range". For scoring purposes, out-ofrange values are given the same points as missing values.

```
sc = modifybins(sc,'CustAge','MinValue',0);
sc = modifybins(sc,'CustIncome','MinValue',0);
```

Display and plot bin information for numeric data for 'CustAge' that includes missing data in a separate bin labelled <missing>.

| Bin | Good | Bad | Odds | WOE | InfoValue |
| :---: | :---: | :---: | :---: | :---: | :---: |
| ' $[0,33$ ) ' | 69 | 52 | 1.3269 | -0.42156 | 0.018993 |
| ' $[33,37)^{\prime}$ | 63 | 45 | 1.4 | -0.36795 | 0.012839 |
| ' $[37,40)^{\prime}$ | 72 | 47 | 1.5319 | -0.2779 | 0.0079824 |
| ' $[40,46$ )' | 172 | 89 | 1.9326 | -0.04556 | 0.0004549 |
| ' $[46,48$ )' | 59 | 25 | 2.36 | 0.15424 | 0.0016199 |
| ' $[48,51$ )' | 99 | 41 | 2.4146 | 0.17713 | 0.0035449 |
| ' $[51,58$ )' | 157 | 62 | 2.5323 | 0.22469 | 0.0088407 |
| '[58,Inf]' | 93 | 25 | 3.72 | 0.60931 | 0.032198 |
| '<missing>' | 19 | 11 | 1.7273 | -0.15787 | 0.00063885 |
| 'Totals' | 803 | 397 | 2.0227 | NaN | 0.087112 |

```
plotbins(sc,'CustAge')
```



Display and plot bin information for categorical data for 'ResStatus' that includes missing data in a separate bin labelled <missing>.
[bi,cg] = bininfo(sc,'ResStatus'); disp(bi)

| Bin | Good | Bad | Odds | WOE | InfoValue |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 'Tenant' | 296 | 161 | 1.8385 | -0.095463 | 0.0035249 |
| 'Home Owner' | 352 | 171 | 2.0585 | 0.017549 | 0.00013382 |
| '0ther' | 128 | 52 | 2.4615 | 0.19637 | 0.0055808 |
| '<missing>' | 27 | 13 | 2.0769 | 0.026469 | $2.3248 \mathrm{e}-05$ |
| 'Totals' | 803 | 397 | 2.0227 | NaN | 0.0092627 |

```
plotbins(sc,'ResStatus')
```

ResStatus


Good $\square \mathrm{Bad}$

For the 'CustAge' and 'ResStatus' predictors, there is missing data ( NaNs and <undefined>) in the training data, and the binning process estimates a WOE value of -0.15787 and 0.026469 respectively for missing data in these predictors, as shown above.

For EmpStatus and CustIncome there is no explicit bin for missing values because the training data has no missing values for these predictors.

```
bi = bininfo(sc,'EmpStatus');
disp(bi)
```



Use fitmodel to fit a logistic regression model using Weight of Evidence (WOE) data. fitmodel internally transforms all the predictor variables into WOE values, using the bins found with the automatic binning process. fitmodel then fits a logistic regression model using a stepwise method (by default). For predictors that have missing data, there is an explicit <missing> bin, with a corresponding WOE value computed from the data. When using fitmodel, the corresponding WOE value for the <missing> bin is applied when performing the WOE transformation.

```
[sc,mdl] = fitmodel(sc);
```

1. Adding CustIncome, Deviance $=1490.8527$, Chi2Stat $=32.588614$, PValue $=1.1387992 \mathrm{e}-\mathrm{C}$
2. Adding TmWBank, Deviance $=1467.1415$, Chi2Stat $=23.711203$, PValue $=1.1192909 \mathrm{e}-06$
3. Adding AMBalance, Deviance $=1455.5715$, Chi2Stat $=11.569967$, PValue $=0.0006702560$
4. Adding EmpStatus, Deviance $=1447.3451$, Chi2Stat $=8.2264038$, PValue $=0.0041285257$
5. Adding CustAge, Deviance $=1442.8477$, Chi2Stat $=4.4974731$, PValue $=0.033944979$
6. Adding ResStatus, Deviance $=1438.9783$, Chi2Stat $=3.86941$, PValue $=0.049173805$
7. Adding OtherCC, Deviance $=1434.9751$, Chi2Stat $=4.0031966$, PValue $=0.045414057$

Generalized linear regression model:
status ~ [Linear formula with 8 terms in 7 predictors]
Distribution = Binomial

Estimated Coefficients:

|  | Estimate |  | SE |  | tStat |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | pValue

```
1200 observations, 1192 error degrees of freedom
Dispersion: 1
Chi^2-statistic vs. constant model: 88.5, p-value = 2.55e-16
```

Scale the scorecard points by the "points, odds, and points to double the odds (PDO)" method using the 'Points0ddsAndPDO' argument of formatpoints. Suppose that you want a score of 500 points to have odds of 2 (twice as likely to be good than to be bad) and that the odds double every 50 points (so that 550 points would have odds of 4 ).

Display the scorecard showing the scaled points for predictors retained in the fitting model.

```
sc = formatpoints(sc,'PointsOddsAndPDO',[500 2 50]);
PointsInfo = displaypoints(sc)
PointsInfo=33\times3 table
Predictors Bin Points
    'CustAge'
    '[0,33)
54.062
    'CustAge'
    '[33,37)
56.282
    'CustAge' '[37,40)' 60.012
    'CustAge' '[40,46)' 69.636
    'CustAge' '[46,48)' 77.912
    'CustAge' '[48,51)' 78.86
    'CustAge' '[51,58)' 80.83
    'CustAge' '[58,Inf]' 96.76
    'CustAge' '<missing>' 64.984
    'ResStatus' 'Tenant' 62.138
    'ResStatus' 'Home Owner' }73.24
    'ResStatus' 'Other' 90.828
```

| 'ResStatus' | '<missing>' | 74.125 |
| :--- | :--- | :--- |
| 'EmpStatus' | 'Unknown' | 58.807 |
| 'EmpStatus' | 'Employed' | 86.937 |
| 'CustIncome' | '[0,29000)' | 29.375 |

Notice that points for the <missing> bin for CustAge and ResStatus are explicitly shown (as 64.9836 and 74.1250 , respectively). These points are computed from the WOE value for the <missing> bin, and the logistic model coefficients.

For predictors that have no missing data in the training set, there is no explicit <missing> bin. By default the points are set to NaN for missing data and they lead to a score of NaN when running score. For predictors that have no explicit <missing> bin, use the name-value argument 'Missing' in formatpoints to indicate how missing data should be treated for scoring purposes.

For the purpose of illustration, take a few rows from the original data as test data and introduce some missing data. Also introduce some invalid, or out-of-range values. For numeric data, values below the minimum (or above the maximum) allowed are considered invalid, such as a negative value for age (recall 'MinValue' was earlier set to 0 for CustAge and CustIncome). For categorical data, invalid values are categories not explicitly included in the scorecard, for example, a residential status not previously mapped to scorecard categories, such as "House", or a meaningless string such as "abc123".

```
tdata = dataMissing(11:18,mdl.PredictorNames); % Keep only the predictors retained in
% Set some missing values
tdata.CustAge(1) = NaN;
tdata.ResStatus(2) = '<undefined>';
tdata.EmpStatus(3) = '<undefined>';
tdata.CustIncome(4) = NaN;
% Set some invalid values
tdata.CustAge(5) = -100;
tdata.ResStatus(6) = 'House';
tdata.EmpStatus(7) = 'Freelancer';
tdata.CustIncome(8) = -1;
disp(tdata)
\begin{tabular}{|c|c|c|c|c|c|}
\hline CustAge & ResStatus & EmpStatus & CustIncome & TmWBank & OtherCC \\
\hline NaN & Tenant & Unknown & 34000 & 44 & Yes \\
\hline
\end{tabular}
```

| 48 | <undefined> | Unknown | 44000 | 14 | Yes | 403.6 |
| ---: | :--- | :--- | ---: | ---: | ---: | ---: |
| 65 | Home Owner | <undefined> | 48000 | 6 | No | 111.8 |
| 44 | Other | Unknown | NaN | 35 | No | 436.6 |
| -100 | Other | Employed | 46000 | 16 | Yes | 162.2 |
| 33 | House | Employed | 36000 | 36 | Yes | 845.6 |
| 39 | Tenant | Freelancer | 34000 | 40 | Yes | 756.2 |
| 24 | Home Owner | Employed | -1 | 19 | Yes | 449.6 |

Score the new data and see how points are assigned for missing CustAge and ResStatus, because we have an explicit bin with points for <missing>. However, for EmpStatus and CustIncome the score function sets the points to NaN .

```
[Scores,Points] = score(sc,tdata);
disp(Scores)
```

    481.2231
    520.8353
        NaN
        NaN
    551.7922
    487.9588
        NaN
        NaN
    disp(Points)

| CustAge | ResStatus | EmpStatus | CustIncome | TmWBank | Othercc | AMBalance |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 64.984 | 62.138 | 58.807 | 67.893 | 61.858 | 75.622 | 89.922 |
| 78.86 | 74.125 | 58.807 | 82.439 | 61.061 | 75.622 | 89.922 |
| 96.76 | 73.248 | NaN | 96.969 | 51.132 | 50.914 | 89.922 |
| 69.636 | 90.828 | 58.807 | NaN | 61.858 | 50.914 | 89.922 |
| 64.984 | 90.828 | 86.937 | 82.439 | 61.061 | 75.622 | 89.922 |
| 56.282 | 74.125 | 86.937 | 70.107 | 61.858 | 75.622 | 63.028 |
| 60.012 | 62.138 | NaN | 67.893 | 61.858 | 75.622 | 63.028 |
| 54.062 | 73.248 | 86.937 | NaN | 61.061 | 75.622 | 89.922 |

Use the name-value argument 'Missing ' in formatpoints to choose how to assign points to missing values for predictors that do not have an explicit <missing> bin. In this example, use the 'MinPoints ' option for the 'Missing' argument. The minimum points for EmpStatus in the scorecard displayed above are 58.8072, and for CustIncome the minimum points are 29.3753.

```
sc = formatpoints(sc,'Missing','MinPoints');
[Scores,Points] = score(sc,tdata);
disp(Scores)
    481.2231
    520.8353
    517.7532
    451.3405
    551.7922
    487.9588
    449.3577
    470.2267
disp(Points)
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline CustAge & ResStatus & EmpStatus & CustIncome & TmWBank & OtherCC & AMBalance \\
\hline 64.984 & 62.138 & 58.807 & 67.893 & 61.858 & 75.622 & 89.922 \\
\hline 78.86 & 74.125 & 58.807 & 82.439 & 61.061 & 75.622 & 89.922 \\
\hline 96.76 & 73.248 & 58.807 & 96.969 & 51.132 & 50.914 & 89.922 \\
\hline 69.636 & 90.828 & 58.807 & 29.375 & 61.858 & 50.914 & 89.922 \\
\hline 64.984 & 90.828 & 86.937 & 82.439 & 61.061 & 75.622 & 89.922 \\
\hline 56.282 & 74.125 & 86.937 & 70.107 & 61.858 & 75.622 & 63.028 \\
\hline 60.012 & 62.138 & 58.807 & 67.893 & 61.858 & 75.622 & 63.028 \\
\hline 54.062 & 73.248 & 86.937 & 29.375 & 61.061 & 75.622 & 89.922 \\
\hline
\end{tabular}
```


## Score a New Dataset

This example shows how to use score to obtain scores for a new dataset (for example, a validation or a test dataset) using the optional 'data' input in the score function.

Create a creditscorecard object using the CreditCardData.mat file to load the data (using a dataset from Refaat 2011). Use the 'IDVar' argument in creditscorecard to indicate that 'CustID' contains ID information and should not be included as a predictor variable.

```
load CreditCardData
sc = creditscorecard(data,'IDVar','CustID');
```

Perform automatic binning to bin for all predictors.

```
sc = autobinning(sc);
```

Fit a linear regression model using default parameters.

```
sc = fitmodel(sc);
```

1. Adding CustIncome, Deviance = 1490.8527, Chi2Stat = 32.588614, PValue = 1.1387992e-
2. Adding TmWBank, Deviance $=1467.1415$, Chi2Stat $=23.711203$, PValue $=1.1192909 \mathrm{e}-06$
3. Adding AMBalance, Deviance $=1455.5715$, Chi2Stat $=11.569967$, PValue $=0.0006702560$
4. Adding EmpStatus, Deviance $=1447.3451$, Chi2Stat $=8.2264038$, PValue $=0.0041285257$
5. Adding CustAge, Deviance $=1441.994$, Chi2Stat $=5.3511754$, PValue $=0.020708306$
6. Adding ResStatus, Deviance $=1437.8756$, Chi2Stat $=4.118404$, PValue $=0.042419078$
7. Adding OtherCC, Deviance = 1433.707, Chi2Stat = 4.1686018, PValue $=0.041179769$

Generalized linear regression model:
status ~ [Linear formula with 8 terms in 7 predictors] Distribution = Binomial

Estimated Coefficients:

| Estimate | SE | tStat |  |  |
| ---: | ---: | ---: | ---: | ---: | pValue

1200 observations, 1192 error degrees of freedom
Dispersion: 1
Chi^2-statistic vs. constant model: 89.7, p-value = 1.4e-16
For the purpose of illustration, suppose that a few rows from the original data are our "new" data. Use the optional data input argument in the score function to obtain the scores for the newdata.
newdata = data(10:20,:);
Scores = score(sc, newdata)
Scores = $11 \times 1$
0.8252
0.6553
1.2443
0.9478
0.5690
1.6192
0.4899
0.3824
0.2945
1.4401

## Input Arguments

## sc - Credit scorecard model <br> creditscorecard object

Credit scorecard model, specified as a creditscorecard object. Use creditscorecard to create a creditscorecard object.
data - Dataset to be scored
table
(Optional) Dataset to be scored, specified as a MATLAB table where each row corresponds to individual observations. The data must contain columns for each of the predictors in the creditscorecard object.

## Output Arguments

## Scores - Scores for each observation <br> vector

Scores for each observation, returned as a vector.

## Points - Points per predictor for each observation table

Points per predictor for each observation, returned as a table.

## Algorithms

The score of an individual $i$ is given by the formula

```
Score(i) = Shift + Slope*(b0 + b1*WOE1(i) + b2*WOE2(i)+ ... +bp*WOEp(i))
```

where $b j$ is the coefficient of the $j$-th variable in the model, and WOEj(i) is the Weight of Evidence (WOE) value for the $i$-th individual corresponding to the $j$-th model variable. Shift and Slope are scaling constants that can be controlled with formatpoints.

If the data for individual $i$ is in the $i$-th row of a given dataset, to compute a score, the data $(i, j)$ is binned using existing binning maps, and converted into a corresponding Weight of Evidence value WOEj(i). Using the model coefficients, the unscaled score is computed as

```
s = b0 + b1*WOE1(i) + ... +bp*WOEp(i).
```

For simplicity, assume in the description above that the $j$-th variable in the model is the $j$ th column in the data input, although, in general, the order of variables in a given dataset does not have to match the order of variables in the model, and the dataset could have additional variables that are not used in the model.

The formatting options can be controlled using formatpoints.

## References

[1] Anderson, R. The Credit Scoring Toolkit. Oxford University Press, 2007.
[2] Refaat, M. Credit Risk Scorecards: Development and Implementation Using SAS. lulu.com, 2011.

## See Also

autobinning|bindata|bininfo|creditscorecard|displaypoints|fitmodel | formatpoints | modifybins | modifypredictor|plotbins | predictorinfo| probdefault|setmodel|table|validatemodel

## Topics

"Case Study for a Credit Scorecard Analysis" on page 8-101
"Credit Scorecard Modeling with Missing Values" on page 8-73
"Troubleshooting Credit Scorecard Results" on page 8-91 "Credit Scorecard Modeling Workflow" on page 8-67
"About Credit Scorecards" on page 8-62

## Introduced in R2014b

## formatpoints

Format scorecard points and scaling

## Syntax

sc = formatpoints(sc,Name,Value)

## Description

sc = formatpoints(sc,Name,Value) modifies the scorecard points and scaling using optional name-value pair arguments. For example, use optional name-value pair arguments to change the scaling of the scores or the rounding of the points.

## Examples

## Scale Points Using Worst and Best Scores

This example shows how to use formatpoints to scale by providing the Worst and Best score values. By using formatpoints to scale, you can put points and scores in a desired range that is more meaningful for practical purposes. Technically, this involves a linear transformation from the unscaled to the scaled points.

Create a creditscorecard object using the CreditCardData.mat file to load the data (using a dataset from Refaat 2011). Use the 'IDVar' argument in creditscorecard to indicate that 'CustID' contains ID information and should not be included as a predictor variable.
load CreditCardData
sc = creditscorecard(data,'IDVar','CustID');
Perform automatic binning to bin for all predictors.
sc = autobinning(sc);
Fit a linear regression model using default parameters.

```
sc = fitmodel(sc);
```

1. Adding CustIncome, Deviance $=1490.8527$, Chi2Stat $=32.588614$, PValue $=1.1387992 \mathrm{e}-\mathrm{C}$
2. Adding TmWBank, Deviance $=1467.1415$, Chi2Stat $=23.711203$, PValue $=1.1192909 \mathrm{e}-06$
3. Adding AMBalance, Deviance $=1455.5715$, Chi2Stat $=11.569967$, PValue $=0.0006702560$
4. Adding EmpStatus, Deviance $=1447.3451$, Chi2Stat $=8.2264038$, PValue $=0.0041285257$
5. Adding CustAge, Deviance $=1441.994$, Chi2Stat $=5.3511754$, PValue $=0.020708306$
6. Adding ResStatus, Deviance $=1437.8756$, Chi2Stat $=4.118404$, PValue $=0.042419078$
7. Adding OtherCC, Deviance $=1433.707$, Chi2Stat $=4.1686018$, PValue $=0.041179769$

Generalized linear regression model:
status ~ [Linear formula with 8 terms in 7 predictors]
Distribution = Binomial
Estimated Coefficients:

| Estimate | SE | tStat | pValue |
| :---: | :---: | :---: | :---: |
| 0.70239 | 0.064001 | 10.975 | 5.0538e-28 |
| 0.60833 | 0.24932 | 2.44 | 0.014687 |
| 1.377 | 0.65272 | 2.1097 | 0.034888 |
| 0.88565 | 0.293 | 3.0227 | 0.0025055 |
| 0.70164 | 0.21844 | 3.2121 | 0.0013179 |
| 1.1074 | 0.23271 | 4.7589 | 1.9464e-06 |
| 1.0883 | 0.52912 | 2.0569 | 0.039696 |
| 1.045 | 0.32214 | 3.2439 | 0.0011792 |

1200 observations, 1192 error degrees of freedom
Dispersion: 1
Chi^2-statistic vs. constant model: 89.7, p-value = 1.4e-16
Display unscaled points for predictors retained in the fitting model and display the minimum and maximum possible unscaled scores.
[PointsInfo,MinScore,MaxScore] = displaypoints(sc)

| PointsInfo=30×3 Predictors | Bin | Points |
| :---: | :---: | :---: |
| 'CustAge' | '[-Inf,33)' | -0.15894 |
| 'CustAge' | '[33,37)' | -0.14036 |
| 'CustAge' | '[37,40)' | -0.060323 |
| 'CustAge' | '[40,46)' | 0.046408 |


| 'CustAge' | '[46,48)' | 0.21445 |
| :--- | :--- | ---: |
| 'CustAge' | '[48,58)' | 0.23039 |
| 'CustAge' | '[58,Inf]' | 0.479 |
| 'ResStatus' | 'Tenant' | -0.031252 |
| 'ResStatus' | 'Home Owner' | 0.12696 |
| 'ResStatus' | 'Other' | 0.37641 |
| 'EmpStatus' | 'Unknown' | -0.076317 |
| 'EmpStatus' | 'Employed' | 0.31449 |
| 'CustIncome' | '[-Inf,29000)' | -0.45716 |
| 'CustIncome' | '[29000,33000)' | -0.10466 |
| 'CustIncome' | '[33000,35000)' | 0.052329 |
| 'CustIncome' | '[35000,40000)' | 0.081611 |

MinScore = -1.3100
MaxScore $=3.0726$
Scale by providing the 'Worst' and 'Best' score values. The range provided below is a common score range. Display the points information again to verify that they are now scaled and also display the scaled minimum and maximum scores.

```
sc = formatpoints(sc,'WorstAndBestScores',[300 850]);
[PointsInfo,MinScore,MaxScore] = displaypoints(sc)
```



```
MinScore = 300
```

MaxScore $=850.0000$

As expected, the values of MinScore and MaxScore correspond to the desired worst and best scores.

## Scale Points Using Shift and Slope

This example shows how to use formatpoints to scale by providing the Shift and Slope values. By using formatpoints to scale, you can put points and scores in a desired range that is more meaningful for practical purposes. Technically, this involves a linear transformation from the unscaled to the scaled points by the formatpoints function.

Create a creditscorecard object using the CreditCardData.mat file to load the data (using a dataset from Refaat 2011). Use the 'IDVar' argument in creditscorecard to indicate that 'CustID' contains ID information and should not be included as a predictor variable.

```
load CreditCardData
sc = creditscorecard(data,'IDVar','CustID');
```

Perform automatic binning to bin for all predictors.

```
sc = autobinning(sc);
```

Fit a linear regression model using default parameters.

```
sc = fitmodel(sc);
```

1. Adding CustIncome, Deviance = 1490.8527, Chi2Stat = 32.588614, PValue = 1.1387992e-c
2. Adding TmWBank, Deviance $=1467.1415$, Chi2Stat $=23.711203$, PValue $=1.1192909 \mathrm{e}-06$
3. Adding AMBalance, Deviance $=1455.5715$, Chi2Stat $=11.569967$, PValue $=0.0006702560$
4. Adding EmpStatus, Deviance $=1447.3451$, Chi2Stat $=8.2264038$, PValue $=0.0041285257$
5. Adding CustAge, Deviance $=1441.994$, Chi2Stat $=5.3511754$, PValue $=0.020708306$
6. Adding ResStatus, Deviance $=1437.8756$, Chi2Stat $=4.118404$, PValue $=0.042419078$
7. Adding OtherCC, Deviance $=1433.707$, Chi2Stat $=4.1686018$, PValue $=0.041179769$

Generalized linear regression model:
status ~ [Linear formula with 8 terms in 7 predictors] Distribution = Binomial

Estimated Coefficients:

| Estimat | SE | tStat | pValue |
| :---: | :---: | :---: | :---: |
| 0.70239 | 0.064001 | 10.975 | 5.0538e-28 |
| 0.60833 | 0.24932 | 2.44 | 0.014687 |
| 1.377 | 0.65272 | 2.1097 | 0.034888 |
| 0.88565 | 0.293 | 3.0227 | 0.0025055 |
| 0.70164 | 0.21844 | 3.2121 | 0.0013179 |
| 1.1074 | 0.23271 | 4.7589 | $1.9464 \mathrm{e}-06$ |
| 1.0883 | 0.52912 | 2.0569 | 0.039696 |
| 1.045 | 0.32214 | 3.2439 | 0.0011792 |

1200 observations, 1192 error degrees of freedom
Dispersion: 1
Chi^2-statistic vs. constant model: 89.7, p-value $=1.4 \mathrm{e}-16$
Display unscaled points for predictors retained in the fitting model and display the minimum and maximum possible unscaled scores.

```
[PointsInfo,MinScore,MaxScore] = displaypoints(sc)
```

| PointsInfo=30×3 Predictors | Bin | Points |
| :---: | :---: | :---: |
| 'CustAge ' | '[-Inf,33)' | -0.15894 |
| 'CustAge' | '[33,37)' | -0.14036 |
| 'CustAge' | '[37,40) ' | -0.060323 |
| 'CustAge' | '[40, 46) ' | 0.046408 |
| 'CustAge' | ' $[46,48$ ) ' | 0.21445 |
| 'CustAge' | ' $[48,58)$ ' | 0.23039 |
| 'CustAge' | '[58,Inf]' | 0.479 |
| 'ResStatus' | 'Tenant' | -0.031252 |
| 'ResStatus' | 'Home Owner' | 0.12696 |
| 'ResStatus' | '0ther ${ }^{\text {' }}$ | 0.37641 |
| 'EmpStatus' | 'Unknown' | -0.076317 |
| 'EmpStatus' | 'Employed' | 0.31449 |
| 'CustIncome' | '[-Inf, 29000)' | -0.45716 |

```
'CustIncome' '[29000,33000)' -0.10466
'CustIncome' '[33000,35000)' 0.052329
'CustIncome' '[35000,40000)' 0.081611
```

MinScore = - 1.3100
MaxScore = 3.0726

Scale by providing the 'Shift' and 'Slope' values. In this example, there is an arbitrary choice of shift and slope. Display the points information again to verify that they are now scaled and also display the scaled minimum and maximum scores.

```
sc = formatpoints(sc,'ShiftAndSlope',[300 6]);
[PointsInfo,MinScore,MaxScore] = displaypoints(sc)
PointsInfo=30\times3 table
    Predictors Bin Points
    'CustAge'
    '[-Inf,33)'
    41.904
    'CustAge'
    '[33,37)'
    4 2 . 0 1 5
    'CustAge'
    'CustAge'
    'CustAge'
    'CustAge'
    'CustAge'
    'ResStatus'
    'ResStatus'
    'ResStatus'
    '[37,40)
    '[ [40,46)'
    42.495
    43.136
    '[46,48)' 44.144
    '[48,58)' 44.239
    '[58,Inf]
    4 5 . 7 3 1
        4 3 . 6 1 9
    'Unknown'
        45.116
    'EmpStatus'
    'Tenant' 42.67
        'Home Owner'
    42.399
    'EmpStatus' 'Employed' 44.744
    'CustIncome' '[-Inf,29000)' 40.114
    'CustIncome' '[29000,33000)' 42.229
    'CustIncome' '[33000,35000)' 43.171
    'CustIncome' '[35000,40000)' 43.347
MinScore = 292.1401
MaxScore = 318.4355
```


## Scale Points Using Points, Odds Levels, and PDO

This example shows how to use formatpoints to scale by providing the points, odds levels, and PDO (points to double the odds). By using formatpoints to scale, you can put points and scores in a desired range that is more meaningful for practical purposes. Technically, this involves a linear transformation from the unscaled to the scaled points by the formatpoints function.

Create a creditscorecard object using the CreditCardData.mat file to load the data (using a dataset from Refaat 2011). Use the 'IDVar' argument in creditscorecard to indicate that 'CustID' contains ID information and should not be included as a predictor variable.

```
load CreditCardData
sc = creditscorecard(data,'IDVar','CustID');
```

Perform automatic binning to bin for all predictors.

```
sc = autobinning(sc);
```

Fit a linear regression model using default parameters.

```
sc = fitmodel(sc);
```

1. Adding CustIncome, Deviance = 1490.8527, Chi2Stat $=32.588614$, PValue $=1.1387992 \mathrm{e}-\mathrm{C}$
2. Adding TmWBank, Deviance $=1467.1415$, Chi2Stat $=23.711203$, PValue $=1.1192909 \mathrm{e}-06$
3. Adding AMBalance, Deviance $=1455.5715$, Chi2Stat $=11.569967$, PValue $=0.0006702560$
4. Adding EmpStatus, Deviance $=1447.3451$, Chi2Stat $=8.2264038$, PValue $=0.0041285257$
5. Adding CustAge, Deviance $=1441.994$, Chi2Stat $=5.3511754$, PValue $=0.020708306$
6. Adding ResStatus, Deviance $=1437.8756$, Chi2Stat $=4.118404$, PValue $=0.042419078$
7. Adding OtherCC, Deviance $=1433.707$, Chi2Stat $=4.1686018$, PValue $=0.041179769$
Generalized linear regression model:
status ~ [Linear formula with 8 terms in 7 predictors]
Distribution = Binomial
Estimated Coefficients:

| Estimate | SE | tStat |  |  |
| ---: | ---: | ---: | ---: | ---: | pValue


| CustIncome | 0.70164 | 0.21844 | 3.2121 | 0.0013179 |
| :--- | ---: | ---: | ---: | ---: |
| TmWBank | 1.1074 | 0.23271 | 4.7589 | $1.9464 \mathrm{e}-06$ |
| OtherCC | 1.0883 | 0.52912 | 2.0569 | 0.039696 |
| AMBalance | 1.045 | 0.32214 | 3.2439 | 0.0011792 |

```
1200 observations, }1192\mathrm{ error degrees of freedom
Dispersion: 1
Chi^2-statistic vs. constant model: 89.7, p-value = 1.4e-16
```

Display unscaled points for predictors retained in the fitting model and display the minimum and maximum possible unscaled scores.

```
[PointsInfo,MinScore,MaxScore] = displaypoints(sc)
```

PointsInfo=30×3 table
Predictors Bin
'CustAge'
'[-Inf,33)'
-0. 15894
'CustAge'
' $[33,37$ )
' $[37,40$ )
-0.14036
$\begin{array}{lr}\text { '[37,40)' } & -0.060323\end{array}$
'CustAge'
'CustAge'
'[40,46)'
0.046408
'[46,48)' 0.21445
'CustAge'
$\begin{array}{ll} \\ \\ {[48,58)} & 0.23039\end{array}$
'CustAge'
'ResStatus'
'[58,Inf] '
0.479
'Tenant'
$-0.031252$
'ResStatus'
'Home Owner'
0.12696
'ResStatus' 'Other' 0.37641
'EmpStatus'
'Unknown'
-0.076317
'EmpStatus'
'Employed '
0.31449
'CustIncome
'[-Inf, 29000)'
-0.45716
'CustIncome' '[29000,33000)' -0.10466
'CustIncome' '[33000,35000)' 0.052329
'CustIncome' '[35000,40000)' 0.081611
MinScore = - 1.3100
MaxScore $=3.0726$

Scale by providing the points, odds levels, and PDO (points to double the odds). Suppose that you want a score of 500 points to have odds of 2 (twice as likely to be good than to be bad) and that the odds double every 50 points (so that 550 points would have odds of 4).

```
sc = formatpoints(sc,'PointsOddsAndPDO',[500 2 50]);
[PointsInfo,MinScore,MaxScore] = displaypoints(sc)
PointsInfo=30\times3 table
    Predictors
    'CustAge'
    'CustAge'
    'CustAge'
    'CustAge'
    'CustAge'
    'CustAge'
    'CustAge'
    'ResStatus'
    'ResStatus'
    'ResStatus'
    'EmpStatus'
    'EmpStatus
    'CustIncome' '[-Inf,29000)' 31.309
    'CustIncome' '[29000,33000)'
    'CustIncome' '[33000,35000)'
    'CustIncome'
    '[-Inf,33)
    '[33,37)
    '[37,40)'
    '[40,46)'
    '[46,48)'
    '[48,58)
    '[58,Inf]
    'Tenant'
    'Home Owner'
    'Other'
        Bin
Predictors
    'Unknown'
    'Employed'
    'CustIncome' '[-Inf,29000)' 31.309
        56.736
    6 8 . 0 6
    '[33000, 35000)
        :
    '[35000,40000)'
    70.173
MinScore = 355.5051
MaxScore = 671.6403
```


## Report Base Points Separately

This example shows how to use formatpoints to separate the base points from the rest of the points assigned to each predictor variable. The formatpoints name-value pair argument 'BasePoints' serves this purpose.

Create a creditscorecard object using the CreditCardData.mat file to load the data (using a dataset from Refaat 2011). Use the 'IDVar' argument in creditscorecard to indicate that 'CustID' contains ID information and should not be included as a predictor variable.

```
load CreditCardData
sc = creditscorecard(data,'IDVar','CustID');
```

Perform automatic binning to bin for all predictors.
sc = autobinning(sc);
Fit a linear regression model using default parameters.

```
sc = fitmodel(sc);
```

1. Adding CustIncome, Deviance $=1490.8527$, Chi2Stat $=32.588614$, PValue $=1.1387992 \mathrm{e}-1$
2. Adding TmWBank, Deviance $=1467.1415$, Chi2Stat $=23.711203$, PValue $=1.1192909 \mathrm{e}-06$
3. Adding AMBalance, Deviance $=1455.5715$, Chi2Stat $=11.569967$, PValue $=0.0006702560$
4. Adding EmpStatus, Deviance $=1447.3451$, Chi2Stat $=8.2264038$, PValue $=0.0041285257$
5. Adding CustAge, Deviance $=1441.994$, Chi2Stat $=5.3511754$, $\mathrm{PValue}=0.020708306$
6. Adding ResStatus, Deviance $=1437.8756$, Chi2Stat $=4.118404$, PValue $=0.042419078$
7. Adding OtherCC, Deviance $=1433.707$, Chi2Stat $=4.1686018$, PValue $=0.041179769$

Generalized linear regression model:
status ~ [Linear formula with 8 terms in 7 predictors] Distribution = Binomial

Estimated Coefficients:

| Estimate | SE | tStat | pValue |
| :---: | :---: | :---: | :---: |
| 0.70239 | 0.064001 | 10.975 | 5.0538e-28 |
| 0.60833 | 0.24932 | 2.44 | 0.014687 |
| 1.377 | 0.65272 | 2.1097 | 0.034888 |
| 0.88565 | 0.293 | 3.0227 | 0.0025055 |
| 0.70164 | 0.21844 | 3.2121 | 0.0013179 |
| 1.1074 | 0.23271 | 4.7589 | $1.9464 \mathrm{e}-06$ |
| 1.0883 | 0.52912 | 2.0569 | 0.039696 |
| 1.045 | 0.32214 | 3.2439 | 0.0011792 |

1200 observations, 1192 error degrees of freedom
Dispersion: 1
Chi^2-statistic vs. constant model: 89.7, p -value $=1.4 \mathrm{e}-16$
Display unscaled points for predictors retained in the fitting model and display the minimum and maximum possible unscaled scores.

```
[PointsInfo,MinScore,MaxScore] = displaypoints(sc)
```

```
PointsInfo=30\times3 table
    Predictors Bin
    'CustAge'
    '[-Inf,33)'
    '[33,37)'
        -0.14036
    'CustAge'
    'CustAge'
    'CustAge'
    'CustAge'
    'CustAge'
    'CustAge'
    'ResStatus'
    'ResStatus'
    'ResStatus'
    'EmpStatus'
    'EmpStatus'
    'CustIncome'
    'CustIncome'
    'CustIncome'
    'CustIncome'
        :
    -0.15894
    '[37,40)'
        -0.060323
    '[40,46)'
    0.046408
    '[46,48)'
        0.21445
    '[48,58)' 0.23039
    '[58,Inf]'
        0.479
    'Tenant'
        Points
        Pred
    'Home Owner'
        -0.031252
    'Other' 0.37641
        0.12696
    'Unknown'
    -0.076317
    ' Employed'
        0.31449
    '[-Inf,29000)'
    ' [29000, 33000)
        -0.45716
    -0.10466
    ' [33000, 35000)'
        0.052329
    ' [35000,40000)'
    0.081611
MinScore = -1.3100
MaxScore = 3.0726
```

By setting the name-value pair argument BasePoints to true, the points information table reports the base points separately in the first row. The minimum and maximum possible scores are not affected by this option.

```
sc = formatpoints(sc,'BasePoints',true);
[PointsInfo,MinScore,MaxScore] = displaypoints(sc)
```

PointsInfo=31×3 table
Predictors Bin Points
'BasePoints'
'BasePoints'

Points
'CustAge'
'[-Inf,33)'
0.70239
'CustAge'
' $[33,37)$
-0. 25928
'CustAge'

|  |  |
| :--- | :--- |
| $[37,40)$ | -0.16066 |

'CustAge'
'[40,46)' -0.053933
'CustAge' '[46,48)' 0.11411
$\begin{array}{lll}\text { 'CustAge } & \text { ' }[48,58)^{\prime} & 0.13005\end{array}$

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| 'CustAge' | '[58,Inf]' | 0.37866 |
| :--- | :--- | ---: |
| 'ResStatus' | 'Tenant' | -0.13159 |
| 'ResStatus' | 'Home Owner' | 0.026616 |
| 'ResStatus' | 'Other' | 0.27607 |
| 'EmpStatus' | 'Unknown' | -0.17666 |
| 'EmpStatus' | 'Employed' | 0.21415 |
| 'CustIncome' | '[-Inf,29000)' | -0.5575 |
| 'CustIncome' | '[29000,33000)' | -0.205 |
| 'CustIncome' | '[33000,35000)' | -0.048013 |

```
MinScore = -1.3100
MaxScore = 3.0726
```


## Round Points

This example shows how to use formatpoints to round points. Rounding is usually applied after scaling, otherwise, if the points for a particular predictor are all in a small range, rounding could cause the rounded points for different bins to be the same. Also, rounding all the points may slightly change the minimum and maximum total points.

Create a creditscorecard object using the CreditCardData.mat file to load the data (using a dataset from Refaat 2011). Use the 'IDVar' argument in creditscorecard to indicate that 'CustID' contains ID information and should not be included as a predictor variable.
load CreditCardData
sc = creditscorecard(data,'IDVar','CustID');
Perform automatic binning to bin for all predictors.
sc = autobinning(sc);
Fit a linear regression model using default parameters.

```
sc = fitmodel(sc);
```

1. Adding CustIncome, Deviance $=1490.8527$, Chi2Stat $=32.588614$, PValue $=1.1387992 \mathrm{e}-\mathrm{C}$
2. Adding TmWBank, Deviance $=1467.1415$, Chi2Stat $=23.711203$, PValue $=1.1192909 \mathrm{e}-06$
3. Adding AMBalance, Deviance $=1455.5715$, Chi2Stat $=11.569967$, PValue $=0.0006702560$
4. Adding EmpStatus, Deviance $=1447.3451$, Chi2Stat $=8.2264038$, $\mathrm{PValue}=0.0041285257$
5. Adding CustAge, Deviance $=1441.994$, Chi2Stat $=5.3511754$, PValue $=0.020708306$
6. Adding ResStatus, Deviance $=1437.8756$, Chi2Stat $=4.118404$, PValue $=0.042419078$
7. Adding OtherCC, Deviance $=1433.707$, Chi2Stat $=4.1686018$, PValue $=0.041179769$

Generalized linear regression model:
status ~ [Linear formula with 8 terms in 7 predictors]
Distribution = Binomial

Estimated Coefficients:

| Estimate | SE | tStat | pValue |
| :---: | :---: | :---: | :---: |
| 0.70239 | 0.064001 | 10.975 | 5.0538e-28 |
| 0.60833 | 0.24932 | 2.44 | 0.014687 |
| 1.377 | 0.65272 | 2.1097 | 0.034888 |
| 0.88565 | 0.293 | 3.0227 | 0.0025055 |
| 0.70164 | 0.21844 | 3.2121 | 0.0013179 |
| 1.1074 | 0.23271 | 4.7589 | $1.9464 \mathrm{e}-06$ |
| 1.0883 | 0.52912 | 2.0569 | 0.039696 |
| 1.045 | 0.32214 | 3.2439 | 0.001179 |

1200 observations, 1192 error degrees of freedom Dispersion: 1
Chi^2-statistic vs. constant model: 89.7, p-value = 1.4e-16
Display unscaled points for predictors retained in the fitting model and display the minimum and maximum possible unscaled scores.

```
[PointsInfo,MinScore,MaxScore] = displaypoints(sc)
```

| PointsInfo=30×3 Predictors | Bin | Points |
| :---: | :---: | :---: |
| 'CustAge ' | '[-Inf,33)' | -0.15894 |
| 'CustAge' | '[33,37)' | -0.14036 |
| 'CustAge' | '[37,40)' | -0.060323 |
| 'CustAge' | '[40, 46) ' | 0.046408 |
| 'CustAge' | ' $[46,48$ )' | 0.21445 |
| 'CustAge' | ' $[48,58$ )' | 0.23039 |
| 'CustAge' | '[58,Inf]' | 0.479 |
| 'ResStatus' | 'Tenant' | -0.031252 |
| 'ResStatus' | 'Home Owner' | 0.12696 |

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| 'ResStatus' | 'Other' | 0.37641 |
| :--- | :--- | ---: |
| 'EmpStatus' | 'Unknown' | -0.076317 |
| 'EmpStatus' | 'Employed' | 0.31449 |
| 'CustIncome' | '[-Inf,29000)' | -0.45716 |
| 'CustIncome' | '[29000,33000)' | -0.10466 |
| 'CustIncome' | '[33000,35000)' | 0.052329 |
| 'CustIncome' | '[35000,40000)' | 0.081611 |

MinScore $=-1.3100$
MaxScore $=3.0726$
Scale points, and display the points information. By default, no rounding is applied.

```
sc = formatpoints(sc,'WorstAndBestScores',[300 850]);
PointsInfo = displaypoints(sc)
PointsInfo=30\times3 table
Predictors Bin Points
    'CustAge' '[-Inf,33)' 46.396
    'CustAge' '[33,37)' 48.727
    'CustAge' '[37,40)' 58.772
    'CustAge' '[40,46)' 72.167
    'CustAge' '[46,48)' 93.256
    'CustAge' '[48,58)' 95.256
    'CustAge' '[58,Inf]' 126.46
    'ResStatus' 'Tenant' 62.421
    'ResStatus' 'Home Owner' 82.276
    'ResStatus' 'Other' 113.58
    'EmpStatus' 'Unknown' 56.765
    'EmpStatus' 'Employed' 105.81
    'CustIncome' '[-Inf,29000)' 8.9706
    'CustIncome' '[29000,33000)' 53.208
    'CustIncome' '[33000,35000)' 72.91
    'CustIncome' '[35000,40000)' 76.585
```

Use the name-value pair argument Round to apply rounding for all points and then display the points information again.

```
sc = formatpoints(sc,'Round','AllPoints');
PointsInfo = displaypoints(sc)
PointsInfo=30\times3 table
    Predictors Bin Points
    'CustAge'
    'CustAge' '[33,37)' 49
    [ -Inf,33)46
    'CustAge' '[37,40)' 59
    'CustAge' '[40,46)' }7
    'CustAge' '[46,48)' 93
    'CustAge' '[48,58)' 95
    'CustAge'
    'ResStatus'
    'ResStatus
    'EmpStatus' 'Unknown' 57
    'EmpStatus' 'Employed' 106
    'CustIncome' '[-Inf,29000)' }
    'CustIncome' '[29000,33000)' 53
    'CustIncome' '[33000,35000)' 73
    'CustIncome' '[35000,40000)' 77
```


## Scores for Missing or Out-of-Range Data

This example shows how to use formatpoints to score missing or out-of-range data. When data is scored, some observations can be either missing ( NaN , or undefined) or out of range. You will need to decide whether or not points are assigned to these cases. Use the name-value pair argument Missing to do so.

Create a creditscorecard object using the CreditCardData.mat file to load the data (using a dataset from Refaat 2011). Use the 'IDVar' argument in creditscorecard to indicate that 'CustID ' contains ID information and should not be included as a predictor variable.
load CreditCardData
sc = creditscorecard(data,'IDVar','CustID');
Perform automatic binning to bin for all predictors.

```
sc = autobinning(sc);
```

Indicate that the minimum allowed value for 'CustAge' is zero. This makes any negative values for age invalid or out-of-range.
sc = modifybins(sc,'CustAge','MinValue',0);
Fit a linear regression model using default parameters.

```
sc = fitmodel(sc);
```

1. Adding CustIncome, Deviance $=1490.8527$, Chi2Stat $=32.588614$, PValue $=1.1387992 \mathrm{e}-0$
2. Adding TmWBank, Deviance $=1467.1415$, Chi2Stat $=23.711203$, PValue $=1.1192909 \mathrm{e}-06$
3. Adding AMBalance, Deviance $=1455.5715$, Chi2Stat $=11.569967$, PValue $=0.0006702560$
4. Adding EmpStatus, Deviance $=1447.3451$, Chi2Stat $=8.2264038$, PValue $=0.0041285257$
5. Adding CustAge, Deviance $=1441.994$, Chi2Stat $=5.3511754$, PValue $=0.020708306$
6. Adding ResStatus, Deviance $=1437.8756$, Chi2Stat $=4.118404$, PValue $=0.042419078$
7. Adding OtherCC, Deviance = 1433.707, Chi2Stat = 4.1686018, PValue $=0.041179769$

Generalized linear regression model:
status ~ [Linear formula with 8 terms in 7 predictors]
Distribution = Binomial
Estimated Coefficients:

| Estimate | SE | tStat |  |  |
| ---: | ---: | ---: | ---: | ---: | pValue

1200 observations, 1192 error degrees of freedom
Dispersion: 1
Chi^2-statistic vs. constant model: 89.7, p-value $=1.4 \mathrm{e}-16$
Suppose there are missing or out of range observations in the data that you want to score. Notice that by default, the points and score assigned to the missing value is NaN .
\% Set up a data set with missing and out of range data for illustration purposes newdata $=$ data(1:5,:);

```
newdata.CustAge(1) = NaN; % missing
newdata.CustAge(2) = -100; % invalid
newdata.ResStatus(3) = '<undefined>'; % missing
newdata.ResStatus(4) = 'House'; % invalid
disp(newdata)
\begin{tabular}{crclllll} 
CustID & CustAge & \multicolumn{2}{c}{ TmAtAddress } & & ResStatus & & EmpStatus
\end{tabular}
```

```
[Scores,Points] = score(sc,newdata);
```

[Scores,Points] = score(sc,newdata);
disp(Scores)
disp(Scores)
NaN
NaN
NaN
NaN
1.4535
disp(Points)

| CustAge | ResStatus | EmpStatus | CustIncome | TmWBank | OtherCC | AMBalar |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| NaN | -0.031252 | -0.076317 | 0.43693 | 0.39607 | 0.15842 | -0.017 |
| NaN | 0.12696 | 0.31449 | 0.43693 | -0.033752 | 0.15842 | -0.017 |
| 0.21445 | NaN | 0.31449 | 0.081611 | 0.39607 | -0.19168 | -0.017 |
| 0.23039 | NaN | 0.31449 | 0.43693 | -0.044811 | 0.15842 | 0.35 |
| 0.479 | 0.12696 | 0.31449 | 0.43693 | -0.044811 | 0.15842 | -0.017 |

Use the name-value pair argument Missing to replace NaN with points corresponding to a zero Weight-of-Evidence (WOE).

```
```

sc = formatpoints(sc,'Missing','ZeroWOE');

```
sc = formatpoints(sc,'Missing','ZeroWOE');
[Scores,Points] = score(sc,newdata);
[Scores,Points] = score(sc,newdata);
disp(Scores)
disp(Scores)
0.9667
1.0859
0.8978
```

1.5513
1.4535
disp(Points)

| CustAge | ResStatus | EmpStatus | CustIncome | TmWBank | OtherCC | AMBala |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.10034 | -0.031252 | -0.076317 | 0.43693 | 0.39607 | 0.15842 | -0.017 |
| 0.10034 | 0.12696 | 0.31449 | 0.43693 | -0.033752 | 0.15842 | -0.017 |
| 0.21445 | 0.10034 | 0.31449 | 0.081611 | 0.39607 | -0.19168 | -0.017 |
| 0.23039 | 0.10034 | 0.31449 | 0.43693 | -0.044811 | 0.15842 | 0.35 |
| 0.479 | 0.12696 | 0.31449 | 0.43693 | -0.044811 | 0.15842 | -0.017 |

Alternatively, use the name-value pair argument Missing to replace the missing value with the minimum points for the predictors that have the missing values.

```
sc = formatpoints(sc,'Missing','MinPoints');
[Scores,Points] = score(sc,newdata);
disp(Scores)
```

    0.7074
    0.8266
    0.7662
    1.4197
    1.4535
    disp(Points)

| CustAge | ResStatus | EmpStatus | CustIncome | TmWBank | OtherCC | AMBala |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -0.15894 | -0.031252 | -0.076317 | 0.43693 | 0.39607 | 0.15842 | -0.01 |
| -0.15894 | 0.12696 | 0.31449 | 0.43693 | -0.033752 | 0.15842 | -0.01 |
| 0.21445 | -0.031252 | 0.31449 | 0.081611 | 0.39607 | -0.19168 | -0.01 |
| 0.23039 | -0.031252 | 0.31449 | 0.43693 | -0.044811 | 0.15842 | 0.3 |
| 0.479 | 0.12696 | 0.31449 | 0.43693 | -0.044811 | 0.15842 | -0.01 |

As a third alternative, use the name-value pair argument Missing to replace the missing value with the maximum points for the predictors that have the missing values.

```
sc = formatpoints(sc,'Missing','MaxPoints');
[Scores,Points] = score(sc,newdata);
disp(Scores)
```

1.3454
1.4646
1.1739
1.8273
1.4535
disp(Points)

| CustAge | ResStatus | EmpStatus | CustIncome | TmWBank | OtherCC | AMBala |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.479 | -0.031252 | -0.076317 | 0.43693 | 0.39607 | 0.15842 | -0.017 |
| 0.479 | 0.12696 | 0.31449 | 0.43693 | -0.033752 | 0.15842 | -0.017 |
| 0.21445 | 0.37641 | 0.31449 | 0.081611 | 0.39607 | -0.19168 | -0.017 |
| 0.23039 | 0.37641 | 0.31449 | 0.43693 | -0.044811 | 0.15842 | 0.35 |
| 0.479 | 0.12696 | 0.31449 | 0.43693 | -0.044811 | 0.15842 | -0.017 |

Verify that the minimum and maximum points assigned to the missing data correspond to the minimum and maximum points for the corresponding predictors. The points for 'CustAge ' are reported in the first seven rows of the points information table. For 'ResStatus' the points are in rows 8 through 10.

```
PointsInfo = displaypoints(sc);
```

PointsInfo(1:7,:)

```
ans=7\times3 table
    Predictors Bin Points
    'CustAge' '[0,33)' -0.15894
    'CustAge' '[33,37)' -0.14036
    'CustAge' '[37,40)' -0.060323
    'CustAge' '[40,46)' 0.046408
    'CustAge' '[46,48)' 0.21445
    'CustAge' '[48,58)' 0.23039
    'CustAge' '[58,Inf]' 0.479
min(PointsInfo.Points(1:7))
ans = -0.1589
max(PointsInfo.Points(1:7))
ans = 0.4790
PointsInfo(8:10,:)
```

```
ans=3\times3 table
    Predictors
    Bin
    'ResStatus'
    'Tenant'
    'Home Owner'
        'Other'
    'ResStatus'
    'ResStatus'
    0.12696
    0.37641
min(PointsInfo.Points(8:10))
ans = -0.0313
max(PointsInfo.Points(8:10))
ans = 0.3764
```


## Scores for Missing or Out-of-Range Data When Using the 'BinMissingData' Option

This example describes the assignment of points for missing data when the 'BinMissingData' option is set to true.

- Predictors that have missing data in the training set have an explicit bin for <missing> with corresponding points in the final scorecard. These points are computed from the Weight-of-Evidence (WOE) value for the <missing> bin and the logistic model coefficients. For scoring purposes, these points are assigned to missing values and to out-of-range values.
- Predictors with no missing data in the training set have no <missing> bin, therefore no WOE can be estimated from the training data. By default, the points for missing and out-of-range values are set to NaN , and this leads to a score of NaN when running score. For predictors that have no explicit <missing> bin, use the name-value argument 'Missing' in formatpoints to indicate how missing data should be treated for scoring purposes.

Create a creditscorecard object using the CreditCardData.mat file to load the dataMissing with missing values.
load CreditCardData.mat
head(dataMissing,5)

```
ans=5\times11 table
    CustID CustAge TmAtAddress ResStatus EmpStatus CustIncome TmWBa
    <
\begin{tabular}{lrr}
1 & 53 & 62 \\
2 & 61 & 22 \\
3 & 47 & 30 \\
4 & NaN & 75 \\
5 & 68 & 56
\end{tabular}
\begin{tabular}{lllc} 
ResStatus & & EmpStatus & \\
& & & CustIncome \\
<undefined> & & Unknown & \\
Home Owner & & Employed & \\
Tenant & & 520000 \\
Homployed Owner & & & 37000 \\
Home Owner & Employed & & 53000 \\
Homed & & 53000
\end{tabular}
```

fprintf('Number of rows: %d\n',height(dataMissing))

```
fprintf('Number of rows: %d\n',height(dataMissing))
Number of rows: 1200
fprintf('Number of missing values CustAge: %d\n',sum(ismissing(dataMissing.CustAge)))
Number of missing values CustAge: 30
fprintf('Number of missing values ResStatus: %d\n',sum(ismissing(dataMissing.ResStatus
Number of missing values ResStatus: 40
```

Use creditscorecard with the name-value argument 'BinMissingData' set to true to bin the missing numeric or categorical data in a separate bin. Apply automatic binning.

```
sc = creditscorecard(dataMissing,'IDVar','CustID','BinMissingData',true);
sc = autobinning(sc);
disp(sc)
```

    creditscorecard with properties:
                        GoodLabel: 0
            ResponseVar: 'status'
                    WeightsVar: ''
                            VarNames: \{1x11 cell\}
        NumericPredictors: \(\{1 \times 6\) cell\}
        CategoricalPredictors: \{'ResStatus' 'EmpStatus' 'OtherCC'\}
                        BinMissingData: 1
                            IDVar: 'CustID'
            PredictorVars: \{1x9 cell\}
                            Data: [1200x11 table]
    Set a minimum value of zero for CustAge and CustIncome. With this, any negative age or income information becomes invalid or "out-of-range". For scoring purposes, out-ofrange values are given the same points as missing values.
sc = modifybins(sc,'CustAge','MinValue',0);
sc = modifybins(sc,'CustIncome','MinValue',0);
Display and plot bin information for numeric data for 'CustAge' that includes missing data in a separate bin labelled <missing>.

```
[bi,cp] = bininfo(sc,'CustAge');
disp(bi)
```

| Bin | Good | Bad | Odds | WOE | InfoValue |
| :---: | :---: | :---: | :---: | :---: | :---: |
| ' $[0,33$ ) ' | 69 | 52 | 1.3269 | -0.42156 | 0.018993 |
| ' $[33,37)^{\prime}$ | 63 | 45 | 1.4 | -0.36795 | 0.012839 |
| ' $[37,40$ )' | 72 | 47 | 1.5319 | -0.2779 | 0.0079824 |
| '[40,46)' | 172 | 89 | 1.9326 | -0.04556 | 0.0004549 |
| ' $[46,48$ )' | 59 | 25 | 2.36 | 0.15424 | 0.0016199 |
| ' $[48,51$ )' | 99 | 41 | 2.4146 | 0.17713 | 0.0035449 |
| ' $[51,58$ ) ' | 157 | 62 | 2.5323 | 0.22469 | 0.0088407 |
| '[58,Inf]' | 93 | 25 | 3.72 | 0.60931 | 0.032198 |
| '<missing>' | 19 | 11 | 1.7273 | -0.15787 | 0.00063885 |
| 'Totals' | 803 | 397 | 2.0227 | NaN | 0.087112 |

```
plotbins(sc,'CustAge')
```



Display and plot bin information for categorical data for 'ResStatus ' that includes missing data in a separate bin labelled <missing>.
[bi,cg] = bininfo(sc,'ResStatus'); disp(bi)

| Bin | Good | Bad | Odds | WOE | InfoValue |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 'Tenant ' | 296 | 161 | 1.8385 | -0.095463 | 0.0035249 |
| 'Home Owner' | 352 | 171 | 2.0585 | 0.017549 | 0.00013382 |
| 'Other' | 128 | 52 | 2.4615 | 0.19637 | 0.0055808 |
| '<missing>' | 27 | 13 | 2.0769 | 0.026469 | 2.3248e-05 |
| 'Totals' | 803 | 397 | 2.0227 | NaN | 0.0092627 |

```
plotbins(sc,'ResStatus')
```

ResStatus


Good $\square$ Bad

For the 'CustAge' and 'ResStatus' predictors, there is missing data (NaNs and <undefined>) in the training data, and the binning process estimates a WOE value of - 0.15787 and 0.026469 respectively for missing data in these predictors, as shown above.

For EmpStatus and CustIncome there is no explicit bin for missing values because the training data has no missing values for these predictors.

```
bi = bininfo(sc,'EmpStatus');
disp(bi)
```

| Bin | Good | Bad | Odds | WOE |  | InfoValue |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 'Unknown ' | 396 | 239 |  |  | 0. 19947 | $7 \quad 0.0$ |  |
| 'Employed' | 407 | 158 |  |  | 0.2418 | 80.0 |  |
| 'Totals' | 803 | 397 |  |  | NaN | $N \quad 0.0$ |  |
| ```bi = bininfo(sc,'CustIncome'); disp(bi)``` |  |  |  |  |  |  |  |
| Bin | Good |  | Bad | Odds | WOE |  | InfoValue |
| ' $[0,29000)^{\prime}$ |  | 53 | 58 | 0.91379 |  | -0.79457 | 0.06364 |
| ' $[29000,33000)$ |  | 74 | 49 | 1.5102 |  | -0.29217 | 0.0091366 |
| ' [33000, 35000) |  | 68 | 36 | 1.8889 |  | -0.06843 | 0.00041042 |
| ' [35000, 40000) |  | 193 | 98 | 1.9694 |  | 0.026696 | 0.00017359 |
| ' [40000, 42000) |  | 68 | 34 |  | -0 | 0.011271 | $1.0819 \mathrm{e}-05$ |
| ' [42000, 47000) |  | 164 | 66 | 2.4848 |  | 0.20579 | 0.0078175 |
| '[47000, Inf]' |  | 183 | 56 | 3.2679 |  | 0.47972 | 0.041657 |
| 'Totals ' |  | 803 | 397 | 2.0227 |  | NaN | 0.12285 |

Use fitmodel to fit a logistic regression model using Weight of Evidence (WOE) data. fitmodel internally transforms all the predictor variables into WOE values, using the bins found with the automatic binning process. fitmodel then fits a logistic regression model using a stepwise method (by default). For predictors that have missing data, there is an explicit <missing> bin, with a corresponding WOE value computed from the data. When using fitmodel, the corresponding WOE value for the <missing> bin is applied when performing the WOE transformation.

```
[sc,mdl] = fitmodel(sc);
```

1. Adding CustIncome, Deviance $=1490.8527$, Chi2Stat $=32.588614$, PValue $=1.1387992 \mathrm{e}-\mathrm{C}$
2. Adding TmWBank, Deviance $=1467.1415$, Chi2Stat $=23.711203$, PValue $=1.1192909 \mathrm{e}-06$
3. Adding AMBalance, Deviance $=1455.5715$, Chi2Stat $=11.569967$, PValue $=0.0006702560$
4. Adding EmpStatus, Deviance $=1447.3451$, Chi2Stat $=8.2264038$, PValue $=0.0041285257$
5. Adding CustAge, Deviance $=1442.8477$, Chi2Stat $=4.4974731$, PValue $=0.033944979$
6. Adding ResStatus, Deviance $=1438.9783$, Chi2Stat $=3.86941$, PValue $=0.049173805$
7. Adding OtherCC, Deviance $=1434.9751$, Chi2Stat $=4.0031966$, PValue $=0.045414057$

Generalized linear regression model:
status ~ [Linear formula with 8 terms in 7 predictors]
Distribution = Binomial

Estimated Coefficients:

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|  | Estimate | SE | tStat | pValue |
| :---: | :---: | :---: | :---: | :---: |
| (Intercept) | 0.70229 | 0.063959 | 10.98 | 4.7498e-28 |
| CustAge | 0.57421 | 0.25708 | 2.2335 | 0.025513 |
| ResStatus | 1.3629 | 0.66952 | 2.0356 | 0.04179 |
| EmpStatus | 0.88373 | 0.2929 | 3.0172 | 0.002551 |
| CustIncome | 0.73535 | 0.2159 | 3.406 | 0.00065929 |
| TmWBank | 1.1065 | 0.23267 | 4.7556 | 1.9783e-06 |
| OtherCC | 1.0648 | 0.52826 | 2.0156 | 0.043841 |
| AMBalance | 1.0446 | 0.32197 | 3.2443 | 0.0011775 |

```
1200 observations, 1192 error degrees of freedom
Dispersion: 1
Chi^2-statistic vs. constant model: 88.5, p-value = 2.55e-16
```

Scale the scorecard points by the "points, odds, and points to double the odds (PDO)" method using the 'Points0ddsAndPDO' argument of formatpoints. Suppose that you want a score of 500 points to have odds of 2 (twice as likely to be good than to be bad) and that the odds double every 50 points (so that 550 points would have odds of 4).

Display the scorecard showing the scaled points for predictors retained in the fitting model.

```
sc = formatpoints(sc,'PointsOddsAndPDO',[500 2 50]);
PointsInfo = displaypoints(sc)
PointsInfo=33\times3 table
Predictors Bin Points
    ' CustAge'
'[0,33)
54.062
    'CustAge' '[33,37)' 56.282
    'CustAge' '[37,40)' 60.012
    'CustAge' '[40,46)' 69.636
    'CustAge' '[46,48)' }77.91
    'CustAge' '[48,51)' 78.86
    'CustAge' '[51,58)' 80.83
    'CustAge' '[58,Inf]' 96.76
    'CustAge' '<missing>' 64.984
    'ResStatus' 'Tenant' 62.138
    'ResStatus' 'Home Owner' 73.248
    'ResStatus' 'Other' 90.828
```

| 'ResStatus' | '<missing>' | 74.125 |
| :--- | :--- | :--- |
| 'EmpStatus' | 'Unknown' | 58.807 |
| 'EmpStatus' | 'Employed' | 86.937 |
| 'CustIncome' | '[0,29000)' | 29.375 |

Notice that points for the <missing> bin for CustAge and ResStatus are explicitly shown (as 64.9836 and 74.1250, respectively). These points are computed from the WOE value for the <missing> bin, and the logistic model coefficients.

For predictors that have no missing data in the training set, there is no explicit <missing> bin. By default the points are set to NaN for missing data and they lead to a score of NaN when running score. For predictors that have no explicit <missing> bin, use the name-value argument 'Missing' in formatpoints to indicate how missing data should be treated for scoring purposes.

For the purpose of illustration, take a few rows from the original data as test data and introduce some missing data. Also introduce some invalid, or out-of-range values. For numeric data, values below the minimum (or above the maximum) allowed are considered invalid, such as a negative value for age (recall 'MinValue' was earlier set to 0 for CustAge and CustIncome). For categorical data, invalid values are categories not explicitly included in the scorecard, for example, a residential status not previously mapped to scorecard categories, such as "House", or a meaningless string such as "abc123".

```
tdata = dataMissing(11:18,mdl.PredictorNames); % Keep only the predictors retained in
% Set some missing values
tdata.CustAge(1) = NaN;
tdata.ResStatus(2) = '<undefined>';
tdata.EmpStatus(3) = '<undefined>';
tdata.CustIncome(4) = NaN;
% Set some invalid values
tdata.CustAge(5) = -100;
tdata.ResStatus(6) = 'House';
tdata.EmpStatus(7) = 'Freelancer';
tdata.CustIncome(8) = -1;
disp(tdata)
\begin{tabular}{|c|c|c|c|c|c|}
\hline CustAge & ResStatus & EmpStatus & CustIncome & TmWBank & OtherCC \\
\hline NaN & Tenant & Unk & 34000 & 44 & Yes \\
\hline
\end{tabular}
```

| 48 | <undefined> | Unknown | 44000 | 14 | Yes | 403.6 |
| ---: | :--- | :--- | ---: | ---: | ---: | ---: |
| 65 | Home Owner | <undefined> | 48000 | 6 | No | 111.8 |
| 44 | Other | Unknown | NaN | 35 | No | 436.6 |
| -100 | Other | Employed | 46000 | 16 | Yes | 162.2 |
| 33 | House | Employed | 36000 | 36 | Yes | 845.6 |
| 39 | Tenant | Freelancer | 34000 | 40 | Yes | 756.2 |
| 24 | Home Owner | Employed | -1 | 19 | Yes | 449.6 |

Score the new data and see how points are assigned for missing CustAge and ResStatus, because we have an explicit bin with points for <missing>. However, for EmpStatus and CustIncome the score function sets the points to NaN .

```
[Scores,Points] = score(sc,tdata);
disp(Scores)
```

    481.2231
    520.8353
        NaN
        NaN
    551.7922
    487.9588
        NaN
        NaN
    disp(Points)

| CustAge | ResStatus | EmpStatus | CustIncome | TmWBank | Othercc | AMBalance |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 64.984 | 62.138 | 58.807 | 67.893 | 61.858 | 75.622 | 89.922 |
| 78.86 | 74.125 | 58.807 | 82.439 | 61.061 | 75.622 | 89.922 |
| 96.76 | 73.248 | NaN | 96.969 | 51.132 | 50.914 | 89.922 |
| 69.636 | 90.828 | 58.807 | NaN | 61.858 | 50.914 | 89.922 |
| 64.984 | 90.828 | 86.937 | 82.439 | 61.061 | 75.622 | 89.922 |
| 56.282 | 74.125 | 86.937 | 70.107 | 61.858 | 75.622 | 63.028 |
| 60.012 | 62.138 | NaN | 67.893 | 61.858 | 75.622 | 63.028 |
| 54.062 | 73.248 | 86.937 | NaN | 61.061 | 75.622 | 89.922 |

Use the name-value argument 'Missing' in formatpoints to choose how to assign points to missing values for predictors that do not have an explicit <missing> bin. In this example, use the 'MinPoints ' option for the 'Missing' argument. The minimum points for EmpStatus in the scorecard displayed above are 58.8072, and for CustIncome the minimum points are 29.3753.

```
sc = formatpoints(sc,'Missing','MinPoints');
[Scores,Points] = score(sc,tdata);
disp(Scores)
    481.2231
    520.8353
    517.7532
    451.3405
    551.7922
    487.9588
    449.3577
    470.2267
```

disp(Points)

| CustAge | ResStatus | EmpStatus | CustIncome | TmWBank | OtherCC | AMBalance |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 64.984 | 62.138 | 58.807 | 67.893 | 61.858 | 75.622 | 89.922 |
| 78.86 | 74.125 | 58.807 | 82.439 | 61.061 | 75.622 | 89.922 |
| 96.76 | 73.248 | 58.807 | 96.969 | 51.132 | 50.914 | 89.922 |
| 69.636 | 90.828 | 58.807 | 29.375 | 61.858 | 50.914 | 89.922 |
| 64.984 | 90.828 | 86.937 | 82.439 | 61.061 | 75.622 | 89.922 |
| 56.282 | 74.125 | 86.937 | 70.107 | 61.858 | 75.622 | 63.028 |
| 60.012 | 62.138 | 58.807 | 67.893 | 61.858 | 75.622 | 63.028 |
| 54.062 | 73.248 | 86.937 | 29.375 | 61.061 | 75.622 | 89.922 |

## Input Arguments

## sc - Credit scorecard model

creditscorecard object
Credit scorecard model, specified as a creditscorecard object. Use creditscorecard to create a creditscorecard object.

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1,Value1, ... , NameN, ValueN.

```
Example: sc =
formatpoints(sc,'BasePoints',true,'Round','AllPoints','WorstAndBestS
cores',[100, 700])
```

Note ShiftAndSlope, Points0ddsAndPDO, and WorstAndBestScores are scaling methods and you can use only one of these name-value pair arguments at one time. The other three name-value pair arguments (BasePoints, Missing, and Round) are not scaling methods and can be used together or with any one of the three scaling methods.

## BasePoints - Indicator for separating base points

false (default) | logical scalar
Indicator for separating base points, specified as the comma-separated pair consisting of 'BasePoints ' and a logical scalar. If true, the scorecard explicitly separates base points. If false, the base points are spread across all variables in the creditscorecard object.

## Data Types: char

## Missing - Indicator for points assigned to missing or out-of-range information when scoring

NoScore (default) | character vector with values NoScore, ZeroW0E, MinPoints, and MaxPoints

Indicator for points assigned to missing or out-of-range information when scoring, specified as the comma-separated pair consisting of 'Missing' and a character vector with a value for NoScore, ZeroW0E, MinPoints, or MaxPoints, where:

- NoScore - Missing and out-of-range data do not get points assigned and points are set to NaN . Also, the total score is set to NaN .
- ZeroWOE - Missing or out-of-range data get assigned a zero Weight-of-Evidence (WOE) value.
- MinPoints - Missing or out-of-range data get the minimum possible points for that predictor. This penalizes the score if higher scores are better.
- MaxPoints - Missing or out-of-range data get the maximum possible points for that predictor. This penalizes the score if lower scores are better.

Note When using the creditscorecard name-value argument 'BinMissingData' with a value of 'true', missing data for numeric and categorical predictors is binned
in a separate bin labeled <missing>. The <missing> bin only contains missing values for a predictor and does not contain invalid or out-of-range values for a predictor.

## Data Types: char

## Round - Indicator whether to round points or scores

'None' (default)| character vector with values'AllPoints ', 'FinalScore'
Indicator whether to round points or scores, specified as the comma-separated pair consisting of 'Round ' and a character vector with values 'AllPoints', 'FinalScore' or 'None', where:

- None - No rounding is applied.
- AllPoints - Apply rounding to each predictor's points before adding up the total score.
- FinalScore - Round the final score only (rounding is applied after all points are added up).

Data Types: char

## ShiftAndSlope - Indicator for shift and slope scaling parameters

## [0,1] (default)| numeric array with two elements [Shift, Slope]

Indicator for shift and slope scaling parameters for the credit scorecard, specified as the comma-separated pair consisting of 'ShiftAndSlope' and a numeric array with two elements [Shift, Slope]. Slope cannot be zero. The ShiftAndSlope values are used scale the scoring model.

Note ShiftAndSlope, Points0ddsAndPDO, and WorstAndBestScores are scaling methods and you can use only one of these name-value pair arguments at one time. The other three name-value pair arguments (BasePoints, Missing, and Round) are not scaling methods and can be used together or with any one of the three scaling methods.

To remove a previous scaling and revert to unscaled scores, set ShiftAndSlope to $[0,1]$.

## PointsOddsAndPDO - Indicator for target points for given odds and double odds level

 numeric array with three elements [Points, Odds, PDO]Indicator for target points (Points) for a given odds level (Odds) and the desired number of points to double the odds (PDO), specified as the comma-separated pair consisting of 'Points0ddsAndPDO' and a numeric array with three elements [Points,Odds, PDO]. Odds must be a positive number. The Points0ddsAndPD0 values are used to find scaling parameters for the scoring model.

Note The points to double the odds (PDO) may be positive or negative, depending on whether higher scores mean lower risk, or vice versa.

ShiftAndSlope, Points0ddsAndPD0, and WorstAndBestScores are scaling methods and you can use only one of these name-value pair arguments at one time. The other three name-value pair arguments (BasePoints, Missing, and Round) are not scaling methods and can be used together or with any one of the three scaling methods.

To remove a previous scaling and revert to unscaled scores, set ShiftAndSlope to $[0,1]$.

## Data Types: double

## WorstAndBestScores - Indicator for worst (highest risk) and best (lowest risk) scores in scorecard

numeric array with two elements [WorstScore, BestScore]
Indicator for worst (highest risk) and best (lowest risk) scores in the scorecard, specified as the comma-separated pair consisting of 'WorstAndBestScores' and a numeric array with two elements [WorstScore, BestScore]. WorstScore and BestScore must be different values. These WorstAndBestScores values are used to find scaling parameters for the scoring model.

Note WorstScore means the riskiest score, and its value could be lower or higher than the 'best' score. In other words, the 'minimum' score may be the 'worst' score or the 'best' score, depending on the desired scoring scale.

ShiftAndSlope, Points0ddsAndPDO, and WorstAndBestScores are scaling methods and you can use only one of these name-value pair arguments at one time. The other
three name-value pair arguments (BasePoints, Missing, and Round) are not scaling methods and can be used together or with any one of the three scaling methods.

To remove a previous scaling and revert to unscaled scores, set ShiftAndSlope to $[0,1]$.

Data Types: double

## Output Arguments

## sc - Credit scorecard model

creditscorecard object
Credit scorecard model returned as an updated creditscorecard object. For more information on using the creditscorecard object, see creditscorecard.

## Algorithms

The score of an individual $i$ is given by the formula

```
Score(i) = Shift + Slope*(b0 + b1*WOE1(i) + b2*WOE2(i)+ ... +bp*WOEp(i))
```

where $b j$ is the coefficient of the $j$ th variable in the model, and $\mathrm{WOEj}(i)$ is the Weight of Evidence (WOE) value for the $i$ th individual corresponding to the $j$ th model variable. Shift and Slope are scaling constants further discussed below. The scaling constant can be controlled with formatpoints.

If the data for individual $i$ is in the $i$-th row of a given dataset, to compute a score, the data $(i, j)$ is binned using existing binning maps, and converted into a corresponding Weight of Evidence value WOEj(i). Using the model coefficients, the unscaled score is computed as

```
s = b0 + b1*W0E1(i) + ... +bp*W0Ep(i).
```

For simplicity, assume in the description above that the $j$-th variable in the model is the $j$ th column in the data input, although, in general, the order of variables in a given dataset does not have to match the order of variables in the model, and the dataset could have additional variables that are not used in the model.

The formatting options can be controlled using formatpoints. When the base points are reported separately (see the formatpoints parameter BasePoints), the base points are given by

Base Points = Shift + Slope*b0,
and the points for the $j$-th predictor, $i$-th row are given by

```
Points_ji = Slope*(bj*WOEj(i))).
```

By default, the base points are not reported separately, in which case

```
Points_ji = (Shift + Slope*b0)/p + Slope*(bj*WOEj(i)),
```

where $p$ is the number of predictors in the scorecard model.
By default, no rounding is applied to the points by the score function (Round is None). If Round is set to AllPoints using formatpoints, then the points for individual $i$ for variable $j$ are given by

```
points if rounding is 'AllPoints': round( Points_ji )
```

and, if base points are reported separately, the are also rounded. This yields integervalued points per predictor, hence also integer-valued scores. If Round is set to FinalScore using formatpoints, then the points per predictor are not rounded, and only the final score is rounded

```
score if rounding is 'FinalScore': round(Score(i)).
```

Regarding the scaling parameters, the Shift parameter, and the Slope parameter can be set directly with the ShiftAndSlope parameter of formatpoints. Alternatively, you can use the formatpoints parameter for WorstAndBestScores. In this case, the parameters Shift and Slope are found internally by solving the system

```
Shift + Slope*smin = WorstScore,
Shift + Slope*smax = BestScore,
```

where WorstScore and BestScore are the first and second elements in the formatpoints parameter for WorstAndBestScores and smin and smax are the minimum and maximum possible unscaled scores:

```
smin = b0 + min(b1*WOE1) + ... +min(bp*WOEp),
smax = b0 + max(b1*W0E1) + ... +max(bp*W0Ep).
```

A third alternative to scale scores is the Points0ddsAndPD0 parameter in formatpoints. In this case, assume that the unscaled score $s$ gives the log-odds for a row, and the Shift and Slope parameters are found by solving the following system

Points $=$ Shift + Slope*log(Odds)
Points + PDO $=$ Shift + Slope*log(2*0dds)
where Points, Odds, and PDO ("points to double the odds") are the first, second, and third elements in the Points0ddsAndPDO parameter.

Whenever a given dataset has a missing or out-of-range value data ( $i, j$ ), the points for predictor $j$, for individual $i$, are set to NaN by default, which results in a missing score for that row (a NaN score). Using the Missing parameter for formatpoints, you can modify this behavior and set the corresponding Weight-of-Evidence (WOE) value to zero, or set the points to the minimum points, or the maximum points for that predictor.

## References

[1] Anderson, R. The Credit Scoring Toolkit. Oxford University Press, 2007.
[2] Refaat, M. Credit Risk Scorecards: Development and Implementation Using SAS. lulu.com, 2011.

## See Also

autobinning|bindata|bininfo|creditscorecard|displaypoints|fitmodel | modifybins|modifypredictor|plotbins | predictorinfo|probdefault| score| setmodel|validatemodel

## Topics

"Case Study for a Credit Scorecard Analysis" on page 8-101
"Credit Scorecard Modeling with Missing Values" on page 8-73
"Troubleshooting Credit Scorecard Results" on page 8-91
"Credit Scorecard Modeling Workflow" on page 8-67
"About Credit Scorecards" on page 8-62

## Introduced in R2014b

## displaypoints

Return points per predictor per bin

## Syntax

PointsInfo = displaypoints(sc)
[PointsInfo,MinScore,MaxScore] = displaypoints(sc)

## Description

PointsInfo = displaypoints(sc) returns a table of points for all bins of all predictor variables used in the creditscorecard object after a linear logistic regression model is fit using fitmodel to the Weight of Evidence data. The PointsInfo table displays information on the predictor name, bin labels, and the corresponding points per bin.
[PointsInfo,MinScore,MaxScore] = displaypoints(sc) returns a table of points for all bins of all predictor variables used in the creditscorecard object after a linear logistic regression model is fit (fitmodel) to the Weight of Evidence data. The PointsInfo table displays information on the predictor name, bin labels, and the corresponding points per bin and displaypoints. In addition, the optional MinScore and MaxScore values are returned.

## Examples

## Display Unscaled Points

This example shows how to use displaypoints after a model is fitted to compute the unscaled points per bin, for a given predictor in the creditscorecard model.

Create a creditscorecard object using the CreditCardData.mat file to load the data (using a dataset from Refaat 2011). Use the 'IDVar' argument in the creditscorecard function to indicate that 'CustID ' contains ID information and should not be included as a predictor variable.
load CreditCardData
sc = creditscorecard(data,'IDVar','CustID');
Perform automatic binning to bin for all predictors.
sc = autobinning(sc);

Fit a linear regression model using default parameters.
sc = fitmodel(sc);

1. Adding CustIncome, Deviance $=1490.8527$, Chi2Stat $=32.588614$, PValue $=1.1387992 \mathrm{e}-$
2. Adding TmWBank, Deviance $=1467.1415$, Chi2Stat $=23.711203$, PValue $=1.1192909 \mathrm{e}-06$
3. Adding AMBalance, Deviance $=1455.5715$, Chi2Stat $=11.569967$, PValue $=0.0006702560$
4. Adding EmpStatus, Deviance $=1447.3451$, Chi2Stat $=8.2264038$, PValue $=0.0041285257$
5. Adding CustAge, Deviance $=1441.994$, Chi2Stat $=5.3511754$, PValue $=0.020708306$
6. Adding ResStatus, Deviance $=1437.8756$, Chi2Stat $=4.118404$, PValue $=0.042419078$
7. Adding OtherCC, Deviance $=1433.707$, Chi2Stat $=4.1686018$, $\mathrm{PValue}=0.041179769$

Generalized linear regression model:
status ~ [Linear formula with 8 terms in 7 predictors] Distribution = Binomial

Estimated Coefficients:

|  | Estimate | SE | tStat | pValue |
| :---: | :---: | :---: | :---: | :---: |
| (Intercept) | 0.70239 | 0.064001 | 10.975 | $5.0538 \mathrm{e}-28$ |
| CustAge | 0.60833 | 0.24932 | 2.44 | 0.014687 |
| ResStatus | 1.377 | 0.65272 | 2.1097 | 0.034888 |
| EmpStatus | 0.88565 | 0.293 | 3.0227 | 0.0025055 |
| CustIncome | 0.70164 | 0.21844 | 3.2121 | 0.0013179 |
| TmWBank | 1.1074 | 0.23271 | 4.7589 | $1.9464 \mathrm{e}-06$ |
| OtherCC | 1.0883 | 0.52912 | 2.0569 | 0.039696 |
| AMBalance | 1.045 | 0.32214 | 3.2439 | 0.0011792 |

1200 observations, 1192 error degrees of freedom
Dispersion: 1
Chi^2-statistic vs. constant model: 89.7, p -value $=1.4 \mathrm{e}-16$
Display unscaled points for predictors retained in the fitting model.

```
PointsInfo = displaypoints(sc)
```

```
PointsInfo=30\times3 table
    Predictors Bin
    'CustAge'
'[-Inf,33)'
    -0.15894
'CustAge'
'CustAge'
'CustAge'
'CustAge'
'CustAge'
'CustAge'
'ResStatus'
'ResStatus'
'ResStatus'
'EmpStatus'
'EmpStatus'
'CustIncome'
'CustIncome'
'CustIncome'
'CustIncome'
```

!

## Display Unscaled Points When Using Missing Data

Create a creditscorecard object using the CreditCardData.mat file to load the data with missing values.

```
load CreditCardData.mat
head(dataMissing,5)
```

| CustID | CustAge | TmAtAddress | ResStatus | EmpStatus | CustIncome |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 53 | 62 | <undefined> | Unknown | 50000 |
| 2 | 61 | 22 | Home Owner | Employed | 52000 |
| 3 | 47 | 30 | Tenant | Employed | 37000 |
| 4 | NaN | 75 | Home Owner | Employed | 53000 |
| 5 | 68 | 56 | Home Owner | Employed | 53000 |

```
fprintf('Number of rows: %d\n',height(dataMissing))
Number of rows: 1200
fprintf('Number of missing values CustAge: %d\n',sum(ismissing(dataMissing.CustAge)))
Number of missing values CustAge: 30
fprintf('Number of missing values ResStatus: %d\n',sum(ismissing(dataMissing.ResStatus
Number of missing values ResStatus: 40
```

Use creditscorecard with the name-value argument 'BinMissingData' set to true to bin the missing numeric or categorical data in a separate bin. Apply automatic binning.

```
sc = creditscorecard(dataMissing,'IDVar','CustID','BinMissingData',true);
sc = autobinning(sc);
disp(sc)
```

    creditscorecard with properties:
                        GoodLabel: 0
                ResponseVar: 'status'
                    WeightsVar: ''
                            VarNames: \{1x11 cell\}
            NumericPredictors: \{1x6 cell\}
        CategoricalPredictors: \{'ResStatus' 'EmpStatus' 'OtherCC'\}
            BinMissingData: 1
                            IDVar: 'CustID'
                        PredictorVars: \{1x9 cell\}
                                Data: [1200x11 table]
    Display and plot bin information for numeric data for 'CustAge' that includes missing data in a separate bin labelled <missing>.

```
[bi,cp] = bininfo(sc,'CustAge');
disp(bi)
```

| Bin | Good | Bad |  | Odds |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | | WOE |
| :--- |


| '[40,46)' | 172 | 89 | 1.9326 | -0.04556 | 0.0004549 |
| :--- | ---: | ---: | ---: | ---: | ---: |
| '[46,48)' | 59 | 25 | 2.36 | 0.15424 | 0.0016199 |
| '[48,51)' | 99 | 41 | 2.4146 | 0.17713 | 0.0035449 |
| '[51,58)' | 157 | 62 | 2.5323 | 0.22469 | 0.0088407 |
| '[58,Inf]' | 93 | 25 | 3.72 | 0.60931 | 0.032198 |
| '<missing>' | 19 | 11 | 1.7273 | -0.15787 | 0.00063885 |
| 'Totals' $^{\text {'Tota }}$ | 803 | 397 | 2.0227 | NaN | 0.087112 |

```
plotbins(sc,'CustAge')
```


$\square$ Good $\square \mathrm{Bad}$

Display and plot bin information for categorical data for 'ResStatus ' that includes missing data in a separate bin labelled <missing>.
[bi,cg] = bininfo(sc,'ResStatus');
disp(bi)

| Bin | Good | Bad | 0dds | WOE | InfoValue |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 'Tenant ${ }^{\text {' }}$ | 296 | 161 | 1.8385 | -0.095463 | 0.0035249 |
| 'Home Owner' | 352 | 171 | 2.0585 | 0.017549 | 0.00013382 |
| 'Other' | 128 | 52 | 2.4615 | 0.19637 | 0.0055808 |
| '<missing>' | 27 | 13 | 2.0769 | 0.026469 | 2.3248e-05 |
| 'Totals' | 803 | 397 | 2.0227 | NaN | 0.0092627 |



For the 'CustAge' and 'ResStatus' predictors, there is missing data (NaNs and <undefined>) in the training data, and the binning process estimates a WOE value of
-0.15787 and 0.026469 respectively for missing data in these predictors, as shown above.

Use fitmodel to fit a logistic regression model using Weight of Evidence (WOE) data. fitmodel internally transforms all the predictor variables into WOE values, using the bins found with the automatic binning process. fitmodel then fits a logistic regression model using a stepwise method (by default). For predictors that have missing data, there is an explicit <missing> bin, with a corresponding WOE value computed from the data. When using fitmodel, the corresponding WOE value for the <missing> bin is applied when performing the WOE transformation.

```
[sc,mdl] = fitmodel(sc);
```

1. Adding CustIncome, Deviance $=1490.8527$, Chi2Stat $=32.588614$, PValue $=1.1387992 \mathrm{e}-\mathrm{C}$
2. Adding TmWBank, Deviance $=1467.1415$, Chi2Stat $=23.711203$, PValue $=1.1192909 \mathrm{e}-06$
3. Adding AMBalance, Deviance $=1455.5715$, Chi2Stat $=11.569967$, PValue $=0.0006702560$
4. Adding EmpStatus, Deviance $=1447.3451$, Chi2Stat $=8.2264038$, PValue $=0.0041285257$
5. Adding CustAge, Deviance $=1442.8477$, Chi2Stat $=4.4974731$, PValue $=0.033944979$
6. Adding ResStatus, Deviance $=1438.9783$, Chi2Stat $=3.86941$, PValue $=0.049173805$
7. Adding OtherCC, Deviance $=1434.9751$, Chi2Stat $=4.0031966$, PValue $=0.045414057$

Generalized linear regression model:
status ~ [Linear formula with 8 terms in 7 predictors]
Distribution = Binomial
Estimated Coefficients:

| Estimate | SE | tStat |  |  |
| :--- | ---: | :--- | ---: | :--- | pValue

1200 observations, 1192 error degrees of freedom
Dispersion: 1
Chi^2-statistic vs. constant model: 88.5, p -value $=2.55 \mathrm{e}-16$

Display unscaled points for predictors retained in the fitting model (to scale points use formatpoints).

```
PointsInfo = displaypoints(sc)
```

PointsInfo=33×3 table
Predictors Bin Points
'CustAge'
'[-Inf,33)'
-0. 14173
'CustAge' '[33,37)' -0.11095
'CustAge' '[37,40)' -0.059244
'CustAge' '[40,46)' 0.074167
'CustAge' '[46,48)' 0.1889
'CustAge' '[48,51)' 0.20204
'CustAge' '[51,58)' 0.22935
'CustAge' '[58,Inf]' 0.45019
'CustAge' '<missing>' 0.0096749
'ResStatus' 'Tenant' -0.029778
'ResStatus' 'Home Owner' 0.12425
'ResStatus' 'Other' 0.36796
'ResStatus' '<missing>' 0.1364
'EmpStatus' 'Unknown' -0.075948
'EmpStatus' 'Employed' 0.31401
'CustIncome' '[-Inf,29000)' -0.48396

Notice that points for the <missing> bin for CustAge and ResStatus are explicitly shown. These points are computed from the WOE value for the <missing> bin and the logistic model coefficients.

For predictors that have no missing data in the training set, there is no explicit <missing> bin, and by default the points are set to NaN for missing data, and they lead to a score of NaN when running score. For predictors that have no explicit <missing> bin, use the name-value argument 'Missing' in formatpoints to indicate how missing data should be treated for scoring purposes.

## Display Scaled Points

This example shows how to use formatpoints after a model is fitted to format scaled points, and then use displaypoints to display the scaled points per bin, for a given predictor in the creditscorecard model.

Points become scaled when a range is defined. Specifically, a linear transformation from the unscaled to the scaled points is necessary. This transformation is defined either by supplying a shift and slope or by specifying the worst and best scores possible. (For more information, see formatpoints.)

Create a creditscorecard object using the CreditCardData.mat file to load the data (using a dataset from Refaat 2011). Use the 'IDVar' argument in the creditscorecard function to indicate that 'CustID' contains ID information and should not be included as a predictor variable.

```
load CreditCardData
sc = creditscorecard(data,'IDVar','CustID');
```

Perform automatic binning to bin for all predictors.

```
sc = autobinning(sc);
```

Fit a linear regression model using default parameters.

```
sc = fitmodel(sc);
```

1. Adding CustIncome, Deviance $=1490.8527$, Chi2Stat $=32.588614$, PValue $=1.1387992 \mathrm{e}-\mathrm{C}$
2. Adding TmWBank, Deviance $=1467.1415$, Chi2Stat $=23.711203$, PValue $=1.1192909 \mathrm{e}-06$
3. Adding AMBalance, Deviance $=1455.5715$, Chi2Stat $=11.569967$, PValue $=0.0006702560$
4. Adding EmpStatus, Deviance $=1447.3451$, Chi2Stat $=8.2264038$, PValue $=0.0041285257$
5. Adding CustAge, Deviance $=1441.994$, Chi2Stat $=5.3511754$, $\mathrm{PValue}=0.020708306$
6. Adding ResStatus, Deviance $=1437.8756$, Chi2Stat $=4.118404$, PValue $=0.042419078$
7. Adding OtherCC, Deviance $=1433.707$, Chi2Stat $=4.1686018$, PValue $=0.041179769$
Generalized linear regression model:
status ~ [Linear formula with 8 terms in 7 predictors]
Distribution = Binomial
Estimated Coefficients:

| Estimate | SE | tStat | pValue |
| :---: | :---: | :---: | :---: |
| 0.70239 | 0.064001 | 10.975 | $5.0538 \mathrm{e}-2$ |


| CustAge | 0.60833 | 0.24932 | 2.44 | 0.014687 |
| :--- | ---: | ---: | ---: | ---: |
| ResStatus | 1.377 | 0.65272 | 2.1097 | 0.034888 |
| EmpStatus | 0.88565 | 0.293 | 3.0227 | 0.0025055 |
| CustIncome | 0.70164 | 0.21844 | 3.2121 | 0.0013179 |
| TmWBank | 1.1074 | 0.23271 | 4.7589 | $1.9464 \mathrm{e}-06$ |
| OtherCC | 1.0883 | 0.52912 | 2.0569 | 0.039696 |
| AMBalance | 1.045 | 0.32214 | 3.2439 | 0.0011792 |

1200 observations, 1192 error degrees of freedom Dispersion: 1
Chi^2-statistic vs. constant model: 89.7, p-value $=1.4 \mathrm{e}-16$
Use the formatpoints function to scale providing the 'Worst' and 'Best' score values. The range provided below is a common score range.
sc = formatpoints(sc,'WorstAndBestScores',[300 850]);
Display the points information again to verify that the points are now scaled and also display the scaled minimum and maximum scores.

```
sc = formatpoints(sc,'WorstAndBestScores',[300 850]);
[PointsInfo,MinScore,MaxScore] = displaypoints(sc)
PointsInfo=30\times3 table
```


## Predictors

'CustAge'
'CustAge'
'CustAge'
'CustAge
'CustAge
'CustAge
'CustAge
'ResStatus'
'ResStatus'
'ResStatus'
'EmpStatus'
'EmpStatus'
'CustIncome'
'CustIncome'
'CustIncome'
'CustIncome'

Bin
[ - Inf, 33)
'[33,37)
'[37,40)
' $[40,46$ )
$\begin{array}{ll} \\ {[46,48) '} & 93.256\end{array}$
'[48,58)' 95.256
'[58,Inf]' 126.46
'Tenant' 62.421
'Home Owner' 82.276
'Other' 113.58
'Unknown' 56.765
'Employed' 105.81
'[-Inf,29000)' 8.9706
' $[29000,33000)^{\prime} \quad 53.208$
'[33000,35000)' 72.91
'[35000,40000)' 76.585

```
MinScore = 300
MaxScore = 850.0000
```

Notice that, as expected, the values of MinScore and MaxScore correspond to the worst and best possible scores.

## Separate the Base Points From the Total Points

This example shows how to use displaypoints after a model is fitted to separate the base points from the rest of the points assigned to each predictor variable. The namevalue pair argument 'BasePoints' in the formatpoints function is a boolean that serves this purpose. By default, the base points are spread across all variables in the scorecard.

Create a creditscorecard object using the CreditCardData.mat file to load the data (using a dataset from Refaat 2011). Use the 'IDVar' argument in the creditscorecard function to indicate that 'CustID' contains ID information and should not be included as a predictor variable.

```
load CreditCardData
sc = creditscorecard(data,'IDVar','CustID');
```

Perform automatic binning to bin for all predictors.

```
sc = autobinning(sc);
```

Fit a linear regression model using default parameters.

```
sc = fitmodel(sc);
```

1. Adding CustIncome, Deviance $=1490.8527$, Chi2Stat $=32.588614$, PValue $=1.1387992 \mathrm{e}-\mathrm{C}$
2. Adding TmWBank, Deviance $=1467.1415$, Chi2Stat $=23.711203$, PValue $=1.1192909 \mathrm{e}-06$
3. Adding AMBalance, Deviance $=1455.5715$, Chi2Stat $=11.569967$, PValue $=0.0006702560$
4. Adding EmpStatus, Deviance $=1447.3451$, Chi2Stat $=8.2264038$, PValue $=0.0041285257$
5. Adding CustAge, Deviance $=1441.994$, Chi2Stat $=5.3511754$, PValue $=0.020708306$
6. Adding ResStatus, Deviance $=1437.8756$, Chi2Stat $=4.118404$, PValue $=0.042419078$
7. Adding OtherCC, Deviance $=1433.707$, Chi2Stat $=4.1686018$, PValue $=0.041179769$

| ```Generalized linear regression model: status ~ [Linear formula with 8 terms in 7 predictors] Distribution = Binomial``` |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Estimated Coefficients: |  |  |  |  |
|  | Estimate | SE | tStat | pValue |
| (Intercept) | 0.70239 | 0.064001 | 10.975 | $5.0538 \mathrm{e}-28$ |
| CustAge | 0.60833 | 0.24932 | 2.44 | 0.014687 |
| ResStatus | 1.377 | 0.65272 | 2.1097 | 0.034888 |
| EmpStatus | 0.88565 | 0.293 | 3.0227 | 0.0025055 |
| CustIncome | 0.70164 | 0.21844 | 3.2121 | 0.0013179 |
| TmWBank | 1.1074 | 0.23271 | 4.7589 | $1.9464 \mathrm{e}-06$ |
| OtherCC | 1.0883 | 0.52912 | 2.0569 | 0.039696 |
| AMBalance | 1.045 | 0.32214 | 3.2439 | 0.0011792 |

1200 observations, 1192 error degrees of freedom Dispersion: 1
Chi^2-statistic vs. constant model: 89.7, p -value $=1.4 \mathrm{e}-16$
Use the formatpoints function to separate the base points by providing the 'BasePoints' name-value pair argument.

```
sc = formatpoints(sc,'BasePoints',true);
```

Display the base points, separated out from the other points, for predictors retained in the fitting model.

```
PointsInfo = displaypoints(sc)
PointsInfo=31\times3 table
    Predictors Bin
'BasePoints'
    'CustAge'
    'CustAge'
    'CustAge'
    'CustAge'
    'CustAge'
    'CustAge' '[48,58)' 0.13005
    'CustAge' '[58,Inf]' 0.37866
    'ResStatus' 'Tenant' -0.13159
'BasePoints'
'[-Inf, 33)'
'[33, 37)'
'[37, 40)'
'[40,46)'
'[46,48)'
'[48,58)'
'[58,Inf]'
'Tenant'
                                    Points
                                0.70239
'[-Inf 33)'
    -0.25928
    '[33,37)
    -0.24071
    '[37,40)
    -0.16066
    '[40,46)' -0.053933
    '[46,48)' 0.11411
```

| 'ResStatus' | 'Home Owner' | 0.026616 |
| :--- | :--- | ---: |
| 'ResStatus' | 'Other' | 0.27607 |
| 'EmpStatus' | 'Unknown' | -0.17666 |
| 'EmpStatus' | 'Employed' | 0.21415 |
| 'CustIncome' | '[-Inf,29000)' | -0.5575 |
| 'CustIncome' | '[29000,33000)' | -0.205 |
| 'CustIncome' | '[33000,35000)' | -0.048013 |

## Display Points After Modifying Bin Labels

This example shows how to use displaypoints after a model is fitted and the modifybins function is used to provide user-defined bin labels for a numeric predictor.

Create a creditscorecard object using the CreditCardData.mat file to load the data (using a dataset from Refaat 2011). Use the 'IDVar' argument in the creditscorecard function to indicate that 'CustID' contains ID information and should not be included as a predictor variable.

```
load CreditCardData
sc = creditscorecard(data,'IDVar','CustID');
```

Perform automatic binning to bin for all predictors.

```
sc = autobinning(sc);
```

Fit a linear regression model using default parameters.

```
sc = fitmodel(sc);
1. Adding CustIncome, Deviance = 1490.8527, Chi2Stat = 32.588614, PValue = 1.1387992e-
2. Adding TmWBank, Deviance = 1467.1415, Chi2Stat = 23.711203, PValue = 1.1192909e-06
3. Adding AMBalance, Deviance = 1455.5715, Chi2Stat = 11.569967, PValue = 0.0006702560
4. Adding EmpStatus, Deviance = 1447.3451, Chi2Stat = 8.2264038, PValue = 0.0041285257
5. Adding CustAge, Deviance = 1441.994, Chi2Stat = 5.3511754, PValue = 0.020708306
6. Adding ResStatus, Deviance = 1437.8756, Chi2Stat = 4.118404, PValue = 0.042419078
7. Adding OtherCC, Deviance = 1433.707, Chi2Stat = 4.1686018, PValue = 0.041179769
Generalized linear regression model:
    status ~ [Linear formula with 8 terms in 7 predictors]
    Distribution = Binomial
```

Estimated Coefficients:

|  | Estimate | SE | tStat | pValue |
| :---: | :---: | :---: | :---: | :---: |
| (Intercept) | 0.70239 | 0.064001 | 10.975 | 5.0538e-28 |
| CustAge | 0.60833 | 0.24932 | 2.44 | 0.014687 |
| ResStatus | 1.377 | 0.65272 | 2.1097 | 0.034888 |
| EmpStatus | 0.88565 | 0.293 | 3.0227 | 0.0025055 |
| CustIncome | 0.70164 | 0.21844 | 3.2121 | 0.0013179 |
| TmWBank | 1.1074 | 0.23271 | 4.7589 | 1.9464e-06 |
| OtherCC | 1.0883 | 0.52912 | 2.0569 | 0.039696 |
| AMBalance | 1.045 | 0.32214 | 3.2439 | 0.001179 |

1200 observations, 1192 error degrees of freedom
Dispersion: 1
Chi^2-statistic vs. constant model: 89.7, p-value $=1.4 \mathrm{e}-16$
Use the displaypoints function to display point information.
[PointsInfo,MinScore,MaxScore] = displaypoints(sc)
PointsInfo=30×3 table

Predictors Bin
'CustAge'
'CustAge'
'CustAge
'CustAge
'CustAge'
'CustAge'
'CustAge
'ResStatus'
'ResStatus'
'ResStatus'
'EmpStatus'
'EmpStatus'
'CustIncome'
'CustIncome'
'CustIncome'
'CustIncome'

| [-Inf,33)' | -0.15894 |
| :---: | :---: |
| [ 33,37$)^{\prime}$ | -0.14036 |
| '[37,40)' | -0.060323 |
| '[40,46)' | 0.046408 |
| ' $[46,48$ )' | 0.21445 |
| ' $[48,58$ )' | 0.23039 |
| [58,Inf]' | 0.479 |
| Tenant' | -0.031252 |
| Home Owner' | 0.12696 |
| Other' | 0.37641 |
| Unknown' | -0.076317 |
| Employed' | 0.31449 |
| ' [-Inf, 29000) ' | -0.45716 |
| ' [29000, 33000) | -0.10466 |
| '[33000, 35000) | 0.052329 |
| [35000, 40000) | 0.08161 |

Points
$\qquad$
-0. 15894
-0.14036
0.060323
. 23039
0.031252
0.12696
0.37641
0.076317
0.
0.081611

MinScore $=-1.3100$

MaxScore $=3.0726$
Use the modifybins function to specify user-defined bin labels for 'CustAge' so that the bin ranges are described in natural language.
labels $=\left\{' U p\right.$ to $32^{\prime}, ' 33$ to $36^{\prime}, ' 37$ to $39^{\prime}, ' 40$ to $45^{\prime}, ' 46$ to $47{ }^{\prime}, ' 48$ to $57{ }^{\prime}, ' A t$ least sc = modifybins(sc,'CustAge','BinLabels',labels);

Rerun displaypoints to verify the updated bin labels.

```
[PointsInfo,MinScore,MaxScore] = displaypoints(sc)
```

| PointsInfo $=30 \times 3$ Predictors | Bin | Points |
| :---: | :---: | :---: |
| 'CustAge ' | 'Up to 32' | -0.15894 |
| 'CustAge' | '33 to 36' | -0.14036 |
| 'CustAge' | '37 to 39' | -0.060323 |
| 'CustAge' | '40 to 45' | 0.046408 |
| 'CustAge' | '46 to $47{ }^{\prime}$ | 0.21445 |
| 'CustAge' | '48 to 57' | 0.23039 |
| 'CustAge' | 'At least 58' | 0.479 |
| 'ResStatus' | 'Tenant' | -0.031252 |
| 'ResStatus' | 'Home Owner' | 0.12696 |
| 'ResStatus' | 'Other' | 0.37641 |
| 'EmpStatus' | 'Unknown ' | -0.076317 |
| 'EmpStatus' | 'Employed' | 0.31449 |
| 'CustIncome' | '[-Inf, 29000)' | -0.45716 |
| 'CustIncome' | '[29000, 33000) ' | -0.10466 |
| 'CustIncome' | '[33000, 35000)' | 0.052329 |
| 'CustIncome' | '[35000, 40000) ' | 0.081611 |

```
MinScore = -1.3100
```

MaxScore $=3.0726$

## Compute the Predictor Weights

This example shows how to use a credit scorecard to compute the weights of the predictors. The weights of the predictors are determined from the range of points of each predictor, divided by the total range of points for the scorecard. The points for the scorecard not only take into consideration the betas, but also implicitly the binning of the predictor values and the corresponding weights of evidence.

Create a scorecard.

```
load CreditCardData.mat
sc = creditscorecard(data,'IDVar','CustID');
sc = autobinning(sc);
sc = fitmodel(sc);
```

1. Adding CustIncome, Deviance = 1490.8527, Chi2Stat = 32.588614, PValue = 1.1387992e-(
2. Adding TmWBank, Deviance $=1467.1415$, Chi2Stat $=23.711203$, PValue $=1.1192909 \mathrm{e}-06$
3. Adding AMBalance, Deviance $=1455.5715$, Chi2Stat $=11.569967$, PValue $=0.0006702560$
4. Adding EmpStatus, Deviance $=1447.3451$, Chi2Stat $=8.2264038$, PValue $=0.0041285257$
5. Adding CustAge, Deviance $=1441.994$, Chi2Stat $=5.3511754$, PValue $=0.020708306$
6. Adding ResStatus, Deviance $=1437.8756$, Chi2Stat $=4.118404$, PValue $=0.042419078$
7. Adding OtherCC, Deviance $=1433.707$, Chi2Stat $=4.1686018$, PValue $=0.041179769$
Generalized linear regression model:
status ~ [Linear formula with 8 terms in 7 predictors]
Distribution = Binomial

Estimated Coefficients:

| Estimate | SE | tStat | pValue |
| :---: | :---: | :---: | :---: |
| 0.70239 | 0.064001 | 10.975 | 5.0538e-28 |
| 0.60833 | 0.24932 | 2.44 | 0.014687 |
| 1.377 | 0.65272 | 2.1097 | 0.034888 |
| 0.88565 | 0.293 | 3.0227 | 0.0025055 |
| 0.70164 | 0.21844 | 3.2121 | 0.0013179 |
| 1.1074 | 0.23271 | 4.7589 | $1.9464 \mathrm{e}-06$ |
| 1.0883 | 0.52912 | 2.0569 | 0.039696 |
| 1.045 | 0.32214 | 3.2439 | 0.001179 |

1200 observations, 1192 error degrees of freedom
Dispersion: 1
Chi^2-statistic vs. constant model: 89.7, p-value $=1.4 \mathrm{e}-16$

Compute scorecard points and the MinPts and MaxPts scores.

```
sc = formatpoints(sc,'Points0ddsAndPDO',[500 2 50]);
[PointsTable,MinPts,MaxPts] = displaypoints(sc);
PtsRange = MaxPts-MinPts;
disp(PointsTable(1:10,:));
'CustAge'
'CustAge'
'CustAge'
'CustAge'
'CustAge'
'CustAge'
'CustAge'
'ResStatus'
'ResStatus'
'ResStatus'
```

Predictors
$\qquad$

Bin
[-Inf,33)
' $[33,37$ )
' $[37,40$ )
' $[40,46$ ) '
' $[46,48$ )'
' $[48,58$ )
'[58,Inf]'
'Tenant'
'Home Owner'
'Other'

Points
52.821
54.161
59.934
67.633
79.755
80.905
98.838
62.031
73.444
91.438

```
fprintf('Min points: %g, Max points: %g\n',MinPts,MaxPts);
```

```
fprintf('Min points: %g, Max points: %g\n',MinPts,MaxPts);
```

```
Min points: 355.505, Max points: 671.64
```

```
Min points: 355.505, Max points: 671.64
```

Compute the predictor weights.

```
Predictor = unique(PointsTable.Predictors,'stable');
NumPred = length(Predictor);
Weight = zeros(NumPred,1);
for ii=1:NumPred
    Ind = cellfun(@(x)strcmpi(Predictor{ii},x),PointsTable.Predictors);
    MaxPtsPred = max(PointsTable.Points(Ind));
    MinPtsPred = min(PointsTable.Points(Ind));
    Weight(ii) = 100*(MaxPtsPred-MinPtsPred)/PtsRange;
end
PredictorWeights = table(Predictor,Weight);
PredictorWeights(end+1,:) = PredictorWeights(end,:);
PredictorWeights.Predictor{end} = 'Total';
PredictorWeights.Weight(end) = sum(Weight);
disp(PredictorWeights)
    Predictor Weight
```

| 'CustAge' | 14.556 |
| :--- | ---: |
| 'ResStatus ' | 9.302 |
| 'EmpStatus' | 8.9174 |
| 'CustIncome' | 20.401 |
| 'TmWBank ' | 25.884 |
| 'OtherCC' $^{\text {Other }}$ | 7.9885 |
| 'AMBalance' | 12.951 |
| 'Total' | 100 |

The weights are defined as the range of points for the predictor divided by the range of points for the scorecard.

## Input Arguments

## sc - Credit scorecard model

creditscorecard object
Credit scorecard model, specified as a creditscorecard object. Use creditscorecard to create a creditscorecard object.

## Output Arguments

PointsInfo - One row per bin, per predictor, with the corresponding points table

One row per bin, per predictor, with the corresponding points, returned as a table. For example:

| Predictors | Bin | Points |
| :--- | :--- | :--- |
| Predictor_1 | Bin_11 | Points_11 |
|  | $\ldots$ | $\ldots$ |
| Predictor_2 | Bin_21 | Points_21 |
|  | $\ldots$ | $\ldots$ |
| Predictor $j$ | Bin_ji | Points $j i$ |
|  | $\ldots$ | $\ldots$ |

When base points are reported separately (see formatpoints), the first row of the returned PointsInfo table contains the base points.

## MinScore - Minimum possible total score <br> scalar

Minimum possible total score, returned as a scalar.

Note Minimum score is the lowest possible total score in the mathematical sense, independently of whether a low score means high risk or low risk.

## MaxScore - Maximum possible total score

scalar
Maximum possible total score, returned as a scalar.

Note Maximum score is the highest possible total score in the mathematical sense, independently of whether a high score means high risk or low risk.

## Algorithms

The points for predictor $j$ and bin $i$ are, by default, given by
Points_ji = (Shift + Slope*b0)/p + Slope*(bj*WOEj(i))
where $b j$ is the model coefficient of predictor $j, p$ is the number of predictors in the model, and $\operatorname{WOEj}(i)$ is the Weight of Evidence (WOE) value for the $i$-th bin corresponding to the $j$ th model predictor. Shift and Slope are scaling constants.

When the base points are reported separately (see the formatpoints name-value pair argument BasePoints), the base points are given by

Base Points $=$ Shift + Slope*b0,
and the points for the $j$-th predictor, $i$-th row are given by
Points_ji = Slope*(bj*WOEj(i))).
By default, the base points are not reported separately.

The minimum and maximum scores are:

```
MinScore = Shift + Slope*b0 + min(Slope*b1*W0E1) + ... +min(Slope*bp*W0Ep)),
MaxScore = Shift + Slope*b0 + max(Slope*b1*W0E1) + ... +max(Slope*bp*W0Ep)).
```

Use formatpoints to control the way points are scaled, rounded, and whether the base points are reported separately. See formatpoints for more information on format parameters and for details and formulas on these formatting options.

## References

[1] Anderson, R. The Credit Scoring Toolkit. Oxford University Press, 2007.
[2] Refaat, M. Credit Risk Scorecards: Development and Implementation Using SAS. lulu.com, 2011.

## See Also

autobinning|bindata|bininfo|creditscorecard|fitmodel|formatpoints | modifybins |modifypredictor|plotbins |predictorinfo|probdefault| score | setmodel | validatemodel

## Topics

"Case Study for a Credit Scorecard Analysis" on page 8-101
"Credit Scorecard Modeling with Missing Values" on page 8-73
"Troubleshooting Credit Scorecard Results" on page 8-91
"Credit Scorecard Modeling Workflow" on page 8-67
"About Credit Scorecards" on page 8-62

Introduced in R2014b

## fitmodel

Fit logistic regression model to Weight of Evidence (WOE) data

## Syntax

```
sc = fitmodel(sc)
```

[sc,mdl] = fitmodel(sc)
[sc,mdl] = fitmodel( __, Name, Value)

## Description

sc = fitmodel(sc) fits a logistic regression model to the Weight of Evidence (WOE) data and stores the model predictor names and corresponding coefficients in the creditscorecard object.
fitmodel internally transforms all the predictor variables into WOE values, using the bins found with the automatic or manual binning process. The response variable is mapped so that "Good" is 1 , and "Bad" is 0 . This implies that higher (unscaled) scores correspond to better (less risky) individuals (smaller probability of default).

Alternatively, you can use setmodel to provide names of the predictors that you want in the logistic regression model, along with their corresponding coefficients.
[sc,mdl] = fitmodel(sc) fits a logistic regression model to the Weight of Evidence (WOE) data and stores the model predictor names and corresponding coefficients in the creditscorecard object. fitmodel returns an updated creditscorecard object and a GeneralizedLinearModel object containing the fitted model.
fitmodel internally transforms all the predictor variables into WOE values, using the bins found with the automatic or manual binning process. The response variable is mapped so that "Good" is 1 , and "Bad" is 0 . This implies that higher (unscaled) scores correspond to better (less risky) individuals (smaller probability of default).

Alternatively, you can use setmodel to provide names of the predictors that you want in the logistic regression model, along with their corresponding coefficients.
[sc,mdl] = fitmodel(__ ,Name, Value) fits a logistic regression model to the Weight of Evidence (WOE) data using optional name-value pair arguments and stores the model predictor names and corresponding coefficients in the creditscorecard object. Using name-value pair arguments, you can select which Generalized Linear Model to fit the data. fitmodel returns an updated creditscorecard object and a GeneralizedLinearModel object containing the fitted model.

## Examples

## Fit a Stepwise Logistic Model

Create a creditscorecard object using the CreditCardData.mat file to load the data (using a dataset from Refaat 2011).
load CreditCardData

```
sc = creditscorecard(data,'IDVar','CustID')
```

Sc =
creditscorecard with properties:
GoodLabel: 0
ResponseVar: 'status'
WeightsVar:
VarNames: \{1x11 cell\}
NumericPredictors: \{1x6 cell\}
CategoricalPredictors: \{'ResStatus' 'EmpStatus' 'OtherCC'\}
BinMissingData: 0
IDVar: 'CustID'
PredictorVars: \{1x9 cell\}
Data: [1200x11 table]

Perform automatic binning.

```
sc = autobinning(sc)
```

sc =
creditscorecard with properties:

GoodLabel: 0
ResponseVar: 'status'

```
    WeightsVar: ''
    VarNames: {1x11 cell}
    NumericPredictors: {1x6 cell}
CategoricalPredictors: {'ResStatus' 'EmpStatus' 'OtherCC'}
BinMissingData: 0
                            IDVar: 'CustID'
    PredictorVars: {1x9 cell}
                            Data: [1200x11 table]
```

Use fitmodel to fit a logistic regression model using Weight of Evidence (WOE) data. fitmodel internally transforms all the predictor variables into WOE values, using the bins found with the automatic binning process. fitmodel then fits a logistic regression model using a stepwise method (by default).

```
sc = fitmodel(sc);
```

1. Adding CustIncome, Deviance = 1490.8527, Chi2Stat $=32.588614$, PValue $=1.1387992 \mathrm{e}-$
2. Adding TmWBank, Deviance $=1467.1415$, Chi2Stat $=23.711203$, PValue $=1.1192909 \mathrm{e}-06$ 3. Adding AMBalance, Deviance $=1455.5715$, Chi2Stat $=11.569967$, $\mathrm{PValue}=0.0006702560$ 4. Adding EmpStatus, Deviance $=1447.3451$, Chi2Stat $=8.2264038$, PValue $=0.0041285257$ 5. Adding CustAge, Deviance $=1441.994$, Chi2Stat $=5.3511754$, PValue $=0.020708306$
3. Adding ResStatus, Deviance $=1437.8756$, Chi2Stat $=4.118404$, PValue $=0.042419078$
4. Adding OtherCC, Deviance $=1433.707$, Chi2Stat $=4.1686018$, PValue $=0.041179769$

Generalized linear regression model:
status ~ [Linear formula with 8 terms in 7 predictors]
Distribution = Binomial
Estimated Coefficients:
Estimate SE tStat pValue

| (Intercept) | 0.70239 | 0.064001 | 10.975 | $5.0538 \mathrm{e}-28$ |
| :--- | ---: | ---: | ---: | ---: |
| CustAge | 0.60833 | 0.24932 | 2.44 | 0.014687 |
| ResStatus | 1.377 | 0.65272 | 2.1097 | 0.034888 |
| EmpStatus | 0.88565 | 0.293 | 3.0227 | 0.0025055 |
| CustIncome | 0.70164 | 0.21844 | 3.2121 | 0.0013179 |
| TmWBank | 1.1074 | 0.23271 | 4.7589 | $1.9464 \mathrm{e}-06$ |
| OtherCC | 1.0883 | 0.52912 | 2.0569 | 0.039696 |
| AMBalance | 1.045 | 0.32214 | 3.2439 | 0.0011792 |

1200 observations, 1192 error degrees of freedom

Dispersion: 1
Chi^2-statistic vs. constant model: 89.7, p -value $=1.4 \mathrm{e}-16$

## Fit a Stepwise Logistic Model For a creditscorecard Object Containing Weights

Use the CreditCardData.mat file to load the data (dataWeights) that contains a column (RowWeights) for the weights (using a dataset from Refaat 2011).
load CreditCardData

```
Create a creditscorecard object using the optional name-value pair argument for
'WeightsVar'.
sc = creditscorecard(dataWeights,'IDVar','CustID','WeightsVar','RowWeights')
sc =
    creditscorecard with properties:
```

                        GoodLabel: 0
                            ResponseVar: 'status'
                            WeightsVar: 'RowWeights'
                            VarNames: \{1x12 cell\}
            NumericPredictors: \{1x6 cell\}
        CategoricalPredictors: \{'ResStatus' 'EmpStatus' 'OtherCC'\}
            BinMissingData: 0
                            IDVar: 'CustID'
            PredictorVars: \{1x9 cell\}
                Data: [1200x12 table]
    Perform automatic binning.
sc = autobinning(sc)
sc =
creditscorecard with properties:
GoodLabel: 0
ResponseVar: 'status'
WeightsVar: 'RowWeights'
VarNames: $\{1 \times 12$ cell\}
NumericPredictors: \{1x6 cell\}

```
CategoricalPredictors: {'ResStatus' 'EmpStatus' 'OtherCC'}
    BinMissingData: 0
    IDVar: 'CustID'
    PredictorVars: {1x9 cell}
        Data: [1200x12 table]
```

Use fitmodel to fit a logistic regression model using Weight of Evidence (WOE) data. fitmodel internally transforms all the predictor variables into WOE values, using the bins found with the automatic binning process. fitmodel then fits a logistic regression model using a stepwise method (by default). When the optional name-value pair argument 'WeightsVar' is used to specify observation (sample) weights, the mdl output uses the weighted counts with stepwiseglm and fitglm.
[sc,mdl] = fitmodel(sc);

1. Adding CustIncome, Deviance $=764.3187$, Chi2Stat $=15.81927$, PValue $=6.968927 \mathrm{e}-05$
2. Adding TmWBank, Deviance $=751.0215$, Chi2Stat $=13.29726$, PValue $=0.0002657942$
3. Adding AMBalance, Deviance $=743.7581$, Chi2Stat $=7.263384$, PValue $=0.007037455$

Generalized linear regression model:
logit(status) ~ 1 + CustIncome + TmWBank + AMBalance
Distribution = Binomial
Estimated Coefficients:

| Estimate | SE | tStat |  |  |
| ---: | ---: | ---: | ---: | ---: | | pValue |
| :--- |
|  |
|  |
| 0.70642 |

1200 observations, 1196 error degrees of freedom
Dispersion: 1
Chi^2-statistic vs. constant model: 36.4, p -value $=6.22 \mathrm{e}-08$

## Fit a Logistic Model with All Predictors

Create a creditscorecard object using the CreditCardData.mat file to load the data (using a dataset from Refaat 2011).

```
load CreditCardData
sc = creditscorecard(data,'IDVar','CustID')
SC =
    creditscorecard with properties:
```

                            GoodLabel: 0
                ResponseVar: 'status'
                    WeightsVar: ''
                            VarNames: \{1x11 cell\}
            NumericPredictors: \{1x6 cell\}
        CategoricalPredictors: \{'ResStatus' 'EmpStatus' 'OtherCC'\}
                        BinMissingData: 0
                            IDVar: 'CustID'
                            PredictorVars: \{1x9 cell\}
                            Data: [1200x11 table]
    Perform automatic binning.

```
sc = autobinning(sc,'Algorithm','EqualFrequency')
sc =
    creditscorecard with properties:
                    GoodLabel: 0
                ResponseVar: 'status'
                    WeightsVar: ''
                            VarNames: {1x11 cell}
            NumericPredictors: {1x6 cell}
        CategoricalPredictors: {'ResStatus' 'EmpStatus' 'OtherCC'}
            BinMissingData: 0
                            IDVar: 'CustID'
            PredictorVars: {1x9 cell}
                            Data: [1200x11 table]
```

Use fitmodel to fit a logistic regression model using Weight of Evidence (WOE) data. fitmodel internally transforms all the predictor variables into WOE values, using the bins found with the automatic binning process. Set the VariableSelection name-value pair argument to FullModel to specify that all predictors must be included in the fitted logistic regression model.

```
sc = fitmodel(sc,'VariableSelection','FullModel');
```

```
Generalized linear regression model:
    status ~ [Linear formula with 10 terms in 9 predictors]
    Distribution = Binomial
Estimated Coefficients:
\begin{tabular}{lll} 
Estimate \(\quad\) SE \(\quad\) tStat \(\quad\) pValue \\
\hline
\end{tabular}
\begin{tabular}{lrrrr} 
(Intercept) & 0.70262 & 0.063862 & 11.002 & \(3.734 \mathrm{e}-28\) \\
CustAge & 0.57683 & 0.27064 & 2.1313 & 0.033062 \\
TmAtAddress & 1.0653 & 0.55233 & 1.9287 & 0.053762 \\
ResStatus & 1.4189 & 0.65162 & 2.1775 & 0.029441 \\
EmpStatus & 0.89916 & 0.29217 & 3.0776 & 0.002087 \\
CustIncome & 0.77506 & 0.21942 & 3.5323 & 0.0004119 \\
TmWBank & 1.0826 & 0.26583 & 4.0727 & \(4.648 \mathrm{e}-05\) \\
OtherCC & 1.1354 & 0.52827 & 2.1493 & 0.031612 \\
AMBalance & 0.99315 & 0.32642 & 3.0425 & 0.0023459 \\
UtilRate & 0.16723 & 0.55745 & 0.29999 & 0.76419
\end{tabular}
1200 observations, 1190 error degrees of freedom
Dispersion: 1
Chi^2-statistic vs. constant model: 85.6, p-value \(=1.25 \mathrm{e}-14\)
```


## Fit a Stepwise Logistic Model When Using Missing Data

Create a creditscorecard object using the CreditCardData.mat file to load the dataMissing with missing values.

```
load CreditCardData.mat
head(dataMissing,5)
\begin{tabular}{|c|c|c|c|c|c|}
\hline CustID & CustAge & TmAtAddress & ResStatus & EmpStatus & CustIncome \\
\hline 1 & 53 & 62 & <undefined> & Unknown & 50000 \\
\hline 2 & 61 & 22 & Home Owner & Employed & 52000 \\
\hline 3 & 47 & 30 & Tenant & Employed & 37000 \\
\hline 4 & NaN & 75 & Home Owner & Employed & 53000 \\
\hline 5 & 68 & 56 & Home Owner & Employed & 53000 \\
\hline
\end{tabular}
```

```
fprintf('Number of rows: %d\n',height(dataMissing))
Number of rows: 1200
fprintf('Number of missing values CustAge: %d\n',sum(ismissing(dataMissing.CustAge)))
Number of missing values CustAge: 30
fprintf('Number of missing values ResStatus: %d\n',sum(ismissing(dataMissing.ResStatus
Number of missing values ResStatus: 40
```

Use creditscorecard with the name-value argument 'BinMissingData' set to true to bin the missing numeric or categorical data in a separate bin.

```
sc = creditscorecard(dataMissing,'IDVar','CustID','BinMissingData',true);
sc = autobinning(sc);
disp(sc)
```

    creditscorecard with properties:
                        GoodLabel: 0
                ResponseVar: 'status'
                    WeightsVar: ''
                            VarNames: \{1x11 cell\}
            NumericPredictors: \{1x6 cell\}
        CategoricalPredictors: \{'ResStatus' 'EmpStatus' 'OtherCC'\}
            BinMissingData: 1
                            IDVar: 'CustID'
                        PredictorVars: \{1x9 cell\}
                                Data: [1200x11 table]
    Display and plot bin information for numeric data for 'CustAge' that includes missing data in a separate bin labelled <missing>.

```
[bi,cp] = bininfo(sc,'CustAge');
disp(bi)
```

| Bin | Good | Bad | Odds | WOE | InfoValue |
| :---: | :---: | :---: | :---: | :---: | :---: |
| '[-Inf,33)' | 69 | 52 | 1.3269 | -0.42156 | 0.018993 |
| ' $[33,37)$ ' | 63 | 45 | 1.4 | -0.36795 | 0.012839 |
| '[37,40) ' | 72 | 47 | 1.5319 | -0.2779 | 0.0079824 |


| '[40,46)' | 172 | 89 | 1.9326 | -0.04556 | 0.0004549 |
| :--- | ---: | ---: | ---: | ---: | ---: |
| '[46,48)' | 59 | 25 | 2.36 | 0.15424 | 0.0016199 |
| '[48,51)' | 99 | 41 | 2.4146 | 0.17713 | 0.0035449 |
| '[51,58)' | 157 | 62 | 2.5323 | 0.22469 | 0.0088407 |
| '[58,Inf]' | 93 | 25 | 3.72 | 0.60931 | 0.032198 |
| '<missing>' | 19 | 11 | 1.7273 | -0.15787 | 0.00063885 |
| 'Totals' $^{\text {'Tota }}$ | 803 | 397 | 2.0227 | NaN | 0.087112 |

```
plotbins(sc,'CustAge')
```


$\square$ Good $\square \mathrm{Bad}$

Display and plot bin information for categorical data for 'ResStatus ' that includes missing data in a separate bin labelled <missing>.
[bi,cg] = bininfo(sc,'ResStatus');
disp(bi)

| Bin | Good | Bad | 0dds | WOE | InfoValue |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 'Tenant' | 296 | 161 | 1.8385 | -0.095463 | 0.0035249 |
| 'Home Owner' | 352 | 171 | 2.0585 | 0.017549 | 0.00013382 |
| 'Other' | 128 | 52 | 2.4615 | 0.19637 | 0.0055808 |
| '<missing>' | 27 | 13 | 2.0769 | 0.026469 | 2.3248e-05 |
| 'Totals' | 803 | 397 | 2.0227 | NaN | 0.0092627 |

ResStatus


Use fitmodel to fit a logistic regression model using Weight of Evidence (WOE) data. fitmodel internally transforms all the predictor variables into WOE values, using the bins found with the automatic binning process. fitmodel then fits a logistic regression
model using a stepwise method (by default). For predictors that have missing data, there is an explicit <missing> bin, with a corresponding WOE value computed from the data. When using fitmodel, the corresponding WOE value for the <missing> bin is applied when performing the WOE transformation. For example, a missing value for customer age (CustAge) is replaced with -0.15787 which is the WOE value for the <missing> bin for the CustAge predictor. However when 'BinMissingData' is false, a missing value for CustAge remains as missing ( NaN ) when applying the WOE transformation.

```
[sc,mdl] = fitmodel(sc);
```

1. Adding CustIncome, Deviance $=1490.8527$, Chi2Stat $=32.588614$, PValue $=1.1387992 \mathrm{e}-\mathrm{C}$
2. Adding TmWBank, Deviance $=1467.1415$, Chi2Stat $=23.711203$, PValue $=1.1192909 \mathrm{e}-06$
3. Adding AMBalance, Deviance $=1455.5715$, Chi2Stat $=11.569967$, PValue $=0.0006702560$
4. Adding EmpStatus, Deviance = 1447.3451, Chi2Stat = 8.2264038, PValue $=0.0041285257$
5. Adding CustAge, Deviance $=1442.8477$, Chi2Stat $=4.4974731$, PValue $=0.033944979$
6. Adding ResStatus, Deviance $=1438.9783$, Chi2Stat $=3.86941$, PValue $=0.049173805$
7. Adding OtherCC, Deviance $=1434.9751$, Chi2Stat $=4.0031966$, PValue $=0.045414057$

Generalized linear regression model:
status ~ [Linear formula with 8 terms in 7 predictors]
Distribution = Binomial
Estimated Coefficients:

| Estimate | SE | tStat | pValue |
| :---: | :---: | :---: | :---: |
| 0.70229 | 0.063959 | 10.98 | 4.7498e-28 |
| 0.57421 | 0.25708 | 2.2335 | 0.025513 |
| 1.3629 | 0.66952 | 2.0356 | 0.04179 |
| 0.88373 | 0.2929 | 3.0172 | 0.002551 |
| 0.73535 | 0.2159 | 3.406 | 0.00065929 |
| 1.1065 | 0.23267 | 4.7556 | $1.9783 \mathrm{e}-06$ |
| 1.0648 | 0.52826 | 2.0156 | 0.043841 |
| 1.0446 | 0.32197 | 3.2443 | 0.0011775 |

```
1200 observations, 1192 error degrees of freedom
Dispersion: 1
Chi^2-statistic vs. constant model: 88.5, p-value \(=2.55 \mathrm{e}-16\)
```


## Input Arguments

## sc - Credit scorecard model

creditscorecard object
Credit scorecard model, specified as a creditscorecard object. Use creditscorecard to create a creditscorecard object.

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

```
Example:[sc,mdl] = fitmodel(sc,'VariableSelection','FullModel')
```


## PredictorVars - Predictor variables for fitting creditscorecard object all predictors in the creditscorecard object (default) | cell array of character vectors

Predictor variables for fitting the creditscorecard object, specified as the commaseparated pair consisting of 'PredictorVars' and a cell array of character vectors. When provided, the creditscorecard object property PredictorsVars is updated. When not provided, the predictors used to create the creditscorecard object (by using creditscorecard) are used.

```
VariableSelection - Variable selection method to fit logistic regression model
'Stepwise' (default)| character vector with values 'Stepwise', 'FullModel'
```

The variable selection method to fit the logistic regression model, specified as the commaseparated pair consisting of 'VariableSelection' and a character vector with values 'Stepwise' or 'FullModel':

- Stepwise - Uses a stepwise selection method which calls the Statistics and Machine Learning Toolbox function stepwiseglm. Only variables in PredictorVars can potentially become part of the model and uses the StartingModel name-value pair argument to select the starting model.
- FullModel - Fits a model with all predictor variables in the PredictorVars namevalue pair argument and calls fitglm.

Note Only variables in the PredictorVars property of the creditscorecard object can potentially become part of the logistic regression model and only linear terms are included in this model with no interactions or any other higher-order terms.

The response variable is mapped so that "Good" is 1 and "Bad" is 0 .

## Data Types: char

## StartingModel - Initial model for Stepwise variable selection 'Constant ' (default)|character vector with values 'Constant', 'Linear'

Initial model for the Stepwise variable selection method, specified as the commaseparated pair consisting of 'StartingModel' and a character vector with values 'Constant' or 'Linear'. This option determines the initial model (constant or linear) that the Statistics and Machine Learning Toolbox function stepwiseglm starts with.

- Constant - Starts the stepwise method with an empty (constant only) model.
- Linear - Starts the stepwise method from a full (all predictors in) model.

Note StartingModel is used only for the Stepwise option of VariableSelection and has no effect for the FullModel option of VariableSelection.

## Data Types: char

## Display - Indicator to display model information at command line

'On' (default) | character vector with values 'On', 'Off'
Indicator to display model information at command line, specified as the commaseparated pair consisting of 'Display' and a character vector with value 'On' or 'Off'.

Data Types: char

## Output Arguments

sc - Credit scorecard model

creditscorecard object

Credit scorecard model, returned as an updated creditscorecard object. The creditscorecard object contains information about the model predictors and coefficients used to fit the WOE data. For more information on using the creditscorecard object, see creditscorecard.

## mdl - Fitted logistic model

GeneralizedLinearModel object
Fitted logistic model, retuned as an object of type GeneralizedLinearModel containing the fitted model. For more information on a GeneralizedLinearModel object, see GeneralizedLinearModel.

Note When creating the creditscorecard object with creditscorecard, if the optional name-value pair argument WeightsVar was used to specify observation (sample) weights, then mdl uses the weighted counts with stepwiseglm and fitglm.

## Definitions

## Using fitmodel with Weights

When observation weights are provided in the credit scorecard data, the weights are used to calibrate the model coefficients.

The underlying Statistics and Machine Learning Toolboxfunctionality for stepwiseglm and fitglm supports observation weights. The weights also affect the logistic model through the WOE values. The WOE transformation is applied to all predictors before fitting the logistic model. The observation weights directly impact the WOE values. For more information, see "Using bininfo with Weights" on page 19-2162 and "Credit Scorecard Modeling Using Observation Weights" on page 8-70.

Therefore, the credit scorecard points and final score depend on the observation weights through both the logistic model coefficients and the WOE values.

## Algorithms

For the logistic regression model used in the creditscorecard object, the probability of being "Bad" is given by

ProbBad $=\exp (-s) /(1+\exp (-s))$.

## References

[1] Anderson, R. The Credit Scoring Toolkit. Oxford University Press, 2007.
[2] Refaat, M. Credit Risk Scorecards: Development and Implementation Using SAS. lulu.com, 2011.

## See Also

GeneralizedLinearModel |autobinning | bindata|bininfo|creditscorecard |displaypoints|fitglm|formatpoints|modifybins|modifypredictor| plotbins|predictorinfo|probdefault|score|setmodel|stepwiseglm| validatemodel

## Topics

"Case Study for a Credit Scorecard Analysis" on page 8-101
"Credit Scorecard Modeling with Missing Values" on page 8-73
"Troubleshooting Credit Scorecard Results" on page 8-91
"Credit Scorecard Modeling Workflow" on page 8-67
"About Credit Scorecards" on page 8-62
"Credit Scorecard Modeling Using Observation Weights" on page 8-70
"What Are Generalized Linear Models?" (Statistics and Machine Learning Toolbox)

## Introduced in R2014b

## setmodel

Set model predictors and coefficients

## Syntax

sc = setmodel(sc,ModelPredictors,ModelCoefficients)

## Description

sc = setmodel(sc,ModelPredictors,ModelCoefficients) sets the predictors and coefficients of a linear logistic regression model fitted outside the creditscorecard object and returns an updated creditscorecard object. The predictors and coefficients are used for the computation of scorecard points. Use setmodel in lieu of fitmodel, which fits a linear logistic regression model, because setmodel offers increased flexibility. For example, when a model fitted with fitmodel needs to be modified, you can use setmodel. For more information, see "Workflows for Using setmodel" on page 192081.

Note When using setmodel, the following assumptions apply:

- The model coefficients correspond to a linear logistic regression model (where only linear terms are included in the model and there are no interactions or any other higher-order terms).
- The model was previously fitted using Weight of Evidence (WOE) data with the response mapped so that 'Good' is 1 and 'Bad' is 0.


## Examples

## Modify a GLM Model Fitted with fitmodel

This example shows how to use setmodel to make modifications to a logistic regression model initially fitted using the fitmodel function, and then set the new logistic regression model predictors and coefficients back into the creditscorecard object.

Create a creditscorecard object using the CreditCardData.mat file to load the data (using a dataset from Refaat 2011).

```
load CreditCardData
sc = creditscorecard(data,'IDVar','CustID')
sc =
    creditscorecard with properties:
```

```
                        GoodLabel: 0
            ResponseVar: 'status'
                    WeightsVar: ''
                            VarNames: {1x11 cell}
            NumericPredictors: {1x6 cell}
        CategoricalPredictors: {'ResStatus' 'EmpStatus' 'OtherCC'}
                BinMissingData: 0
                            IDVar: 'CustID'
                PredictorVars: {1x9 cell}
                            Data: [1200x11 table]
```

Perform automatic binning.
sc = autobinning(sc);
The standard workflow is to use the fitmodel function to fit a logistic regression model using a stepwise method. However, fitmodel only supports limited options regarding the stepwise procedure. You can use the optional mdl output argument from fitmodel to get a copy of the fitted GeneralizedLinearModel object, to later modify.

```
[sc,mdl] = fitmodel(sc);
```

1. Adding CustIncome, Deviance = 1490.8527, Chi2Stat = 32.588614, PValue = 1.1387992e-
2. Adding TmWBank, Deviance $=1467.1415$, Chi2Stat $=23.711203$, PValue $=1.1192909 \mathrm{e}-06$
3. Adding AMBalance, Deviance $=1455.5715$, Chi2Stat $=11.569967$, PValue $=0.0006702560$
4. Adding EmpStatus, Deviance $=1447.3451$, Chi2Stat $=8.2264038$, PValue $=0.0041285257$
5. Adding CustAge, Deviance $=1441.994$, Chi2Stat $=5.3511754$, PValue $=0.020708306$
6. Adding ResStatus, Deviance $=1437.8756$, Chi2Stat $=4.118404$, PValue $=0.042419078$
7. Adding OtherCC, Deviance $=1433.707$, Chi2Stat $=4.1686018$, PValue $=0.041179769$
```
Generalized linear regression model:
    status ~ [Linear formula with 8 terms in 7 predictors]
    Distribution = Binomial
Estimated Coefficients:
\begin{tabular}{|c|c|c|c|}
\hline Estimate & SE & tStat & pValue \\
\hline 0.70239 & 0.064001 & 10.975 & 5.0538e-28 \\
\hline 0.60833 & 0.24932 & 2.44 & 0.014687 \\
\hline 1.377 & 0.65272 & 2.1097 & 0.034888 \\
\hline 0.88565 & 0.293 & 3.0227 & 0.0025055 \\
\hline 0.70164 & 0.21844 & 3.2121 & 0.0013179 \\
\hline 1.1074 & 0.23271 & 4.7589 & \(1.9464 \mathrm{e}-06\) \\
\hline 1.0883 & 0.52912 & 2.0569 & 0.039696 \\
\hline 1.045 & 0.32214 & 3.2439 & 0.001179 \\
\hline
\end{tabular}
```

1200 observations, 1192 error degrees of freedom Dispersion: 1
Chi^2-statistic vs. constant model: 89.7, p -value $=1.4 \mathrm{e}-16$
Suppose you want to include, or "force," the predictor 'UtilRate' in the logistic regression model, even though the stepwise method did not include it in the fitted model. You can add 'UtilRate' to the logistic regression model using the GeneralizedLinearModel object mdl directly.

```
mdl = mdl.addTerms('UtilRate')
mdl =
Generalized linear regression model:
    status ~ [Linear formula with 9 terms in 8 predictors]
    Distribution = Binomial
Estimated Coefficients:
Estimate SE tStat pValue
\begin{tabular}{lrrrr} 
(Intercept) & 0.70239 & 0.064001 & 10.975 & \(5.0538 \mathrm{e}-28\) \\
CustAge & 0.60843 & 0.24936 & 2.44 & 0.014687 \\
ResStatus & 1.3773 & 0.6529 & 2.1096 & 0.034896 \\
EmpStatus & 0.88556 & 0.29303 & 3.0221 & 0.0025103 \\
CustIncome & 0.70146 & 0.2186 & 3.2089 & 0.0013324 \\
TmWBank & 1.1071 & 0.23307 & 4.7503 & \(2.0316 e-06\)
\end{tabular}
```

```
\begin{tabular}{lrrrr} 
OtherCC & 1.0882 & 0.52918 & 2.0563 & 0.03975 \\
AMBalance & 1.0413 & 0.36557 & 2.8483 & 0.004395 \\
UtilRate & 0.013157 & 0.60864 & 0.021618 & 0.98275
\end{tabular}
1200 observations, }1191\mathrm{ error degrees of freedom
Dispersion: 1
Chi^2-statistic vs. constant model: 89.7, p-value = 5.26e-16
```

Use setmodel to update the model predictors and model coefficients in the creditscorecard object. The ModelPredictors input argument does not explicitly include a string for the intercept. However, the ModelCoefficients input argument does have the intercept information as its first element.

```
    {'CustAge' }
    {'ResStatus' }
    {'EmpStatus' }
    {'CustIncome'}
    {'TmWBank' }
    {'OtherCC' }
    {'AMBalance' }
    {'UtilRate' }
```

ModelPredictors = mdl.PredictorNames
ModelPredictors = 8x1 cell array
ModelCoefficients = mdl.Coefficients.Estimate
ModelCoefficients = 9×1
0.7024
0.6084
1.3773
0.8856
0.7015
1.1071
1.0882
1.0413
0.0132
sc = setmodel(sc,ModelPredictors,ModelCoefficients);

Verify that 'UtilRate' is part of the scorecard predictors by displaying the scorecard points.
!

| '[-Inf,33)' | -0.17152 |
| :---: | :---: |
| ' $[33,37)^{\prime}$ | -0.15295 |
| ' $[37,40$ )' | -0.072892 |
| ' $[40,46$ )' | 0.033856 |
| ' $[46,48$ )' | 0.20193 |
| ' $[48,58$ )' | 0.21787 |
| '[58,Inf]' | 0.46652 |
| 'Tenant' | -0.043826 |
| 'Home Owner' | 0.11442 |
| 'Other' | 0.36394 |
| 'Unknown' | -0.088843 |
| 'Employed' | 0.30193 |
| '[-Inf, 29000)' | -0.46956 |
| '[29000, 33000)' | -0.11715 |
| [33000, 35000)' | 0.039798 |
| [35000, 40000) | 0.069073 |

Bi

```
pi = displaypoints(sc)
```

pi = displaypoints(sc)
pi=33\times3 table
pi=33\times3 table
Predictors
Predictors
'CustAge'
'CustAge'
'CustAge'
'CustAge'
'CustAge'
'CustAge'
'CustAge'
'CustAge'
'CustAge'
'CustAge'
'CustAge'
'CustAge'
'CustAge'
'CustAge'
'ResStatus'
'ResStatus'
'ResStatus'
'ResStatus'
'ResStatus'
'ResStatus'
'EmpStatus'
'EmpStatus'
'EmpStatus'
'EmpStatus'
'CustIncome
'CustIncome
'CustIncome
'CustIncome
'CustIncome' '[33000,35000)' 0.039798
'CustIncome' '[33000,35000)' 0.039798
'CustIncome
'CustIncome
'CustAge'
'CustAge'
'[29000,33000)' -0.11715
'[29000,33000)' -0.11715
' [35000, 40000)
' [35000, 40000)
Points
Points
Bin

```
        Bin
```


## Fit a Logistic Regression Model Outside of the creditscorecard Object

This example shows how to use setmodel to fit a logistic regression model directly, without using the fitmodel function, and then set the new model predictors and coefficients back into the creditscorecard object. This approach gives more flexibility regarding options to control the stepwise procedure. This example fits a logistic regression model with a nondefault value for the 'PEnter' parameter, the criterion to admit a new predictor in the logistic regression model during the stepwise procedure.

Create a creditscorecard object using the CreditCardData.mat file to load the data (using a dataset from Refaat 2011). Use the 'IDVar' argument to indicate that 'CustID ' contains ID information and should not be included as a predictor variable.

```
load CreditCardData
sc = creditscorecard(data,'IDVar','CustID')
SC =
    creditscorecard with properties:
                    GoodLabel: 0
                    ResponseVar: 'status'
                    WeightsVar:
                            VarNames: {1x11 cell}
            NumericPredictors: {1x6 cell}
        CategoricalPredictors: {'ResStatus' 'EmpStatus' 'OtherCC'}
                BinMissingData: 0
                                    IDVar: 'CustID'
            PredictorVars: {1x9 cell}
                        Data: [1200x11 table]
```

Perform automatic binning.

```
sc = autobinning(sc);
```

The logistic regression model needs to be fit with Weight of Evidence (WOE) data. The WOE transformation is a special case of binning, since the data first needs to be binned, and then the binned information is mapped to the corresponding WOE values. This transformation is done using the bindata function. bindata has an argument that prepares the data for the model fitting step. By setting the bindata name-value pair argument for 'OutputType' to WOEModelInput ':

- All predictors are converted to WOE values.
- The output contains only predictors and response (no 'IDVar' or any unused variables).
- Predictors with infinite or undefined (NaN) WOE values are discarded.
- The response values are mapped so that "Good" is 1 and "Bad" is 0 (this implies that higher unscaled scores correspond to better, less risky customers).

```
bd = bindata(sc,'OutputType','WOEModelInput');
```

For example, the first ten rows in the original data for the variables 'CustAge ',
'ResStatus', 'CustIncome', and 'status' (response variable) look like this:
data(1:10,\{'CustAge' 'ResStatus' 'CustIncome' 'status'\})

| CustAge | ResStatus | CustIncome | status |
| :---: | :---: | :---: | :---: |
| 53 | Tenant | 50000 | 0 |
| 61 | Home Owner | 52000 | 0 |
| 47 | Tenant | 37000 | 0 |
| 50 | Home Owner | 53000 | 0 |
| 68 | Home Owner | 53000 | 0 |
| 65 | Home Owner | 48000 | 0 |
| 34 | Home Owner | 32000 | 1 |
| 50 | Other | 51000 | 0 |
| 50 | Tenant | 52000 | 1 |
| 49 | Home Owner | 53000 | 1 |

Here is how the same ten rows look after calling bindata with the name-value pair argument 'OutputType' set to 'WOEModelInput':

```
bd(1:10,{'CustAge' 'ResStatus' 'CustIncome' 'status'})
ans=10\times4 table
    CustAge ResStatus CustIncome status
\begin{tabular}{rrrr}
0.21378 & -0.095564 & 0.47972 & 1 \\
0.62245 & 0.019329 & 0.47972 & 1 \\
0.18758 & -0.095564 & -0.026696 & 1 \\
0.21378 & 0.019329 & 0.47972 & 1 \\
0.62245 & 0.019329 & 0.47972 & 1 \\
0.62245 & 0.019329 & 0.47972 & 1 \\
-0.39568 & 0.019329 & -0.29217 & 0 \\
0.21378 & 0.20049 & 0.47972 & 1 \\
0.21378 & -0.095564 & 0.47972 & 0 \\
0.21378 & 0.019329 & 0.47972 & 0
\end{tabular}
```

Fit a logistic linear regression model using a stepwise method with the Statistics and Machine Learning Toolbox ${ }^{\mathrm{TM}}$ function stepwiseglm, but use a nondefault value for the 'PEnter' and 'PRemove' optional arguments. The predictors 'ResStatus' and ' OtherCC' would normally be included in the logistic linear regression model using default options for the stepwise procedure.

```
mdl = stepwiseglm(bd,'constant','Distribution','binomial',...
'Upper','linear','PEnter',0.025,'PRemove',0.05)
```

1. Adding CustIncome, Deviance = 1490.8527, Chi2Stat = 32.588614, PValue = 1.1387992e-
2. Adding TmWBank, Deviance $=1467.1415$, Chi2Stat $=23.711203$, PValue $=1.1192909 \mathrm{e}-06$
3. Adding AMBalance, Deviance $=1455.5715$, Chi2Stat $=11.569967$, PValue $=0.0006702560$
4. Adding EmpStatus, Deviance $=1447.3451$, Chi2Stat $=8.2264038$, PValue $=0.0041285257$
5. Adding CustAge, Deviance $=1441.994$, Chi2Stat $=5.3511754$, PValue $=0.020708306$
mdl =
Generalized linear regression model:
logit(status) ~ 1 + CustAge + EmpStatus + CustIncome + TmWBank + AMBalance Distribution = Binomial

Estimated Coefficients:

| Estimate | SE | tStat | pValue |
| :---: | :---: | :---: | :---: |
| 0.70263 | 0.063759 | 11.02 | 3.0544e-28 |
| 0.57265 | 0.2482 | 2.3072 | 0.021043 |
| 0.88356 | 0.29193 | 3.0266 | 0.002473 |
| 0.70399 | 0.21781 | 3.2321 | 0.001229 |
| 1.1 | 0.23185 | 4.7443 | $2.0924 \mathrm{e}-06$ |
| 1.0313 | 0.32007 | 3.2221 | 0.0012724 |

1200 observations, 1194 error degrees of freedom
Dispersion: 1
Chi^2-statistic vs. constant model: 81.4, p-value $=4.18 \mathrm{e}-16$
Use setmodel to update the model predictors and model coefficients in the creditscorecard object. The ModelPredictors input argument does not explicitly include a string for the intercept. However, the ModelCoefficients input argument does have the intercept information as its first element.

```
ModelPredictors = mdl.PredictorNames
ModelPredictors = 5x1 cell array
    {'CustAge' }
    {'EmpStatus' }
    {'CustIncome'}
    {'TmWBank' }
    {'AMBalance' }
```

ModelCoefficients = mdl.Coefficients.Estimate
ModelCoefficients = 6×1
0.7026
0.5726
0.8836
0.7040
1.1000
1.0313

```
sc = setmodel(sc,ModelPredictors,ModelCoefficients);
```

Verify that the desired model predictors are part of the scorecard predictors by displaying the scorecard points.

```
pi = displaypoints(sc)
pi=25\times3 table
    Predictors
Points
```

Bin
'CustAge
'[-Inf,33)
'CustAge'
'CustAge
'CustAge
'CustAge'
'CustAge
'CustAge
'EmpStatus'
'EmpStatus'
'CustIncome'
'CustIncome'
' $[33,37)^{\prime}$
-0.10354
'[37,40)' -0.010713
$\begin{array}{ll}{[40,46)} & 0.089757\end{array}$
$\begin{array}{ll} \\ \\ {[46,48) '} & 0.24794\end{array}$
$\begin{array}{ll} \\ \\ {[48,58)} & 0.26294\end{array}$
'[58,Inf]' 0.49697
'Unknown' -0.035716
'Employed' 0.35417
'[-Inf,29000)' -0.41884
'[29000,33000)' -0.065161
'CustIncome'
'[33000,35000)' 0.092353
'CustIncome'

|  |  |
| :--- | :--- | :--- |
| $[35000,40000) '$ | 0.12173 |

'CustIncome'
$[40000,42000)$$\quad 0.13259$
'CustIncome'
' $[42000,47000$ ) '
0.2854
'CustIncome'
'[47000,Inf]'
0.47824

## Input Arguments

## sc - Credit scorecard model

creditscorecard object

Credit scorecard model, specified as a creditscorecard object. Use creditscorecard to create a creditscorecard object.

ModelPredictors - Predictor names included in fitted model
cell array of character vectors with predictor values
\{'PredictorName1','PredictorName2',...\}
Predictor names included in the fitted model, specified as a cell array of character vectors as \{'PredictorName1','PredictorName2', ...\}. The predictor names must match predictor variable names in the creditscorecard object.

Note Do not include a character vector for the constant term in ModelPredictors, setmodel internally handles the ' (Intercept) ' term based on the number of model coefficients (see ModelCoefficients).

## Data Types: cell

ModelCoefficients - Model coefficients corresponding to model predictors numeric array with values [coeff1, coeff2,..]

Model coefficients corresponding to the model predictors, specified as a numeric array of model coefficients, [coeff1, coeff2,..]. If $N$ is the number of predictor names provided in ModelPredictors, the size of ModelCoefficients can be $N$ or $N+1$. If ModelCoefficients has $N+1$ elements, then the first coefficient is used as the ' (Intercept)' of the fitted model. Otherwise, the ' (Intercept)' is set to 0 .
Data Types: double

## Output Arguments

## sc - Credit scorecard model

creditscorecard object
Credit scorecard model, returned as an updated creditscorecard object. The creditscorecard object contains information about the model predictors and coefficients of the fitted model. For more information on using the creditscorecard object, see creditscorecard.

## Definitions

## Workflows for Using setmodel

When using setmodel, there are two possible workflows to set the final model predictors and model coefficients into a creditscorecard object.

The first workflow is:

- Use fitmodel to get the optional output argument mdl. This is a GeneralizedLinearModel object and you can add and remove terms, or modify the parameters of the stepwise procedure. Only linear terms can be in the model (no interactions or any other higher-order terms).
- Once the GeneralizedLinearModel object is satisfactory, set the final model predictors and model coefficients into the creditscorecard object using the setmodel input arguments for ModelPredictors and ModelCoefficients.

An alternate workflow is:

- Obtain the Weight of Evidence (WOE) data using bindata. Use the 'WOEModel Input ' option for the 'OutputType' name-value pair argument in bindata to ensure that:
- The predictors data is transformed to WOE.
- Only predictors whose bins have finite WOE values are included.
- The response variable is placed in the last column.
- The response variable is mapped ("Good" is 1 and "Bad" is 0 ).
- Use the data from the previous step to fit a linear logistic regression model (only linear terms in the model, no interactions, or any other higher-order terms). See, for example, stepwiseglm.
- Once the GeneralizedLinearModel object is satisfactory, set the final model predictors and model coefficients into the creditscorecard object using the setmodel input arguments for ModelPredictors and ModelCoefficients.


## References

[1] Anderson, R. The Credit Scoring Toolkit. Oxford University Press, 2007.
[2] Refaat, M. Credit Risk Scorecards: Development and Implementation Using SAS. lulu.com, 2011.

## See Also

GeneralizedLinearModel|autobinning|bindata|bininfo|creditscorecard |displaypoints|fitglm|fitmodel|formatpoints|modifybins| modifypredictor|plotbins|predictorinfo|probdefault|score| stepwiseglm|validatemodel

## Topics

"Case Study for a Credit Scorecard Analysis" on page 8-101
"Troubleshooting Credit Scorecard Results" on page 8-91
"Credit Scorecard Modeling Workflow" on page 8-67
"About Credit Scorecards" on page 8-62

Introduced in R2014b

## bindata

Binned predictor variables

## Syntax

```
bdata = bindata(sc)
bdata = bindata(sc,data)
bdata = bindata(sc,Name,Value)
```


## Description

bdata $=$ bindata(sc) binned predictor variables returned as a table. This is a table of the same size as the data input, but only the predictors specified in the creditscorecard object's PredictorVars property are binned and the remaining ones are unchanged.
bdata $=$ bindata(sc,data) returns a table of binned predictor variables. bindata returns a table of the same size as the creditscorecard data, but only the predictors specified in the creditscorecard object's PredictorVars property are binned and the remaining ones are unchanged.
bdata $=$ bindata(sc,Name, Value) binned predictor variables returned as a table using optional name-value pair arguments. This is a table of the same size as the data input, but only the predictors specified in the creditscorecard object's PredictorVars property are binned and the remaining ones are unchanged.

## Examples

## Bin creditscorecard Data as Bin Numbers, Categories, or WOE Values

This example shows how to use the bindata function to simply bin or discretize data.
Suppose bin ranges of

- '0 to 30'
- '31 to 50'
- '51 and up'
are determined for the age variable (via manual or automatic binning). If a data point with age 41 is given, binning this data point means placing it in the bin for 41 years old, which is the second bin, or the ' 31 to 50 ' bin. Binning is then the mapping from the original data, into discrete groups or bins. In this example, you can say that a 41-year old is mapped into bin number 2, or that it is binned into the '31 to 50' category. If you know the Weight of Evidence (WOE) value for each of the three bins, you could also replace the data point 41 with the WOE value corresponding to the second bin. bindata supports the three binning formats just mentioned:
- Bin number (where the 'OutputType' name-value pair argument is set to 'BinNumber'); this is the default option, and in this case, 41 is mapped to bin 2.
- Categorical (where the 'OutputType ' name-value pair argument is set to 'Categorical' ); in this case, 41 is mapped to the '31 to 50' bin.
- WOE value (where the 'OutputType' name-value pair argument is set to 'WOE'); in this case, 41 is mapped to the WOE value of bin number 2.

Create a creditscorecard object using the CreditCardData.mat file to load the data (using a dataset from Refaat 2011). Use the 'IDVar' argument to indicate that
'CustID ' contains ID information and should not be included as a predictor variable.

```
load CreditCardData
sc = creditscorecard(data,'IDVar','CustID')
sc =
    creditscorecard with properties:
```

                    GoodLabel: 0
                    ResponseVar: 'status'
                    WeightsVar:
                            VarNames: \{1×11 cell\}
            NumericPredictors: \{1x6 cell\}
        CategoricalPredictors: \{'ResStatus' 'EmpStatus' 'OtherCC'\}
            BinMissingData: 0
                    IDVar: 'CustID'
            PredictorVars: \{1x9 cell\}
                    Data: [1200x11 table]
    Perform automatic binning.

```
sc = autobinning(sc);
```

Show the bin information for 'CustAge'.

```
bininfo(sc,'CustAge')
```

ans $=8 \times 6$ table
Bin
-
'[-Inf,33)'
'[(33,37)'
'([37,40)'
'(40,46)'
'[46,48)'
'[48,58)'
'[58,Inf]'
'Totals'

According to the bin information, the first age should be mapped into the fourth bin, the second age into the fifth bin, etc. These are the first 10 binned ages, in bin-number format.

```
bdata(1:10,'CustAge')
ans=10\times1 table
    CustAge
6
7
5
6
7
7
2
6
6
6
```

Bin the scorecard data and show their bin labels. To do this, set the bindata name-value pair argument for 'OutputType' to 'Categorical'.

```
bdata = bindata(sc,'OutputType','Categorical');
```

These are the first 10 binned ages, in categorical format.

```
bdata(1:10,'CustAge')
ans=10\times1 table
    CustAge
    [48,58)
    [58,Inf]
    [46,48)
    [48,58)
    [58,Inf]
    [58,Inf]
    [33,37)
    [48,58)
    [48,58)
```

Convert the scorecard data to WOE values. To do this, set the bindata name-value pair argument for 'OutputType' to 'WOE'.

```
bdata = bindata(sc,'OutputType','WOE');
```

These are the first 10 binned ages, in WOE format. The ages are mapped to the WOE values that are internally displayed using the bininfo function.

```
bdata(1:10,'CustAge')
ans=10\times1 table
    CustAge
```

        0.21378
        0.62245
        0.18758
        0.21378
        0.62245
        0.62245
        -0. 39568
        0.21378
        0.21378
        0.21378
    
## Bin Additional "Test" Data

This example shows how to use the bindata function's optional input for the data to bin. If not provided, bindata bins the creditscorecard training data. However, if a different dataset needs to be binned, for example, some "test" data, this can be passed into bindata as an optional input.

Create a creditscorecard object using the CreditCardData.mat file to load the data (using a dataset from Refaat 2011). Use the 'IDVar' argument to indicate that 'CustID ' contains ID information and should not be included as a predictor variable.
load CreditCardData
sc = creditscorecard(data,'IDVar','CustID')

```
sc =
    creditscorecard with properties:
                    GoodLabel: 0
            ResponseVar: 'status'
            WeightsVar: '
                    VarNames: {1x11 cell}
        NumericPredictors: {1x6 cell}
        CategoricalPredictors: {'ResStatus' 'EmpStatus' 'OtherCC'}
            BinMissingData: 0
                    IDVar: 'CustID'
            PredictorVars: {1x9 cell}
                        Data: [1200x11 table]
```

Perform automatic binning.

```
sc = autobinning(sc);
```

Show the bin information for 'CustAge'.

| $\begin{gathered} \text { ans }=8 \times 6 \text { table } \\ \text { Bin } \end{gathered}$ | Good | Bad | 0dds | WOE | InfoValue |
| :---: | :---: | :---: | :---: | :---: | :---: |
| '[-Inf,33)' | 70 | 53 | 1.3208 | -0.42622 | 0.019746 |
| ' $[33,37)^{\prime}$ | 64 | 47 | 1.3617 | -0.39568 | 0.015308 |
| ' $[37,40$ )' | 73 | 47 | 1.5532 | -0.26411 | 0.0072573 |
| ' $[40,46$ )' | 174 | 94 | 1.8511 | -0.088658 | 0.001781 |
| ' $[46,48$ )' | 61 | 25 | 2.44 | 0.18758 | 0.0024372 |
| ' $[48,58$ )' | 263 | 105 | 2.5048 | 0.21378 | 0.013476 |
| '[58,Inf]' | 98 | 26 | 3.7692 | 0.62245 | 0.0352 |
| 'Totals' | 803 | 397 | 2.0227 | NaN | 0.095205 |

For the purpose of illustration, take a few rows from the original data as "test" data and display the first 10 age values in the test data.

```
tdata = data(101:110,:);
tdata(1:10,'CustAge')
ans=10\times1 table
    CustAge
```

```
34
59
6 4
6 1
28
6 5
55
37
4 9
5 1
```

Convert the test data to WOE values. To do this, set the bindata name-value pair argument for 'OutputType ' to 'WOE', passing the test data (tdata) as an optional input.

| $\begin{gathered} \text { bdata }=10 \times 11 \\ \text { CustID } \end{gathered}$ | table CustAge | TmAtAddress | ResStatus | EmpStatus | CustIncome | TmWBar |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 101 | -0.39568 | -0.087767 | -0.095564 | 0.2418 | -0.011271 | 0.768 |
| 102 | 0.62245 | 0.14288 | 0.019329 | -0.19947 | 0.20579 | -0.13 |
| 103 | 0.62245 | 0.02263 | 0.019329 | 0.2418 | 0.47972 | -0.12 |
| 104 | 0.62245 | 0.02263 | -0.095564 | 0.2418 | 0.47972 | -0.12 |
| 105 | -0.42622 | 0.02263 | 0.019329 | 0.2418 | -0.06843 | 0.768 |
| 106 | 0.62245 | 0.02263 | 0.019329 | -0.19947 | 0.20579 | -0.13 |
| 107 | 0.21378 | -0.087767 | -0.095564 | 0.2418 | 0.47972 | 0.26 |
| 108 | -0.26411 | -0.087767 | 0.019329 | -0.19947 | -0.29217 | -0.13 |
| 109 | 0.21378 | -0.087767 | -0.095564 | 0.2418 | -0.026696 | -0.13 |
| 110 | 0.21378 | -0.087767 | 0.019329 | 0.2418 | 0.20579 | -0.13 |

These are the first 10 binned ages, in WOE format. The ages are mapped to the WOE values displayed internally by bininfo.

```
bdata(1:10,'CustAge')
ans=10\times1 table
    CustAge
```

-0. 039568
0.62245
0.62245
0.62245
0.42622
0.62245
0.21378
-0.26411
0.21378
0.21378

## Bin Additional "Test" Data When Using the 'BinMissingData' Option

Create a creditscorecard object using the CreditCardData.mat file to load the data with missing values. The variables CustAge and ResStatus have missing values.
load CreditCardData.mat
head(dataMissing,5)

| CustID | CustAge | TmAtAddress | ResStatus | EmpStatus | CustIncome |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 53 | 62 | <undefined> | Unknown | 50000 |
| 2 | 61 | 22 | Home Owner | Employed | 52000 |
| 3 | 47 | 30 | Tenant | Employed | 37000 |
| 4 | NaN | 75 | Home Owner | Employed | 53000 |
| 5 | 68 | 56 | Home Owner | Employed | 53000 |

Use creditscorecard with the name-value argument 'BinMissingData' set to true to bin the missing numeric or categorical data in a separate bin. Apply automatic binning.

```
sc = creditscorecard(dataMissing,'IDVar','CustID','BinMissingData',true);
sc = autobinning(sc);
disp(sc)
    creditscorecard with properties:
```

```
            ResponseVar: 'status'
            WeightsVar: '
            VarNames: {1x11 cell}
    NumericPredictors: {1x6 cell}
CategoricalPredictors: {'ResStatus' 'EmpStatus' 'OtherCC'}
            BinMissingData: 1
                        IDVar: 'CustID'
            PredictorVars: {1x9 cell}
                            Data: [1200x11 table]
```

Display and plot bin information for numeric data for 'CustAge' that includes missing data in a separate bin labelled <missing>.

```
[bi,cp] = bininfo(sc,'CustAge');
disp(bi)
```

| Bin | Good | Bad | Odds | WOE | InfoValue |
| :---: | :---: | :---: | :---: | :---: | :---: |
| '[-Inf,33)' | 69 | 52 | 1.3269 | -0.42156 | 0.018993 |
| ' $[33,37)^{\prime}$ | 63 | 45 | 1.4 | -0.36795 | 0.012839 |
| ' $[37,40$ )' | 72 | 47 | 1.5319 | -0.2779 | 0.0079824 |
| '[40,46) ' | 172 | 89 | 1.9326 | -0.04556 | 0.0004549 |
| ' $[46,48$ )' | 59 | 25 | 2.36 | 0.15424 | 0.0016199 |
| ' $[48,51$ )' | 99 | 41 | 2.4146 | 0.17713 | 0.0035449 |
| ' $[51,58$ )' | 157 | 62 | 2.5323 | 0.22469 | 0.0088407 |
| '[58, Inf]' | 93 | 25 | 3.72 | 0.60931 | 0.032198 |
| '<missing>' | 19 | 11 | 1.7273 | -0.15787 | 0.00063885 |
| 'Totals' | 803 | 397 | 2.0227 | NaN | 0.087112 |

```
plotbins(sc,'CustAge')
```



Display and plot bin information for categorical data for 'ResStatus' that includes missing data in a separate bin labelled <missing>.
[bi,cg] = bininfo(sc,'ResStatus'); disp(bi)

| Bin | Good | Bad | Odds | WOE | InfoValue |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 'Tenant ' | 296 | 161 | 1.8385 | -0.095463 | 0.0035249 |
| 'Home Owner' | 352 | 171 | 2.0585 | 0.017549 | 0.00013382 |
| 'Other' | 128 | 52 | 2.4615 | 0.19637 | 0.0055808 |
| '<missing>' | 27 | 13 | 2.0769 | 0.026469 | 2.3248e-05 |
| 'Totals' | 803 | 397 | 2.0227 | NaN | 0.0092627 |

```
plotbins(sc,'ResStatus')
```

ResStatus


For the 'CustAge' and 'ResStatus' predictors, there is missing data (NaNs and <undefined>) in the training data, and the binning process estimates a WOE value of - 0.15787 and 0.026469 respectively for missing data in these predictors, as shown above.

For the purpose of illustration, take a few rows from the original data as test data and introduce some missing data.

```
tdata = dataMissing(11:14,:);
tdata.CustAge(1) = NaN;
tdata.TmAtAddress(2) = NaN;
tdata.ResStatus(3) = '<undefined>';
```

| CustID | CustAge | TmAtAddress | ResStatus | EmpStatus | CustIncome |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 11 | NaN | 24 | Tenant | Unknown | 34000 |
| 12 | 48 | NaN | Other | Unknown | 44000 |
| 13 | 65 | 63 | <undefined> | Unknown | 48000 |
| 14 | 44 | 75 | Other | <undefined> | 41000 |

Convert the test data to WOE values. To do this, set the bindata name-value pair argument for 'OutputType ' to 'WOE', passing the test data tdata as an optional input.

```
bdata = bindata(sc,tdata,'OutputType','WOE');
disp(bdata)
```

| CustID | CustAge | TmAtAddress | ResStatus | EmpStatus | CustIncome | TmWBaı |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 11 | -0.15787 | 0.02263 | -0.095463 | -0.19947 | -0.06843 | -0.12 |
| 12 | 0.17713 | NaN | 0.19637 | -0.19947 | 0.20579 | -0.13 |
| 13 | 0.60931 | 0.02263 | 0.026469 | -0.19947 | 0.47972 | -0.25 |
| 14 | -0.04556 | 0.02263 | 0.19637 | NaN | -0.011271 | -0.12 |

For the 'CustAge' and 'ResStatus' predictors, because there is missing data in the training data, the missing values in the test data get mapped to the WOE value estimated for the <missing> bin. Therefore, a missing value for 'CustAge' is replaced with -0.15787, and a missing value for 'ResStatus' is replaced with 0.026469.

For 'TmAtAddress ' and 'EmpStatus', the training data has no missing values, therefore there is no bin for missing data, and there is no way to estimate a WOE value for missing data. Therefore, for these predictors, the WOE transformation leaves missing values as missing (that is, sets a WOE value of NaN).

These rules apply when 'OutputType' is set to 'WOE' or 'WOEModelInput'. The rationale is that if a data-based WOE value exists for missing data, it should be used for the WOE transformation and for subsequent steps (for example, fitting a logistic model or scoring).

On the other hand, when 'OutputType' is set to 'BinNumber' or 'Categorical', bindata leaves missing values as missing, since this allows you to subsequently treat the missing data as you see fit.


## Apply a Weight of Evidence (WOE) Transformation to Data

bindata supports the following types of WOE transformation:

- When the 'OutputType' name-value argument is set to 'W0E', bindata simply applies the WOE transformation to all predictors and keeps the rest of the variables in the original data in place and unchanged.
- When the 'OutputType' name-value pair argument is set to 'WOEModelInput', bindata returns a table that can be used directly as an input for fitting a logistic regression model for the scorecard. In this case, bindata:
- Applies WOE transformation to all predictors.
- Returns predictor variables, but no IDVar or unused variables are included in the output.
- Includes the mapped response variable as the last column.
- The fitmodel function calls bindata internally using the 'WOEModelInput ' option to fit the logistic regression model for the creditscorecard model.

Create a creditscorecard object using the CreditCardData.mat file to load the data (using a dataset from Refaat 2011). Use the 'IDVar' argument to indicate that 'CustID ' contains ID information and should not be included as a predictor variable.

```
load CreditCardData
sc = creditscorecard(data,'IDVar','CustID')
sc =
    creditscorecard with properties:
                    GoodLabel: 0
            ResponseVar: 'status'
                    WeightsVar:
                            VarNames: {1x11 cell}
        NumericPredictors: {1x6 cell}
        CategoricalPredictors: {'ResStatus' 'EmpStatus' 'OtherCC'}
        BinMissingData: 0
                            IDVar: 'CustID'
            PredictorVars: {1x9 cell}
                        Data: [1200x11 table]
```

Perform automatic binning.

```
sc = autobinning(sc);
```

Show the bin information for 'CustAge'.

| $\begin{gathered} \text { ans }=8 \times 6 \text { table } \\ \text { Bin } \end{gathered}$ | Good | Bad | Odds | WOE | InfoValue |
| :---: | :---: | :---: | :---: | :---: | :---: |
| '[-Inf,33)' | 70 | 53 | 1.3208 | -0.42622 | 0.019746 |
| ' $[33,37)$ ' | 64 | 47 | 1.3617 | -0.39568 | 0.015308 |
| ' $[37,40)^{\prime}$ | 73 | 47 | 1.5532 | -0.26411 | 0.0072573 |
| '[40,46)' | 174 | 94 | 1.8511 | -0.088658 | 0.001781 |
| ' $[46,48$ )' | 61 | 25 | 2.44 | 0.18758 | 0.0024372 |
| ' $[48,58$ ) ' | 263 | 105 | 2.5048 | 0.21378 | 0.013476 |


| '[58,Inf]' | 98 | 26 | 3.7692 | 0.62245 | 0.0352 |
| :--- | ---: | ---: | ---: | ---: | ---: |
| 'Totals' | 803 | 397 | 2.0227 | NaN | 0.095205 |

These are the first 10 age values in the original data, used to create the creditscorecard object.

```
data(1:10,'CustAge')
```

ans $=10 \times 1$ table
CustAge
53
61
47
50
68
65
34
50
50
49

Convert the test data to WOE values. To do this, set the bindata name-value pair argument for 'OutputType' to 'WOE'.
bdata = bindata(sc,'OutputType','WOE');
These are the first 10 binned ages, in WOE format. The ages are mapped to the WOE values displayed internally by bininfo.

```
bdata(1:10,'CustAge')
```

```
ans=10\times1 table
```

    CustAge
    0.21378
0.62245
0.18758
0.21378
0.62245
0.62245
-0. 39568
0.21378
0.21378
0.21378

These are the first 10 binned ages, in WOE format. The ages are mapped to the WOE values displayed internally by bininfo.

```
bdata(1:10,'CustAge')
ans=10\times1 table
    CustAge
        0.21378
        0.62245
        0.18758
        0.21378
        0.62245
        0.62245
        -0.39568
        0.21378
        0.21378
        0.21378
```

The size of the original data and the size of bdata output are the same because bindata leaves unused variables (such as 'IDVar') unchanged and in place.

```
whos data bdata
```

| Name | Size | Bytes | Class Attributes |
| :--- | ---: | ---: | :--- | ---: |
| bdata | $1200 \times 11$ | 109204 | table |
| data | $1200 \times 11$ | 84876 | table |

The response values are the same in the original data and in the binned data because, by default, bindata does not modify response values.

```
disp([data.status(1:10) bdata.status(1:10)])
```

$0 \quad 0$
$0 \quad 0$
$0 \quad 0$

| 0 | 0 |
| :--- | :--- |
| 0 | 0 |
| 0 | 0 |
| 1 | 1 |
| 0 | 0 |
| 1 | 1 |
| 1 | 1 |

When fitting a logistic regression model with WOE data, set the 'OutputType' namevalue pair argument to 'WOEModelInput'.
bdata = bindata(sc,'OutputType','WOEModelInput');
The binned predictor data is the same as when the 'OutputType' name-value pair argument is set to 'WOE '.

```
bdata(1:10,'CustAge')
```

```
ans=10\times1 table
```

    CustAge
    0.21378
0.62245
0. 18758
0.21378
0.62245
0.62245
-0. 39568
0.21378
0.21378
0.21378

However, the size of the original data and the size of bdata output are different. This is because bindata removes unused variables (such as 'IDVar').

| Name | Size | Bytes | Class | Attributes |
| :---: | :---: | :---: | :---: | :---: |
| bdata | $1200 \times 10$ | 99368 | table |  |
| data | $1200 \times 11$ | 84876 | table |  |

The response values are also modified in this case and are mapped so that "Good" is 1 and "Bad" is 0 .

```
disp([data.status(1:10) bdata.status(1:10)])
0 1
0}
0 1
0 1
0}
0}
10
0 1
1 0
1 0
```


## Input Arguments

## sc - Credit scorecard model

creditscorecard object
Credit scorecard model, specified as a creditscorecard object. Use creditscorecard to create a creditscorecard object.

## data - Data to bin given the rules set in creditscorecard object table

Data to bin given the rules set in the creditscorecard object, specified using a table. By default, data is set to the creditscorecard object's raw data.

Before creating a creditscorecard object, perform a data preparation task to have an appropriately structured data as input to a creditscorecard object.

Data Types: table

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1,Value1, . . . , NameN, ValueN.

Example: bdata =
bindata(sc,'OutputType','W0E','ResponseFormat','Mapped')

## OutputType - Output format

'BinNumber' (default) | character vector with values 'BinNumber', 'Categorical', 'WOE'

Output format, specified as the comma-separated pair consisting of 'OutputType ' and a character vector with the following values:

- BinNumber - Returns the bin numbers corresponding to each observation.
- Categorical - Returns the bin label corresponding to each observation.
- WOE - Returns the Weight of Evidence (WOE) corresponding to each observation.
- WOEModelInput - Use this option when fitting a model. This option:
- Returns the Weight of Evidence (WOE) corresponding to each observation.
- Returns predictor variables, but no IDVar or unused variables are included in the output.
- Discards any predictors whose bins have Inf or NaN WOE values.
- Includes the mapped response variable as the last column.

> Note When the bindata name-value pair argument 'OutputType' is set to 'WOEModel Input', the bdata output only contains the columns corresponding to predictors whose bins do not have Inf or NaN Weight of Evidence (WOE) values, and bdata includes the mapped response as the last column.

> Missing data (if any) are included in the bdata output as missing data as well, and do not influence the rules to discard predictors when 'OutputType' is set to 'WOEModelInput'.

## Data Types: char

## ResponseFormat - Response values format

'RawData' (default)|character vector with values 'RawData', 'Mapped '
Response values format, specified as the comma-separated pair consisting of 'ResponseFormat' and a character vector with the following values:

- RawData - The response variable is copied unchanged into the bdata output.
- Mapped - The response values are modified (if necessary) so that "Good" is mapped to 1 , and "Bad" is mapped to 0 .

Data Types: char

## Output Arguments

## bdata - Binned predictor variables <br> table

Binned predictor variables, returned as a table. This is a table of the same size (see exception in the following Note) as the data input, but only the predictors specified in the creditscorecard object's PredictorVars property are binned and the remaining ones are unchanged.

Note When the bindata name-value pair argument 'OutputType' is set to 'WOEModelInput', the bdata output only contains the columns corresponding to predictors whose bins do not have Inf or NaN Weight of Evidence (WOE) values, and bdata includes the mapped response as the last column.

Missing data (if any) are included in the bdata output as missing data as well, and do not influence the rules to discard predictors when 'OutputType' is set to 'WOEModelInput'.

## References

[1] Anderson, R. The Credit Scoring Toolkit. Oxford University Press, 2007.
[2] Refaat, M. Credit Risk Scorecards: Development and Implementation Using SAS. lulu.com, 2011.

## See Also

autobinning|bininfo|creditscorecard|displaypoints|fitmodel| formatpoints|modifybins|modifypredictor|plotbins|predictorinfo| probdefault|score|setmodel|validatemodel

## Topics

"Case Study for a Credit Scorecard Analysis" on page 8-101
"Credit Scorecard Modeling with Missing Values" on page 8-73
"Troubleshooting Credit Scorecard Results" on page 8-91 "Credit Scorecard Modeling Workflow" on page 8-67
"About Credit Scorecards" on page 8-62

## Introduced in R2014b

## plotbins

Plot histogram counts for predictor variables

## Syntax

```
plotbins(sc,PredictorName)
hFigure = plotbins(sc,PredictorName)
hFigure = plotbins(___,Name,Value)
```


## Description

plotbins(sc, PredictorName) plots histogram counts for given predictor variables. When a predictor's bins are modified using modifybins or autobinning, rerun plotbins to update the figure to reflect the change.
hFigure $=$ plotbins(sc,PredictorName) returns a handle to the figure. plotbins plots histogram counts for given predictor variables. When a predictor's bins are modified using modifybins or autobinning, rerun plotbins to update the figure to reflect the change.
hFigure $=$ plotbins( $\qquad$ , Name, Value) returns a handle to the figure. plotbins plots histogram counts for given predictor variables using optional name-value pair arguments. When a predictor's bins are modified using modifybins or autobinning, rerun plotbins to update the figure to reflect the change.

## Examples

## Plot a Histogram for Bin Information

Create a creditscorecard object using the CreditCardData. mat file to load the data (using a dataset from Refaat 2011).
load CreditCardData
sc = creditscorecard(data);

Perform automatic binning for the PredictorName input argument for CustIncome using the defaults for the algorithm Monotone.

```
sc = autobinning(sc, 'CustIncome')
sc =
    creditscorecard with properties:
                    GoodLabel: 0
                ResponseVar: 'status'
                    WeightsVar: ''
                            VarNames: {1x11 cell}
        NumericPredictors: {1x7 cell}
    CategoricalPredictors: {'ResStatus' 'EmpStatus' 'OtherCC'}
        BinMissingData: 0
                        IDVar: ''
            PredictorVars: {1x10 cell}
                        Data: [1200x11 table]
```

Use bininfo to display the autobinned data.

```
[bi, cp] = bininfo(sc, 'CustIncome')
```

bi=8×6 table
Bin Good
Bad Odds
WOE

InfoValue
0.91379
$1.5102-0.29217$
$1.8889 \quad-0.06843 \quad 0.00041042$
$1.9694 \quad-0.026696 \quad 0.00017359$
2 -0.011271 1.0819e-05
$2.4848 \quad 0.20579 \quad 0.0078175$
$3.2679 \quad 0.47972 \quad 0.041657$
2.0227 NaN 0.12285
$c p=6 \times 1$
29000
33000
35000
40000
42000

Manually remove the second cut point (the boundary between the second and third bins) to merge bins two and three. Use the modifybins function to update the scorecard and then display updated bin information.

```
cp(2) = [];
sc = modifybins(sc,'CustIncome','CutPoints',cp);
bi = bininfo(sc,'CustIncome')
\begin{tabular}{|c|c|c|c|c|c|}
\hline Bin & Good & Bad & 0dds & WOE & InfoValue \\
\hline ' [-Inf, 29000) ' & 53 & 58 & 0.91379 & -0.79457 & 0.06364 \\
\hline '[29000, 35000) ' & 142 & 85 & 1.6706 & -0.19124 & 0.0071274 \\
\hline '[35000, 40000) ' & 193 & 98 & 1.9694 & -0.026696 & 0.00017359 \\
\hline '[40000, 42000)' & 68 & 34 & 2 & -0.011271 & 1.0819e-05 \\
\hline '[42000, 47000) ' & 164 & 66 & 2.4848 & 0.20579 & 0.0078175 \\
\hline '[47000,Inf]' & 183 & 56 & 3.2679 & 0.47972 & 0.041657 \\
\hline 'Totals' & 803 & 397 & 2.0227 & NaN & 0.12043 \\
\hline
\end{tabular}
```

Plot the histogram count for updated bin information for the PredictorName called CustIncome.
plotbins(sc,'CustIncome')
xtickangle(30)
legend('Location','north')


Plot a Histogram for Bin Information Using Name-Value Pair Arguments
Create a creditscorecard object using the CreditCardData.mat file to load the data (using a dataset from Refaat 2011).
load CreditCardData
sc = creditscorecard(data);
Perform automatic binning for the PredictorName input argument for CustIncome using the defaults for the algorithm Monotone.
sc = autobinning(sc, 'CustIncome')

```
sc =
    creditscorecard with properties:
                    GoodLabel: 0
            ResponseVar: 'status'
            WeightsVar: ''
                    VarNames: {1x11 cell}
        NumericPredictors: {1x7 cell}
        CategoricalPredictors: {'ResStatus' 'EmpStatus' 'OtherCC'}
            BinMissingData: 0
            IDVar: ''
            PredictorVars: {1x10 cell}
                        Data: [1200x11 table]
```

Use bininfo to display the autobinned data.

```
[bi, cp] = bininfo(sc, 'CustIncome')
bi=8\times6 table
\begin{tabular}{llll} 
Bin & Good & Bad \(\quad\) Odds & \(\square\)
\end{tabular}
\begin{tabular}{|c|c|c|c|c|c|}
\hline '[-Inf, 29000) ' & 53 & 58 & 0.91379 & -0.79457 & 0.06364 \\
\hline '[29000, 33000) ' & 74 & 49 & 1.5102 & -0.29217 & 0.0091366 \\
\hline '[33000, 35000)' & 68 & 36 & 1.8889 & -0.06843 & 0.00041042 \\
\hline '[35000, 40000)' & 193 & 98 & 1.9694 & -0.026696 & 0.00017359 \\
\hline '[40000, 42000)' & 68 & 34 & 2 & -0.011271 & 1.0819e-05 \\
\hline '[42000, 47000)' & 164 & 66 & 2.4848 & 0.20579 & 0.0078175 \\
\hline '[47000,Inf]' & 183 & 56 & 3.2679 & 0.47972 & 0.041657 \\
\hline 'Totals' & 803 & 397 & 2.0227 & NaN & 0.12285 \\
\hline
\end{tabular}
cp = 6x1
```

    29000
    33000
    35000
    40000
    42000
    47000
    Plot the bin information for Cust Income without the Weight of Evidence (WOE) line and without a legend by setting the 'WOE' and 'Legend ' name-value arguments to 'Off'.

Also, set the 'BinText ' name-value pair argument to 'PercentRows' to show as text over the plot bars for the proportion of "Good" and "Bad" within each bin, that is, the probability of "Good" and "Bad" within each bin.
plotbins(sc,'CustIncome','WOE','Off','Legend','Off','BinText','PercentRows') xtickangle(30)

## Custincome



## Plot Bin Information When Using Missing Data

Create a creditscorecard object using the CreditCardData.mat file to load the data with missing values.

```
load CreditCardData.mat
head(dataMissing,5)
```



```
fprintf('Number of rows: %d\n',height(dataMissing))
Number of rows: 1200
```

```
fprintf('Number of missing values CustAge: %d\n',sum(ismissing(dataMissing.CustAge)))
```

fprintf('Number of missing values CustAge: %d\n',sum(ismissing(dataMissing.CustAge)))
Number of missing values CustAge: 30
fprintf('Number of missing values ResStatus: %d\n',sum(ismissing(dataMissing.ResStatus)
Number of missing values ResStatus: 40
Use creditscorecard with the name-value argument 'BinMissingData' set to true to bin the missing numeric or categorical data in a separate bin.

```
```

sc = creditscorecard(dataMissing,'IDVar','CustID','BinMissingData',true);

```
sc = creditscorecard(dataMissing,'IDVar','CustID','BinMissingData',true);
sc = autobinning(sc);
sc = autobinning(sc);
disp(sc)
disp(sc)
    creditscorecard with properties:
    creditscorecard with properties:
                        GoodLabel: 0
                ResponseVar: 'status'
                        WeightsVar:
                            VarNames: {1x11 cell}
        NumericPredictors: {1x6 cell}
        CategoricalPredictors: {'ResStatus' 'EmpStatus' 'OtherCC'}
            BinMissingData: 1
                            IDVar: 'CustID'
            PredictorVars: {1x9 cell}
                                    Data: [1200x11 table]
```

Display and plot bin information for numeric data for 'CustAge' that includes missing data in a separate bin labelled <missing>.

```
[bi,cp] = bininfo(sc,'CustAge');
disp(bi)
\begin{tabular}{|c|c|c|c|c|c|}
\hline Bin & Good & Bad & Odds & WOE & InfoValue \\
\hline '[-Inf,33)' & 69 & 52 & 1.3269 & -0.42156 & 0.018993 \\
\hline ' \([33,37)^{\prime}\) & 63 & 45 & 1.4 & -0.36795 & 0.012839 \\
\hline '[37,40)' & 72 & 47 & 1.5319 & -0.2779 & 0.0079824 \\
\hline '[40,46) ' & 172 & 89 & 1.9326 & -0.04556 & 0.0004549 \\
\hline ' \([46,48\) )' & 59 & 25 & 2.36 & 0.15424 & 0.0016199 \\
\hline ' \([48,51\) ) ' & 99 & 41 & 2.4146 & 0.17713 & 0.0035449 \\
\hline ' \([51,58\) )' & 157 & 62 & 2.5323 & 0.22469 & 0.0088407 \\
\hline '[58,Inf]' & 93 & 25 & 3.72 & 0.60931 & 0.032198 \\
\hline '<missing>' & 19 & 11 & 1.7273 & -0.15787 & 0.00063885 \\
\hline 'Totals' & 803 & 397 & 2.0227 & NaN & 0.087112 \\
\hline
\end{tabular}
plotbins(sc,'CustAge')
```



Display and plot bin information for categorical data for 'ResStatus' that includes missing data in a separate bin labelled <missing>.
[bi,cg] = bininfo(sc,'ResStatus'); disp(bi)

| Bin | Good | Bad | Odds | WOE | InfoValue |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 'Tenant ' | 296 | 161 | 1.8385 | -0.095463 | 0.0035249 |
| 'Home Owner' | 352 | 171 | 2.0585 | 0.017549 | 0.00013382 |
| 'Other' | 128 | 52 | 2.4615 | 0.19637 | 0.0055808 |
| '<missing>' | 27 | 13 | 2.0769 | 0.026469 | 2.3248e-05 |
| 'Totals' | 803 | 397 | 2.0227 | NaN | 0.0092627 |

```
plotbins(sc,'ResStatus')
```



## Input Arguments

## sc - Credit scorecard model

creditscorecard object
Credit scorecard model, specified as a creditscorecard object. Use creditscorecard to create a creditscorecard object.

## PredictorName - Name of one or more predictors to plot

character vector with predictor name | cell array of character vectors with predictor names

Name of one or more predictors to plot, specified using a character vector or cell array of character vectors containing one or more names of the predictors.
Data Types: char | cell

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Example: plotbins(sc,PredictorName,'BinText','Count','WOE','On')

## BinText - Information to display on top of plotted bin counts

'None' (default)| character vector with values 'None', 'Count', 'PercentRows', 'PercentCols', 'PercentTotal'

Information to display on top of plotted bin counts, specified as the comma-separated pair consisting of 'BinText' and a character vector with values:

- None - No text is displayed on top of the bins.
- Count - For each bin, displays the count for "Good" and "Bad."
- PercentRows - For each bin, displays the count for "Good" and "Bad" as a percentage of the number of observations in the bin.
- PercentCols - For each bin, displays the count for "Good" and "Bad" as a percentage of the total "Good" and total "Bad" in the entire sample.
- PercentTotal - For each bin, displays the count for "Good" and "Bad" as a percentage of the total number of observations in the entire sample.


## Data Types: char

## WOE - Indicator for Weight of Evidence (WOE)

'On' (default)| character vector with values 'On', 'Off'

Indicator for Weight of Evidence (WOE) line, specified as the comma-separated pair consisting of 'WOE' and a character vector with values On or Off. When set to On, the WOE line is plotted on top of the histogram.

## Data Types: char

## Legend - Indicator for legend on plot

'On' (default) | character vector with values 'On', 'Off'
Indicator for legend on the plot, specified as the comma-separated pair consisting of ' Legend ' and a character vector with values On or Off.

## Data Types: char

## Output Arguments

## hFigure - Figure handle for histogram plot for predictor variables

figure object
Figure handle for histogram plot for predictor variables, returned as figure object or array of figure objects if more than one PredictorName is specified as an input.

## References

[1] Anderson, R. The Credit Scoring Toolkit. Oxford University Press, 2007.
[2] Refaat, M. Credit Risk Scorecards: Development and Implementation Using SAS. lulu.com, 2011.

## See Also

autobinning|bindata|bininfo|creditscorecard|displaypoints|fitmodel | formatpoints |modifybins |modifypredictor | predictorinfo|probdefault | score|setmodel|validatemodel

## Topics

"Case Study for a Credit Scorecard Analysis" on page 8-101
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"About Credit Scorecards" on page 8-62

## Introduced in R2014b

## modifybins

Modify predictor's bins

## Syntax

sc = modifybins(sc,PredictorName,Name,Value)

## Description

sc = modifybins(sc,PredictorName,Name,Value) manually modifies predictor bins for numeric predictors or categorical predictors using optional name-value pair arguments. For numeric predictors, minimum value, maximum value, and cut points can be specified. For categorical predictors, category groupings can be specified. Bin labels can be specified for both types of predictors.

## Examples

## Modify Predictor Bins for Numeric Data

Create a creditscorecard object using the CreditCardData.mat file to load the data (using a dataset from Refaat 2011).

```
load CreditCardData
sc = creditscorecard(data);
```

The predictor CustIncome is numeric. By default, each value of a predictor is placed in a separate bin.

```
bi = bininfo(sc,'CustIncome')
bi=46\times6 table
    Bin Good Bad Odds WOE InfoValue
    '18000'
        2
        3
        0.66667
```

WOE
-1. 1099

InfoValue
0.0056227

| ' $19000{ }^{\prime}$ | 1 | 2 | 0.5 | -1.3976 | 0.0053002 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| '20000' | 4 | 2 | 2 | -0.011271 | 6.3641e-07 |
| '21000' | 6 | 3 | 2 | -0.011271 | 9.5462e-07 |
| '22000' | 4 | 2 | 2 | -0.011271 | 6.3641e-07 |
| '23000' | 4 | 4 | 1 | -0.70442 | 0.0035885 |
| '24000' | 5 | 5 | 1 | -0.70442 | 0.0044856 |
| '25000' | 4 | 9 | 0.44444 | -1.5153 | 0.026805 |
| '26000' | 4 | 11 | 0.36364 | -1.716 | 0.038999 |
| '27000' | 6 | 6 | 1 | -0.70442 | 0.0053827 |
| '28000' | 13 | 11 | 1.1818 | -0.53736 | 0.0061896 |
| '29000' | 11 | 10 | 1.1 | -0.60911 | 0.0069988 |
| '30000' | 18 | 16 | 1.125 | -0.58664 | 0.010493 |
| '31000' | 24 | 8 | 3 | 0.39419 | 0.0038382 |
| '32000' | 21 | 15 | 1.4 | -0.36795 | 0.0042797 |
| '33000' | 35 | 19 | 1.8421 | -0.093509 | 0.00039951 |

Use modifybins to set a minimum value of 0 , and cut points every 10000, from 20000 to 60000. Display updated bin information, including cut points.

```
sc = modifybins(sc,'CustIncome','MinValue',0,'CutPoints',20000:10000:60000);
[bi,cp] = bininfo(sc,'CustIncome')
bi=7\times6 table
Bin
        '[0,20000)'
    '[20000,30000)'
    '[30000,40000)'
    '[40000,50000)'
    '[50000,60000)'
    '[60000,Inf]'
    'Totals'
        Good
        Bad
```

0dds

- 0.6
0.6
0.96825
1.8728
2.4715
3.2188

8
2.0227
$\qquad$

| -1.2152 | 0.010765 |
| ---: | ---: |
| -0.73668 | 0.060942 |
| -0.076967 | 0.0024846 |
| 0.20042 | 0.013781 |
| 0.46457 | 0.022144 |
| 1.375 | 0.010235 |
| NaN | 0.12035 |

```
cp = 5×1
20000
30000
40000
50000
60000
```

The first and last bins contain very few points. To merge the first bin into the second one, remove the first cut point. Similarly, to merge the last bin into the second-to-last one, remove the last cut point. Then use modifybins to update the scorecard, and display updated bin information.

```
cp(1)=[];
cp(end)=[];
sc = modifybins(sc,'CustIncome','CutPoints',cp);
bi = bininfo(sc,'CustIncome')
\begin{tabular}{|c|c|c|c|c|c|}
\hline Bin & Good & Bad & 0dds & WOE & InfoValue \\
\hline ' \([0,30000)^{\prime}\) & 64 & 68 & 0.94118 & -0.76504 & 0.070065 \\
\hline '[30000, 40000) ' & 324 & 173 & 1.8728 & -0.076967 & 0.0024846 \\
\hline '[40000, 50000) ' & 304 & 123 & 2.4715 & 0. 20042 & 0.013781 \\
\hline '[50000,Inf]' & 111 & 33 & 3.3636 & 0.5086 & 0.028028 \\
\hline 'Totals' & 803 & 397 & 2.0227 & NaN & 0.11436 \\
\hline
\end{tabular}
```


## Modify Predictor Bins for Categorical Data

Create a creditscorecard object using the CreditCardData.mat file to load the data (using a dataset from Refaat 2011).

```
load CreditCardData
sc = creditscorecard(data);
```

The binning map or rules for categorical data are summarized in a "category grouping" table, returned as an optional output. By default, each category is placed in a separate bin. Here is the information for the predictor ResStatus.

```
[bi,cg] = bininfo(sc,'ResStatus')
\begin{tabular}{|c|c|c|c|c|c|}
\hline Bin & Good & Bad & Odds & WOE & InfoValue \\
\hline 'Home Owner' & 365 & 177 & 2.0621 & 0.019329 & 0.0001682 \\
\hline 'Tenant' & 307 & 167 & 1.8383 & -0.095564 & 0.0036638 \\
\hline '0ther' & 131 & 53 & 2.4717 & 0.20049 & 0.0059418 \\
\hline
\end{tabular}
```

$\begin{array}{llllll}\text { 'Totals' } & 803 & 397 & 2.0227 & \text { NaN } 0.0097738\end{array}$

| Category | BinNumber |
| :---: | :---: |
| 'Home Owner' | 1 |
| 'Tenant' | 2 |
| 'Other' | 3 |

To group categories 'Tenant ' and 'Other', modify the category grouping table cg , so the bin number for 'Other' is the same as the bin number for 'Tenant'. Then use modifybins to update the scorecard.

```
cg.BinNumber(3) = 2;
sc = modifybins(sc,'ResStatus','CatGrouping',cg);
```

Display the updated bin information. Note that the bin labels has been updated and that the bin membership information is contained in the category grouping cg .

```
[bi,cg] = bininfo(sc,'ResStatus')
\begin{tabular}{|c|c|c|c|c|c|}
\hline Bin & Good & Bad & Odds & WOE & InfoValue \\
\hline 'Group1' & 365 & 177 & 2.0621 & 0.019329 & 0.0001682 \\
\hline 'Group2' & 438 & 220 & 1.9909 & -0.015827 & 0.00013772 \\
\hline 'Totals' & 803 & 397 & 2.0227 & NaN & 0.00030592 \\
\hline
\end{tabular}
```

```
cg=3\times2 table
    Category BinNumber
    'Home Owner' 1
    'Tenant' 2
    'Other' 2
```


## Merge Bins for Numerical and Categorical Predictors

Create a creditscorecard object (using a dataset from Refaat 2011).
load CreditCardData
sc = creditscorecard(data,'IDVar','CustID','GoodLabel',0);
For the numerical predictor CustAge, use the modifybins function to set the following cut points:

```
cp = [25 37 49 65];
sc = modifybins(sc,'CustAge','CutPoints',cp,'MinValue',0,'MaxValue',75);
bininfo(sc,'CustAge')
\begin{tabular}{|c|c|c|c|c|c|}
\hline Bin & Good & Bad & Odds & WOE & InfoValue \\
\hline ' \(\left[0,25\right.\) ) \({ }^{\prime}\) & 9 & 8 & 1.125 & -0.58664 & 0.0052464 \\
\hline ' \([25,37)^{\prime}\) & 125 & 92 & 1.3587 & -0.39789 & 0.030268 \\
\hline ' \([37,49\) )' & 340 & 183 & 1.8579 & -0.084959 & 0.0031898 \\
\hline ' \([49,65\) )' & 298 & 108 & 2.7593 & 0.31054 & 0.030765 \\
\hline '[65,75]' & 31 & 6 & 5.1667 & 0.93781 & 0.022031 \\
\hline 'Totals' & 803 & 397 & 2.0227 & NaN & 0.0915 \\
\hline
\end{tabular}
```

Use the modifybins function to merge the 2nd and 3rd bins.

| $\begin{gathered} \text { ans }=5 \times 6 \text { table } \\ \text { Bin } \end{gathered}$ | Good | Bad | Odds | WOE | InfoValue |
| :---: | :---: | :---: | :---: | :---: | :---: |
| ' $[0,25)^{\prime}$ | 9 | 8 | 1.125 | -0.58664 | 0.0052464 |
| ' $[25,49$ ) ' | 465 | 275 | 1.6909 | -0.17915 | 0.020355 |
| ' $[49,65$ )' | 298 | 108 | 2.7593 | 0.31054 | 0.030765 |
| ' $[65,75]$ ' | 31 | 6 | 5.1667 | 0.93781 | 0.022031 |
| 'Totals' | 803 | 397 | 2.0227 | NaN | 0.078397 |

Display bin information for the categorical predictor ResStatus.
[bi,cg] = bininfo(sc,'ResStatus');
disp(bi)

| Bin | Good | Bad | Odds | WOE | InfoValue |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 'Home Owner' | 365 | 177 | 2.0621 | 0.019329 | 0.0001682 |
| 'Tenant' | 307 | 167 | 1.8383 | -0.095564 | 0.0036638 |
| '0ther' | 131 | 53 | 2.4717 | 0.20049 | 0.0059418 |
| 'Totals' | 803 | 397 | 2.0227 | NaN | 0.0097738 |

Use the modifybins function to merge categories 2 and 3.

```
cg.BinNumber(3) = 2;
sc = modifybins(sc,'ResStatus','CatGrouping',cg);
bininfo(sc,'ResStatus')
```



## Split Bins for Numerical and Categorical Predictors

Create a creditscorecard object (using a dataset from Refaat 2011).

```
load CreditCardData
sc = creditscorecard(data,'IDVar','CustID','GoodLabel',0)
sc =
    creditscorecard with properties:
                        GoodLabel: 0
                ResponseVar: 'status'
                            WeightsVar: ''
                            VarNames: {1x11 cell}
            NumericPredictors: {1x6 cell}
        CategoricalPredictors: {'ResStatus' 'EmpStatus' 'OtherCC'}
            BinMissingData: 0
                            IDVar: 'CustID'
                        PredictorVars: {1x9 cell}
```


## Data: [1200x11 table]

For the numerical predictor TmAtAddress, use the modifybins function to set the following cut points:

```
cp = [30 80 120];
sc = modifybins(sc,'TmAtAddress','CutPoints',cp,'MinValue',0,'MaxValue',210);
bininfo(sc,'TmAtAddress')
\begin{tabular}{|c|c|c|c|c|c|}
\hline Bin & Good & Bad & Odds & WOE & InfoValue \\
\hline ' \([0,30\) ) ' & 330 & 154 & 2.1429 & 0.057722 & 0.0013305 \\
\hline '[30,80)' & 379 & 201 & 1.8856 & -0.070187 & 0.0024086 \\
\hline ' \([80,120)^{\prime}\) & 78 & 36 & 2.1667 & 0.068771 & 0.00044396 \\
\hline '[120,210]' & 16 & 6 & 2.6667 & 0.27641 & 0.0013301 \\
\hline 'Totals' & 803 & 397 & 2.0227 & NaN & 0.0055131 \\
\hline
\end{tabular}
```

Use the modifybins function to split the 2nd bin.

| $\begin{gathered} \text { ans }=6 \times 6 \text { table } \\ \text { Bin } \end{gathered}$ | Good | Bad | 0dds | WOE | InfoValue |
| :---: | :---: | :---: | :---: | :---: | :---: |
| '[0,30) ' | 330 | 154 | 2.1429 | 0.057722 | 0.0013305 |
| '[30,50) | 211 | 104 | 2.0288 | 0.0030488 | 2.4387e-06 |
| ' $[50,80$ ) ' | 168 | 97 | 1.732 | -0.15517 | 0.005449 |
| ' $[80,120$ ) ' | 78 | 36 | 2.1667 | 0.068771 | 0.00044396 |
| '[120,210]' | 16 | 6 | 2.6667 | 0.27641 | 0.0013301 |
| 'Totals' | 803 | 397 | 2.0227 | NaN | 0.0085559 |

Display bin information for the categorical predictor ResStatus.

```
[bi,cg] = bininfo(sc,'ResStatus')
```

bi $=4 \times 6$ table
Bin Good Bad Odds W0E InfoValue

| 'Home Owner' | 365 | 177 | 2.0621 | 0.019329 | 0.0001682 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 'Tenant' | 307 | 167 | 1.8383 | -0.095564 | 0.0036638 |
| '0ther' | 131 | 53 | 2.4717 | 0.20049 | 0.0059418 |
| 'Totals' | 803 | 397 | 2.0227 | NaN | 0.0097738 |
| $\mathrm{cg}=3 \times 2$ table |  |  |  |  |  |
| Category | BinNumber |  |  |  |  |
| 'Home Owner' |  |  |  |  |  |
| 'Tenant' |  |  |  |  |  |
| '0ther' |  |  |  |  |  |

Use the modifybins function to merge categories 2 and 3.

```
cg.BinNumber(3) = 2;
sc = modifybins(sc,'ResStatus','CatGrouping',cg);
bininfo(sc,'ResStatus')
```



Use the modifybins function to split bin 2 and put 0ther under bin 3.

```
cg.BinNumber(3) = 3;
sc = modifybins(sc,'ResStatus','CatGrouping',cg);
[bi,cg] = bininfo(sc,'ResStatus')
\begin{tabular}{|c|c|c|c|c|c|}
\hline Bin & Good & Bad & Odds & WOE & InfoValue \\
\hline 'Home Owner' & 365 & 177 & 2.0621 & 0.019329 & 0.0001682 \\
\hline 'Tenant' & 307 & 167 & 1.8383 & -0.095564 & 0.0036638 \\
\hline 'Other' & 131 & 53 & 2.4717 & 0.20049 & 0.0059418 \\
\hline 'Totals' & 803 & 397 & 2.0227 & NaN & 0.0097738 \\
\hline
\end{tabular}
```

```
cg=3\times2 table
    Category BinNumber
    'Home Owner' 1
    'Tenant' 2
    'Other' 3
```


## Modify Bin Labels

Create a creditscorecard object using the CreditCardData.mat file to load the data (using a dataset from Refaat 2011).

```
load CreditCardData
```

sc = creditscorecard(data);

Use modifybins to reset the minimum value and create three bins for the predictor CustIncome and display updated bin information.

```
sc = modifybins(sc,'CustIncome','MinValue',0,'CutPoints',[30000 50000]);
bi = bininfo(sc,'CustIncome')
\begin{tabular}{|c|c|c|c|c|c|}
\hline Bin & Good & Bad & Odds & WOE & InfoValue \\
\hline ' [0, 30000) ' & 64 & 68 & 0.94118 & -0.76504 & 0.070065 \\
\hline ' [30000, 50000) ' & 628 & 296 & 2.1216 & 0.047762 & 0.0017421 \\
\hline '[50000,Inf]' & 111 & 33 & 3.3636 & 0.5086 & 0.028028 \\
\hline 'Totals' & 803 & 397 & 2.0227 & NaN & 0.099836 \\
\hline
\end{tabular}
```

Modify the bin labels and display updated bin information.

```
NewLabels = {'Up to 30k','30k to 50k','50k and more'};
sc = modifybins(sc,'CustIncome','BinLabels',NewLabels);
bi = bininfo(sc,'CustIncome')
bi=4\times6 table Good Bad Odds WOE InfoValue
```

| 'Up to 30k' | 64 | 68 | 0.94118 | -0.76504 | 0.070065 |
| :--- | ---: | ---: | ---: | ---: | ---: |
| '30k to 50k' | 628 | 296 | 2.1216 | 0.047762 | 0.0017421 |
| '50k and more' | 111 | 33 | 3.3636 | 0.5086 | 0.028028 |
| 'Totals' | 803 | 397 | 2.0227 | NaN | 0.099836 |

Bin labels should be the last bin-modification step. As in this example, user-defined bin labels often contain information about the cut points, minimum, or maximum values for numeric data, or information about category groupings for categorical data. To prevent situations where user-defined labels and cut points are inconsistent (and labels are misleading), the creditscorecard object overrides user-defined labels every time the bins are modified using modifybins.

To illustrate modifybins overriding user-defined labels every time the bins are modified, reset the first cut point to 31000 and display updated bin information. Note that the bin labels are reset to their default format and accurately reflect the change in the cut points.

```
sc = modifybins(sc,'CustIncome','CutPoints',[31000 50000]);
bi = bininfo(sc,'CustIncome')
\begin{tabular}{|c|c|c|c|c|c|}
\hline Bin & Good & Bad & 0dds & WOE & InfoValue \\
\hline '[0,31000)' & 82 & 84 & 0.97619 & -0.72852 & 0.079751 \\
\hline '[31000, 50000) ' & 610 & 280 & 2.1786 & 0.074251 & 0.0040364 \\
\hline '[50000,Inf]' & 111 & 33 & 3.3636 & 0.5086 & 0.028028 \\
\hline 'Totals' & 803 & 397 & 2.0227 & NaN & 0.11182 \\
\hline
\end{tabular}
```


## Modify Bin Information When Using Missing Data

Create a creditscorecard object using the CreditCardData.mat file to load the dataMissing with missing values.

```
load CreditCardData.mat
head(dataMissing,5)
ans=5\times11 table CustID CustAge TmAtAddress ResStatus EmpStatus CustIncome
```

```
\begin{tabular}{rrr}
1 & 53 & 62 \\
2 & 61 & 22 \\
3 & 47 & 30 \\
4 & NaN & 75 \\
5 & 68 & 56
\end{tabular}
```

```
fprintf('Number of rows: %d\n',height(dataMissing))
```

fprintf('Number of rows: %d\n',height(dataMissing))
Number of rows: 1200
fprintf('Number of missing values CustAge: %d\n',sum(ismissing(dataMissing.CustAge)))
Number of missing values CustAge: 30
fprintf('Number of missing values ResStatus: %d\n',sum(ismissing(dataMissing.ResStatus
Number of missing values ResStatus: 40
Use creditscorecard with the name-value argument 'BinMissingData' set to true to bin the missing data in a separate bin.

```
```

sc = creditscorecard(dataMissing,'IDVar','CustID','BinMissingData',true);

```
sc = creditscorecard(dataMissing,'IDVar','CustID','BinMissingData',true);
sC = autobinning(sc);
sC = autobinning(sc);
disp(sc)
disp(sc)
    creditscorecard with properties:
                        GoodLabel: 0
                ResponseVar: 'status'
                    WeightsVar: ''
                            VarNames: {1x11 cell}
            NumericPredictors: {1x6 cell}
        CategoricalPredictors: {'ResStatus' 'EmpStatus' 'OtherCC'}
            BinMissingData: 1
                            IDVar: 'CustID'
            PredictorVars: {1x9 cell}
                            Data: [1200x11 table]
```

Display bin information for numeric data for 'CustAge' that includes missing data in a separate bin labelled <missing>.

```
[bi,cp] = bininfo(sc,'CustAge');
disp(bi)
\begin{tabular}{|c|c|c|c|c|c|}
\hline Bin & Good & Bad & Odds & WOE & InfoValue \\
\hline '[-Inf,33)' & 69 & 52 & 1.3269 & -0.42156 & 0.018993 \\
\hline ' \([33,37)\) ' & 63 & 45 & 1.4 & -0.36795 & 0.012839 \\
\hline ' \([37,40\) )' & 72 & 47 & 1.5319 & -0.2779 & 0.0079824 \\
\hline ' \([40,46\) )' & 172 & 89 & 1.9326 & -0.04556 & 0.0004549 \\
\hline ' \([46,48\) )' & 59 & 25 & 2.36 & 0.15424 & 0.0016199 \\
\hline ' \([48,51\) )' & 99 & 41 & 2.4146 & 0.17713 & 0.0035449 \\
\hline ' \([51,58\) ) ' & 157 & 62 & 2.5323 & 0.22469 & 0.0088407 \\
\hline '[58,Inf]' & 93 & 25 & 3.72 & 0.60931 & 0.032198 \\
\hline '<missing>' & 19 & 11 & 1.7273 & -0.15787 & 0.00063885 \\
\hline 'Totals' & 803 & 397 & 2.0227 & NaN & 0.087112 \\
\hline
\end{tabular}
plotbins(sc,'CustAge')
```



For the numeric predictor CustAge, remove cut points 48 and 51 and then use modifybins to define a 'MinValue' of 0 to manually change the binning and notice that this does not affect the data in the <missing> bin and the <missing> bin remains at the end.

```
cp(cp==48) = [];
cp(cp==51) = [];
sc = modifybins(sc,'CustAge','CutPoints',cp,'MinValue',0);
bi = bininfo(sc,'CustAge');
disp(bi)
\begin{tabular}{lllll} 
Bin Good Bad Odds WOE \\
\hline
\end{tabular}
```

| ' $[0,33$ ) ' | 69 | 52 | 1.3269 | -0.42156 | 0.018993 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| '[33,37)' | 63 | 45 | 1.4 | -0.36795 | 0.012839 |
| ' $[37,40)^{\prime}$ | 72 | 47 | 1.5319 | -0.2779 | 0.0079824 |
| ' $[40,46$ )' | 172 | 89 | 1.9326 | -0.04556 | 0.0004549 |
| ' $[46,58$ )' | 315 | 128 | 2.4609 | 0.19612 | 0.013701 |
| '[58,Inf]' | 93 | 25 | 3.72 | 0.60931 | 0.032198 |
| '<missing>' | 19 | 11 | 1.7273 | -0.15787 | 0.00063885 |
| 'Totals' | 803 | 397 | 2.0227 | NaN | 0.086808 |

plotbins(sc,'CustAge')


Display bin information for categorical data for 'ResStatus' that includes missing data in a separate bin labelled <missing>.


ResStatus

$\square$ Good $\square \mathrm{Bad}$

For the categorical predictor ResStatus, use modifybins to manually merge 'HomeOwner' and 'Other' into a single group by assigning the same bin number to these categories. Notice that this does not affect the data in the <missing> bin and the <missing> bin remains at the end.
cg.BinNumber(3) = 2;
sc = modifybins(sc,'ResStatus','CatGrouping', cg);
[bi,cg] = bininfo(sc,'ResStatus');
disp(bi)

| Bin | Good | Bad | Odds | WOE | InfoValue |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 'Group1' | 296 | 161 | 1.8385 | -0.095463 | 0.0035249 |
| 'Group2' | 480 | 223 | 2.1525 | 0.062196 | 0.0022419 |
| '<missing>' | 27 | 13 | 2.0769 | 0.026469 | 2.3248e-05 |
| 'Totals' | 803 | 397 | 2.0227 | NaN | 0.00579 |

## disp(cg)

Category
BinNumber
'Tenant' 1
'Home Owner' 2
'Other' 2

## Input Arguments

## sc - Credit scorecard model

## creditscorecard object

Credit scorecard model, specified as a creditscorecard object. Use creditscorecard to create a creditscorecard object.

## PredictorName - Name of predictor

character vector
Name of predictor, specified as a character vector containing the name of the predictor. PredictorName is case-sensitive.

Data Types: char

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1,Value1, . . . , NameN, ValueN.

Example: sc = modifybins(sc,PredictorName,'MinValue',10,'CutPoints', [23, 44, 66, 88])

## MinValue - Minimum acceptable value (numeric predictors only) - Inf (default) | numeric

Minimum acceptable value, specified as the comma-separated pair consisting of 'MinValue' and a numeric value (for numeric predictors only). Values below this number are considered out of range.

Data Types: double

## MaxValue - Maximum acceptable value (numeric predictors only) Inf (default) | numeric

Maximum acceptable value, specified as the comma-separated pair consisting of 'MaxValue' and a numeric value (for numeric predictors only). Values above this number are considered out of range.

Data Types: double

## CutPoints - Split points between bins

each observed value of the predictor is placed in a separate bin (default) | nondecreasing numeric array

Split points between bins, specified as the comma-separated pair consisting of 'CutPoints ' and a nondecreasing numeric array. If there are NumBins bins, there are $n$ $=$ NumBins -1 cut points so that $\mathrm{C} 1, \mathrm{C} 2, \ldots, \mathrm{Cn}$ describe the bin boundaries with the following convention:

- The first bin includes any values $>=$ MinValue, but $<\mathrm{C} 1$.
- The second bin includes any values $>=\mathrm{C} 1$, but $<\mathrm{C} 2$.
- The last bin includes any values $>=C n$, and $<=$ MaxValue.

Note Cut points do not include MinValue or MaxValue.

By default, cut points are defined so that each observed value of the predictor is placed in a separate bin. If the sorted observed values are V1, ..., VM, the default cut points are V2, ..., VM, which define $M$ bins.

## Data Types: double

CatGrouping - Table with two columns named Category and BinNumber each category is placed in a separate bin (default) | table with two columns named Category and BinNumber

Table with two columns named Category and BinNumber, specified as the commaseparated pair consisting of 'CatGrouping' and a table, where the first column contains an exhaustive list of categories for the predictor, and the second column contains the bin number to which each category belongs.

By default, each category is placed in a separate bin. If the observed categories are 'Cat1 '..., 'CatM' , the default category grouping is as follows.

| Category | BinNumber |
| :--- | :--- |
| 'Cat1' | 1 |
| ' ${ }^{\prime}$ 'at2 ' | 2 |
| $\ldots$ | $\ldots$ |
| ' ${\text { Cat } M^{\prime}}^{\prime}$ | $M$ |

Data Types: double

## BinLabels - Bin labels for each bin

automatically generated bin labels depending on the predictor's type (default) | cell array of character vectors

Bin labels for each bin, specified as the comma-separated pair consisting of 'BinLabels ' and a cell array of character vectors with bin label names.

Note 'BinLabels' does not support a value of <missing>.

Bin labels are used to tag the bins in different object functions such as bininfo, plotbins, and displaypoints. A creditscorecard object automatically sets default bins whenever bins are modified. The default format for bin labels depends on the predictor's type.

## The format for BinLabels is:

- Numeric data - Before any manual or automatic modification of the predictor bins, there is a bin for each observed predictor value by default. In that case, the bin labels simply show the predictor values. Once the predictor bins have been modified, there are nondefault values for MinValue or MaxValue, or nondefault cut points $\mathrm{C} 1, \mathrm{C} 2, \ldots$, Cn . In that case, the bin labels are:
- Bin 1 label: ' [MinValue, C1)'
- Bin 2 label: ' [C1, C2)'
- Last bin label: ' [Cn, MaxValue]'

For example, if there are three bins, MinValue is 0 and MaxValue is 40 , and cut point 1 is 20 and cut point 2 is 30 , then the corresponding three bin labels are:
' $[0,20$ ) '
'[20,30)'
'[30,40]'

- Categorical data - For categorical data, before any modification of the predictor bins, there is one bin per category. In that case, the bin labels simply show the predictor categories. Once the bins have been modified, the labels are set to 'Groupl' ', 'Group2 ' , and so on, for bin 1, bin 2, and so on, respectively. For example, suppose that we have the following category grouping

| Category | BinNumber |
| :--- | :--- |
| 'Cat1 ' | 1 |
| 'Cat2 ' | 2 |
| 'Cat3' | 2 |

Bin 1 contains 'Cat1' only and its bin label is set to 'Group1'. Bin 2 contains 'Cat2' and 'Cat3' and its bin label is set to 'Group2'.

Tip Using BinLabels should be the last step (if needed) in modifying bins. BinLabels definitions are overridden each time that the bins are modified using the modifybins or autobinning functions.

## Data Types: cell

## Output Arguments

## sc - Credit scorecard model

creditscorecard object
Credit scorecard model, returned as an updated creditscorecard object. For more information on using the creditscorecard object, see creditscorecard.

## References

[1] Anderson, R. The Credit Scoring Toolkit. Oxford University Press, 2007.
[2] Refaat, M. Credit Risk Scorecards: Development and Implementation Using SAS. lulu.com, 2011.

## See Also

autobinning|bindata|bininfo|creditscorecard|displaypoints|fitmodel | formatpoints|modifypredictor|plotbins|predictorinfo|probdefault| score | setmodel | validatemodel

## Topics

"Case Study for a Credit Scorecard Analysis" on page 8-101
"Credit Scorecard Modeling with Missing Values" on page 8-73
"Troubleshooting Credit Scorecard Results" on page 8-91
"Credit Scorecard Modeling Workflow" on page 8-67
"About Credit Scorecards" on page 8-62

## Introduced in R2014b

## modifypredictor

Set properties of credit scorecard predictors

## Syntax

```
sc = modifypredictor(sc,PredictorName)
sC = modifypredictor(
```

$\qquad$

``` ,Name, Value)
```


## Description

sc = modifypredictor(sc,PredictorName) sets the properties of the credit scorecard predictors.
sc = modifypredictor( $\qquad$ ,Name, Value) sets the properties of the credit scorecard predictors using optional name-value pair arguments.

## Examples

## Modify a Predictor to Change the Predictor Type from Numeric to Categorical

Create a creditscorecard object using the CreditCardData.mat file to load the data (using a dataset from Refaat 2011). In practice, categorical data many times is represented with numeric values. To show the case where categorical data is given as numeric data, the data for the variable 'ResStatus ' is intentionally converted to numeric values.

```
load CreditCardData
data.ResStatus = double(data.ResStatus);
sc = creditscorecard(data,'IDVar','CustID')
SC =
    creditscorecard with properties:
```

```
        ResponseVar: 'status'
        WeightsVar:
            VarNames: {1x11 cell}
    NumericPredictors: {1x7 cell}
    CategoricalPredictors: {'EmpStatus' 'OtherCC'}
        BinMissingData: 0
            IDVar: 'CustID'
            PredictorVars: {1x9 cell}
                        Data: [1200x11 table]
[T,Stats] = predictorinfo(sc,'ResStatus')
T=1\times2 table
            PredictorType LatestBinning
    ResStatus 'Numeric' 'Original Data'
Stats=4\times1 table
                        Value
    Min 1
    Max 3
    Mean 1.7017
    Std 0.71833
```

Note that 'ResStatus ' appears as part of the NumericPredictors property. Assume that you want 'ResStatus' to be treated as categorical data. For example, you may want to allow automatic binning algorithms to reorder the categories. Use modifypredictor to change the 'PredictorType' of the PredictorName 'ResStatus' from numeric to categorical.

```
sc = modifypredictor(sc,'ResStatus','PredictorType','Categorical')
sc =
    creditscorecard with properties:
            GoodLabel: 0
            ResponseVar: 'status'
            WeightsVar: ''
            VarNames: {1x11 cell}
```

```
    NumericPredictors: {1x6 cell}
    CategoricalPredictors: {'ResStatus' 'EmpStatus' 'OtherCC'}
        BinMissingData: 0
            IDVar: 'CustID'
        PredictorVars: {1x9 cell}
                        Data: [1200x11 table]
[T,Stats] = predictorinfo(sc,'ResStatus')
T=1\times3 table
\begin{tabular}{llll} 
PredictorType & Ordinal & & LatestBinning \\
'Categorical' & false & 'Original Data'
\end{tabular}
```

```
Stats=3\times1 table
```

Stats=3\times1 table
Count
Count
C1 542
C1 542
C2 474
C2 474
C3 184

```
    C3 184
```

Notice that 'ResStatus ' now appears as part of the 'Categorical' predictors.

## Input Arguments

## sc - Credit scorecard model

creditscorecard object
Credit scorecard model, specified as a creditscorecard object. Use creditscorecard to create a creditscorecard object.

## PredictorName - Predictor name

character vector | cell array of character vectors
Predictor name, specified using a character vector or cell array of character vectors containing the names of the credit scorecard predictors. PredictorName is casesensitive.

Data Types: char |cell

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, ... , NameN, ValueN.
Example: sc = modifypredictor(sc, \{'CustAge','CustIncome'\},'PredictorType','Categorical','Ordinal',tru e)

PredictorType - Predictor type that one or more predictors are converted to
' ' no conversion occurs (default)| character vector with values 'Numeric',
'Categorical'
Predictor type that one or more predictors are converted to, specified as the commaseparated pair consisting of 'PredictorType' and a character vector. Possible values are:

- ' ' - No conversion occurs.
- 'Numeric' - The predictor data specified by PredictorName is converted to numeric.
- 'Categorical' - The predictor data specified by PredictorName is converted to categorical.


## Data Types: char

## Ordinal - Indicator for whether predictors being converted to categorical are ordinal <br> false (default) | logical with values true, false

Indicator for whether predictors being converted to categorical or existing categorical predictors are treated as ordinal data, specified as the comma-separated pair consisting of 'Ordinal' and a logical with values true or false.

Note This optional input parameter is only used for predictors of type 'Categorical '.

Data Types: logical

## Output Arguments

sc - Credit scorecard model
creditscorecard object
Credit scorecard model, returned as an updated creditscorecard object.

See Also<br>bininfo|creditscorecard|modifybins|predictorinfo<br>Introduced in R2015b

## predictorinfo

Summary of credit scorecard predictor properties

## Syntax

[T,Stats] = predictorinfo(sc,PredictorName)

## Description

[T,Stats] = predictorinfo(sc,PredictorName) returns a summary of credit scorecard predictor properties and some basic predictor statistics.

## Examples

## Obtain Information for a Specified PredictorName

Create a creditscorecard object using the CreditCardData.mat file to load the data (using a dataset from Refaat 2011).

```
load CreditCardData
sc = creditscorecard(data,'IDVar','CustID')
sc =
    creditscorecard with properties:
```

                                    GoodLabel: 0
                    ResponseVar: 'status'
                        WeightsVar:
                            VarNames: \{1×11 cell\}
            NumericPredictors: \{1×6 cell\}
        CategoricalPredictors: \{'ResStatus' 'EmpStatus' 'OtherCC'\}
            BinMissingData: 0
                            IDVar: 'CustID'
            PredictorVars: \{1x9 cell\}
    
## Data: [1200x11 table]

Obtain the predictor statistics for the PredictorName of CustAge.
[T,Stats] = predictorinfo(sc,'CustAge')
$\mathrm{T}=1 \times 2$ table
$\begin{array}{ll}\text { PredictorType } & \text { LatestBinning } \\ \text { 'Numeric' } & \text { 'Original Data' }\end{array}$

Stats=4×1 table
Value

Min 21
$\begin{array}{ll}\text { Max } & 74\end{array}$
Mean 45.174
Std 9.8302

Obtain the predictor statistics for the PredictorName of ResStatus.
[T,Stats] = predictorinfo(sc,'ResStatus')
$\mathrm{T}=1 \times 3$ table

|  |  |  |  |
| :---: | :---: | :---: | :---: |
|  | PredictorType | Ordinal | LatestBinning |
| ResStatus | 'Categorical' | false | 'Original Data' |

## Stats=3×1 table

Count

Home Owner 542
Tenant 474
Other 184

## Input Arguments

## sc - Credit scorecard model

creditscorecard object
Credit scorecard model, specified as a creditscorecard object. Use creditscorecard to create a creditscorecard object.

## PredictorName - Predictor name

## character vector

Predictor name, specified using a character vector containing the names of the credit scorecard predictor of interest. PredictorName is case-sensitive.

Data Types: char

## Output Arguments

## T - Summary information for specified predictor table

Summary information for specified predictor, returned as table with the following columns:

- 'PredictorType' - 'Numeric' or 'Categorical'.
- 'Ordinal' - For categorical predictors, a boolean indicating whether it is ordinal.
- 'LatestBinning' - Character vector indicating the last applied algorithm for the input argument PredictorName. The values are:
- 'Original Data' - When no binning is applied to the predictor.
- 'Automatic / BinningName' - Where 'BinningName' is one of the following: Monotone, Equal Width, or Equal Frequency.
- 'Manual' - After each call of modifybins, where either 'CutPoints', 'CatGrouping', 'MinValue', or 'MaxValue' are modified.

The predictor's name is used as a row name in the table that is returned.

## Stats - Summary statistics for the input PredictorName table

Summary statistics for the input PredictorName, returned as a table. The corresponding value is stored in the 'Value' column.

The table's row names indicate the relevant statistics for numeric predictors:

- 'Min' - Minimum value in the sample.
- 'Max' - Maximum value in the sample.
- 'Mean' - Mean value in the sample.
- 'Std' - Standard deviation of the sample.

Note For data types other than 'double' or 'single', numeric precision may be lost for the standard deviation. Data types other than 'double' or 'single' are cast as 'double' before computing the standard deviation.

For categorical predictors, the row names contain the names of the categories, with corresponding total count in the 'Count ' column.

## See Also

bininfo|creditscorecard|modifybins | modifypredictor Introduced in R2015b

## bininfo

Return predictor's bin information

## Syntax

bi = bininfo(sc,PredictorName)
bi = bininfo( $\qquad$ ,Name, Value)
[bi,bm] = bininfo(sc,PredictorName,Name,Value)
[bi,bm,mv] = bininfo(sc,PredictorName,Name,Value)

## Description

bi = bininfo(sc,PredictorName) returns information at bin level, such as frequencies of "Good," "Bad," and bin statistics for the predictor specified in PredictorName.
bi = bininfo( ___ ,Name, Value) adds optional name-value arguments.
[bi,bm] = bininfo(sc,PredictorName,Name, Value) adds optional name-value arguments.bininfo also optionally returns the binning map (bm) or bin rules in the form of a vector of cut points for numeric predictors, or a table of category groupings for categorical predictors.
[bi,bm,mv] = bininfo(sc,PredictorName,Name, Value) returns information at bin level, such as frequencies of "Good," "Bad," and bin statistics for the predictor specified in PredictorName using optional name-value pair arguments. bininfo optionally returns the binning map or bin rules in the form of a vector of cut points for numeric predictors, or a table of category groupings for categorical predictors. In addition, optional name-value pair arguments mv returns a numeric array containing the minimum and maximum values, as set (or defined) by the user. The mv output argument is set to an empty array for categorical predictors.

## Examples

## Display Bin Information Using Default Options

Create a creditscorecard object using the CreditCardData.mat file to load the data (using a dataset from Refaat 2011).

```
load CreditCardData
```

sc = creditscorecard(data);

Display bin information for the categorical predictor ResStatus.

```
bi = bininfo(sc,'ResStatus')
```



## Display Bin Information For a creditscorecard Object Containing Weights

Use the CreditCardData.mat file to load the data (dataWeights) that contains a column (RowWeights) for the weights (using a dataset from Refaat 2011).
load CreditCardData
Create a creditscorecard object using the optional name-value pair argument for 'WeightsVar'.

```
sc = creditscorecard(dataWeights,'WeightsVar','RowWeights')
SC =
    creditscorecard with properties:
```

GoodLabel: 0

```
        ResponseVar: 'status'
        WeightsVar: 'RowWeights'
            VarNames: {1x12 cell}
    NumericPredictors: {1x7 cell}
CategoricalPredictors: {'ResStatus' 'EmpStatus' 'OtherCC'}
        BinMissingData: 0
            IDVar: ''
        PredictorVars: {1x10 cell}
                            Data: [1200x12 table]
```

Display bin information for the numerical predictor 'CustIncome'. When the optional name-value pair argument 'WeightsVar' is used to specify observation (sample) weights, the bi table contains weighted counts.

```
bi = bininfo(sc,'CustIncome');
bi(1:10,:)
\begin{tabular}{|c|c|c|c|c|c|}
\hline Bin & Good & Bad & Odds & WOE & InfoValue \\
\hline ' 18000 ' & 0.94515 & 1.496 & 0.63179 & -1.1667 & 0.0059198 \\
\hline '19000' & 0.47588 & 0.80569 & 0.59065 & -1.2341 & 0.0034716 \\
\hline '20000' & 2.1671 & 1.4636 & 1.4806 & -0.31509 & 0.00061392 \\
\hline '21000' & 3.2522 & 0.88064 & 3.693 & 0.59889 & 0.0021303 \\
\hline '22000' & 1.5438 & 1.2714 & 1.2142 & -0.51346 & 0.0012913 \\
\hline '23000' & 1.787 & 2.7529 & 0.64913 & -1.1397 & 0.010509 \\
\hline '24000' & 3.4111 & 2.2538 & 1.5135 & -0.29311 & 0.00082663 \\
\hline '25000' & 2.2333 & 6.1383 & 0.36383 & -1.7186 & 0.042642 \\
\hline '26000' & 2.1246 & 4.4754 & 0.47474 & -1.4525 & 0.024526 \\
\hline '27000' & 3.1058 & 3.528 & 0.88032 & -0.83501 & 0.0082144 \\
\hline
\end{tabular}
```


## Display Bin Information Using Name-Value Arguments

Create a creditscorecard object using the CreditCardData.mat file to load the data (using a dataset from Refaat 2011).
load CreditCardData
sc = creditscorecard(data);

Display customized bin information for the categorical predictor ResStatus, keeping only the WOE column. The Weight-of-Evidence (WOE) is defined bin by bin, but there is no concept of "total WOE", therefore the last element in the 'Totals ' row is set to NaN.

```
bi = bininfo(sc,'ResStatus','Statistics','W0E');
disp(bi)
\begin{tabular}{|c|c|c|c|}
\hline Bin & Good & Bad & WOE \\
\hline Home Owner' & 365 & 177 & 0.019329 \\
\hline 'Tenant' & 307 & 167 & -0.095564 \\
\hline Other' & 131 & 53 & 0.20049 \\
\hline 'Totals' & 803 & 397 & NaN \\
\hline
\end{tabular}
```

Display customized bin information for the categorical predictor ResStatus, keeping only the Odds and WOE columns, without the Totals row.

```
bi = bininfo(sc,'ResStatus','Statistics',{'Odds','W0E'},'Totals','Off');
disp(bi)
\begin{tabular}{|c|c|c|c|c|}
\hline Bin & Good & Bad & Odds & WOE \\
\hline 'Home Owner' & 365 & 177 & 2.0621 & 0.019329 \\
\hline 'Tenant' & 307 & 167 & 1.8383 & -0.095564 \\
\hline 'Other' & 131 & 53 & 2.4717 & 0.20049 \\
\hline
\end{tabular}
```

Display information value, entropy, Gini, and chi square statistics. For more information on these statistics, see "Statistics for a Credit Scorecard" on page 19-2160.

For information value, entropy and Gini, the value reported at a bin level is the contribution of the bin to the total value. The total information value, entropy, and Gini measures are in the 'Totals' row.

For chi square, if there are $N$ bins, the first $N-1$ values in the 'Chi2' column report pairwise chi square statistics for contiguous bins. For example, this pairwise measure is also used by the 'Merge' algorithm in autobinning to determine if two contiguous bins should be merged. In this example, the first value in the 'Chi2 ' column (1.0331) is the chi square statistic of bins 1 and 2 ('Home Owner' and 'Tenant'), and the second value in the column $(2.5103)$ is the chi square statistic of bins 2 and 3 ('Tenant ' and ' Other'). There are no more pairwise chi square values to compute in this example, so the third element of the 'Chi2' column is set to NaN . The chi square value reported in the 'Totals' row is the chi square statistic computed over all bins.

```
bi = bininfo(sc,'ResStatus','Statistics',{'InfoValue','Entropy','Gini','Chi2'});
disp(bi)
```

| Bin | Good | Bad | InfoValue | Entropy | Gini | Chi2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 'Home Owner' | 365 | 177 | 0.0001682 | 0.91138 | 0.43984 | 1.0331 |
| 'Tenant' | 307 | 167 | 0.0036638 | 0.93612 | 0.45638 | 2.5103 |
| '0ther' | 131 | 53 | 0.0059418 | 0.86618 | 0.41015 | NaN |
| 'Totals' | 803 | 397 | 0.0097738 | 0.91422 | 0.44182 | 2.5549 |

## Display Bin Information and Binning Map for Categorical Data

Create a creditscorecard object using the CreditCardData.mat file to load the data (using a dataset from Refaat 2011).

```
load CreditCardData
sc = creditscorecard(data);
```

The binning map or rules for categorical data are summarized in a "category grouping" table, returned as an optional output. By default, each category is placed in a separate bin. Here is the information for the predictor ResStatus.

```
[bi,cg] = bininfo(sc,'ResStatus')
bi=4\times6 table
    Bin Good Bad
- Bood
    'Home Owner'
    365 177
                                2.0621
\begin{tabular}{rlr}
\multicolumn{1}{c}{ WOE } & & InfoValue \\
\cline { 1 - 1 } 0.019329 & & 0.0001682 \\
-0.095564 & & 0.0036638 \\
0.20049 & & 0.0059418 \\
NaN & 0.0097738
\end{tabular}
```

```
cg=3\times2 table
```

cg=3\times2 table
Category
Category
BinNumber
BinNumber
'Home Owner' 1
'Home Owner' 1
'Tenant' 2
'Tenant' 2
'Other' 3

```
    'Other' 3
```

To group categories Tenant and Other, modify the category grouping table cg so that the bin number for 0ther is the same as the bin number for Tenant. Then use the modifybins function to update the scorecard.

```
cg.BinNumber(3) = 2;
sc = modifybins(sc,'ResStatus','CatGrouping',cg);
```

Display the updated bin information. The bin labels have been updated and that the bin membership information is contained in the category grouping cg.

```
[bi,cg] = bininfo(sc,'ResStatus')
bi=3\times6 table
        Bin Good Bad Odds
```

Odds
. 0621
1.9909
2.0227

WOE
0.019329
0.0001682
'Group1' 365
'Group2' 438
'Totals' 803397

BinNumber
'Home Owner' 1
'Tenant' 2
'Other' 2
-

```
cg=3\times2 table
    Category
            1
    2
```


## Display Bin Information and Binning Map for Numeric Data

Create a creditscorecard object using the CreditCardData.mat file to load the data (using a dataset from Refaat 2011).

```
load CreditCardData
sc = creditscorecard(data);
```

The predictor CustIncome is numeric. By default, each value of the predictor is placed in a separate bin.

```
bi = bininfo(sc,'CustIncome')
```

| Bin | Good | Bad | 0dds | WOE | InfoValue |
| :---: | :---: | :---: | :---: | :---: | :---: |
| '18000' | 2 | 3 | 0.66667 | -1.1099 | 0.0056227 |
| '19000' | 1 | 2 | 0.5 | -1.3976 | 0.0053002 |
| '20000' | 4 | 2 | 2 | -0.011271 | $6.3641 e-07$ |
| '21000' | 6 | 3 | 2 | -0.011271 | 9.5462e-07 |
| '22000' | 4 | 2 | 2 | -0.011271 | 6.3641e-07 |
| '23000' | 4 | 4 | 1 | -0.70442 | 0.0035885 |
| '24000' | 5 | 5 | 1 | -0.70442 | 0.0044856 |
| '25000' | 4 | 9 | 0.44444 | -1.5153 | 0.026805 |
| '26000' | 4 | 11 | 0.36364 | -1.716 | 0.038999 |
| '27000' | 6 | 6 | 1 | -0.70442 | 0.0053827 |
| '28000' | 13 | 11 | 1.1818 | -0.53736 | 0.0061896 |
| '29000' | 11 | 10 | 1.1 | -0.60911 | 0.0069988 |
| '30000' | 18 | 16 | 1.125 | -0.58664 | 0.010493 |
| '31000' | 24 | 8 | 3 | 0.39419 | 0.0038382 |
| '32000' | 21 | 15 | 1.4 | -0.36795 | 0.0042797 |
| '33000' | 35 | 19 | 1.8421 | -0.093509 | 0.00039951 |

Reduce the number of bins using the autobinning function (the modifybins function can also be used).
sc = autobinning(sc,'CustIncome');
Display the updated bin information. The binning map or rules for numeric data are summarized as "cut points," returned as an optional output (cp).

| $\begin{array}{r} \text { bi=8×6 table } \\ \text { Bin } \end{array}$ | Good | Bad | Odds | WOE | InfoValue |
| :---: | :---: | :---: | :---: | :---: | :---: |
| '[-Inf, 29000) ' | 53 | 58 | 0.91379 | -0.79457 | 0.06364 |
| '[29000, 33000)' | 74 | 49 | 1.5102 | -0.29217 | 0.0091366 |
| '[33000, 35000) ' | 68 | 36 | 1.8889 | -0.06843 | 0.00041042 |
| '[35000, 40000) ' | 193 | 98 | 1.9694 | -0.026696 | 0.00017359 |
| '[40000, 42000)' | 68 | 34 | 2 | -0.011271 | 1.0819e-05 |
| ' [42000, 47000) ' | 164 | 66 | 2.4848 | 0.20579 | 0.0078175 |
| '[47000, Inf]' | 183 | 56 | 3.2679 | 0.47972 | 0.041657 |

```
    'Totals' 803 397 2.0227 NaN 0.12285
cp = 6\times1
    29000
    33000
    35000
    4 0 0 0 0
    4 2 0 0 0
    4 7 0 0 0
```

Manually remove the second cut point (the boundary between the second and third bins) to merge bins two and three. Use the modifybins function to update the scorecard.

```
cp(2) = [];
sc = modifybins(sc,'CustIncome','CutPoints',cp,'MinValue',0);
```

Display the updated bin information.
[bi,cp,mv] = bininfo(sc,'CustIncome')
bi=7×6 table
Bin
$\qquad$
Good
Bad
0dds


InfoValue

$$
-0.79457
$$

0.06364
[0,29000)' 53
'[35000,40000)' 193
'[40000,42000)' 68

.0071274
[4200, 47000 .
164
183
803397

## 66

56
3.2679
2.0227
0.47972
0.00017359
'[42000, 47000) '
58
85
98
34
66
56
397

1. 6706
-0. 19124
'[47000,Inf]'
都
$c p=5 \times 1$
29000
35000
40000
42000
47000

19-2153
$\mathrm{mv}=1 \times 2$

0 Inf

Note, it is recommended to avoid having bins with frequencies of zero because they lead to infinite or undefined ( NaN ) statistics. Use the modifybins or autobinning functions to modify bins.

## Display Bin Information for Missing Data

Create a creditscorecard object using the CreditCardData.mat file to load the dataMissing with missing values.
load CreditCardData.mat
head(dataMissing,5)

```
ans=5\times11 table
CustID CustAge TmAtAddress ResStatus EmpStatus CustIncome
\begin{tabular}{lrr} 
& & \\
& 53 & 62 \\
2 & 61 & 22 \\
3 & 47 & 30 \\
4 & NaN & 75 \\
5 & 68 & 56
\end{tabular}
```

<undefined>
Home Owner Tenant
Home Owner
Home Owner

```
Unknown
Employed
Employed
Employed
Employed
50000
fprintf('Number of rows: \%d\n',height(dataMissing))
Number of rows: 1200
fprintf('Number of missing values CustAge: \%d\n',sum(ismissing(dataMissing.CustAge)))
Number of missing values CustAge: 30
fprintf('Number of missing values ResStatus: \%d\n',sum(ismissing(dataMissing.ResStatus)
Number of missing values ResStatus: 40
```

Use creditscorecard with the name-value argument 'BinMissingData' set to true to bin the missing data in a separate bin.

```
sc = creditscorecard(dataMissing,'IDVar','CustID','BinMissingData',true);
sc = autobinning(sc);
disp(sc)
    creditscorecard with properties:
                        GoodLabel: 0
                ResponseVar: 'status'
                    WeightsVar:
                            VarNames: {1x11 cell}
        NumericPredictors: {1x6 cell}
    CategoricalPredictors: {'ResStatus' 'EmpStatus' 'OtherCC'}
        BinMissingData: 1
                            IDVar: 'CustID'
                PredictorVars: {1x9 cell}
                            Data: [1200x11 table]
```

Display bin information for numeric data for 'CustAge' that includes missing data in a separate bin labelled <missing>.

```
bi = bininfo(sc,'CustAge');
disp(bi)
\begin{tabular}{|c|c|c|c|c|c|}
\hline Bin & Good & Bad & Odds & WOE & InfoValue \\
\hline '[-Inf,33)' & 69 & 52 & 1.3269 & -0.42156 & 0.018993 \\
\hline '[33,37)' & 63 & 45 & 1.4 & -0.36795 & 0.012839 \\
\hline ' \([37,40\) )' & 72 & 47 & 1.5319 & -0.2779 & 0.0079824 \\
\hline '[40,46)' & 172 & 89 & 1.9326 & -0.04556 & 0.0004549 \\
\hline ' \([46,48\) )' & 59 & 25 & 2.36 & 0.15424 & 0.0016199 \\
\hline ' \([48,51\) )' & 99 & 41 & 2.4146 & 0.17713 & 0.0035449 \\
\hline '[51,58)' & 157 & 62 & 2.5323 & 0.22469 & 0.0088407 \\
\hline '[58,Inf]' & 93 & 25 & 3.72 & 0.60931 & 0.032198 \\
\hline '<missing>' & 19 & 11 & 1.7273 & -0.15787 & 0.00063885 \\
\hline 'Totals' & 803 & 397 & 2.0227 & NaN & 0.087112 \\
\hline
\end{tabular}
plotbins(sc,'CustAge')
```



Display bin information for categorical data for 'ResStatus' that includes missing data in a separate bin labelled <missing>.
[bi,cg] = bininfo(sc,'ResStatus'); disp(bi)

| Bin | Good | Bad | 0dds | WOE | InfoValue |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 'Tenant ' | 296 | 161 | 1.8385 | -0.095463 | 0.0035249 |
| 'Home Owner' | 352 | 171 | 2.0585 | 0.017549 | 0.00013382 |
| 'Other' | 128 | 52 | 2.4615 | 0.19637 | 0.0055808 |
| '<missing>' | 27 | 13 | 2.0769 | 0.026469 | 2.3248e-05 |
| 'Totals' | 803 | 397 | 2.0227 | NaN | 0.0092627 |

```
disp(cg)
    Category
'Tenant' 1
'Home Owner' 2
'Other' 3
```

Note that the category grouping table does not include <missing> because this is a reserved bin and users cannot interact directly with the <missing> bin.
plotbins(sc,'ResStatus')


## Input Arguments

sc - Credit scorecard model

creditscorecard object

Credit scorecard model, specified as a creditscorecard object. Use creditscorecard to create a creditscorecard object.

## PredictorName - Predictor name

character vector
Predictor name, specified using a character vector containing the name of the predictor. PredictorName is case-sensitive.
Data Types: char

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, ... , NameN, ValueN.

Example: bi = bininfo(sc, PredictorName,'Statistics','W0E','Totals','On')

## Statistics - List of statistics to include for bin information

\{'Odds','WOE','InfoValue'\} (default)| character vector with values 'Odds', 'WOE', 'InfoValue','Entropy', 'Gini', 'Chi2' | cell array of character vectors with values 'Odds', 'WOE', 'InfoValue', 'Entropy','Gini', 'Chi2'

List of statistics to include in the bin information, specified as the comma-separated pair consisting of 'Statistics' and a character vector or a cell array of character vectors. For more information, see "Statistics for a Credit Scorecard" on page 19-2160. Possible values are:

- 'Odds ' - Odds information is the ratio of "Goods" over "Bads."
- 'WOE ' - Weight of Evidence. The WOE Statistic measures the deviation between the distribution of "Goods" and "Bads."
- 'InfoValue' - Information value. Closely tied to the WOE, it is a statistic used to determine how strong a predictor is to use in the fitting model. It measures how
strong the deviation is between the distributions of "Goods" and "Bads." However, bins with only "Good" or only "Bad" observations do lead to an infinite Information Value. Consider modifying the bins in those cases by using modifybins or autobinning.
- 'Entropy ' - Entropy is a measure of unpredictability contained in the bins. The more the number of "Goods" and "Bads" differ within the bins, the lower the entropy.
- 'Gini ' - Measure of statistical dispersion or inequality within a sample of data.
- 'Chi2' - Measure of statistical difference and independence between groups.

Note Avoid having bins with frequencies of zero because they lead to infinite or undefined ( NaN ) statistics. Use modifybins or autobinning to modify bins.

## Data Types: char | cell

## Totals - Indicator to include row of totals at bottom information table

'On' (default)| character vector with values 'On', 'Off'
Indicator to include a row of totals at the bottom of the information table, specified as the comma-separated pair consisting of 'Totals' and a character vector with values 0 n or Off.

Data Types: char

## Output Arguments

## bi - Bin information

table
Bin information, returned as a table. The bin information table contains one row per bin and a row of totals. The columns contain bin descriptions, frequencies of "Good" and "Bad," and bin statistics. Avoid having bins with frequencies of zero because they lead to infinite or undefined ( NaN ) statistics. Use modifybins or autobinning to modify bins.

Note When creating the creditscorecard object with creditscorecard, if the optional name-value pair argument WeightsVar was used to specify observation (sample) weights, then the bi table contains weighted counts.

## bm - Binning map or rules

vector of cut points for numeric predictors | table of category groupings for categorical predictors

Binning map or rules, returned as a vector of cut points for numeric predictors, or a table of category groupings for categorical predictors. For more information, see modifybins.

## mv - Binning minimum and maximum values

numeric array
Binning minimum and maximum values (as set or defined by the user), returned as a numeric array. The mv output argument is set to an empty array for categorical predictors.

## Definitions

## Statistics for a Credit Scorecard

Weight of Evidence (WOE) is a measure of the difference of the distribution of "Goods" and "Bads" within a bin.

Suppose the predictor's data takes on $M$ possible values $\mathrm{b} 1, \ldots, \mathrm{~b} M$. For binned data, $M$ is a small number. The response takes on two values, "Good" and "Bad." The frequency table of the data is given by:

|  | Good | Bad | Total |
| :--- | :--- | :--- | :--- |
| $\mathrm{b} 1:$ | n11 | n12 | n1 |
| $\mathrm{b} 2:$ | n21 | n22 | n2 |
| $\mathrm{bM}:$ | nM1 | nM2 | nM |
| Total: | nGood | nBad | nTotal |

The Weight of Evidence (WOE) is defined for each data value bi as

```
WOE(i) = log((ni1/nGood)/(ni2/nBad)).
```

If you define

```
pGood(i) = nil/nGood, pBad(i) = ni2/nBad
```

then $\operatorname{pGood}(i)$ is the proportion of "Good" observations that take on the value bi, and similarly for $\mathrm{pBad}(i)$. In other words, $\mathrm{pGood}(i)$ gives the distribution of good observations over the $M$ observed values of the predictor, and similarly for pBad(i). With this, an equivalent formula for the WOE is

WOE(i) $=\log ($ pGood(i)/pBad(i)).
Using the same frequency table, the odds for row $i$ are defined as
Odds(i) = nil / ni2,
and the odds for the sample are defined as
OddsTotal = nGood / nBad.
For each row $i$, you can also compute its contribution to the total Information Value, given by

```
InfoValue(i) = (pGood(i) - pBad(i)) * WOE(i),
```

and the total Information Value is simply the sum of all the InfoValuel(i) terms. (A nansum is returned to discard contributions from rows with no observations at all.)

Likewise, for each row $i$, we can compute its contribution to the total Entropy, given by

```
    Entropy(i) = -1/log(2)*(ni1/ni*log(ni1/ni)+ni2/ni*log(ni2/ni),
```

and the total Entropy is simply the weighted sum of the row entropies,

```
Entropy = sum(ni/nTotal * Entropy(i)), i = 1...M.
```

Chi2 is computed pairwise for each pair of bins and measures the statistical difference between two groups when splitting or merging bins and is defined as:

```
Chi2 = sum(sum((Aij - Eij)^2/Eij , j=1..k), i=m,m+1).
```

For more information on splitting and merging bins, see Split on page 19-2193 and Merge on page 19-2196.

Gini ratio is a measure of the parent node, that is, of the given bins/categories prior to splitting or merging. The Gini ratio is defined as:

```
Gr = 1-G_hat/Gp
```

G_hat is the weighted Gini measure for the current split or merge:

G_hat $=\operatorname{Sum}((\mathrm{nj} / \mathrm{N}) * \mathrm{Gj}, \mathrm{j}=1 . \mathrm{m})$.
For more information on splitting and merging bins, see Split on page 19-2193 and Merge on page 19-2196.

## Using bininfo with Weights

When observation weights are defined using the optional WeightsVar argument when creating a creditscorecard object, instead of counting the rows that are good or bad in each bin, the bininfo function accumulates the weight of the rows that are good or bad in each bin.

The "frequencies" reported are no longer the basic "count" of rows, but the "cumulative weight" of the rows that are good or bad and fall in a particular bin. Once these "weighted frequencies" are known, all other relevant statistics (Good, Bad, Odds, WOE, and InfoValue) are computed with the usual formulas. For more information, see "Credit Scorecard Modeling Using Observation Weights" on page 8-70.

## References

[1] Anderson, R. The Credit Scoring Toolkit. Oxford University Press, 2007.
[2] Refaat, M. Credit Risk Scorecards: Development and Implementation Using SAS. lulu.com, 2011.

## See Also

autobinning|bindata|creditscorecard|displaypoints|fitmodel| formatpoints|modifybins |modifypredictor|plotbins|predictorinfo| probdefault|score|setmodel|validatemodel

## Topics

"Case Study for a Credit Scorecard Analysis" on page 8-101
"Credit Scorecard Modeling with Missing Values" on page 8-73
"Troubleshooting Credit Scorecard Results" on page 8-91
"Credit Scorecard Modeling Workflow" on page 8-67
"About Credit Scorecards" on page 8-62
"Credit Scorecard Modeling Using Observation Weights" on page 8-70

## Introduced in R2014b

## autobinning

Perform automatic binning of given predictors

## Syntax

```
sc = autobinning(sc)
sc = autobinning(sc,PredictorNames)
sc = autobinning( ___ ,Name,Value)
```


## Description

sc = autobinning(sc) performs automatic binning of all predictors.
Automatic binning finds binning maps or rules to bin numeric data and to group categories of categorical data. The binning rules are stored in the creditscorecard object. To apply the binning rules to the creditscorecard object data, or to a new dataset, use bindata.
sc = autobinning(sc, PredictorNames) performs automatic binning of the predictors given in PredictorNames.

Automatic binning finds binning maps or rules to bin numeric data and to group categories of categorical data. The binning rules are stored in the creditscorecard object. To apply the binning rules to the creditscorecard object data, or to a new dataset, use bindata.
sc = autobinning( $\qquad$ , Name, Value) performs automatic binning of the predictors given in PredictorNames using optional name-value pair arguments. See the namevalue argument Algorithm for a description of the supported binning algorithms.

Automatic binning finds binning maps or rules to bin numeric data and to group categories of categorical data. The binning rules are stored in the creditscorecard object. To apply the binning rules to the creditscorecard object data, or to a new dataset, use bindata.

## Examples

## Perform Automatic Binning Using the Defaults

Create a creditscorecard object using the CreditCardData.mat file to load the data (using a dataset from Refaat 2011).

```
load CreditCardData
sc = creditscorecard(data,'IDVar','CustID');
```

Perform automatic binning using the default options. By default, autobinning bins all predictors and uses the Monotone algorithm.

```
sc = autobinning(sc);
```

Use bininfo to display the binned data for the predictor CustAge.

```
bi = bininfo(sc, 'CustAge')
\begin{tabular}{|c|c|c|c|c|c|}
\hline Bin & Good & Bad & 0dds & WOE & InfoValue \\
\hline '[-Inf,33)' & 70 & 53 & 1.3208 & -0.42622 & 0.019746 \\
\hline ' \([33,37)\) ' & 64 & 47 & 1.3617 & -0.39568 & 0.015308 \\
\hline ' \([37,40\) )' & 73 & 47 & 1.5532 & -0.26411 & 0.0072573 \\
\hline ' \([40,46\) ) ' & 174 & 94 & 1.8511 & -0.088658 & 0.001781 \\
\hline ' \([46,48)\) ' & 61 & 25 & 2.44 & 0.18758 & 0.0024372 \\
\hline ' \([48,58\) )' & 263 & 105 & 2.5048 & 0.21378 & 0.013476 \\
\hline '[58,Inf]' & 98 & 26 & 3.7692 & 0.62245 & 0.0352 \\
\hline 'Totals' & 803 & 397 & 2.0227 & NaN & 0.095205 \\
\hline
\end{tabular}
```

Use plotbins to display the histogram and WOE curve for the predictor CustAge.

```
plotbins(sc,'CustAge')
```



## Perform Automatic Binning with a Named Predictor Using the Defaults

Create a creditscorecard object using the CreditCardData.mat file to load the data (using a dataset from Refaat 2011).
load CreditCardData
sc = creditscorecard(data);
Perform automatic binning for the predictor CustIncome using the default options. By default, autobinning uses the Monotone algorithm.

```
sc = autobinning(sc,'CustIncome');
```

Use bininfo to display the binned data.

| bi=8×6 table Bin | Good | Bad | Odds | WOE | InfoValue |
| :---: | :---: | :---: | :---: | :---: | :---: |
| '[-Inf, 29000) ' | 53 | 58 | 0.91379 | -0.79457 | 0.06364 |
| '[29000, 33000)' | 74 | 49 | 1.5102 | -0.29217 | 0.0091366 |
| '[33000, 35000)' | 68 | 36 | 1.8889 | -0.06843 | 0.00041042 |
| '[35000, 40000)' | 193 | 98 | 1.9694 | -0.026696 | 0.00017359 |
| '[40000, 42000) ' | 68 | 34 | 2 | -0.011271 | 1.0819e-05 |
| '[42000, 47000) ' | 164 | 66 | 2.4848 | 0.20579 | 0.0078175 |
| '[47000,Inf]' | 183 | 56 | 3.2679 | 0.47972 | 0.041657 |
| 'Totals' | 803 | 397 | 2.0227 | NaN | 0.12285 |

## Perform Automatic Binning Using Two Name-Value Pair Arguments

Create a creditscorecard object using the CreditCardData.mat file to load the data (using a dataset from Refaat 2011).

```
load CreditCardData
sc = creditscorecard(data);
```

Perform automatic binning for the predictor CustIncome using the Monotone algorithm with the initial number of bins set to 20 . This example explicitly sets both the Algorithm and the AlgorithmOptions name-value arguments.

```
AlgoOptions = {'InitialNumBins',20};
```

sc = autobinning(sc,'CustIncome','Algorithm','Monotone','Algorithm0ptions', ...
AlgoOptions);

Use bininfo to display the binned data. Here, the cut points, which delimit the bins, are also displayed.

```
[bi,cp] = bininfo(sc,'CustIncome')
```

```
bi=11\times6 table Good Bad Odds WOE InfoValue
```

| '[-Inf, 19000) ' | 2 | 3 | 0.66667 | -1.1099 | 0.0056227 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| '[19000, 29000) ' | 51 | 55 | 0.92727 | -0.77993 | 0.058516 |
| ' $[29000,31000$ )' | 29 | 26 | 1.1154 | -0.59522 | 0.017486 |
| ' $[31000,34000)^{\prime}$ | 80 | 42 | 1.9048 | -0.060061 | 0.0003704 |
| '[34000, 35000)' | 33 | 17 | 1.9412 | -0.041124 | 7.095e-05 |
| '[35000, 40000)' | 193 | 98 | 1.9694 | -0.026696 | 0.00017359 |
| '[40000, 42000)' | 68 | 34 | 2 | -0.011271 | 1.0819e-05 |
| '[42000, 43000)' | 39 | 16 | 2.4375 | 0.18655 | 0.001542 |
| '[43000, 47000)' | 125 | 50 | 2.5 | 0.21187 | 0.0062972 |
| '[47000,Inf]' | 183 | 56 | 3.2679 | 0.47972 | 0.041657 |
| 'Totals' | 803 | 397 | 2.0227 | NaN | 0.13175 |

19000
29000
31000
34000
35000
40000
42000
43000
47000

## Perform Automatic Binning Using Multiple Name-Value Pair Arguments

This example shows how to use the autobinning default Monotone algorithm and the AlgorithmOptions name-value pair arguments associated with the Monotone algorithm. The AlgorithmOptions for the Monotone algorithm are three name-value pair parameters: 'InitialNumBins', 'Trend', and 'SortCategories'.
'InitialNumBins' and 'Trend' are applicable for numeric predictors and 'Trend ' and 'SortCategories' are applicable for categorical predictors.

Create a creditscorecard object using the CreditCardData.mat file to load the data (using a dataset from Refaat 2011).

```
load CreditCardData
sc = creditscorecard(data,'IDVar','CustID');
```

Perform automatic binning for the numeric predictor Cust Income using the Monotone algorithm with 20 bins. This example explicitly sets both the Algorithm argument and the Algorithm0ptions name-value arguments for 'InitialNumBins' and 'Trend'.

```
AlgoOptions = {'InitialNumBins',20,'Trend','Increasing'};
```

```
sc = autobinning(sc,'CustIncome','Algorithm','Monotone',...
```

sc = autobinning(sc,'CustIncome','Algorithm','Monotone',...
'AlgorithmOptions',AlgoOptions);

```

Use bininfo to display the binned data.
\begin{tabular}{|c|c|c|c|c|c|}
\hline \[
\begin{array}{r}
\text { bi=11×6 table } \\
\text { Bin }
\end{array}
\] & Good & Bad & 0dds & WOE & InfoValue \\
\hline '[-Inf, 19000) ' & 2 & 3 & 0.66667 & -1.1099 & 0.0056227 \\
\hline '[19000, 29000)' & 51 & 55 & 0.92727 & -0.77993 & 0.058516 \\
\hline ' [29000, 31000) ' & 29 & 26 & 1.1154 & -0.59522 & 0.017486 \\
\hline '[31000, 34000) ' & 80 & 42 & 1.9048 & -0.060061 & 0.0003704 \\
\hline ' [34000, 35000) ' & 33 & 17 & 1.9412 & -0.041124 & 7.095e-05 \\
\hline '[35000, 40000)' & 193 & 98 & 1.9694 & -0.026696 & 0.00017359 \\
\hline ' [40000, 42000) ' & 68 & 34 & 2 & -0.011271 & 1.0819e-05 \\
\hline '[42000, 43000) ' & 39 & 16 & 2.4375 & 0.18655 & 0.001542 \\
\hline '[43000, 47000)' & 125 & 50 & 2.5 & 0.21187 & 0.0062972 \\
\hline '[47000, Inf]' & 183 & 56 & 3.2679 & 0.47972 & 0.041657 \\
\hline 'Totals' & 803 & 397 & 2.0227 & NaN & 0.13175 \\
\hline
\end{tabular}

\section*{Perform Automatic Binning for Multiple Predictors}

Create a creditscorecard object using the CreditCardData.mat file to load the data (using a dataset from Refaat 2011).
load CreditCardData
sc = creditscorecard(data,'IDVar','CustID');
Perform automatic binning for the predictor CustIncome and CustAge using the default Monotone algorithm with AlgorithmOptions for InitialNumBins and Trend.
```

AlgoOptions = {'InitialNumBins',20,'Trend','Increasing'};

```
```

sc = autobinning(sc,{'CustAge','CustIncome'},'Algorithm','Monotone',...
'AlgorithmOptions',AlgoOptions);

```

Use bininfo to display the binned data.
```

bil = bininfo(sc, 'CustIncome')

```

```

bi2 = bininfo(sc, 'CustAge')

```
\begin{tabular}{|c|c|c|c|c|c|}
\hline \[
\begin{gathered}
\times 6 \text { table } \\
\text { Bin }
\end{gathered}
\] & Good & Bad & 0dds & WOE & InfoValue \\
\hline '[-Inf,35)' & 93 & 76 & 1.2237 & -0.50255 & 0.038003 \\
\hline ' \([35,40\) )' & 114 & 71 & 1.6056 & -0.2309 & 0.0085141 \\
\hline ' \([40,42\) )' & 52 & 30 & 1.7333 & -0.15437 & 0.0016687 \\
\hline ' \([42,44\) )' & 58 & 32 & 1.8125 & -0.10971 & 0.00091888 \\
\hline ' \([44,47)^{\prime}\) & 97 & 51 & 1.902 & -0.061533 & 0.00047174 \\
\hline ' \([47,62\) ) ' & 333 & 130 & 2.5615 & 0.23619 & 0.020605 \\
\hline '[62,Inf]' & 56 & 7 & 8 & 1.375 & 0.071647 \\
\hline 'Totals' & 803 & 397 & 2.0227 & NaN & 0.14183 \\
\hline
\end{tabular}

\section*{Perform Automatic Binning for a Categorical Predictor Using the Defaults}

Create a creditscorecard object using the CreditCardData.mat file to load the data (using a dataset from Refaat 2011).
```

load CreditCardData

```
sc = creditscorecard(data);

Perform automatic binning for the predictor that is a categorical predictor called ResStatus using the default options. By default, autobinning uses the Monotone algorithm.
```

sc = autobinning(sc,'ResStatus');

```

Use bininfo to display the binned data.
```

bi = bininfo(sc, 'ResStatus')

| Bin | Good | Bad | Odds | WOE | InfoValue |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 'Tenant ' | 307 | 167 | 1.8383 | -0.095564 | 0.0036638 |
| 'Home Owner' | 365 | 177 | 2.0621 | 0.019329 | 0.0001682 |
| 'Other' | 131 | 53 | 2.4717 | 0.20049 | 0.0059418 |
| 'Totals' | 803 | 397 | 2.0227 | NaN | 0.0097738 |

```

\section*{Perform Automatic Binning for a Categorical Predictor Using Name-Value Pair Arguments}

This example shows how to modify the data (for this example only) to illustrate binning categorical predictors using the Monotone algorithm.

Create a creditscorecard object using the CreditCardData.mat file to load the data (using a dataset from Refaat 2011).
load CreditCardData
Add two new categories and updating the response variable.
```

newdata = data;
rng('default'); %for reproducibility

```
```

Predictor = 'ResStatus';
Status = newdata.status;
NumObs = length(newdata.(Predictor));
Ind1 = randi(NumObs,100,1);
Ind2 = randi(NumObs,100,1);
newdata.(Predictor)(Ind1) = 'Subtenant';
newdata.(Predictor)(Ind2) = 'CoOwner';
Status(Ind1) = randi(2,100,1)-1;
Status(Ind2) = randi(2,100,1)-1;
newdata.status = Status;

```

Update the creditscorecard object using the newdata and plot the bins for a later comparison.
```

scnew = creditscorecard(newdata,'IDVar','CustID');
[bi,cg] = bininfo(scnew,Predictor)

| Bin | Good | Bad | Odds | WOE | InfoValue |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 'Home Owner' | 308 | 154 | 2 | 0.092373 | 0.0032392 |
| 'Tenant' | 264 | 136 | 1.9412 | 0.06252 | 0.0012907 |
| 'Other' | 109 | 49 | 2.2245 | 0.19875 | 0.0050386 |
| 'Subtenant' | 42 | 42 | 1 | -0.60077 | 0.026813 |
| 'CoOwner' | 52 | 44 | 1.1818 | -0.43372 | 0.015802 |
| 'Totals' | 775 | 425 | 1.8235 | NaN | 0.052183 |

```
cg=5×2 table

    Category BinNumber
    'Home Owner' 1
    'Tenant' 2
    'Other' 3
    'Subtenant' 4
    'CoOwner' 5
plotbins(scnew, Predictor)


Perform automatic binning for the categorical Predictor using the default Monotone algorithm with the Algorithm0ptions name-value pair arguments for 'SortCategories' and 'Trend'.

AlgoOptions = \{'SortCategories','Goods','Trend','Increasing'\};
scnew = autobinning(scnew,Predictor,'Algorithm','Monotone',...
'AlgorithmOptions',AlgoOptions);
Use bininfo to display the bin information. The second output parameter ' cg ' captures the bin membership, which is the bin number that each group belongs to.
[bi,cg] = bininfo(scnew,Predictor)
```

bi=4\times6 table

| Bin | Good | Bad | Odds | WOE | InfoValue |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 'Group1' | 42 | 42 | 1 | -0.60077 | 0.026813 |
| 'Group2' | 52 | 44 | 1.1818 | -0.43372 | 0.015802 |
| 'Group3' | 681 | 339 | 2.0088 | 0.096788 | 0.0078459 |
| 'Totals' | 775 | 425 | 1.8235 | NaN | 0.05046 |

```
```

cg=5\times2 table

```
cg=5\times2 table
    Category BinNumber
    Category BinNumber
    Subtenant
    Subtenant
                                1
                                1
    'CoOwner' 2
    'CoOwner' 2
    'Other' 3
    'Other' 3
    'Tenant' 3
    'Tenant' 3
    'Home Owner' 3
```

    'Home Owner' 3
    ```

Plot bins and compare with the histogram plotted pre-binning.
```

plotbins(scnew,Predictor)

```


\section*{Perform Automatic Binning When Using Missing Data}

Create a creditscorecard object using the CreditCardData.mat file to load the dataMissing with missing values.
```

load CreditCardData.mat

```
head(dataMissing,5)
```

ans=5\times11 table
CustID CustAge TmAtAddress ResStatus EmpStatus CustIncome

```

19-2175
```

| 1 | 53 | 62 |
| ---: | ---: | ---: |
| 2 | 61 | 22 |
| 3 | 47 | 30 |
| 4 | NaN | 75 |
| 5 | 68 | 56 |

fprintf('Number of rows: %d\n',height(dataMissing))
Number of rows: 1200
fprintf('Number of missing values CustAge: %d\n',sum(ismissing(dataMissing.CustAge)))
Number of missing values CustAge: 30
fprintf('Number of missing values ResStatus: %d\n',sum(ismissing(dataMissing.ResStatus)
Number of missing values ResStatus: 40

```

Use creditscorecard with the name-value argument 'BinMissingData' set to true to bin the missing numeric and categorical data in a separate bin.
```

sc = creditscorecard(dataMissing,'BinMissingData',true);
disp(sc)
creditscorecard with properties:
GoodLabel: 0
ResponseVar: 'status'
WeightsVar: ''
VarNames: {1x11 cell}
NumericPredictors: {1x7 cell}
CategoricalPredictors: {'ResStatus' 'EmpStatus' 'OtherCC'}
BinMissingData: 1
IDVar: ''
PredictorVars: {1x10 cell}
Data: [1200x11 table]

```

Perform automatic binning using the Merge algorithm.
```

sc = autobinning(sc,'Algorithm','Merge');

```

Display bin information for numeric data for 'CustAge' that includes missing data in a separate bin labelled <missing> and this is the last bin. No matter what binning
algorithm is used in autobinning, the algorithm operates on the non-missing data and the bin for the <missing> numeric values for a predictor is always the last bin.
```

[bi,cp] = bininfo(sc,'CustAge');
disp(bi)

```
\begin{tabular}{|c|c|c|c|c|c|}
\hline Bin & Good & Bad & 0dds & WOE & InfoValue \\
\hline '[-Inf,32) ' & 56 & 39 & 1.4359 & -0.34263 & 0.0097643 \\
\hline '[32,33)' & 13 & 13 & 1 & -0.70442 & 0.011663 \\
\hline ' \([33,34\) )' & 9 & 11 & 0.81818 & -0.90509 & 0.014934 \\
\hline ' \([34,65\) )' & 677 & 317 & 2.1356 & 0.054351 & 0.002424 \\
\hline '[65,Inf]' & 29 & 6 & 4.8333 & 0.87112 & 0.018295 \\
\hline '<missing>' & 19 & 11 & 1.7273 & -0.15787 & 0.00063885 \\
\hline
\end{tabular}
plotbins(sc,'CustAge')


Display bin information for categorical data for 'ResStatus' that includes missing data in a separate bin labelled <missing> and this is the last bin. No matter what binning algorithm is used in autobinning, the algorithm operates on the non-missing data and the bin for the <missing> categorical values for a predictor is always the last bin.
```

[bi,cg] = bininfo(sc,'ResStatus');
disp(bi)

| Bin | Good | Bad | Odds | WOE | InfoValue |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 'Group1' | 648 | 332 | 1.9518 | -0.035663 | 0.0010449 |
| 'Group2' | 128 | 52 | 2.4615 | 0.19637 | 0.0055808 |

```


\section*{Perform Automatic Binning Using the Split Algorithm}

This example demonstrates using the 'Split ' algorithm with categorical and numeric predictors. Load the CreditCardData.mat dataset and modify so that it contains four categories for the predictor 'ResStatus ' to demonstrate how the split algorithm works.
```

load CreditCardData.mat
x = data.ResStatus;
Ind = find(x == 'Tenant');
Nx = length(Ind);
x(Ind(1:floor(Nx/3))) = 'Subletter';
data.ResStatus = x;

```

Create a creditscorecard and use bininfo to display the 'Statistics'.
sc = creditscorecard(data,'IDVar','CustID');
[bil,cgl] = bininfo(sc,'ResStatus','Statistics',\{'Odds','WOE','InfoValue'\});
disp(bil)
Bin
'Home Owner'
'Tenant ..... 204'0ther''Subletter''Totals'365131103GoodBad Odds

WOE

177
1.8214
\(\begin{array}{ll}53 & 2.4717 \\ 55 & 1.8727\end{array}\)
112
0.20049
\(1.8727-0.077023 \quad 0.00079103\)
803397
\begin{tabular}{r}
\hline 0.0001682 \\
0.0029415 \\
0.0059418 \\
0.00079103 \\
0.0098426
\end{tabular}
\begin{tabular}{r}
\hline 0.0001682 \\
0.0029415 \\
0.0059418 \\
0.00079103 \\
0.0098426
\end{tabular}
InfoValue

\section*{disp(cg1)}
Category BinNumber
'Home Owner' ..... 1
'Tenant' ..... 2
'Other' ..... 3
'Subletter' ..... 4

\section*{Using the Split Algorithm with a Categorical Predictor}

Apply presorting to the 'ResStatus' category using the default sorting by 'Odds ' and specify the 'Split' algorithm.
```

sc = autobinning(sc,'ResStatus', 'Algorithm', 'split','AlgorithmOptions',...
{'Measure','gini','SortCategories','odds','Tolerance',1e-4});
[bi2,cg2] = bininfo(sc,'ResStatus','Statistics',{'Odds','WOE','InfoValue'});
disp(bi2)

```
\begin{tabular}{|c|c|c|c|c|c|}
\hline Bin & Good & Bad & 0dds & WOE & InfoValue \\
\hline Group1' & 307 & 167 & 1.8383 & -0.095564 & 0. 0036638 \\
\hline
\end{tabular}
\begin{tabular}{rrrrrr} 
'Group2' & 365 & 177 & 2.0621 & 0.019329 & 0.0001682 \\
'Group3' & 131 & 53 & 2.4717 & 0.20049 & 0.0059418 \\
'Totals' & 803 & 397 & 2.0227 & NaN & 0.0097738
\end{tabular}

\section*{disp(cg2)}
\begin{tabular}{lll} 
Category & & \multicolumn{2}{c}{ BinNumber } \\
& & \\
'Tenant' & \\
'Subletter' & & 1 \\
'Home Owner' & 2 \\
'Other' & 3
\end{tabular}

\section*{Using the Split Algorithm with a Numeric Predictor}

To demonstrate a split for the numeric predictor, 'TmAtAddress ' , first use autobinning with the default 'Monotone' algorithm.
```

sc = autobinning(sc,'TmAtAddress');
bi3 = bininfo(sc,'TmAtAddress','Statistics',{'Odds','WOE','InfoValue'});
disp(bi3)

| Bin | Good | Bad | Odds | WOE | InfoValue |
| :---: | :---: | :---: | :---: | :---: | :---: |
| '[-Inf,23)' | 239 | 129 | 1.8527 | -0.087767 | 0.0023963 |
| ' $[23,83$ )' | 480 | 232 | 2.069 | 0.02263 | 0.00030269 |
| '[83, Inf]' | 84 | 36 | 2.3333 | 0.14288 | 0.00199 |
| 'Totals' | 803 | 397 | 2.0227 | NaN | 0.004689 |

```

Then use autobinning with the 'Split' algorithm.
```

sc = autobinning(sc,'TmAtAddress','Algorithm', 'Split');
bi4 = bininfo(sc,'TmAtAddress','Statistics',{'Odds','WOE','InfoValue'});
disp(bi4)

```
\begin{tabular}{|c|c|c|c|c|c|}
\hline Bin & Good & Bad & Odds & WOE & InfoValue \\
\hline '[-Inf,4)' & 20 & 12 & 1.6667 & -0.19359 & 0.0010299 \\
\hline ' \([4,5\) )' & 4 & 7 & 0.57143 & -1.264 & 0.015991 \\
\hline ' \([5,23\) ) & 215 & 110 & 1.9545 & -0.034261 & 0.00031973 \\
\hline ' \([23,33\) )' & 130 & 39 & 3.3333 & 0.49955 & 0.0318 \\
\hline
\end{tabular}
\begin{tabular}{llllrr} 
'[33,Inf]' & 434 & 229 & 1.8952 & -0.065096 & 0.0023664 \\
'Totals' & 803 & 397 & 2.0227 & NaN & 0.051507
\end{tabular}

\section*{Perform Automatic Binning Using the Merge Algorithm}

Load the CreditCardData.mat dataset. This example demonstrates using the 'Merge' algorithm with categorical and numeric predictors.

\section*{load CreditCardData.mat}

\section*{Using the Merge Algorithm with a Categorical Predictor}

To merge a categorical predictor, create a creditscorecard using default sorting by 'Odds' and then use bininfo on the categorical predictor 'ResStatus'.
```

sc = creditscorecard(data,'IDVar','CustID');
[bil,cg1] = bininfo(sc,'ResStatus','Statistics',{'Odds','WOE','InfoValue'});
disp(bil);

```
\begin{tabular}{llllllll}
\multicolumn{2}{c}{ Bin } & Good & \multicolumn{2}{c}{ Bad } & Odds & & \multicolumn{1}{c}{ WOE }
\end{tabular}

\section*{disp(cg1);}

Category BinNumber
'Home Owner' 1
'Tenant' 2
'Other' 3
Use autobinning and specify the 'Merge' algorithm.
```

sc = autobinning(sc,'ResStatus','Algorithm', 'Merge');
[bi2,cg2] = bininfo(sc,'ResStatus','Statistics',{'Odds','WOE','InfoValue'});
disp(bi2)

```
Bin Good Bad Odds WOE InfoValue
\begin{tabular}{rrrrrr} 
'Group1' & 672 & 344 & 1.9535 & -0.034802 & 0.0010314 \\
'Group2' & 131 & 53 & 2.4717 & 0.20049 & 0.0059418 \\
'Totals' & 803 & 397 & 2.0227 & NaN & 0.0069732
\end{tabular}

\section*{disp(cg2)}

Category BinNumber
\(\qquad\)
'Tenant' 1
'Home Owner' 1
'Other' 2

\section*{Using the Merge Algorithm with a Numeric Predictor}

To demonstrate a merge for the numeric predictor, 'TmAtAddress ' , first use autobinning with the default 'Monotone' algorithm.
```

sc = autobinning(sc,'TmAtAddress');
bi3 = bininfo(sc,'TmAtAddress','Statistics',{'Odds','WOE','InfoValue'});
disp(bi3)

```
\begin{tabular}{|c|c|c|c|c|c|}
\hline Bin & Good & Bad & 0dds & WOE & InfoValue \\
\hline '[-Inf,23)' & 239 & 129 & 1.8527 & -0.087767 & 0.0023963 \\
\hline ' \([23,83\) )' & 480 & 232 & 2.069 & 0.02263 & 0.00030269 \\
\hline '[83, Inf]' & 84 & 36 & 2.3333 & 0.14288 & 0.00199 \\
\hline 'Totals' & 803 & 397 & 2.0227 & NaN & 0.004689 \\
\hline
\end{tabular}

Then use autobinning with the 'Merge' algorithm.
```

sc = autobinning(sc,'TmAtAddress','Algorithm', 'Merge');
bi4 = bininfo(sc,'TmAtAddress','Statistics',{'Odds','WOE','InfoValue'});
disp(bi4)

```
\begin{tabular}{crrrrrr} 
Bin & Good & \multicolumn{2}{c}{ Bad } & \multicolumn{1}{c}{ Odds } & & \multicolumn{1}{c}{ WOE }
\end{tabular}
\begin{tabular}{lrrrrr} 
'[106,Inf]' & 34 & 14 & 2.4286 & 0.18288 & 0.0012942 \\
'Totals' & 803 & 397 & 2.0227 & NaN & 0.072466
\end{tabular}

\section*{Input Arguments}

\section*{sc - Credit scorecard model}
creditscorecard object
Credit scorecard model, specified as a creditscorecard object. Use creditscorecard to create a creditscorecard object.

\section*{PredictorNames - Predictor or predictors names to automatically bin}
character vector | cell array of character vectors
Predictor or predictors names to automatically bin, specified as a character vector or a cell array of character vectors containing the name of the predictor or predictors. PredictorNames are case-sensitive and when no PredictorNames are defined, all predictors in the PredictorVars property of the creditscorecard object are binned.
Data Types: char | cell

\section*{Name-Value Pair Arguments}

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, ... , NameN, ValueN.

Example: sc = autobinning(sc,'Algorithm','EqualFrequency')

\section*{Algorithm - Algorithm selection}
'Monotone' (default)|character vector with values 'Monotone', 'Split', 'Merge', 'EqualFrequency', 'EqualWidth'

Algorithm selection, specified as the comma-separated pair consisting of 'Algorithm' and a character vector indicating which algorithm to use. The same algorithm is used for all predictors in PredictorNames. Possible values are:
- 'Monotone' - (default) Monotone Adjacent Pooling Algorithm (MAPA), also known as Maximum Likelihood Monotone Coarse Classifier (MLMCC). Supervised optimal binning algorithm that aims to find bins with a monotone Weight-Of-Evidence (WOE)
trend. This algorithm assumes that only neighboring attributes can be grouped. Thus, for categorical predictors, categories are sorted before applying the algorithm (see 'SortCategories ' option for Algorithm0ptions). For more information, see "Monotone" on page 19-2190.
- 'Split' - Supervised binning algorithm, where a measure is used to split the data into bins. The measures supported by 'Split' are gini, chi2, infovalue, and entropy. The resulting split must be such that the gain in the information function is maximized. For more information on these measures, see AlgorithmOptions and "Split" on page 19-2193.
- 'Merge' - Supervised automatic binning algorithm, where a measure is used to merge bins into buckets. The measures supported by 'Merge' are chi2, gini, infovalue, and entropy. The resulting merging must be such that any pair of adjacent bins is statistically different from each other, according to the chosen measure. For more information on these measures, see Algorithm0ptions and "Merge" on page 19-2196.
- 'EqualFrequency ' - Unsupervised algorithm that divides the data into a predetermined number of bins that contain approximately the same number of observations. This algorithm is also known as "equal height" or "equal depth." For categorical predictors, categories are sorted before applying the algorithm (see 'SortCategories' option for Algorithm0ptions). For more information, see "Equal Frequency" on page 19-2200.
- 'EqualWidth' - Unsupervised algorithm that divides the range of values in the domain of the predictor variable into a predetermined number of bins of "equal width." For numeric data, the width is measured as the distance between bin edges. For categorical data, width is measured as the number of categories within a bin. For categorical predictors, categories are sorted before applying the algorithm (see 'SortCategories ' option for Algorithm0ptions). For more information, see "Equal Width" on page 19-2201.

\section*{Data Types: char}

\section*{AlgorithmOptions - Algorithm options for selected Algorithm}
\{'InitialNumBins',10,'Trend','Auto','SortCategories','Odds'\} for Monotone (default) | cell array with \{ 'OptionName' ,OptionValue\} for Algorithm options

Algorithm options for the selected Algorithm, specified as the comma-separated pair consisting of 'AlgorithmOptions' and a cell array. Possible values are:
- For Monotone algorithm:
- \{'InitialNumBins ',\(n\}\) - Initial number ( \(n\) ) of bins (default is 10 ). 'InitialNumBins ' must be an integer \(>2\). Used for numeric predictors only.
- \{'Trend','TrendOption'\} - Determines whether the Weight-Of-Evidence (WOE) monotonic trend is expected to be increasing or decreasing. The values for 'TrendOption ' are:
- 'Auto ' - (Default) Automatically determines if the WOE trend is increasing or decreasing.
- 'Increasing' - Look for an increasing WOE trend.
- 'Decreasing ' - Look for a decreasing WOE trend.

The value of the optional input parameter 'Trend ' does not necessarily reflect that of the resulting WOE curve. The parameter 'Trend ' tells the algorithm to "look for" an increasing or decreasing trend, but the outcome may not show the desired trend. For example, the algorithm cannot find a decreasing trend when the data actually has an increasing WOE trend. For more information on the 'Trend ' option, see "Monotone" on page 19-2190.
- \{'SortCategories', 'Sort0ption'\} - Used for categorical predictors only. Used to determine how the predictor categories are sorted as a preprocessing step before applying the algorithm. The values of 'SortOption' are:
- 'Odds' - (default) The categories are sorted by order of increasing values of odds, defined as the ratio of "Good" to "Bad" observations, for the given category.
- 'Goods ' - The categories are sorted by order of increasing values of "Good."
- 'Bads ' - The categories are sorted by order of increasing values of "Bad."
- 'Totals' - The categories are sorted by order of increasing values of total number of observations ("Good" plus "Bad").
- 'None' - No sorting is applied. The existing order of the categories is unchanged before applying the algorithm. (The existing order of the categories can be seen in the category grouping optional output from bininfo.)

For more information, see Sort Categories on page 19-2202
- For Split algorithm:
- \(\{\) 'InitialNumBins ',\(n\}\) - Specifies an integer that determines the number ( \(n\) \(>0\) ) of bins that the predictor is initially binned into before splitting. Valid for numeric predictors only. Default is 50 .
- \{ 'Measure', MeasureName\} - Specifies the measure where 'MeasureName' is one of the following:' Gini' (default), 'Chi2', 'InfoValue', or 'Entropy '.
- \{'MinBad ',\(n\}\) - Specifies the minimum number \(n(n>=0)\) of Bads per bin. The default value is 1 , to avoid pure bins.
- \{ 'MaxBad' , \(n\}\) - Specifies the maximum number \(n\) ( \(n>=0\) ) of Bads per bin. The default value is Inf.
- \{'MinGood ',\(n\}\) - Specifies the minimum number \(n(n>=0)\) of Goods per bin. The default value is 1 , to avoid pure bins.
- \{ 'MaxGood ',\(n\}\) - Specifies the maximum number \(n(n>=0)\) of Goods per bin. The default value is Inf.
- \{'MinCount ',\(n\}\) - Specifies the minimum number \(n(n>=0)\) of observations per bin. The default value is 1 , to avoid empty bins.
- \(\{\) 'MaxCount ',\(n\}\) - Specifies the maximum number \(n(n>=0)\) of observations per bin. The default value is Inf.
- \(\{\) 'MaxNumBins ',\(n\}\) - Specifies the maximum number \(n(n>=2)\) of bins resulting from the splitting. The default value is 5 .
- \{'Tolerance', Tol\} - Specifies the minimum gain ( \(>0\) ) in the information function, during the iteration scheme, to select the cut-point that maximizes the gain. The default is \(1 \mathrm{e}-4\).
- \{'Significance' , \(n\}\) - Significance level threshold for the chi-square statistic, above which splitting happens. Values are in the interval [ 0,1 ]. Default is 0.9 ( \(90 \%\) significance level).
- \{'SortCategories', 'Sort0ption'\} - Used for categorical predictors only. Used to determine how the predictor categories are sorted as a preprocessing step before applying the algorithm. The values of 'Sort0ption' are:
- 'Goods ' - The categories are sorted by order of increasing values of "Good."
- 'Bads ' - The categories are sorted by order of increasing values of "Bad."
- 'Odds' - (default) The categories are sorted by order of increasing values of odds, defined as the ratio of "Good" to "Bad" observations, for the given category.
- 'Totals ' - The categories are sorted by order of increasing values of total number of observations ("Good" plus "Bad").
- 'None' - No sorting is applied. The existing order of the categories is unchanged before applying the algorithm. (The existing order of the categories can be seen in the category grouping optional output from bininfo.)

For more information, see Sort Categories on page 19-2202
- For Merge algorithm:
- \{'InitialNumBins',\(n\}\) - Specifies an integer that determines the number ( \(n\) \(>0\) ) of bins that the predictor is initially binned into before merging. Valid for numeric predictors only. Default is 50.
- \{'Measure', MeasureName\} - Specifies the measure where 'MeasureName' is one of the following: 'Chi2' (default), 'Gini', 'InfoValue', or 'Entropy'.
- \{'MinNumBins',\(n\}\) - Specifies the minimum number \(n(n>=2)\) of bins that result from merging. The default value is 2 .
- \{'MaxNumBins',\(n\}\) - Specifies the maximum number \(n(n>=2)\) of bins that result from merging. The default value is 5 .
- \(\{\) 'Tolerance ',\(n\}\) - Specifies the minimum threshold below which merging happens for the information value and entropy statistics. Valid values are in the interval (0.1). Default is \(1 \mathrm{e}-3\).
- \{'Significance' \(n\}\) - Significance level threshold for the chi-square statistic, below which merging happens. Values are in the interval [ 0,1 ]. Default is 0.9 ( \(90 \%\) significance level).
- \{'SortCategories','Sort0ption'\} - Used for categorical predictors only. Used to determine how the predictor categories are sorted as a preprocessing step before applying the algorithm. The values of 'Sort0ption' are:
- 'Goods ' - The categories are sorted by order of increasing values of "Good."
- 'Bads ' - The categories are sorted by order of increasing values of "Bad."
- 'Odds ' - (default) The categories are sorted by order of increasing values of odds, defined as the ratio of "Good" to "Bad" observations, for the given category.
- 'Totals ' - The categories are sorted by order of increasing values of total number of observations ("Good" plus "Bad").
- 'None' - No sorting is applied. The existing order of the categories is unchanged before applying the algorithm. (The existing order of the categories can be seen in the category grouping optional output from bininfo.)

For more information, see Sort Categories on page 19-2202
- For EqualFrequency algorithm:
- \(\{\) 'NumBins ',\(n\}\) - Specifies the desired number ( \(n\) ) of bins. The default is \{'NumBins',5\} and the number of bins must be a positive number.
- \{'SortCategories','SortOption'\} - Used for categorical predictors only. Used to determine how the predictor categories are sorted as a preprocessing step before applying the algorithm. The values of 'Sort0ption' are:
- ' Odds ' - (default) The categories are sorted by order of increasing values of odds, defined as the ratio of "Good" to "Bad" observations, for the given category.
- 'Goods ' - The categories are sorted by order of increasing values of "Good."
- 'Bads ' - The categories are sorted by order of increasing values of "Bad."
- 'Totals' - The categories are sorted by order of increasing values of total number of observations ("Good" plus "Bad").
- 'None' - No sorting is applied. The existing order of the categories is unchanged before applying the algorithm. (The existing order of the categories can be seen in the category grouping optional output from bininfo.)

For more information, see Sort Categories on page 19-2202
- For EqualWidth algorithm:
- \(\{\) 'NumBins ',\(n\}\) - Specifies the desired number ( \(n\) ) of bins. The default is \{'NumBins',5\} and the number of bins must be a positive number.
- \{'SortCategories', 'Sort0ption'\} - Used for categorical predictors only. Used to determine how the predictor categories are sorted as a preprocessing step before applying the algorithm. The values of 'SortOption' are:
- 'Odds ' - (default) The categories are sorted by order of increasing values of odds, defined as the ratio of "Good" to "Bad" observations, for the given category.
- 'Goods ' - The categories are sorted by order of increasing values of "Good."
- 'Bads ' - The categories are sorted by order of increasing values of "Bad."
- 'Totals' - The categories are sorted by order of increasing values of total number of observations ("Good" plus "Bad").
- 'None' - No sorting is applied. The existing order of the categories is unchanged before applying the algorithm. (The existing order of the categories can be seen in the category grouping optional output from bininfo.)

For more information, see Sort Categories on page 19-2202
```

Example: SC =
autobinning(sc,'CustAge','Algorithm','Monotone','AlgorithmOptions',
{'Trend','Increasing'})
Data Types: cell

```
```

Display - Indicator to display information on status of the binning process at command line
'Off' (default)| character vector with values 'On', 'Off'

```

Indicator to display the information on status of the binning process at command line, specified as the comma-separated pair consisting of 'Display' and a character vector with a value of 'On' or 'Off'.

Data Types: char

\section*{Output Arguments}

\section*{sc - Credit scorecard model}
creditscorecard object
Credit scorecard model, returned as an updated creditscorecard object containing the automatically determined binning maps or rules (cut points or category groupings) for one or more predictors. For more information on using the creditscorecard object, see creditscorecard.

Note If you have previously used the modifybins function to manually modify bins, these changes are lost when running autobinning because all the data is automatically binned based on internal autobinning rules.

\section*{Definitions}

\section*{Monotone}

The 'Monotone' algorithm is an implementation of the Monotone Adjacent Pooling Algorithm (MAPA), also known as Maximum Likelihood Monotone Coarse Classifier (MLMCC); see Anderson or Thomas in the "References" on page 19-2203.

\section*{Preprocessing}

During the preprocessing phase, preprocessing of numeric predictors consists in applying equal frequency binning, with the number of bins determined by the 'InitialNumBins ' parameter (the default is 10 bins). The preprocessing of categorical predictors consists in sorting the categories according to the 'SortCategories' criterion (the default is to sort by odds in increasing order). Sorting is not applied to ordinal predictors. See the "Sort Categories" on page 19-2202 definition or the description of AlgorithmOptions option for 'SortCategories' for more information.

\section*{Main Algorithm}

The following example illustrates how the 'Monotone ' algorithm arrives at cut points for numeric data.
\begin{tabular}{|l|l|l|l|l|l|l|}
\hline Bin & Good & Bad & Iteration1 & Iteration2 & Iteration3 & Iteration4 \\
\hline \begin{tabular}{l} 
' [ - Inf , \\
\(33000) '\)
\end{tabular} & 127 & 107 & 0.543 & & & \\
\hline \begin{tabular}{l} 
'[33000, \\
\(38000)^{\prime}\)
\end{tabular} & 194 & 90 & 0.620 & 0.683 & & \\
\hline \begin{tabular}{l} 
'[38000, \\
\(42000) '\)
\end{tabular} & 135 & 78 & 0.624 & 0.662 & & \\
\hline \begin{tabular}{l} 
'[42000, \\
\(47000) '\)
\end{tabular} & 164 & 66 & 0.645 & 0.678 & 0.713 & \\
\hline \begin{tabular}{l} 
'[47000, \\
Inf] '
\end{tabular} & 183 & 56 & 0.669 & 0.700 & 0.740 & 0.766 \\
\hline
\end{tabular}

Initially, the numeric data is preprocessed with an equal frequency binning. In this example, for simplicity, only the five initial bins are used. The first column indicates the equal frequency bin ranges, and the second and third columns have the "Good" and "Bad" counts per bin. (The number of observations is 1,200 , so a perfect equal frequency binning would result in five bins with 240 observations each. In this case, the observations per bin do not match 240 exactly. This is a common situation when the data has repeated values.)

Monotone finds break points based on the cumulative proportion of "Good" observations. In the 'Iteration 1 ' column, the first value (0.543) is the number of "Good" observations in the first bin (127), divided by the total number of observations in the bin \((127+107)\). The second value ( 0.620 ) is the number of "Good" observations in bins 1 and

2, divided by the total number of observations in bins 1 and 2 . And so forth. The first cut point is set where the minimum of this cumulative ratio is found, which is in the first bin in this example. This is the end of iteration 1.

Starting from the second bin (the first bin after the location of the minimum value in the previous iteration), cumulative proportions of "Good" observations are computed again. The second cut point is set where the minimum of this cumulative ratio is found. In this case, it happens to be in bin number 3, therefore bins 2 and 3 are merged.

The algorithm proceeds the same way for two more iterations. In this particular example, in the end it only merges bins 2 and 3 . The final binning has four bins with cut points at \(33,000,42,000\), and 47,000.

For categorical data, the only difference is that the preprocessing step consists in reordering the categories. Consider the following categorical data:
\begin{tabular}{|l|l|l|l|}
\hline Bin & Good & Bad & Odds \\
\hline 'Home Owner' & 365 & 177 & 2.062 \\
\hline 'Tenant' & 307 & 167 & 1.838 \\
\hline 'Other' & 131 & 53 & 2.474 \\
\hline
\end{tabular}

The preprocessing step, by default, sorts the categories by 'Odds '. (See the "Sort Categories" on page 19-2202 definition or the description of AlgorithmOptions option for 'SortCategories' for more information.) Then, it applies the same steps described above, shown in the following table:
\begin{tabular}{|l|l|l|l|l|l|l|}
\hline Bin & Good & Bad & Odds & Iteration1 & Iteration2 & Iteration3 \\
\hline 'Tenant' & 307 & 167 & 1.838 & 0.648 & & \\
\hline \begin{tabular}{l} 
'Home \\
Owner'
\end{tabular} & 365 & 177 & 2.062 & 0.661 & 0.673 & \\
\hline 'Other' & 131 & 53 & 2.472 & 0.669 & 0.683 & 0.712 \\
\hline
\end{tabular}

In this case, the Monotone algorithm would not merge any categories. The only difference, compared with the data before the application of the algorithm, is that the categories are now sorted by 'Odds '.

In both the numeric and categorical examples above, the implicit 'Trend ' choice is 'Increasing'. (See the description of AlgorithmOptions option for the 'Monotone'
'Trend ' option.) If you set the trend to 'Decreasing', the algorithm looks for the
maximum (instead of the minimum) cumulative ratios to determine the cut points. In that case, at iteration 1, the maximum would be in the last bin, which would imply that all bins should be merged into a single bin. Binning into a single bin is a total loss of information and has no practical use. Therefore, when the chosen trend leads to a single bin, the Monotone implementation rejects it, and the algorithm returns the bins found after the preprocessing step. This state is the initial equal frequency binning for numeric data and the sorted categories for categorical data. The implementation of the Monotone algorithm by default uses a heuristic to identify the trend ('Auto' option for 'Trend').

\section*{Split}

Split is a supervised automatic binning algorithm, where a measure is used to split the data into buckets. The supported measures are gini, chi2, infovalue, and entropy.

Internally, the split algorithm proceeds as follows:
1 All categories are merged into a single bin.
2 At the first iteration, all potential cutpoint indices are tested to see which one results in the maximum increase in the information function (Gini, InfoValue, Entropy, or Chi2). That cutpoint is then selected, and the bin is split.
3 The same procedure is reiterated for the next sub-bins.
4 The algorithm stops when the maximum number of bins is reached or when the splitting does not result in any additional change in the information change function.

The following table for a categorical predictor summarizes the values of the change function at each iteration. In this example, 'Gini ' is the measure of choice, such that the goal is to see a decrease of the Gini measure at each iteration.
\begin{tabular}{|l|l|l|l|l|l|l|l|l|}
\hline \begin{tabular}{l} 
Iteratio \\
\(\mathbf{n ~ 0 ~ B i n ~}\) \\
Number
\end{tabular} & \begin{tabular}{l} 
Membe \\
\(\mathbf{r}\)
\end{tabular} & \(\mathbf{G i n i}\) & \begin{tabular}{l} 
Iteratio \\
\(\mathbf{n} \mathbf{1}\) Bin \\
Number
\end{tabular} & \begin{tabular}{l} 
Membe \\
\(\mathbf{r}\)
\end{tabular} & Gini & \begin{tabular}{l} 
Iteratio \\
\(\mathbf{n} \mathbf{2}\) Bin \\
Number
\end{tabular} & \begin{tabular}{l} 
Membe \\
\(\mathbf{r}\)
\end{tabular} & Gini \\
\hline 1 & 'Tenant' & & 1 & 'Tenant' \(^{\prime}\) & & 1 & 'Tenant' & 0.45638 \\
\hline 1 & \begin{tabular}{l} 
'Sublette \\
\(\mathrm{r}^{\prime}\)
\end{tabular} & & 1 & \begin{tabular}{l} 
'Sublette \\
\(\mathrm{r}^{\prime}\)
\end{tabular} & 0.44789 & 1 & \begin{tabular}{l} 
'Sublette \\
\(\mathrm{r}^{\prime}\)
\end{tabular} & \\
\hline 1 & \begin{tabular}{l} 
'Home \\
Owner'
\end{tabular} & & 1 & \begin{tabular}{l} 
'Home \\
Owner'
\end{tabular} & & 2 & \begin{tabular}{l} 
'Home \\
Owner'
\end{tabular} & 0.43984 \\
\hline 1 & 'Other' & & 2 & 'Other' & 0.41015 & 3 & 'Other' & 0.41015 \\
\hline
\end{tabular}
\begin{tabular}{|c|c|c|c|c|c|c|c|c|}
\hline Iteratio n 0 Bin Number & Membe r & Gini & Iteratio n 1 Bin Number & Membe \(r\) & Gini & Iteratio n 2 Bin Number & Membe r & Gini \\
\hline Total Gini & & \[
\begin{aligned}
& 0.44276 \\
& 5
\end{aligned}
\] & & & \[
\begin{aligned}
& 0.44210 \\
& 2
\end{aligned}
\] & & & \[
\begin{aligned}
& 0.44182 \\
& 2
\end{aligned}
\] \\
\hline Relative Change & & 0 & & & \[
\begin{aligned}
& 0.00149 \\
& 8
\end{aligned}
\] & & & \[
\begin{aligned}
& 0.00212 \\
& 8
\end{aligned}
\] \\
\hline
\end{tabular}

The relative change at iteration \(i\) is with respect to the Gini measure of the entire bins at iteration \(i-1\). The final result corresponds to that from the last iteration which, in this example, is iteration 2.

The following table for a numeric predictor summarizes the values of the change function at each iteration. In this example, 'Gini ' is the measure of choice, such that the goal is to see a decrease of the Gini measure at each iteration. Since most numeric predictors in datasets contain many bins, there is a preprocessing step where the data is pre-binned into 50 equal-frequency bins. This makes the pool of valid cutpoints to choose from for splitting smaller and more manageable.
\begin{tabular}{|c|c|c|c|c|c|c|c|c|}
\hline Iteratio n 0 Bin Number & Membe \(r\) & Gini & Iteratio n 1 Bin Number & Gini & Iteratio n 2 Bin Number & Gini & Iteratio n 3 Bin Number & Gini \\
\hline 1 & '21' & & \[
\begin{aligned}
& 1[- \\
& \text { Inf, } \\
& 47]
\end{aligned}
\] & \[
\begin{array}{|l|}
\hline 0.47389 \\
7
\end{array}
\] & \[
\begin{aligned}
& \hline \text { [ }- \\
& \text { Inf }, \\
& 47]
\end{aligned}
\] & \[
\begin{aligned}
& 0.47389 \\
& 7
\end{aligned}
\] & \[
\begin{aligned}
& \prime[- \\
& \text { Inf, } \\
& 35]
\end{aligned}
\] & \[
\begin{aligned}
& 0.49494 \\
& 1
\end{aligned}
\] \\
\hline 1 & '22' & & \[
\begin{aligned}
& \text { '[47, I } \\
& \text { nf]' }
\end{aligned}
\] & \[
\begin{aligned}
& 0.38523 \\
& 8
\end{aligned}
\] & \[
\begin{aligned}
& 1[47,6 \\
& 1] '
\end{aligned}
\] & \[
\begin{aligned}
& 0.40707 \\
& 2
\end{aligned}
\] & \[
\begin{aligned}
& \text { '[35, } \\
& 47] \text { ' }
\end{aligned}
\] & \[
\begin{aligned}
& 0.46320 \\
& 1
\end{aligned}
\] \\
\hline 1 & '23' & & & & \[
\begin{aligned}
& \text { '[61, I } \\
& \text { nf] }
\end{aligned}
\] & \[
\begin{aligned}
& 0.20879 \\
& 5
\end{aligned}
\] & \[
\begin{aligned}
& \text { '[47, } \\
& 61] \text { ' }
\end{aligned}
\] & \[
\begin{aligned}
& 0.40707 \\
& 2
\end{aligned}
\] \\
\hline 1 & '74' & & & 0 & & & \[
\begin{aligned}
& \text { '[61,I } \\
& \text { nf]' }
\end{aligned}
\] & \[
\begin{aligned}
& 0.20879 \\
& 5
\end{aligned}
\] \\
\hline \begin{tabular}{l}
Total \\
Gini
\end{tabular} & & \[
\begin{aligned}
& 0.44276 \\
& 5
\end{aligned}
\] & & \[
\begin{aligned}
& 0.43503 \\
& 5
\end{aligned}
\] & & \[
\begin{aligned}
& 0.43204 \\
& 8
\end{aligned}
\] & & \[
\begin{aligned}
& 0.43051 \\
& 1
\end{aligned}
\] \\
\hline Relative Change & & 0 & & 0.01746 & & \[
\begin{aligned}
& 0.00686 \\
& 7
\end{aligned}
\] & & 0.0356 \\
\hline
\end{tabular}

The resulting split must be such that the information function (content) increases. As such, the best split is the one that results in the maximum information gain. The information functions supported are:
- Gini: Each split results in an increase in the Gini Ratio, defined as:

G_r = 1- G_hat/G_p
G_p is the Gini measure of the parent node, that is, of the given bins/categories prior to splitting. G_hat is the weighted Gini measure for the current split:

G_hat \(=\operatorname{Sum}((n j / N) * \operatorname{Gini}(j), j=1 . . m)\)
where
\(n j\) is the total number of observations in the \(j\) th bin.
N is the total number of observations in the dataset.
\(m\) is the number of splits for the given variable.
Gini( j ) is the Gini measure for the \(j\) th bin.
The Gini measure for the split/node \(j\) is:
Gini(j) = 1 - ( \(\left.\mathrm{Gj}^{\wedge} 2+\mathrm{Bj}{ }^{\wedge} 2\right) /(\mathrm{nj})^{\wedge} 2\)
where \(\mathrm{Gj}, \mathrm{Bj}=\) Number of Goods and Bads for bin \(j\).
- InfoValue: The information value for each split results in an increase in the total information. The split that is retained is the one which results in the maximum gain, within the acceptable gain tolerance. The Information Value (IV) for a given observation \(j\) is defined as:

IV \(=\operatorname{sum}\left(\right.\) ( \(\left.\left.p G \_i-p B \_i\right) ~ * ~ l o g\left(p G \_i / p B \_i\right), i=1 . . n\right)\)
where
pG_i is the distribution of Goods at observation \(i\), that is Goods(i)/Total_Goods.
\(p B_{-} i\) is the distribution of Bads at observation \(i\), that is Bads (i)/Total_Bads.
n is the total number of bins.
- Ent ropy: Each split results in a decrease in entropy variance defined as:
```

E = -sum(ni * Ei, i=1..n)

```
where
\(n i\) is the total count for bin \(i\), that is ( \(n i=G i+B i\) ).
Ei is the entropy for row (or bin) \(i\), defined as:
```

Ei = -sum(Gi*log2(Gi/ni) + Bi*log2(Bi/ni))/N,
i=1..n

```
- Chi2: Chi2 is computed pairwise for each pair of bins and measures the statistical difference between two groups. Splitting is selected at a point (cutpoint or category indexing) where the maximum Chi2 value is:
```

Chi2 = sum(sum((Aij - Eij)^2/Eij , j=1..k), i=m,m+1)

```
where
m takes values from \(1 \ldots \mathrm{n}-1\), where n is the number of bins.
k is the number of classes. Here \(\mathrm{k}=2\) for the (Goods, Bads).
Aij is the number of observations in bin \(i, j\) th class.
Eij is the expected frequency of Aij , which is equal to \(\left(\mathrm{Ri}^{*} \mathrm{Cj}\right) / \mathrm{N}\).
\(R i\) is the number of observations in bin \(i\), which is equal to sum ( \(\mathrm{Aij}, \mathrm{j}=1 . \mathrm{k}\) ).
Cj is the number of observations in the j th class, which is equal to sum(Aij, \(\mathrm{I}=\) \(m, m+1\) ).
\(N\) is the total number of observations, which is equal to \(\operatorname{sum}(C j, j=1 . . k)\).
The Chi2 measure for the entire sample (as opposed to the pairwise Chi2 measure for adjacent bins) is:
```

Chi2 = sum(sum((Aij - Eij)^2/Eij , j=1..k), i=1..n)

```

\section*{Merge}

Merge is a supervised automatic binning algorithm, where a measure is used to merge bins into buckets. The supported measures are chi2, gini, infovalue, and entropy.

Internally, the merge algorithm proceeds as follows:
1 All categories are initially in separate bins.
2 The user selected information function (Chi2, Gini, InfoValue or Entropy) is computed for any pair of adjacent bins.
3 At each iteration, the pair with the smallest information change measured by the selected information function is merged.

4 The merging continues until either:
a All pairwise information values are greater than the threshold set by the significance level or the relative change is smaller than the tolerance.
b If at the end, the number of bins is still greater than the MaxNumBins allowed, merging is forced until there are at most MaxNumBins bins. Similarly, merging stops when there are only MinNumBins bins.
5 For categorical, original bins/categories are pre-sorted according to the sorting of choice set by the user. For numeric data, the data is preprocessed to get IntialNumBins bins of equal frequency before the merging algorithm starts.

The following table for a categorical predictor summarizes the values of the change function at each iteration. In this example, 'Chi2' is the measure of choice. The default sorting by Odds is applied as a preprocessing step. The Chi2 value reported below at row \(i\) is for bins \(i\) and \(i+1\). The significance level is \(0.9(90 \%)\), so that the inverse Chi2 value is 2.705543 . This is the threshold below which adjacent pairs of bins are merged. The minimum number of bins is 2 .
\begin{tabular}{|c|c|c|c|c|c|c|c|c|}
\hline Iteratio n 0 Bin Number & Membe \(r\) & Chi2 & Iteratio n 1 Bin Number & Membe r & Chi2 & Iteratio n 2 Bin Number & \begin{tabular}{l}
Membe \\
\(r\)
\end{tabular} & Chi2 \\
\hline 1 & 'Tenant' & \[
\begin{aligned}
& 1.00761 \\
& 3
\end{aligned}
\] & 1 & 'Tenant' & \[
\begin{aligned}
& 0.79592 \\
& 0
\end{aligned}
\] & 1 & 'Tenant' & \\
\hline 2 & 'Sublette \(r^{\prime}\) & \[
\begin{aligned}
& 0.25734 \\
& 7
\end{aligned}
\] & 2 & 'Sublette \(r^{\prime}\) & & 1 & \begin{tabular}{l}
'Sublette \\
r'
\end{tabular} & \\
\hline 3 & 'Home Owner' & \[
\begin{aligned}
& 1.56633 \\
& 0
\end{aligned}
\] & 2 & 'Home Owner' & \[
\begin{aligned}
& 1.52291 \\
& 4
\end{aligned}
\] & 1 & 'Home Owner' & \[
\begin{aligned}
& 1.79739 \\
& 5
\end{aligned}
\] \\
\hline 4 & 'Other' & & 3 & 'Other' & & 2 & 'Other' & \\
\hline \[
\begin{aligned}
& \text { Total } \\
& \text { Chi2 }
\end{aligned}
\] & & \[
\begin{aligned}
& 2.57394 \\
& 3
\end{aligned}
\] & & & \[
\begin{aligned}
& 2.31771 \\
& 7
\end{aligned}
\] & & & \[
\begin{aligned}
& 1.79739 \\
& 5
\end{aligned}
\] \\
\hline
\end{tabular}

The following table for a numeric predictor summarizes the values of the change function at each iteration. In this example, 'Chi2' is the measure of choice.


The resulting merging must be such that any pair of adjacent bins is statistically different from each other, according to the chosen measure. The measures supported for Merge are:
- Chi2: Chi2 is computed pairwise for each pair of bins and measures the statistical difference between two groups. Merging is selected at a point (cutpoint or category indexing) where the maximum Chi2 value is:
```

Chi2 = sum(sum((Aij - Eij)^2/Eij , j=1..k), i=m,m+1)

```
where
m takes values from \(1 \ldots \mathrm{n}-1\), and n is the number of bins.
k is the number of classes. Here \(\mathrm{k}=2\) for the (Goods, Bads).
Aij is the number of observations in bin \(i, j\) th class.
Eij is the expected frequency of Aij , which is equal to \(\left(\mathrm{Ri}^{*} \mathrm{Cj}\right) / \mathrm{N}\).
Ri is the number of observations in bin \(i\), which is equal to sum( \(A i j, j=1 . . k\) ).
Cj is the number of observations in the j th class, which is equal to sum(Aij, \(\mathrm{I}=\) \(m, m+1\) ).
\(N\) is the total number of observations, which is equal to \(\operatorname{sum}(C j, j=1 . . k)\).
The Chi2 measure for the entire sample (as opposed to the pairwise Chi2 measure for adjacent bins) is:
```

Chi2 = sum(sum((Aij - Eij)^2/Eij , j=1..k), i=1..n)

```
- Gini: Each merge results in a decrease in the Gini Ratio, defined as:

G_r = 1- G_hat/G_p
G_p is the Gini measure of the parent node, that is, of the given bins/categories prior to merging. G_hat is the weighted Gini measure for the current merge:

G_hat \(=\operatorname{Sum}((\mathrm{nj} / \mathrm{N}) * \operatorname{Gini}(\mathrm{j}), \mathrm{j}=1 . . \mathrm{m})\)
where
nj is the total number of observations in the \(j\) th bin.
N is the total number of observations in the dataset.
\(m\) is the number of merges for the given variable.
Gini( j ) is the Gini measure for the \(j\) th bin.
The Gini measure for the merge/node \(j\) is:

Gini(j) = \(1-\left(G j^{\wedge} 2+B j^{\wedge} 2\right) /(n j)^{\wedge} 2\)
where \(\mathrm{Gj}, \mathrm{Bj}=\) Number of Goods and Bads for bin \(j\).
- InfoValue: The information value for each merge will result in a decrease in the total information. The merge that is retained is the one which results in the minimum gain, within the acceptable gain tolerance. The Information Value (IV) for a given observation \(j\) is defined as:

IV \(=\operatorname{sum}\left(\left(p G_{-} i-p B \_i\right) * \log \left(p G_{-} i / p B \_i\right), i=1 . . n\right)\)
where
pG_i is the distribution of Goods at observation i, that is Goods(i)/Total_Goods.
\(p B \_i\) is the distribution of Bads at observation \(i\), that is Bads (i)/Total_Bads.
n is the total number of bins.
- Entropy: Each merge results in an increase in entropy variance defined as:
```

E = -sum(ni * Ei, i=1..n)

```
where
\(n i\) is the total count for bin \(i\), that is ( \(n i=G i+B i\) ).
Ei is the entropy for row (or bin) \(i\), defined as:
```

    Ei = -sum(Gi*log2(Gi/ni) + Bi*log2(Bi/ni))/N,
    i=1..n

```

Note When using the Merge algorithm, if there are pure bins (bins that have either zero count of Goods or zero count of Bads), the statistics such as Information Value and Entropy have non-finite values. To account for this, a frequency shift of . 5 is applied for computing various statistics whenever the algorithm finds pure bins.

\section*{Equal Frequency}

Unsupervised algorithm that divides the data into a predetermined number of bins that contain approximately the same number of observations.

EqualFrequency is defined as:

Let \(\mathrm{v}[1], \mathrm{v}[2], \ldots, \mathrm{v}[\mathrm{N}]\) be the sorted list of different values or categories observed in the data. Let \(\mathrm{f}[i]\) be the frequency of \(\mathrm{v}[i]\). Let \(\mathrm{F}[k]=\mathrm{f}[1]+\ldots+\mathrm{f}[k]\) be the cumulative sum of frequencies up to the \(k\) th sorted value. Then \(\mathrm{F}[N]\) is the same as the total number of observations.

Define AvgFreq \(=\mathrm{F}[N] /\) NumBins, which is the ideal average frequency per bin after binning. The \(n\)th cut point index is the index \(k\) such that the distance \(\operatorname{abs}(\mathrm{F}[k]\) \(\left.n^{*} A v g F r e q\right)\) is minimized.

This rule attempts to match the cumulative frequency up to the \(n\)th bin. If a single value contains too many observations, equal frequency bins are not possible, and the above rule yields less than NumBins total bins. In that case, the algorithm determines NumBins bins by breaking up bins, in the order in which the bins were constructed.

The preprocessing of categorical predictors consists in sorting the categories according to the 'SortCategories ' criterion (the default is to sort by odds in increasing order). Sorting is not applied to ordinal predictors. See the "Sort Categories" on page 19-2202 definition or the description of AlgorithmOptions option for 'SortCategories' for more information.

\section*{Equal Width}

Unsupervised algorithm that divides the range of values in the domain of the predictor variable into a predetermined number of bins of "equal width." For numeric data, the width is measured as the distance between bin edges. For categorical data, width is measured as the number of categories within a bin.

The EqualWidth option is defined as:
For numeric data, if MinValue and MaxValue are the minimum and maximum data values, then
```

Width = (MaxValue - MinValue)/NumBins

```
and the CutPoints are set to MinValue + Width, MinValue + 2 *Width, ... MaxValue Width. If a MinValue or MaxValue have not been specified using the modifybins function, the EqualWidth option sets MinValue and MaxValue to the minimum and maximum values observed in the data.

For categorical data, if there are NumCats numbers of original categories, then
Width = NumCats / NumBins,
and set cut point indices to the rounded values of Width, \(2 *\) Width, ..., NumCats - Width, plus 1.

The preprocessing of categorical predictors consists in sorting the categories according to the 'SortCategories' criterion (the default is to sort by odds in increasing order). Sorting is not applied to ordinal predictors. See the "Sort Categories" on page 19-2202 definition or the description of AlgorithmOptions option for 'SortCategories' for more information.

\section*{Sort Categories}

As a preprocessing step for categorical data, 'Monotone', 'EqualFrequency', and 'EqualWidth' support the 'SortCategories' input. This serves the purpose of reordering the categories before applying the main algorithm. The default sorting criterion is to sort by ' Odds '. For example, suppose that the data originally looks like this:
\begin{tabular}{|l|l|l|l|}
\hline Bin & Good & Bad & Odds \\
\hline 'Home Owner' & 365 & 177 & 2.062 \\
\hline 'Tenant' & 307 & 167 & 1.838 \\
\hline 'Other' & 131 & 53 & 2.472 \\
\hline
\end{tabular}

After the preprocessing step, the rows would be sorted by ' Odds ' and the table looks like this:
\begin{tabular}{|l|l|l|l|}
\hline Bin & Good & Bad & Odds \\
\hline 'Tenant' & 307 & 167 & 1.838 \\
\hline 'Home Owner' & 365 & 177 & 2.062 \\
\hline 'Other' & 131 & 53 & 2.472 \\
\hline
\end{tabular}

The three algorithms only merge adjacent bins, so the initial order of the categories makes a difference for the final binning. The 'None' option for 'SortCategories' would leave the original table unchanged. For a description of the sorting criteria supported, see the description of the AlgorithmOptions option for 'SortCategories'.

Upon the construction of a scorecard, the initial order of the categories, before any algorithm or any binning modifications are applied, is the order shown in the first output
of bininfo. If the bins have been modified (either manually with modifybins or automatically with autobinning), use the optional output (cg, 'category grouping') from bininfo to get the current order of the categories.

The 'SortCategories' option has no effect on categorical predictors for which the 'Ordinal' parameter is set to true (see the 'Ordinal' input parameter in MATLAB categorical arrays for categorical. Ordinal data has a natural order, which is honored in the preprocessing step of the algorithms by leaving the order of the categories unchanged. Only categorical predictors whose 'Ordinal' parameter is false (default option) are subject to reordering of categories according to the 'SortCategories' criterion.

\section*{Using autobinning with Weights}

When observation weights are defined using the optional WeightsVar argument when creating a creditscorecard object, instead of counting the rows that are good or bad in each bin, the autobinning function accumulates the weight of the rows that are good or bad in each bin.

The "frequencies" reported are no longer the basic "count" of rows, but the "cumulative weight" of the rows that are good or bad and fall in a particular bin. Once these "weighted frequencies" are known, all other relevant statistics (Good, Bad, Odds, W0E, and InfoValue) are computed with the usual formulas. For more information, see "Credit Scorecard Modeling Using Observation Weights" on page 8-70.

\section*{References}
[1] Anderson, R. The Credit Scoring Toolkit. Oxford University Press, 2007.
[2] Kerber, R. "ChiMerge: Discretization of Numeric Attributes." AAAI-92 Proceedings. 1992.
[3] Liu, H., et. al. Data Mining, Knowledge, and Discovery. Vol 6. Issue 4. October 2002, pp. 393-423.
[4] Refaat, M. Data Preparation for Data Mining Using SAS. Morgan Kaufmann, 2006.
[5] Refaat, M. Credit Risk Scorecards: Development and Implementation Using SAS. lulu.com, 2011.
[6] Thomas, L., et al. Credit Scoring and Its Applications. Society for Industrial and Applied Mathematics, 2002.

\section*{See Also}
bindata|bininfo|creditscorecard|displaypoints|fitmodel| formatpoints|modifybins|modifypredictor|plotbins|predictorinfo| probdefault|score|setmodel|validatemodel

\section*{Topics}
"Case Study for a Credit Scorecard Analysis" on page 8-101
"Credit Scorecard Modeling with Missing Values" on page 8-73
"Troubleshooting Credit Scorecard Results" on page 8-91
"Credit Scorecard Modeling Workflow" on page 8-67
"About Credit Scorecards" on page 8-62
"Credit Scorecard Modeling Using Observation Weights" on page 8-70

\section*{Introduced in R2014b}

\section*{probdefault}

Likelihood of default for given data set

\section*{Syntax}
```

pd = probdefault(sc)
pd = probdefault(sc,data)

```

\section*{Description}
pd = probdefault(sc) computes the probability of default for sc, the data used to build the creditscorecard object.
pd = probdefault(sc,data) computes the probability of default for a given data set specified using the optional argument data.

By default, the data used to build the creditscorecard object are used. You can also supply input data, to which the same computation of probability of default is applied.

\section*{Examples}

\section*{Compute Probability for Default Using Credit ScoreCard Data}

Create a creditscorecard object using the CreditCardData.mat file to load the data (using a dataset from Refaat 2011).
```

load CreditCardData
sc = creditscorecard(data,'IDVar','CustID')
SC =
creditscorecard with properties:

```
```

    WeightsVar: '
    VarNames: {1x11 cell}
    NumericPredictors: {1x6 cell}
    CategoricalPredictors: {'ResStatus' 'EmpStatus' 'OtherCC'}
BinMissingData: 0
IDVar: 'CustID'
PredictorVars: {1x9 cell}
Data: [1200x11 table]

```

Perform automatic binning using the default options. By default, autobinning uses the Monotone algorithm.
```

sc = autobinning(sc);

```

Fit the model.
```

sc = fitmodel(sc);

```
1. Adding CustIncome, Deviance = 1490.8527, Chi2Stat = 32.588614, PValue = 1.1387992e-
2. Adding TmWBank, Deviance \(=1467.1415\), Chi2Stat \(=23.711203\), PValue \(=1.1192909 \mathrm{e}-06\) 3. Adding AMBalance, Deviance = 1455.5715, Chi2Stat = 11.569967, PValue = 0.0006702560 4. Adding EmpStatus, Deviance \(=1447.3451\), Chi2Stat \(=8.2264038\), PValue \(=0.0041285257\) 5. Adding CustAge, Deviance \(=1441.994\), Chi2Stat \(=5.3511754\), \(\operatorname{PValue}=0.020708306\)
6. Adding ResStatus, Deviance \(=1437.8756\), Chi2Stat \(=4.118404\), PValue \(=0.042419078\)
7. Adding OtherCC, Deviance \(=1433.707\), Chi2Stat \(=4.1686018\), PValue \(=0.041179769\)

Generalized linear regression model:
status ~ [Linear formula with 8 terms in 7 predictors]
Distribution = Binomial
Estimated Coefficients:
\begin{tabular}{rrrrr} 
Estimate & \multicolumn{1}{c}{ SE } & & \multicolumn{1}{c}{ tStat } &
\end{tabular} pValue
```

1200 observations, 1192 error degrees of freedom
Dispersion: 1
Chi^2-statistic vs. constant model: 89.7, p-value = 1.4e-16

```

Compute the probability of default.
```

pd = probdefault(sc);
disp(pd(1:15,:))

```
0.2503
0.1878
0.3173
0.1711
0.1895
0.1307
0.5218
0.2848
0.2612
0.3047
0.3418
0.2237
0.2793
0.3615
0.1653

\section*{Compute Probability for Default Using Credit ScoreCard Data When Using the 'BinMissingData' Option}

This example describes both the assignment of points for missing data when the 'BinMissingData' option is set to true, and the corresponding computation of probabilities of default.
- Predictors that have missing data in the training set have an explicit bin for <missing> with corresponding points in the final scorecard. These points are computed from the Weight-of-Evidence (WOE) value for the <missing> bin and the logistic model coefficients. For scoring purposes, these points are assigned to missing values and to out-of-range values, and the final score is mapped to a probability of default when using probdefault.
- Predictors with no missing data in the training set have no <missing> bin, therefore no WOE can be estimated from the training data. By default, the points for missing and out-of-range values are set to NaN , and this leads to a score of NaN when running
score. For predictors that have no explicit <missing> bin, use the name-value argument 'Missing' in formatpoints to indicate how missing data should be treated for scoring purposes. The final score is then mapped to a probability of default when using probdefault.

Create a creditscorecard object using the CreditCardData.mat file to load the dataMissing with missing values.
load CreditCardData.mat
head(dataMissing,5)
\begin{tabular}{|c|c|c|c|c|c|}
\hline Custid & CustAge & TmAtAddress & ResStatus & EmpStatus & CustIncome \\
\hline 1 & 53 & 62 & <undefined> & Unknown & 50000 \\
\hline 2 & 61 & 22 & Home Owner & Employed & 52000 \\
\hline 3 & 47 & 30 & Tenant & Employed & 37000 \\
\hline 4 & NaN & 75 & Home Owner & Employed & 53000 \\
\hline 5 & 68 & 56 & Home Owner & Employed & 53000 \\
\hline
\end{tabular}

Use creditscorecard with the name-value argument 'BinMissingData' set to true to bin the missing numeric or categorical data in a separate bin. Apply automatic binning.
```

sc = creditscorecard(dataMissing,'IDVar','CustID','BinMissingData',true);
sc = autobinning(sc);
disp(sc)
creditscorecard with properties:

```
                    GoodLabel: 0
            ResponseVar: 'status'
                    WeightsVar:
                            VarNames: \{1x11 cell\}
        NumericPredictors: \{1x6 cell\}
        CategoricalPredictors: \{'ResStatus' 'EmpStatus' 'OtherCC'\}
            BinMissingData: 1
                            IDVar: 'CustID'
            PredictorVars: \{1x9 cell\}
                    Data: [1200x11 table]

Set a minimum value of 0 for CustAge and CustIncome. With this, any negative age or income information becomes invalid or "out-of-range". For scoring and probability of default computations, out-of-range values are given the same points as missing values.
```

sc = modifybins(sc,'CustAge','MinValue',0);
sc = modifybins(sc,'CustIncome','MinValue',0);

```

Display bin information for numeric data for 'CustAge' that includes missing data in a separate bin labelled <missing>.
```

bi = bininfo(sc,'CustAge');
disp(bi)

| Bin | Good | Bad | Odds | WOE | InfoValue |
| :---: | :---: | :---: | :---: | :---: | :---: |
| ' $[0,33)^{\prime}$ | 69 | 52 | 1.3269 | -0.42156 | 0.018993 |
| ' $[33,37)^{\prime}$ | 63 | 45 | 1.4 | -0.36795 | 0.012839 |
| ' $[37,40$ ) ' | 72 | 47 | 1.5319 | -0.2779 | 0.0079824 |
| ' $[40,46$ )' | 172 | 89 | 1.9326 | -0.04556 | 0.0004549 |
| ' $[46,48$ )' | 59 | 25 | 2.36 | 0.15424 | 0.0016199 |
| ' $[48,51$ )' | 99 | 41 | 2.4146 | 0.17713 | 0.0035449 |
| ' $[51,58$ )' | 157 | 62 | 2.5323 | 0.22469 | 0.0088407 |
| '[58,Inf]' | 93 | 25 | 3.72 | 0.60931 | 0.032198 |
| '<missing>' | 19 | 11 | 1.7273 | -0.15787 | 0.00063885 |
| 'Totals' | 803 | 397 | 2.0227 | NaN | 0.087112 |

```

Display bin information for categorical data for 'ResStatus ' that includes missing data in a separate bin labelled <missing>.
\begin{tabular}{|c|c|c|c|c|c|}
\hline Bin & Good & Bad & Odds & WOE & InfoValue \\
\hline 'Tenant' & 296 & 161 & 1.8385 & -0.095463 & 0.0035249 \\
\hline 'Home Owner' & 352 & 171 & 2.0585 & 0.017549 & 0.00013382 \\
\hline 'Other' & 128 & 52 & 2.4615 & 0.19637 & 0.0055808 \\
\hline '<missing>' & 27 & 13 & 2.0769 & 0.026469 & 2.3248e-05 \\
\hline 'Totals' & 803 & 397 & 2.0227 & NaN & 0.0092627 \\
\hline
\end{tabular}

For the 'CustAge' and 'ResStatus ' predictors, there is missing data (NaNs and <undefined>) in the training data, and the binning process estimates a WOE value of -0.15787 and 0.026469 respectively for missing data in these predictors, as shown above.

For EmpStatus and CustIncome there is no explicit bin for missing values because the training data has no missing values for these predictors.
```

bi = bininfo(sc,'EmpStatus');
disp(bi)

| Bin | Good | Bad |  | Odds |  |  | WOE |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

bi = bininfo(sc,'CustIncome');
disp(bi)

| Bin | Good | Bad | Odds | WOE | InfoValue |
| :---: | :---: | :---: | :---: | :---: | :---: |
| '[0,29000)' | 53 | 58 | 0.91379 | -0.79457 | 0.06364 |
| ' [29000, 33000) ' | 74 | 49 | 1.5102 | -0.29217 | 0.0091366 |
| '[33000, 35000) ' | 68 | 36 | 1.8889 | -0.06843 | 0.00041042 |
| '[35000, 40000)' | 193 | 98 | 1.9694 | -0.026696 | 0.00017359 |
| '[40000, 42000)' | 68 | 34 | 2 | -0.011271 | $1.0819 \mathrm{e}-05$ |
| '[42000, 47000) ' | 164 | 66 | 2.4848 | 0.20579 | 0.0078175 |
| '[47000,Inf]' | 183 | 56 | 3.2679 | 0.47972 | 0.041657 |
| 'Totals' | 803 | 397 | 2.0227 | NaN | 0.12285 |

```

Use fitmodel to fit a logistic regression model using Weight of Evidence (WOE) data. fitmodel internally transforms all the predictor variables into WOE values, using the bins found with the automatic binning process. fitmodel then fits a logistic regression model using a stepwise method (by default). For predictors that have missing data, there is an explicit <missing> bin, with a corresponding WOE value computed from the data. When using fitmodel, the corresponding WOE value for the <missing> bin is applied when performing the WOE transformation.
```

[sc,mdl] = fitmodel(sc);

```
1. Adding CustIncome, Deviance = 1490.8527, Chi2Stat = 32.588614, PValue = 1.1387992e-
2. Adding TmWBank, Deviance = 1467.1415, Chi2Stat = 23.711203, PValue = 1.1192909e-06
3. Adding AMBalance, Deviance = 1455.5715, Chi2Stat = 11.569967, PValue = 0.0006702560:
4. Adding EmpStatus, Deviance \(=1447.3451\), Chi2Stat \(=8.2264038\), PValue \(=0.0041285257\)
5. Adding CustAge, Deviance \(=1442.8477\), Chi2Stat \(=4.4974731\), PValue \(=0.033944979\)
6. Adding ResStatus, Deviance \(=1438.9783\), Chi2Stat \(=3.86941\), \(\mathrm{PValue}=0.049173805\)
7. Adding OtherCC, Deviance \(=1434.9751\), Chi2Stat \(=4.0031966\), \(\mathrm{PValue}=0.045414057\)
```

Generalized linear regression model:
status ~ [Linear formula with 8 terms in 7 predictors]
Distribution = Binomial
Estimated Coefficients:

| Estimate | SE $\quad$ tStat $\quad$ pValue |
| :--- | :--- | :--- |


| (Intercept) | 0.70229 | 0.063959 | 10.98 | $4.7498 \mathrm{e}-28$ |
| :--- | ---: | ---: | ---: | ---: |
| CustAge | 0.57421 | 0.25708 | 2.2335 | 0.025513 |
| ResStatus | 1.3629 | 0.66952 | 2.0356 | 0.04179 |
| EmpStatus | 0.88373 | 0.2929 | 3.0172 | 0.002551 |
| CustIncome | 0.73535 | 0.2159 | 3.406 | 0.00065929 |
| TmWBank | 1.1065 | 0.23267 | 4.7556 | $1.9783 e-06$ |
| OtherCC | 1.0648 | 0.52826 | 2.0156 | 0.043841 |
| AMBalance | 1.0446 | 0.32197 | 3.2443 | 0.0011775 |

1200 observations, 1192 error degrees of freedom Dispersion: 1
Chi^2-statistic vs. constant model: 88.5, p-value $=2.55 \mathrm{e}-16$

```

Scale the scorecard points by the "points, odds, and points to double the odds (PDO)" method using the 'Points0ddsAndPDO' argument of formatpoints. Suppose that you want a score of 500 points to have odds of 2 (twice as likely to be good than to be bad) and that the odds double every 50 points (so that 550 points would have odds of 4 ).

Display the scorecard showing the scaled points for predictors retained in the fitting model.
```

sc = formatpoints(sc,'PointsOddsAndPDO',[500 2 50]);
PointsInfo = displaypoints(sc)
PointsInfo=33\times3 table
Predictors Bin Points
'CustAge' '[0,33)' 54.062
'CustAge' '[33,37)' 56.282
'CustAge' '[37,40)' 60.012
'CustAge' '[40,46)' 69.636
'CustAge' '[46,48)' }77.91
'CustAge' '[48,51)' }78.8

```
\begin{tabular}{llr} 
'CustAge' & '[51,58)' & 80.83 \\
'CustAge' & '[58,Inf]' & 96.76 \\
'CustAge' & '<missing>' & 64.984 \\
'ResStatus' & 'Tenant' & 62.138 \\
'ResStatus' & 'Home Owner' & 73.248 \\
'ResStatus' & '0ther' & 90.828 \\
'ResStatus' & '<missing>' & 74.125 \\
'EmpStatus' & 'Unknown' & 58.807 \\
'EmpStatus' & 'Employed' & 86.937 \\
'CustIncome' & '[0,29000)' & 29.375
\end{tabular}

Notice that points for the <missing> bin for CustAge and ResStatus are explicitly shown (as 64.9836 and 74.1250 , respectively). These points are computed from the WOE value for the <missing> bin and the logistic model coefficients.

For predictors that have no missing data in the training set, there is no explicit <missing> bin. By default the points are set to NaN for missing data, and they lead to a score of NaN when running score. For predictors that have no explicit <missing> bin, use the name-value argument 'Missing' in formatpoints to indicate how missing data should be treated for scoring purposes.

For the purpose of illustration, take a few rows from the original data as test data and introduce some missing data. Also introduce some invalid, or out-of-range, values. For numeric data, values below the minimum (or above the maximum) allowed are considered invalid, such as a negative value for age (recall 'MinValue' was earlier set to 0 for CustAge and CustIncome). For categorical data, invalid values are categories not explicitly included in the scorecard, for example, a residential status not previously mapped to scorecard categories, such as "House", or a meaningless string such as "abc123".
```

tdata = dataMissing(11:18,mdl.PredictorNames); % Keep only the predictors retained in
% Set some missing values
tdata.CustAge(1) = NaN;
tdata.ResStatus(2) = '<undefined>';
tdata.EmpStatus(3) = '<undefined>';
tdata.CustIncome(4) = NaN;
% Set some invalid values
tdata.CustAge(5) = -100;
tdata.ResStatus(6) = 'House';
tdata.EmpStatus(7) = 'Freelancer';
tdata.CustIncome(8) = -1;
disp(tdata)

```
\begin{tabular}{rllllllll} 
CustAge & ResStatus & EmpStatus & & CustIncome & & TmWBank & & OtherCC
\end{tabular} AMBala

Score the new data and see how points are assigned for missing CustAge and ResStatus, because we have an explicit bin with points for <missing>. However, for EmpStatus and CustIncome the score function sets the points to NaN. The corresponding probabilities of default are also set to NaN .
```

[Scores,Points] = score(sc,tdata);
disp(Scores)

```
```

481.2231
520.8353
NaN
NaN
551.7922
487.9588
NaN
NaN

```
disp(Points)
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline CustAge & ResStatus & EmpStatus & CustIncome & TmWBank & OtherCC & AMBalance \\
\hline 64.984 & 62.138 & 58.807 & 67.893 & 61.858 & 75.622 & 89.922 \\
\hline 78.86 & 74.125 & 58.807 & 82.439 & 61.061 & 75.622 & 89.922 \\
\hline 96.76 & 73.248 & NaN & 96.969 & 51.132 & 50.914 & 89.922 \\
\hline 69.636 & 90.828 & 58.807 & NaN & 61.858 & 50.914 & 89.922 \\
\hline 64.984 & 90.828 & 86.937 & 82.439 & 61.061 & 75.622 & 89.922 \\
\hline 56.282 & 74.125 & 86.937 & 70.107 & 61.858 & 75.622 & 63.028 \\
\hline 60.012 & 62.138 & NaN & 67.893 & 61.858 & 75.622 & 63.028 \\
\hline 54.062 & 73.248 & 86.937 & NaN & 61.061 & 75.622 & 89.922 \\
\hline
\end{tabular}
pd \(=\) probdefault(sc,tdata);
disp(pd)

19-2213
0.3934
0.2725

NaN
NaN
0.1961
0.3714

NaN
NaN
Use the name-value argument 'Missing ' in formatpoints to choose how to assign points to missing values for predictors that do not have an explicit <missing> bin. In this example, use the 'MinPoints ' option for the 'Missing' argument. The minimum points for EmpStatus in the scorecard displayed above are 58.8072, and for CustIncome the minimum points are 29.3753. All rows now have a score and a corresponding probability of default.
```

sc = formatpoints(sc,'Missing','MinPoints');
[Scores,Points] = score(sc,tdata);
disp(Scores)
481.2231
520.8353
517.7532
451.3405
551.7922
487.9588
449.3577
470.2267

```
disp(Points)
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline CustAge & ResStatus & EmpStatus & CustIncome & TmWBank & OtherCC & AMBalance \\
\hline 64.984 & 62.138 & 58.807 & 67.893 & 61.858 & 75.622 & 89.922 \\
\hline 78.86 & 74.125 & 58.807 & 82.439 & 61.061 & 75.622 & 89.922 \\
\hline 96.76 & 73.248 & 58.807 & 96.969 & 51.132 & 50.914 & 89.922 \\
\hline 69.636 & 90.828 & 58.807 & 29.375 & 61.858 & 50.914 & 89.922 \\
\hline 64.984 & 90.828 & 86.937 & 82.439 & 61.061 & 75.622 & 89.922 \\
\hline 56.282 & 74.125 & 86.937 & 70.107 & 61.858 & 75.622 & 63.028 \\
\hline 60.012 & 62.138 & 58.807 & 67.893 & 61.858 & 75.622 & 63.028 \\
\hline 54.062 & 73.248 & 86.937 & 29.375 & 61.061 & 75.622 & 89.922 \\
\hline
\end{tabular}
pd \(=\) probdefault(sc,tdata);
disp(pd)
0.3934
0.2725
0.2810
0.4954
0.1961
0.3714
0.5022
0.4304

\section*{Input Arguments}

\section*{sc - Credit scorecard model}
creditscorecard object
Credit scorecard model, specified as a creditscorecard object. To create this object, use creditscorecard.

\section*{data - Dataset to apply probability of default rules \\ table}
(Optional) Dataset to apply probability of default rules, specified as a MATLAB table, where each row corresponds to individual observations. The data must contain columns for each of the predictors in the creditscorecard object.
Data Types: table

\section*{Output Arguments}

\section*{pd - Probability of default}
array
Probability of default, returned as a Num0bs-by-1 numerical array of default probabilities.

\section*{Definitions}

\section*{Default Probability}

After the unscaled scores are computed (see "Algorithms for Computing and Scaling Scores" on page 19-1999), the probability of the points being "Good" is represented by the following formula:
```

ProbGood = 1./(1 + exp(-UnscaledScores))

```

Thus, the probability of default is
```

pd = 1 - ProbGood

```

\section*{References}
[1] Refaat, M. Credit Risk Scorecards: Development and Implementation Using SAS. lulu.com, 2011.

\section*{See Also}
bindata|bininfo|creditscorecard|displaypoints|fitmodel| formatpoints|modifybins|modifypredictor|plotbins|predictorinfo| score | setmodel|table|validatemodel

\section*{Topics}
"Case Study for a Credit Scorecard Analysis" on page 8-101
"Troubleshooting Credit Scorecard Results" on page 8-91
"Credit Scorecard Modeling Workflow" on page 8-67
"About Credit Scorecards" on page 8-62

\section*{Introduced in R2015a}

\section*{validatemodel}

Validate quality of credit scorecard model

\section*{Syntax}
```

Stats = validatemodel(sc)
Stats = validatemodel(sc,data)
[Stats,T] = validatemodel(sc,Name,Value)
[Stats,T,hf] = validatemodel(sc,Name,Value)

```

\section*{Description}

Stats = validatemodel(sc) validates the quality of the creditscorecard model.
By default, the data used to build the creditscorecard object is used. You can also supply input data to which the validation is applied.

Stats = validatemodel(sc,data) validates the quality of the creditscorecard model for a given data set specified using the optional argument data.
[Stats,T] = validatemodel(sc,Name,Value) validates the quality of the creditscorecard model using the optional name-value pair arguments, and returns Stats and T outputs.
[Stats,T,hf] = validatemodel(sc,Name,Value) validates the quality of the creditscorecard model using the optional name-value pair arguments, and returns the figure handle hf to the CAP, ROC, and KS plots.

\section*{Examples}

\section*{Validate a Credit Scorecard Model}

Create a creditscorecard object using the CreditCardData.mat file to load the data (using a dataset from Refaat 2011).
```

load CreditCardData
sc = creditscorecard(data, 'IDVar','CustID')
sc =
creditscorecard with properties:
GoodLabel: 0
ResponseVar: 'status'
WeightsVar: ''
VarNames: {1x11 cell}
NumericPredictors: {1x6 cell}
CategoricalPredictors: {'ResStatus' 'EmpStatus' 'OtherCC'}
BinMissingData: 0
IDVar: 'CustID'
PredictorVars: {1x9 cell}
Data: [1200x11 table]

```

Perform automatic binning using the default options. By default, autobinning uses the Monotone algorithm.
```

sc = autobinning(sc);

```

Fit the model.
```

sc = fitmodel(sc);

```
1. Adding CustIncome, Deviance \(=1490.8527\), Chi2Stat \(=32.588614\), PValue \(=1.1387992 e-(\)
2. Adding TmWBank, Deviance \(=1467.1415\), Chi2Stat \(=23.711203\), PValue \(=1.1192909 \mathrm{e}-06\)
3. Adding AMBalance, Deviance \(=1455.5715\), Chi2Stat \(=11.569967\), PValue \(=0.0006702560\)
4. Adding EmpStatus, Deviance \(=1447.3451\), Chi2Stat \(=8.2264038\), PValue \(=0.0041285257\)
5. Adding CustAge, Deviance \(=1441.994\), Chi2Stat \(=5.3511754\), \(\operatorname{PValue}=0.020708306\)
6. Adding ResStatus, Deviance \(=1437.8756\), Chi2Stat \(=4.118404\), PValue \(=0.042419078\)
7. Adding OtherCC, Deviance \(=1433.707\), Chi2Stat \(=4.1686018\), PValue \(=0.041179769\)

Generalized linear regression model:
status ~ [Linear formula with 8 terms in 7 predictors]
Distribution = Binomial
Estimated Coefficients:
\begin{tabular}{|c|c|c|c|}
\hline Estimate & SE & tStat & pValue \\
\hline 0.70239 & 0.064001 & 10.975 & 5.0538e-28 \\
\hline 0.60833 & 0.24932 & 2.44 & 0.014687 \\
\hline
\end{tabular}
\begin{tabular}{lrrrr} 
ResStatus & 1.377 & 0.65272 & 2.1097 & 0.034888 \\
EmpStatus & 0.88565 & 0.293 & 3.0227 & 0.0025055 \\
CustIncome & 0.70164 & 0.21844 & 3.2121 & 0.0013179 \\
TmWBank & 1.1074 & 0.23271 & 4.7589 & \(1.9464 \mathrm{e}-06\) \\
OtherCC & 1.0883 & 0.52912 & 2.0569 & 0.039696 \\
AMBalance & 1.045 & 0.32214 & 3.2439 & 0.0011792
\end{tabular}
```

1200 observations, 1192 error degrees of freedom
Dispersion: 1
Chi^2-statistic vs. constant model: 89.7, p-value = 1.4e-16

```

Format the unscaled points.
sc = formatpoints(sc, 'Points0ddsAndPDO',[500,2,50]);
Score the data.
scores = score(sc);
Validate the credit scorecard model by generating the CAP, ROC, and KS plots.
[Stats,T] = validatemodel(sc,'Plot',\{'CAP','ROC','KS'\});



\begin{abstract}
K-S Plot

disp(Stats)
\begin{tabular}{lll}
\multicolumn{1}{c}{ Measure } & & Value \\
& \\
'Accuracy Ratio' & & 0.32258 \\
'Area under ROC curve' & 0.66129 \\
'KS statistic' & 0.2246 \\
'KS score' & 499.62
\end{tabular}
```

disp(T(1:15,:))

```

Scores ProbDefault
TrueBads
FalseBads
TrueGoods
FalseGoods
\end{abstract}
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline 369.54 & 0.75313 & 0 & 1 & 802 & 397 & \\
\hline 378.19 & 0.73016 & 1 & 1 & 802 & 396 & 0.00 \\
\hline 380.28 & 0.72444 & 2 & 1 & 802 & 395 & 0.00 \\
\hline 391.49 & 0.69234 & 3 & 1 & 802 & 394 & 0.00 \\
\hline 395.57 & 0.68017 & 4 & 1 & 802 & 393 & 0.0 \\
\hline 396.14 & 0.67846 & 4 & 2 & 801 & 393 & 0.0 \\
\hline 396.45 & 0.67752 & 5 & 2 & 801 & 392 & 0.0 \\
\hline 398.61 & 0.67094 & 6 & 2 & 801 & 391 & 0.0 \\
\hline 398.68 & 0.67072 & 7 & 2 & 801 & 390 & 0.0 \\
\hline 401.33 & 0.66255 & 8 & 2 & 801 & 389 & 0.0 \\
\hline 402.66 & 0.65842 & 8 & 3 & 800 & 389 & 0.0 \\
\hline 404.25 & 0.65346 & 9 & 3 & 800 & 388 & 0. \\
\hline 404.73 & 0.65193 & 9 & 4 & 799 & 388 & 0. \\
\hline 405.53 & 0.64941 & 11 & 4 & 799 & 386 & 0.0 \\
\hline 405.7 & 0.64887 & 11 & 5 & 798 & 386 & 0.0 \\
\hline
\end{tabular}

\section*{Validate a Credit Score Card Model With Weights}

Use the CreditCardData.mat file to load the data (dataWeights) that contains a column (RowWeights) for the weights (using a dataset from Refaat 2011).
load CreditCardData
Create a creditscorecard object using the optional name-value pair argument for 'WeightsVar'.
```

sc = creditscorecard(dataWeights,'IDVar','CustID','WeightsVar','RowWeights')
SC =
creditscorecard with properties:

```
                        GoodLabel: 0
            ResponseVar: 'status'
                    WeightsVar: 'RowWeights'
                            VarNames: \{1x12 cell\}
        NumericPredictors: \{1x6 cell\}
        CategoricalPredictors: \{'ResStatus' 'EmpStatus' 'OtherCC'\}
            BinMissingData: 0
                            IDVar: 'CustID'
                PredictorVars: \{1x9 cell\}
                            Data: [1200x12 table]

Perform automatic binning.
```

sc = autobinning(sc)
sc =
creditscorecard with properties:
GoodLabel: 0
ResponseVar: 'status'
WeightsVar: 'RowWeights'
VarNames: {1x12 cell}
NumericPredictors: {1x6 cell}
CategoricalPredictors: {'ResStatus' 'EmpStatus' 'OtherCC'}
BinMissingData: 0
IDVar: 'CustID'
PredictorVars: {1x9 cell}
Data: [1200x12 table]

```

\section*{Fit the model.}
```

sc = fitmodel(sc);

```
1. Adding CustIncome, Deviance \(=764.3187\), Chi2Stat \(=15.81927\), PValue \(=6.968927 \mathrm{e}-05\)
2. Adding TmWBank, Deviance \(=751.0215\), Chi2Stat \(=13.29726\), PValue \(=0.0002657942\)
3. Adding AMBalance, Deviance \(=743.7581\), Chi2Stat \(=7.263384\), PValue \(=0.007037455\)

Generalized linear regression model:
logit(status) ~ 1 + CustIncome + TmWBank + AMBalance Distribution = Binomial

Estimated Coefficients:
Estimate \(\quad\)\begin{tabular}{l} 
SE \(\quad\) tStat \(\quad\) pValue
\end{tabular}
\begin{tabular}{lrrrr} 
(Intercept) & 0.70642 & 0.088702 & 7.964 & \(1.6653 \mathrm{e}-15\) \\
CustIncome & 1.0268 & 0.25758 & 3.9862 & \(6.7132 \mathrm{e}-05\) \\
TmWBank & 1.0973 & 0.31294 & 3.5063 & 0.0004543 \\
AMBalance & 1.0039 & 0.37576 & 2.6717 & 0.0075464
\end{tabular}

1200 observations, 1196 error degrees of freedom
Dispersion: 1
Chi^2-statistic vs. constant model: 36.4, \(p\)-value \(=6.22 \mathrm{e}-08\)
Format the unscaled points.
sc = formatpoints(sc, 'PointsOddsAndPDO',[500,2,50]);
Score the data.
```

scores = score(sc);

```

Validate the credit scorecard model by generating the CAP, ROC, and KS plots. When the optional name-value pair argument 'WeightsVar' is used to specify observation (sample) weights, the T table uses statistics, sums, and cumulative sums that are weighted counts.
```

[Stats,T] = validatemodel(sc,'Plot',{'CAP','ROC','KS'});

```




Stats
```

Stats=4\times2 table
Measure
Value

```
'Accuracy Ratio'
'Area under ROC curve'
'KS statistic'
\(\begin{array}{lr}\text { 'KS statistic } & 0.23215 \\ \text { 'KS score' } & 505.41\end{array}\)

T(1:10,:)


\section*{Validate a Credit Score Card Model When Using the 'BinMissingData' Option}

This example describes both the assignment of points for missing data when the 'BinMissingData' option is set to true, and the corresponding computation of model validation statistics.
- Predictors that have missing data in the training set have an explicit bin for <missing> with corresponding points in the final scorecard. These points are computed from the Weight-of-Evidence (WOE) value for the <missing> bin and the logistic model coefficients. For scoring purposes, these points are assigned to missing values and to out-of-range values, and the final score is used to compute model validation statistics with validatemodel.
- Predictors with no missing data in the training set have no <missing> bin, therefore no WOE can be estimated from the training data. By default, the points for missing and out-of-range values are set to NaN , and this leads to a score of NaN when running score. For predictors that have no explicit <missing> bin, use the name-value argument 'Missing' in formatpoints to indicate how missing data should be treated for scoring purposes. The final score is used to compute model validation statistics with validatemodel.

Create a creditscorecard object using the CreditCardData. mat file to load the dataMissing with missing values.
```

load CreditCardData.mat
head(dataMissing,5)

| CustID | CustAge | TmAtAddress | ResStatus | EmpStatus | CustIncome |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 53 | 62 | <undefined> | Unknown | 50000 |
| 2 | 61 | 22 | Home Owner | Employed | 52000 |
| 3 | 47 | 30 | Tenant | Employed | 37000 |
| 4 | NaN | 75 | Home Owner | Employed | 53000 |
| 5 | 68 | 56 | Home Owner | Employed | 53000 |

```

Use creditscorecard with the name-value argument 'BinMissingData' set to true to bin the missing numeric or categorical data in a separate bin. Apply automatic binning.
```

sc = creditscorecard(dataMissing,'IDVar','CustID','BinMissingData',true);
sc = autobinning(sc);
disp(sc)

```
    creditscorecard with properties:
                        GoodLabel: 0
            ResponseVar: 'status'
                    WeightsVar:
                            VarNames: \{1x11 cell\}
        NumericPredictors: \{1x6 cell\}
    CategoricalPredictors: \{'ResStatus' 'EmpStatus' 'OtherCC'\}
        BinMissingData: 1
                            IDVar: 'CustID'
        PredictorVars: \{1x9 cell\}
                            Data: [1200x11 table]

Set a minimum value of zero for CustAge and CustIncome. With this, any negative age or income information becomes invalid or "out-of-range". For scoring and probability of default computations, out-of-range values are given the same points as missing values.
```

sc = modifybins(sc,'CustAge','MinValue',0);
sc = modifybins(sc,'CustIncome','MinValue',0);

```

Display bin information for numeric data for 'CustAge' that includes missing data in a separate bin labelled <missing>.
\begin{tabular}{|c|c|c|c|c|c|}
\hline Bin & Good & Bad & Odds & WOE & InfoValue \\
\hline ' \([0,33\) ) ' & 69 & 52 & 1.3269 & -0.42156 & 0.018993 \\
\hline ' \([33,37)^{\prime}\) & 63 & 45 & 1.4 & -0.36795 & 0.012839 \\
\hline ' \([37,40\) ) ' & 72 & 47 & 1.5319 & -0.2779 & 0.0079824 \\
\hline '[40,46) ' & 172 & 89 & 1.9326 & -0.04556 & 0.0004549 \\
\hline ' \([46,48\) )' & 59 & 25 & 2.36 & 0.15424 & 0.0016199 \\
\hline ' \([48,51\) )' & 99 & 41 & 2.4146 & 0.17713 & 0.0035449 \\
\hline ' \([51,58\) ) ' & 157 & 62 & 2.5323 & 0.22469 & 0.0088407 \\
\hline '[58,Inf]' & 93 & 25 & 3.72 & 0.60931 & 0.032198 \\
\hline '<missing>' & 19 & 11 & 1.7273 & -0.15787 & 0.00063885 \\
\hline 'Totals' & 803 & 397 & 2.0227 & NaN & 0.087112 \\
\hline
\end{tabular}

Display bin information for categorical data for 'ResStatus' that includes missing data in a separate bin labelled <missing>.
```

bi = bininfo(sc,'ResStatus');
disp(bi)

| Bin | Good | Bad | 0dds | WOE | InfoValue |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 'Tenant' | 296 | 161 | 1.8385 | -0.095463 | 0.0035249 |
| 'Home Owner' | 352 | 171 | 2.0585 | 0.017549 | 0.00013382 |
| '0ther' | 128 | 52 | 2.4615 | 0.19637 | 0.0055808 |
| '<missing>' | 27 | 13 | 2.0769 | 0.026469 | $2.3248 \mathrm{e}-05$ |
| 'Totals' | 803 | 397 | 2.0227 | NaN | 0.0092627 |

```

For the 'CustAge' and 'ResStatus' predictors, there is missing data ( NaNs and <undefined>) in the training data, and the binning process estimates a WOE value of -0.15787 and 0.026469 respectively for missing data in these predictors, as shown above.

For EmpStatus and CustIncome there is no explicit bin for missing values, because the training data has no missing values for these predictors.
```

bi = bininfo(sc,'EmpStatus');
disp(bi)

```
\begin{tabular}{lllll} 
Bin & Good & Bad \(\quad\) Odds & W0E & InfoValue \\
\hline
\end{tabular}


Use fitmodel to fit a logistic regression model using Weight of Evidence (WOE) data. fitmodel internally transforms all the predictor variables into WOE values, using the bins found with the automatic binning process. fitmodel then fits a logistic regression model using a stepwise method (by default). For predictors that have missing data, there is an explicit <missing> bin, with a corresponding WOE value computed from the data. When using fitmodel, the corresponding WOE value for the <missing> bin is applied when performing the WOE transformation.
```

[sc,mdl] = fitmodel(sc);

```
1. Adding CustIncome, Deviance \(=1490.8527\), Chi2Stat \(=32.588614\), PValue \(=1.1387992 e-(\)
2. Adding TmWBank, Deviance \(=1467.1415\), Chi2Stat \(=23.711203\), PValue \(=1.1192909 \mathrm{e}-06\)
3. Adding AMBalance, Deviance \(=1455.5715\), Chi2Stat \(=11.569967\), PValue \(=0.0006702560\)
4. Adding EmpStatus, Deviance \(=1447.3451\), Chi2Stat \(=8.2264038\), PValue \(=0.0041285257\)
5. Adding CustAge, Deviance \(=1442.8477\), Chi2Stat \(=4.4974731\), PValue \(=0.033944979\)
6. Adding ResStatus, Deviance \(=1438.9783\), Chi2Stat \(=3.86941\), PValue \(=0.049173805\)
7. Adding OtherCC, Deviance \(=1434.9751\), Chi2Stat \(=4.0031966\), PValue \(=0.045414057\)

Generalized linear regression model:
status ~ [Linear formula with 8 terms in 7 predictors]
Distribution = Binomial

Estimated Coefficients:
Estimate SE tStat pValue

19-2231
\begin{tabular}{lrrrr} 
(Intercept) & 0.70229 & 0.063959 & 10.98 & \(4.7498 \mathrm{e}-28\) \\
CustAge & 0.57421 & 0.25708 & 2.2335 & 0.025513 \\
ResStatus & 1.3629 & 0.66952 & 2.0356 & 0.04179 \\
EmpStatus & 0.88373 & 0.2929 & 3.0172 & 0.002551 \\
CustIncome & 0.73535 & 0.2159 & 3.406 & 0.00065929 \\
TmWBank & 1.1065 & 0.23267 & 4.7556 & \(1.9783 \mathrm{e}-06\) \\
OtherCC & 1.0648 & 0.52826 & 2.0156 & 0.043841 \\
AMBalance & 1.0446 & 0.32197 & 3.2443 & 0.0011775
\end{tabular}
```

1200 observations, }1192\mathrm{ error degrees of freedom
Dispersion: 1
Chi^2-statistic vs. constant model: 88.5, p-value = 2.55e-16

```

Scale the scorecard points by the "points, odds, and points to double the odds (PDO)" method using the 'Points0ddsAndPDO' argument of formatpoints. Suppose that you want a score of 500 points to have odds of 2 (twice as likely to be good than to be bad) and that the odds double every 50 points (so that 550 points would have odds of 4 ).

Display the scorecard showing the scaled points for predictors retained in the fitting model.
```

sc = formatpoints(sc,'PointsOddsAndPDO',[500 2 50]);
PointsInfo = displaypoints(sc)
PointsInfo=33\times3 table
Predictors Bin Points
'CustAge' '[0,33)' 54.062
'CustAge' '[33,37)' 56.282
'CustAge' '[37,40)' 60.012
'CustAge' '[40,46)' 69.636
'CustAge' '[46,48)' 77.912
'CustAge' '[48,51)' }78.8
'CustAge' '[51,58)' 80.83
'CustAge' '[58,Inf]' 96.76
'CustAge' '<missing>' 64.984
'ResStatus' 'Tenant' 62.138
'ResStatus' 'Home Owner' }73.24
'ResStatus' 'Other' 90.828
'ResStatus' '<missing>' }74.12
'EmpStatus' 'Unknown' 58.807

```
\begin{tabular}{lll} 
'EmpStatus' & 'Employed' & 86.937 \\
'CustIncome' & '[0,29000)' & 29.375
\end{tabular}

Notice that points for the <missing> bin for CustAge and ResStatus are explicitly shown (as 64.9836 and 74.1250 , respectively). These points are computed from the WOE value for the <missing> bin, and the logistic model coefficients.

For predictors that have no missing data in the training set, there is no explicit <missing> bin. By default the points are set to NaN for missing data, and they lead to a score of NaN when running score. For predictors that have no explicit <missing> bin, use the name-value argument 'Missing' in formatpoints to indicate how missing data should be treated for scoring purposes.

For the purpose of illustration, take a few rows from the original data as test data and introduce some missing data. Also introduce some invalid, or out-of-range, values. For numeric data, values below the minimum (or above the maximum) allowed are considered invalid, such as a negative value for age (recall 'MinValue' was earlier set to 0 for CustAge and CustIncome). For categorical data, invalid values are categories not explicitly included in the scorecard, for example, a residential status not previously mapped to scorecard categories, such as "House", or a meaningless string such as "abc123".

This is a very small validation data set, only used to illustrate the scoring of rows with missing and out-of-range values, and its relationship with model validation.
```

tdata = dataMissing(11:18,mdl.PredictorNames); % Keep only the predictors retained in
tdata.status = dataMissing.status(11:18); % Copy the response variable value, needed f
% Set some missing values
tdata.CustAge(1) = NaN;
tdata.ResStatus(2) = '<undefined>';
tdata.EmpStatus(3) = '<undefined>';
tdata.CustIncome(4) = NaN;
% Set some invalid values
tdata.CustAge(5) = -100;
tdata.ResStatus(6) = 'House';
tdata.EmpStatus(7) = 'Freelancer';
tdata.CustIncome(8) = -1;
disp(tdata)
CustAge ResStatus EmpStatus CustIncome TmWBank OtherCC AMBala

```
\begin{tabular}{rllrrrr} 
NaN & Tenant & Unknown & 34000 & 44 & Yes & 119 \\
48 & <undefined> & Unknown & 44000 & 14 & Yes & 403.6 \\
65 & Home Owner & <undefined> & 48000 & 6 & No & 111.8 \\
44 & Other & Unknown & NaN & 35 & No & 436.6 \\
-100 & Other & Employed & 46000 & 16 & Yes & 162.2 \\
33 & House & Employed & 36000 & 36 & Yes & 845.1 \\
39 & Tenant & Freelancer & 34000 & 40 & Yes & 756.2 \\
24 & Home Owner & Employed & -1 & 19 & Yes & 449.6
\end{tabular}

Score the new data and see how points are assigned for missing CustAge and ResStatus, because we have an explicit bin with points for <missing>. However, for EmpStatus and CustIncome the score function sets the points to NaN .

The validation results are unreliable, the scores with NaN values are kept (see the validation table ValTable below), but it is unclear what impact these NaN values have in the validation statistics (ValStats). This is a very small validation data set, but NaN scores could still influence the validation results on a larger data set.
```

[Scores,Points] = score(sc,tdata);
disp(Scores)
481.2231
520.8353
NaN
NaN
551.7922
487.9588
NaN
NaN
disp(Points)

| CustAge | ResStatus | EmpStatus | CustIncome | TmWBank | OtherCC | AMBalance |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 64.984 | 62.138 | 58.807 | 67.893 | 61.858 | 75.622 | 89.922 |
| 78.86 | 74.125 | 58.807 | 82.439 | 61.061 | 75.622 | 89.922 |
| 96.76 | 73.248 | NaN | 96.969 | 51.132 | 50.914 | 89.922 |
| 69.636 | 90.828 | 58.807 | NaN | 61.858 | 50.914 | 89.922 |
| 64.984 | 90.828 | 86.937 | 82.439 | 61.061 | 75.622 | 89.922 |
| 56.282 | 74.125 | 86.937 | 70.107 | 61.858 | 75.622 | 63.028 |
| 60.012 | 62.138 | NaN | 67.893 | 61.858 | 75.622 | 63.028 |
| 54.062 | 73.248 | 86.937 | NaN | 61.061 | 75.622 | 89.922 |

```


Use the name-value argument 'Missing' in formatpoints to choose how to assign points to missing values for predictors that do not have an explicit <missing> bin. In this example, use the 'MinPoints ' option for the 'Missing' argument. The minimum points for EmpStatus in the scorecard displayed above are 58.8072 and for CustIncome the minimum points are 29.3753.

The validation results are no longer influenced by NaN values, since all rows now have a score.
```

sc = formatpoints(sc,'Missing','MinPoints');
[Scores,Points] = score(sc,tdata);
disp(Scores)

```
481.2231
520.8353
517.7532
451.3405
551.7922
487.9588
```

449.3577
470.2267

```

\section*{disp(Points)}
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline CustAge & ResStatus & EmpStatus & CustIncome & TmWBank & OtherCC & AMBalance \\
\hline 64.984 & 62.138 & 58.807 & 67.893 & 61.858 & 75.622 & 89.922 \\
\hline 78.86 & 74.125 & 58.807 & 82.439 & 61.061 & 75.622 & 89.922 \\
\hline 96.76 & 73.248 & 58.807 & 96.969 & 51.132 & 50.914 & 89.922 \\
\hline 69.636 & 90.828 & 58.807 & 29.375 & 61.858 & 50.914 & 89.922 \\
\hline 64.984 & 90.828 & 86.937 & 82.439 & 61.061 & 75.622 & 89.922 \\
\hline 56.282 & 74.125 & 86.937 & 70.107 & 61.858 & 75.622 & 63.028 \\
\hline 60.012 & 62.138 & 58.807 & 67.893 & 61.858 & 75.622 & 63.028 \\
\hline 54.062 & 73.248 & 86.937 & 29.375 & 61.061 & 75.622 & 89.922 \\
\hline
\end{tabular}

\section*{[ValStats,ValTable] = validatemodel(sc,tdata);} disp(ValStats)
\begin{tabular}{lll}
\multicolumn{1}{c}{ Measure } & & \multicolumn{1}{c}{ Value } \\
& \\
'Accuracy Ratio' & & \\
'Area under ROC curve' & 0.66667 \\
'KS statistic' & 0.83333 \\
'KS score' & 0.66667 \\
& 481.22
\end{tabular}

\section*{disp(ValTable)}
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline Scores & ProbDefault & TrueBads & FalseBads & TrueGoods & FalseGoods & Sensi \\
\hline 449.36 & 0.50223 & 1 & 0 & 6 & 1 & 0 \\
\hline 451.34 & 0.49535 & 1 & 1 & 5 & 1 & 0 \\
\hline 470.23 & 0.43036 & 1 & 2 & 4 & 1 & 0 \\
\hline 481.22 & 0.39345 & 2 & 2 & 4 & 0 & \\
\hline 487.96 & 0.3714 & 2 & 3 & 3 & 0 & \\
\hline 517.75 & 0.28105 & 2 & 4 & 2 & 0 & \\
\hline 520.84 & 0.2725 & 2 & 5 & 1 & 0 & \\
\hline 551.79 & 0.19605 & 2 & 6 & 0 & 0 & \\
\hline
\end{tabular}

\section*{Input Arguments}

\section*{sc - Credit scorecard model}

\section*{creditscorecard object}

Credit scorecard model, specified as a creditscorecard object. To create this object, use creditscorecard.

\section*{data - Validation data}
table
(Optional) Validation data, specified as a MATLAB table, where each table row corresponds to individual observations. The data must contain columns for each of the predictors in the credit scorecard model. The columns of data can be any one of the following data types:
- Numeric
- Logical
- Cell array of character vectors
- Character array
- Categorical
- String
- String array

In addition, the table must contain a binary response variable.

Note When observation weights are defined using the optional WeightsVar name-value pair argument when creating a creditscorecard object, the weights stored in the WeightsVar column are used when validating the model on the training data. If a different validation data set is provided using the optional data input, observation weights for the validation data must be included in a column whose name matches WeightsVar, otherwise unit weights are used for the validation data. For more information, see "Using validatemodel with Weights" on page 19-2242.

\section*{Name-Value Pair Arguments}

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.
Example: sc = validatemodel(sc,data,'AnalysisLevel','Deciles','Plot','CAP')

\section*{AnalysisLevel - Type of analysis level}
'Scores' (default)| character vector with values 'Deciles', 'Scores'
Type of analysis level, specified as the comma-separated pair consisting of 'AnalysisLevel' and a character vector with one of the following values:
- 'Scores' - Returns the statistics (Stats) at the observation level. Scores are sorted from riskiest to safest, and duplicates are removed.
- 'Deciles ' - Returns the statistics (Stats) at decile level. Scores are sorted from riskiest to safest and binned with their corresponding statistics into 10 deciles (10\%, 20\%, ..., 100\%).

\section*{Data Types: char}

\section*{Plot - Type of plot}
'None' (default) | character vector with values 'None', 'CAP', 'ROC', 'KS' | cell array of character vectors with values 'None', 'CAP', 'ROC', 'KS'

Type of plot, specified as the comma-separated pair consisting of 'Plot ' and a character vector with one of the following values:
- 'None' - No plot is displayed.
- 'CAP' - Cumulative Accuracy Profile. Plots the fraction of borrowers up to score " s " versus the fraction of defaulters up to score "s" ('Pct0bs' versus 'Sensitivity' columns of T optional output argument). For more details, see "Cumulative Accuracy Profile (CAP)" on page 19-2240.
- 'ROC' - Receiver Operating Characteristic. Plots the fraction of non-defaulters up to score "s" versus the fraction of defaulters up to score "s" ('FalseAlarm' versus 'Sensitivity ' columns of T optional output argument). For more details, see "Receiver Operating Characteristic (ROC)" on page 19-2241.
- 'KS ' - Kolmogorov-Smirnov. Plots each score "s" versus the fraction of defaulters up to score "s," and also versus the fraction of non-defaulters up to score "s" ( 'Scores ' versus both 'Sensitivity' and 'FalseAlarm' columns of the optional output argument T). For more details, see "Kolmogorov-Smirnov statistic (KS)" on page 192241.

Tip For the Kolmogorov-Smirnov statistic option, you can enter 'KS' or 'K-S'.
Data Types: char | cell

\section*{Output Arguments}

\section*{Stats - Validation measures \\ table}

Validation measures, returned as a 4-by-2 table. The first column, 'Measure', contains the names of the following measures:
- Accuracy ratio (AR)
- Area under the ROC curve (AUROC)
- The KS statistic
- KS score

The second column, 'Value', contains the values corresponding to these measures.

\section*{T - Validation statistics data \\ array}

Validation statistics data, returned as an N-by-9 table of validation statistics data, sorted, by score, from riskiest to safest. When AnalysisLevel is set to 'Deciles', N is equal to 10 . Otherwise, N is equal to the total number of unique scores, that is, scores without duplicates.

The table T contains the following nine columns, in this order:
- 'Scores ' - Scores sorted from riskiest to safest. The data in this row corresponds to all observations up to, and including the score in this row.
- 'ProbDefault' - Probability of default for observations in this row. For deciles, the average probability of default for all observations in the given decile is reported.
- 'TrueBads ' - Cumulative number of "bads" up to, and including, the corresponding score.
- 'FalseBads ' - Cumulative number of "goods" up to, and including, the corresponding score.
- 'TrueGoods ' - Cumulative number of "goods" above the corresponding score.
- 'FalseGoods ' - Cumulative number of "bads" above the corresponding score.
- 'Sensitivity' - Fraction of defaulters (or the cumulative number of "bads" divided by total number of "bads"). This is the distribution of "bads" up to and including the corresponding score.
- 'FalseAlarm' - Fraction of non-defaulters (or the cumulative number of "goods" divided by total number of "goods"). This is the distribution of "goods" up to and including the corresponding score.
- 'Pct0bs' - Fraction of borrowers, or the cumulative number of observations, divided by total number of observations up to and including the corresponding score.

Note When creating the creditscorecard object with creditscorecard, if the optional name-value pair argument WeightsVar was used to specify observation (sample) weights, then the T table uses statistics, sums, and cumulative sums that are weighted counts.

\section*{hf - Handle to the plotted measures \\ figure handle}

Figure handle to plotted measures, returned as a figure handle or array of handles. When Plot is set to 'None', hf is an empty array.

\section*{Definitions}

\section*{Cumulative Accuracy Profile (CAP)}

CAP is generally a concave curve and is also known as the Gini curve, Power curve, or Lorenz curve.

The scores of given observations are sorted from riskiest to safest. For a given fraction M ( \(0 \%\) to \(100 \%\) ) of the total borrowers, the height of the CAP curve is the fraction of
defaulters whose scores are less than or equal to the maximum score of the fraction M , also known as "Sensitivity."

The area under the CAP curve, known as the AUCAP, is then compared to that of the perfect or "ideal" model, leading to the definition of a summary index known as the accuracy ratio \((A R)\) or the Gini coefficient:
\[
A R=\frac{A_{R}}{A_{P}}
\]
where \(A_{R}\) is the area between the CAP curve and the diagonal, and \(A_{P}\) is the area between the perfect model and the diagonal. This represents a "random" model, where scores are assigned randomly and therefore the proportion of defaulters and non-defaulters is independent of the score. The perfect model is the model for which all defaulters are assigned the lowest scores, and therefore, perfectly discriminates between defaulters and nondefaulters. Thus, the closer to unity \(A R\) is, the better the scoring model.

\section*{Receiver Operating Characteristic (ROC)}

To find the receiver operating characteristic (ROC) curve, the proportion of defaulters up to a given score "s," or "Sensitivity," is computed.

This proportion is known as the true positive rate (TPR). Additionally, the proportion of nondefaulters up to score "s," or "False Alarm Rate," is also computed. This proportion is also known as the false positive rate (FPR). The ROC curve is the plot of the "Sensitivity" vs. the "False Alarm Rate." Computing the ROC curve is similar to computing the equivalent of a confusion matrix at each score level.

Similar to the CAP, the ROC has a summary statistic known as the area under the ROC curve (AUROC). The closer to unity, the better the scoring model. The accuracy ratio (AR) is related to the area under the curve by the following formula:
\[
A R=2(A U R O C)-1
\]

\section*{Kolmogorov-Smirnov statistic (KS)}

The Kolmogorov-Smirnov (KS) plot, also known as the fish-eye graph, is a common statistic used to measure the predictive power of scorecards.

The KS plot shows the distribution of defaulters and the distribution of non-defaulters on the same plot. For the distribution of defaulters, each score " \(s\) " is plotted versus the proportion of defaulters up to "s," or "Sensitivity." For the distribution of non-defaulters, each score "s" is plotted versus the proportion of non-defaulters up to "s," or "False Alarm." The statistic of interest is called the KS statistic and is the maximum difference between these two distributions ("Sensitivity" minus "False Alarm"). The score at which this maximum is attained is also of interest.

\section*{Using validatemodel with Weights}

Model validation statistics incorporate observation weights when these are provided by the user.

Without weights, the validation statistics are based on how many good and bad observations fall below a particular score. On the other hand, when observation weights are provided, the weight (not the count) is accumulated for the good and the bad observations that fall below a particular score.

When observation weights are defined using the optional WeightsVar name-value pair argument when creating a creditscorecard object, the weights stored in the WeightsVar column are used when validating the model on the training data. When a different validation data set is provided using the optional data input, observation weights for the validation data must be included in a column whose name matches WeightsVar, otherwise unit weights are used for the validation data set.

Not only the validation statistics, but the credit scorecard scores themselves depend on the observation weights of the training data. For more information, see "Using fitmodel with Weights" on page 19-2069 and "Credit Scorecard Modeling Using Observation Weights" on page 8-70.

\section*{References}
[1] "Basel Committee on Banking Supervision: Studies on the Validation of Internal Rating Systems." Working Paper No. 14, February 2005.
[2] Refaat, M. Credit Risk Scorecards: Development and Implementation Using SAS. lulu.com, 2011.
[3] Loeffler, G. and Posch, P. N. Credit Risk Modeling Using Excel and VBA. Wiley Finance, 2007.

\title{
See Also \\ bindata|bininfo|creditscorecard|displaypoints |fitmodel| formatpoints |modifybins | modifypredictor|plotbins | predictorinfo| probdefault|score|setmodel|table
}

\section*{Topics}
"Case Study for a Credit Scorecard Analysis" on page 8-101
"Credit Scorecard Modeling with Missing Values" on page 8-73
"Troubleshooting Credit Scorecard Results" on page 8-91
"Credit Scorecard Modeling Workflow" on page 8-67
"About Credit Scorecards" on page 8-62
"Credit Scorecard Modeling Using Observation Weights" on page 8-70

\section*{Introduced in R2015a}

\section*{creditscorecard}

Create creditscorecard object to build credit scorecard model

\section*{Description}

Build a credit scorecard model by creating a creditscorecard object and specify input data in a table format.

After creating a creditscorecard object, you can use the associated object functions to bin the data and perform logistic regression analysis to develop a credit scorecard model to guide credit decisions. This workflow shows how to develop a credit scorecard model.

1 Create a creditscorecard object (see "Create creditscorecard" on page 19-2244 and "Properties" on page 19-2248).
2 Bin the data.
3 Fit a logistic regression model.
4 Review and format the credit scorecard points.
5 Score the data.
6 Calculate the probabilities of default for the data.
7 Validate the quality of the credit scorecard model.
For more detailed information on this workflow, see "Credit Scorecard Modeling Workflow" on page 8-67.

\section*{Creation}

\section*{Syntax}
```

sc = creditscorecard(data)
sc = creditscorecard(

```
\(\qquad\)
``` ,Name, Value)
```


## Description

sc = creditscorecard(data) creates a creditscorecard object by specifying data. The credit scorecard model, returned as a creditscorecard object, contains the binning maps or rules (cut points or category groupings) for one or more predictors.
sc = creditscorecard (__ ,Name, Value) sets Properties on page 19-2248 using name-value pairs and any of the arguments in the previous syntax. For example, sc = creditscorecard(data,'GoodLabel',
0,'IDVar','CustID','ResponseVar','status','PredictorVars', \{'CustAge','CustIncome'\},'WeightsVar','RowWeights','BinMissingData', true). You can specify multiple name-value pairs.

Note To use observation (sample) weights in the credit scorecard workflow, when creating a creditscorecard object, you must use the optional name-value pair WeightsVar to define which column in the data contains the weights.

## Input Arguments

## data - Data for creditscorecard object

table
Data for the creditscorecard object, specified as a MATLAB table, where each column of data can be any one of the following data types:

- Numeric
- Logical
- Cell array of character vectors
- Character array
- Categorical
- String

In addition, the table must contain a binary response variable. Before creating a creditscorecard object, perform a data preparation task to have appropriately structured data as input to a creditscorecard object. The data input sets the Data on page 19-0 property.

Data Types: table

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Example: sc = creditscorecard(data,' GoodLabel', 0,'IDVar','CustAge','ResponseVar','status','PredictorVars', \{'CustID','CustIncome'\},'WeightsVar','RowWeights','BinMissingData',t rue)

## GoodLabel - Indicator for which of two possible values in response variable correspond to "Good" observations

set to the response value with the highest count (default) | character vector | numeric scalar | logical

Indicator for which of the two possible values in the response variable correspond to "Good" observations, specified as the comma-separated pair consisting of 'GoodLabel' and a numeric scalar, logical, or character vector. The GoodLabel name-value pair argument sets the GoodLabel on page 19-0 property.

When specifying GoodLabel, follow these guidelines.

| If Response Is... | GoodLabel Must be... |
| :--- | :--- |
| numeric | numeric |
| logical | logical or numeric |
| cell array of character vectors | character vector |
| character array | character vector |
| categorical | character vector |

If not specified, GoodLabel is set to the response value with the highest count. However, if the optional WeightsVar argument is provided when creating the creditscorecard object, then counts are replaced with weighted frequencies. For more information, see "Credit Scorecard Modeling Using Observation Weights" on page 8-70.

GoodLabel can only be set when creating the creditscorecard object. This parameter cannot be set using dot notation.

Data Types: char|double

## IDVar - Variable name used as ID or tag for observations <br> empty character vector ' ' (default) | character vector

Variable name used as ID or tag for the observations, specified as the comma-separated pair consisting of 'IDVar' and a character vector. The IDVar data could be an ordinal number (for example, $1,2,3 \ldots$ ), a Social Security number. This is provided as a convenience to remove this column from the predictor variables. IDVar is case-sensitive. The IDVar name-value pair argument sets the IDVar on page 19-0 property.

You can set this optional parameter using the creditscorecard function or by using dot notation at the command line, as follows.

Example: sc.IDVar = 'CustID'
Data Types: char

## ResponseVar - Response variable name for "Good" or "Bad" indicator last column of the data input (default) | character vector

Response variable name for the "Good" or "Bad" indicator, specified as the commaseparated pair consisting of 'ResponseVar' and a character vector. The response variable data must be binary. The ResponseVar name-value pair argument sets the ResponseVar on page 19-0 property.

If not specified, ResponseVar is set to the last column of the data input. ResponseVar can only be set when creating the creditscorecard object using the creditscorecard function. ResponseVar is case-sensitive.

Data Types: char

## WeightsVar - Weights variable name

empty character vector ' ' (default) | character vector
Weights variable name, specified as the comma-separated pair consisting of 'WeightsVar' and a character vector to indicate which column name in the data table contains the row weights. WeightsVar is case-sensitive. The WeightsVar name-value pair argument sets the WeightsVar on page 19-0 property, and this property can only be set at the creation of a creditscorecard object. If the name-value pair argument WeightsVar is not specified when creating a creditscorecard object, then observation weights are set to unit weights by default.

The WeightsVar values are used in the credit scorecard workflow by autobinning, bininfo, fitmodel, and validatemodel. For more information, see "Credit Scorecard Modeling Using Observation Weights" on page 8-70.

## Data Types: char

## BinMissingData - Indicates if missing data is removed or displayed in a separate bin <br> false (missing data removed) (default) | logical with value true or false

Indicates if missing data is removed or displayed in a separate bin, specified as the comma-separated pair consisting of 'BinMissingdata' and a logical scalar with a value of true or false. If BinMissingData is true, the missing data for a predictor is displayed in a separate bin labeled <missing>.

Data Types: logical

## Properties

## Data - Data used to create the creditscorecard object

table
Data used to create the creditscorecard object, specified as a table when creating a creditscorecard object. In the Data property, categorical predictors are stored as categorical arrays.
Example: sc.Data(1:10,:)
Data Types: table

## IDVar - Name of the variable used as ID or tag for the observations empty character vector ' ' (default) | character vector

Name of the variable used as ID or tag for the observations, specified as a character vector. This property can be set as an optional parameter when creating a creditscorecard object or by using dot notation at the command line. IDVar is casesensitive.

Example: sc.IDVar = 'CustID'
Data Types: char

## VarNames - All variable names from the data input

VarNames come directly from data input to creditscorecard object (default)
This property is read-only.

VarNames is a cell array of character vectors containing the names of all variables in the data. The VarNames come directly from the data input to the creditscorecard object. VarNames is case-sensitive.

## Data Types: cell

## ResponseVar - Name of the response variable, "Good" or "Bad" indicator

last column of the data input (default) | character vector
Name of the response variable, "Good" or "Bad" indicator, specified as a character vector. The response variable data must be binary. If not specified, ResponseVar is set to the last column of the data input. This property can only be set with an optional parameter when creating a creditscorecard object. ResponseVar is case-sensitive.

## Data Types: char

## WeightsVar - Name of the variable used as ID or tag for weights empty character vector ' ' (default)| character vector

Name of the variable used as ID or tag to indicate which column name in the data table contains the row weights, specified as a character vector. This property can be set as an optional parameter (WeightsVar) when creating a creditscorecard object. WeightsVar is case-sensitive.

Data Types: char

## GoodLabel - Indicator for which of the two possible values in the response variable correspond to "Good" observations <br> set to the response value with the highest count (default) | character vector | numeric scalar | logical

Indicator for which of the two possible values in the response variable correspond to "Good" observations. When specifying GoodLabel, follow these guidelines:

| If Response is... | GoodLabel must be: |
| :--- | :--- |
| numeric | numeric |
| logical | logical or numeric |
| cell array of character vectors | character vector |
| character array | character vector |
| categorical | character vector |

If not specified, GoodLabel is set to the response value with the highest count. This property can only be set with an optional parameter when creating a creditscorecard object. This property cannot be set using dot notation.
Data Types: char | double
PredictorVars - Predictor variable names
set difference between VarNames and \{IDVar,ResponseVar\} (default) | cell array of character vectors containing names

Predictor variable names, specified using a cell array of character vectors containing names. By default, when you create a creditscorecard object, all variables are predictors except for IDVar and ResponseVar. This property can be modified using a name-value pair argument for the fitmodel function or by using dot notation.
PredictorVars is case-sensitive and the predictor variable name cannot be the same as the IDVar or ResponseVar.

Example: sc.PredictorVars = \{'CustID','CustIncome'\}
Data Types: cell

```
NumericPredictors - Name of numeric predictors
empty character vector ' ' (default)| character vector
```

Name of numeric predictors, specified as a character vector. This property cannot be set by using dot notation at the command line. It can only be modified using the modifypredictor function.

Data Types: char

## CategoricalPredictors - Name of categorical predictors

empty character vector ' ' (default) | character vector
Name of categorical predictors, specified as a character vector. This property cannot be set by using dot notation at the command line. It can only be modified using the modifypredictor function.
Data Types: char

## BinMissingData - Indicates if missing data is removed or displayed in a separate bin <br> false (missing data removed) (default) | logical with value true or false

Indicates if missing data is removed or displayed in a separate bin, specified as the comma-separated pair consisting of 'BinMissingdata' and a logical scalar with a value
of true or false. If BinMissingData is true, the missing data for a predictor is displayed in a separate bin labeled <missing>. For more information on working with missing data, see "Credit Scorecard Modeling with Missing Values" on page 8-73.

## Data Types: logical

| creditscorecard <br> Property | Set/Modify <br> Property from <br> Command Line <br> Using <br> creditscorecard <br> Function | Modify Property <br> Using Dot Notation | Property Not User- <br> Defined and Value <br> Is Defined <br> Internally |
| :--- | :--- | :--- | :--- |
| Data | No | No | Yes, copy of data <br> input |
| IDVar | Yes | Yes | No, but the user <br> specifies this |
| VarNames | No | No | Yes |
| ResponseVar | No | If not specified, set <br> to last column of <br> data input |  |
| WeightsVar | No | No | Yes |
| GoodLabel | No not specified, set |  |  |
| to response value |  |  |  |
| with highest count |  |  |  |\(\left|\begin{array}{l}Yes, but the user can <br>


modify this\end{array}\right|\)| PredictorVars |
| :--- |
| Yes (also modifiable <br> using fitmodel <br> function) |
| Yes |
| NumericPredictor <br> s |
| No (can only be <br> modified using <br> modifypredictor <br> function) |
| CategoricalPredi <br> ctors |
| No (can only be <br> modified using <br> modifypredictor <br> function) |
| No |


| creditscorecard <br> Property | Set/Modify <br> Property from <br> Command Line <br> Using <br> creditscorecard <br> Function | Modify Property <br> Using Dot Notation | Property Not User- <br> Defined and Value <br> Is Defined <br> Internally |
| :--- | :--- | :--- | :--- |
| BinMissingData | Yes | No | False by default, but <br> the user can modify <br> this |

## Object Functions

autobinning
bininfo predictorinfo modifypredictor modifybins bindata plotbins fitmodel setmodel displaypoints formatpoints score probdefault validatemodel

Perform automatic binning of given predictors
Return predictor's bin information
Summary of credit scorecard predictor properties
Set properties of credit scorecard predictors Modify predictor's bins
Binned predictor variables
Plot histogram counts for predictor variables
Fit logistic regression model to Weight of Evidence (WOE) data
Set model predictors and coefficients
Return points per predictor per bin
Format scorecard points and scaling
Compute credit scores for given data
Likelihood of default for given data set
Validate quality of credit scorecard model

## Examples

## Create a creditscorecard Object

Create a creditscorecard object using the CreditCardData.mat file to load the data (using a dataset from Refaat 2011).
load CreditCardData
sc = creditscorecard(data)

```
SC =
    creditscorecard with properties:
```

                        GoodLabel: 0
            ResponseVar: 'status'
                            WeightsVar:
                            VarNames: \{1x11 cell\}
        NumericPredictors: \(\{1 \times 7\) cell\}
    CategoricalPredictors: \{'ResStatus' 'EmpStatus' 'OtherCC'\}
        BinMissingData: 0
                            IDVar: ''
            PredictorVars: \(\{1 \times 10\) cell\}
                        Data: [1200x11 table]
    
## Create a creditscorecard Object Containing Weights

Use the CreditCardData.mat file to load the data (dataWeights) that contains a column (RowWeights) for the weights (using a dataset from Refaat 2011).

```
load CreditCardData
```

Create a creditscorecard object using the optional name-value pair argument for 'WeightsVar'.

```
sc = creditscorecard(dataWeights,'WeightsVar','RowWeights')
sc =
    creditscorecard with properties:
                    GoodLabel: 0
                    ResponseVar: 'status'
                    WeightsVar: 'RowWeights'
                            VarNames: {1x12 cell}
            NumericPredictors: {1x7 cell}
        CategoricalPredictors: {'ResStatus' 'EmpStatus' 'OtherCC'}
            BinMissingData: 0
                            IDVar: ''
            PredictorVars: {1x10 cell}
                            Data: [1200x12 table]
```


## Display creditscorecard Object Properties

Create a creditscorecard object using the CreditCardData.mat file to load the data (using a dataset from Refaat 2011).

```
load CreditCardData
```

sc = creditscorecard(data)
sc =
creditscorecard with properties:

GoodLabel: 0
ResponseVar: 'status'
WeightsVar: ''
VarNames: $\{1 \times 11$ cell\}
NumericPredictors: \{1×7 cell\}
CategoricalPredictors: \{'ResStatus' 'EmpStatus' 'OtherCC'\}
BinMissingData: 0
IDVar: ''
PredictorVars: \{1x10 cell\}
Data: [1200x11 table]

To display the creditscorecard object properties, use dot notation.
sc.PredictorVars
ans $=1 \times 10$ cell array
Columns 1 through 4
\{'CustID'\} \{'CustAge'\} \{'TmAtAddress'\} \{'ResStatus'\}
Columns 5 through 8
\{'EmpStatus'\} \{'CustIncome'\} \{'TmWBank'\} \{'OtherCC'\}
Columns 9 through 10
\{'AMBalance'\} \{'UtilRate'\}
sc.VarNames

```
ans = 1x11 cell array
    Columns 1 through 4
        {'CustID'} {'CustAge'} {'TmAtAddress'} {'ResStatus'}
    Columns 5 through 8
        {'EmpStatus'} {'CustIncome'} {'TmWBank'} {'OtherCC'}
    Columns 9 through 11
        {'AMBalance'} {'UtilRate'} {'status'}
```


## Change a Property Value for a creditscorecard Object

Create a creditscorecard object using the CreditCardData.mat file to load the data (using a dataset from Refaat 2011).

```
load CreditCardData
sc = creditscorecard(data)
SC =
    creditscorecard with properties:
                        GoodLabel: 0
            ResponseVar: 'status'
                    WeightsVar:
                            VarNames: {1x11 cell}
        NumericPredictors: {1x7 cell}
        CategoricalPredictors: {'ResStatus' 'EmpStatus' 'OtherCC'}
            BinMissingData: 0
                            IDVar: '
            PredictorVars: {1x10 cell}
                            Data: [1200x11 table]
```

Since the IDVar property has public access, you can change its value at the command line.

```
sc.IDVar = 'CustID'
```

```
sc =
    creditscorecard with properties:
            GoodLabel: 0
            ResponseVar: 'status'
                        WeightsVar:
                            VarNames: {1xll cell}
        NumericPredictors: {1x6 cell}
        CategoricalPredictors: {'ResStatus' 'EmpStatus' 'OtherCC'}
            BinMissingData: 0
                            IDVar: 'CustID'
            PredictorVars: {1x9 cell}
                        Data: [1200x11 table]
```


## Create a creditscorecard Object

Create a creditscorecard object using the CreditCardData.mat file to load the data (using a dataset from Refaat 2011).

```
load CreditCardData
sc = creditscorecard(data)
sc =
    creditscorecard with properties:
                        GoodLabel: 0
            ResponseVar: 'status'
                            WeightsVar:
                            VarNames: {1x11 cell}
        NumericPredictors: {1x7 cell}
        CategoricalPredictors: {'ResStatus' 'EmpStatus' 'OtherCC'}
            BinMissingData: 0
                            IDVar: ''
            PredictorVars: {1x10 cell}
                Data: [1200x11 table]
```

In this example, the default values for the properties ResponseVar, PredictorVars and GoodLabel are assigned when this object is created. By default, the property ResponseVar is set to the variable name that is in the last column of the input data ('status' in this example). The property PredictorVars contains the names of all the
variables that are in VarNames, but excludes IDVar and ResponseVar. Also, by default in the previous example, GoodLabel is set to 0 , since it is the value in the response variable (ResponseVar) with the highest count.

Display the creditscorecard object properties using dot notation.

```
sc.PredictorVars
ans = lx10 cell array
    Columns 1 through 4
        {'CustID'} {'CustAge'} {'TmAtAddress'} {'ResStatus'}
    Columns 5 through 8
        {'EmpStatus'} {'CustIncome'} {'TmWBank'} {'OtherCC'}
    Columns 9 through 10
        {'AMBalance'} {'UtilRate'}
sc.VarNames
ans = lx11 cell array
    Columns 1 through 4
        {'CustID'} {'CustAge'} {'TmAtAddress'} {'ResStatus'}
    Columns 5 through 8
        {'EmpStatus'} {'CustIncome'} {'TmWBank'} {'0therCC'}
    Columns 9 through 11
        {'AMBalance'} {'UtilRate'} {'status'}
```

Since IDVar and PredictorVars have public access, you can change their values at the command line.

```
sc.IDVar = 'CustID'
SC =
    creditscorecard with properties:
```

```
                        GoodLabel: 0
        ResponseVar: 'status'
            WeightsVar:
                            VarNames: {1x11 cell}
        NumericPredictors: {1x6 cell}
    CategoricalPredictors: {'ResStatus' 'EmpStatus' 'OtherCC'}
        BinMissingData: 0
                            IDVar: 'CustID'
        PredictorVars: {1x9 cell}
                            Data: [1200x11 table]
sc.PredictorVars = {'CustIncome','ResStatus','AMBalance'}
sc =
    creditscorecard with properties:
                        GoodLabel: 0
            ResponseVar: 'status'
                    WeightsVar:
                            VarNames: {1x11 cell}
            NumericPredictors: {'CustIncome' 'AMBalance'}
        CategoricalPredictors: {'ResStatus'}
            BinMissingData: 0
                            IDVar: 'CustID'
                PredictorVars: {'CustIncome' 'ResStatus' 'AMBalance'}
                            Data: [1200x11 table]
```

```
disp(sc)
```

disp(sc)
creditscorecard with properties:
GoodLabel: 0
ResponseVar: 'status'
WeightsVar: ''
VarNames: {1x11 cell}
NumericPredictors: {'CustIncome' 'AMBalance'}
CategoricalPredictors: {'ResStatus'}
BinMissingData: 0
IDVar: 'CustID'
PredictorVars: {'CustIncome' 'ResStatus' 'AMBalance'}
Data: [1200x11 table]

```

\section*{Create a creditscorecard Object and Set GoodLabel and ResponseVar}

Create a creditscorecard object using the CreditCardData.mat file to load the data (using a dataset from Refaat 2011). Then use name-value pair arguments for creditscorecard to define GoodLabel and ResponseVar.
```

load CreditCardData
sc = creditscorecard(data,'IDVar','CustID','GoodLabel',0,'ResponseVar','status')
sc =
creditscorecard with properties:

```
                    GoodLabel: 0
                    ResponseVar: 'status'
                    WeightsVar:
                            VarNames: \{1x11 cell\}
            NumericPredictors: \{1x6 cell\}
        CategoricalPredictors: \{'ResStatus' 'EmpStatus' 'OtherCC'\}
            BinMissingData: 0
                            IDVar: 'CustID'
            PredictorVars: \{1x9 cell\}
                Data: [1200x11 table]

GoodLabel and ResponseVar can only be set (enforced) when creating a creditscorecard object using creditscorecard.

\section*{Create a creditscorecard Object and Set the 'BinMissingData' Property}

Create a creditscorecard object using the CreditCardData.mat file to load the dataMissing with missing values.
```

load CreditCardData
head(dataMissing,5)

```


62

ResStatus
<undefined>

EmpStatus

Unknown

CustIncome 50000
```

ans=5\times11 table

```
ans=5\times11 table
    CustID CustAge TmAtAddress ResStatus EmpStatus CustIncome
    CustID CustAge TmAtAddress ResStatus EmpStatus CustIncome
        1 53
```

        1 53
    ```
```

| 2 | 61 | 22 |
| ---: | ---: | ---: |
| 3 | 47 | 30 |
| 4 | NaN | 75 |
| 5 | 68 | 56 |

fprintf('Number of rows: %d\n',height(dataMissing))
Number of rows: 1200
fprintf('Number of missing values CustAge: %d\n',sum(ismissing(dataMissing.CustAge)))
Number of missing values CustAge: 30
fprintf('Number of missing values ResStatus: %d\n',sum(ismissing(dataMissing.ResStatus
Number of missing values ResStatus: 40

```

Use creditscorecard with the name-value argument 'BinMissingData' set to true to bin the missing data in a separate bin.
```

sc = creditscorecard(dataMissing,'IDVar','CustID','BinMissingData',true);
sc = autobinning(sc);
disp(sc)
creditscorecard with properties:
GoodLabel: 0
ResponseVar: 'status'
WeightsVar:
VarNames: {1x11 cell}
NumericPredictors: {1x6 cell}
CategoricalPredictors: {'ResStatus' 'EmpStatus' 'OtherCC'}
BinMissingData: 1
IDVar: 'CustID'
PredictorVars: {1x9 cell}
Data: [1200x11 table]

```

Display bin information for numeric data for 'CustAge' that includes missing data in a separate bin labelled <missing>.
```

bi = bininfo(sc,'CustAge');
disp(bi)

| Bin | Good | Bad | Odds | W0E |
| :--- | :--- | :--- | :--- | :--- |

```
\begin{tabular}{|c|c|c|c|c|c|}
\hline '[-Inf,33)' & 69 & 52 & 1.3269 & -0.42156 & 0.018993 \\
\hline ' \([33,37)\) ' & 63 & 45 & 1.4 & -0.36795 & 0.012839 \\
\hline ' \([37,40\) ) ' & 72 & 47 & 1.5319 & -0.2779 & 0.0079824 \\
\hline '[40,46)' & 172 & 89 & 1.9326 & -0.04556 & 0.0004549 \\
\hline ' \([46,48\) )' & 59 & 25 & 2.36 & 0.15424 & 0.0016199 \\
\hline ' \([48,51\) )' & 99 & 41 & 2.4146 & 0.17713 & 0.0035449 \\
\hline ' \([51,58\) )' & 157 & 62 & 2.5323 & 0.22469 & 0.0088407 \\
\hline '[58,Inf]' & 93 & 25 & 3.72 & 0.60931 & 0. 032198 \\
\hline '<missing>' & 19 & 11 & 1.7273 & -0.15787 & 0.00063885 \\
\hline 'Totals' & 803 & 397 & 2.0227 & NaN & 0.087112 \\
\hline
\end{tabular}
plotbins(sc,'CustAge')


Display bin information for categorical data for 'ResStatus' that includes missing data in a separate bin labelled <missing>.
```

bi = bininfo(sc,'ResStatus');
disp(bi)

```
Bin
'Tenant'
'Home Owner'
'Other'
'<missing>'
'Totals'
Good

WOE
WOE

296
352
128
27
803

Bad
Odds


0dds 161 171 52 13 397
1.8385
2.0585
2.4615
2.0769
2.0227
\(-0.095463\)
0.0035249
0.00013382
0.0055808
\(2.3248 \mathrm{e}-05\)
0.0092627
plotbins(sc,'ResStatus')


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[1] Anderson, R. The Credit Scoring Toolkit. Oxford University Press, 2007.
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\section*{See Also}

\section*{Functions}
autobinning|bindata|bininfo|displaypoints|fitmodel|formatpoints| modifybins|modifypredictor|plotbins|predictorinfo|probdefault| score| setmodel|table|validatemodel

\section*{Apps \\ Binning Explorer}

\section*{Topics}
"Case Study for a Credit Scorecard Analysis" on page 8-101
"Using Credit Scorecards with Constrained Logistic Regression Coefficients"
"Credit Scorecard Modeling with Missing Values" on page 8-73
"Troubleshooting Credit Scorecard Results" on page 8-91
"Binning Explorer Case Study Example" (Risk Management Toolbox)
"Credit Scorecard Modeling Workflow" on page 8-67
on page 8-62
"Credit Scorecard Modeling Using Observation Weights" on page 8-70
"Overview of Binning Explorer" (Risk Management Toolbox)

\section*{External Websites}

Credit Risk Modeling with MATLAB (53 min 10 sec )

\section*{Introduced in R2014b}

\section*{Bibliography}

\section*{Bibliography}
"Bond Pricing and Yields" on page A-2
"Term Structure of Interest Rates" on page A-3
"Derivatives Pricing and Yields" on page A-3
"Portfolio Analysis" on page A-3
"Investment Performance Metrics" on page A-3
"Financial Statistics" on page A-4
"Standard References" on page A-5
"Credit Risk Analysis" on page A-6
"Portfolio Optimization" on page A-7
"Stochastic Differential Equations" on page A-8
"Life Tables" on page A-8

Note For the well-known algorithms and formulas used in Financial Toolbox software (such as how to compute a loan payment given principal, interest rate, and length of the loan), no references are given here. The references here pertain to less common formulas.

\section*{Bond Pricing and Yields}

The pricing and yield formulas for fixed-income securities come from:
[1] Golub, B.W. and L.M. Tilman. Risk Management: Approaches for Fixed Income Markets. Wiley, 2000.
[2] Martellini, L., P. Priaulet, and S. Priaulet. Fixed Income Securities. Wiley, 2003.
[3] Mayle, Jan. Standard Securities Calculation Methods. New York: Securities Industry Association, Inc. Vol. 1, 3rd ed., 1993, ISBN 1-882936-01-9. Vol. 2, 1994, ISBN 1-882936-02-7.
[4] Tuckman, B. Fixed Income Securities: Tools for Today's Markets. Wiley, 2002.
In many cases these formulas compute the price of a security given yield, dates, rates, and other data. These formulas are nonlinear, however; so when solving for an independent variable within a formula, Financial Toolbox software uses Newton's method. See any elementary numerical methods textbook for the mathematics underlying Newton's method.

\section*{Term Structure of Interest Rates}

The formulas and methodology for term structure functions come from:
[5] Fabozzi, Frank J. "The Structure of Interest Rates." Ch. 6 in Fabozzi, Frank J. and T. Dessa Fabozzi, eds. The Handbook of Fixed Income Securities. 4th ed. New York, Irwin Professional Publishing, 1995, ISBN 0-7863-0001-9.
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\section*{Portfolio Analysis}

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\section*{Investment Performance Metrics}

The risk and ratio formulas for investment performance metrics come from:
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[24] Meng, Xiao-Li, and Donald B. Rubin. "Maximum Likelihood Estimation via the ECM Algorithm." Biometrika. Vol. 80, No. 2, 1993, pp. 267-278.
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Standard references include:
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\section*{Portfolio Optimization}

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\begin{tabular}{ll} 
Glossary & and \\
active return & \begin{tabular}{l} 
Amount of return achieved in excess of the return \\
produced by an appropriate benchmark (for example, an \\
index portfolio).
\end{tabular} \\
active risk & \begin{tabular}{l} 
Standard deviation of the active return. Also known as the \\
tracking error on page Glossary-0
\end{tabular} \\
American option & \begin{tabular}{l} 
An option that can be exercised any time until its \\
expiration date. Contrast with European option.
\end{tabular} \\
amortization & \begin{tabular}{l} 
Reduction in value of an asset over some period for \\
accounting purposes. Generally used with intangible \\
assets. Depreciation is the term used with fixed or \\
tangible assets.
\end{tabular} \\
annuity & \begin{tabular}{l} 
A series of payments over a period of time. The payments \\
are usually in equal amounts and usually at regular \\
intervals such as quarterly, semiannually, or annually.
\end{tabular} \\
antithetic sampling & \begin{tabular}{l} 
A variance reduction technique that pairs a sequence of \\
independent normal random numbers with a second \\
sequence obtained by negating the random numbers of \\
the first. The first sequence simulates increments of one \\
path of Brownian motion, and the second sequence \\
simulates increments of its reflected, or antithetic, path.
\end{tabular} \\
These two paths form an antithetic pair independent of \\
any other pair.
\end{tabular}
coupon period. Supported day count conventions and basis values are:
\begin{tabular}{|c|c|}
\hline B
a
si
s
s
V
al
u
e & Day Count Convention \\
\hline 0 & actual/actual (default) - Number of days in both a period and a year is the actual number of days. \\
\hline 1 & 30/360 SIA - Year fraction is calculated based on a 360 day year with 30-day months, after applying the following rules: If the first date and the second date are the last day of February, the second date is changed to the 30th. If the first date falls on the 31st or is the last day of February, it is changed to the 30th. If after the preceding test, the first day is the 30th and the second day is the 31 st, then the second day is changed to the 30th. \\
\hline
\end{tabular}

Glossary-2
\(\left.\begin{array}{|l|l|}\hline \text { B } \\ \mathbf{a} \\ \mathbf{s i} \\ \mathbf{s} \\ \mathbf{V} \\ \text { al }\end{array} \quad \begin{array}{l}\text { Day Count } \\ \text { Convention } \\ \mathbf{u}\end{array}\right)\)

Glossary-3
\begin{tabular}{|c|c|}
\hline B
a
si
si
s
V
al
u
e & Day Count Convention \\
\hline 5 & 30/360 ISDA Number of days in every month is set to 30, except for February where it is the actual number of days. If the start date of the period is the 31st of a month, the start date is set to the 30th while if the start date is the 30th of a month and the end date is the 31st, the end date is set to the 30th. The number of days in a year is 360 . \\
\hline 6 & 30E /360 - Number of days in every month is set to 30 except for February where it is equal to the actual number of days. If the start date or the end date of the period is the 31st of a month, that date is set to the 30th. The number of days in a year is 360 . \\
\hline
\end{tabular}

Glossary-4
\begin{tabular}{|c|c|}
\hline \[
\begin{aligned}
& \hline \text { B } \\
& \mathbf{a} \\
& \mathbf{s i} \\
& \mathbf{s} \\
& \mathbf{V} \\
& \text { al } \\
& \mathbf{u} \\
& \mathbf{e}
\end{aligned}
\] & Day Count Convention \\
\hline 7 & actual/365 Japanese Number of days in a period is equal to the actual number of days, except for leap days (29th February) which are ignored. The number of days in a year is 365 (even in a leap year). \\
\hline 8 & actual/actual ICMA Number of days in both a period and a year is the actual number of days and the compounding frequency is annual. \\
\hline 9 & actual/360 ICMA Number of days in a period is equal to the actual number of days, however the number of days in a year is 360 and the compounding frequency is annual. \\
\hline
\end{tabular}
\begin{tabular}{|c|c|}
\hline \begin{tabular}{l}
\hline B \\
\hline a \\
si \\
s \\
v \\
al \\
u \\
\hline \\
\hline
\end{tabular} & Day Count Convention \\
\hline \[
\begin{aligned}
& 1 \\
& 0
\end{aligned}
\] & actual/365 ICMA Number of days in a period is equal to the actual number of days, however the number of days in a year is 365 (even in a leap year) and the compounding frequency is annual. \\
\hline 1 & 30/360 ICMA Number of days in every month is set to 30, except for February where it is equal to the actual number of days. If the start date or the end date of the period is the 31st of a month, that date is set to the 30th. The number of days in a year is 360 and the compounding frequency is annual. \\
\hline
\end{tabular}
\begin{tabular}{|l|l|}
\hline \(\mathbf{B}\) & Day Count \\
\(\mathbf{a}\) & Convention \\
\(\mathbf{s i}\) & \(\mathbf{s}\) \\
\(\mathbf{V}\) & \\
\(\mathbf{a l}\) \\
\(\mathbf{u}\) & \\
\(\mathbf{e}\) & \\
\hline 1 & \begin{tabular}{l} 
actual/365 ISDA - The \\
day count fraction is \\
calculated using the \\
following formula: \\
(Actual number of \\
days in period \\
that fall in a \\
leap year / 366) \\
+ (Actual number \\
of days in period
\end{tabular} \\
that fall in a \\
normal year / \\
365 ). \\
\hline 1 & \begin{tabular}{l} 
bus/252 - The \\
number of days in a \\
period is equal to the \\
actual number of \\
business days. The \\
number of business \\
days in a year is 252.
\end{tabular} \\
\hline
\end{tabular}

\section*{beta}

\section*{binomial model}

The price volatility of a financial instrument relative to the price volatility of a market or index as a whole. Beta is commonly used with respect to equities. A high-beta instrument is riskier than a low-beta instrument.

A method of pricing options or other equity derivatives in which the probability over time of each possible price follows a binomial distribution. The basic assumption is that prices can move to only two values (one higher and one lower) over any short time period.
\(\left.\begin{array}{ll}\text { Black-Scholes model } & \begin{array}{l}\text { The first complete mathematical model for pricing options, } \\ \text { developed by Fischer Black and Myron Scholes. It } \\ \text { examines market price, strike price, volatility, time to } \\ \text { expiration, and interest rates. It is limited to only certain } \\ \text { kinds of options. }\end{array} \\ \text { Bollinger band chart } & \begin{array}{l}\text { A financial chart that plots actual asset data along with } \\ \text { three other bands of data: the upper band is two standard } \\ \text { deviations above a user-specified moving average; the } \\ \text { lower band is two standard deviations below that moving } \\ \text { average; and the middle band is the moving average itself. }\end{array} \\ \text { bootstrapping, } & \begin{array}{l}\text { An arithmetic method for backing an implied zero curve } \\ \text { out of the par yield curve. }\end{array} \\ \text { bootstrap method } & \begin{array}{l}\text { A zero-mean continuous-time stochastic process with } \\ \text { independent increments (also known as a Wiener process). }\end{array} \\ \text { brownian motion } & \begin{array}{l}\text { For a binomial option model: plotting the two possible } \\ \text { short-term price-changes values, and then the subsequent } \\ \text { two values each, and then the subsequent two values } \\ \text { each, and so on over time, is known as "building a }\end{array} \\ \text { binomial tree." See also binomial model on page }\end{array}\right\}\)

The first complete mathematical model for pricing options, developed by Fischer Black and Myron Scholes. It examines market price, strike price, volatility, time to expiration, and interest rates. It is limited to only certain kinds of options.

A financial chart that plots actual asset data along with three other bands of data: the upper band is two standard deviations above a user-specified moving average; the lower band is two standard deviations below that moving average; and the middle band is the moving average itself.

An arithmetic method for backing an implied zero curve out of the par yield curve.

A zero-mean continuous-time stochastic process with independent increments (also known as a Wiener process).

For a binomial option model: plotting the two possible short-term price-changes values, and then the subsequent two values each, and then the subsequent two values each, and so on over time, is known as "building a binomial tree." See also binomial model on page Glossary-0 .
a. An option to buy a certain quantity of a stock or commodity for a specified price within a specified time. See also put on page Glossary-0 . b. A demand to submit bonds to the issuer for redemption before the maturity date. c. A demand for payment of a debt. d. A demand for payment due on stock bought on margin.

A bond that allows the issuer to buy back the bond at a predetermined price at specified future dates. The bond contains an embedded call option; that is, the holder has sold a call option to the issuer. See also puttable bond on page Glossary-0 .

A financial chart usually used to plot the high, low, open, "candle" is the region between the open and close price of
cap

\section*{cash flow}
clean price
collar
convexity

\section*{correlation}
correlation coefficient
coupon

\section*{coupon dates}

\section*{coupon rate}
the security. Thin vertical lines extend up to the high and down to the low, respectively. If the open price is greater than the close price, the body is empty. If the close price is greater than the open price, the body is filled. See also high-low-close chart on page Glossary-0

Interest-rate option that guarantees that the rate on a floating-rate loan will not exceed a certain level.

Cash received and paid over time.
The price of a bond excluding any interest that has accrued since issue or the most recent coupon payment.

Interest-rate option that guarantees that the rate on a floating-rate loan will not exceed a certain upper level nor fall below a lower level. It is designed to protect an investor against wide fluctuations in interest rates.

A measure of the rate of change in duration; measured in time. The greater the rate of change, the more the duration changes as yield changes.

The simultaneous change in value of two random numeric variables.

A statistic in which the covariance is scaled to a value between minus one (perfect negative correlation) and plus one (perfect positive correlation).

Detachable certificate attached to a bond that shows the amount of interest payable at regular intervals, usually semiannually. Originally coupons were actually attached to the bonds and had to be cut off or "clipped" to redeem them and receive the interest payment.

The dates when the coupons are paid. Typically a bond pays coupons annually or semiannually.

The nominal interest rate that the issuer promises to pay the buyer of a bond.
\begin{tabular}{ll} 
covariance & \begin{tabular}{l} 
A measure of the degree to which returns on two assets \\
move in tandem. A positive covariance means that asset \\
returns move together; a negative covariance means they \\
vary inversely.
\end{tabular} \\
credit rating & \begin{tabular}{l} 
A credit rating evaluates a potential borrower's ability to \\
repay debt.
\end{tabular} \\
day count convention & \begin{tabular}{l} 
A convention used to determine the number of days \\
between two coupon dates, which is important in \\
calculating accrued interest and present value when the \\
next coupon payment is less than a full coupon period \\
away. See also basis on page Glossary-0
\end{tabular} \\
delta & \begin{tabular}{l} 
The rate of change of the price of a derivative security \\
relative to the price of the underlying asset; that is, the \\
first derivative of the curve that relates the price of the \\
derivative to the price of the underlying security.
\end{tabular} \\
depreciation & \begin{tabular}{l} 
Reduction in value of fixed or tangible assets over some \\
period for accounting purposes. See also amortization on \\
page Glossary-0 .
\end{tabular} \\
derivative & \begin{tabular}{l} 
A financial instrument that is based on some underlying \\
asset. For example, an option is a derivative instrument \\
based on the right to buy or sell an underlying instrument.
\end{tabular} \\
diffusion & \begin{tabular}{l} 
The function that characterizes the random (stochastic) \\
portion of a stochastic differential equation. See also \\
stochastic differential equation on page Glossary-0
\end{tabular} \\
dirty price & \begin{tabular}{l} 
The price of a bond including the accrued interest.
\end{tabular} \\
discount curve & \begin{tabular}{l} 
The curve of discount rates versus maturity dates for
\end{tabular} \\
of an investment or fund.
\end{tabular}

Glossary-10
\begin{tabular}{ll} 
drift & \begin{tabular}{l} 
The function that characterizes the deterministic portion \\
of a stochastic differential equation. See also stochastic \\
differential equation on page Glossary-0
\end{tabular} \\
duration & \begin{tabular}{l} 
The expected life of a fixed-income security considering its \\
coupon yield, interest payments, maturity, and call \\
features. As market interest rates rise, the duration of a \\
financial instrument decreases. See also Macaulay \\
duration on page Glossary-0.
\end{tabular} \\
efficient frontier & \begin{tabular}{l} 
A graph representing a set of portfolios that maximizes \\
expected return at each level of portfolio risk. See also \\
Markowitz model on page Glossary-0 .
\end{tabular} \\
efficient portfolio & \begin{tabular}{l} 
Portfolios satisfying the criteria of minimum risk for a \\
given level of return and maximum return for a given level \\
of risk. See also Markowitz model on page Glossary-0
\end{tabular} \\
elasticity & \begin{tabular}{l} 
See Lambda on page Glossary-0
\end{tabular} \\
Euler approximation & \begin{tabular}{l} 
A simulation technique that provides a discrete-time \\
approximation of a continuous-time stochastic process.
\end{tabular} \\
European option & \begin{tabular}{l} 
An option that can be exercised only on its expiration date. \\
Contrast with American option.
\end{tabular} \\
ex-ante & \begin{tabular}{l} 
Referring to future events, such as the future price of a \\
stock.
\end{tabular} \\
ex-post & \begin{tabular}{l} 
Referring to past events, when uncertainty of the result \\
has been eliminated.
\end{tabular} \\
exercise price & \begin{tabular}{l} 
The price set for buying an asset (call) or selling an asset \\
(put). The strike price.
\end{tabular} \\
floore value & \begin{tabular}{l} 
The maturity value of a security. Also known as par value, \\
principal value, or redemption value.
\end{tabular} \\
A security that pays a specified cash flow over a specific \\
period. Bonds are typical fixed-income securities.
\end{tabular}
\begin{tabular}{ll} 
forward curve & \begin{tabular}{l} 
The curve of forward interest rates versus maturity dates \\
for bonds.
\end{tabular} \\
forward rate & \begin{tabular}{l} 
The future interest rate of a bond inferred from the term \\
structure, especially from the yield curve of zero-coupon \\
bonds, calculated from the growth factor of an investment \\
in a zero held until maturity.
\end{tabular} \\
future value & \begin{tabular}{l} 
The value that a sum of money (the present value) earning \\
compound interest will have in the future.
\end{tabular} \\
gamma & \begin{tabular}{l} 
The rate of change of delta for a derivative security \\
relative to the price of the underlying asset; that is, the \\
second derivative of the option price relative to the \\
security price.
\end{tabular} \\
greeks & \begin{tabular}{l} 
Collectively, "greeks" refer to the financial measures beta, \\
delta, gamma, lambda, rho, theta, and vega, which are \\
sensitivity measures used in evaluating derivatives.
\end{tabular} \\
ISDA & \begin{tabular}{l} 
International Swaps and Derivatives Association.
\end{tabular} \\
ISMA & \begin{tabular}{l} 
International Securities Market Association.
\end{tabular} \\
hedge & \begin{tabular}{l} 
A securities transaction that reduces or offsets the risk on \\
an existing investment position.
\end{tabular} \\
high-low-close chart & \begin{tabular}{l} 
A financial chart usually used to plot the high, low, open, \\
and close price of a security over time. Plots are vertical \\
lines whose top is the high, bottom is the low, open is a \\
short horizontal tick to the left, and close is a short \\
horizontal tick to the right.
\end{tabular} \\
implied volatility & \begin{tabular}{l} 
For an option, the variance that makes a call option price \\
equal to the market price. Given the option price, strike \\
price, and other factors, the Black-Scholes model \\
computes implied volatility.
\end{tabular} \\
internal rate of return & \begin{tabular}{l} 
The ratio of relative return to relative risk.
\end{tabular} \\
a. The average annual yield earned by an investment \\
during the period held. b. The effective rate of interest on
\end{tabular}

Glossary-12
\begin{tabular}{ll} 
issue date & \begin{tabular}{l} 
The date a security is first offered for sale. That date \\
usually determines when interest payments, known as \\
coupons, are made.
\end{tabular} \\
Ito process & \begin{tabular}{l} 
Statistical assumptions about the behavior of security \\
prices. For details, see the book by Hull in "Derivatives \\
Pricing and Yields" on page A-3.
\end{tabular} \\
key rate duration & \begin{tabular}{l} 
Key rate duration measures the sensitivity of a portfolio's \\
(or security's) value in relation to changes in specific \\
maturities of the zero or spot curve.
\end{tabular} \\
Lambda & \begin{tabular}{l} 
The percentage change in the price of an option relative to \\
a 1\% change in the price of the underlying security. Also \\
known as elasticity.
\end{tabular} \\
long position & \begin{tabular}{l} 
Outright ownership of a security or financial instrument. \\
The owner expects the price to rise in order to make a \\
profit on some future sale.
\end{tabular} \\
long rate & \begin{tabular}{l} 
The yield on a zero-coupon Treasury bond.
\end{tabular} \\
lower partial moment & \begin{tabular}{l} 
A model for the moments of asset returns that fall below a \\
minimum acceptable level of return.
\end{tabular} \\
Macaulay duration & \begin{tabular}{l} 
A widely used measure of price sensitivity to yield changes \\
developed by Frederick Macaulay in 1938. It is measured \\
in years and is a weighted average-time-to-maturity of an \\
instrument. The Macaulay duration of an income stream,
\end{tabular} \\
such as a coupon bond, measures how long, on average, \\
the owner waits before receiving a payment. It is the \\
weighted average of the times payments are made, with \\
the weights at time T equal to the present value of the \\
money received at time T.
\end{tabular}

Glossary-13
\begin{tabular}{ll} 
Markowitz model & \begin{tabular}{l} 
A model for selecting an optimum investment portfolio, \\
devised by H. M. Markowitz. It uses a discrete-time, \\
continuous-outcome approach for modeling investment \\
problems, often called the mean-variance paradigm. See \\
also efficient portfolio on page Glossary-0 and \\
efficient frontier on page Glossary-0 .
\end{tabular} \\
maturity date & \begin{tabular}{l} 
The date when the issuer returns the final face value of a \\
bond to the buyer.
\end{tabular} \\
mean & \begin{tabular}{l} 
a. A number that typifies a set of numbers, such as a \\
geometric mean or an arithmetic mean. b. The average \\
value of a set of numbers.
\end{tabular} \\
modified duration & \begin{tabular}{l} 
The Macaulay duration discounted by the per-period \\
interest rate; that is, divided by (1+rate/frequency).
\end{tabular} \\
Monte-Carlo simulation & \begin{tabular}{l} 
A mathematical modeling process. For a model that has \\
several parameters with statistical properties, pick a set of \\
random values for the parameters and run a simulation. \\
Then pick another set of values, and run it again. Run it \\
many times (often 10,000 times) and build up a statistical \\
distribution of outcomes of the simulation. This \\
distribution of outcomes is then used to answer whatever
\end{tabular} \\
question you are asking.
\end{tabular}

Glossary-14
on-the-run treasury
bonds
option
par value
par yield curve
point and figure chart
present value
principal value
proportional sampling

PSA
purchase price

Instead, the coupon is prorated according to how long the bond is held during that period.

The most recently auctioned issue of a U.S. Treasury bond or note of a particular maturity.

A right to buy or sell specific securities or commodities at a stated price (exercise or strike price) within a specified time. An option is a type of derivative.

The maturity or face value of a security or other financial instrument.

The yield curve of bonds selling at par, or face, value.
A financial chart usually used to plot asset price data. Upward price movements are plotted as X's and downward price movements are plotted as O's.

Today's value of an investment that yields some future value when invested to earn compounded interest at a known interest rate; that is, the future value at a known period in time discounted by the interest rate over that time period.

See par value on page Glossary-0 .
A stratified sampling technique that ensures that the proportion of random draws matches its theoretical probability. One of the most common examples of proportional sampling involves stratifying the terminal value of a price process in which each sample path is associated with a single stratified terminal value such that the number of paths equals the number of strata.

See also stratified sampling on page Glossary-0
Public Securities Association.
Price actually paid for a security. Typically the purchase price of a bond is not the same as the redemption value.
\begin{tabular}{ll} 
put & \begin{tabular}{l} 
An option to sell a stipulated amount of stock or securities \\
within a specified time and at a fixed exercise price. See \\
also call on page Glossary-0
\end{tabular} \\
puttable bond & \begin{tabular}{l} 
A bond that allows the holder to redeem the bond at a \\
predetermined price at specified future dates. The bond \\
contains an embedded put option; that is, the holder has \\
bought a put option. See also callable bond on page \\
Glossary-0 .
\end{tabular} \\
Quant & \begin{tabular}{l} 
A quantitative analyst; someone who does numerical \\
analysis of financial information in order to detect \\
relationships, disparities, or patterns that can lead to \\
making money.
\end{tabular} \\
redemption value & \begin{tabular}{l} 
See par value on page Glossary-0
\end{tabular} \\
regression analysis & \begin{tabular}{l} 
Statistical analysis techniques that quantify the \\
relationship between two or more variables. The intent is \\
quantitative prediction or forecasting, particularly using a \\
small population to forecast the behavior of a large \\
population.
\end{tabular} \\
rho & \begin{tabular}{l} 
The rate of change in a derivative's price relative to the \\
underlying security's risk-free interest rate.
\end{tabular} \\
return proxy & \begin{tabular}{l} 
The proxy for return is a function that characterizes either \\
the gross benefits or net benefits associated with portfolio \\
choices.
\end{tabular} \\
settlement date & \begin{tabular}{l} 
The proxy for risk is a function that characterizes either \\
the variability or losses associated with portfolio choices.
\end{tabular} \\
sensitivity & \begin{tabular}{l} 
The "what if" relationship between variables; the degree \\
to which changes in one variable cause changes in another \\
variable. A specific synonym is volatility.
\end{tabular} \\
muyer actually pays for a security. It need not coincide a \\
with the issue date.
\end{tabular}
\begin{tabular}{ll} 
Sharpe ratio & \begin{tabular}{l} 
The ratio of the excess return of an asset divided by the \\
asset's standard deviation of returns.
\end{tabular} \\
short rate & \begin{tabular}{l} 
The annualized one-period interest rate.
\end{tabular} \\
\begin{tabular}{l} 
short sale, short \\
position
\end{tabular} & \begin{tabular}{l} 
The sale of a security or financial instrument not owned, \\
in anticipation of a price decline and making a profit by \\
purchasing the instrument later at a lower price, and then \\
delivering the instrument to complete the sale. See also \\
long position on page Glossary-0
\end{tabular} \\
SIA & \begin{tabular}{l} 
Securities Industry Association.
\end{tabular} \\
spot curve, spot yield \\
curve & \begin{tabular}{l} 
See zero curve, zero-coupon yield curve on page \\
Glossary-0
\end{tabular} \\
spot rate & \begin{tabular}{l} 
The current interest rate appropriate for discounting a \\
cash flow of some given maturity.
\end{tabular} \\
spread & \begin{tabular}{l} 
For options, a combination of call or put options on the \\
same stock with differing exercise prices or maturity \\
dates.
\end{tabular} \\
standard deviation & \begin{tabular}{l} 
A measure of the variation in a distribution, equal to the \\
square root of the arithmetic mean of the squares of the \\
deviations from the arithmetic mean; the square root of \\
the variance.
\end{tabular} \\
stochastic & \begin{tabular}{l} 
Involving or containing a random variable or variables; \\
involving chance or probability.
\end{tabular} \\
stochastic differential & \begin{tabular}{l} 
A generalization of an ordinary differential equation, with \\
the addition of a noise process, that yields random \\
variables as solutions.
\end{tabular} \\
equation & \begin{tabular}{l} 
A strategy used in trading options or futures. It involves \\
simultaneously purchasing put and call options with the \\
same exercise price and expiration date, and it is most \\
profitable when the price of the underlying security is very \\
volatile.
\end{tabular} \\
See stratified sampling on page Glossary-0
\end{tabular}
\begin{tabular}{ll} 
stratified sampling & \begin{tabular}{l} 
A variance reduction technique that constrains a \\
proportion of sample paths to specific subsets (or strata) \\
of the sample space.
\end{tabular} \\
strike & \begin{tabular}{l} 
Exercise a put or call option.
\end{tabular} \\
strike price & See exercise price on page Glossary-0 \\
swap & \begin{tabular}{l} 
A contract between two parties to exchange cash flows in \\
the future according to some formula.
\end{tabular} \\
swaption & \begin{tabular}{l} 
A swap option; an option on an interest-rate swap. The \\
option gives the holder the right to enter into a contracted \\
interest-rate swap at a specified future date. See also \\
swap on page Glossary-0 .
\end{tabular} \\
term structure & \begin{tabular}{l} 
The relationship between the yields on fixed-interest \\
securities and their maturity dates. Expectation of \\
changes in interest rates affects term structure, as do \\
liquidity preferences and hedging pressure. A yield curve \\
is one representation in the term structure.
\end{tabular} \\
theta & \begin{tabular}{l} 
The rate of change in the price of a derivative security \\
relative to time. Theta is usually very small or negative \\
since the value of an option tends to drop as it approaches \\
maturity.
\end{tabular} \\
tracking error & \begin{tabular}{l} 
See active risk on page Glossary-0 .
\end{tabular} \\
treasury bill & \begin{tabular}{l} 
Short-term U.S. government security issued at a discount \\
from the face value and paying the face value at maturity.
\end{tabular} \\
vegariance & \begin{tabular}{l} 
Long-term debt obligation of the U.S. government that \\
makes coupon payments semiannually and is sold at or
\end{tabular} \\
near par value in \$1000 denominations or higher. Face \\
value is paid at maturity.
\end{tabular}

Glossary-18

\section*{volatility}

\section*{Wiener process}
yield
yield curve
yield to maturity
zero curve, zero-coupon yield curve

\section*{zero-coupon bond, or} zero
vega is large, the security is sensitive to small changes in volatility.
a. Another general term for sensitivity. b. The standard deviation of the annualized continuously compounded rate of return of an asset. c. A measure of uncertainty or risk.

See Brownian motion on page Glossary-0 .
a. Measure of return on an investment, stated as a percentage of price. Yield can be computed by dividing return by purchase price, current market value, or other measure of value. \(\mathbf{b}\). Income from a bond expressed as an annualized percentage rate. c. The nominal annual interest rate that gives a future value of the purchase price equal to the redemption value of the security. Any coupon payments determine part of that yield.

Graph of yields (vertical axis) of a particular type of security versus the time to maturity (horizontal axis). This curve usually slopes upward, indicating that investors usually expect to receive a premium for securities that have a longer time to maturity. The benchmark yield curve is for U.S. Treasury securities with maturities ranging from three months to 30 years. See also term structure on page Glossary-0 .

A measure of the average rate of return that will be earned on a bond if held to maturity.

A yield curve for zero-coupon bonds; zero rates versus maturity dates. Since the maturity and duration (Macaulay duration) are identical for zeros, the zero curve is a pure depiction of supply/demand conditions for loanable funds across a continuum of durations and maturities. Also known as spot curve or spot yield curve.

A bond that, instead of carrying a coupon, is sold at a discount from its face value, pays no interest during its life, and pays the principal only at maturity.```


[^0]:    load CAPMuniverse
    time = datetime(Dates,'ConvertFrom','datenum');
    stockTT = array2timetable(Data,'RowTimes',time, 'VariableNames', Assets);
    stockTT. Properties
    \% Notice that GOOG has missing data, because it was not listed before Aug 2004
    head(stockTT, 5);

[^1]:    sampleTotals(Years==1993).totalsVec
    sampleTotals(Years==1993). totalsMat sampleTotals(Years==1993).algorithm

[^2]:    Data Types: double | function

[^3]:    Data Types: double | function

[^4]:    'jackel2016' (default) | character vector with values 'search' or 'jackel2016' | string with values "search" or "jackel2016"

[^5]:    "Greek-Neutral Portfolios of European Stock Options" on page 10-18 "Plotting Sensitivities of an Option" on page 10-30
    "Plotting Sensitivities of a Portfolio of Options" on page 10-33
    Introduced before R2006a

[^6]:    InterpMethod - Interpolation method used to obtain points from zero curve
    'linear' (default)|'cubic', 'pchip'

[^7]:    Note By default, SIA bases (0-7) and BUS/252 use a semiannual compounding convention and ICMA bases (8-12) use an annual compounding convention.

[^8]:    0 (actual/actual) (default) | numeric with value 0 through $13 \mid$ vector of numerics with values 0 through 13

[^9]:    Topics
    "Analyzing and Computing Cash Flows" on page 2-18

    Introduced before R2006a

[^10]:    [] (default) | matrix

[^11]:    [ ] (default) | matrix

[^12]:    [] (default) | matrix

[^13]:    [ ] (default) | matrix

[^14]:    "Convert Financial Time Series Objects fints to Timetables" on page 12-2 Introduced before R2006a

[^15]:    Topics
    "Data Transformation and Frequency Conversion" on page 13-13

    Introduced before R2006a

[^16]:    Data Types: char

[^17]:    Topics
    "Data Transformation and Frequency Conversion" on page 13-13

    Introduced before R2006a

[^18]:    Topics
    "Data Transformation and Frequency Conversion" on page 13-13

    Introduced before R2006a

